IOP Conf. Series: Journal of Physics: Conf. Series **1489** (2020) 012017 doi:10.1088/1742-6596/1489/1/012017

State estimation for dynamic weighing using Kalman filter

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Abstract. Dynamic weighing has become an essential requirement in a diverse range of industries. In dynamic weighing, loadcell based weighing mechanisms are employed in determining the weight of the products while they are in motion. This paper proposes a method of weight ascertainment based on state estimation theory. A simplified time domain response of the weighing system is modelled as an output error model and the 1-D Kalman filter is used in two stages to determine the weight of the fruit. The dependency of the weight with the change of speed is taken into account in the calibration stage. The validity of the method is tested using the data provided by Compac sorting equipment, Auckland, New Zealand.

1. Introduction

Compac Sorting Equipment (Auckland, New Zealand) is a leading supplier of packhouse technology for the produce industry. Their high-speed sorting systems sort fruit and vegetables by size, colour, defects and sweetness. Compac requires an improved signal processing method for determining the true weight of fruit. As a part of the expected improvement, the settling period of the signal is required to be reduced so that the signal reaches the steady state prior to the data sampled for weight assessment.
Time series data of the load cell readings of fruit passing over the load cell are provided from a number of fruit at three different speeds.

Dynamic weighing is different from static weighing in that static weighing involves determining the weight while the product being weighed is stationary whereas dynamic weighing weighs the products while they are moving. Force sensors are commonly used in these weighing systems. In static weighing, the weighed object is placed stationary on the platform and the steady state of the sensor signal is used to assess the weight. However, in dynamic weighing the sensor signal may not reach the steady state during the brief time of weighing, hence the weight is assessed, for example, by averaging the tail end of the signal after it has been through a low-pass filter. The resulting mass estimates can be inaccurate for faster, heavier items. It is useful to consider better ways of estimating the true weight in igh speed weighing applications

The proposed method is to employ the 1-D Kalman filter algorithm to estimate the optimal state of the signal. The improved steady state signal is then used in weight estimation. The proposed method has been tested using data collected from a loadcell when different masses pass over the loadcell. The results show a significant improvement in the filtered signal quality which is then used to improve the weight assessment.

2. Checkweigher

A checkweigher is the machine that checks the weight of products. It is integrated into the production line and usually found at the end of the production line. There are several different designs depending

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on their application requirements. The primary components of a checkweigher are an in-feed section, weighing platform and an out-feed section.

The items are fed into the weighing conveyer from the in-feed conveyor which is typically mounted on a force transducer. A signal processor receives a signal from the force sensor and estimates a value of the weight for the product that is being passed over the weigh table.

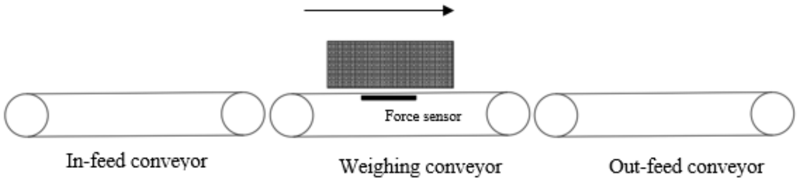


Figure (1). Checkweigher arrangement

3. Mathematical model

In the existing system available at Compac, fruit are transported in individual carriers. Each carrier is pulled across a weighing station which is equipped with a dual loadcell system. When the fruit moves onto the weighing station, the weight of the fruit applies a force ($m'g$) on the platform mounted on two strain gauge loadcells as depicted in Figure (2).

It is assumed that the load is equally shared between the two loadcells.

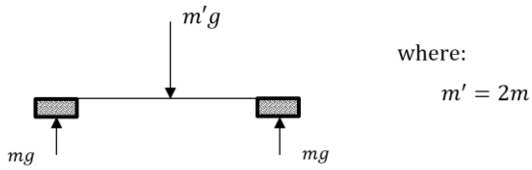


Figure (2). Schematic Diagram of the dual load cell arrangement.

The output voltage of the load cell is amplified and filtered by a fifth order analogue Butterworth filter and sampled at a rate around 4 kHz by a 12 bit 'analogue to digital converter' (ADC).

