IOP Publishing Journal of Physics G: Nuclear and Particle Physics

J. Phys. G: Nucl. Part. Phys. **47** (2020) 055109 (9pp) https://doi.org/10.1088/1361-6471/ab7ec1

How much two-photon exchange is needed to resolve the proton form factor discrepancy?

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Received 3 January 2020, revised 5 March 2020
Accepted for publication 11 March 2020
Published 3 April 2020

Abstract

One possible explanation for the proton form factor discrepancy is a contribution to the elastic electron–proton cross section from hard two-photon exchange (TPE), a typically neglected radiative correction. Hard TPE cannot be calculated in a model-independent way, but it can be determined experimentally by looking for deviations from unity in the ratio of positron-proton to electron–proton cross sections. Three recent experiments have measured this cross section ratio to quantify hard TPE. To interpret the results of these experiments, it is germane to ask: 'How large of a deviation from unity is necessary to fully resolve the form factor discrepancy?' With a minimal set of assumptions and using global fits to unpolarized and polarized elastic scattering data, I estimate the necessary size of the TPE correction in the kinematics of the three recent experiments and compare to their measurements. I find wide variation when using different global fits, implying that the magnitude of the form factor discrepancy is not well-constrained. The recent hard TPE measurements can easily accommodate the hypothesis that TPE underlies the proton form factor discrepancy.

Keywords: form factors, two-photon exchange, electron–proton scattering

(Some figures may appear in colour only in the online journal)

Introduction

There is a considerable discrepancy between unpolarized Rosenbluth measurements and polarized measurements of the proton's electromagnetic form factor ratio, $R_{FF} \equiv \mu_p G_E / G_M$.

Hard two-photon exchange (TPE), a previously neglected radiative effect, has been suggested as a possible explanation for the discrepancy [1, 2]. While the effect of hard TPE cannot be calculated in a model-independent way, it can be determined experimentally, by measuring the deviation from unity in $R_{2\gamma} \equiv \sigma_{e^+p}/\sigma_{e^-p}$, the ratio of the positron-proton to electron–proton elastic cross sections. Three recent experiments measured $R_{2\gamma}$ over a range of squared four-momentum transfer, Q^2, up to $2\,\mathrm{GeV}^2$ [3–5] (working in units where the speed of light, $c = 1$), but the results showed only modest hard TPE, leaving open the question whether or not hard TPE is in fact the cause of the proton form factor discrepancy. See [6] for a recent review.

To interpret the results of these new experiments, it is helpful to ask the question: 'How much TPE is needed to resolve the proton form factor discrepancy?' This question does not have a precise answer. One challenge is estimating the size of the discrepancy itself. There have been dozens of experimental determinations of R_{FF} at many different values of Q^2 and these results must be combined, averaged, and interpolated. Fortunately, there have been several global fits to both polarized and unpolarized form factor data, and while they may differ slightly in methodology or included data, they can provide a parameterization for R_{FF} as determined by the two different techniques. A second challenge is that the exact kinematic dependence of the hard TPE effect is unknown. There is not a unique way to translate from the size of the discrepancy at a given value of Q^2 to the necessary value of $R_{2\gamma}$ as a function of both Q^2 and ϵ, the virtual photon polarization parameter. Many model-dependent calculations of TPE have been made (see, for example, [7–9], and others), and these provide a valuable guide for interpreting experimental results. However, rather than adopt any model or calculation framework, I propose a method of estimating the TPE contribution necessary to resolve the form factor discrepancy from form factor data alone, relying on three reasonable assumptions:

1. Polarized measurements accurately determine R_{FF}, i.e. they are unaffected by hard TPE. This is the general consensus of the community [1, 6, 10, 11].
2. Hard TPE makes no contribution to the elastic cross section in the limit $\epsilon \to 1$. This is supported by the majority of theoretical calculations of hard TPE (see [7–9] as examples, as well as the discussion in [12]).
3. Hard TPE preserves the linearity of Rosenbluth plots. This may not be true, especially at extreme kinematics, but is very-well supported by previous unpolarized data (and thoroughly studied in [13]).

