# Hybrid numerical solution for unsteady state of constant accelerated MHD in a third-grade fluid with a rotation

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**Abstract**. The aim of this research is to investigate the problem related to the constant accelerated of unsteady MHD third grade fluid in a rotating frame. New numerical approach will be used in order to solve the problem. Hybrid numerical approach of finite difference method and asymptotic interpolation method is introduced. This method is suitable for solving unbounded domain where the domain of the problems tends to infinity. Validation has been made with other analytical method; Homotopy Analysis Method to show that this hybrid method is acceptable. The equation of unsteady state MHD third grade fluid in a rotation about z-axis is derived. The nonlinear equation will be discretized by using finite difference method and couple with asymptotic interpolation to fulfil the unbounded domain of boundary condition. The effect of various values of parameters such as MHD, rotation, time, second and third grade are being tested and discussed. This study concludes that the velocity of distribution decreased when the value of MHD and rotation increased. Meanwhile a contrary result occurs when the factor of time increased. The velocity profile for real part also will be increased and imaginary part will be decreased when the parameter of second and third grade increased.

## 1. Introduction

The complexity of non-Newtonian fluid becomes interesting subject among the researchers as there are many kinds of equations related to the fluid flow can be developed. Non-Newtonian fluids can be seen in our daily foods like ketchup and yogurt. Besides that, mucus and blood that have viscous-elastic behavior also belongs to this type of fluid.

In a past few years, several models of fluid flow problem have been discussed numerically and experimentally such as the problem related to the unsteady flow of non-Newtonian third grade fluids in a rotating frame [1,2], steady state of MHD flow for a third grade fluid in rotating and porous space [3], MHD rotating flow of a second grade fluid in a porous space [4], MHD oscillating flows of rotating second grade fluid in a porous medium [5] and Oldroyd-B fluid with the effects of magnetic and porosity [6].

In this paper, magnetohydrodynamic (MHD) and rotation are highlighted as the main factors of the study. MHD is the dynamics of electrically conducting fluids in the presence of magnetic field [7]. The application of MHD can be seen in various industrial area and engineering fields. Prior researchers have been discussed about the rotation that appears in the fields. The imaginary part which is found on the axis that perpendicular (90°) to the real axis will make the rotation exist on a plane. In other words, multiplying by a complex number has same meaning as make a rotation around the origin by the complex number's argument, followed by a scaling and its magnitude.

Many of the mathematical solution have been applied to the fluid flow problems. Homotopy Analysis Method (HAM) is one of the analytical methods which has been used to solve the unsteady flow problem of non-Newtonian third grade fluid in a rotating frame [2]. HAM also is applied to the problem of steady flow of a third grade in a porous plate [8].

Previous researchers also had produced an exact solution of Oldroyd-B fluid with the effects of magnetic and porosity by using Laplace Transform [6]. Numerical Inversion of the Laplace Transform has been applied to the unsteady problems of the MHD flow in a porous medium, MHD flow in non-porous space with Hall currents and flow over a flat rigid plate with porous medium [9].

MHD nonlinear equation of a fourth-grade fluid due to noncoaxial rotations of a porous disk with fluid at infinity is solved by applying numerical method of finite difference method and successive under relaxation scheme [10]. The same numerical method can be seen in the fluid flow problem solved by [11]. Finite element method has been used to solve the problem related to fourth grade fluid subject to no-slip condition and slip condition [12]. Previous researchers also had solved the non - linear differential system of Stokes' first problem by employing Newton method [1]. The non-linear partial differential equation which arise from the problem of unsteady MHD third grade fluid with heat transfer has been discretized by using implicit finite difference scheme and hence used damped Newton method to solve the non-linear algebraic system [13]. Furthermore, finite difference method and Newton iterative method are used to solve the model of nonlinear thin film flow velocity [14].

Hybrid method is a method which combines between two or more methods either in analytical, numerical or both. Hybrid homotopy analysis method which combine between homotopy analysis and shooting method are used to solve the two-dimensional boundary layer flow of third order fluid over a lubricated layer of variable thickness [15]. Hybrid block methods is used to solve the first order of ordinary differential equations [16]. Moreover, hybrid method of Chebyshev wavelet finite difference method is employed to solve the system of higher order boundary value problem [17]. Other works of proposed hybrid method can be seen in computation of the temperature and moisture in porous medium [18]. Numerical solution of hybrid method which couples between finite difference method and asymptotic interpolation has been proposed to solve the variable accelerated problem of third grade fluid in a rotating frame [19].

In this research, the hybrid numerical solution of finite difference method and asymptotic interpolation method is chosen to solve the differential type of non-Newtonian fluid. The constitutive equation and the derivation of unsteady state problem of constant accelerated MHD third grade fluid in a rotating frame is shown under mathematical formulation. The parameters involved are MHD, rotation, third grade fluid and time.