The readings in the filtered signal are averaged over a set 'weighing window'. This window is a given percentage of the full cycle. It is also given that the current system achieves a repeatability standard deviation of less than 0.5 when 200g moving at 10 fruit per second.

3.1. Loadcell modelling

The loadcell is cantilevered to allow deflection. As the fruit and the carrier apply the weight force on the load cell, it deflects and is set into oscillations. In literature, the dynamics of the load cells are modelled as a spring-mass-damper system, and mathematically represented by a second order differential equation [3],[4].

The mathematical model is developed for a single loadcell. The model also assumes that there is no relative movement between the fruit and the loadcell, i.e. the fruit, carrier and the load cell oscillate as a single unit.

The input function is a step function with a magnitude of mg which is half of the total weight of the fruit.

$$(M+m)\ddot{x}+c\dot{x}+kx=mg\,U(t) \tag{1}$$

where,
c: Damping coefficient
k: Spring constant
m: mass of the fruit
M: mass of the career
U(t) : the unit step function

The transfer function is obtained by taking the Laplace transform[1] of equation (1),

$$(M+m)(X(s)s^2 - sx(0) - \dot{x}(0)) + c(sX(s) - x(0)) + kX(s) = mg\,U(s)$$

$$X(s)\big((M+m)s^2 + cs + k\big) = mg\,U(s) + (M+m)\big(sx(0) + \dot{x}(0)\big) + cx(0)$$

The transfer function of the system,

$$H(s) = X(s) = \frac{mg}{s((M+m)s^2+cs+k)} + \frac{(M+m)\big(sx(0)+\dot{x}(0)\big)+cx(0)}{s((M+m)s^2+cs+k)} \qquad 2)$$

Using the initial value theorem: $\lim_{t\to 0} x(t) = \lim_{s\to\infty} sX(s)$, the value of the transfer function:

$$\lim_{s\to\infty} sX(s) = \lim_{s\to\infty} s\left[\frac{mg}{s((M+m)s^2+cs+k)} + \frac{(M+m)(sx(0)+\dot{x}(0))+cx(0)}{s((M+m)s^2+cs+k)}\right] = 0$$

Using the final value theorem, $\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$, the value of the transfer function:

$$\lim_{s\to 0} sX(s) = \lim_{s\to 0} s\left[\frac{mg}{s((M+m)s^2+cs+k)} + \frac{(M+m)(sx(0)+\dot{x}(0))+cx(0)}{s((M+m)s^2+cs+k)}\right]$$

$$\lim_{s\to 0} sX(s) = \lim_{s\to 0} s\left[\frac{mg}{s\big((M+m)s^2+cs+k\big)} + \frac{(M+m)\big(sx(0)+\dot{x}(0)\big)+cx(0)}{s\big((M+m)s^2+cs+k\big)}\right]$$

$$\lim_{s\to 0} sX(s) = \frac{mg}{k} \qquad (3)$$

These limits describe the system response to a step function when t = 0 (the initial value) and as $t \to \infty$ (the final value). The final value (mg/k) is also the ***steady state value*** of the system response.

Assuming that $\dot{x}(0) = 0, and\ using\ \ x(0) = 0$, the equation (2) can be re-written as,

$$H_L(s) = \frac{(\frac{mg}{k})\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)} \qquad (4)$$

where natural frequency of the fruit and the career, $\omega_n = \sqrt{\frac{k}{M+m}}\ and$ the damping ratio, $\xi = \frac{c\omega_n}{2k}$

The time domain behaviour of the step response is obtained using the partial fraction and inverse Laplace transform of equation (4).

[1] Laplace transform converts a time domain signal into a frequency domain where the magnitude and the phase angle of the signal is determined. The transform also converts differential equations into a simple algebraic equations. (Bateson, 1999, p. 120).

$$x(t) = \frac{mg}{k} - \frac{mg}{k}\frac{(2\xi\omega_n+1)}{\omega\sqrt{1-\xi^2}}e^{-\xi\omega t\sqrt{1-\xi^2}} \quad (5)$$

The time domain response essentially consists of two parts: a constant component of mg/k and a decaying oscillatory component. The response suddenly increases to a constant value of mg/k . The expected time domain behaviour of the load cell output represented by the second order differential equation is shown in Figure (3).