These three assumptions, combined with global fits to unpolarized measurements of G_E, unpolarized measurements of G_M, and polarized measurements of R_{FF} are sufficient to define the value of $R_{2\gamma}$ that would fully explain the form factor discrepancy.

In this paper I use three different global fits to unpolarized measurements to make predictions of the hard TPE effect necessary to resolve the form factor discrepancy. I compare these predictions to the results of the recent TPE experiments, at VEPP-3 [3], at CLAS [4, 14], and the OLYMPUS Experiment [5]. I find that in the Q^2 range relevant for these experiments (up to $2\,\mathrm{GeV}^2$), the spread in predictions from using different global fits is very large, indicating that the size of the form factor discrepancy is not well constrained. The recent TPE measurements fall within the spread of predictions, indicating consistency with the hypothesis that TPE is the origin of the form factor discrepancy.

Derivation

To preserve the linearity of Rosenbluth plots, hard TPE must correct the reduced cross section in a way that satisfies:

$$G_M^2(Q^2) + \frac{\epsilon}{\tau}G_E^2(Q^2) - \delta(Q^2)(1-\epsilon) = \tilde{G}_M^2(Q^2) + \frac{\epsilon}{\tau}\tilde{G}_E^2(Q^2), \quad (1)$$

where G_E and G_M represent the true form factors, $\tilde{G}_E$ and $\tilde{G}_M$ represent the form factors extracted from unpolarized Rosenbluth separation without accounting for hard TPE, $\tau \equiv Q^2/4m_p^2$, where m_p is the proton mass, and $\delta(Q^2)$ represents a lepton charge-odd modification due to hard TPE. Given the ϵ dependence of both sides of equation (1), two relationships must hold at every value of Q^2:

$$G_E^2 = \tilde{G}_E^2 - \tau\delta \quad (2)$$

$$G_M^2 = \tilde{G}_M^2 + \delta. \quad (3)$$

Dividing the two, one finds that

$$R_{FF}^2 = \frac{\mu_p^2(\tilde{G}_E^2 - \tau\delta)}{\tilde{G}_M^2 + \delta}, \quad (4)$$

which can be solved for δ:

$$\delta = \frac{\mu_p^2\tilde{G}_E^2 - R_{FF}^2\tilde{G}_M^2}{R_{FF}^2 + \mu_p^2\tau}. \quad (5)$$

By using global fits to unpolarized data to supply $\tilde{G}_E$ and $\tilde{G}_M$ and a global fit to polarized data to supply R_{FF}, an estimate of the value of $R_{2\gamma}$ needed to resolve the discrepancy can be made:

$$R_{2\gamma} = 1 + \frac{2\delta(1-\epsilon)}{\tilde{G}_M^2 + \frac{\epsilon}{\tau}\tilde{G}_E^2}. \quad (6)$$

This approach of estimating $R_{2\gamma}$ from the size of the form factor discrepancy has been employed by many others in the past starting from a range of assumptions and using a variety of assumed functional forms [1, 12, 15–22]. In [18], Borisyuk and Kobushkin derived an expression that is mathematically equivalent to that of equation (6) though using a slightly different set of assumptions. In [20], Qattan *et al* employ the expression of [18] to extract TPE from several Rosenbluth separation data sets. In this work, I use global fit models of $\tilde{G}_E$ and $\tilde{G}_M$ to estimate the size of the TPE correction to resolve the discrepancy for the kinematics of the three recent $R_{2\gamma}$ measurements.

Global fit models

For this method, suitable global fits of $\tilde{G}_E$ and $\tilde{G}_M$ must consider only unpolarized cross section measurements and not include any hard TPE corrections, either on the cross sections or in the fit parameterization. Many well known proton form factor parameterizations (e.g. [23–25]) are therefore not suitable. I consider three suitable fits to exclusively unpolarized elastic electron–proton cross sections:

- Bosted (1995) [26],

- Arrington (2004), unpolarized [12],
- Bernauer *et al* (2013), unpolarized [21].

