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# Joint Power Allocation and Relay Beam-forming in Orbital Angular Momentum Amplify-and-Forward Relay Networks

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**Abstract.** Orbital Angular Momentum (OAM) is a promising technology that can effectively improve the spectral efficiency of wireless communications. However, it is important for OAM to keep the antenna alignment of transmitter and receiver. The alignment requirements greatly increases the difficulty of the practical application of OAM, especially when there is a barrier or a large turning angle between uniform circular arrays (UCAs). In this letter, we studied the application of amplify-and-forward (AF) relay in multiple-input multiple-output (MIMO) OAM system to solve the problem caused by the existence of UCAs corners. In this work, our goal is to achieve the maximum achievable rate by jointly optimizing the source power allocation matrix and the relay pre-coding matrix, constrained by the power of the source and the relay. The problem is expressed as a convex problem by mathematical operations, and then based on the Lagrange dual method, we solved it by proposing an iterative algorithm. Numerical simulations show the superiority of the algorithm.

## 1. Introduction

With the rapid development of 5G communication technology and the universal application of terminal equipment, it is urgent to find a way to efficiently use spectrum resources. As a relatively new wireless communication mode, OAM is receiving increasing attention due to its great potential in improving the spectral efficiency (SE) of wireless communications. In short-range line of sight (LOS) applications, OAM is playing a more and more important role.

In order to ensure the performance of the OAM system, the source and destination antenna arrays are required to be fully aligned. If this requirement cannot be strictly enforced, the OAM system cannot be easily commercialized. In [1], the author models the UAM-based OAM channel and analyses the effect of the tilt angle on the OAM channel capacity.

A relay communication network is a technology that utilizes one or more relay terminals to obtain spatial diversity to improve transmission performance. In the AF relay strategy, the relay amplifies the received signal and then forwards it to the destination, which has lower complexity [2]. As far as we know, the application of AF technology to the OAM system to solve the problem of UCAs misalignment has not been analysed yet.

In this paper, a three node MIMO AF relay system, including a source, an AF relay and a destination is investigated to maximize the information rate by jointly optimizing the source power allocation matrix and the relay pre-coding matrix, constrained by the power of the source and the relay. The problem is expressed as a convex problem by mathematical operations, and then solved by an iterative algorithm based on the Lagrange dual method. The rest of the paper is organized as follows. In the second section, we introduce the system model and analyse and simplify the problem. The third section proposes an iterative algorithm for power allocation. The simulation results are given in the fourth section, and the article is summarized in the fifth section.

Notations: Boldface lowercase and uppercase letters denote vectors and matrices, respectively. The transpose, conjugate transpose and Frobenius norm of matrix $\mathbf{A}$ are denoted as $\mathbf{A}^T$, $\mathbf{A}^\dagger$ and $\|\mathbf{A}\|$, respectively. $\mathcal{CN}\left(0,\mathbf{I}\right)$ denotes the distribution of a circularly symmetric complex Gaussian vector with mean 0 and covariance **I**.

## 2. System Model and Problem Formulation

In this section, we present the considered OAM MIMO AF relay system mode and formulate the power allocation problem to maximize the achievable rate of the destination.

### *2.1. OAM MIMO AF Relay System*

We consider a three node MIMO AF relay system, including a source, a relay and a destination. Assuming that all the nodes are equipped UCAs for signal transmitting, but the source's UCAs can't aligned with and the destination's because of the obstacle or the deployment restrictions of antenna installation sites.

To solve this problem, we can introduce an AF relay to realize the function of turn the signal in OAM system with UCAs used. The source's transmitting UCAs are aligned with the receiving UCAs of the AF relay, and the distance between the centres of them is $d_1$. As well as the transmitting UCAs of the AF relay are aligned with the receiving UCAs of the destination, and the distance between the centres of them is $d_2$. All the UCAs are equipped with $N$ elements and the radii of UCAs for source, transmitter of relay, receiver of relay and destination are given by $r_s$, $r_1$, $r_2$ and $r_d$, respectively. There can generate $N$ different OAM modes, counting the model number with $l$, where $l \in \{0,1,...,N-1\}$ [3].

