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A Carrier Frequency Offset Estimation for OFDM System via Local Searching Capon Algorithm

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Abstract. Orthogonal frequency division multiplexing (OFDM) technology is an important modulation technology in wireless communication. One of the main disadvantages of OFDM system is that it is very sensitive to carrier frequency deviation. Therefore, estimation of carrier frequency deviation is one of the key technologies of OFDM system. In this paper, the global search Capon algorithm is improved, and a local search Capon algorithm is proposed. Simulation results show that the algorithm has low complexity and high performance. In the local search, P spectral peaks are superimposed, which can effectively overcome the problem that the estimation performance of traditional Capon algorithm is seriously reduced due to the unapparent spectral peaks or the missing spectral peaks in the case of low SNR., and it has a good application prospect in power wireless OFDM communication system. Since contemporary information-retrieval systems rely heavily on the content of titles and abstracts to identify relevant articles in literature searches, great care should be taken in constructing both.

1. Introduction

OFDM, started in the 1990s. The characteristic of this technology is to make full use of the orthogonality of subcarriers to carry out multi-carrier modulation. At present, OFDM system has been widely used in HDTV signal transmission, digital video transmission, wireless LAN and other fields, and applied in wireless wide area network and the fourth generation cellular communication network [1-2]. Compared with other systems, OFDM system has many advantages, such as high spectrum utilization and good anti-frequency selective fading ability [3]. However, OFDM system is particularly strict on the orthogonality of each subcarrier self-test [4-5]. Any point of carrier frequency offset (CFO) will affect the orthogonality between subcarriers. Therefore, the CFO in OFDM system needs to be accurately estimated and compensated to ensure the performance of the system. CFO estimation methods currently fall into two categories: non-blind CFO estimation and blind CFO estimation [6-7]. Non-blind CFO estimation methods include pilot based method and cyclic prefix based method. Blind CFO estimation adopts traditional spectral estimation methods, such as MUSIC algorithm, ESPRIT algorithm based on rotation-invariant signal parameters estimation, and CFO estimation algorithm based on kurtosis. Compared with the non-blind CFO estimation method, blind CFO estimation method has higher frequency band utilization and some algorithms have higher CFO estimation performance [8-9].

Compared with the non-blind estimation method, the global search Capon algorithm involved in this paper has better estimation performance, but has higher computational complexity. The local Capon algorithm proposed in this paper is improved on the basis of global search, and only one subcarrier is searched, so it has lower computational complexity. According to the simulation results, when the SNR is high, the local Capon algorithm proposed in this paper has the CFO estimation performance close to that of the global search Capon algorithm on the basis of low complexity. In the case of low SNR, the CFO estimation performance of global search Capon algorithm is improved.

2. System Model

Consider an uplink OFDM system containing N subcarriers, P of them are used for data transmission, and the rest N-P ones are virtual carriers. A looped prefix with length L is used to eliminate inter channel interference caused by multipath effect, where L is greater than the maximum delay of signal. After inserting the cyclic prefix, the signal is transmitted through multiple weak channels. After CP is removed, the output signal of the receiver can be denoted as

$$\boldsymbol{x}(k) = \boldsymbol{E}\boldsymbol{F}_P diag(\boldsymbol{h})\boldsymbol{s}(k)e^{j2\pi\Delta f(k-1)(N+L)} \tag{1}$$

Where $\boldsymbol{E} = diag(1, e^{j2\pi\Delta f/N}, \dots, e^{j2\pi\Delta f(N-1)/N})$ is the CFO matrix, Δf is CFO, $\boldsymbol{F}_P$ consists of the first P columns of the inverse discrete Fourier transform Matrix. The channel vector of frequency is $\boldsymbol{h} = [\boldsymbol{H}(1), \boldsymbol{H}(2), \dots, \boldsymbol{H}(P)]^T$, $\boldsymbol{H}(n) = \sum_{l=0}^{L_m-1} h(l)e^{-j2\pi nl/N}$. $\boldsymbol{s}(k) = [s_1(k), s_2(k), \dots, s_P(k)]^T$ is the *k*th symbol of transmit signals. According to (1), it follows that

