IOP Conf. Series: Materials Science and Engineering **719** (2020) 012035 doi:10.1088/1757-899X/719/1/012035

DOA Estimation for Unfolded Coprime Arrays: Successive-MUSIC Algorithm

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Abstract. In this paper, we investigate the problem of direction of arrival (DOA) estimation of unfolded coprime arrays with successive-MUSIC algorithm which cascades multiple signal classification (MUSIC) algorithm and rotational invariance techniques (ESPRIT).We propose An algorithm with low computational complexity and high estimation accuracy, which first use ESPRIT algorithm for initial estimation, then according to the results of initial estimation narrow the search scope, finally, more accurate estimation results are obtained by MUSIC algorithm. Compared with the traditional MUSIC algorithm, this algorithm only needs to conduct a small range of angle search, which greatly reduces the algorithm complexity. Compared with the traditional ESPRIT algorithm, this algorithm also effectively improves the angle estimation performance of the algorithm. Moreover, comparing with traditional coprime linear arrays, the enhanced degrees of freedom (DOFs) can be achieved due to larger array aperture, Simulations are presented to validate the superiority of the proposed algorithm.

1. Introduction

In recent years, a new array structure coprime array [1-3] has aroused wide interest, its advantage is to provide a larger array aperture [4], improved estimation performance [5] and enhanced DOFs [6]. A large number of effective algorithms are applied to DOA estimation in coprime array. Such as ESPRIT, MUSIC [7], PARAFAC algorithm as well as others. The effective estimation algorithms of coprime array are mainly defuzzification methods and spatial smoothing algorithms [8]. The DOA estimation of two submatrices is carried out by defuzzification method respectively, and the only DOA estimation value is obtained by comparing the estimation results with the mutual quality characteristics of subarray elements. Although the method is easy to implement, the space degree of freedom is greatly reduced by the method of estimating two submatrices separately.

In this paper, we investigate the problem of DOA estimation of unfolded coprime arrays. The unfolded coprime array is obtained by reversing one of the coprime linear arrays (CLA) to the right of the origin. We proposed a low complexity and high DOA estimation precision of algorithm, by combining the ESPRIT - MUSIC cascading algorithm and unfolded coprime array. on the one hand the proposed algorithm can play coprime array on the advantages of effectively improve spatial degrees of freedom, on the other hand, the proposed algorithm can fully combine the advantages of MUSIC algorithm and ESPRIT algorithm, improve the estimation accuracy of the algorithm while effectively reducing the complexity of the algorithm. numerical simulation verified the effectiveness of the proposed algorithm.

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2. Data Model

Consider unfolded coprime arrays configuration which consists of two uniform linear subarrays, The spacing of the subarray to the left of the origin is $d_1 = N\lambda/2$, the number of subarray elements is M, the spacing of the subarray to the left of the origin is $d_2 = M\lambda/2$, the number of subarray elements is N, where M and N are coprime integers and λ denotes the signal wavelength. Its array topology is shown in figure 1.

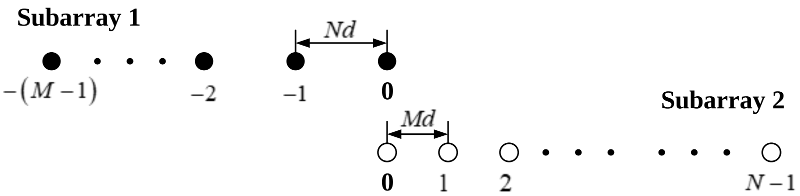


Figure 1. Unfold coprime arrays structure

The unfolded coprime arrays composed of subarray 1 and subarray 2 overlaps at the origin, and the positions of other subarrays elements do not overlap except at the origin. The position of each array element on the expanded matrix can be expressed as

$$Ls = \{(-m_1 d_1, 0), | m_1 = 0,1,2 \cdots M-1\} \cup \{(0, m_2 d_2), | m_2 = 0,1,2 \cdots N-1\} \tag{1}$$

The origin of the arrays can be Shared by two subarrays, so the number of unfolded coprime arrays is $M + N - 1$, but can provide $O\ (MN)$ degrees of freedom.

