

Cosmological constant problem: deflation during inflation

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Abstract. We argue that the discrepancy between the Planck mass scale and the observed value of the cosmological constant can be largely attenuated, if those quantities are understood as a result of effective, and thus scale-dependent, couplings. We exemplify this mechanism for the early inflationary epoch of the universe by solving the corresponding effective gap equations, subject to an energy condition. Several non-trivial checks and extensions are discussed. A comparison of our results to the renormalization group flow, obtained within the asymptotic safety program reveals a stunning agreement.

Keywords: gravity, inflation, modified gravity

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1 Introduction

The cosmological constant problem (CCP) seems to point towards deep misconceptions in our current understanding of the interplay between quantum field theory and general relativity [1–3]. In this article, we want to point out that one possible misconception is that, one makes reference to gravitational couplings at different epochs of the universe, in particular for the most dramatic version of the CCP. This reference to different epochs of the Universe is however inconsistent if there is an epoch where the gravitational couplings are not constant, as it is expected for example from quantum effects which can induce an effective running of the fundamental couplings (e.g. Newtons coupling $G_0 \rightarrow G_k$). Such effects were first considered in [4]. After this seminal paper of Dirac, several models emerged implementing this idea in different ways [5–7]. More recently, the justification of such running of the coupling constants can also be attributed to quantum features which are present in any (non-conformal) quantum field theory. It is clear that for quantum effects to be notable one needs a violent epoch of the universe. Due to our current understanding the early inflationary period [8, 9] is the best candidate for such non-classical corrections to be relevant. In particular, during inflation the quantum effects and thus the scale-dependence of effective couplings can be expected to be much stronger than during other later evolution periods. There are two arguments that support this idea. First, since the very early universe is typically related to Planck-time and Planck-distances it is natural to expect important quantum gravity corrections [10–17], such as scale-dependence of the effective couplings. Second, in these very early times, other scales such as particle masses, which might moderate the scale-dependence are still irrelevant. During later times, to the contrary, there are no Planck distances involved and there are also logarithmically running particle masses contributing which could diminish the scale-dependence of the gravitational couplings further. Thus, it makes sense to focus this type of study on the first inflationary period of the Universe. Under such extreme conditions the gravitational couplings including the cosmological “constant” can not be expected to be

constant [18] and thus one has to take care when realizing a comparison of those couplings at different times.

In the following sections we will elaborate a scale-dependent vacuum dominated model which incorporates the following features, which are in agreement with the ideas mentioned above: i) scale-dependence within effective gap equations ii) inflationary behavior of the cosmological scale factor $a(t)$ iii) non-trivial evolution of the vacuum energy density and the gravitational coupling and iv) exponential suppression of the CCP during the inflationary period. This is achieved by applying methods that have been successfully used in order to include quantum correction into gravitational theories, black hole backgrounds, wormholes, and particular cosmological problems [19–57].

1.1 Classical Λ dominated universe

Based on the cosmological principle and assuming a flat Universe, one uses the line element

$$ds^2 = -dt^2 + a(t)^2 [dr^2 + r^2 d\Omega_2^2], \quad (1.1)$$

as ansatz for the metric field, where $d\Omega_2^2 = d\theta^2 + \sin^2(\theta)d\phi^2$. The scale factor gives

$$a(t) = a_i e^{t/\tau}, \quad (1.2)$$

where

$$\tau = \pm \sqrt{\frac{3}{\Lambda_0}}. \quad (1.3)$$

We will only use the positive sign in (1.3), which corresponds to an expanding universe.

1.2 The cosmological constant problem in the very early universe

There are different versions of the CCP to be found in the literature. Most of them can be cast in the dimensionless ratio between the highest UV cut-off M_{UV} of the quantum field theory one is using to calculate the vacuum energy density and the energy density $\rho = \Lambda_0/G$ observed due to the accelerated expansion of the universe [3]

$$r \approx \frac{M_{UV}^4}{\rho} \approx \frac{M_{UV}^4 G}{\Lambda_0}. \quad (1.4)$$

For a good theoretical prediction, the value of (1.4) is supposed to be of order one. Clearly, there is a dramatic discrepancy.