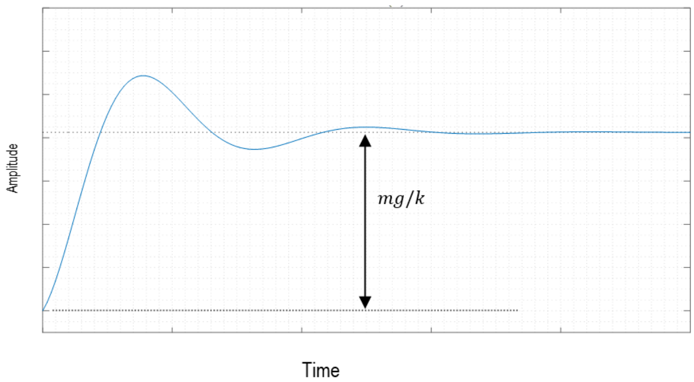
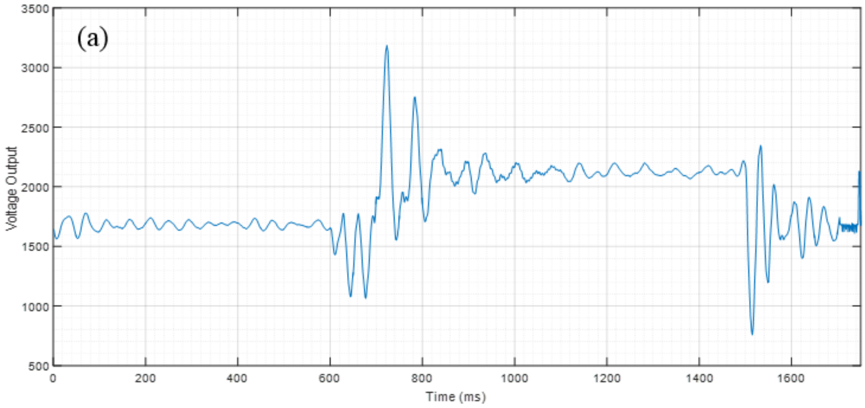


Figure (3). Step response to a second order differential equation

The actual voltage output of the unfiltered signal for 200 g at 0.5 m/s and 573.1g at 0.5 m/s 1.5 m/s are shown in figure (4 (a)) and (4 (b)).

The model response and actual data plots shows that the values suddenly increase to a new value (a constant component of the values) where it settles as the time progresses. The model settles faster after two dominant peaks. The unfiltered data for 200g at 0.5m/s, shows a similar characteristic to that of model response, however, the presence of extra oscillations makes the graph fluctuate around a constant value. 573.1g at 0.5 m/s, demonstrates the constant component. The signal is highly oscillatory during the entire signal time. In comparison, the actual data plots exhibit some similarities to the model behaviour with some deviations.

The second order system equation is developed based on Newton's second Law therefore it precisely represents the system dynamics. Deviation from this expected pattern implies the presence of other elements such as system disturbances, interference and other vibrations noise in the system.



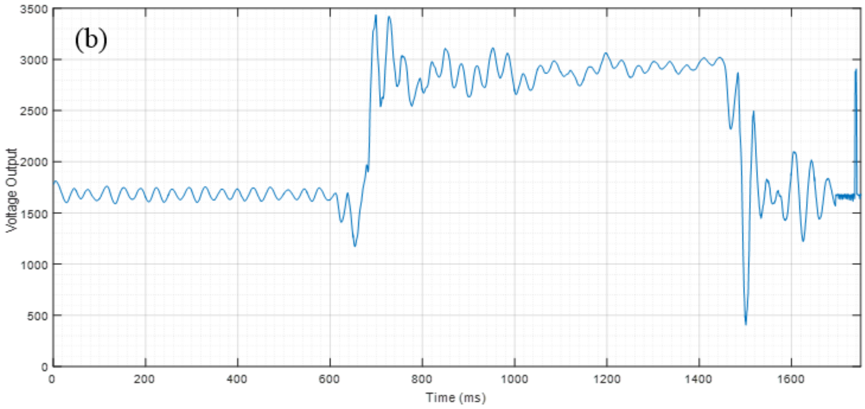


Figure (4). The response of the load cells for calibrated mass of 200 g (a) at 0.5 m/s (b) at 1.5 m/s for a single carrier

4. Kalman filter algorithm

The Kalman filter is an optimal estimation algorithm named after the Rudolf E. Kalman who developed the algorithm. The recursive optimal state estimation algorithm essentially estimates the unmeasured states of linear dynamic systems or processes from noisy observations. Extensions of the algorithm were also developed for non-linear systems [7].

