IOP Conf. Series: Journal of Physics: Conf. Series **1489** (2020) 012005 doi:10.1088/1742-6596/1489/1/012005

Algorithms to determine fuzzy chromatic number of cartesian product and join of fuzzy graphs

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Abstract. Throughout this paper, we use the concept of fuzzy chromatic number (FCN) of fuzzy graphs based on δ-fuzzy independent vertex sets with $\delta \in [0,1]$ published in 2015. In this paper, we construct two type algorithms to determine fuzzy chromatic number of cartesian product and join of fuzzy graphs. The first type algorithm is constructed to find FCN of specific fuzzy graphs based on formulas in the theorems. The second type algorithm is designed to establish FCN of cartesian product and join of any fuzzy graphs based on the concepts of the cartesian product and join of fuzzy graphs and fuzzy chromatic algorithm. Experimental results of the algorithms are also discussed.

1. Introduction

Since many real life problems contain indeterminate phenomena, we need tools to model the problems. A fuzzy graph is one of effective tools to model a real life problem because it can deal with indeterminate phenomena. Many researchers have proposed applications of some notions in fuzzy graphs, among others Munoz et al. [1] and Rosyida et al. [2] illustrated applications of fuzzy graph coloring in traffic light problems. Akram gave an application of interval-valued fuzzy line graphs in neural networks [3]. Malik et al. [4] proposed an application of the concept of fuzzy incidence graphs for solving a human trafficking problem. Mathew and Mordeson [5] discussed fuzzy incidence blocks and their applications in illegal migration problems, and so on.

The methods for coloring of fuzzy graphs and finding their chromatic number (CN) have been given by several researchers. Cioban [6] introduced a coloring of fuzzy graphs through fuzzy independent vertex sets (FIVS) depended on values δ in [0,1] interval. Further, a fuzzy graph coloring by means of maximal FIVS has been put forward by Bershtein and Bozhenuk [7]. Munoz et al. [1] created a coloring method by way of α-cuts of the fuzzy graphs. Tahmasbpour and Borzooei [8] investigated CN of bipolar fuzzy graphs. In 2015, Rosyida et al. [9] constructed a fuzzy chromatic number (FCN) of fuzzy graphs through δ-FIVS. Moreover, FCN of some operations of fuzzy graphs, i.e. join, union, and cartesian product have been investigated in [10], [2], and [11].

Many researchers also investigated algorithms based on the concepts in fuzzy graphs. Dong et al. [12] invented hierarchical clustering algorithm based on fuzzy graph connectedness. Shiono et al. [13] introduced an algorithm for drawing intelligible and comprehensive fuzzy graphs using a partition tree. A new algorithm to get fuzzy Hamilton cycle in a fuzzy network by means of adjacency matrix and minimum vertex degree has been introduced by Ghani and Latha [14]. Kishore and Sunitha [15] gave an algorithm to obtain CN of fuzzy graphs via α-cuts of the fuzzy graph. Recently, Thakur et al. [16] initiated an efficient coloring algorithm applied for time detraction of sign image segmentation by way of fuzzy graph theory. Rosyida et al. [2] constructed an algorithm to determine FCN of union of fuzzy graphs.

Since a fuzzy graph coloring has many applications in real life problems, it is needed algorithms and computation techniques to solve coloring problems in fuzzy graphs. As a continuation of the previous works, we design algorithms to determine FCN of join and cartesian product of fuzzy graphs based on the properties given in [10] and [11]. Further, we also construct general algorithms for finding FCN of join and cartesian product of fuzzy graphs in this paper. As far as we know, these are novelties in the research of fuzzy graph coloring because there is no one who investigates algorithms and computation techniques for determining FCN of operation of two fuzzy graphs.

The structure of this paper is as follows: Section 2 discuss basic theories used in the next sections. Algorithms to determine FCN of cartesian product and join of fuzzy graphs are presented in Section 3. In Section 4, experimental results are discussed. Finally, conclusions are provided in Section 5.

