

Generating project risk membership functions based on experts' estimates and alpha-cut variations

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Abstract. This paper presents an approach for generating project risk membership function (MFS) based on experts' estimates and α -level variations using simulations. The proposed algorithm employs combination of computer and mathematics application in the area of risk assessment. The determination of appropriate MFS plays a substantial role in the performance of a fuzzy system. Most of the discussions in the previous literature on MFS generated, the assumptions that the risks are outlooked from similar perspective of the experts. However, this would be unlikely true in the real life when there is more than one expert from different background and experience. Proposed simulation method focuses on characteristics of MFS as well as the fuzzy numbers generation incorporating uncertainties in the experts' inputs. Furthermore, results of set of fuzzy numbers of triangular MFS generated is presented in the fuzzy probability distribution and fuzzy cumulative distribution functions.

1. Introduction

In recent years, the application of fuzzy sets theory (FST) has been widely used in various area of decision making including in engineering [1][2], portfolio selection [3], education [4][5] and supplier chain [6] to name the least. Many real-world examples involving the judgement of human in its decision-making such as inherent uncertainty, ambiguities and vagueness is generally unavoidable. A highly reliable and effective performance method is essential in decision making environments. Researchers use FST integrating with other methods to make analysis more profound and thus contributing to the body of knowledge. The key backbone of any fuzzy system performance is empirically dependence on its membership functions i.e. the degree at which element of X is mapped onto the value of 0 to 1 [7][8][9].

Membership function (MFS) comes in various shapes such as triangular, trapezoidal, Gaussian and bell-shaped that can be chosen, and it is demonstrably based on the case study under consideration and its selected input parameters. Moreover, these membership functions can be modified, hybridized, or customized its shape as to give the best fit of the fuzziness of inputs and hence this generates maximum accuracy [10]. The most commonly used and effective membership functions is the triangular membership function [11][12] and it is said to be an alternative method to model fuzzy risk factor for project constructions. Issues on empirical measurement of constructing and generating MFS to develop the best MFS, has been debated over years by many researchers. However, it was left implicitly unanswerable. There is no absolute correct or wrong method in generating MFS [13]. Many



came to a conclusion to use the guesstimating approach and apply the existing measurement in the literatures.

Since more input parameters involves human intuition i.e. expert opinion, it is therefore important to articulate a suitable method incorporating the uncertainties in the judgement. It was found that many researchers and practitioners apply the probabilistic estimation in uncertainty analysis [14][15][16] which on the other hand, very few articles are interested in exploring the possibility theory in the assessment. The glitches in probability distributions on field data can be solved by the use of simulations. These concerns the data limitations for first-timers of construction projects on the vagueness, imprecision and subjectivity in activity risk factor.

Motivation on this paper is an attempt to integrate the probability approach and the possibility theory in constructing MFS. Furthermore, the determination of MFS will be looked into the distribution of MFS on project risk-level based on multiple experts' opinions and the alpha-level variations. A series of simulation experiments are executed and compared when probability distributions are in use, articulating the features of the fuzzy simulation. Risk parameter in this study is on project risk level (RL) and the terms of the linguistic MFS are T, L, M, H, and IN representing tolerable, low, medium, high and intolerable level of risk respectively.

2. Fuzzy sets theory

Fuzzy sets can determine ambiguous notions mathematically. According to the set theory concept, in a set; an object or element may or may not be part of the set, whereby elements of a fuzzy set theory may belong to a variety of membership degrees of a given set. To comprehend further on the theory of fuzzy sets, the main concepts of said theory will be further explained below.

2.1 Main snapshots of fuzzy logic

Given a set S and a universal set X .

In theoretical term, **crisp set** is known that x is an element of set X ($x \in X$). Every element may or may not be part of set S , such as $S \subseteq X$. However, if it is true based on the first statement " x belongs to S ", the latter would be false [17]. In short, crisp sets can be defined in the following ways [18]:

- The elements that belong to set S can be numbered in an order list. Let s_1, s_2, \dots, s_n be a group of set S . The given set S can be expressed as follows:

$$S = [s_1, s_2, \dots, s_n]$$

- A set is defined as a property satisfied by its members known as the rule method. It can be written as:

$$S = \{x|P(x)\},$$

Denoting that S is a set of all elements of X for which the proposition $P(x)$ holds true. For any given $x \in X$, the proposition $P(x)$ is either true or false.