As explained in the introduction, in this paper we will focus on the early expansion of the universe. During this epoch, governed by pure gravity, the only dimensionful quantity is actually the Planck mass given by $m_P^{-1} = \sqrt{8\pi G}$. If one uses this scale as UV scale $M_{UV}^2 \approx 1/G$ to estimate the vacuum energy density, the ratio (1.4) simplifies to

$$r_{infl} \approx \frac{1}{G \cdot \Lambda_0} \approx 10^{122}. \quad (1.5)$$

The fact that (1.5) differs from one by more than one hundred orders of magnitude is known as the cosmological constant problem in its worst form. There have been numerous attempts to either solve, alleviate, or to deny this problem [16, 58–68], but up to now the quarrel continues and no agreement has been achieved.

Since our purely gravitational approach ceases to be valid as soon as further mass scales such as the top mass appear, we only can attempt to explain the discrepancy of 66 orders of magnitude spanned by the value of the ratio (1.5) between the top mass and the Planck mass.

2 Scale-dependence in gravity

A very powerful tool in quantum field theory is the use of effective actions, which incorporate quantum effects in a scale-dependent modification of the classical action. This type of scenario has been proposed and studied in the context of gravity in different ways and with different methods [69–78]. In the so-called Einstein-Hilbert truncation the effective action reads

$$\Gamma[g_{\mu\nu}, k] = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G_k} (R - 2\Lambda_k) + \mathcal{L}_M \right], \quad (2.1)$$

where Λ_k and G_k are the cosmological and Newton scale-dependent couplings, respectively. A great advantage of working with effective quantum actions like (2.1) is that they already incorporate the effects of quantum fluctuations like those being responsible for the CCP. Thus, if the problem is properly addressed in terms of a background solution of this action, no additional quantum corrections need to be added. In order to obtain background solutions for this effective action one has to derive the corresponding gap equations. Thus, varying the effective action respect to the inverse metric field, one obtains the corresponding Einstein field equations [75]

$$G_{\mu\nu} = T_{\mu\nu}^{\text{eff}} = -\Lambda(t)g_{\mu\nu} - \Delta t_{\mu\nu}, \quad (2.2)$$

where

$$\Delta t_{\mu\nu} = G(t) \left[g_{\mu\nu} \nabla^\alpha \nabla_\alpha - \nabla_\mu \nabla_\nu \right] G(t)^{-1}. \quad (2.3)$$

In order to get physical information out of those equations one has to set the renormalization scale in terms of the physical variables of the system under consideration $k \rightarrow k(x, \dots)$.

2.1 Scale setting

For most types of scale settings [73, 76, 79–102] the identification $k \rightarrow k(x, \dots)$ breaks the reparametrization symmetry (2.2). There are, however, approaches which allow to maintain the symmetries and thus the consistency of the system. We will invoke a procedure which invokes the variational principle for k

$$\frac{\delta \Gamma_k}{\delta k} = 0. \quad (2.4)$$

This approach has been proposed in [103–105] and it is a straightforward implementation of background scale independence [106–113]. Relation (2.4) is not a fixed point condition but rather a scale setting condition, it even can be used when the renormalization group flow has already approached its fixed point [114].

Further, for highly symmetric systems with a small number of couplings one can attempt to solve (2.2) for $g_{\mu\nu}(x)$, $G(x)$, and $\Lambda(x)$ directly by the use of a special ansatz [41, 42, 44, 48, 50–52, 54, 115] motivated by the structure of the classical solution [116], or by imposing an energy condition [115].

In this paper we will apply both strategies, the variational scale setting (2.4) and the energy condition mentioned above.

2.2 Modified Friedmann equations

Within the ansatz (1.1) the dynamical variable $a = a(t)$ does only depend on time and not on spatial directions, the same will be assumed for the two additional variables $G = G(t)$

and $\Lambda = \Lambda(t)$. With this, the gap equations (2.2) take the form of generalized Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{\Lambda}{3} = \frac{8\pi G}{3}\rho(t), \quad (2.5)$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 - \Lambda = -(8\pi G)p(t), \quad (2.6)$$

where an effective density (ρ) and pressure (p), given by the scale-dependent framework have been identified with the density and pressure parameters of a perfect fluid

$$\frac{8\pi G}{3}\rho(t) \equiv \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{G}}{G}\right), \quad (2.7)$$

$$-(8\pi G)p(t) \equiv -2\left(\frac{\dot{G}}{G}\right)^2 + \left(\frac{\ddot{G}}{G}\right) + 2\left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{G}}{G}\right). \quad (2.8)$$

Further, one has the scale setting relation (2.4)

$$\frac{d}{dt} \left[\frac{1}{16\pi G(t)} (R - 2\Lambda(t)) \right] = 0. \quad (2.9)$$

There are three equations, namely the two cosmological gap equations (2.5), (2.6) and the scale setting condition (2.9). There are also three unknown functions ($a(t)$, $G(t)$, $\Lambda(t)$). However, the system respects general covariance, which implies that $\nabla_\mu G^{\mu\nu} = 0$ and thus one of the three equations (2.5), (2.6), (2.9) is just a consequence of the other two. This means that one needs one additional relation or condition in order to determine a solution. In the next subsection, we will invoke an energy condition in order to provide the missing equation.