2. Preliminaries

Throughout this paper, we use basic notions in fuzzy sets cited from [17] and some concepts of fuzzy graphs in [18]. Given a non-empty set V and E is a subset of $V \times V$. A graph which consists of a crisp set V (vertex set) and a fuzzy set $\tilde{E}$ (fuzzy edge set) is named fuzzy graph, symbolized by $\tilde{G}(V,\tilde{E})$, where $\mu: V \times V \to [0,1]$ is a membership function of $\tilde{E}$. Thereafter, the graph $G(V,E)$ is called as a crisp graph. A graph $G^*(V,E^*)$ with $E^* = \{uv \mid \mu(uv) > 0\}$ is named as an underlying graph of $\tilde{G}(V,\tilde{E})$.

Given fuzzy graphs $\tilde{G}_1(V_1,\tilde{E}_1)$ and $\tilde{G}_2(V_2,\tilde{E}_2)$ where $V_1 \cap V_2 = \varnothing$, $\tilde{E}_1, \tilde{E}_2$ have membership functions μ_1, μ_2, respectively. A product $\tilde{G}_1 \square \tilde{G}_2$ is called a cartesian product if its vertex set is $V = V \times V$ and its edge set is $\tilde{E}$ that is defined as $\{(x,y_1)(x,y_2) \mid x \in V_1; y_1y_2 \in \tilde{E}_2\} \cup \{(x_1,y)(x_2,y) \mid y \in V_2; x_1x_2 \in \tilde{E}_1\}$, with membership degrees $\mu_{\tilde{E}}((x,y_1)(x,y_2)) = \mu_2(y_1y_2)$, and and $\mu_{\tilde{E}}((x_1,y)(x_2,y)) = \mu_1(x_1x_2)$. Meanwhile, operation $\tilde{G}_1 + \tilde{G}_2$ is called join of fuzzy graphs $\tilde{G}_1$ and $\tilde{G}_2$ if its vertex set is $V = V_1 \cup V_2$ and the edge set is $\tilde{E} = \tilde{E}_1 \cup \tilde{E}_2 \cup \tilde{E}'$ with $\tilde{E}' = \{(xy, \mu'(xy)) \mid x \in V_1, y \in V_2\}, \mu_{\tilde{E}_1 \cup \tilde{E}_2 \cup \tilde{E}'}(xy) = \max\{\mu_{\tilde{E}_1}(xy), \mu_{\tilde{E}_2}(xy)\}$ when $xy \in \tilde{E}_1 \cup \tilde{E}_2$, and $\mu_{\tilde{E}_1 \cup \tilde{E}_2 \cup \tilde{E}'}(xy) = 1$ when $xy \in \tilde{E}'$ [19].

The concepts of coloring fuzzy graphs depended on values δ was discussed in [6]. Let $\delta \in [0.1]$, a set $\mathcal{A} \subseteq V$ is named a fuzzy independent vertex set (FIVS) of $\tilde{G}(V,\tilde{E})$, if it satisfies $\mu(x,y) \le \delta, \forall x,y \in \mathcal{A}$. The FIVS $\mathcal{A}$ can be symbolized by $\mathcal{A}^{\delta}$. The k-coloring of $\tilde{G}$ was constructed by partitioning V into δ-FIVS $\{\mathcal{A}_1^{\delta}, \mathcal{A}_2^{\delta} \ldots, \mathcal{A}_k^{\delta}\}$ for which $\mathcal{A}_i^{\delta} \cap \mathcal{A}_j^{\delta} = \varnothing, \forall i \ne j$ and $\mathcal{A}_1^{\delta} \cup \mathcal{A}_2^{\delta} \ldots \cup \mathcal{A}_k^{\delta} = V$. The value k (minimum) required in the k-coloring of $\tilde{G}$ is named a δ-chromatic number, represented as $\chi^{\delta}(\tilde{G})$.

Further, we discuss a concept of fuzzy chromatic number (FCN) of fuzzy graphs proposed in [9]. Let $\tilde{G}(V,\tilde{E})$, be a fuzzy graph with n vertices. A fuzzy set $\tilde{\chi}(\tilde{G}) = \{(k, L_{\tilde{\chi}}(k)) \mid k = 1,2,\ldots,n\}$ is said to be FCN of $\tilde{G}$ if $L_{\tilde{\chi}}(k) = \max\{1-\delta \mid \delta \in [0,1], \chi^{\delta}(\tilde{G}) = k\}$. The notation $L_{\tilde{\chi}}(k)$ stands for a membership degree of k in $\tilde{\chi}$. The formula for FCN of cartesian product of fuzzy graphs, especially path and complete fuzzy graphs provided in [11], is presented in the following theorem.