- The elements of X can be defined by a characteristic function μ_S , such, that $\mu_S(x) \in \{0,1\}$. The characteristic function declares which elements are members of a set S and which are not, as follows:

$$\mu_S(x) = \begin{cases} 1 & \text{for } x \in S \\ 0 & \text{for } x \notin S \end{cases}$$

- For each $x \in X$, when $\mu_S(x) = 1$, x is a member of S ; when $\mu_S(x) = 0$, x is a non-member of S .

Unlike crisp set, **fuzzy set** (known as uncertain set) is characterised by its membership function. The values of membership function are from 0 to 1 instead of $\{0, 1\}$ as in the binary set. Given set X , the membership function permits various measures of membership for every element [19].

Let F be a fuzzy set that belongs to X . Then fuzzy set F is expressed by a membership function $\mu_F(x) \in [0,1]$ which states that the elements of X belong to F with a level located in $[0, 1]$

A **fuzzy number** is a specific case of a fuzzy set defined by real numbers, R . A fuzzy number F is a fuzzy set defined by a membership function in the form of $\mu_F: R \rightarrow [0,1]$, in which satisfies the following definitions [20]:

- F is said to be normal when there exists a real number m , such that $\mu_F(m) = 1$.
- For any pair x, y , belongs to support (F), F is said to be convex fuzzy set if for all $\lambda \in [0,1]$; $\mu_F(\lambda x + (1-\lambda)y) \geq \min\{\mu_F(x), \mu_F(y)\}$, where support (F) is the support of F and support (F) = $\{x \in R \mid \mu(x) > 0\}$.
- F is said to be upper semi-continuous for each $\alpha \in (0,1)$, if both α -level set $[F]_\alpha = \{x \in R \mid \mu_F(x) \geq \alpha\}$ and $[F]_\alpha = \{x \in R \mid \mu(x) > \alpha\}$ is closed.

According to [21], the definition of a fuzzy number is based on the following membership function:

$$\mu_F(x) = \begin{cases} L\left(\frac{m-x}{e^L}\right), & x \leq m, e^L \geq 0 \\ R\left(\frac{x-m}{e^R}\right), & x \geq m, e^R \geq 0 \end{cases}$$

Where: $x \in R$;

$L(\cdot), R(\cdot)$ = left and right reference functions of the membership function, respectively;

m = mode, most likely values of the fuzzy number;

e^L, e^R = left and right spreads of the fuzzy number, respectively.

Fuzzy numbers denoted in this form of equation are called L-R type fuzzy numbers. Figure 1 shows the shape where **triangular fuzzy number** (TFN) is termed, as one particular case of semi-symmetric L-R fuzzy number.

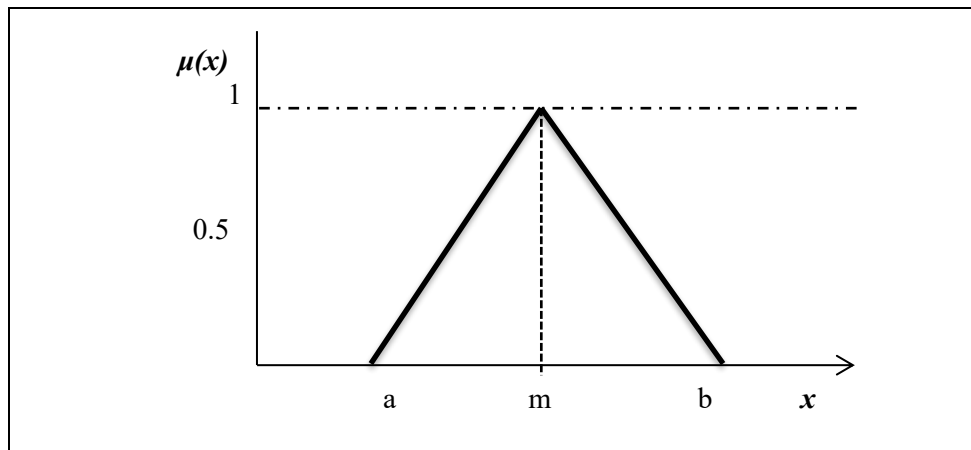


Figure 1. Membership function of a triangular fuzzy number (a, m, b).