By inputting the same signal $x_l$ to all the antenna unit of the UCA and performing phase shift sequentially, we can get number $l$ OAM wave. Denote the input signal as $\boldsymbol{x} \in \mathbb{C}^{N\times 1}$, where $\boldsymbol{x} = \left[0,1,\ldots,N-1\right]^T$. So the transmission vector the OAM wave with all the modes activated can be expressed as

$$\boldsymbol{s} = \mathbf{WP}\boldsymbol{x} \tag{1}$$

where $\boldsymbol{x} \sim \mathcal{CN}\left(0,\mathbf{I}\right)$, $\mathbf{P} = \mathrm{diag}\left(\left[p_0, p_1, \ldots, p_{\mathrm{N-1}}\right]\right)$ is the power allocation matrix with $p_l$ as the power allocation factor of mode $l$, $\mathbf{W} \in \mathbb{C}^{N\times N}$ is the discrete Fourier transform (DFT) matrix with $w_{nl}$ being the element in the n-th row and l-th column of **W**, where

$$w_{\mathrm{nl}} = \frac{1}{\sqrt{N}} \exp\left(-\mathrm{j}2\pi \frac{nl}{N}\right) \tag{2}$$

In this paper, we use the free-space propagation model to simulate the channel as

$$h(d_{\mathrm{ij}}) = \beta \frac{\lambda}{4\pi d_{\mathrm{ij}}} \exp\left(-\mathrm{j}2\pi \frac{d_{\mathrm{ij}}}{\lambda}\right) \tag{3}$$

where $\beta$ and $\lambda$ representing attenuation and phase rotation, which are complex constant associated with the carrier frequency, respectively. Given the complete alignment of the transmit and receive UCAs, the distance between the antenna elements denoted as $d_{\mathrm{ij}}, i \in \{0,1,\ldots,N\text{-}1\}, j \in \{0,1,\ldots,N\text{-}1\}$ is [4]:

$$d_{\mathrm{ij}}^{\mathrm{sr}} = (d_1^2 + r_{\mathrm{s}}^2 + r_{\mathrm{r1}}^2 - 2r_{\mathrm{s}}r_{\mathrm{r1}}\cos\phi_{\mathrm{ij}})^{1/2} \tag{4}$$

$$d_{\mathrm{ij}}^{\mathrm{rd}} = (d_2^2 + r_{\mathrm{d}}^2 + r_{\mathrm{r2}}^2 - 2r_{\mathrm{d}}r_{\mathrm{r2}}\cos\phi_{\mathrm{ij}})^{1/2} \tag{5}$$

where $\phi_{\mathrm{ij}} = \frac{2\pi(i-j)}{N}$ .

Then the channel matrix between source and relay , and between relay and destination, denoted as $\mathbf{H}_{\mathrm{sr}} \in \mathbb{C}^{N\times N}$ , $\mathbf{H}_{\mathrm{rd}} \in \mathbb{C}^{N\times N}$ , respectively, can be expressed as $\mathbf{H}_{\mathrm{sr}} = \left[\left(h(d_{\mathrm{ij}}^{\mathrm{sr}})\right)\right]$, $\mathbf{H}_{\mathrm{rd}} = \left[\left(h(d_{\mathrm{ij}}^{\mathrm{rd}})\right)\right]$, where $\mathbf{H} = \left[\left(h(d_{\mathrm{ij}})\right)\right]$ means that $h(d_{ij})$ is the element of the i-th row and j-th column of **H**. We can find that $\mathbf{H}_{\mathrm{sr}}$ , $\mathbf{H}_{\mathrm{rd}}$ are circulant matrixes because that the UCAs are used on both the source and the destination and there exists a precise symmetry.