These fits differ in their parameterization, but more significantly in the input data that are considered. Bosted fits a representative sample of elastic scattering data, which are described in [27]. The Arrington fit, whose procedure is described in [28], includes newer high-Q^2 data from Jefferson Lab [29–31], as well as additional low-Q^2 data from Mainz [32, 33] and Saskatchewan [34]. The Bernauer *et al* fit includes the 2010 Mainz measurements [35], comprising approximately 1400 new data points up to $Q^2 = 1$ GeV^2, in addition to previous world data at larger Q^2. For comparison with these global fits, I also consider the standard dipole parameterization

$$G_E(Q^2) \approx \frac{1}{\mu_p} G_M(Q^2) \approx \left(1 + \frac{Q^2}{0.71\ \mathrm{GeV}^2}\right)^{-2}. \tag{7}$$

A suitable global fit for R_{FF} should consider only polarization measurements, without any incorporation of TPE-corrected unpolarized cross section measurements. This type of fit has not yet been of significant interest so no extremely sophisticated fits of this kind have been published. Gayou *et al* perform a linear fit in the range of $0.5 < Q^2 < 5.6$ GeV^2 [36]. For this paper, I will also use a linear model that is consistent with the world polarization data:

- $R_{FF}(Q^2) = 1 - (0.12\ \mathrm{GeV}^{-2})Q^2$.

Bernauer *et al* conveniently report uncertainty estimates on $\tilde{G}_E$, $\tilde{G}_M$, and their ratio, allowing me to make an uncertainty estimate on the extracted $R_{2\gamma}$. My estimate is approximate; the uncertainties on $\tilde{G}_E$ and $\tilde{G}_M$ are correlated, and these correlations are not reported. Except at very low Q^2 however, the uncertainty on $\tilde{G}_E$ is dominant. Therefore, my uncertainty estimates on $R_{2\gamma}$ extracted using the Bernauer *et al* fits are based on the uncertainty on $\tilde{G}_E$ only. I am further neglecting any contribution to the uncertainty from my linear model of R_{FF}, making my uncertainty estimates an underestimate. It should be noted that, since the Bosted and Arrington fits are based on fewer data than the Bernauer fits, one would expect them to have uncertainties that are at least as large as those of Bernauer *et al*, and significantly larger for $Q^2 < 1$ GeV^2.

Figure 1 shows R_{FF} as predicted by the three unpolarized global fits as a function of Q^2, as well as the Q^2 coverage of the three recent TPE experiments. The proton form factor discrepancy is essentially the deviation between the unpolarized and polarized predictions of R_{FF}. The size of the discrepancy varies considerably between the different unpolarized fits.

One remark must be made regarding the consistency of radiative corrections. Bosted, Arrington, and Bernauer *et al* make explicit efforts to make sure that consistent radiative corrections were re-applied to all input cross sections before fitting. However, they chose to apply different correction prescriptions. Bosted and Arrington follow a prescription based on Mo and Tsai [37], with some improvements detailed in the appendices of [27]. By contrast, Bernauer *et al* adopted the prescription by Maximon and Tjon [38] and further exponentiated the correction to account for radiation at all orders. In addition, Bernauer *et al*, also chose to apply a Feshbach correction for Coulomb distortion. The exact choice of radiative corrections prescription may seem like a small technical detail, but it will alter the form factors obtained from a fit (see, for example, [25]). The choice of prescription amounts to an assertion of what corrections are necessary to make the measured reduced cross sections linear in ϵ for fixed Q^2. However, without redoing the fits, it is difficult to assess the magnitude of the effect on G_E/G_M—since these corrections are nonlinear in ϵ, the effect depends on the ϵ coverage of the input cross section data. Therefore, I make no attempting to correct the Bernauer *et al*