2.1.1 α -cut. With a fuzzy set P , we can associate a collection of crisp set known as α -cuts or level sets of P . α - cuts of fuzzy set P denoted as P_α is defined as: $P_\alpha = \{x \in X \mid \mu_P(x) \geq \alpha\}$

A fuzzy set P can be represented by interval as below. $P_\alpha = [p_1^{(\alpha)}, p_2^{(\alpha)}]$

The α -level sets can be obtained as follows:

Let $X = [a, b, c] > 0$ be a fuzzy number. Then $X = [(b-a)\alpha + a, c - (c-b)\alpha]$ is the α -cut of the fuzzy numbers of X .

$$f(x; a, b, c) = \max \left(\min \left(\frac{x-a}{b-a}, \frac{c-x}{c-b} \right), 0 \right)$$

2.1.2 Fuzzy arithmetic. Fuzzy set theory portrays the extension principle as one of the most practical principles [19]. It is an extension of the concepts of traditional mathematics technique in providing general method to a fuzzy domain. Fuzzy numbers can be manipulated by fuzzy arithmetic. Addition and scalar multiplication of fuzzy numbers is the main research structure which explains the next discussion.

2.1.3 Fuzzy addition. If P and Q are triangular fuzzy numbers such that $P=(a,b,c)$ and $Q=(d,e,f)$, then $P(+)Q$ is also a triangular fuzzy numbers. Then the addition operation of two fuzzy set is given by

$$P + Q = (a,b,c) + (d,e,f) = (a + d, b + e, c + f).$$

Given k is a crisp number ($k \in R$), then their multiplication is given by

$$\begin{aligned} k \cdot P &= (ka, kb, kc), & k > 0 \\ &= (kc, kb, ka), & k < 0 \\ &= (0,0,0), & k = 0 \end{aligned}$$

3. Membership functions

3.1 Related works

There are variety of methods used by researchers in constructing and generating membership function of a fuzzy system. Designing the fuzzy sets requires certain input or historical data that comes from the knowledge of acquisition phase. Sources such as domain experts, procedure manuals, articles and relevant documents are needed.

Earlier works on fuzzy system are sought in the study by [9] whom proposed a fuzzy learning algorithm according to the α -cuts of the relations that corresponds to the α -cuts of fuzzy sets. The construction of MFs of the pairs input-output variables of fuzzy rules were conducted from the numerical training dataset using a Pentium PC program in MATLAB. The results obtained from the experiment indicated that the proposed method has a higher average of classification ratio and therefore can reduce the huge number of rules generated as compared to the existing algorithm.

There are several researches focused on an automatic generating membership function. [22] developed a method of automatic and MFS generation that is dependable to the connection of the fuzzy usage and later examines its application in several datasets. The implementation was carried out by representing input values for problem presentation and the result are in different ways, one of which the functions were obtained from an expert. Unfortunately, the proposed method is rather inefficient and unobtainable or available, hence automatic membership function and definition is extremely desirable. [23] was interested in ranking the fuzzy numbers. The researchers proposed ranking techniques using numerical examples and found out that the approach outlined could eventually overcome the shortcoming of the existing fuzzy ranking approach. There were also developments on several membership function construction in engineering knowledge. More works on fuzzy system has shown that the use of probability distributions to generate these membership functions. [7] incorporated Monte Carlo simulations in study of variations in MFS. The researchers outlined their areas of the studies: (1) there is a minor variation in the centre points of the MFS; (2) variation in the widths of MFS; and lastly the addition of “white noise” to MFS. However, the use of mathematical statistics will yield different results for different applications.

In recent years, method of self-generating MFS has been prevalently utilized by several researchers and scholars in the said area. According to [8], semi unsupervised learning method is proposed as one of the methods for self-generating the membership functions. As the starting point, the study was carried out by taking the emergence of triangular and trapezoidal MFS using neural network clustering

approach. On top of that, underlying data was clustered in order to obtain the cluster centres. And these cluster centres were then used to formulate MFS centres. The overlapping MFS end vertex is formulated by approximating boundary values of each cluster obtained. Finally, a fuzzy inference system is developed based on MFS generated using classification problem.