Theorem 1 [11]. Assuming $\tilde{G}_1(V_1,\tilde{E}_1)$ and $\tilde{G}_2(V_2,\tilde{E}_2)$ are fuzzy graphs where $G_1^*(V_1,E_1^*), G_2^*(V_2,E_2^*)$ are their associated underlying graphs in the form of path and complete graphs. Let $|V_1| = n_1$ and $|V_2| = n_2$. If $\tilde{G} = \tilde{G}_1 \square \tilde{G}_2$ is a cartesian product, then $\tilde{\chi}(\tilde{G}) = \{(k, L_{\tilde{\chi}}(k))\}$ is FCN of the cartesian product with

$$L_{\tilde{\chi}}(k) = \begin{cases} \min\left\{L_{\tilde{\chi}_1}(k), L_{\tilde{\chi}_2}(k)\right\}, & \text{if } 1 \le k < n_2, \\ 1, & \text{if } n_2 \le k \le n_1 \times n_2. \end{cases}$$

Further, we construct an algorithm for determining FCN of cartesian product of path and cycle fuzzy graphs based on formulas given in Theorem 1.

3. Main Results

3.1. Algorithms to determine fuzzy chromatic number (FCN) of cartesian product of fuzzy graphs

We present two algorithms for determining FCN of cartesian product of fuzzy graphs. The type I algorithm (Algorithm 1) is designed based on the fuzzy chromatic algorithm [2] and the concept of cartesian product of two fuzzy graphs. It can be applied to cartesian product of any two fuzzy graphs. Let $\tilde{G}_1(V_1,\tilde{E}_1)$ and $\tilde{G}_2(V_2,\tilde{E}_2)$ be fuzzy graphs with the underlying graphs $G_1^*(V_1,E_1^*), G_2^*(V_2,E_2^*)$, $|V_1| = n_1$ and $|V_2| = n_2$.

The first step in type I algorithm is to construct cartesian product $\tilde{G} = \tilde{G}_1 \square \tilde{G}_2$ based on the definition. The second step is to find FCN of cartesian product of fuzzy graphs based on fuzzy chromatic algorithm. The inputs to be processed in Algorithm 1 are the vertex set $V = V \times V$ with cardinality $n_1.n_2$ and the edge set with cardinality $(n_1.|E_2^*|)+(n_2.|E_1^*|)$. Therefore, the size of inputs to be processed in Algorithm 1 are large enough.

Whereas, the type II algorithm (Algorithm 2) is constructed based on the properties in Theorem 1. It can be applied to cartesian product of path and complete fuzzy graphs. There are two steps in Algorithm 2. The first step is to invent FCN of $\tilde{G}_1$ and $\tilde{G}_2$ by using fuzzy chromatic algorithm. The second step is to compute FCN of cartesian product $\tilde{G} = \tilde{G}_1 \square \tilde{G}_2$ according to formulas given in the theorem. The inputs to be processed are $E_1^*, W_1, V_1, E_2^*, W_2, V_2$ where W_1 and W_2 are the sets of membership degrees of edges in $\tilde{E}_1$ and $\tilde{E}_2$. It means that the size of inputs used in Algorithm 2 are smaller than the size of inputs in Algorithm 1 and it will yield less running time as explained in the experimental results.

Table 1. Algorithm 1 (To determine FCN of cartesian product of any two fuzzy graphs).

	Commands
Input	SS1 = load('data E_1, W_1'); SS2=load('data E_2, W_2')
Output	FCN of $\tilde{G} = \tilde{G}_1 \square \tilde{G}_2$
Step 1	Set: $x1 = SS1.E1; x2 = SS1.W1; z1 = SS2.E2; z2 = SS2.W2;$ V1 = unique(x1(:)); nV1 = length(V1); V2 = unique(z1(:)); nV2 = length(V2); [X,Y] = meshgrid(V1,V2); VV = [X(:) Y(:)]; nVV=length(VV); Vnum=[1:nVV]';
Step 2	Create cartesian product $\tilde{G} = \tilde{G}_1 \square \tilde{G}_2$: $[Eo, Wo]$ = fuzzycartesian_function $(x1, x2, z1, z2)$; Cartesian_FG = table(Eo, Wo, 'VariableNames', {'Edge', 'Weight'})
Step 3	Set: V = VV; n = length(V); $E = \{(u,y_1)(u,y_2) \mid y_1y_2 \in z1\} \cup \{(u_1,y)(u_2,y) \mid u_1u_2 \in x1\}$; W = [x2; z2]; delta = [0, W]; m = length(delta);