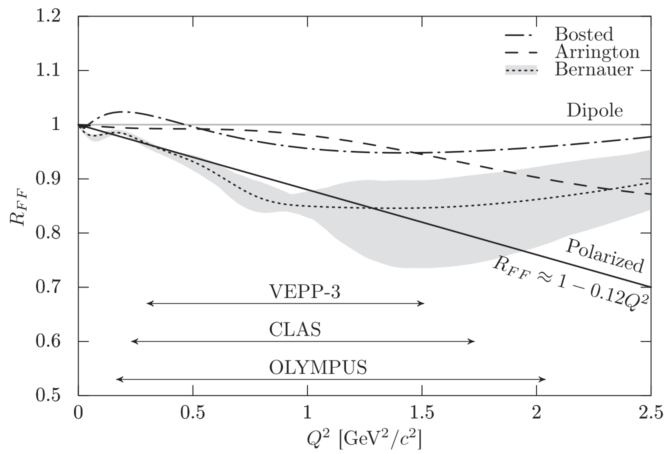

Figure 1. The proton form factor ratio, R_{FF}, is shown as a function of Q^2 for the fits employed in this work. The Bernauer *et al* unpolarized fit predicts a significantly smaller form factor discrepancy than the fits of Arrington and Bosted. The Q^2 coverage of the three recent TPE experiments is shown with arrows.

extractions of the form factors to unify approaches with Arrington and Bosted. I use the global fits as published and the results should be interpreted with this caveat in mind.

The choice of radiative correction prescriptions also affects the interpretation of measurements of $R_{2\gamma}$, since, for example, Maximon and Tjon use a different definition of soft TPE than do Mo and Tsai. The $R_{2\gamma}$ data shown in this work all use the Mo and Tsai definition (OLYMPUS has published results for multiple prescriptions [5]).

Results

Figures 2–4 show predictions for $R_{2\gamma}$ based on equation (6) as functions of ϵ in the kinematics of the VEPP-3, CLAS, and OLYMPUS TPE experiments, compared with their respective results. As a general trend, the measured data fall within or slightly above the uncertainty band using the Bernauer fits, but below the predictions using the Bosted and Arrington fits.

The results of the two runs of the VEPP-3 TPE experiment are shown in figure 2. The inner error bars show the statistical uncertainty, while the outer error bars show the statistical and systematic uncertainties added in quadrature. Arrows mark the luminosity normalization points (LNPs), the kinematic point to which $R_{2\gamma}$ was normalized. In comparing the data to predictions, the measured values of $R_{2\gamma}$ can float relative to the value of $R_{2\gamma}$ at the LNP. The band associated with the prediction using Bernauer *et al* indicates the uncertainty arising from the statistical and systematic uncertainty (added in quadrature) on $\tilde{G}_E$. The data from both beam energies show an increasing $R_{2\gamma}$ with decreasing ϵ, which is the correct sign for explaining the discrepancy. The magnitude of this increase falls between the prediction of the Bernauer fits and those of the Bosted and Arrington fits.

The results from the CLAS TPE experiment, using their constant Q^2 binning scheme [14], are shown in figure 3. Inner error bars show the statistical uncertainty, while outer error bars show the statistical and systematic uncertainties added in quadrature. A normalization uncertainty of 0.003 is not shown. Like with the VEPP-3 results, the CLAS data are below the

