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Edge Irregular Reflexive Labeling on Corona of Path and Other Graphs

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Abstract. Let $G(V,E)$ be an undirected and simple graph with vertex set V and edge set E Define a k-labeling f on G such that the element belong to E are labeled with integers $\{1,2,\ldots,k_e\}$ and the element belong to V are labeled with even integers $\{0,2,\ldots,2k_v\}$, where $k=\max\{k_e,2k_v\}$. A k-labeling f is mentioned as an edge irregular reflexive k-labeling if distinct edges have distinct weight. The weight of edge xy is denoted by $wt(xy)$ and defined as $wt(xy)=f(x)+f(xy)+f(y)$. A minimum k for which G has an edge irregular reflexive k-labeling is called reflexive edge strength of G and denoted by $\text{res}(G)$. This paper contains investigation of edge irregular reflexive k-labeling for corona of path and other graphs and determination of their reflexive edge strengths.

1. Introduction

All graphs considered in this paper are simple, undirected and connected. A graph *G(V,E)* or to be shorter, the notation can be written as G, contains vertex set V and edge set E. A labeling is a mapping which is has a component graph as a domain and an integers as a co-domain (Wallis [7]). Bača et al. [1] defined a total k-labeling as a labeling with the domain is all vertices and edges and the codomain is integer positive from 1 until k. If there is a different weight for all edges, then the labeling is called edge irregular total k-labeling. The weight of edge xy, notated by $wt(xy)$ is defined as a sum of label of x, label of xy and label of y. The minimum k for which G has an edge irregular total k-labeling is defined as total edge irregularity strength of G, symbolized by $tes(G)$.

There are various type of labeling [4]. In this paper we discuss another kind of labeling, namely edge irregular reflexive k-labeling, which is introduced by Ryan et al. ([6], [2]). The definition of edge weight and irregularity of edge weight are similar with the terminology in the edge irregular total k-labeling. Disparity of these two kinds of labeling is in the codomain of mapping. If f is an edge irregular reflexive k-labeling, then $f(V)$ is an even integers from 0 up to $2k_v$ while $f(E)$ is integers from 1 until k_e. Moreover, $k=\max\{2k_v,k_e\}$, is mentioned as reflexive edge strength of graph G, notated by $res(G)$. The vertex label must be even non negative integers, because it represents the fact that every vertex is allowed to have a loop/loops which is one loop contributes 2 to the vertex degree. Vertex label 0 is permissible as representing that there is no loop in a vertex. The label of edge xy represents the number of parallel edges between two vertices x and y. Thus, firstly there is a multi graph with loop and parallel edges allowed, then the multi graph can be changed to simple graph by giving the label on its vertices and edges, depend on the number of loop in each vertex and the number of parallel edge between two vertices, respectively. Let us recall the following lemma proven by Ryan et al. [2].

Lemma 1. For every graph G,

$$res(G) \geq \begin{cases} \left\lceil \frac{|E(G)|}{3} \right\rceil & , if\ |E(G)| \not\equiv 2,3, mod(6), \\ \left\lceil \frac{|E(G)|}{3} \right\rceil + 1, & if\ |E(G)| \equiv 2,3, mod(6). \end{cases}$$

Some results in $res(G)$ have been obtained, for example Bača et al. [2] have determined the $res(G)$ for generalized friendship graph $f_{n,m}, n = 3,4,5, m \geq 1$, Tanna et al. [6] have established for prisms, wheels, baskets and fans. Then, Zhang et al. [8] have decided for the disjoint association of m copies of some wheel-related graphs, specially gear graph and prism graphs. Guirao et al. [5] have obtained for disjoint union of generalized Petersen graph. In this paper, we discuss the $res(G)$ for corona of path and other graph, namely K_1 and P_2 graphs.

2. The corona of path and complete graph K_1

Based on definition of corona by Frucht and Harary [3], the corona of path $P_n, n \geq 2$ and complete graph K_1, denoted by $P_n \odot K_1$ is a graph constructed by taking one copy of P_n and n copies of K_1, and then connecting the i th vertex of P_n with an edge to every vertex in the i th copy of K_1. Thus, this graph has $2n$ vertices and $2n-1$ edges. The vertex set of this graph is $V(P_n \odot K_1) = \{v_i, v_{i1} : 1 \leq i \leq n\}$ and the set of edge is $E(P_n \odot K_1) = \{v_i v_{i1} : 1 \leq i \leq n\} \cup \{v_i v_{i+1} : 1 \leq i \leq n-1\}$. The *reflexive edge strength* of this graph can be got through Theorem 1.

Theorem 1. For the corona of path Pn and $Kn, n \geq 2$,

$$res(P_n \odot K_1) = \begin{cases} \left\lceil \frac{2n-1}{3} \right\rceil & , for\ n \equiv 0, 1\ mod(3), \\ \left\lceil \frac{2n-1}{3} \right\rceil + 1, & for\ n \equiv 2\ mod(3). \end{cases}$$

Proof.