The basic notion of the Kalman filter is presented for the convenience of the reader.

The system dynamics that evolves in time is given by the system equation,

$$x_{t+1} = F_t x_t + w_t \; ; \; w_t \sim N\,(0,\, \sigma_w^2)$$

Where,

x_t : state vector at time, t . (the vector to be estimated) (n x 1).

F_t : state transition matrix that describes the effect of the current state x_t on x_{t+1} , the updated state ($n\ x\ n$)

w_t: process noise vector associated with x_t (n x 1). The process noise is assumed to be Gaussian with zero mean and a known variance of σ_w^2.

The measurement equation,

$$y_t = H_t x_t + v_t \;\; ; v_t \;\sim N\,(v_t; 0,\;\sigma_v^2)$$

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H_t: the transformation vector that maps the system states into the measurement domain $(m \; x \; n)$.

v_t : is observation noise matrix $(m \; x \; 1)$, distributed according to Gaussian distribution with zero mean and a known variance of σ_v^2.

$$\mathrm{v_t} \text{ and } \mathrm{w_t} \text{ are uncorrelated so that } \mathrm{E}[\mathrm{v_j}, \mathrm{w_k}] = 0.$$

The Kalman filter algorithm is summarised below.

The unbiased estimate of the state at time t is $\hat{x}_t$ and P_t is the square error of the priori estimate, i.e., $P_t = E[(\hat{x}_t - x_t)^2]$.
The time updates of the estimate of the state, $\hat{x}_{t+1}$ and P_{t+1} are given by,

$$\hat{x}_{t+1} = F_t\hat{x}_t + K_t(y_t - H_t\hat{x}_t),$$
$$P_{t+1} = (F_t - k_tH_t)P_tF_t^T + \sigma_w^2$$

where, $K_t = F_tP_tH_t^T(H_tP_tH_t^T + \sigma_v^2)^{-1}$

K_t is the most important parameter of the filter and is called *the **Kalman Gain***. The Kalman Gain is calculated in every iteration of the recursive algorithm in such a way that the variance of the new estimate is minimum. When a new observation y_t becomes available a new estimate for the state vector and the variance vector is calculated. Initial values of the state vector x_o and the variance vector P_o must be defined before the filter is implemented. However, the results show that the initial values do not have a significant impact on the filter outcome.

5. *1 – D* Kalman filter approach

In one dimensional state space, the system is characterised by scalar quantities.

x_t - state to be estimated
F_t - state transition coefficient
w_t - process noise associated with x_t and is a scalar variable.

The measurement equation,

$$y_t = H_tx_t + v_t \quad \sim v_t: N\,(0,\; \sigma_v^2)$$

H_t - transformation coefficient that maps the system states into the measurement domain.
v_t - observation noise, a scalar variable zero mean and a known variance of σ_v^2.

The equation (5) presented in Section 3 of this paper shows that the time domain response of the system consists of two components and system noise. The sudden increase of the signal value is responsible for the weight of the fruit. The focus in this approach is to obtain the increase of the signal value by using a simplified mode of the response as shown below.