2.3 Null energy condition (NEC)

In an empty universe, a light-like signal traveling on a null geodesic with velocity vector $\ell^\mu = dx^\mu/dt$ should not perceive any energy density of the background. This should be true for other vacuum contributions to the effective stress energy tensor, such as $\Delta t_{\mu\nu}$, introduced by the scale-dependence of the gravitational coupling

$$T_{\mu\nu}^{\text{effec}} \ell^\mu \ell^\nu = -\Delta t_{\mu\nu} \ell^\mu \ell^\nu = 0. \quad (2.10)$$

This is reminiscent of the null energy condition [117]. The above relation is the scale setting condition which we will impose in addition to the cosmological gap equations (2.5), (2.6). The vector field ℓ^μ satisfying the geodesic equation can be written as $\ell^\mu = C_0 a^{-1}(1, (1 - \kappa r^2)^{1/2} a^{-1}, 0, 0)$.

Replacing this in (2.10) gives the NEC for the cosmological model

$$-2\left(\frac{\dot{G}}{G}\right)^2 + \left(\frac{\ddot{G}}{G}\right) - \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{G}}{G}\right) = 0. \quad (2.11)$$

Please note that the above null vector ℓ^μ was chosen for convenience, but any other direction of the null vector does give the same physical result in terms of the relation (2.11).

3 Results and discussion

The solution of (2.5), (2.6), and (2.11) that describes an expanding universe is

$$a(t) = a_i e^{\frac{t}{\tau}}, \quad (3.1)$$

$$G(t) = G_0 \left[1 + \tilde{\xi} \int_{t_0}^t a(t') dt' \right]^{-1} = \frac{G_0}{1 + \xi a(t)}, \quad (3.2)$$

$$\Lambda(t) = \Lambda_0 \left[\frac{1 + 2\xi a(t)}{1 + \xi a(t)} \right]. \quad (3.3)$$

It is direct to see that (3.1) takes the same form as the classical counterpart (1.2). A generalization to non zero values of κ is straight forward (see version one of [118]). The solution involves four integration constants which we have labeled $(a_i, G_0, \Lambda_0, \xi)$. They are set by the conditions

$$\lim_{t \rightarrow 0} a(t) = a_i, \quad \lim_{\xi \rightarrow 0} G(t) = G_0, \quad \lim_{\xi \rightarrow 0} \Lambda(t) = \Lambda_0. \quad (3.4)$$

One notes that the parameter ξ plays the role of a control parameter, When setting ξ to zero, the time-dependent couplings $G(t)$ and $\Lambda(t)$ are reduced to their constant and thus “classical” counterparts (G_0 and Λ_0). Please note that the expansion rate is given by $\tau = \pm \sqrt{3/\Lambda_0}$, which looks exactly like the classical rate (1.3). There is however an important conceptual difference since now, Λ_0 is one of the integration constants, while in (1.3) it was given as input parameter.

3.1 Equation of state of scale-dependent $G(t)$

Is there a meaningful equation of state $p = \omega \rho$ for the effective type of pressure and energy density? Subtracting (2.6) from $3 \times (2.5)$, one finds for the solution (3.1), (3.2)

$$p(t) + \rho(t) = 0. \quad (3.5)$$

Thus, the effective parameters take the form of a “classical” Dark Energy equation of state with $\omega = -1$. Thus, the effective pressure and the effective energy density cancel each other, which explains why the scale-dependence parameter ξ does not appear in the solution for $a(t)$.