## 2. Mathematical formulation

The incompressible fluid of differential type of grade $n$ obeying the following constitutive equation as follow

$$\mathbf{T} = -p\mathbf{I} + \sum_{j=1}^{n} \mathbf{S}_j \tag{1}$$

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where $\mathbf{T}$ is a Cauchy stress tensor, $p$ is pressure of hydrostatic while $\mathbf{I}$ is an identity tensor. The first three tensors $\mathbf{S}_j$ is given by $\mathbf{S}_1 = \mu\mathbf{A}_1$, $\mathbf{S}_2 = \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2$ and $\mathbf{S}_3 = \beta_1\mathbf{A}_3 + \beta_2\left(\mathbf{A}_2\mathbf{A}_1 + \mathbf{A}_1\mathbf{A}_2\right) + \beta_3\left(tr\mathbf{A}_1^2\right)\mathbf{A}_1$ where $\mu$ is shear viscosity, $\alpha_i\left(i=1,2\right)$ and $\beta_i\left(i=1,2,3\right)$ are material constants and $\mathbf{A}_n(n=1,2,3)$ is the Rivlin-Ericson tensors. After considering the first three tensor $\mathbf{S}_j$ into equation (1), the constitutive equation for third grade fluid is given by

$$\mathbf{T} = -p\mathbf{I} + \left[\mu + \beta_3\left(tr\mathbf{A}_1^2\right)\right]\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 \tag{2}$$

Next, momentum equation and continuity equation (for fluid density is constant) is given by $\rho\dfrac{\partial\mathbf{V}}{\partial t} = \rho b + div\mathbf{T}$ and $\nabla\cdot\mathbf{V} = 0$ where $\rho$ is density, $\dfrac{\partial}{\partial t}$ is material derivatives, $\mathbf{V}$ is velocity, $\rho b$ is body forces and $div\mathbf{T}$ is surface forces. Following [1-2], the momentum equation involving rotation is given as follow

$$\rho\left(\frac{\partial\mathbf{V}}{\partial t} + \left(\mathbf{V}\cdot\nabla\right)\mathbf{V} + 2\Omega\times\mathbf{V} + \Omega\times\left(\Omega\times r\right)\right) = -\nabla p + div\mathbf{T} \tag{3}$$

where $\Omega$ is angular velocity, $r$ is radial coordinate with $r^2 = x^2 + y^2$, $\rho\left(2\Omega\times\mathbf{V}\right)$ is Cariolis and $\rho\left(\Omega\times\left(\Omega\times r\right)\right)$ is centripetal acceleration. Meanwhile, the momentum equation involving magnetic field is given by

$$\rho\left(\frac{\partial\mathbf{V}}{\partial t}\right) = -\nabla p + div\mathbf{T} + \mathbf{J}\times\mathbf{B} \tag{4}$$

where $\mathbf{J}$ is current density, $\mathbf{B}$ is total magnetic field and therefore $\mathbf{J}\times\mathbf{B} = -\sigma B_0^2\mathbf{V}$ is Lorentz force per unit volume.

The nonlinear equation of unsteady MHD third grade fluid in a rotation about z-axis is introduced as follows

$$\rho\left(\frac{\partial u}{\partial t} - 2\Omega v\right) = \mu\frac{\partial^2 u}{\partial z^2} + \alpha_1\frac{\partial^3 u}{\partial z^2\partial t} + 2\beta_3\frac{\partial}{\partial z}\left[\frac{\partial u}{\partial z}\left(\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right)\right] - \sigma B_0^{\,2}u \tag{5}$$

$$\rho\left(\frac{\partial v}{\partial t} + 2\Omega u\right) = \mu\frac{\partial^2 v}{\partial z^2} + \alpha_1\frac{\partial^3 v}{\partial z^2\partial t} + 2\beta_3\frac{\partial}{\partial z}\left[\frac{\partial v}{\partial z}\left(\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right)\right] - \sigma B_0^{\,2}v \tag{6}$$

The initial and boundary conditions for constant accelerated [2] are given as follow

$$u(z,0) = 0,\ v(z,0) = 0 \text{ for } z > 0$$
$$u(0,t) = At,\ v(0,t) = 0, \text{ for } t > 0$$

$$u(z,t)\to 0,\ v(z,t)\to 0 \text{ as } z\to\infty \text{ for every } t \tag{7}$$

Equation (6) is multiplied with complex $i$ and then will be added with the equation (5). Thus, the equation is shown as follow

$$\frac{\partial F}{\partial t}+2i\Omega F=v\frac{\partial^2 F}{\partial z^2}+\frac{\alpha_1}{\rho}\frac{\partial^3 F}{\partial z^2\partial t}+\frac{2\beta_3}{\rho}\frac{\partial}{\partial z}\left[\left(\frac{\partial F}{\partial z}\right)^2\left(\frac{\partial \overline{F}}{\partial z}\right)\right]-\frac{\sigma}{\rho}B_0{}^2F \tag{8}$$

where $F=u+iv$ and $\overline{F}=u-iv$. Consider the following dimensionless parameter to normalize the equation (8)

$$f=\frac{F}{(vA)^{\frac{1}{3}}},\ \eta=z\left(\frac{A}{v^2}\right)^{\frac{1}{3}},\ \tau=t\left(\frac{A^2}{v}\right)^{\frac{1}{3}},\ \ \Omega=R\left(\frac{A^2}{v}\right)^{\frac{1}{3}} \text{ and } M=\frac{\sigma B_0{}^2}{\rho}\left(\frac{v}{A^2}\right)^{1/3}. \tag{9}$$