[24] proposed an algorithm which utilizes the ability of the statistical techniques: to analyse probabilistic or non-deterministic systems. The program known as GA based entropy function optimization algorithm were proposed to obtain “Optimized Support” subjected to maximum collective fuzzy entropy of the fuzzy system. In order to generate initial benchmark for optimization process, it requires several assumptions made in the model distribution errors. More advanced method proposed by [25] illustrated in the PPP-BOT case study. The method employs hybrid simulation for risk and uncertainty assessment. The Fuzzy randomness–Monte Carlo simulation (FR-MCS) method extends the conventional Monte Carlo simulation (MCS) by comparing between possibility and probability distributions. The approach developed a new algorithm in generating fuzzy random variables based on α -level set. It can be deduced that the proposed method is ordinarily time-effective and more flexible for decision-makers. The approach by [26] on the other hand, focus on the relationship between risk degrees and risk indexes using three different MFS. The study assumed that risk degrees will not necessarily indicate a linear relationship with risk indexes and finally came up with the conclusion that the relationship between the two risks elements possesses a nonlinear increasing MFS as the best selection.

4. Implementations

4.1 Research design

The membership function generation literature as previously discussed, were mostly under the assumption of similar risk reviewed among the experts. However, in practical reality regarding the differences of understanding and experience between experts, a credible estimation is hardly attainable. It is therefore necessary to aggregate several views appropriately. There are some methods used towards the said aggregation, in which one of it is by adjusting the membership functions in the model input while the other is by regulating the model output by way of aggregating. The multiple inputs extracted can be produced using various both tools qualitatively, quantitatively or by using graphical data representations [27]. Resolution of weights are extracted by the expert's past knowledge, confidence in his/her opinion, information of the matter inspected and the accuracy of past estimation.

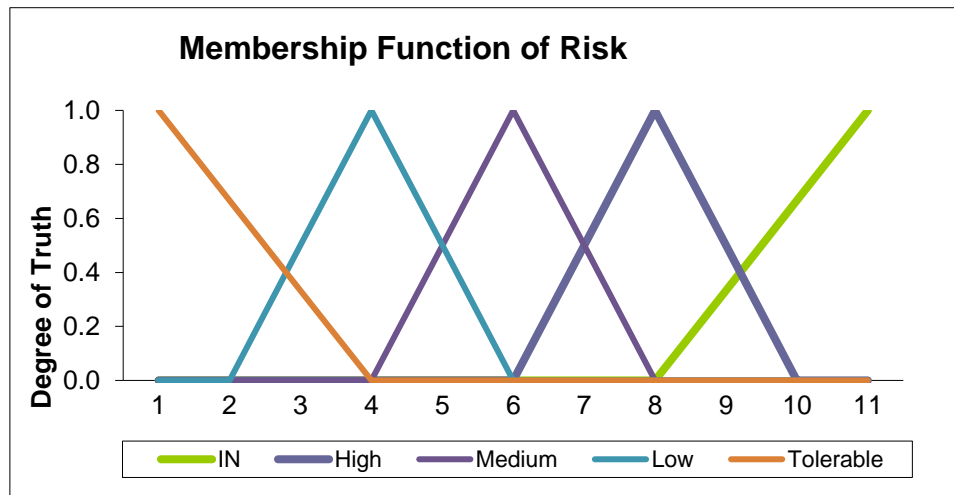
In this study, a risk management team consists of four experts with more than 15 years' experience in the subject is formed to manage risks arising in the office renovation. This study incorporated certain procedures demonstrated by [28] with some modification regarding the case study model, to produce membership functions of the variables. The procedural steps are as follows:

Step 1: The linguistic terms that the project experts frequently use were determined to evaluate the level of risk of the variables from the model as shown in Table 1. Consecutive meeting with experienced personnel in construction industries were conducted, discussing the risk variables valuation. Assumptions were made based on triangular distribution of the risk level i.e. lowest possible value, the most possible value, and the highest possible value of risk level in a subjective scale of 1 to 10. Triangular membership functions were chosen due to its simplicity towards the analysis and revealed better results comparatively. The domains for each risk variables given by four experts were determined based on percentages and index scores. The four experts had agreed in the evaluation of the construction project's risk level, to use the 5-level scale as standard practice i.e. tolerable (T), low (L), medium (M), high (H) and intolerable (IN). The project risk level is treated as uncertain parameter in this illustrative example.