$$\boldsymbol{X} = \boldsymbol{A} diag(\boldsymbol{h})\boldsymbol{B}^T \tag{2}$$

Where $\boldsymbol{B} = diag(1, e^{j2\pi\Delta f(N+L)}, \dots, e^{j2\pi\Delta f(K-1)(N+L)})\boldsymbol{S}$; $\boldsymbol{S} = [\boldsymbol{s}(1), \boldsymbol{s}(2), \dots \boldsymbol{s}(K)]^T$ is all K data blocks transferred by P channels. The vandermonde Matrix **A** is

$$\boldsymbol{A} = \boldsymbol{E}\boldsymbol{F}_P = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{j2\pi\left(\frac{0}{N}+\Delta f\right)} & e^{j2\pi\left(\frac{1}{N}+\Delta f\right)} & \dots & e^{j2\pi\left(\frac{P-1}{N}+\Delta f\right)} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j2\pi(N-1)\left(\frac{0}{N}+\Delta f\right)} & e^{j2\pi(N-1)\left(\frac{1}{N}+\Delta f\right)} & \dots & e^{j2\pi(N-1)\left(\frac{P-1}{N}+\Delta f\right)} \end{bmatrix} \tag{3}$$

In the case of noise, the received signal can be denoted as

$$\widetilde{\boldsymbol{X}} = \boldsymbol{A}\boldsymbol{S} + \boldsymbol{W} \tag{4}$$

Where **W** is the noise in the receiving process, $\boldsymbol{S} = diag(\boldsymbol{h})\boldsymbol{B}^T$.

3. Local Search Capon Algorithm for CFO estimation

3.1. Calculate the Estimated Value of the Signal Covariance Matrix

The estimated value of the covariance matrix is as follows

$$\widehat{\boldsymbol{R}}_x = \widetilde{\boldsymbol{X}}\widetilde{\boldsymbol{X}}^H/K \tag{5}$$

The basic algorithm of this paper is global search Capon algorithm, which is as follows:
The Capon algorithm is also known as the least variance method(MVM), the optimization problem can be expressed as

$$\varphi = arg\min_{\omega} P(\omega) \tag{6}$$

The constraint conditions are, $\omega^H \boldsymbol{a}(\varphi) = 1$, $P(\omega) = E[\boldsymbol{Y}^2] = \omega^H E[\boldsymbol{X}\boldsymbol{X}^H]\omega = \omega^H \boldsymbol{R}\omega$ is the power of the output signal.

The optimal weighted vector can be solved by Lagrange multiplier method, the result is

$$\omega_{Cap} = \frac{\widehat{\boldsymbol{R}}_x^{-1}\boldsymbol{a}(\varphi)}{\boldsymbol{a}^H(\varphi)\widehat{\boldsymbol{R}}_x^{-1}\boldsymbol{a}(\varphi)} \tag{7}$$

Where $\widehat{\boldsymbol{R}}_x$ is the received signal covariance matrix. For the data model in (4), its data covariance matrix can be estimated by $\hat{R}_x = \tilde{X}\tilde{X}^H/K$. Combining equation (6) and constraint conditions, the spatial spectrum of Capon algorithm can be obtained, and the CFO estimate can be obtained by searching the spectral peak of the spectral function

$$P_{Capon}(\varphi) = \frac{1}{\boldsymbol{a}^H(\varphi)\hat{\boldsymbol{R}}_x^{-1}\boldsymbol{a}(\varphi)} \tag{8}$$

Where $a(\varphi) = \left[1, e^{j2\pi\varphi}, \dots, e^{j2\pi(N-1)\varphi}\right]$. The global spectral peak search is carried out for the spectral function, search area is $[0,2\pi P/N]$, P spectral peaks can be obtained, and the corresponding frequency offset respectively is$(\frac{0}{N} + \Delta f, \frac{1}{N} + \Delta f, \dots, \frac{P-1}{N} + \Delta f)$.

