

Urban Subway Vehicle Dynamic Modelling and Simulation

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Abstract. With the continuous development of urban rail transit, urban rail transit project needs to be analysed under different factors. Accurately and quickly calculate the real-time operating results could support the research, planning, construction and operation of urban rail transit. This paper used Matlab/Simulink programming language to develop the transit dynamics model and UI design.

1. Introduction

International study on railway operation simulation system started in the 1970s and a quite few companies have launched relatively mature railway operation software till now. [1] For instance, RALSIM software of the United States has set TPC (Train Performance Calculator) as its core function; RAILSYS software of Germany was developed on the basis of micro simulation for rail road net; Japan has developed UTRAS system with the purpose of generality. China has also developed GTMSS system that accords with “train operation calculation regulations” recently [2][3]. These applications lay emphasis on different aspects, but they are all composed of the following three components: the vehicle and route data, the operation process calculation, and the result analysis. Therefore, this paper focuses on these three factors.

2. Urban Railway Train Modelling

2.1. Urban Railway Train Dynamics Modeling

The train moves due to the forces acting on it through the wheel-rail interaction and the forces from the external environment, such as the wind resistance and gravity. In the paper the train is taken as the single particle with all the concentrating mass for during the modelling and all forces acting on the particle change as the train moves regardless of internal force of the train [4]. The external forces act on the particle mainly consist of the train, tractive effort, braking force, basic resistance, and additional resistance.

Traction pushes forward the train via the reactive force of the rail when adhesive force is generated by connecting the driving wheel with the rail through mechanical energy conversion device. In practical traction calculation, it can be worked out through traction characteristic curve. When a train speed is given, the particularly tractive effort can be calculated under linear interpolation method from the curve graph.



$$w_x = w_1 + \frac{(v_x - v_1)(w_2 - w_1)}{v_2 - v_1} \quad (1)$$

where w_x is the traction force of single motor at the required point, kN. Calculation method for traction of the entire motor train unit is:

$$W_x = n_m \times w_x = n_m \times \left[w_1 + \frac{(v_x - v_1)(w_2 - w_1)}{v_2 - v_1} \right] \quad (2)$$

Where n_m is the number of motors in motor train unit and W_x is the traction force of motor train for the required point.

Migration force of unit weight is:

$$w'_x = \frac{W_x \times 1000}{(n_m m_m + n_t m_t)g} \quad (3)$$

Where w'_x is the traction per unit weight at the required point, m_m is the weight of motor train, m_t is the weight of trailer, n_t is the number of trailers in the motor train; g is the acceleration of gravity, m/s^2

2.2. Resistance Model

Resistance of motor train unit during operation includes basic resistance and additional resistance: basic resistance is caused by running resistance between the bullet train and its components, air resistance during operation, friction between the wheel and the rail; additional resistance is caused by route, bridge and tunnel, including additional resistance for gradient factor, additional resistance for curve factor, and additional resistance for tunnel factor. Basic resistance is based on the resistance formula raised by Davis:

$$F_{r0} = A + B \cdot v + C \cdot D \cdot v^2 \quad (4)$$

A, B, C are parameters of this function. v is the velocity of motor train.

Additional resistance is caused due to comprehensive influence of ramp, camber and tunnel. It can be divided into additional resistance for gradient, additional resistance for curve, and additional resistance for tunnel.

In the practical route, included angle θ of the ramp is small, often less than 60%. Therefore, $\tan \theta$ can be treated as $\sin \theta$, thus the calculation method for gradient resistance is:

$$w_i = \frac{W_i}{(n_m m_m + n_t m_t)g} \times 1000 = 1000 \sin \theta = 1000 \tan \theta = i \quad (5)$$

According to Regulations on Railway Train Traction Calculation [5], the calculation formula of additional resistance for curve of upper curve central angle and curve length is:

$$w_r = \frac{600}{R} = \frac{600}{\left(\frac{360}{2\pi}\right)\left(\frac{L_r}{\alpha}\right)} = \frac{10.5\alpha}{L_r} (N/kN) \quad (6)$$

w_r is the additional resistance for curve units, R is the curve radius, m.