IOP Conf. Series: Journal of Physics: Conf. Series **1489** (2020) 012005 doi:10.1088/1742-6596/1489/1/012005

Step 4	Determine FCN of the cartesian product: $[\mathrm{Voo}, \mathrm{Loo}] = \mathrm{fchrom\_fcartesian}(\mathrm{V}, \mathrm{E}, \mathrm{W}, \mathrm{delta}, \mathrm{m});$ FCN_cartesian_FG = table(Voo, Loo, 'VariableNames', {'k', 'L_k'})

Meanwhile, the second algorithm is presented in Table 2.

Table 2. Algorithm 2 (Determining FCN of Cartesian Product of Path and Complete Fuzzy Graphs)

	Commands
Input	SS1 = load('data E_1, W_1'); SS2=load('data E_2, W_2')
Output	FCN of cartesian product of path and complete fuzzy graph $\tilde{G} = \tilde{G}_1 \square \tilde{G}_2$
Step 1	Set: $x1 = SS1.E1; x2 = SS1.W1; z1 = SS2.E2; z2 = SS2.W2;$ $\mathrm{V1} = \mathrm{unique}(\mathrm{x1}(:));$ $\mathrm{nV1} = \mathrm{length}(\mathrm{V1}); \mathrm{V2} = \mathrm{unique}(\mathrm{z1}(:));$ $\mathrm{nV2} = \mathrm{length}(\mathrm{V2});$ Path_fuzzygraph = table(x1, x2, 'VariableNames', {'Edge', 'Weight'}). Complete_fuzzygraph = table(z1, z2, 'VariableNames', {'Edge', 'Weight'}).
Step 2	Determine FCN of path and complete fuzzy graphs: $[Vo, Lo] =$ fchrom_fuzzypath_function $(x1, x2)$; $[V1, L1] =$ fchrom_fuzzycomplete_function $(z1, z2)$;
Step 3	Set $N1 = \min(nV1, nV2)$; $N2 = \max(nV1, nV2)$;
Step 4	for $k = 1$ to length $(N1)$ do
Step 5	$L2(k) = \min(Lo(k), L1(k))$;
Step 6	**end**
Step 7	for $k = N1 + 1$ to length $(N2)$ do
Step 8	$L3(k) = L1(k)$;
Step 9	**end**
Step 10	for $k = N2 + 1$ to length $(nV1 * nV2)$ do
Step 11	$L4(k) = 1$;
Step 12	**end**
Step 13	Cartesian product: $[Eo, Wo] =$ fuzzycartesian_function $(x1, x2, z1, z2)$; Cartesian_Path_Complete_FG = table (Eo, Wo).
Step 14	Set $L = [L2\ \ L3(1, nV1 + 1{:}\,nV2)\ \ L4(1, nV2 + 1{:}\,nV1 * nV2)]$; FCN_fcartesian_Path_Complete = table $((1{:}\,(nV1 * nV2))', (L)',$ 'VariableNames', {'k', 'L_k'}).

3.2. Algorithms to determine FCN of join of fuzzy graphs

A property on fuzzy chromatic number of join of fuzzy graphs given in [10] be valid for join operation of any two fuzzy graphs, except:

- the join $\tilde{G}_1+\tilde{G}_2$ contains complete fuzzy subgraphs with even number of vertices (greater than 4) or $\tilde{G}_1$ and $\tilde{G}_2$ contains complete fuzzy subgraphs with odd number of vertices;
- the fuzzy graph $\tilde{G}_1$ contains a complete fuzzy subgraph with number of vertices $n \geq 5$ and $\tilde{G}_2$ contains a cycle fuzzy subgraph or vice versa.

We modify the property in [10] by the proposed remark.