Therefore, the equation of the system is presented as follow

$$\frac{\partial f}{\partial \tau}+\left(2iR+M\right)f=\frac{\partial^2 f}{\partial \eta^2}+a\frac{\partial^3 f}{\partial \eta^2\partial \tau}+2b\left(2\left(\frac{\partial^2 f}{\partial \eta^2}\right)\left(\frac{\partial f}{\partial \eta}\right)\left(\frac{\partial \overline{f}}{\partial \eta}\right)+\left(\frac{\partial f}{\partial \eta}\right)^2\left(\frac{\partial^2 \overline{f}}{\partial \eta^2}\right)\right) \tag{10}$$

where $a=\frac{\alpha_1}{\rho}\left(\frac{A^2}{v^4}\right)^{\frac{1}{3}}$ and $b=\frac{\beta_3}{\rho}\left(\frac{A^4}{v^5}\right)^{\frac{1}{3}}$ are parameters.

## 3. Method of solution

### 3.1 Finite Difference Method

The nonlinear equation (10) is discretized by using forward and central finite difference method as follow

$$\begin{aligned}\frac{f_i^{n+1}-f_i^n}{k}+\left(2iR+M\right)f_i^n&=\frac{f_{i+1}^n-2f_i^n+f_{i-1}^n}{h^2}+\frac{a}{h^2}\left(\frac{\left(f_{i+1}^{n+1}-2f_i^{n+1}+f_{i-1}^{n+1}\right)-\left(f_{i+1}^{n-1}-2f_i^{n-1}+f_{i-1}^{n-1}\right)}{2k}\right)\\&+\frac{b}{h^4}\left(f_{i+1}^n-2f_i^n+f_{i-1}^n\right)\left(f_{i+1}^n-f_{i-1}^n\right)\left(\overline{f}_{i+1}^n-\overline{f}_{i-1}^n\right)+\frac{b}{2h^4}\left(f_{i+1}^n-f_{i-1}^n\right)^2\left(\overline{f}_{i+1}^n-2\overline{f}_i^n+\overline{f}_{i-1}^n\right)\end{aligned} \tag{11}$$

The initial and boundary conditions in (7) becomes (if the index start from 1)

$$\begin{gathered}f_i^1=0 \text{ for } i=1,2,3,\ldots,N+1\\ f_1^n=A(n-1)\Delta t \text{ for } n=1,2,3,\ldots\\ f_L^n=0 \text{ for every } n=1,2,3,\ldots,\ L=L_1,L_2,L_3.\end{gathered} \tag{12}$$

Rearrange the equation (11) with $n+1$ terms move to the left side while $n$ and $n-1$ terms are move to the right side as follow

$$\begin{aligned}
&\frac{f_i^{n+1}}{k}+\frac{a}{h^2}\left(\frac{f_{i+1}^{n+1}-2f_i^{n+1}+f_{i-1}^{n+1}}{2k}\right)\\
&=\frac{f_i^n}{k}-\left(2iR+M\right)f_i^n+\frac{f_{i+1}^n-2f_i^n+f_{i-1}^n}{h^2}-\frac{a}{h^2}\left(\frac{f_{i+1}^{n-1}-2f_i^{n-1}+f_{i-1}^{n-1}}{2k}\right)\\
&+\frac{b}{h^4}\left(f_{i+1}^n-2f_i^n+f_{i-1}^n\right)\left(f_{i+1}^n-f_{i-1}^n\right)\left(\overline{f}_{i+1}^n-\overline{f}_{i-1}^n\right)+\frac{b}{2h^4}\left(f_{i+1}^n-f_{i-1}^n\right)^2\left(\overline{f}_{i+1}^n-2\overline{f}_i^n+\overline{f}_{i-1}^n\right)
\end{aligned} \tag{13}$$

At grid point $i=2$, the equation (13) becomes

$$\begin{aligned}
&\frac{a}{2h^2k}f_1^{n+1}+\left(\frac{1}{k}-\frac{a}{h^2k}\right)f_2^{n+1}+\frac{a}{2h^2k}f_3^{n+1}\\
&=\frac{f_2^n}{k}-\left(2iR+M\right)f_2^n+\frac{f_3^n-2f_2^n+f_1^n}{h^2}-\frac{a}{h^2}\left(\frac{f_3^{n-1}-2f_2^{n-1}+f_1^{n-1}}{2k}\right)\\
&+\frac{b}{h^4}\left(f_3^n-2f_2^n+f_1^n\right)\left(f_3^n-f_1^n\right)\left(\overline{f}_3^n-\overline{f}_1^n\right)+\frac{b}{2h^4}\left(f_3^n-f_1^n\right)^2\left(\overline{f}_3^n-2\overline{f}_2^n+\overline{f}_1^n\right)
\end{aligned} \tag{14}$$