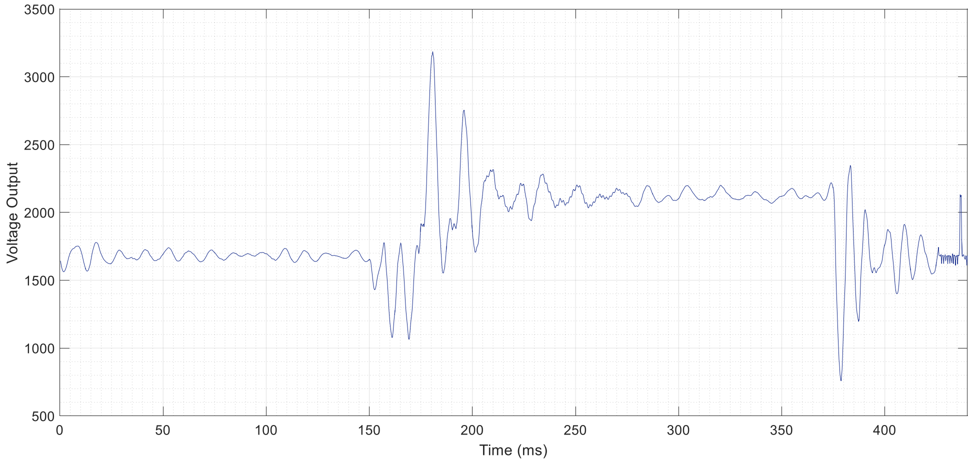

Figure (5). Graph of unfiltered data for 200 g moving at 0.5 m/s

Dashed line marked with black is the simplified response that shows the change in voltage when the load cell is loaded with a fruit (mass).

The data is separated into two sets:
Between A and B, unloaded response
Between C and D, loaded response (constant state of the step response).

The proposed method uses a selected set of data. Two optical position sensors are to be used to start and end weight sampling. When the first sensor is blocked by the cup, the signal for collection of weight data is triggered and when the second sensor is blocked, the weight sampling is to be ended. The data sampled between the sensor signals is used in weight estimation.

The loaded and unloaded responses are treated as two sets of time series data. A steady state value of each is estimated separately, and the difference between the two steady state values can be used to estimate the mass of the fruit.

The time response consists of oscillatory dynamics, constant dynamics and system noise. The 1-d Kalman filter is used to estimate the underlying constant dynamics and this approach assumes that the oscillatory dynamics and noise is the stochastic input to the Kalman filter.

A discrete time state space model is employed for time series data sampled at regular intervals. Development of the state space model is explained below.

In order to obtain the constant state, a constant dynamic model is used as the system model. The process noise variance, σ_w was set to zero assuming that there are no mismatches between the model and the expected constant state.

$$x_{t+1} = x_t + w_t; \ w_t \sim N(0, \ \sigma_w), \text{ where } \ \sigma_w = 0,$$

The measurement equation of the 1- dimensional state space model was given by equation.

$$y_t = x_t + v_t: \ v_t \sim N\,(0, \ \sigma_v)$$

In summary, the state space model proposed is,

$x_{t+1} = x_t$
$y_t = x_t + v_t: \ v_t \sim N\,(0, \ \sigma_v)$

σ_v : The measurement uncertainty was obtained from the sensor data sheets.
x_0 : The first reading of the data sampled between position sensors
P_0 : The square error between the first and the second readings

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6. Results and simulations

The Kalman filter algorithm is tested on numbers of data sets provided by Compac. The results and simulations for loaded data are listed shown in this section. The results of unloaded data exhibit similar performance.

The filter performances are measured by directly comparing the settling times and the standard deviation of the Kalman filter with that of the 5th order Butterworth filter.

Results of number of tests were presented in two sections;

- Constant mass at varying speeds.
- Varying masses at constant speed.

Summary of the results are listed in table (1) and table (2) below.

Table (1). Summary of results for 200 g mass moving at 0.5 m/s, 1.0 m/s and 1.5 m/s. note: some offsets in voltages have been introduced to the Butterworth filter signal values to clearly view the signals.

Conveyor speed (m/s)	**Butterworth**			**Kalman**		
	Mean	Std[2]	Settling time (ms)	Mean	Std	Settling time (ms)
0.5	1419.7	34.1	112	2363.0	7.7	28
1.0	1487.1	57.2	67	2458.0	9.0	47
1.5	1703.3	48.5	-	2822.5	15.8	27

Table (2). Summary of results for 200 g and 573.1 g masses moving at 0.5 m/s. note: some offsets in voltages have been introduced to the Butterworth filter signal values to clearly view the signals.

Mass (g)	**Butterworth**			**Kalman**		
	Mean	Std	Settling time (ms)	Mean	Std	Settling time (ms)
200.0	1562.8	9.8	112	2130.2	1.7	28
573.1	2323.1	43.9	172	2874.1	9.0	45
573.1	2363.0	36.7	158	2924.9	5.0	30
573.1	2343.9	68.9	-	2891.9	11.1	40
573.1	2286.5	35.9	174	2846.4	4.0	35
573.1	2315.1	52.8	174	2884.8	6.3	35

The comparison showed in the Table (1) and Table (2) shows that the Kalman filter outperforms the Butterworth low pass filter in many aspects.