Remark 1. Let $\tilde{G}_1(V_1,\tilde{E}_1)$ and $\tilde{G}_2(V_2,\tilde{E}_2)$ be fuzzy graphs. Let G_1^*, G_2^* be their underlying graphs with chromatic numbers χ_1^* and χ_2^*, respectively. The join $\tilde{G}=\tilde{G}_1+\tilde{G}_2$ has fuzzy chromatic number $\tilde{\chi}(\tilde{G})=\{(k,L_{\tilde{\chi}}(k))\}$, where

$$L_{\tilde{\chi}}(k)=\begin{cases}\min\left(L_{\tilde{\chi}_1}(\frac{k}{2}),L_{\tilde{\chi}_2}(\frac{k}{2})\right), & \text{if } k \text{ is even}, 2\leq k<\chi_1^*+\chi_2^*,\\ \max\left(L_{\tilde{\chi}_1}\left(\left\lfloor\frac{k}{2}\right\rfloor\right),L_{\tilde{\chi}_2}\left(\left\lfloor\frac{k}{2}\right\rfloor\right)\right), & \text{if } k \text{ is odd}, 3\leq k<\chi_1^*+\chi_2^*,\\ 0, & \text{if } k=1,\\ 1, & \text{if } \chi_1^*+\chi_2^*\leq k\leq n_1+n_2.\end{cases} \tag{1}$$

In this section, we present two algorithms for finding fuzzy chromatic number of join of fuzzy graphs. The type I algorithm (Algorithm 3) is constructed based on fuzzy chromatic algorithm [2] and the concept of join of fuzzy graphs. The first step in type I algorithm is to produce the join operation $\tilde{G}=\tilde{G}_1+\tilde{G}_2$ based on the definition of join of fuzzy graphs. The second step is to invent FCN of join of fuzzy graphs based on fuzzy chromatic algorithm. Let $\tilde{G}_1(V_1,\tilde{E}_1)$ and $\tilde{G}_2(V_2,\tilde{E}_2)$ be fuzzy graphs with the underlying graphs $G_1^*(V_1,E_1^*), G_2^*(V_2,E_2^*)$ where $|V_1|=n_1$ and $|V_2|=n_2$. The inputs that will be processed in Algorithm 3 are the vertex set $V=V_1\cup V_2$ with cardinality n_1+n_2 and the edge set $\tilde{E}=\tilde{E}_1\cup\tilde{E}_2\cup\tilde{E}'$ with cardinality $|E_1^*|+|E_2^*|+(n_1.n_2)$. Therefore, the size of inputs used in Algorithm 3 are large enough.

Meanwhile, the type II algorithm (Algorithm 4) is designed for specific fuzzy graphs based on Remark 1. There are two steps in the second algorithm. The first step is to determine FCN of fuzzy graph $\tilde{G}_1$ and $\tilde{G}_2$ by using fuzzy chromatic algorithm. The second step is to compute fuzzy chromatic number of join $\tilde{G}=\tilde{G}_1+\tilde{G}_2$ according to formulas in Remark 1. The inputs to be processed in Algorithm 4 are $E_1^*, W_1, V_1, E_2^*, W_2, V_2$ where W_1 and W_2 are the sets of membership degrees of edges in $\tilde{E}_1$ and $\tilde{E}_2$. It is clear that the size of inputs used in Algorithm 4 are smaller than the inputs in Algorithm 3 and it will produce less running time (this fact is shown in the experimental results). Algorithm 3 is presented in Table 3.

Table 3. Algorithm 3 (To determine FCN of join of any two fuzzy graphs).

	Commands
Input	S1 = load('data E_1, W_1'); S2=load('data E_2, W_2')
Output	FCN of join $\tilde{G}=\tilde{G}_1+\tilde{G}_2$
Step 1	Set: $x1 = S1.E1; x2 = S1.W1;$ $z1 = S2.E2; z2 = S2.W2;$ V1 = unique(x1(:)); nV1 = length(V1);