At grid point $i=3$,

$$\begin{aligned}
&\frac{a}{2h^2k}f_2^{n+1}+\left(\frac{1}{k}-\frac{a}{h^2k}\right)f_3^{n+1}+\frac{a}{2h^2k}f_4^{n+1}\\
&==\frac{f_3^n}{k}-\left(2iR+M\right)f_3^n+\frac{f_4^n-2f_3^n+f_2^n}{h^2}-\frac{a}{h^2}\left(\frac{f_4^{n-1}-2f_3^{n-1}+f_2^{n-1}}{2k}\right)\\
&+\frac{b}{h^4}\left(f_4^n-2f_3^n+f_2^n\right)\left(f_4^n-f_2^n\right)\left(\overline{f}_4^n-\overline{f}_2^n\right)+\frac{b}{2h^4}\left(f_4^n-f_2^n\right)^2\left(\overline{f}_4^n-2\overline{f}_3^n+\overline{f}_2^n\right)
\end{aligned} \tag{15}$$

At grid point $i=N-1$,

$$\begin{aligned}
&\frac{f_{N-1}^{n+1}}{k}+\frac{a}{h^2}\left(\frac{f_N^{n+1}-2f_{N-1}^{n+1}+f_{N-2}^{n+1}}{2k}\right)\\
&=\frac{f_{N-1}^n}{k}-\left(2iR+M\right)f_{N-1}^n+\frac{f_N^n-2f_{N-1}^n+f_{N-2}^n}{h^2}-\frac{a}{h^2}\left(\frac{f_N^{n-1}-2f_{N-1}^{n-1}+f_{N-2}^{n-1}}{2k}\right)\\
&+\frac{b}{h^4}\left(f_N^n-2f_{N-1}^n+f_{N-2}^n\right)\left(f_N^n-f_{N-2}^n\right)\left(\overline{f}_N^n-\overline{f}_{N-2}^n\right)+\frac{b}{2h^4}\left(f_N^n-f_{N-2}^n\right)^2\left(\overline{f}_N^n-2\overline{f}_{N-1}^n+\overline{f}_{N-2}^n\right)
\end{aligned} \tag{16}$$

The equation (14)-(16) will produce a tridiagonal matrix

$$[A]_{(n-2)\times(n-2)}[F]_{(n-2)\times 1}=[b]_{(n-2)\times 1} \tag{17}$$

and hence the results are obtained from $F = A^{-1}b$.

### 3.2 Asymptotic Interpolation Method

The unbounded domain of boundary conditions are taken into consideration whereby the asymptotic interpolation method is embedded into the system by introducing a special function $y = a_0 + a_1 e^{-a_2{}^2 L}$ where $L$ represents different length as $L \to \infty$. In the unbounded domain, as $L = 6,12,18$, the concept of limits is applied and the solution is represented by $a_0$ as shown below:

$$\lim_{L\to\infty} y = \lim_{L\to\infty}\left(a_0 + a_1 e^{-a_2{}^2 L}\right) = a_0 + \frac{a_1}{e^{a_2{}^2(\infty)}} = a_0 + \frac{a_1}{\infty} = a_0 \,. \tag{18}$$

In this study, we couple the finite difference method with asymptotic interpolation method by using minimization technique. Figure 1 shows the graphical representation of solution after inserting the asymptotic function. It also shows that the solution will converge to the horizontal asymptote. $F_1$, $F_2$ and $F_3$ are the results of $L$=6, $L$=12 and $L$=18, respectively while the results of hybrid method are obtained from the least square curve fitting.

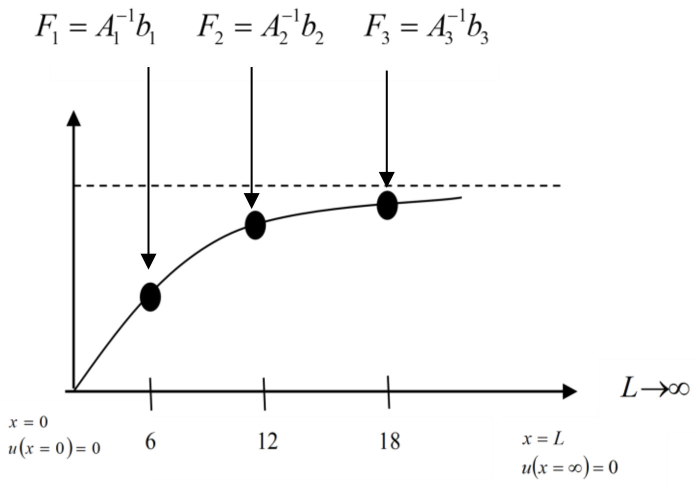



**Figure 1**. Graphical representation of hybrid method

The MATLAB software is used to run the algorithms and plot the functions. The validation of hybrid method has been made by comparing with other method such as analytical method; homotopy analysis methods [19]. Here, the absolute error and relative error are calculated using the formula given as follows

$$\text{Absolute error} = \left|\text{Exact solution} - \text{Hybrid method}\right|. \quad (19)$$

$$\text{Relative error } (\%) = \frac{\text{Absolute error}}{\text{Exact solution}} \times 100\%. \quad (20)$$

Based on the calculation using equation (20), the relative error for this hybrid method is presented in Table 1. Figure 2 shows the graphs of hybrid method and exact solution at $a = b = R = M = \tau = 1$ and it is clearly showing that both graphs are closed to each other.