Considering unfolded coprime arrays, it is divided into subarray 1 and subarray 2. The array manifolds of subarray 1 and subarray 2 are defined as follows

$$\boldsymbol{a}_1(\theta_k) = [e^{j2\pi(M-1)d_1 \sin\theta_k/\lambda}, \cdots, e^{j2\pi d_1 \sin\theta_k/\lambda}, 1]^T \tag{2}$$

$$\boldsymbol{a}_2(\theta_k) = [1, e^{-j2\pi d_2 \sin\theta_k/\lambda}, \cdots, e^{-j2\pi(M-1)d_2 \sin\theta_k/\lambda}]^T \tag{3}$$

The direction matrix of subarray i is defined as

$$\boldsymbol{A}_i = [\boldsymbol{a}_i(\theta_1), \boldsymbol{a}_i(\theta_2), \cdots, \boldsymbol{a}_i(\theta_k)] \tag{4}$$

The source matrix is defined as

$$\boldsymbol{S} = [\boldsymbol{s_1}, \boldsymbol{s_2}, \cdots, \boldsymbol{s_K}]^{\boldsymbol{T}} \tag{5}$$

Among, $\boldsymbol{s}_k = [s_k(1), s_k(2), \cdots, s_k(J)]$, $(k = 1,2, \cdots, K)$, J represents snapshots.
Then, the received signal of the subarray i is defined as

$$\boldsymbol{X}_i = \boldsymbol{A}_i S + N_i \tag{6}$$

Among, $\boldsymbol{N}_i$ represents noise matrix.

Considering the whole arrays, it is no longer divided into subarray 1 and subarray 2. The array manifold of the unfolded coprime arrays is defined as

$$\boldsymbol{a}(\theta_k) = [e^{\frac{j2\pi(M-1)d_1 \sin\theta_k}{\lambda}}, \cdots, e^{\frac{j2\pi d_1 \sin\theta_k}{\lambda}}, 1]^T 1, e^{-\frac{j2\pi d_2 \sin\theta_k}{\lambda}}, \cdots, e^{-\frac{j2\pi(N-1)d_2 \sin\theta_k}{\lambda}}]^T \tag{7}$$

The direction matrix is defined as

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \cdots, \boldsymbol{a}(\theta_k)] \tag{8}$$

Then, the received signal of the arrays is defined as

$$\boldsymbol{X} = \boldsymbol{AS} + \boldsymbol{N} \tag{9}$$

Among, $\boldsymbol{N} \in C^{(M+N-1)\times J}$ represents noise matrix.

Get J snapshot to obtain unfolded coprime arrays covariance matrix $\widehat{\boldsymbol{R}}$ estimated

$$\widehat{\boldsymbol{R}} = \frac{1}{J}\sum_{j=1}^{J}\boldsymbol{X}(j)\boldsymbol{X}^H(j) \tag{10}$$

The eigenvalue decomposition of the signal covariance matrix can be expressed as

$$\widehat{\boldsymbol{R}} = \boldsymbol{E}_S\boldsymbol{D}_s\boldsymbol{E}_S^H + \boldsymbol{E}_N\boldsymbol{D}_N\boldsymbol{E}_N^H \tag{11}$$

3. The Proposed Algorithm

In this section, we first use ESPRIT algorithm to make the initial estimation, and then use the result of the initial estimation to determine the angle search range. Finally, we use MUSIC algorithm to complete the signal angle parameter estimation.

