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Experimental analysis of a compound pendulum with variable suspension point

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Abstract

A compound pendulum with variable point of suspension—and hence variable moment of inertia—is experimentally analysed. In particular, the period of the small oscillations as a function of position of the suspension point is measured using three different methods: a smartphone used both as an independent tool or as a data-logger and commercial photo-gate. The experimental results are successfully compared with two theoretical calculations—one of them taking all the dimensions into account and the other using a simplified geometry—based on the addition of moments of inertia and the parallel axis theorem.

Compound pendulum

Concepts about inertia and torque are of paramount importance in almost all introductory Physics courses in high school and introductory university level. In particular, one of the typical examples is the compound pendulum (also known as physical pendulum) which consists of a rigid body that can freely rotate around a horizontal axis through a fixed center of suspension. The period of the small oscillations, T, depends on the mass M, the distance from the suspension point to the center of mass R and the moment of inertia I as

$$T = 2\pi\sqrt{\frac{I}{MgR}} \qquad (1)$$

where g is the gravitational acceleration.

In the current study, small oscillations of a compound pendulum are analysed using different modern technologies [1–4]. The period of oscillation is measured as a function of its point of suspension, which affects its moment of inertia. As it involves key concepts in classical mechanics and can be readily implemented virtually in any Physics laboratory, the present experiment could encourage students' interest and motivation to experiment by themselves.

Experimental implementation

The experimental setup, depicted in figure 1, consists of a rigid metallic bar with equispaced holes and a smartphone. As there are several possible suspension points, the moment of inertia depends on the selected point. The dimensions of the bar, with holes made at points separated a uniform distance of 1.0 cm, are $L = 1.199$ m and $w = 0.024$ m, and the mass $M = 0.2518$ kg. The smartphone is a Nexus 5, with mass $m = 0.1311$ kg, length $L_s = 0.135$ m and $w_s = 0.068$ cm. The distance from the suspension point, O, to the center of mass of the bar, C, is indicated with z, while the distance from C to the center of mass of the smartphone is z_s.

Using equation (1) and the geometrical characteristics of the systems the period of the physical pendulum in the regime of small oscillations can be written as

$$T = 2\pi\sqrt{\frac{I}{((M+m)\,z + mz_s)\,g}} \qquad (2)$$

where the moment of inertia, I is obtained as the sum of the contributions from the bar and the smartphone $I = I_{\text{bar}} + I_{\text{s}}$. Applying the well-known parallel axis (also known as Steiner) theorem the moments of inertia can be expressed as

$$I_{\text{bar}} = \frac{1}{12}M\left(L^2 + w^2\right) + Mz^2 \qquad (3)$$

and

$$I_{\text{s}} = \frac{1}{12}m\left(L_s^2 + w_s^2\right) + m\left(z + z_s\right)^2. \qquad (4)$$

To get a deeper insight into the orders of magnitude, a simplified theoretical model neglecting the width of the bar and assuming the smartphone as a point-like particle is also considered.

Methods and analysis

The period of physical pendulum was measured by three different experimental methods:

- **Phyphox:** direct measurement on the smartphone screen using the *Pendulum* tool of the *Phyphox app*. From a temporal series of the angular velocity (about 30 s) provided by the gyroscope sensor, the app automatically calculates the period by means of an autocorrelation function (similar to a FFT but taking the correlation between maximums of several cycles). The advantage is that it is a direct measure for which only the smartphone with the free app is needed.
- **Vernier:** direct measurements using a LabQuest interface and an optical barrier. The advantage is that direct values are obtained even if there is cell phone mounted on the bar. The disadvantage is that it is necessary to employ a commercial interface or some equivalent home-made system.
- **Androsensor:** smartphone as a data-logger and data processing on a PC. In this procedure, a temporal series of the gyroscope sensor (z axis) was taken for 10 s, and saved in the smartphone using the *Androsensor app*. All generated *.csv* files (one for each suspension point) were exported to the PC and analyzed using a simple software package (in this case, *Scilab*) and the period obtained by means of a sinusoidal fit. This method can be useful for experimental courses in which students must develop data management techniques and implement signal processing algorithms in different languages such as Python or Matlab.

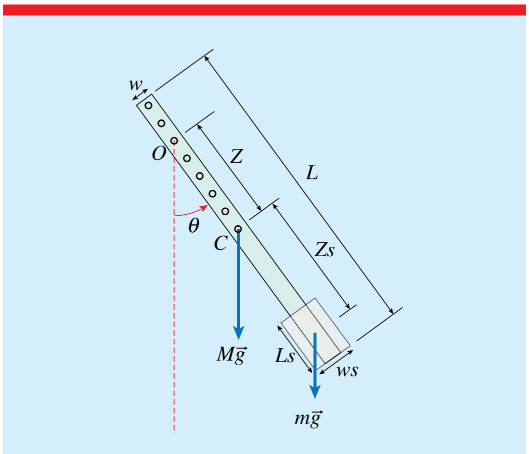

Figure 1. Experimental setup consisting of a bar with equispaced suspension points and a smartphone. The labels are described in the text.

Experimental results using the three methods (symbols) and the theoretical predictions (continuous lines), plotted in figure 2, display a great agreement. Several aspects are worth discussing. The difference between the full model (red line) and the simplified model (blue line)—neglecting the width of the bar and considering the smartphone as a point-like particle—is less than 2.0 ms (less than 0.12%).

The root-mean-square deviations with respect to the full theoretical model, also indicated in the figure caption, reveal that the three methods, each one with *pros* and *cons* are valid and possible implementations of the present experiment. Remarkable, the period displays a minimum for a particular distance to the center of the bar. These points could lead to interesting classroom discussions. To sum up, we presented a simple experiment that involves relevant concepts of classical mechanics using moderns technologies that can be readily implemented in a Physics laboratory.

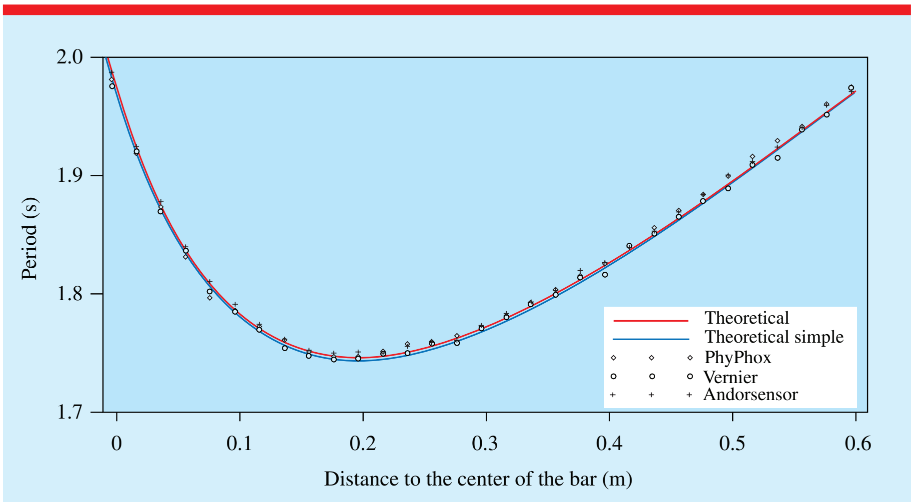

Figure 2. Experimental results (symbols) and models (continuous lines) for the period of the small oscillations as a function of the distance of the suspension point to the center of the bar. The root-mean-square deviation with respect to the full theoretical model are: 5.0 ms (Phyphox), 4.0 ms (Vernier) and 3.7 ms (Androsensor).

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