The estimated true frequency deviation can be calculated from the corresponding frequency deviation values of P spectral peaks

$$\Delta f = \frac{\sum_{i=1}^{P}\phi_i - \sum_{j=1}^{P}\frac{i-1}{N}}{P} \tag{9}$$

Where ϕ_i is the frequency deviation value corresponding to the *i*th spectral peak.

3.2. CFO Estimation

In order to solve the problem of traditional global search Capon algorithm, we can use an improved local search approach for CFO estimates. The CFO corresponding to P spectral peaks can be expressed as

$$\phi_p = 2\pi\left(\Delta f + \frac{p-1}{N}\right), p = 1,2, \dots, P \tag{10}$$

So, similar to (8), we can construct the following spectral function to obtain CFO estimate through local search

$$P_{LS-Capon} = \frac{1}{\sum_{p=1}^{P}\boldsymbol{a}(\varphi+\frac{p-1}{N})^H\hat{\boldsymbol{R}}_x^{-1}\boldsymbol{a}(\varphi+\frac{p-1}{N})} \tag{11}$$

Where $\boldsymbol{a}(\varphi + \frac{p-1}{N}) = \left[1, e^{j2\pi(\varphi+\frac{p-1}{N})}, \dots, e^{j2\pi(N-1)(\varphi+\frac{p-1}{N})}\right]$. The spectrum peak search range is $[0,2\pi/N]$.

It can be seen from (10) and (11), the search range of local Capon algorithm is greatly reduced, so the computation goes down. In addition, since the spectral function of this algorithm is superimposed by P spectral peaks, even if the individual spectral peaks are not obvious or the spectral peaks are missing, the true frequency deviation of the obtained spectral function still corresponds to the unique spectral peak in the case of low SNR.

The steps for local search of Capon CFO estimation are as follows:

Step 1. Calculate the estimated value of the signal covariance matrix $\hat{R}_x = \tilde{X}\tilde{X}^H/K$.

Step 2. According to equation (10), CFO estimate can be obtained through local spectral peak search.

Advantages of the algorithm:

(1) this algorithm adopts a local spectral peak search with a sub-carrier range, while the traditional Capon algorithm adopts a global search with all P sub-carriers, and a larger search range leads to a larger operation.

(2) this algorithm inherits the high estimation performance of traditional Capon algorithm and is superior to ESPRIT algorithm.

(3) this algorithm superimposes P spectral peaks in the local search, which can effectively overcome the problem that the estimation performance of traditional MUSIC algorithm is seriously reduced due to the lack of spectral peaks or spectral peaks in the case of low SNR.

3.3. Complexity Analysis

The complexity of each algorithm is shown in table 1, where N and P are the number of subcarriers and the number of channels used by the system respectively, and K represents the number of snapshots. n_g represents the number of searches by the MUSIC algorithm or Capon global search, n_l, represents the search times of local search Capon algorithm.

Table 1. Complexity of different methods

algorithm	complexity
PM	$O(KN^2 + PN(P+N) + 3P^2(P-1) + P^3)$
ESPRIT	$O(KN^2 + N^3 + 3P^2(N-1) + P^3)$
MUSIC	$O(KN^2 + N^3 + n_g(2N^2 + N))$
Capon	$O(KN^2 + n_g N^3)$
Local search Capon	$O(KN^2 + n_l(N^3 + N^2P + P^2N)$

General Capon algorithm adopts global spectral peak search, n_g is large and complex. The Capon algorithm proposed in this letter adopts local spectral peak search, n_l is much less than n_g, greatly reducing the complexity of the algorithm.