First, we proof the lower bound of $res(P_n \odot K_1)$. Because the number of edges is $2n-1$, then by Lemma 1, we get

$$res(G) \geq \begin{cases} \left\lceil \frac{2n-1}{3} \right\rceil & , if\ (2n-1) \not\equiv 2, 3\ mod(6), \\ \left\lceil \frac{|E(G)|}{3} \right\rceil + 1, & if\ (2n-1) \equiv 2, 3\ mod(6). \end{cases}$$

This equivalent with the statement:

$$res(P_n \odot K_1) = \begin{cases} \left\lceil \frac{2n-1}{3} \right\rceil & , for\ n \equiv 0,1\ mod(3), \\ \left\lceil \frac{2n-1}{3} \right\rceil + 1, & for\ n \equiv 2\ mod(3). \end{cases}$$

To obtain the upper bound, we construct the θ -labeling on $P_n \odot K_1$ as follows.

$$\theta(v_{i1}) = 2\left\lceil \frac{i-1}{3} \right\rceil,\ for\ i \geq 1.$$

$$\theta(v_i) = \begin{cases} 0 & , for\ i = 1, 2, \\ 2\left\lceil \frac{i-1}{3} \right\rceil & , for\ i \geq 3. \end{cases}$$

$$\theta(v_i v_{i1}) = \begin{cases} 1 & , for\ i = 2, \\ 1 + 2\left\lceil \frac{i-3}{3} \right\rceil, & for\ i \equiv 0\ mod(3), \\ 1 + 2\left\lceil \frac{i-1}{3} \right\rceil, & for\ i \equiv 1\ mod(3), \\ 1 + 2\left\lceil \frac{i-5}{3} \right\rceil, & for\ i \equiv 2\ mod(3), i \neq 2. \end{cases}$$

$$\theta(v_i v_{i+1}) = \begin{cases} 2 & , for\ i = 1, 2, \\ 2\left\lceil \frac{i-2}{3} \right\rceil & , for\ i \geq 3. \end{cases}$$

Then, we know that the label of vertices is even integer and the maximum label of edges and vertices is $\left\lceil \frac{2n-1}{3} \right\rceil$ for $n \equiv 0,1\ (mod\ 3)$ and $\left\lceil \frac{2n-1}{3} \right\rceil + 1$ for $n \equiv 2\ (mod\ 3)$. Computing the weight of edges, we get:

$$wt(v_i v_{i1}) = 2i - 1,\ for\ 1 \leq i \leq n.$$
$$wt(v_i v_{i+1}) = 2i,\ for\ 1 \leq i \leq n - 1.$$

By an observation, it can be seen that all edges have different weight. Therefore, θ is an edge irregular reflexive labeling, which is has reflexive edge strength as in the theorem. The proof of theorem is completed.

Figure 1 is an ilustration of an edge irregular reflexive 4-labeling of $P_5 \odot K_1$. The black color is the label of vertices and edges, while the red color is the edge weight.

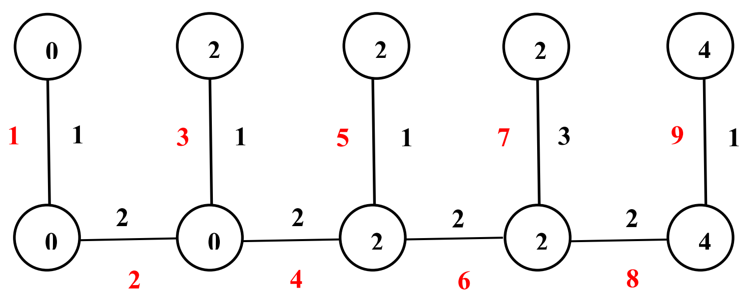

Figure 1. Edge irregular reflexive 4-labeling of $P_5 \odot K_1$

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3. The corona of path and path P_2

Using the similar definition as the above theorem, corona of path $P_n, n \geq 2$ and path P_2, denoted by $P_n \odot P_2$ is a graph formed by linking i th vertex of one copy of P_n to every vertex of i th copy of n copies of P_2. Hence the graph has $3n$ vertices and $4n-1$ edges. Let us consider the vertex set of this graph, $V(P_n \odot P_2) = \{v_i, v_{ij} : 1 \leq i \leq n, 1 \leq j \leq 2\}$ and the set of edge, $E(P_n \odot P_2) = \{v_i v_{ij} : 1 \leq i \leq n, 1 \leq j \leq 2\} \cup \{v_i v_{i+1} : 1 \leq i \leq n-1\}$. The *reflexive edge strength* of this graph can be obtained in Theorem 2.

Theorem 2. Consider the corona of path Pn and P_2, $n \geq 2$,

$$res(P_n \odot P_2) = \begin{cases} \left\lceil \frac{4n-1}{3} \right\rceil & , for\ n \equiv 0, 2\ mod(3), \\ \left\lceil \frac{4n-1}{3} \right\rceil + 1, & for\ n \equiv 1\ mod(3). \end{cases}$$

Proof.