The AF transmission scheme is employed. During the first slot, the source transmits the signal vector $\boldsymbol{s}$. Then the received signal at the relay is

$$\boldsymbol{y}_{\mathrm{r}} = \mathbf{H}_{\mathrm{sr}}\boldsymbol{s} + n_{\mathrm{r}} = \mathbf{H}_{\mathrm{sr}}\mathbf{W}\mathbf{P}\boldsymbol{x} + n_{\mathrm{r}} \tag{6}$$

where $\boldsymbol{n}_{\mathrm{r}} \sim \mathcal{CN}\left(0,\mathbf{I}\right)$ is the additive Gaussian noise at the relay. The transmit power of the source is constrained to $P_S$ as

$$P_{\mathrm{s}} \triangleq \left\|\mathbf{W}\mathbf{P}\right\|^2 = \left\|\mathbf{P}\right\|^2 = \sum_{i=0}^{N-1} p_{\mathrm{i}}^2 \le P_{\mathrm{S}} \tag{7}$$

During the second slot, the relay process the received signal by multiplying a pre-coding matrix $\mathbf{F} \in \mathbb{C}^{N\times N}$ , $\mathbf{F} = \mathrm{diag}\left(\left[f_0, f_1, ..., f_{\mathrm{N-1}}\right]\right)$ and forwards it to the destination. But before the process of pre-coding, we realize the OAM mode de-multiplexing by the multiplying the received signal $\boldsymbol{y}_{\mathrm{r}}$ with the inverse DFT (IDFT) matrix $\mathbf{W}^{\mathrm{H}}$ [5] as

$$\tilde{\boldsymbol{y}}_{\mathrm{r}} = \mathbf{W}^{\mathrm{H}}\boldsymbol{y}_{\mathrm{r}} = \mathbf{W}^{\mathrm{H}}\mathbf{H}_{\mathrm{sr}}\mathbf{W}\mathbf{P}\boldsymbol{x} + \mathbf{W}^{\mathrm{H}}\boldsymbol{n}_{\mathrm{r}} = \boldsymbol{\Lambda}_{\mathrm{sr}}\mathbf{P}\boldsymbol{x} + \mathbf{W}^{\mathrm{H}}\boldsymbol{n}_{\mathrm{r}} \tag{8}$$

where $\boldsymbol{\Lambda}_{\mathrm{sr}} \in \mathbb{C}^{N\times N}$ is a diagonal matrix. We can get it by the diagonalization of matrix $\mathbf{H}_{\mathrm{sr}}$ through the DFT matrix,

$$\boldsymbol{\Lambda}_{\mathrm{sr}} = \mathrm{diag}(\mathrm{eig}(\mathbf{H}_{\mathrm{sr}})) = \sqrt{N}\,\mathrm{diag}(\mathbf{W}\boldsymbol{h}_{\mathrm{sr1}}) \tag{9}$$

where $\boldsymbol{h}_{\mathrm{sr1}}$ is the first column of $\mathbf{H}_{\mathrm{sr}}$ . $\boldsymbol{\Lambda}_{\mathrm{sr}}$ contains the eigenvalues of $\mathbf{H}_{\mathrm{sr}}\left(\xi_0, \xi_1, \ldots, \xi_{\mathrm{N-1}}\right)$. We denote $\boldsymbol{\Lambda}_{\mathrm{sr}} = \mathrm{diag}\left(\left[\lambda_{\mathrm{sr0}}, \lambda_{\mathrm{sr1}}, \ldots, \lambda_{\mathrm{sr\,N-1}}\right]\right)$. Then we multiples the signal $\tilde{\boldsymbol{y}}_{\mathrm{r}}$ with the pre-coding matrix **F**, and after that we take the processing of OAM multiplexing by the DFT matrix **W**. So the received signal at the destination is