Table 1. Five main linguistic scale and fuzzy numbers

		Crisp	Description	Range of values	Fuzzy numbers
Risk level	P	Tolerable (T)	Risk is very low	Less than 20%	(0,1,3)
	Q	Low (L)	Risk is rather low	21% to 30%	(1,3,5)
	R	Medium (M)	Risk is moderately acceptable	31% to 50%	(3,5,7)
	S	High (H)	Risk is significantly high	51% to 70%	(5,7,9)
	T	Intolerable (IN)	Risk is not acceptable	More than 70%	(7,9,10)

Step 2: For each linguistic term of risk level, experts provided a probable range of numerical values (in percentages) and also the least, most likely and highest possible value that they estimated. Each set (linguistic risk level) has been used to generate 1,000 random samples. Samples are used to obtain probability density function (PDF) and cumulative density function (CDF). The aggregation of all four experts' estimates represent the MFS of project risk level. Figure 2 demonstrates the development of membership functions to denote the risk level for a construction project at different experts' level of uncertainties.

**Figure 2.** Membership functions

The numerical fuzzy data can be converted to obtained triangular fuzzy numbers (OTFNs) which are relatively easy and intuitive to use by experts (E_i). The OTFN measured by E_i can be defined by $RL_{ij} = (a_{ij}^l, a_{ij}^m, a_{ij}^u)$ in the interval $[1,10]$. Its corresponding MFS is shown below using predetermined fuzzy numbers.

$$\mu_{\text{tolerable}}(x) = \begin{cases} x & 0 \leq x \leq 1 \\ \frac{3-x}{2} & 1 < x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_{\text{low}}(x) = \begin{cases} \frac{x-1}{2} & 1 \leq x \leq 3 \\ \frac{5-x}{2} & 3 < x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_{\text{medium}}(x) = \begin{cases} \frac{x-3}{2} & 3 \leq x \leq 5 \\ \frac{7-x}{2} & 5 < x \leq 7 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_{\text{high}}(x) = \begin{cases} \frac{x-5}{2} & 5 \leq x \leq 7 \\ \frac{9-x}{2} & 7 < x \leq 9 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_{\text{intolerable}}(x) = \begin{cases} \frac{x-7}{2} & 7 \leq x \leq 9 \\ \frac{10-x}{2} & 9 < x \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

Step 3: MFS fabrication based on variations of α -level set values of 0.05, 0.1, 0.3, 0.5, 0.7 and 0.9 were chosen. A preliminary membership value $\mu_A(x)$ was determined to every linguistic term; used to calculate the respective risk items by the experts' degree of knowledge that concerns the range of values and its values represented. The amount of uncertainty involved in the construction of MFS is represented by the variations of α -cut levels [25]. For each α -cut value, random samples of 1000 were generated. On each sample, minimum and maximum value of CDF's among five sets were determined to get the lower and upper bound of intervals corresponding to each α -cut for fuzzy random results (Figure 3).

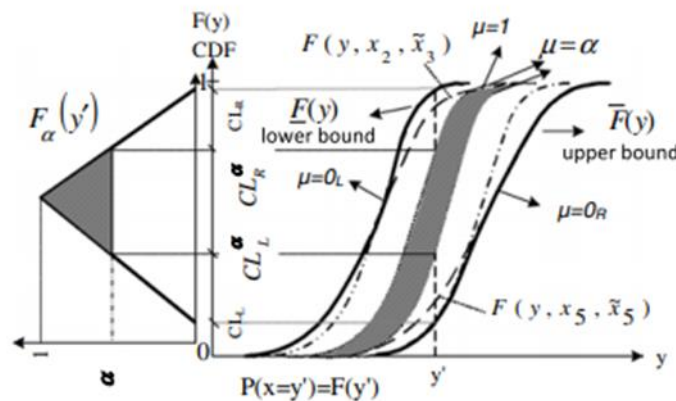


Figure 3. Confidence interval of fuzzy distribution (Source:[25])

Step 4: The interval of confidence at level α is characterized as follows:

$$Y_{\alpha} = [x_{\alpha}^L, x_{\alpha}^R] = [(b-a)\alpha + a, c - (c-b)\alpha]$$