- Observed in Table (1), the standard deviation has a lower value when compared with that of the Butterworth filter in each case. When 200g moving 0.5 m/s the standard deviation decreases from 34.1 to 7.7 (77%) and when the same mass moving at 1.5 m/s, the standard deviation decreases from 48.5 to 15.8 (67.4%).

  Table (2) shows the test results of different masses moving at the same speed of 0.5 m/s. These results also demonstrate similar improvements in standard deviation.

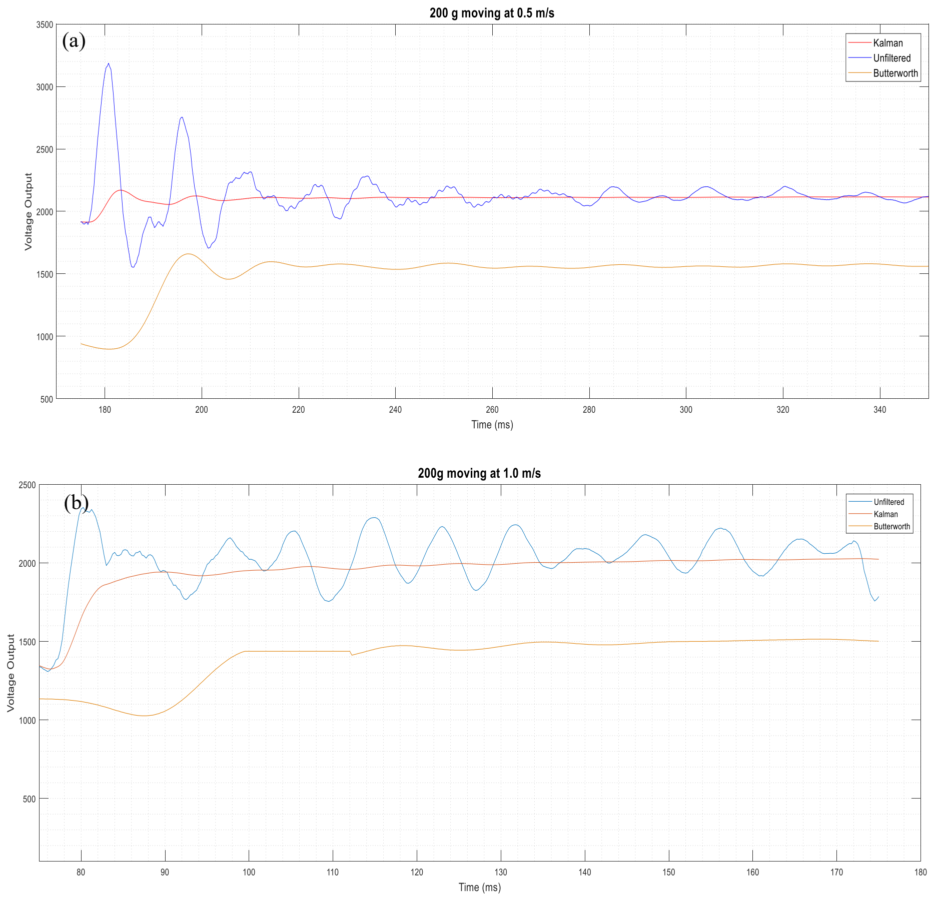
[2] Std- Standard deviation.

The decrease of standard deviation reduces the 95% confidence interval, reducing the margin of error of the filtered signal.

- Settling time tabulated in Table (1) and Table (2), is the time required for the system response curve to reach and stay within 2% about the mean value. The data was examined using spread sheets in MS Excel and a trial and error method was used to estimate the settling time in each case.
  As evident in table (1), the settling time also improves at all the speeds. At 0.5 m/s, the settling time reduces by 75% (112 ms to 28 ms) and at 1.0 m/s, the settling time reduces by 29.8% (from 67 ms to 47 ms). At the highest speed available, i.e. 1.5 m/s, a settling time of 27 ms is observed when using the Kalman filter whereas the existing Butterworth filter response does not reach the steady state prior to the data is sampled for weight assessment.
  This enable the transient response to reach the steady state faster resulting in larger 'averaging window'. This improvement is specifically useful at higher belt speeds where the signal time is shorter, and the signal does not reach steady state.