3.1. Initial Estimation

Firstly, the direction matrix $\boldsymbol{A}_i$ and receiving signal matrix $\boldsymbol{X}_i$ of subarray i are divided into equations (12) and (13) respectively

$$\boldsymbol{A}_i = \begin{bmatrix} a_{i1} \\ \boldsymbol{A}_{i2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{i1} \\ a_{i2} \end{bmatrix} \tag{12}$$

$$\boldsymbol{X}_i = \begin{bmatrix} x_{i1} \\ \boldsymbol{X}_{i2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}_{i1} \\ x_{i2} \end{bmatrix} \tag{13}$$

Among, $\boldsymbol{A}_{i1}$ and $\boldsymbol{A}_{i2}$ are matrices composed of $M_i - 1$ rows before $\boldsymbol{A}_i$ and $M_i - 1$ rows after $\boldsymbol{A}_i$ respectively. $\boldsymbol{X}_{i1}$ and $\boldsymbol{X}_{i2}$ are matrices composed of $M_i - 1$ rows before $\boldsymbol{A}_i$ and $M_i - 1$ rows after $\boldsymbol{A}_i$ respectively. Then the construction matrix $\boldsymbol{Z}_i$ can be obtained by combining the output of $\boldsymbol{X}_{i1}$ and $\boldsymbol{X}_{i2}$.

$$\boldsymbol{Z}_i = \begin{bmatrix} \boldsymbol{X}_{i1} \\ \boldsymbol{X}_{i2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{i1} \\ \boldsymbol{A}_{i2} \end{bmatrix}\boldsymbol{S} + \boldsymbol{N}_i \tag{14}$$

Among, $\boldsymbol{S}$ is source matrix, $\boldsymbol{N}_i$ represents noise matrix.$\boldsymbol{A}_{i2} = \boldsymbol{A}_{i1}\boldsymbol{\Phi}_i$,$\boldsymbol{\Phi}_i = diag\{e^{-j\mu_{i1}}, \cdots, e^{-j\mu_{ik}}\}$. $\boldsymbol{\Phi}_i$ represents the rotation operator matrix.

To estimate direction of arrival (DOA) $\widehat{\theta}_k^{\,ini}$ which is based on matrix $\boldsymbol{Z}_i$. It is necessary to estimate $\boldsymbol{\Phi}_i$.The signal subspace composed of $\boldsymbol{X}_{i1}$ and $\boldsymbol{X}_{i2}$ rotates phase μ_{ik}.
The covariance matrix $\boldsymbol{R}_{iz} = \boldsymbol{Z}_i\boldsymbol{Z}_i^{\,H}$ is constructed for $\boldsymbol{Z}_i$, and the signal subspace $\boldsymbol{E}_{iz}$ is obtained by eigenvalue decomposition. Then there is a matrix $\boldsymbol{T}_i$ that makes the following equation valid

$$\boldsymbol{E}_{iz} = \begin{bmatrix} \boldsymbol{E}_{i1} \\ \boldsymbol{E}_{i2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{i1} \\ \boldsymbol{A}_{i1}\boldsymbol{\Phi}_i \end{bmatrix}\boldsymbol{T}_i \tag{15}$$

Among, $\boldsymbol{E}_{i1}$ is composed of lines one to $M_i - 1$ of $\boldsymbol{E}_{iz}$,and $\boldsymbol{E}_{i1} = \boldsymbol{A}_{i1}\boldsymbol{T}_i$, $\boldsymbol{E}_{i2}$ is composed of lines M_i of $\boldsymbol{E}_{iz}$ to the last line, and $\boldsymbol{E}_{i2} = \boldsymbol{A}_{i1}\boldsymbol{\Phi}_i\boldsymbol{T}_i$ without considering the influence of noise

$$\boldsymbol{E}_{i2} = \boldsymbol{E}_{i1}\boldsymbol{T}_i^{-1}\boldsymbol{\Phi}_i\boldsymbol{T}_i \tag{16}$$

In this case

$$\boldsymbol{T}_i^{-1}\boldsymbol{\Phi}_i\boldsymbol{T}_i = \boldsymbol{E}_{i1}^{+}\boldsymbol{E}_{i2} \tag{17}$$

Comparisons between $\widehat{\theta}_{k1}^{\,ini}$and $\widehat{\theta}_{k2}^{\,ini}$ obtained by subarray 1 and subarray 2 of the unfolded coprime arrays. then get initial estimation $\widehat{\theta}_k^{\,ini}$.