$$\boldsymbol{y}_{\mathrm{d}} = \mathbf{H}_{\mathrm{rd}}\mathbf{W}\mathbf{F}\boldsymbol{\Lambda}_{\mathrm{sr}}\mathbf{P}\boldsymbol{x} + \mathbf{H}_{\mathrm{rd}}\mathbf{W}\mathbf{F}\mathbf{W}^{\mathrm{H}}\boldsymbol{n}_{\mathrm{r}} + \boldsymbol{n}_{\mathrm{d}} \tag{10}$$

where $\boldsymbol{n}_{\mathrm{d}} \sim \mathcal{CN}\left(0,\mathbf{I}\right)$ is the additive Gaussian noise at the destination. The transmit power of the relay is constrained to $P_{\mathrm{R}}$ as

$$P_{\mathrm{r}} \triangleq \left\|\mathbf{W}\mathbf{F}\boldsymbol{\Lambda}_{\mathrm{sr}}\mathbf{P}\right\|^2 + \left\|\mathbf{W}\mathbf{F}\right\|^2 = \sum_{i=0}^{N-1} (\lambda_{\mathrm{sri}}^2 p_{\mathrm{i}}^2 + 1) f_{\mathrm{i}}^2 \le P_{\mathrm{R}} \tag{11}$$

Just like the processing at the relay, at the destination, we realize the OAM mode de-multiplexing by multiplying the received signal $\boldsymbol{y}_{\mathrm{d}}$ with the IDFT matrix $\mathbf{W}^{\mathrm{H}}$ as

$$\begin{aligned}\tilde{\boldsymbol{y}}_{\rm d} &= \mathbf{W}^{\rm H}\boldsymbol{y}_{\rm d} = \mathbf{W}^{\rm H}\mathbf{H}_{\rm rd}\mathbf{W}\mathbf{F}\mathbf{\Lambda}_{\rm sr}\mathbf{P}\boldsymbol{x} + \mathbf{W}^{\rm H}\mathbf{H}_{\rm rd}\mathbf{W}\mathbf{F}\mathbf{W}^{\rm H}\boldsymbol{n}_{\rm r} + \mathbf{W}^{\rm H}\boldsymbol{n}_{\rm d} \\ &= \mathbf{\Lambda}_{\rm rd}\mathbf{F}\mathbf{\Lambda}_{\rm sr}\mathbf{P}\boldsymbol{x} + \mathbf{\Lambda}_{\rm rd}\mathbf{F}\mathbf{W}^{\rm H}\boldsymbol{n}_{\rm r} + \mathbf{W}^{\rm H}\boldsymbol{n}_{\rm d} \\ &= \mathbf{\Lambda}_{\rm rd}\mathbf{F}\mathbf{\Lambda}_{\rm sr}\mathbf{P}\boldsymbol{x} + \mathbf{\Lambda}_{\rm rd}\mathbf{F}\tilde{\boldsymbol{n}}_{\rm r} + \tilde{\boldsymbol{n}}_{\rm d}\end{aligned} \tag{12}$$

where $\mathbf{\Lambda}_{\rm rd} \in \mathbb{C}^{N\times N}$ is a diagonal matrix, like before

$$\mathbf{\Lambda}_{\rm rd} = \mathrm{diag}(\mathrm{eig}(\mathbf{H}_{\rm rd})) = \sqrt{N}\,\mathrm{diag}(\mathbf{W}\boldsymbol{h}_{\rm rd1}) \tag{13}$$

where $\boldsymbol{h}_{\rm rd1}$ is the first column of $\mathbf{H}_{\rm rd}$. We denote $\mathbf{\Lambda}_{\rm rd} = \mathrm{diag}\left(\left[\lambda_{\rm rd0}, \lambda_{\rm rd1}, \ldots, \lambda_{\rm rdN\text{-}1}\right]\right)$.