**Table 1**. Comparison of results between hybrid method and exact solution for $a = b = R = M = \tau = 1$.

| Eta $(\eta)$ | Exact solution (for validation) | Hybrid method | Absolute error | Relative error (%) |
|---|---|---|---|---|
| 0 | 1.0000 | 1.0000 | 0.0000 | 0.0000 |
| 0.2 | 0.7536 | 0.7506 | 0.0030 | 0.3981 |
| 0.4 | 0.5684 | 0.5669 | 0.0015 | 0.2639 |
| 0.6 | 0.4288 | 0.4282 | 0.0006 | 0.1399 |
| 0.8 | 0.3235 | 0.3233 | 0.0002 | 0.0618 |
| 1.0 | 0.2441 | 0.2442 | 0.0001 | 0.0410 |
| 1.2 | 0.1842 | 0.1844 | 0.0002 | 0.1086 |
| 1.4 | 0.1391 | 0.1393 | 0.0002 | 0.1438 |
| 1.6 | 0.1050 | 0.1052 | 0.0002 | 0.1905 |
| 1.8 | 0.0793 | 0.0794 | 0.0001 | 0.1261 |
| 2.0 | 0.0599 | 0.0600 | 0.0001 | 0.1669 |

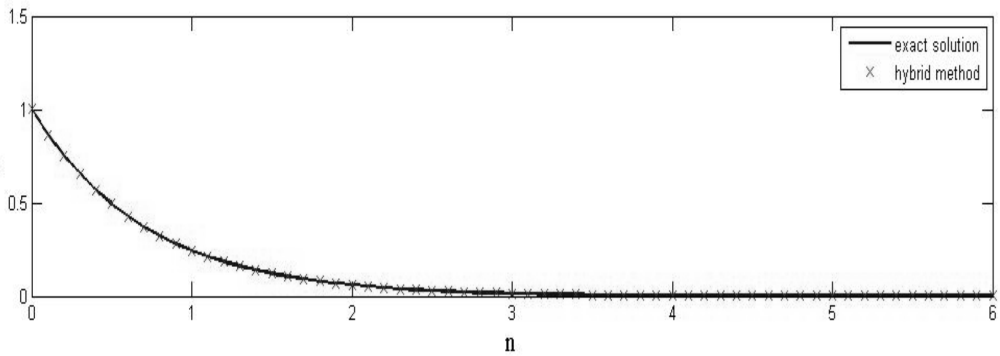



**Figure 2**. Comparison of graphs between hybrid method and exact solution for $a = b = R = M = \tau = 1$.

Figure 3 shows the graphs of percentage error for different values of $k$. In this test, the value of $h$ is fixed while the values of $k$ is varies in order to see the percentage of error for this hybrid method. It is proven that the hybrid method is stable when the value of $k$ decrease.

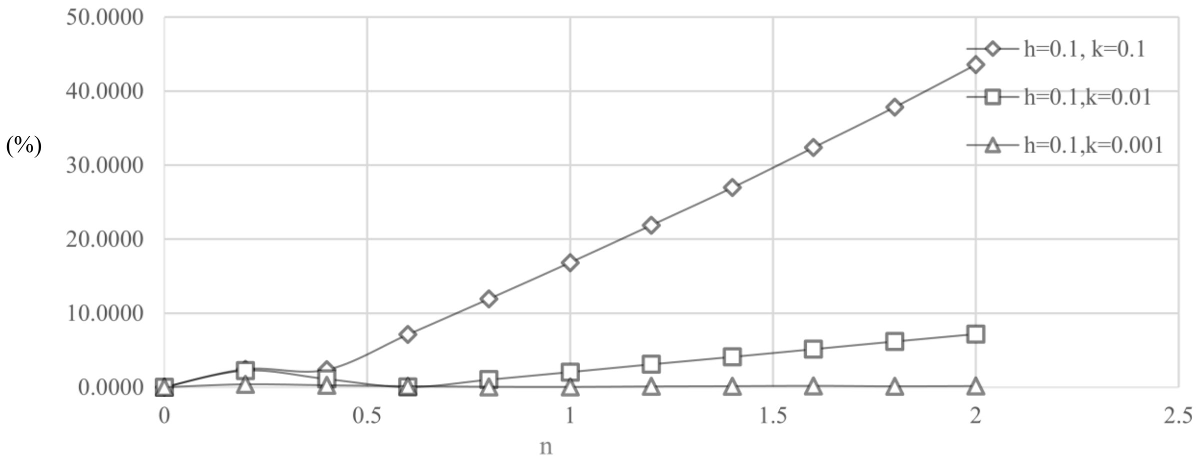



**Figure 3**. Error for different values of $k = dt$ while $h = dx$ keep fixed.

## 4. Results and Discussions

The graphs of velocity distributions are plotted using hybrid method where the finite difference and asymptotic interpolation method are applied as shown in Figure 4 - 7. The parameters involved in the graph development are

i. MHD parameter, $M$
ii. second and third grade fluid parameters, $a$ and $b$ respectively
iii. rotating frame parameter, $R$
iv. time, $\tau$

### *4.1. Effects of Parameter* $M$

The velocity distribution of real part ($u$) and imaginary part ($v$) when $M = 3$ and $a = b = R = \tau = 1$ are kept fixed is illustrated in Figure 4. It shows the results are converges for this infinite problem.