3.2. Accurate Estimation

After get $\widehat{\theta}_k^{\,ini}$, through the use of MUSIC algorithm space spectrum function in the interval$[\widehat{\theta}_k^{\,ini} - \Delta, \widehat{\theta}_k^{\,ini} + \Delta]$ (Δ is a small value) within the search theta or get more precise $\widehat{\theta}_k$.

$$\mathrm{P}_{\mathrm{music}}(\theta_k) = \frac{1}{\boldsymbol{a}^H(\theta_k)\boldsymbol{E}_N\boldsymbol{E}_N^H\boldsymbol{a}(\theta_k)} \tag{18}$$

Among, $\boldsymbol{a}(\theta_k)$ is obtained by $\boldsymbol{E}_N$ according to (11). By looking for the crest to estimate the direction of arrival (DOA), The maximum K peaks relative to $\hat{\theta}_k$ are the Angle parameters of K sources to be estimated.
Remark 1. The number of source searches are required in the case of multiple sources.

3.3. Detailed Steps of the Proposed Algorithm
Step 1. Compute $\widehat{\boldsymbol{R}}$ and perform eigen decomposition of $\widehat{\boldsymbol{R}}$ to obtain $\widehat{\boldsymbol{E}}_n$.
Step 2. Compute ${\hat{\theta}_{k1}}^{ini}$ and ${\hat{\theta}_{k2}}^{ini}$ according ESPRIT algorithm.
Step 3. Obtain source angle initial estimates of ${\hat{\theta}_k}^{ini}$ according to the subarray 1 and subarray 2 coprime relations.
Step 4. Through one-dimensional MUSIC algorithm in $[{\hat{\theta}_k}^{ini}-\Delta, {\hat{\theta}_k}^{ini}+\Delta]$ for spectral peaks searching.
Step 5. Obtain $\hat{\theta}_k$ by the maximum K peaks.

4. Performance Analysis

4.1. Complexity Analysis
In this part, we evaluate the complexity calculation of the proposed algorithm and compare it with other related methods. For ESPRIT algorithm, the major complexity calculation is caused by the covariance matrix calculation, eigenvalue decomposition. Suppose that the number of elements of subarray 1 is M , the number of elements of subarray 2 is N, the covariance matrix calculation needs $O\{J(M^2+N^2)\}$ and eigenvalue decomposition of the covariance matrix requires $O\{M^3+N^3\}$. The complexity required to obtain $\boldsymbol{\Psi}_i$ is $O\{2K^3+3K^2(M+N-2)\}$, and eigenvalue decomposition of $\boldsymbol{\Psi}_i$ requires $O\{2K^3\}$. Consequently, the total complexity of ESPRIT is given by $\{4K^3+M^3+N^3+(M^2+N^2)J+3K^2(M+N-2)\}$. For the proposed algorithm, the major complexity calculation is caused by using ESPRIT for initial estimation and MUSIC for spectral peak searching. The required complexity of local range MUSIC algorithm search is $O\{n_1[(P-K)(2P+1)]\}$, so the total complexity of proposed algorithm is given by $O\{n_1[(P-K)(2P+1)]+4K^3+M^3+N^3+(M^2+N^2)J+3K^2(M+N-2)\}$.For MUSIC algorithm, the calculation of covariance, eigenvalue decomposition and spectral peak searching requires $O\{n[(P-K)(2P+1)]+P^3+P^2J\}$ .Among, total number of elements P equals $M+N-1$, n_1, n is the number of peak searches, K is the number of sources, J represents the number of snapshots. where $n_1=60/\Delta$ and $n=2K/\Delta$ represent the search times corresponding to finding peaks and the search interval of spectral peak is $\Delta=0.01$.