Empirical formula is adopted to calculate additional resistance for tunnel of the train [6]:

$$w_s = 0.00013L_s \quad (7)$$

Where L_s is the tunnel length, m.

2.3. Braking Model

At present, braking method of urban railway often choose electric-pneumatic braking, and its data can be gained via interpolation method by referring to braking force curve. Suppose that (v_1, w_1) and (v_2, w_2) are two given points on the braking characteristic curve, (v_x, w_x) is point to be calculated between these two points, the braking effort can also be calculated when the velocity v_x is given:

$$w_x = w_1 + \frac{(v_x - v_1)(w_2 - w_1)}{v_2 - v_1} (kN) \quad (8)$$

2.4. Train Dynamics Model

When the train is regarded as rigid system, operation state of the train can be obtained via theorem of kinetic energy. Kinetic energy of the train includes two parts: one is kinetic energy during linear motion, and the other one is the rotational kinetic energy of the rotating part:

$$E_k = \frac{mv^2}{2} + \sum \frac{I\omega^2}{2} = \frac{mv^2}{2} + \sum \frac{Iv^2}{2R^2} = \frac{mv^2}{2} (1 + \sum \frac{I}{2mR^2}) = \frac{mv^2}{2} (1 + \gamma) \quad (9)$$

m is the weight of the whole train, v is the speed of train operation, I is the moment of inertia of the rotating part, ω is the angular velocity of the rotating part, γ is the coefficient of rotational mass.

It can be gained through differentiation for (9):

$$dE_k = m(1 + \gamma)dv \quad (10)$$

The calculation formula for acceleration can be gained by transforming (10):

$$a = \frac{dv}{dt} = \frac{C}{m(1 + \gamma)} \frac{W_x - W_r - W_j}{m(1 + \gamma)} \quad (11)$$

Formula for train velocity can be obtained by integration for (11):

$$v = \int \frac{dv}{a} = \int \frac{m(1 + \gamma)}{C} dv \quad (12)$$

Formula for operating range of the train can be gained by integration for (12):

$$S = \int dS = \int \frac{v}{C/m(1 + \gamma)} dv = \int \frac{v}{a} dv \quad (13)$$

2.5. Calculation for Traction Energy Consumption of the Train

The train will consume power network energy during actual operation and the train is pushed forward by converting electric energy into mechanical energy. Energy consumption during train operation is related to engine attribute, route attribute, and other relevant factors [7][8]. Its energy consumption formula can be simplified into (14) and (15):

$$P = F \cdot V \cdot \eta \quad (14)$$

$$I = P / U \times \cos \varphi \quad (15)$$

In the formula: η is the energy conversion efficiency of the train; $\cos \varphi$ is power factor and often set as 0.95.

Energy consumption for traction during operation of the train can be gained via the energy formula by multiplying traction voltage by traction current by time. Therefore, the calculation formula for total energy consumption is:

$$W = \int P dt = \int U \cdot I dt \quad (16)$$

2.6. Route Model

Track parameter data refer to route data, covering serial number of sections, accumulated journey, track speed, distance between the two stations, route plane data, vertical data, and tunnel data. Plane data include curve length and curve central angle. Vertical section includes gradient value and gradient distance. Tunnel data include serial number of tunnel and tunnel length. For convenient input, the same variable was adopted for both curve length and ramp length. The specific value was determined according to curve central angle and gradient size. For instance, if there are two different curvatures at a certain gradient, we can suppose that the gradient size is fixed while curve length and central angle vary, and total length of the curve is equal to the sum of the two curve lengths. Refer to the following table for specific variable names.

Table 1. Track parameters table.