	$\mathrm{V21} = \mathrm{unique}(\mathrm{z1}(:))$; $\mathrm{nV21} = \mathrm{length}(\mathrm{V21})$;
Step 2	for $i = 1$ to $nV21$ do (%Define labels of vertices in $V2$ by continuing labels of vertices in $V1$)
Step 3	$V22(i) = V21(i) + nV1$;
Step 4	**end**
Step 5	Set: $V2 = (V22)'; nV2 = \mathrm{length}(V2); nz1 = \mathrm{length}(z1)$;
Step 6	for $i = 1$ to $nz1$ do (%Define labels of edges in $E2$ based on new labels of vertices in $V2$)
Step 7	$z(i, 1{:}2) = [z1(i, 1) + nV1 \quad z1(i, 2) + nV1]$
Step 8	**end**
Step 9	Create join $\tilde{G} = \tilde{G}_1 + \tilde{G}_2$: $[X, Y] =$ meshgrid $(\mathrm{V1}, \mathrm{V2})$; $\mathrm{VV} = [\mathrm{X}(:)\ \mathrm{Y}(:)]$; nVV=length(VV); $E = [x1; z; VV]$; $V = [V1; V2]$; $n = \mathrm{length}(V)$; $[Eo, Wo] =$ fuzzyjoin_function $(x1, x2, z1, z2)$; Join_FG = table(Eo, Wo, 'VariableNames', {'Edge', 'Weight'})
Step 10	Determine FCN of the join: W = [x2; z2]; delta = [0; W]; m = length(delta); [Voo, Loo] = fchrom_fjoin(V, E, W, delta, m); FCN_join = table(Voo, Loo, 'VariableNames', {'k', 'L_k'})

Further, we present Algorithm 4 in Table 4.

Table 4. Algorithm 4 (Determining FCN of Join of Specific Fuzzy Graphs)

	Command
Input	S1 = load('data E_1, W_1'); S2=load('data E_2, W_2'); $a1$ = chromatic number of underlying graph G_1^*; $a2$ = chromatic number of underlying graph G_2^*;
Output	FCN of Join $\tilde{G} = \tilde{G}_1 \square \tilde{G}_2$.
Step 1	Set $x1 = S1.E1; x2 = S1.W1; z1 = S2.E2; z2 = S2.W2$; $\mathrm{V1} = \mathrm{unique}(\mathrm{x1}(:))$; $\mathrm{nV1} = \mathrm{length}(\mathrm{V1}); \mathrm{n1} = \mathrm{nV1}; \mathrm{VV1} = \{1{:}\mathrm{n1}]'$; $\mathrm{V2} = \mathrm{unique}(\mathrm{z1}(:)); \mathrm{nV2} = \mathrm{length}(\mathrm{V2}); \mathrm{n2} = \mathrm{nV2}; \mathrm{VV2} = \{1{:}\mathrm{n2}]'$; $\mathrm{delta1} = [0; \mathrm{x2}]$; $m1 = \mathrm{length}(\mathrm{delta1}); \mathrm{delta2} = [0; z2]$; $m2 = \mathrm{length}(\mathrm{delta2})$;
Step 2	Determine FCN of $\tilde{G}_1$ and $\tilde{G}_2$: $[V01, L01] =$ fchrom_function $(VV1, x1, x2, \mathrm{delta1}, m1)$; $[V02, L02] =$ fchrom_function $(VV2, z1, z2, \mathrm{delta2}, m2)$;
Step 3	Set $NN = \max(nV1, nV2)$; $x = a1 + a2$; $V0 = [1{:}(nV1 + nV2)]'$;
Step 4	for $k = 1$ to $x - 1$ do
Step 5	if (rem $(k, 2) == 0$) && $\min(2 \le k \le x - 1)$;

Step 6 $NN1 = L01\left(\frac{k}{2}\right); NN2 = L02\left(\frac{k}{2}\right); L001(k) = \min(NN1, NN2);$
Step 7 else
Step 8 $y1 = \text{floor}\left(\frac{k}{2}\right); NN2 = L01(y1); NN3 = L02(y1)$
Step 9 $L002(k) = \max(NN2, NN3);$
Step 10 end
Step 11 end
Step 12 for $k = x$ to $(nV1 + nV2)$ do
Step 13 $L003(k) = 1;$
Step 14 end
Step 15 Set $LL001 = (L001)'; LL002 = (L002)';$
$LL00 = [0; LL001(2); LL002(3); LL001(4); LL002(5); LL001(6);$
$\ldots; (L003(1, x{:}\, nV1 + nV2))']$
FCN_join = table $(V0, LL00, '\text{VariableNames}', \{'k', 'L_k'\})$

4. Experimental Results

We implement Algorithm 1 and Algorithm 2 for finding fuzzy chromatic number of cartesian product of two fuzzy graphs $\tilde{G}_1 \square \tilde{G}_2$ in Figure 1 where $\tilde{G}_1 = (V_1, \tilde{E}_1), V_1 = \{u_1, u_2, u_3, u_4\}$, and $\tilde{E}_2 = \{(v_1v_5, 0.7), (v_1v_4, 0.6), (v_1v_3, 0.5), (v_1v_2, 0.4), (v_2v_5, 0.3), (v_2v_4, 0.2), (v_2v_3, 0.1), (v_3v_5, 0.6), (v_3v_4, 0.4), (v_4v_5, 0.3)\}$. The outcomes of the algorithms for fuzzy chromatic number (FCN) of the cartesian product in Matlab R2016a are shown in Figure 2.