We summarize the computational complexity of above algorithms in Table 1. Moreover, Figure 2 depicts the complexities of the different algorithms versus different N, where $M=4$, $K=3$, $J=500$. The complexity comparison versus snapshots is given in Fig. 3, where $M=4$, $N=5$ and $K=3$. It is observed from Figs. 2 and 3 that the complexity of the proposed algorithm is Significantly lower than MUSIC algorithm but is slightly higher than ESPRIT algorithm. It can be concluded that the complexity of the algorithm mainly comes from peaks search. Because the proposed algorithm greatly reduces the number of peaks searches, the complexity is greatly reduced.

Table 1. Complexity of different methods.

Algorithm	Computational complexity
Proposed	$O\{n_1[(P-K)(2P+1)]+4K^3+M^3+N^3+(M^2+N^2)J+3K^2(M+N-2)\}$
Esprit	$O\{4K^3+M^3+N^3+(M^2+N^2)J+3K^2(M+N-2)\}$
MUSIC	$O\{n[(P-K)(2P+1)]+P^3+P^2J\}$

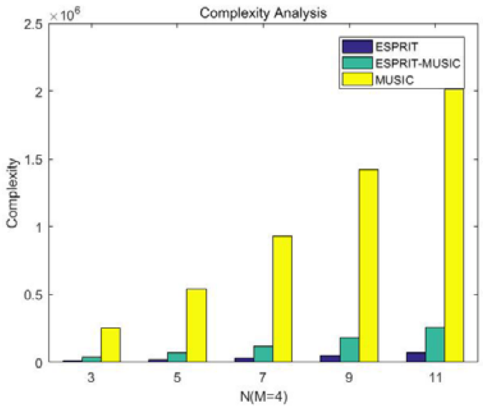


Figure 2. Complexities versus different N

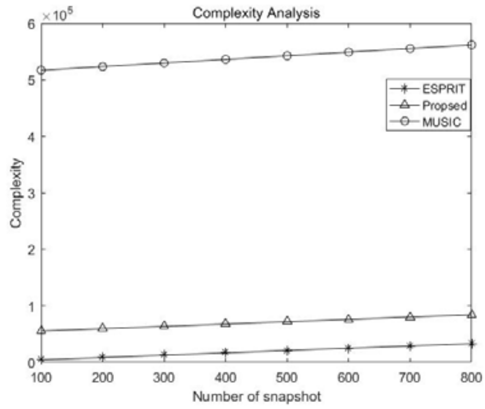


Figure 3. Complexities versus different snapshots

4.2. Advantages

The main advantages of the proposed algorithm are summarized as follows:

1) The proposed algorithm can achieve high resolution one-dimensional DOA estimation.
2) The proposed algorithm compared with the MUSIC algorithm, the complexity is greatly reduced. The performance of DOA estimation is better than ESPRIT algorithm, and very close to the high complexity of the MUSIC algorithm.
3) With the same array element number, the unfolded coprime arrays have a larger array aperture than that of CLA and ULA.

5. Simulation Results

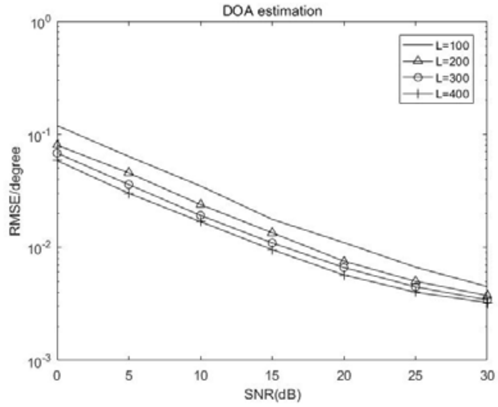
The DOA estimation performance of the MUSIC algorithm, ESPRIT algorithm and proposed algorithm in unfolded coprime arrays was evaluated by 1000 Monte Carlo simulations. The Root Mean Square Error (RMSE) is defined as

$$RMSE = \frac{1}{K}\sum_{k=1}^{K}\sqrt{\frac{1}{1000}\sum_{l=1}^{1000}\left[\left(\hat{\theta}_{k,l}-\theta_k\right)^2\right]} \tag{19}$$

Among, $\hat{\theta}_{k,l}$ is the estimated value of θ_k in Monte Carlo simulation. Assuming that there are three sources in space, the angular information: $\theta_1 = 10°, \theta_2 = 30°$， $\theta_3 = 50°$, M and N represent the number of elements of subarray 1 and subarray 2, $d_1 = N\lambda/2$,$d_2 = M\lambda/2$. d_1 and d_2 represent the spacing of elements of subarray 1 and subarray 2, J represents the number of snapshots, respectively.