The resulting intervals for triangular fuzzy numbers using a pre-defined fuzzy number are characterized as follows:

$$\begin{aligned} P &= [(1-0)\alpha + 0, 3 - (3-1)\alpha] = [\alpha, \alpha] \\ Q &= [(3-1)\alpha + 1, 5 - (5-3)\alpha] = [2\alpha + 1, 3\alpha] \\ R &= [(5-3)\alpha + 3, 7 - (7-5)\alpha] = [2\alpha + 3, 5\alpha] \\ S &= [(7-5)\alpha + 5, 9 - (9-7)\alpha] = [2\alpha + 5, 7\alpha] \\ T &= [(9-7)\alpha + 7, 10 - (10-9)\alpha] = [2\alpha + 7, 9\alpha] \end{aligned}$$

Step 5: Generate random variables from resulted intervals, $Y_\alpha = [x_\alpha^L, x_\alpha^R]$ correspond to each set of α -level $[x_\alpha^m = x_\alpha^L + \text{RAND}() \times (x_\alpha^R - x_\alpha^L)]$. Hence, a set of obtained triangular fuzzy numbers is generated [25].

$$\text{OTFN} = \{x_\alpha^L, x_\alpha^m, x_\alpha^R\}$$

The Excel Risk Simulator add-ins were used for the algorithm described in the previous section. By using the expert input as an illustrative example, the program is then tested.

5. Results and discussions

Experts estimates in this study uses the method of aggregating the PDF of four expert opinions with regards to MFS. The simulation output function can be modelled using the intervals of confidence and fuzzy numbers instead of solely on probabilistic characterization. Based on estimation from experts at three intervals of confidence, results in Table 2 portrays the obtained MFS in numerical form.

Table 2. MFS based on expert's estimates

	Membership Functions	Expert Estimates (pdfs)					
		25%		50%		90%	
		low	high	low	high	low	high
P	$\mu_{\text{tolerable}}(x)$	0.7	0.8	0.7	0.8	0.5	0.9
Q	$\mu_{\text{low}}(x)$	2.2	2.3	2.1	2.3	2.0	2.5
R	$\mu_{\text{medium}}(x)$	4.2	4.3	4.1	4.4	3.9	4.5
S	$\mu_{\text{high}}(x)$	5.7	5.8	5.6	5.9	5.4	6.1
T	$\mu_{\text{intolerable}}(x)$	7.9	8.0	7.9	8.1	7.7	8.3

As can be seen from the Table 2, some conclusions are drawn:

- Given the various parameter estimates and the simulation assumption of a triangular distribution, the estimated MFS for project risk level **TOLERABLE** anticipates a 90% confidence interval is obtained between 0.5 to 0.9.
- The estimated project risk MFS for **LOW** is between 2.0 to 2.5, **MEDIUM** risk level is between 3.9 to 4.5; **HIGH** between 5.4 to 6.1 respectively.
- Based on the two-tail probability, experts estimated that there is 90% probability that project risk MFS for **INTOLERABLE** is between 7.7 to 8.3.
- The MFS corresponds to percentile value of 50% will provide mean MFS of fuzzy random parameter i.e. risk level.
- At the 25% confidence interval, it is estimated that MFS of risk level is in the interval range of **P** [0.7,0.8], **Q** [2.2,2.3], **R** [4.2,4.3], **S** [5.7,5.8] and **T** [7.9,8.0].

Figure 4(a) and Figure 4(b) display simulations of triangular MFS based on multiple experts estimates using the intervals of confidence in PDFs and CDFs formation.