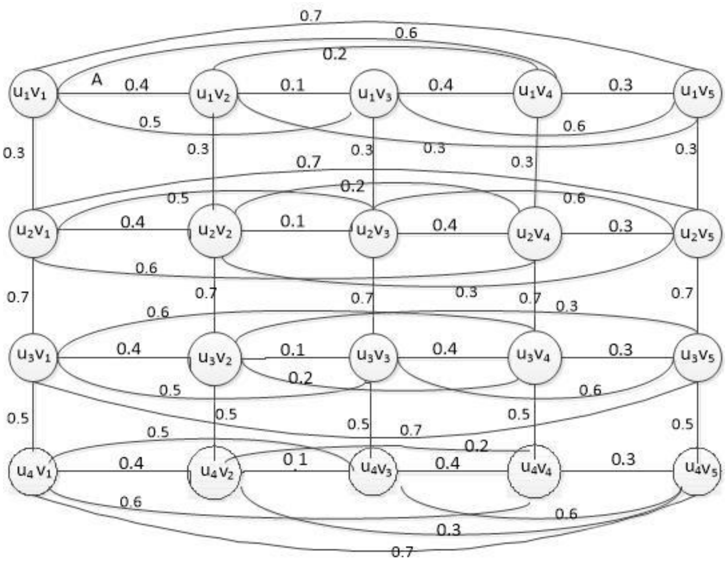

Figure 1. Cartesian product $\tilde{G}_1 \square \tilde{G}_2$.

Fuzzy_chrom_fuzzygraph1

k	L_k
1	0.3
2	1
3	1
4	1

Fuzzychrom_fuzzygraph2

k2	L_k2
1	0.3
2	0.5
3	0.7
4	0.9
5	1

Fuzzychrom_cartesian_pathKn

k3	L_k3
1	0.3
2	0.5
3	0.7
4	0.9
5	1
6	1
7	1
8	1
9	1
10	1
11	1
12	1
13	1
14	1
15	1
16	1
17	1
18	1
19	1
20	1

Elapsed time is 0.296312 seconds.

(a)

Fuzzychrom_fuzzycartesian

k	L_k
1	0.3
2	0.4
3	0.7
4	0.9
5	1
6	1
7	1
8	1
9	1
10	1
11	1
12	1
13	1
14	1
15	1
16	1
17	1
18	1
19	1
20	1

Elapsed time is 57.832365 seconds

(b)

Figure 2. Outcomes of Algorithm 1 (b) and Algorithm 2 (a) for FCN of $\tilde{G}_1 \square \tilde{G}_2$ in Fig.1.

Algorithm 2 which is constructed through formulas given in Theorem 1 has less running time, i.e. average 0.296 seconds, applied for fuzzy graph $\tilde{G}_1$ with 4 vertices and $\tilde{G}_2$ with 5 vertices and the number of edges are 3 and 10 edges. Whereas, the number of vertices and edges to be processed in Algorithm 1 are 20 vertices and 55 edges. It causes longer running time compared with Algorithm 2, i.e. average 57.83 seconds.

Moreover, Algorithm 3 and Algorithm 4 have been implemented for finding fuzzy chromatic number of join of two fuzzy graphs $\tilde{G}_1 \square \tilde{G}_2$ in Figure 3. The experimental results of the algorithms for determining FCN of the join $\tilde{G}_1 \square \tilde{G}_2$ are shown in Figure 4. The inputs to be processed in Algorithm 4 are the number of vertices of V_1 and V_2 that are 5 and 6 vertices and the number of edges, i.e. 10 and 15 edges. It causes less running time, i.e. average 0.149 seconds. Meanwhile, the number of vertices and edges to be processed in Algorithm 3 are larger, i.e. 11 vertices and 55 edges. This conditions cause longer running time compared with Algorithm 4, i.e. average 1.113 seconds.

