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Fermion mass and mixing in the $U(1)_{B-L}$ extension of the standard model with D_4 symmetry

V V Vien

Theoretical Particle Physics and Cosmology Research Group, Advanced Institute of Materials Science, Ton Duc Thang University, Ho Chi Minh City, Vietnam
Faculty of Applied Sciences, Ton Duc Thang University, Ho Chi Minh City, Vietnam

E-mail: vovanvien@tdtu.edu.vn

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Abstract

We propose a renormalizable $B-L$ standard model extension based on D_4 symmetry which accommodates fermion mass and mixing parameters with CP violation. Both normal and inverted neutrino mass ordering as well as the smallness of the active neutrino masses are generated through a type I seesaw mechanism. The obtained physical parameters are well consistent with the global fit of neutrino oscillation in Esteban *et al* (2019 *J. High Energy Phys.* JHEP01(2019)106) while the quark masses are in good agreement with the recent experimental data (Tanabashi *et al* (Particle Data Group) 2018 *Phys. Rev.* D **98** 030001 and 2019 update). The model also predicts an effective neutrino mass parameter of $\langle m_{ee}\rangle = 1.24 \times 10^{-2}$ eV for normal hierarchy and $\langle m_{ee}\rangle = 4.88 \times 10^{-2}$ eV for inverted hierarchy which are all consistent with the recent experimental limits on neutrinoless double beta decay.

Keywords: quark and lepton mass and mixing, extensions of electroweak Higgs sector, non-standard-model neutrinos, right-handed neutrinos, discrete symmetries

(Some figures may appear in colour only in the online journal)

1. Introduction

In recent years, there have been a vast number of proposals that could provide the explanation of the smallness of neutrino masses, for instance, seesaw mechanisms [1–6], the neutrino

Table 1. Neutrino oscillation parameters extracted from the global analysis, taken from [59]. Here, $l = 1$ for NH and $l = 2$ for IH.

Parameter		Δm^2_{21} $(10^{-5}\,\mathrm{eV}^2)$	Δm^2_{3l} $(10^{-3}\,\mathrm{eV}^2)$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δ (°)
Best fit	NH	7.39	2.525	0.310	0.582	0.02240	217
	IH	7.39	−2.512	0.310	0.582	0.02263	280

minimal SM [7–11], Two-Higgs-doublet model [12], the scotogenic model[3] [18], the 3-3-1 models [19–24] and so on. However, the fermion mixings was not explicitly explained in these extensions.

Among the possible extensions of the SM, the extension with an extra $U(1)_{B-L}$ (baryon number minus lepton number) gauge symmetry is one of promising extensions of the SM which has been considered in [25–46] in which the anomalies can be canceled in different ways. In this work, we develop the model proposed by S. Khalil *et al*, where three right-handed neutrinos, two $SU(2)_L$ doublets (H', H'') and two $SU(2)_L$ singlets (ϕ, φ) are introduced in addition to the SM particle content. In this type of model the presence of three right-handed neutrinos (ν_{iR}) is essential to cancel anomalies [31–36], the masses of new boson Z' and three right-handed neutrinos are generated when the $B-L$ gauge symmetry is broken [47, 48], and other phenomena including leptogenesis [30, 49, 50], dark matter [51–56, 33], the muon anomalous magnetic moment [32], gravitational wave radiation [31], inflation [57], etc, are explained, however, the model by itself does not provide a natural explanation for fermion mixings. The implications of the $B-L$ extension to the SM do not change the decay branching ratios and the extra Higgs has relatively small cross sections, but it is accessible at LHC and the searching for Z' is accessible via a clean dilepton signal at LHC [33]. Moreover, the richer TeV phenomenology for the coming LHC and neutrino physics opens new prospects [58].

Experimentally, the best-fit values for neutrino mass squared splittings, leptonic mixing angles and the Dirac CP violating phase, for both mass hierarchies with Super-Kamiokande atmospheric neutrino data (wSK-atm), are given in [59] as shown in table 1.