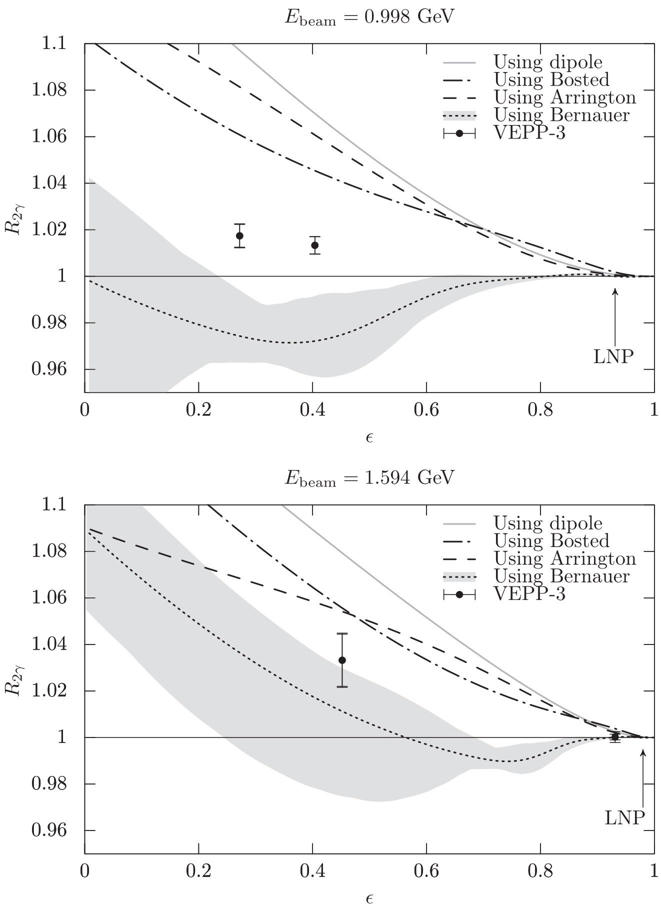

Figure 2. The results from the VEPP-3 TPE experiment [3] for a beam energy of 0.998 GeV (top panel) and 1.594 GeV (bottom panel) fall below predictions based the Bosted and Arrington fits but above the prediction based on the Bernauer fit.

predictions using the Bosted and Arrington fits, but are reasonably consistent with those using the Bernauer fits when accounting for uncertainties.

The results from the OLYMPUS experiment [5], with exponentiated Mo and Tsai radiative corrections, are presented in figure 4. The inner error bars show statistical uncertainty, while the outer error bars show statistical and point-to-point systematic uncertainties added in quadrature. Additional correlated uncertainty ranging from 0.0036 to 0.0045 is not shown.

The OLYMPUS results have a non-zero slope, increasing with decreasing ϵ, indicating a hard TPE contribution. However, at high epsilon, the data fall below $R_{2\gamma} = 1$. The OLYMPUS results are closest to the prediction based on the Bernauer fit, but with less slope. Meanwhile, the predictions based on the Bosted and Arrington are significantly above the OLYMPUS data.