Based on (12), the achievable rate at the destination is

$$\begin{aligned} R &= 0.5\log_{10}\left|\mathbf{I} + (\mathbf{\Lambda}_{\rm rd}\mathbf{F}\mathbf{\Lambda}_{\rm sr}\mathbf{P})^{\dagger}\mathbf{Q}^{-1}\mathbf{\Lambda}_{\rm rd}\mathbf{F}\mathbf{\Lambda}_{\rm sr}\mathbf{P}\right| \\ &= 0.5\log_{10}\left|\mathbf{I} + \mathbf{P}^{\dagger}\mathbf{\Lambda}_{\rm sr}{}^{\dagger}\mathbf{F}^{\dagger}\mathbf{\Lambda}_{\rm rd}{}^{\dagger}\mathbf{Q}^{-1}\mathbf{\Lambda}_{\rm rd}\mathbf{F}\mathbf{\Lambda}_{\rm sr}\mathbf{P}\right| \end{aligned} \tag{14}$$

where $\mathbf{Q} = \mathbf{\Lambda}_{\rm rd}\mathbf{F}(\mathbf{\Lambda}_{\rm rd}\mathbf{F})^{\dagger} + \mathbf{I} = \mathrm{diag}[\lambda_{\rm rd0}\lambda_{\rm rd0}{}^{\dagger}f_0{}^2+1,\ \lambda_{\rm rd1}\lambda_{\rm rd1}{}^{\dagger}f_1^2+1, \ldots,\ \lambda_{\rm rdN\text{-}1}\lambda_{\rm rdN\text{-}1}{}^{\dagger}f_{\rm N\text{-}1}{}^2+1]$. It's obviously that the matrixes in (14) are diagonal matrix, so we can rewrite the equation as

$$\begin{aligned} R &= 0.5\log_{10}\left|\mathrm{diag}\left[\left(SNR_{\rm i}\right)\right]\right| \\ &= 0.5\log_{10}\prod_{i=0}^{N\text{-}1} SNR_{\rm i} \\ &= 0.5\sum_{i=0}^{N\text{-}1}\log_{10} SNR_{\rm i} \end{aligned} \tag{15}$$

where

$$SNR_{\rm i} = 1 + \frac{\tilde{p}_{\rm i}\tilde{f}_{\rm i}\tilde{\lambda}_{\rm sri}\tilde{\lambda}_{\rm rdi}}{\tilde{f}_{\rm i}\tilde{\lambda}_{\rm rdi} + 1} \tag{16}$$

where $\tilde{p}_{\rm i} = p_{\rm i}^2, \tilde{f}_{\rm i} = f_{\rm i}^2$, $\tilde{\lambda}_{rdi} = \lambda_{\rm rdi}\lambda_{\rm rdi}^{\dagger}$, $\tilde{\lambda}_{\rm sri} = \lambda_{\rm sri}\lambda_{\rm sri}^{\dagger}$. And in (13), $\mathrm{diag}\left[\left(SNR_{\rm i}\right)\right]$ means that the diagonal matrix takes $SNR_{\rm i}$ as its i-th element in the diagonal.

### *2.2. Problem Formulation*

In this work, we aim to maximize the achievable rate of the destination, subject to the power constraint of the source and relay, by jointly optimizing the source power allocation matrix **P** and the relay pre-coding matrix **F**. So the problem of the OAM AF system can be formulated as (**P1**)

$$\begin{aligned} (\mathbf{P1})\quad & \max_{\mathbf{P},\mathbf{F}} R = 0.5\sum_{i=0}^{N-1}\log_{10} SNR_{\rm i} \\ \text{s.t.}\quad & P_{\rm s} \le P_{\rm S} \\ & P_{\rm r} \le P_{\rm R} \\ & p_{\rm i} \ge 0, f_{\rm i} \ge 0, \forall i \in \{0,1,\ldots,N-1\} \end{aligned} \tag{17}$$

The problem (**P1**) is convex, because the objective function in (17) is jointly convex with respect to $p_{\rm i}$ and $f_{\rm i}$, and the constraint are linear. We can solve the problem by the tool of CVX.

## 3. Iterative Algorithm

To solve this problem, we propose an iterative algorithm based on the Lagrange dual method.