4. Simulation Results

In order to verify the effectiveness of the algorithm and describe the Angle estimation performance of the algorithm, Monte Carlo simulation is adopted in this section to simulate the algorithm, and the simulation times are all 1000. The definition of root mean square error (RMSE) is shown in equation (1.12).

$$RMSE = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\left(\frac{\Delta\hat{f}^m - \Delta f}{1/N}\right)^2} \quad (12)$$

Figure 1 is the frequency offset estimation graph corresponding to the global search Capon algorithm when SNR=0dB. Figure 2 is the frequency offset estimation graph corresponding to the global search Capon algorithm when SNR=20dB. Among them, the frequency offset Δf=0.4, the subcarrier number N = 32, number of transmission channels P=20, cyclic prefix number L=8.

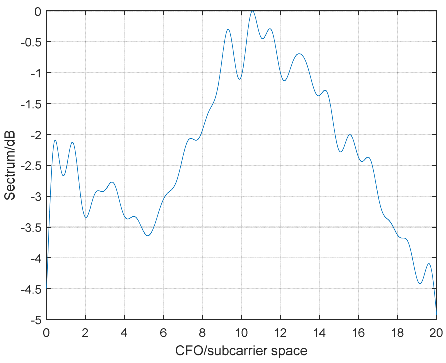


Figure 1. Global search Capon with SNR=0dB

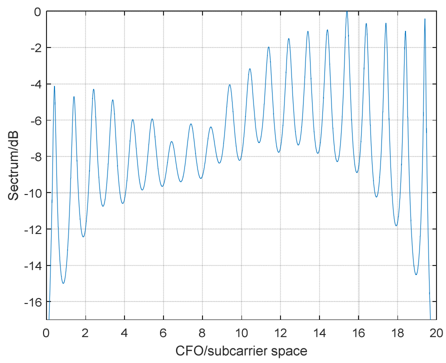
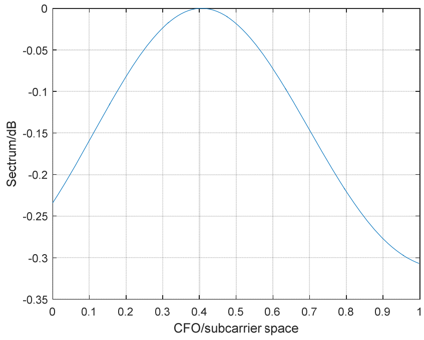


Figure 2. Global search Capon with SNR=20dB

Figure 3 is the frequency offset estimation diagram corresponding to the local search Capon algorithm when SNR=0dB. Among them, the frequency offset Δf=0.4, the subcarrier number N=32, number of transmission channels P=20, cyclic prefix number L=8. Figure 4 is the frequency offset estimation diagram corresponding to the local search Capon algorithm when SNR=20dB. Among them, the frequency offset Δf=0.4, the subcarrier number N=32, number of transmission channels P=20, cyclic prefix number L= 8.



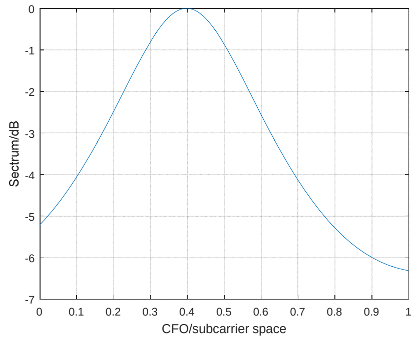
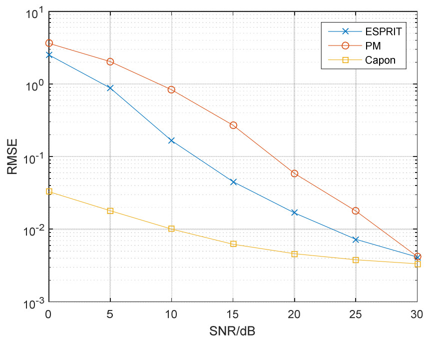


Figure 3. Local search Capon with SNR=0dB **Figure 4.** Local search Capon with SNR=20dB

Figure 5 is the performance comparison diagram of the local search Capon algorithm, ESPRIT algorithm and PM algorithm in this paper. According to the figure, the estimated performance of the algorithm in the present invention is much higher than that of the other two algorithms. Where, N=32, P=20, L=8, K=200, and the number of simulation times is 500. Figure 6 shows CFO estimation performance under different CFO scenarios. According to the figure, the local search Capon algorithm has very similar CFO estimation performance for different CFO. Where N = 32, P =20, K = 200,SNR=20dB and CFO range $[0, \omega]$, $\omega = 2\pi/N$ is normalized subcarrier space.