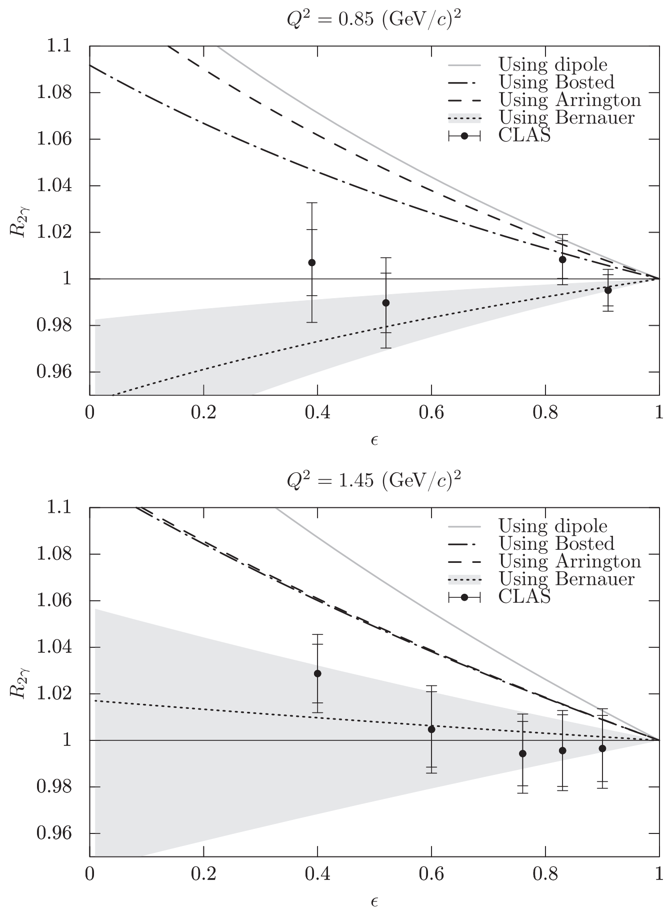

Figure 3. The results from the CLAS TPE experiment [14] for $Q^2 = 0.85$ GeV^2 (top panel) and $Q^2 = 1.45$ GeV^2 also fall below the prediction produced using the Bosted and Arrington fits, but above that coming from the Bernauer fit.

Discussion

There are two general trends that can be seen in figures 2–4. First, the prediction based on the Bernauer fits is significantly different from those based on Bosted, Arrington, and the standard dipole, even given the uncertainty estimate. The inclusion of the 2010 Mainz data set has a large effect on the apparent size of the form factor discrepancy. Looking at figure 1, Bernauer shows no discrepancy up to $Q^2 \approx 1.3$ GeV^2. The R_{FF} difference between the Bernauer fits and the others are driven largely by the differences in G_M at low Q^2. This suggests that as long as there is a lack of consensus on G_M, there will be uncertainty on how big the proton form factor discrepancy actually is, and on how much TPE is needed to resolve it. More unpolarized cross section data, especially at low Q^2 and backward angles, would provide valuable constraints on G_M. New results, such as those from the PRad Experiment [39] will at least allow updated global form factor fits that may help solidify the situation.

Second, the recent TPE data fall below the predictions using the Bosted and Arrington fits, but above the prediction using the Bernauer fits. If the Bosted and Arrington form factor

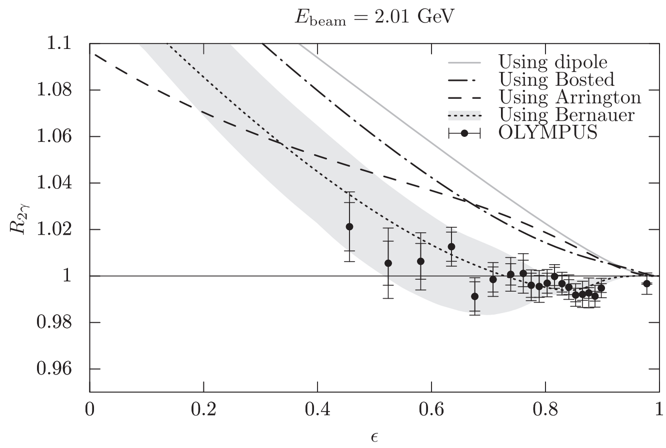

Figure 4. OLYMPUS results [5] are close to the prediction based on the Bernauer fit, but have a smaller slope.

fits are to be believed, the data do not support the hypothesis that TPE is the sole cause of the form factor discrepancy. Judging by the Bernauer fit prediction, there is adequate TPE. The data so far cannot make any definitive claims, and can easily accommodate the TPE hypothesis. As is clear from figure 1, higher Q^2 data are needed for a more definitive test.

Given both the spread in predictions based on different form factor fits, and the large uncertainties indicated by the Bernauer fits, it is clear that a proper and comprehensive uncertainty analysis is needed. Such an analysis must take into account the correlations between fit parameters, the correlations they introduce between G_E and G_M, and the resulting uncertainty on $R_{2\gamma}$.

As new elastic electron–proton scattering data become available, the technique I describe can be used to improve our understanding of the proton form factor discrepancy and the amount of hard TPE needed to resolve it. This can provide valuable context for the interpretation of upcoming experiments, such as MUSE [40], and those being considered at DESY and Mainz.

Acknowledgments

I thank J C Bernauer, R M Milner, and D K Hasell for their helpful advice on this work.

This work was supported by the Office of Nuclear Physics of the US Department of Energy, Grant No. DE-FG02-94ER40818.

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