### *3.1. When Pre-Coding Matrix* **F** *is Given*

When **F** is given, then $f_i$ can be determined, and we can rewrite (**P1**) as (**P2**):

$$\begin{aligned} (\mathbf{P2})\ \max_{\mathbf{P}}\ & 2R=\sum_{i=0}^{N-1}\log_{10}(1+\alpha_i\tilde{p}_i) \\ \text{s.t.}\ & \sum_{i=0}^{N-1}\tilde{p}_i\le P_S \\ & \sum_{i=0}^{N-1}\gamma_i\tilde{p}_i\le\tilde{P}_R \\ & \tilde{p}_i\ge 0,\forall i\in\{0,1,\dots,N-1\} \end{aligned} \tag{18}$$

where $\alpha_i=\frac{\tilde{f}_i\tilde{\lambda}_{sri}\tilde{\lambda}_{rdi}}{\tilde{f}_i\tilde{\lambda}_{rdi}+1},\gamma_i=\tilde{f}_i\tilde{\lambda}_{sri}$, and $\tilde{P}_R=P_R-\sum_{i=0}^{N-1}\tilde{f}_i$ .

We can find that (**P2**) is a convex problem, so we can use the Lagrange dual method to find the closed solution. The Lagrangian of (**P2**) with the Lagrange multiplier $\eta_1\ge 0,\eta_2\ge 0$ is given as

$$L\ (\mathbf{P},\eta_1,\eta_2)=\sum_{i=0}^{N-1}\log_{10}(1+\alpha_i\tilde{p}_i)+\eta_1(P_S-\sum_{i=0}^{N-1}\tilde{p}_i)+\eta_2(\tilde{P}_R-\sum_{i=0}^{N-1}\gamma_i\tilde{p}_i) \tag{19}$$

The derivative for the transmit power $\tilde{p}_i$ is

$$\frac{\partial L}{\partial\tilde{p}_i}=\frac{\alpha_i}{1+\alpha_i\tilde{p}_i}-\eta_1-\eta_2\gamma_i \tag{20}$$

Setting this derivative to zero, we can get that

$$\tilde{p}_i^{\,*}=\left(\frac{1}{\eta_1+\eta_2\gamma_i}-\frac{1}{\alpha_i}\right)^+ \tag{21}$$

Equation (21) is a max-operation $(x)^+=\max(0,x)$ to ensure that the result is non-negative. The Karush-Kuhn-Tucker conditions guarantee the optimality of the solution even when equation (21) has been used.

The dual function is given by

$$f(\eta_1,\eta_2)=\max_{\mathbf{P}}L\ (\mathbf{P},\eta_1,\eta_2)\quad \text{s.t. } \tilde{p}_i\ge 0,\forall i\in\{0,1,\dots,N-1\} \tag{22}$$

The dual problem is

$$(\mathbf{D2}):\min_{\eta_1,\eta_2}=f(\eta_1,\eta_2)\quad \text{s.t. } \eta_1\ge 0,\eta_2\ge 0 \tag{23}$$

Since the dual problem is always convex, but is generally non-differentiable, we can use the sub-gradient-based method, such as the ellipsoid method, to obtain the optimal dual solution as $\eta_1^*,\eta_2^*$ .

### *3.2. When Source Power Matrix* **P** *is Given*

When **P** is given, then $p_i$ can be determined, and we can rewrite (**P1**) as (**P3**).

$$\begin{aligned} (\mathbf{P3})\ \max_{\mathbf{F}}\ & 2R=\sum_{i=0}^{N-1}\log_{10}\left(1+\frac{\theta_i\tilde{f}_i}{\tilde{\lambda}_{rdi}\tilde{f}_i+1}\right) \\ \text{s.t.}\ & \sum_{i=0}^{N-1}\mu_i\tilde{f}_i\le P_R \\ & \tilde{f}_i\ge 0,\forall i\in\{0,1,\dots,N-1\} \end{aligned} \tag{24}$$

where $\theta_{\mathrm{i}} = \widetilde{p}_{\mathrm{i}}\,\widetilde{\lambda}_{\mathrm{sri}}\widetilde{\lambda}_{\mathrm{rdi}}, \mu_{\mathrm{i}} = \widetilde{p}_{\mathrm{i}}\,\widetilde{\lambda}_{\mathrm{sri}} + 1$.