The improvement in the signal quality (standard deviation and the settling times) listed in table (1) and (2) show that the Kalman filter provides improved filtering solution in dynamic weighing system discussed in this paper. The reduction in settling times and the standard deviation results in improved throughput rate and the measurement accuracy respectively.

Some graphs showing the filtered and unfiltered signal are given in figure (6).



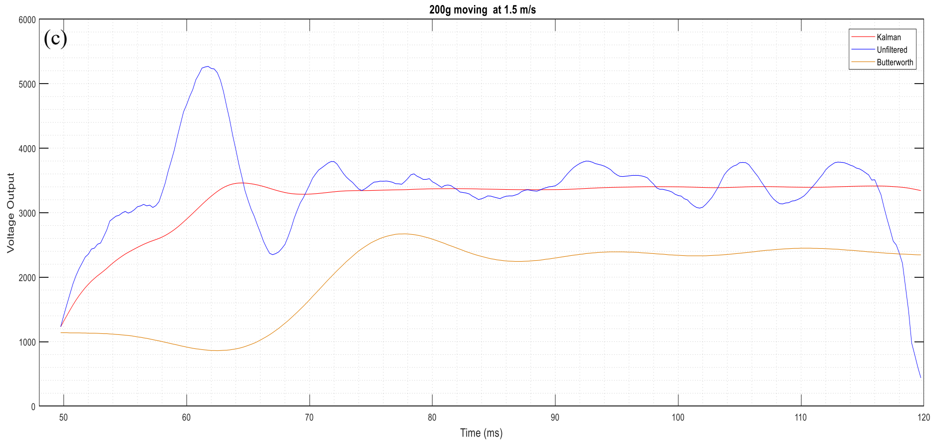


Figure (6). The graphs of filtered and unfiltered data for 'loaded data' for 200g fruit moving at (a) 0.5m/s, (b) 1.0 m/s and (c) 1.5 m/s.

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7. Conclusions

In general, the dynamic weighing systems have limitations in achieving the required throughput and accuracy. An alternative method based on the Kalman filter algorithm is explored in this paper. The Kalman filter was regarded as the optimal solution to data tracking and prediction problems. The filter was constructed as a mean error minimiser in the deterministic derivation. The Kalman filter has many advantages in state estimation in continuously changing systems. The system equation is in the form of a first order difference equation that uses only the information of the previous state. Hence the computation of the states is faster. The filter gain is updated in every iteration of its calculations, minimising the residual with an updated gain factor in every step.

The filter is also adaptive to the data being used. All these features results in a fast response enabling the system reaching the steady state fast.

The continuous time Kalman filter has previously been used in weight estimation in high speed dynamic weighing system using the second order differential equation as the system model [3].

An alternative method of using the one- dimensional Kalman filtering technique has been explored as a possible solution that will enable improved accuracy of dynamic weighing.

The dynamic behaviour of the weighing mechanism was studied and analysed using a mathematical model: a second order differential equation.

The step response of the second order differential equation is given by the equation (5) shown below.

$$x(t) = \frac{mg}{k} - \frac{mg}{k}\frac{(2\xi\omega_n + 1)}{\omega\sqrt{1-\xi^2}}e^{-\xi\omega t\sqrt{1-\xi^2}}$$

$$x(t) = \text{steady state response} + \text{transient response}$$

It consists of a constant state (or a steady state) of magnitude $\frac{mg}{k}$ and a decaying oscillatory component.

The actual data plots exhibit some similarities to the model behaviour with some deviations which imply the presence of other elements such as system disturbances, interference, vibrations and instrumentation noise in the system.

The constant state, or the steady state, that is responsible for the weight of the fruit was estimated using an alternative method, i.e. using the *1- dimensional* Kalman filter algorithm. The step response of the system was considered as two sets of stochastic data; loaded and unloaded data. The mean value of the filtered signal is calculated in each case and the difference between the two mean values are used to estimate the weights of fruit.

The simplicity of the proposed technique and the Kalman filter algorithm will make the proposed method a useful tool in obtaining improved weight measurements in dynamic weighing systems. The method is comparatively less computationally intensive and can be implemented with simple computer resources such as MS-Excel.

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