5.1. RMSE Results of the Proposed Algorithm

Figure 4 gives the parameter estimation performance of the proposed algorithm with unfolded coprime arrays versus snapshots, where $N = 5, M = 4$. As depicted in Fig. 4, The number of snapshots increases, the sampling data increases. The DOA estimation performance of the algorithm becomes better as the snapshots increases. Figure 5 gives the parameter estimation performance of the proposed algorithm with unfolded coprime arrays versus N, where $J = 200, M = 4$. As depicted in Fig. 5, Array element number increases, namely diversity gain increases. The DOA estimation performance of the algorithm becomes better as the array number increases.



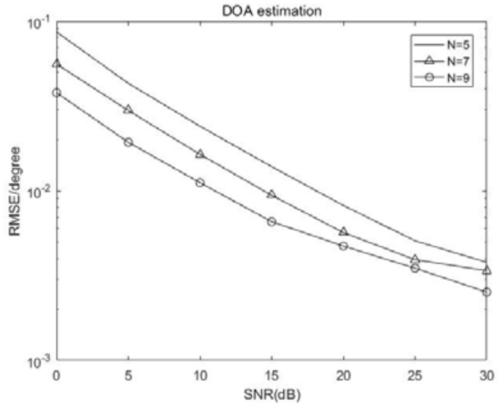


Figure 4. RMSE performance versus snapshots **Figure 5.** RMSE performance versus N

5.2. RMSE Results of Different Methods
Figure 6 gives the parameter estimation performance of the proposed algorithm, MUSIC algorithm and ESPRIT algorithm with unfolded coprime arrays under the same conditions, where $J = 200, M = 4, N = 5$. As depicted in Fig. 6, Array element number increases, namely diversity gain increases. The DOA estimation performance of the algorithm becomes better as the array number increases. the parameter estimation performance the proposed algorithm is very close to MUSIC algorithm but is Significantly higher than ESPRIT algorithm.

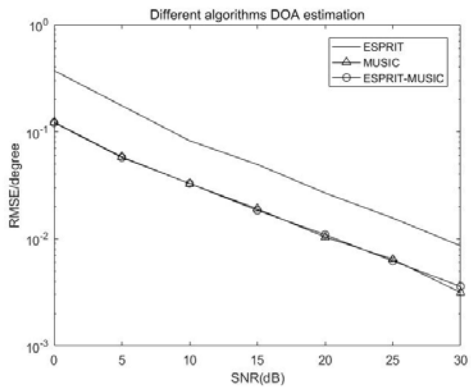


Figure 6. RMSE performance of different algorithms versus SNR

6. Conclusion
In this paper, we propose a low computational complexity and high estimation accuracy algorithm which cascades MUSIC and ESPRIT with unfolded coprime arrays. From the perspective of DOA estimation performance, this algorithm uses spectral peak search in a small range, so the DOA estimation performance of this algorithm is very close to MUSIC algorithm, and better than ESPRIT algorithm using eigenvalue decomposition. In terms of the complexity of the algorithm, since the algorithm uses local search instead of traversing the whole angle range, the complexity of the algorithm is far lower than that of the MUSIC algorithm, but only slightly higher than that of ESPRIT algorithm. Therefore, the algorithm is of great significance for the research on DOA estimation of unfolded coprime arrays. Numerical simulations corroborate the superiority of the algorithm in terms of computational complexity and estimation performance.

7. Acknowledgements
This work is supported by China NSF Grants (No. 61371169, No. 61601167).

8. References

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