The magnitude of the elements of the leptonic mixing matrix obtained from global fit results, at the 3σ CL ranges, are given in [59]:

$$|U^{3\sigma}_{\text{wSK-atm}}| = \begin{pmatrix} 0.797 \to 0.842 & 0.518 \to 0.585 & 0.143 \to 0.156 \\ 0.235 \to 0.484 & 0.458 \to 0.671 & 0.647 \to 0.781 \\ 0.304 \to 0.531 & 0.497 \to 0.699 & 0.607 \to 0.747 \end{pmatrix}. \tag{1}$$

Besides that, the best-fit results for the magnitudes of all nine CKM elements as well as quark masses have now been determined with high accuracy [60]

$$|V^{\text{exp}}_{\text{CKM}}| = \begin{pmatrix} 0.97446 & 0.22452 & 0.00365 \\ 0.22438 & 0.97359 & 0.04214 \\ 0.00896 & 0.04133 & 0.99911 \end{pmatrix}, \tag{2}$$

$$\begin{aligned} m_u &= 2.16\ \text{MeV}, \quad m_c = 1.27\ \text{GeV}, \quad m_t = 172.9\ \text{GeV}, \\ m_d &= 4.67\ \text{MeV}, \quad m_s = 93\ \text{MeV}, \quad\ \ m_b = 4.18\ \text{GeV}, \end{aligned} \tag{3}$$

[3] Depending on the particle content, there exist models which generate an active neutrino mass at one-loop [13], two-loop [14, 15], or three-loop [16, 17] level.

where m_u, m_d are very small compared to other quark masses and the quark mixing angles are very small compared to the lepton mixing ones.

The difference of the lepton mixing angles given in equation (1) and quark mixing angles in equation (2) has stimulated works on discrete symmetries which has shown many outstanding advantages in explaining the observed pattern of SM fermionic masses and mixing angles. There have been many models based on discrete symmetries, see for instance, A_4 [61–75], S_3 [76–82], S_4 [83–103], T' [104–108], etc. However, in these papers, the fermion masses and mixings are generated from non-renormalizable interactions or at loop level. Some extensions of the SM based on D_4 symmetry generate mass spectra and mixing parameters for quarks and/or leptons were presented in [109–111] in which only fermion masses and maximal mixing of the atmospheric neutrino and/or the vanishing θ_{13} was predicted which was ruled out by the recent experimental data [60]. This problem has been improved in our previous work, based on 3-3-1 model, by adding a new triplet ρ put in $\underline{1}''$ under D_4 regarded as a small perturbation [112]. In this work, we build a $B-L$ SM extension, where the D_4 discrete symmetry is supplemented by the Z_4 discrete group, providing a framework capable of reproducing the SM fermion masses and mixings. Three right-handed neutrinos (ν_{iR}), two $SU(2)_L$ doublets H', H'' with $B-L=0$ respectively put in $\underline{1}'$ and $\underline{2}$ under D_4 and two flavons φ, ϕ with $B-L=2$ respectively put in $\underline{1}$ and $\underline{2}$ under D_4 are introduced. We note that D_4 symmetry has not been considered before in this kind of the model[4]. This model is completely different from our previous works [112–114] because the 3-3-1 model itself is an extension of the SM.

In this work, the first generations are put in $\underline{1}$ while the two others are put in $\underline{2}$ under D_4 symmetry. D_4 is the symmetry group of a square. It has four singlets $\underline{1}$, $\underline{1}'$, $\underline{1}''$ and $\underline{1}'''$ and one doublet $\underline{2}$ [115]. For convention, we present briefly the Clebsch–Gordan coefficients of D_4 in appendix A. This paper is organized as follows. In section 2 we present a simple SM extension by adding $U(1)_{B-L}$ and D_4 symmetries. In section 3 we present the lepton sector of the model and introduce necessary Higgs fields responsible masses for the leptons. Section 4 deals with quark masses and mixings. We make conclusions in section 5.

2. The model

In the model under consideration, the electroweak sector of the SM is supplemented by a gauge symmetry $\mathrm{U}(1)_{B-L}$ and a D_4 discrete symmetry. In addition to the SM model particle content, three right-handed neutrinos (ν_{iR}), two $SU(2)_L$ doublets H', H'' with $B-L=0$, respectively, put in $\underline{1}'$ and $\underline{2}$ under D_4 and two flavons φ, ϕ with $B-L=2$ respectively put in $\underline{1}$ and $\underline{2}$ under D_4 are introduced. The particle content of the model is given in tables 2 and 3. In general, the lepton mixing matrix is given by $U=U^+_{eL}U_\nu$ where U_{eL} refers to the left-handed charged-lepton mixing matrix and U_ν is the neutrino mixing matrix. In the basis where the charged lepton mass is diagonal ($U_{eL}=1$), the lepton mixing matrix is that of the neutrinos, $U\equiv U_\nu$. With the fermion content in tables 2 and 3 and the tensor products of D_4 group in appendix A, the charged lepton masses can arise from the couplings of $\bar{\psi}_{(1,\alpha)L}l_{(1,\alpha)R}$ to scalars, where under $SU(3)_c\times SU(2)_L\times U(1)_Y\times U(1)_{B-L}\times D_4$ symmetry, $\bar{\psi}_{1L}l_{1R}$ transforms as $(\mathbf{1},\mathbf{2},-1/2,0,\underline{1})$ which implies that in order to generate the charged-lepton mass matrix, one needs one D_4 singlet transforming as $(\mathbf{1},\mathbf{2},1/2,0,\underline{1})$ to build an invariant under all given symmetries. Similarly, we have $\bar{\psi}_{1L}l_{\alpha R}\sim(\mathbf{1},\mathbf{2},1/2,0,\underline{2})$, $\bar{\psi}_{\alpha L}l_{1R}\sim(\mathbf{1},\mathbf{2},1/2,0,\underline{2})$ and $\bar{\psi}_{\alpha L}l_{\alpha R}\sim(\mathbf{1},\mathbf{2},1/2,0,\underline{1}+\underline{1}'+\underline{1}''+\underline{1}''')$. Therefore, to generate masses for the charged leptons, we need two $SU(2)_L$ scalar