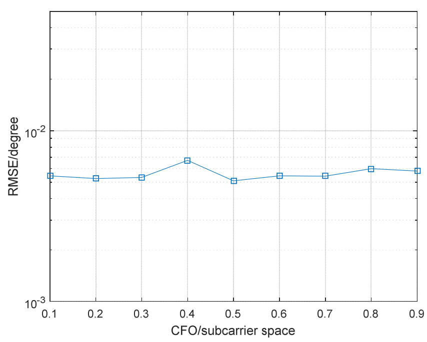


Figure 5. CFO estimation of different algorithms **Figure 6.** CFO estimation with different CFOs

Figure 7 to figure 9 are performance graphs of local search Capon algorithm under different parameters. Figure 7 and figure 8 respectively show the estimation performance of local search Capon algorithm under different subcarrier number N and different snapshot number K. The estimation performance of CFO improves with the increase of K and N. Figure 9 shows the estimated performance of the algorithm under different channel number P, and it can be seen that the performance of CFO algorithm decreases with the increase of P. As the number of channels increases, the interference between channels increases, leading to the CFO estimation of performance.

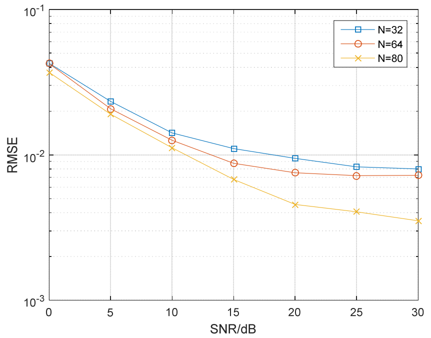


Figure 7. CFO estimation performance versus M

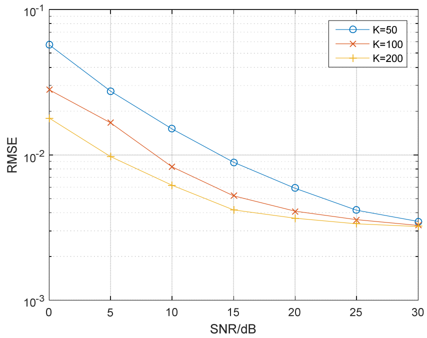


Figure 8. CFO estimation performance versus K

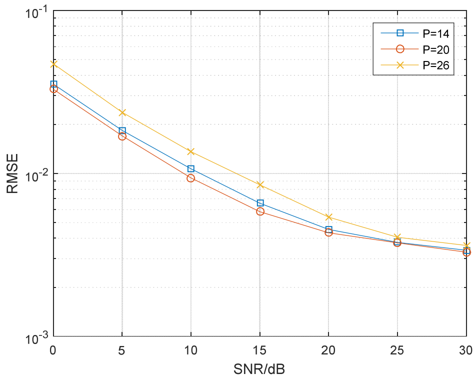


Figure 9. CFO estimation performance versus P

5. Conclusions

In this letter, we present a carrier frequency offset estimation algorithm for OFDM System by utilizing local search Capon. The algorithm adopts local spectral peak search, and the search range is a sub-carrier, which makes the search range small and the operation amount greatly reduced compared with the traditional Capon algorithm. And its estimated performance is better than ESPRIT algorithm. In addition, the superposition of P spectral peaks in local search can effectively overcome the problem that the estimation performance of traditional Capon algorithm is seriously reduced due to the lack of spectral peaks or the absence of spectral peaks in the case of low SNR. Simulation results show that the algorithm is effective.

6. References

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