We can find the objective function is of the form $f(x) = \log_{10}\left(1 + \frac{\theta_{\mathrm{i}} x}{1+\widetilde{\lambda}_{\mathrm{rdi}} x}\right)$, which can be proved to be convex because that the outer function is convex and non-decreasing, the inner function is convex. So (**P3**) is also a convex problem, we can solve the problem as before. The Lagrangian of (**P3**) with the Lagrange multiplier $v_1 \geq 0$ is given as

$$L\left(\mathbf{F}, v_1\right) = \sum_{i=0}^{N-1} \log_{10}\left(1 + \frac{\theta_{\mathrm{i}}\widetilde{f}_{\mathrm{i}}}{\widetilde{\lambda}_{\mathrm{rdi}}\widetilde{f}_{\mathrm{i}} + 1}\right) + v_1\left(P_{\mathrm{R}} - \mu_{\mathrm{i}}\widetilde{f}_{\mathrm{i}}\right) \tag{25}$$

The derivative for the transmit power $\widetilde{f}_{\mathrm{i}}$ is

$$\frac{\partial L}{\partial \widetilde{f}_{\mathrm{i}}} = \frac{\theta_{\mathrm{i}} + \widetilde{\lambda}_{\mathrm{rdi}}}{1 + (\theta_{\mathrm{i}} + \widetilde{\lambda}_{\mathrm{rdi}})\widetilde{f}_{\mathrm{i}}} - \frac{\widetilde{\lambda}_{\mathrm{rdi}}}{1 + \widetilde{\lambda}_{\mathrm{rdi}}} - v_1\mu_{\mathrm{i}} \tag{26}$$

Setting this derivative to zero, we can get that

$$\widetilde{f}_{\mathrm{i}}^{\,*} = \left(\frac{-(\theta_{\mathrm{i}} + 2\widetilde{\lambda}_{\mathrm{rdi}}) + \sqrt{\Delta}}{2(\theta_{\mathrm{i}} + \widetilde{\lambda}_{\mathrm{rdi}})\widetilde{\lambda}_{\mathrm{rdi}}}\right)^{+} \tag{27}$$

where $\Delta = \dfrac{4(\theta_{\mathrm{i}} + \widetilde{\lambda}_{\mathrm{rdi}})\widetilde{\lambda}_{\mathrm{rdi}}\theta_{\mathrm{i}}}{v_1\mu_{\mathrm{i}}} + \theta_{\mathrm{i}}^2$

Also, we can use the same method to solve the dual problem (**D3**) of (**P3**) to obtain the optimal dual solution as $v_1^*$.

We can conclude the alterative algorithm as shown in table 1:

**Table 1.** The proposed iterative algorithm for OAM AF system.

| | |
|---|---|
| **1: Initialize** | n = 0, $\mathbf{F}^{(0)}$ |
| **2:Repeat** | Solve Problem (**P2**) and (**D2**) to obtain $\mathbf{P}^{(\mathbf{n})}$ and $\eta_1$, $\eta_2$ |
| | n:=n+1; |
| | Solve Problem (**P3**) and (**D3**) to obtain $\mathbf{F}^{(\mathbf{n})}$ and $v_1$ |
| **3:Until** | Convergence. |

## 4. Simulation Results

The simulation results verify that our algorithm can improve the achievable rate when using AF relay.

In the simulation, we set the operation frequency $f_{\mathrm{c}}$ = 10 GHz, the wavelength $\lambda$ = 0.03m, the UCA radius of the source, the relay, and the destination as $r_{\mathrm{s}} = r_{\mathrm{r1}} = r_{\mathrm{r2}} = r_{\mathrm{d}} = 20\lambda$ = 0.6m, the UCA configurations as $N$ = 4, the transmission distance $d_1 = 500\lambda$ = 15m, $d_2 = 500\lambda$ = 15m, the average transmission power constraint $P_{\mathrm{S}} = P_{\mathrm{R}}$ are settled from 50dB to 90dB, and the antenna parameter $\beta$ = 1.