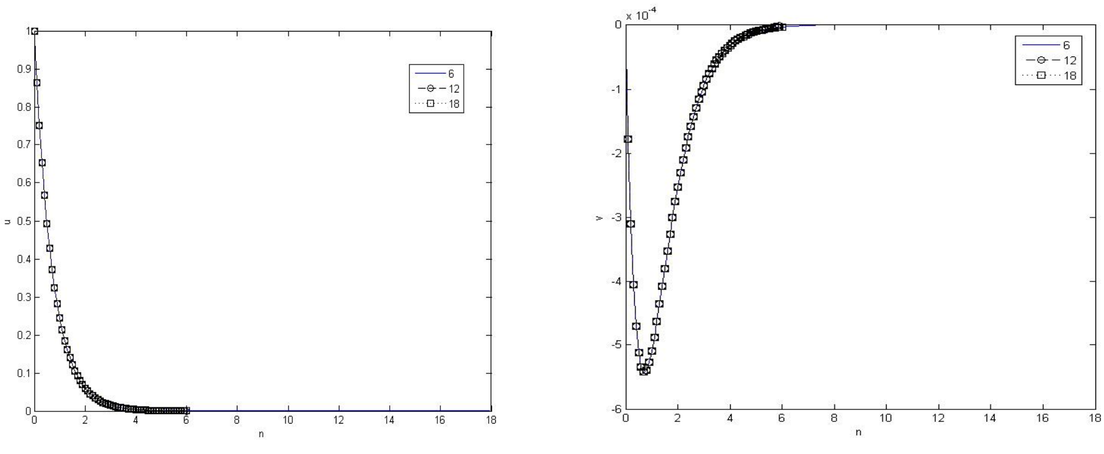



**Figure 4**. Velocity profile for $M = 3$ and $a = b = R = \tau = 1$.

Table 2 presents the influence of different values of parameter $M$ on the velocity distribution. It appears that the velocity profile for real part ($u$) and imaginary part ($v$) are decreased when the values of parameter $M$ increased.

**Table 2**. Effects of parameter $M$ on the velocity distribution for $a = b = R = \tau = 1$.

| $\eta$ | $u$ | | $v$ | |
|---|---|---|---|---|
| | $M = 1$ | $M = 5$ | $M = 1$ | $M = 5$ |
| 0.0 | 1.00000 | 1.00000 | 0.0000000 | 0.0000000 |
| 0.2 | 0.75060 | 0.75020 | -0.0001032 | -0.0005177 |
| 0.4 | 0.56690 | 0.56630 | -0.0001565 | -0.0007838 |
| 0.6 | 0.42820 | 0.42750 | -0.0001776 | -0.0008898 |
| 0.8 | 0.32330 | 0.32260 | -0.0001792 | -0.0008977 |
| 1.0 | 0.24420 | 0.24350 | -0.0001695 | -0.0008491 |

### 4.2. *Effects of Parameter* $R$

Figure 5 shows the result of velocity distribution after considering the unbounded domain. The hybrid method is applied for the different length to show the infinite length and it shows the results are converging. The influence of different values of parameter $R$ on the velocity distribution is shown in Table 3. Decreasing values of velocity profile for real part ($u$) and imaginary part ($v$) occurs when $R$ increased. In physical situation it is related to the Coriolis acceleration where the increment of angular velocity has decrement impact on the velocity.

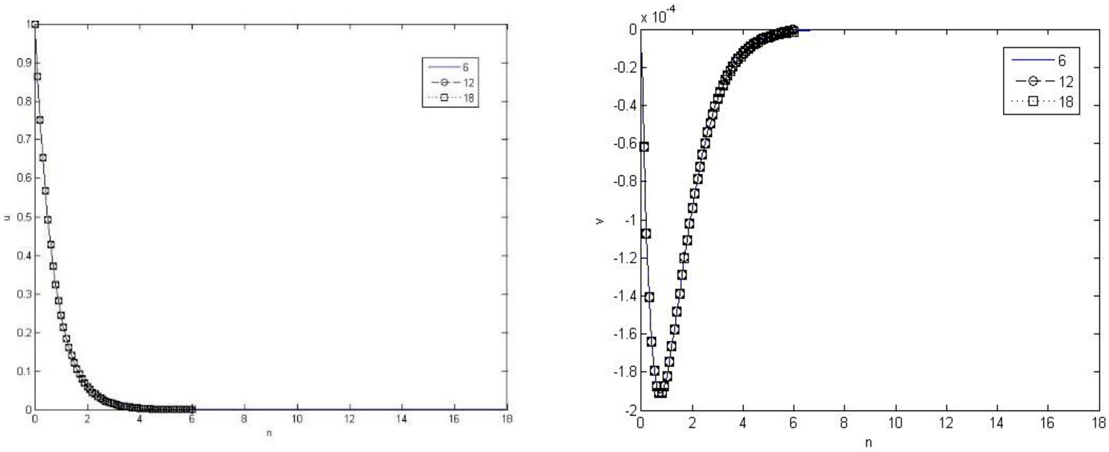



**Figure 5**. Velocity profile for $R = 30$ and $a = b = M = \tau = 1$.

**Table 3**. Effects of parameter $R$ on the velocity distribution for $a = b = M = \tau = 1$.

| $\eta$ | $u$ | | $v$ | |
|---|---|---|---|---|
| | $R = 30$ | $R = 60$ | $R = 30$ | $R = 60$ |
| 0.0 | 1.0000 | 1.0000 | 0.000000 | 0.000000 |
| 0.2 | 0.7505 | 0.7502 | -0.0001071 | -0.0001115 |
| 0.4 | 0.5667 | 0.5661 | -0.0001636 | -0.0001717 |
| 0.6 | 0.4279 | 0.4270 | -0.0001872 | -0.0001982 |