3.2 Deflation of the cosmological constant problem

As explained in the introduction, the CCP in a Planck era universe can be parametrized by the value of $\Lambda \cdot G$ (1.5). However, the quantum gravity calculation makes reference to the very early Planck era t_i , while the measurement makes reference to the more recent history of the universe, long after the end of inflation t_f . scale-dependence can offer alleviation of this problem since it allows for a continuous evolution of $\Lambda(t) \cdot G(t)$ from the Planck era t_i to the end of inflation t_f as shown in the left panel of figure 2. As argued in the introduction, we assume that the most dominant scaling effects occur in this period and that in later times, after t_f , the scaling effects will be dominated by coupling to external particle scales m_i , which only run logarithmically.

In order to show that the CCP can be addressed effectively within our framework, we impose six conditions on the scale-dependent solution (3.1)–(3.3) during inflation. One has to check whether those conditions can be met by a choice of the constants $t_i, t_f, G_0, \Lambda_0, a_i, \xi$.

a) At the initial time $t_i \equiv 0$ one demands

$$a(t_i) = 1. \quad (3.6)$$

and

$$\Lambda(t_i) \cdot G(t_i) \approx 1, \quad (3.7)$$

which meets the theoretical expectation.

b) At the end of inflation t_f one imposes that the gravitational coupling has evolved to a value close to the current measured value

$$G(t_f) \approx G_N, \quad (3.8)$$

and that the cosmological constant problem has gravitationally evolved to

$$\Lambda(t_f) \cdot G(t_f) \approx 5 \cdot 10^{-66}, \quad (3.9)$$

around the magnitude dictated by the heaviest standard model particle.

c) The observed flatness of the universe suggest further that during this epoch (from t_i to t_f) of rapid growth, the universe expanded at least sixty e-folds [119]

$$N_e \geq 60. \quad (3.10)$$

By imposing the conditions (3.6), (3.7), (3.8) one finds

$$a_i = 1, \quad (3.11)$$

$$t_f = N_e \tau, \quad (3.12)$$

$$\Lambda_0 = \frac{(1 + \xi)^2}{G_N(1 + 2\xi)(1 + \xi e^{N_e})}. \quad (3.13)$$

In order to analyze the condition (3.9), we put it into an order of magnitude inequality, accepting five orders of magnitude deviation from the value given in (3.9)

$$10^{-71} \lesssim \Lambda(t_f) \cdot G(t_f) \lesssim 10^{-61}. \quad (3.14)$$

This double inequality puts conditions on the two remaining constants N_e and ξ . The region of allowed values for those two constants, according to (3.14), is shown in the blue contour in figure 1, where we have defined $\xi = e^{\xi_V}$. From this figure one notes that the condition (3.14) never gets in conflict with the observational bound (3.10). It is thus shown that substantial alleviation of the cosmological constant problem within the presented framework can be achieved. For this one does need the inflationary period to last at least $\ln(10^{66}) \approx 150$ e-folds. This is significantly larger than the experimental lower bound, but it does not cause tension with observational data since it is very hard to establish a distinctive upper bound for this quantity [120].

The scale-dependent evolution could be incorporated into more realistic models of inflation [121], but this goes beyond the scope of this article. It is further tempting to speculate whether the suppression of $G(t)$ in the first equality of (3.2) can be used in combination with the scale-dependence of other couplings in the post-inflationary universe [122–131] in order to contribute to the solution of the hierarchy problem [132–134]. This possibility will be explored in future studies.

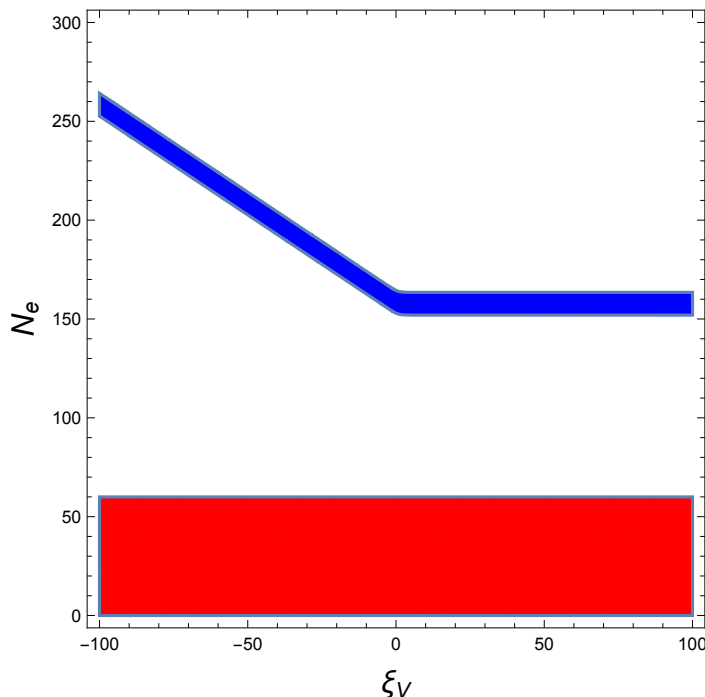


Figure 1. N_e, ξ_V parameter space, where the blue contour is fulfilling both conditions of (3.14) and the red colour is excluded from the observational condition (3.10).