We present the average achievable rates achieved by the proposed scheme, and we compare it with two other situation. Situation 1: the aligned situation, in which the source and the destination are aligned, and they will communicate directly with each other; Situation 2: the non-aligned situation, in which the source and the destination will have an angle offset, but they will ignore the offset and communicate directly with each other, too.

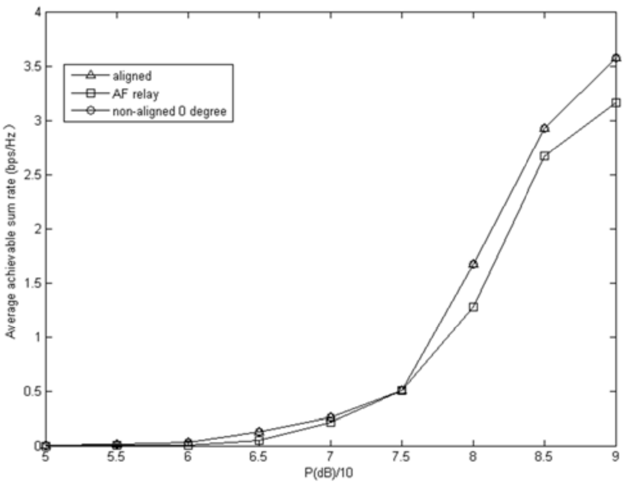


**Figure 1.** The achievable sum rate versus power, obtained by "aligned", "The AF relay" and "0 degree offset".

In Fig. 1, we settle the angle offset as 0 degree. Obviously, the curves of the aligned situation is coincided with the situation 2's curves. And we can find that, situation 1 is better than the AF relay situation, that is caused by the energy cost at the relay.

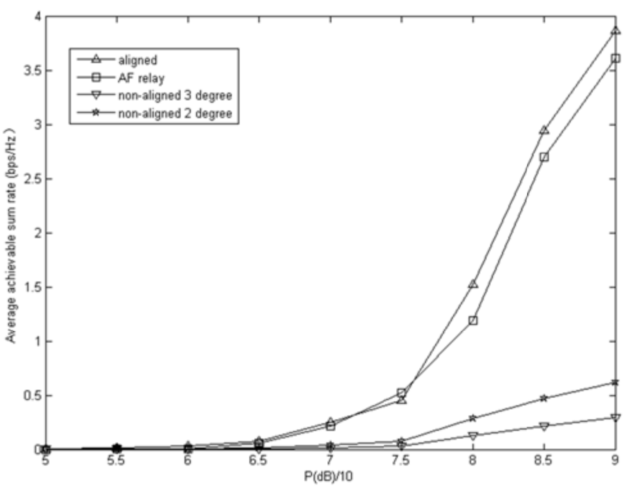


**Figure 2.** The achievable sum rate versus power, obtained by "aligned", "The AF relay" and "3 degree offset", and "2 degree offset".

In Fig. 2, we settle the angle offset as 2 and 3 degree, respectively. Obviously, the curves of the aligned situation is better than all other curves. And we can find that, situation 2 is worse than the AF relay situation, that is because the non-aligned of the UCAs. And, we can find that, with the degree increase of angle offset, the system performance is getting worse without the AF relay.

## 5. Conclusion

In this paper, an OAM MIMO AF relay system is investigated to maximize the information rate by jointly optimizing the source power allocation matrix and the relay pre-coding matrix, constrained by the power of the source and the relay. The problem is expressed as a convex problem by mathematical

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operations, and then solved by an iterative algorithm based on the Lagrange dual method. The simulation results demonstrate the superiority of our proposed scheme.

## 6. References

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