Parameter name	Variable
Accumulated journey	L_m
Track speed	V_{limit}
Distance between the two stations	L
Curve length of plane route	L_r
Curve central angle of plane route	α
Vertical section gradient (positive at upslope)	θ
Vertical section slope length	L_s
Tunnel length	L_s

3. Urban Rail Transit Calculation Program

3.1. Traction Calculation Program

The calculation application should include data input module, simulation calculation module, and result output module. The train data and route data will input in graphical display interface, simulation calculation module will feed back data to result output module during simulation, and states of train will present through the graphical interface.

Simulation calculation module is the heart of the entire metro simulation operation software. Its calculation logic is to form a virtual track via the preloaded track data and then determine the working condition of train braking unit and traction unit by detecting the condition of acceleration switch and brake switch; to obtain acceleration and operation state of the train through resultant force calculation module; to gain real-time velocity of the train through velocity calculation module; then to gain the current running distance of the train via distance calculation module. By directing at difference between simulation period and practical period of Matlab, Level 2 S-Function in Matlab language was utilized to write a timer program “sfun_time” and realize the function of real-time simulation via software.

Simulation calculation module in Simulink is presented in the following figure:

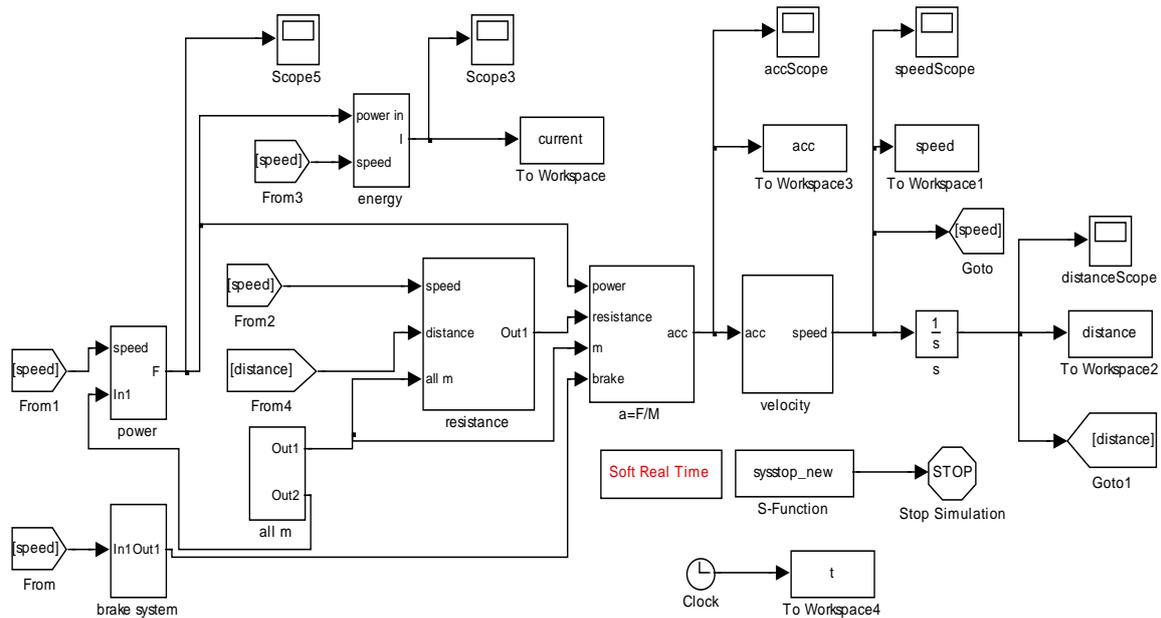
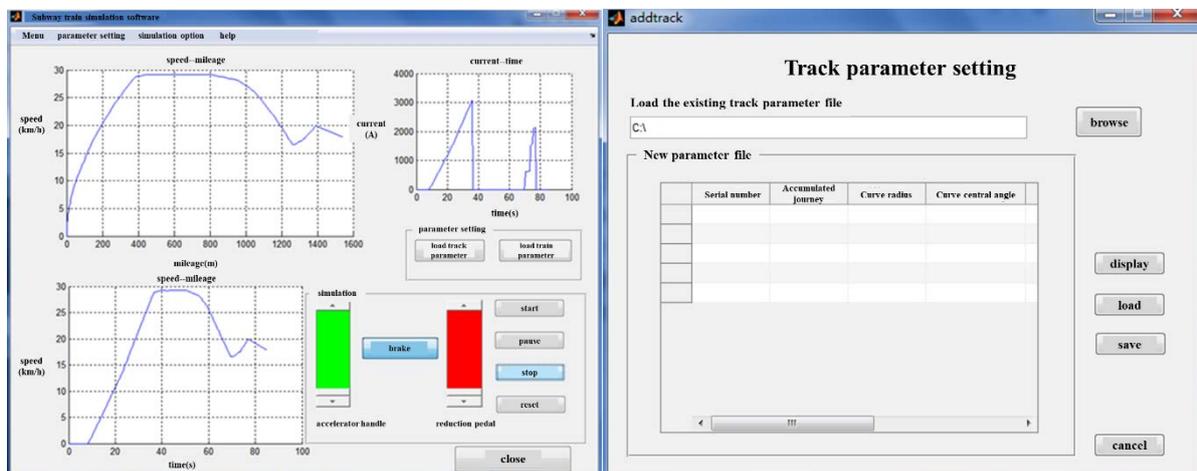