| | | | | |
|---|---|---|---|---|
| 0.8 | 0.3230 | 0.3220 | -0.0001904 | -0.0002032 |
| 1.0 | 0.2438 | 0.2428 | -0.0001815 | -0.0001954 |

### 4.3. *Effects of Parameter* $a, b$

Figure 6 and 7 show the results of velocity profile for real parts ($u$) and imaginary parts ($v$) when the values of parameter $a=1.5$ while $R=b=M=\tau=1$ keep fixed and $b=15$ while $R=a=M=\tau=1$ keep fixed.

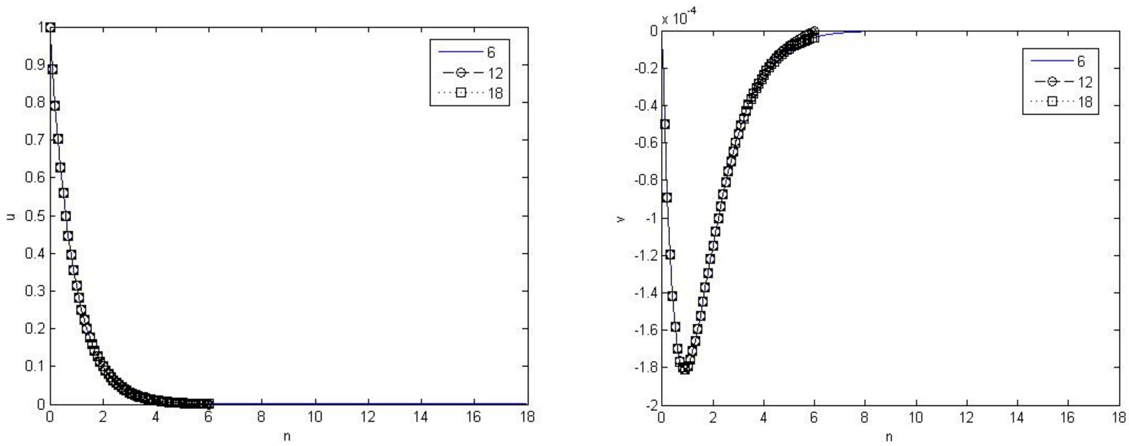



**Figure 6**. Velocity profile for $a=1.5$ and $R=b=M=\tau=1$.

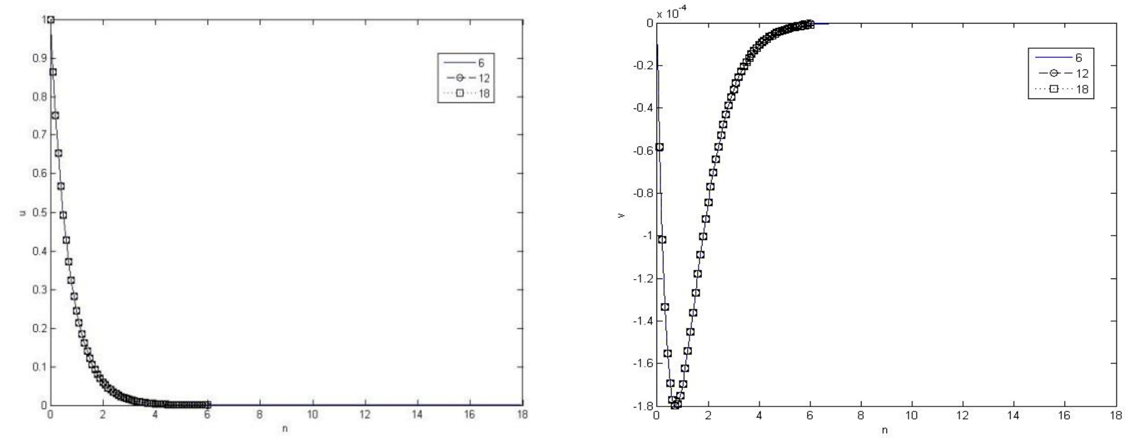



**Figure 7**. Velocity profile for $b=15$ and $R=a=M=\tau=1$.

The results show that the velocity profile for real part increased while the imaginary part decreased when the values of $a$ and $b$ increased as shown in the Table 4 and Table 5.