3.3 Comparison with the functional renormalization group

Even though our calculation never makes use of specific quantum gravity beta functions, it is tempting to make a comparison with the findings in such an approach. The discussion below will be complementary to the findings in [16, 17], where strong effects of the evolution of couplings on the cosmological constant problem have been pointed out in context with a possible infra-red instability of asymptotically safe gravity [135, 136]

Using functional renormalization group (RG) methods, in particular, the Wetterich equation [71, 72, 74], it is possible to obtain a prediction for the scale-dependent couplings in gravitation actions such as (2.1). The dimensionless couplings $\hat{g}(k) = \hat{G}(k)k^2$ and $\hat{\lambda}(k) = \hat{\Lambda}(k)/k^2$ are found for the dimensionless renormalization scale $\hat{t} = \log(k/k_0)$

$$\hat{g}(\hat{t}) = \frac{g_0 e^{2\hat{t}}}{1 + g_0 (e^{2\hat{t}} - 1) / g^*}, \quad (3.15)$$

$$\hat{\lambda}(\hat{t}) = \frac{g^* \lambda_0 + e^{-2\hat{t}} (e^{4\hat{t}} - 1) g_0 \lambda^*}{1 + g_0 (e^{2\hat{t}} - 1) / g^*}, \quad (3.16)$$

where g^* and λ^* are the values of the ultra violet fixed point predicted by the Asymptotic Safety (AS) conjecture [69, 78] and found within the functional RG approach [137–139]. Further, λ_0 and g_0 are initial values defined at an intermediate renormalization scale $\hat{t} = 0$. For the CCP in the very early universe the relevant quantity is the product of the dimensionful couplings $\hat{G}(\hat{t}) \cdot \hat{\Lambda}(\hat{t})$. In four dimensions the product of the dimensionful couplings is equal

to the product of the dimensionless couplings (3.15), (3.16)

$$\hat{G}(\hat{t})\hat{\Lambda}(\hat{t}) = g(\hat{t})\lambda(\hat{t}). \quad (3.17)$$

If one would attempt a comparison of dimension full quantities, such as for example, \hat{G}_k and $G(t)$ instead of the dimensionless product (3.17) one would face a big problem. The problem is that such dimensionful quantities are directly proportional to the arbitrary renormalization scale k^2 and thus any such result will be doomed by the uncertainty of how this scale is chosen. If one insisted on such a comparison of dimensionful quantities, one would find a counter-intuitive $k^2 \sim a(t)$. This is counter-intuitive, but no contradiction with Asymptotic Safety, since such IR behavior has also been found and discussed in the context of the infra-red instability [16, 135] of Asymptotically Safe theories. However, in the following, we will limit the comparison to the naturally dimensionless product (3.17) which allows for more robust statements than a comparison of the individual dimensionful or dimensionless couplings.

The one million dollar question is whether there is a scale setting $\hat{t} = \hat{t}(t)$ for which the product (3.17) obtained from AS agrees, at least qualitatively, with the product $G(t) \cdot \Lambda(t)$ derived in our scale-dependent cosmological solution (3.2), (3.3). In the ultraviolet (UV) and close to the separatrix¹ of the RG flow one can approximate (3.15 and 3.16) for $e^{2\hat{t}}, g^*/g_0 \gg 1$, which gives

$$\hat{g}(\hat{t})\hat{\lambda}(\hat{t}) = g^*\lambda^* \left(\frac{g^*\lambda_0}{g_0\lambda^*} + e^{2\hat{t}} \right) \left(e^{2\hat{t}} + \frac{g^*}{g_0} \right)^{-2}. \quad (3.18)$$

By inspection one realizes that the simple replacements

$$g^*\lambda^* \rightarrow G_0\Lambda_0, \quad g_0 \rightarrow \frac{G_0}{a_i\xi}, \quad \hat{t} \rightarrow -\frac{t}{2\tau}, \quad (3.19)$$

together with the choice $\lambda_0 = 2\lambda^*$, leads to a stunning exact matching

$$\hat{g}(\hat{t})\hat{\lambda}(\hat{t}) \Big|_{(3.19)} = G(t)\Lambda(t). \quad (3.20)$$

This means that for a proper scale setting and a proper choice of the flow trajectory, both approaches show the same type of deflation of the cosmological constant problem. Graphically this behavior is shown in figure 2.