Figure 1. Traction calculation model based on Matlab/ Simulink.

3.2. UI Design

Simple style was adopted for interface of metro train calculation simulation software which is composed of one main window and two secondary windows. The two secondary windows are track parameter loading window and train parameter loading window.



(a) track parameter loading window

(b) train parameter loading window

Figure 2. Interface of the application.

During operation, the train can be controlled through the button of accelerator and decelerator, real-time display of the train operation state can be found in the main interface. The track parameter interface can be opened through parameter setting - load track parameter or by pressing the key button of track parameter. The interface of loading track parameter is for loading route data. The loaded file can be browsed or modified in the table interface. Meanwhile, the interface of loading train parameter is for loading train data. The loaded file can be browsed or modified in the table interface, too.

4. Simulation

Basing upon the planned metro route data, we simulated with type B metro and set the highest velocity is 80km/h [9][10]. The parameters are as follows:

Table 2. Vehicle parameters table.

Parameter name	Variable
Bullet train mass (kg)	49800
Bullet train quantity	4
Trailer mass (kg)	44500
Trailer quantity	2
Designed max speed (km/h)	80
Working voltage (V)	1000
Energy conversion efficiency	0.98
Rotary mass coefficient	0.2
Basic resistance parameter 1	2.43
Basic resistance parameter 2	0.02997
Basic resistance parameter 3	0.00078

Table 3. Traction data table.

Velocity(km/h)	Traction force (kN)
0	93.25
42	93.25
50	83.75
60	70
70	59.5
80	45

Table 4. Braking force data table

Velocity (km/h)	Braking force (kN)
0	-150
5	-150
6	-335
64.5	-335
80	-275

After simulation, the total journey is 4,590.97m. The train starts from the origin station and reaches the first stop after 91 seconds. The station dwell time is 51 seconds. Then after 87.5 seconds, it reaches the second stop, with the station dwell time of 49 seconds. It takes 82.5 seconds from the second stop to the third stop. The total operation time is 361 seconds in which the running time is 261 seconds. The average velocity is 44.78km/h, in which the average running velocity is 61.9km/h. The peak current is 4,863 amperes and the total energy consumption is 38.24kwh.