**Table 4**. Effects of parameter $a$ on the velocity distribution when $b=R=M=\tau=1$.

| $\eta$ | $u$ | | $v$ | |
|---|---|---|---|---|
| | $a=1$ | $a=5$ | $a=1$ | $a=5$ |
| 0.0 | 1.0000 | 1.0000 | 0.000000 | 0.000000 |
| 0.2 | 0.7506 | 0.8761 | -0.0001032 | -0.0000539 |

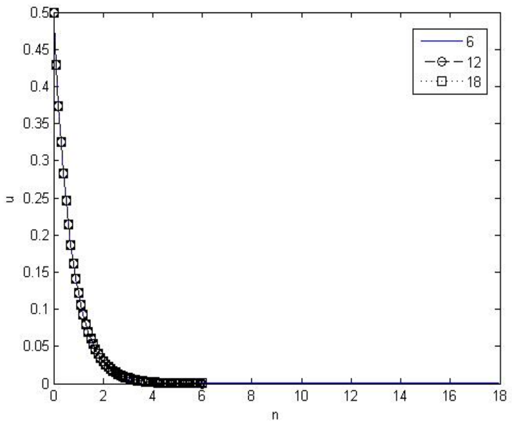
| | | | | |
|---|---|---|---|---|
| 0.4 | 0.5669 | 0.7726 | -0.0001565 | -0.0000954 |
| 0.6 | 0.4282 | 0.6812 | -0.0001776 | -0.0001264 |
| 0.8 | 0.3233 | 0.6007 | -0.0001792 | -0.0001488 |
| 1.0 | 0.2442 | 0.5295 | -0.0001695 | -0.0001642 |

**Table 5**. Effects of parameter $b$ on the velocity distribution for $a = R = M = \tau = 1$.

| $\eta$ | $u$ | | $v$ | |
|---|---|---|---|---|
| | $b=15$ | $b=30$ | $b=15$ | $b=30$ |
| 0.0 | 1.0000 | 1.0000 | 0.000000 | 0.000000 |
| 0.2 | 0.7553 | 0.7599 | -0.0001015 | -0.0000997 |
| 0.4 | 0.5723 | 0.5777 | -0.0001548 | -0.0001532 |
| 0.6 | 0.4330 | 0.4379 | -0.0001765 | -0.0001754 |
| 0.8 | 0.3273 | 0.3314 | -0.0001786 | -0.0001779 |
| 1.0 | 0.2474 | 0.2506 | -0.0001692 | -0.0001688 |

### *4.4. Effects of Parameter $\tau$*

Figure 8 shows the effect of time on the velocity profile when $t = 0.5$ and $a = b = M = R = 1$ for different length.



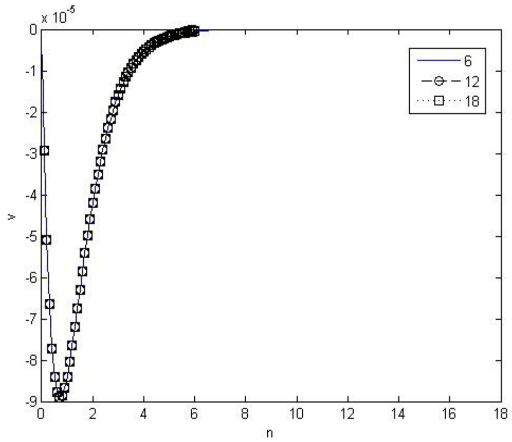




**Figure 8**. Velocity profile for $\tau = 0.5$ and $a = b = M = R = 1$.

Table 6 shows the effect of different time towards the real part and imaginary part of velocity profile. It is shows that the velocity profile increased when the increment of time occurs.

**Table 6**. Effects of parameter $\tau$ on the velocity distribution for $a = b = M = R = 1$.

| $\eta$ | $u$ | | $v$ | |
|---|---|---|---|---|
| | $\tau = 0.75$ | $\tau = 1$ | $\tau = 0.75$ | $\tau = 1$ |
| 0.0 | 0.7500 | 1.0000 | 0.000000 | 0.000000 |
| 0.2 | 0.5621 | 0.7506 | -0.0000769 | -0.0001032 |
| 0.4 | 0.4247 | 0.5669 | -0.0001004 | -0.0001565 |
| 0.6 | 0.3208 | 0.4282 | -0.0001269 | -0.0001776 |

| | | | | |
|---|---|---|---|---|
| 0.8 | 0.2424 | 0.3233 | -0.0001345 | -0.0001792 |
| 1.0 | 0.1831 | 0.2442 | -0.0001309 | -0.0001695 |

## 5. Conclusions

The study of the constant accelerated unsteady state of MHD third grade fluid in a rotating frame has been conducted. A hybrid numerical of finite difference method and asymptotic interpolation method is applied to the problem. Previously, the validation has been made with the other method which is analytical method; HAM to show that this hybrid method is acceptable. In addition, the relative errors also have been calculated and the stability test has been conducted. The parameters involved are MHD, rotation, second-grade, third grade and time. From the results, the velocity profile will be increased and (or) decreased when the values of parameter vary. The study also revealed the following:

i. The increment values of the magnetohydrodynamic and rotating parameters will decrease the velocity profile.
ii. The real part in velocity profile is increased while the imaginary part is decreased when the parameter of second and third grade increased.
iii. The increment of time will increase the velocity profile.

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