Comparing our solutions to the AS predictions, on one hand, we observe that the behavior of the product $G \cdot \Lambda$ as a function of t is exemplified in the figure 2 (*left*). One notes that $G(t) \cdot \Lambda(t)$ experiences a strong suppression. In addition, for smaller values of ξ the evolution of the couplings starts at later times. Apart from this change, the functions are form-invariant under $t \rightarrow t - t_i$ transformations. This means that the value of ξ can be reabsorbed in a shift in the initial or final conditions for $a(t)$, $G(t)$, and $\Lambda(t)$. On the other hand, the same features are shown in 2 (*right*).

Clearly, this agreement can still be a coincidence but it appears sufficiently interesting to us to mention it here in the discussion. There are several comments about the result (3.20) and the identifications (3.19) to be made. The result (3.20) is not the product of a very particular cut-off identification. It is straightforward to check that the same agreement can be obtained from different beta functions arising from different cut-off prescriptions. From (3.19) one sees from the UV/AS perspective that the control parameter ξ is not necessarily small,

¹With “separatrix” it is referred to the RG flow line which connects the trivial fixed point $g = \lambda = 0$ with the non trivial UV fixed point (g^*, λ^*) .

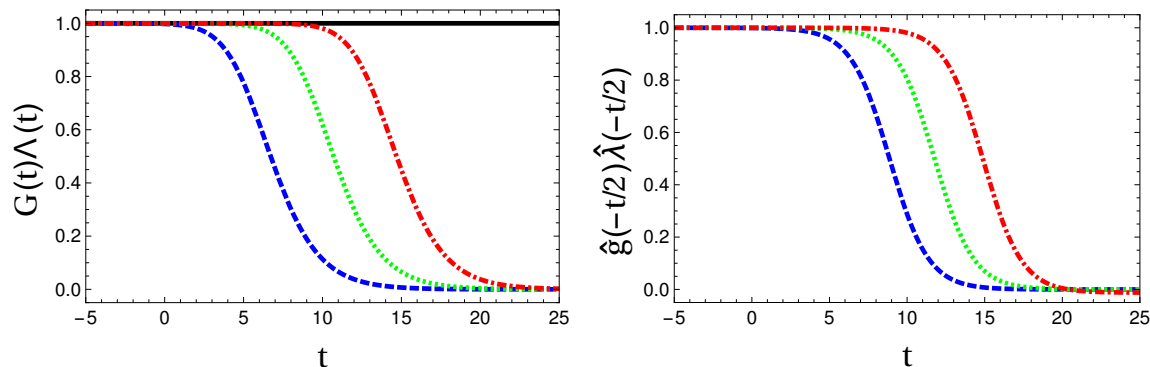


Figure 2. *Left:* evolution of $G(t) \cdot \Lambda(t)$ for $\xi = 0$ (solid black line), $\xi = 0.001$ (dotted-dashed red line), $\xi = 0.01$ (dotted green line) and $\xi = 0.1$ (dashed blue line). The rest of the parameters have been taken to be unity. *Right:* evolution of $\hat{\lambda}(-t/2) \cdot \hat{g}(-t/2)$ as given by AS for $g_0 = 0.001$ (dashed blue line), for $g_0 = 0.01$ (dotted green line), and for $g_0 = 0.1$ (dot-dashed red line). Other constants were set to unity.

but a rather large number. This means that scale-dependence in this regime of high energies is a dominant effect and not just a small correction to classical behavior. One further sees from (3.19) that the product $\Lambda \cdot G$ in the UV is predicted to be just the product of the two fixed points $g^* \cdot \lambda^*$ of the RG flow. The values of those fixed points are typically found to be of order ~ 0.2 , the product is thus not dramatically far from unity, just as we had supposed in our working assumptions. Finally, one notes from (3.19) that the scale identification is in agreement with our intuitive expectations, since it associates very early times $t \rightarrow -\infty$ to very large RG scales $k \rightarrow \infty$ and late times $t \rightarrow \infty$ to small RG scale $k \rightarrow 0$.