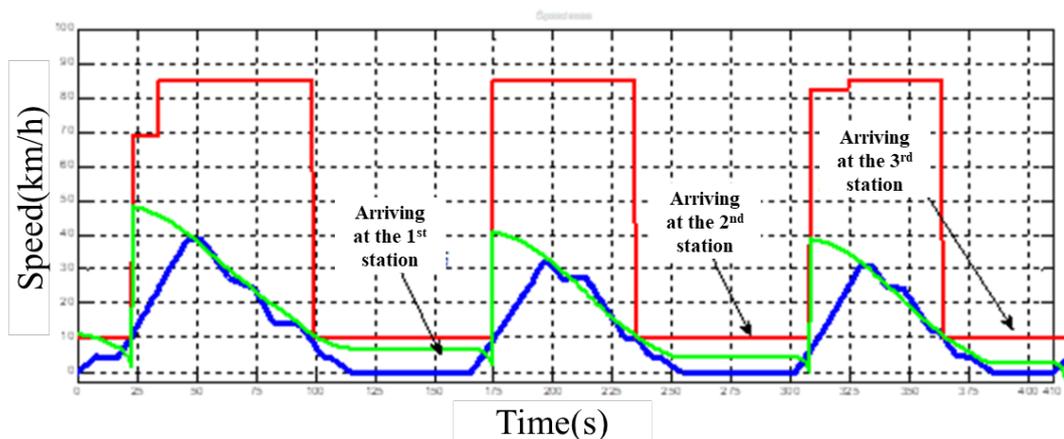


Figure 3. Fastest strategy simulation results.

If energy consumption is considered, the economic targets and velocity targets cannot be paid equal attention to. Therefore, in practical operation of the train, the driver will consciously try to turn the running train into an idle running state within the time allowed by the running schedule, but this will inevitably reduce running velocity of the train. If blocking is further considered, the driver will have no space to consider the economic targets. Therefore, this paper suggests that the economic targets should be brought into the train schedule.

The second group of data shows that the train starts from the origin station and reaches the first stop after 111.3 seconds. The station dwell time is 28.5 seconds. Then after 90.6 seconds, it reaches the second stop, with the station dwell time of 39.5 seconds. It takes 92.5 seconds from the second stop to the third stop. The total operation time is 362.4 seconds in which the running time is 294.4 seconds. The average velocity is 45.60km/h, in which the average running velocity is 56.13km/h. The peak current is 4,206 amperes and the total energy consumption is 22.4kwh.

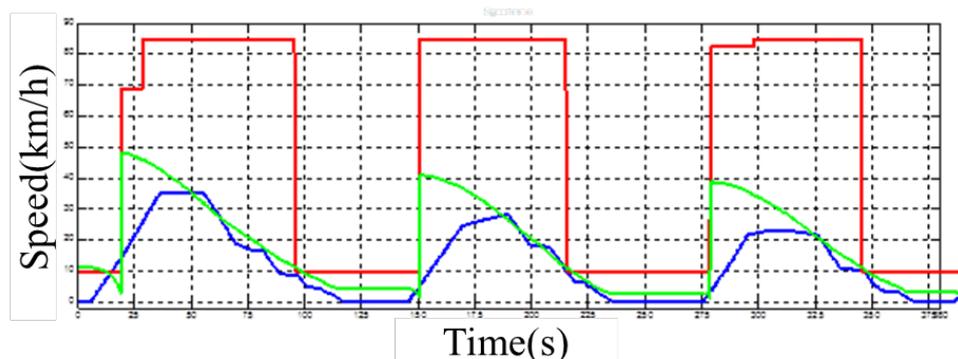


Figure 4. Simulation result based on considering energy costs.

After comparison between these two groups of data, the average running velocity of the second group has decreased by 10.28% when compared with the first group, its energy consumption has reduced by 70.71%, and the peak voltage has decreased by 13.5%.

5. Conclusion

In single-particle model algorithm, metro vehicle simulation software calculates the entire train formation as a whole. The result has proven that the program successfully achieves precise control of a train; it also possesses high simulation efficiency and can be effectively applied to train operation real-time simulation calculation of urban railway system.

6. References

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