Using Lemma 1 and the fact that cardinality of edge set is $4n-1$, we obtain the lower bound of $res(P_n \odot P_2)$ equivalent with the following statement:

$$res(P_n \odot P_2) \geq \begin{cases} \left\lceil \frac{4n-1}{3} \right\rceil & , for\ n \equiv 0, 2\ mod(3), \\ \left\lceil \frac{4n-1}{3} \right\rceil + 1, & for\ n \equiv 1\ mod(3). \end{cases}$$

Then, the upper bound of this graph can be achieved by constructing φ-labeling as below.

$$\varphi(v_i) = \begin{cases} 0 & , for\ i = 1, \\ i + \left\lceil \frac{i}{3} \right\rceil, & for\ i \equiv 0, 1\ mod(3),\ i \neq 1, \\ i + \left\lceil \frac{i-2}{3} \right\rceil, & for\ i \equiv 2\ mod(3). \end{cases}$$

$$\varphi(v_{i1}) = \varphi(v_{i2}) = \varphi(v_i).$$

$$\varphi(v_i v_{i1}) = \begin{cases} 1 & , for\ i = 1, \\ 4\left\lceil \frac{i-1}{3} \right\rceil, & for\ 2 \leq i \leq n. \end{cases}$$

$$\varphi(v_{i1} v_{i2}) = \begin{cases} 2 & , for\ i = 1, \\ 4\left\lceil \frac{i-1}{3} \right\rceil - 2, & for\ 2 \leq i \leq n. \end{cases}$$

$$\varphi(v_i v_{i2}) = \begin{cases} 3 & , for\ i = 1, \\ 4\left\lceil \frac{i-1}{3} \right\rceil - 1, & for\ 2 \leq i \leq n. \end{cases}$$

$$\varphi(v_i v_{i+1}) = \begin{cases} 2 & , for\ i = 1, 2, \\ 2i - 2\left\lceil \frac{i+1}{3} \right\rceil, & for\ 3 \leq i \leq n-1. \end{cases}$$

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Furthermore, the maximum label for edges and vertices is $\left\lceil \frac{4n-1}{3} \right\rceil$ for $n \equiv 0,2\ (mod\,3)$ and $\left\lceil \frac{4n-1}{3} \right\rceil + 1$ for $n \equiv 1\ (mod\,3)$. The label of vertices is even integer. By calculating the edge weight, we obtain:

$$wt(v_i v_{i1}) = 4i - 3,\ for\ 1 \le i \le n.$$
$$wt(v_{i1} v_{i2}) = 4i - 2,\ for\ 1 \le i \le n.$$
$$wt(v_i v_{i2}) = 4i - 1,\ for\ 1 \le i \le n.$$
$$wt(v_i v_{i+1}) = 4i,\ for\ 1 \le i \le n - 1.$$

Then, there is no doubt that the weights of all edge are different. We conclude that φ is an edge irregular reflexive labeling, which its as in the theorem. Then, the proof is finished.

We give an ilustration for an edge irregular reflexive 6-labeling of $P_4 \odot P_2$ in Figure 2. The black color is the label of vertices and edges, while the red color is the edge weight.

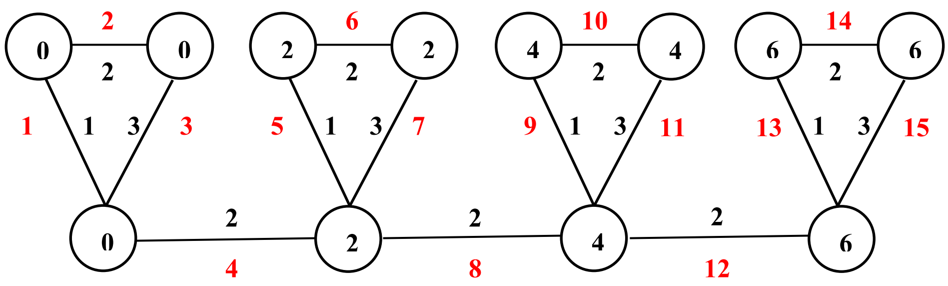

Figure 2. Edge irregular reflexive 6-labeling of $P_4 \odot P_2$

4. Concluding Remark

Repose the discussion, we get the conclusion that the reflexive edge strength of $P_n \odot K_1$ is $\left\lceil \frac{2n-1}{3} \right\rceil$ for $n \equiv 0,1\ (mod\,3)$ and $\left\lceil \frac{2n-1}{3} \right\rceil + 1$ for $n \equiv 2\ (mod\,3)$. While the reflexive edge strength of $P_n \odot P_2$ is $\left\lceil \frac{4n-1}{3} \right\rceil$ for $n \equiv 0,2\ (mod\,3)$ and $\left\lceil \frac{4n-1}{3} \right\rceil + 1$ for $n \equiv 1\ (mod\,3)$. Moreover, we have the following open problems for the direction of further research, which are still in progress.

Open Problem: What is the reflexive edge strength of graph $P_n \odot K_m$ and $P_n \odot P_t$ for $n, t \ge 2$ and $m \ge 1$.

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