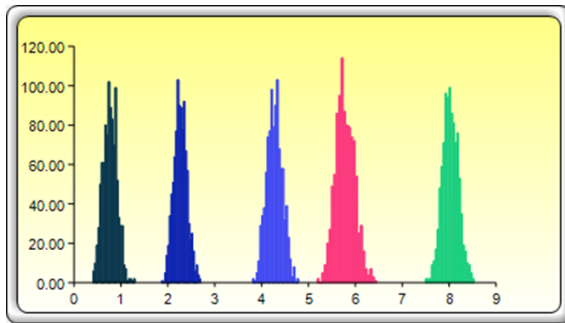


Figure 4(a). PDFs of MFS based on expert estimates

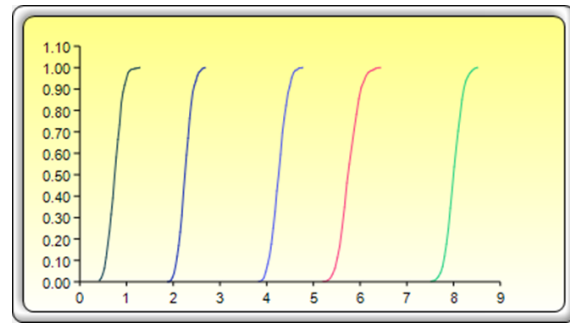


Figure 4(b). CDFs for risk level from multiple experts estimates

5.1 Estimation of uncertainty with MFS at different α -level

The uncertainty measurement was directly derived from the distribution of the resulting risk levels parameters [14] based on expert estimates. In particular, the existence of degree of fuzziness or impreciseness in the RL parameters is analysed to obtain the triangular MFS. The following demonstrates the numerical computation of the lower and upper boundary of MFS. Risk level for *P*, **TOLERABLE** is used as an illustrative example using predefined fuzzy numbers.

$$\begin{aligned}
 P_{0.05} &= [(1 - 0)0.05 + 0.3 - (3 - 1)0.05] \\
 &= [0.05, 2.9] \\
 P_{0.1} &= [(1 - 0)0.1 + 0.3 - (3 - 1)0.1] \\
 &= [0.1, 2.8] \\
 P_{0.3} &= [(1 - 0)0.3 + 0.3 - (3 - 1)0.3] \\
 &= [0.3, 2.4] \\
 P_{0.5} &= [(1 - 0)0.5 + 0.3 - (3 - 1)0.5] \\
 &= [0.5, 2.0] \\
 P_{0.7} &= [(1 - 0)0.7 + 0.3 - (3 - 1)0.7] \\
 &= [0.7, 1.6] \\
 P_{0.9} &= [(1 - 0)0.9 + 0.3 - (3 - 1)0.9] \\
 &= [0.9, 1.2]
 \end{aligned}$$

Figure 5 illustrates distribution of the x-y view of fuzzy numbers resulting from different confidence levels of experts estimates. The combination of a Monte Carlo Simulation with fuzzy probabilistic analysis permits the simultaneous consideration of different types of uncertainty.

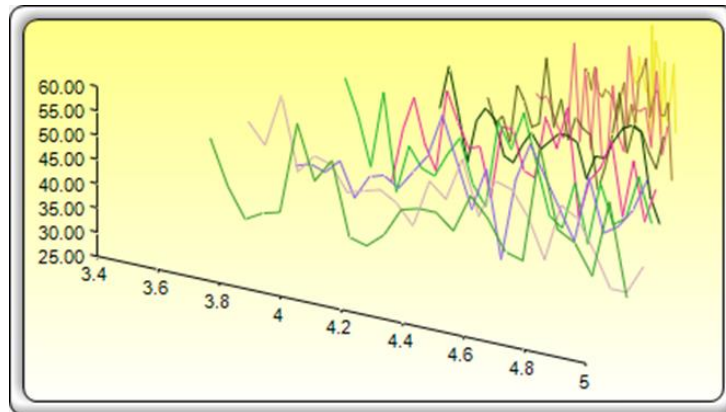
Figure 5. Triangular MFS for various α levels

Table 3 show the lower and upper bound of different MFS level of fuzzy numbers at various α values. The uncertainty of the results is indicated by the width of the fuzzy intervals which provides the decision makers with the information about the vagueness of the resulting values.

Table 3. MFS based on α -level variations

MFS	Interval parameter at different α -level											
	0.05		0.1		0.3		0.5		0.7		0.9	
	x_α^L	x_α^R	x_α^L	x_α^R	x_α^L	x_α^R	x_α^L	x_α^R	x_α^L	x_α^R	x_α^L	x_α^R
P	0.3	1.5	0.3	1.4	0.3	1.2	0.4	1.0	0.4	0.8	0.5	0.6
Q	1.8	3.0	1.8	2.9	1.8	2.7	1.9	2.5	1.9	2.3	2.0	2.1
R	3.5	5.0	3.6	4.9	3.7	4.8	3.9	4.6	4.0	4.5	4.2	4.3
S	5.0	6.7	5.1	6.6	5.2	6.4	5.3	6.1	5.4	5.9	5.5	5.6
T	7.3	8.7	7.3	8.7	7.5	8.5	7.6	8.4	7.8	8.2	7.9	8.1