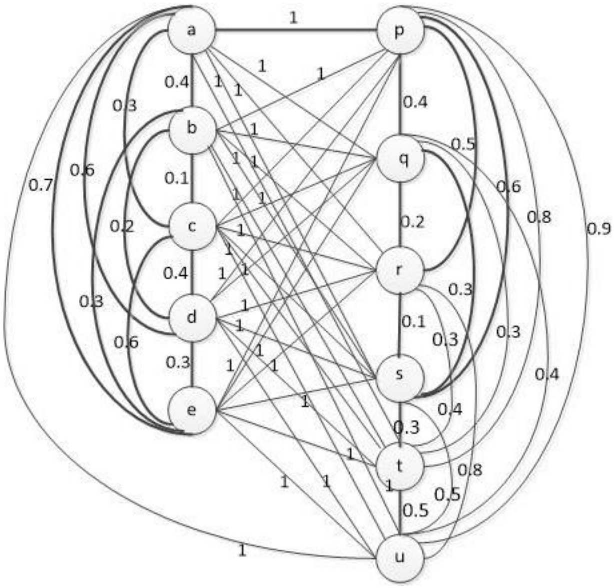

Figure 3. Join $\tilde{G}_1 + \tilde{G}_2$.

Fuzzy_chrom_fgraph1

k1	L_k1
1	0.3
2	0.5
3	0.7
4	0.9
5	1

Fuzzy_chrom_fgraph2

k2	L_k2
1	0.1
2	0.4
3	0.6
4	0.8
5	0.9
6	1

FCN_Join =

k	L_k
1	0
2	0.1
3	0.3
4	0.4
5	0.5
6	0.6
7	0.7
8	0.8
9	0.9
10	0.9
11	1

Elapsed time is 0.149345 seconds.

(a)

FCN_Join =

k	L_k
1	0
2	0.1
3	0.3
4	0.4
5	0.5
6	0.6
7	0.7
8	0.8
9	0.9
10	0.9
11	1

Elapsed time is 1.113433 seconds

(b)

Figure 4. Outcomes of Algorithm 3 (b) and Algorithm 4 (a) for FCN of join in Fig.3.

5. Conclusions

We have constructed two algorithms for determining fuzzy chromatic number (FCN) of cartesian product of fuzzy graphs. The type I algorithm (Algorithm 1) is a general algorithm that can be used for finding FCN of cartesian product of any two fuzzy graphs. The first step in Algorithm 1 is to construct cartesian product of two fuzzy graphs based on the definition. The second step is to find FCN of the cartesian product of fuzzy graphs based on fuzzy chromatic algorithm. Whereas, the type II algorithm (Algorithm 2) is an algorithm to determine FCN of cartesian product of specific fuzzy graphs, i.e., path and complete fuzzy graphs. The steps in Algorithm 2 are constructed based on formulas given in Theorem 1.

Furthermore, two algorithms for finding fuzzy chromatic number (FCN) of join of fuzzy graphs have been constructed. The type I algorithm (Algorithm 3) can be utilized for determining FCN of join of any two fuzzy graphs. The first step in Algorithm 3 is to produce the join operation of two fuzzy graphs based on the definition. The second step is to invent FCN of join of fuzzy graphs based on fuzzy chromatic algorithm. Meanwhile, the type II algorithm (Algorithm 4) is an algorithm to determine FCN of join operation of specific fuzzy graphs according to formulas in Remark 1. In the experimental results, we have shown that the type II algorithms have less average running times compared with the type I algorithms.

In the forthcoming research, we will construct formulas to determine FCN of cartesian product and join of another specific fuzzy graphs, such as fuzzy tree, fuzzy star, etc. Further, we will design algorithms based on the formulas obtained. Moreover, we will investigate properties of FCN of strong product of fuzzy graphs and create algorithms to determine FCN of the strong product based on the properties.

Acknowledgments
This paper is supported by PDUPT-UGM Research grant with contract number No. 2630/UN1.DITLIT/DIT-LIT/LT/2019. Authors wishing to acknowledge to Nurhaida (n.nurhaida@unipa.ac.id) in creating fuzzy chromatic function in Matlab R2016a. Also, authors appreciate to reviewers for their valuable comments.

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