Although the product of gravitational couplings is in agreement with those found in the FRG approach, it is essential to point out that the solution shown in the present manuscript disagrees with some predictions based on the AS program. Our solutions suggest the dominant corrections appear in the opposite limit respect to those encountered using the FRG. As far as we can tell, the difference between formalisms comes from the choice of the renormalization scale k . Also, notice that in the scale-dependent scenario, one does not assume any prescription about k given that we avoid taking a specific functional form of the renormalization scale promoting the functions (which initially depends on k) to depend on the time coordinate t . Recent publications give evidence that a “gravitational instability” could introduce modifications of AS theories in the deep infrared [140, 141]. A comparison shows that this unstable behavior in the infrared limit is quite similar to our results. Therefore, our solutions can be considered the scale-dependent counterpart of the “gravitational infra-red instability” of Asymptotic Safety.

We can not resist to insist that the identity (3.20) is absolutely not trivial. It links two completely different and independent calculations. One is implementing quantum effects into a gravitational theory by means of effective quantum field theory and the other solving the modified Friedmann equations (2.5), (2.6) subject to a simple energy condition (2.11). The support of both approaches is mutual. One can read the agreement of (3.2), (3.3) with (3.15), (3.16) as support of the approach and the solution presented in this paper. One can also conclude that AS is in good shape to contribute to the solution of the CCP, if one chooses a flow line which is in sufficiently close vicinity of the separatrix line ($g_0 \ll 1$).

3.4 Comparison with G-varying theory

It is interesting to point out some similarities (and differences) that our formalism shares with well-known approaches such as G-varying gravity [7] and variable gravity [16]. We will limit the comparison to those two approaches since they are the most similar ones to our scale-dependent theory.

The distinction can be made by the kinetic term of the scalar degree of freedom. In variable gravity, the pre-factor of this term is, by its own, a scale-dependent quantity. In Brans Dicke theory the pre-factor of this term (known as the dimensionless Brans-Dicke constant) is constant and finite by construction. In our scale-dependent approach, the kinetic term is absent (which corresponds to a vanishing Brans-Dicke constant) since the scalar is associated to the unphysical renormalization scale of effective quantum field theories.

This distinction seems to be crucial, since to our knowledge, no exponential suppression of the CCP was reported for [7] or [16] during the epoch of inflation. The reason for this might be that a dynamical scalar field does by its own means contribute to the energy content of the universe and thus to the associated vacuum energy density, while a non-dynamical scale field such a direct effect is absent.

4 Conclusions

In this article, we presented a study on the early inflationary universe in the context of scale-dependence. We give reasons, why it can be expected that the leading gravitational couplings experience scale-, in particular time-dependence in this early epoch. It is then argued that, if one accepts possible scale-dependence of the couplings, then the CCP is ill-defined since it makes an identification of the product $\Lambda(t) \cdot G(t)$ at two different times of the cosmological evolution. We conjecture that this misconception and thus the CCP can be resolved if one takes into account a proper scale-dependence of those couplings.

As a concrete example, we solve the gap equations (2.2) of the scale-dependent gravitational action in the Einstein-Hilbert truncation for the homogenous isotropic line element (1.1) subject to a null energy condition (2.10). It is found that within this solution (3.1)–(3.3) the scale factor $a(t)$ grows exponentially. Further, the couplings $\Lambda(t)$ and $G(t)$ have the expected strong time-dependence. It is shown that the time evolution of $\Lambda(t) \cdot G(t)$ indeed allows accommodating a substantial alleviation of the CCP in agreement with the conditions (3.6)–(3.9) and the observational lower bound (3.10).

We note two particular features of our results. First, the contribution of the scale-dependent coupling $G(t)$, can be written as an ideal fluid obeying an equation of state, which is usually associated with vacuum energy density. This is a strong support for the chosen energy condition (2.10), because it shows that the condition guaranties a vacuum type equation of state for the scale dependent contribution $\Delta t_{\mu\nu}$ of the stress energy tensor. Second, a comparison between the product $\Lambda(t) \cdot G(t)$, deduced in this paper, with the product $\hat{\lambda}(\hat{t}) \cdot \hat{g}(\hat{t})$, obtained from functional RG methods within the AS scenario, reveals highly non-trivial similarities.

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