The estimated interval parameter for fuzzy numbers is summarized in Table 4. It can be stated that at α -level 0.05, the corresponding OTFN are **P** (0.3, 0.9, 1.5), **Q** (1.8, 2.4, 3.0), **R** (3.5, 4.3, 5.0), **S** (5.0, 5.9, 6.7) and **T** (7.3, 8.0, 8.7) respectively. The approach used to generate OTFN somehow must come together with the expert's belief, otherwise he or she can adjust the range of numerical values accordingly based on the project risk under consideration. The results of OTFN is then compared with the agreed pre-sets percentages of each risk level parameter given by experts prior to assessment (see Table 1). Furthermore, the paper mainly aims at proposed algorithms in constructing membership values (x_α^L , x_α^m , x_α^R) on the variations of α directly.

Table 4. Estimated interval parameter of fuzzy numbers

MFS	Interval parameter at different α -level (x_α^L , x_α^m , x_α^R)					
	0.05	0.1	0.3	0.5	0.7	0.9
P	(0.3, 0.9, 1.5)	(0.3, 0.8, 1.4)	(0.3, 0.8, 1.2)	(0.4, 0.7, 1.0)	(0.4, 0.6, 0.8)	(0.5, 0.5, 0.6)
Q	(1.8, 2.4, 3.0)	(1.8, 2.3, 2.9)	(1.8, 2.3, 2.7)	(1.9, 2.2, 2.5)	(1.9, 2.1, 2.3)	(2.0, 2.0, 2.1)
R	(3.5, 4.3, 5.0)	(3.6, 4.3, 4.9)	(3.7, 4.3, 4.8)	(3.9, 4.3, 4.6)	(4.0, 4.3, 4.5)	(4.2, 4.3, 4.3)
S	(5.0, 5.9, 6.7)	(5.1, 5.8, 6.6)	(5.2, 5.8, 6.4)	(5.3, 5.7, 6.1)	(5.4, 5.6, 5.9)	(5.5, 5.5, 5.6)
T	(7.3, 8.0, 8.7)	(7.3, 8.0, 8.7)	(7.5, 8.0, 8.5)	(7.6, 8.0, 8.4)	(7.8, 8.0, 8.2)	(7.9, 8.0, 8.1)

Figure 6 displays the different MFS fuzzy numbers for **TOLERABLE** risk category in which shows a clear narrowing width as line moves to higher level of α . Furthermore, the membership values of the

fuzzy sets at different α value represent the level of possibility of occurrence of a value from an uncertain interval risk. It is observed that as the level of α -value increases, the uncertainty decreases.

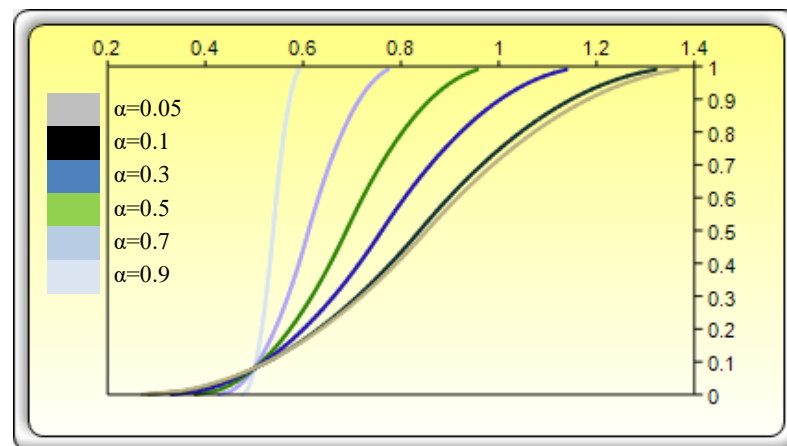


Figure 6. OTFN for risk parameter

6. Conclusions

A fuzzy technique in membership functions construction for risk variable is the main discussion in this paper. It is taken from expert opinion and α -cut variations using simulations. This technique obtained fuzzy probability distributions and fuzzy cumulative distribution functions which have improved the decision making based on conventional simulations by incorporating the uncertainties involved in the experts' estimates. Moreover, the proposed approach is significantly time-effective and less effort is required in developing fuzzy expert system. This study aims as a tool for further progression and able to construct the MFS systematically. The creation for MFS automatically input set-values will further apply for the input-output set of a fuzzy system.

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