[4] In this scenario, fermion masses and mixing angles are generated from renormalizable Yukawa interactions.

Table 2. The fermion content of the model and the charge assignment. Here $\alpha = 2, 3$.

	$\mathrm{SU(3)}_c$	$\mathrm{SU(2)}_L$	$\mathrm{U(1)}_Y$	$\mathrm{U(1)}_{B-L}$	D_4	Z_4
ψ_{1L}	**1**	**2**	$-1/2$	-1	$\underline{1}$	i
$\psi_{\alpha L}$	**1**	**2**	$-1/2$	-1	$\underline{2}$	i
l_{1R}	**1**	**1**	-1	-1	$\underline{1}$	i
$l_{\alpha R}$	**1**	**1**	-1	-1	$\underline{2}$	i
ν_{1R}	**1**	**1**	0	-1	$\underline{1}$	i
$\nu_{\alpha R}$	**1**	**1**	0	-1	$\underline{2}$	i
Q_{1L}	**3**	**2**	$1/6$	$1/3$	$\underline{1}$	i
$Q_{\alpha L}$	**3**	**2**	$1/6$	$1/3$	$\underline{2}$	$-i$
u_{1R}	**3**	**1**	$2/3$	$1/3$	$\underline{1}$	i
$u_{\alpha R}$	**3**	**1**	$2/3$	$1/3$	$\underline{2}$	$-i$
d_{1R}	**3**	**1**	$-1/3$	$1/3$	$\underline{1}$	i
$d_{\alpha R}$	**3**	**1**	$-1/3$	$1/3$	$\underline{2}$	$-i$

Table 3. Scalar content of the model and the charge assignment.

	$\mathrm{SU(3)}_c$	$\mathrm{SU(2)}_L$	$\mathrm{U(1)}_Y$	$\mathrm{U(1)}_{B-L}$	D_4	Z_4
H	**1**	**2**	$1/2$	0	$\underline{1}$	1
H'	**1**	**2**	$1/2$	0	$\underline{1}'$	1
H''	**1**	**2**	$1/2$	0	$\underline{2}$	-1
φ	**1**	**1**	0	2	$\underline{1}$	-1
ϕ	**1**	**1**	0	2	$\underline{2}$	-1

fields H and H' as given in table 3. For the known scalar doublets, available interactions are $(\bar{\psi}_{1L}\widetilde{H})_{\underline{1}}\nu_{1R}$, $(\bar{\psi}_{\alpha L}\widetilde{H})_{\underline{2}}\nu_{\alpha R}$ and $(\bar{\psi}_{\alpha L}\widetilde{H}')_{\underline{2}}\nu_{\alpha R}$ however they only generate the Dirac mass term. In general, neutrinos can have not only Dirac mass term but also Majorana mass term. To generate Majorana mass term for neutrinos we will therefore introduce two new $SU(2)_L$ singlets φ, ϕ with $B-L=2$ respectively put in $\underline{1}$ and $\underline{2}$ under D_4, instead coupling to $\bar{\psi}_{1L}\nu_{1L}$ and $\bar{\psi}_{\alpha L}\nu_{\alpha L}$, responsible for the realistic neutrino masses and mixings.

In the quarks sector, with two $SU(2)_L$ scalar fields H and H', available interactions are $(\bar{Q}_{\beta L}\tilde{H})_{\underline{2}}u_{\beta R}$, $(\bar{Q}_{\beta L}\tilde{H}')_{\underline{2}}u_{\beta R}$, $(\bar{Q}_{3L}\tilde{H})_{\underline{1}''}u_{3R}$, $(\bar{Q}_{\beta L}H)_{\underline{2}}d_{\beta R}$, $(\bar{Q}_{\beta L}H')_{\underline{2}}d_{\beta R}$ and $(\bar{Q}_{3L}H)_{\underline{1}''}d_{3R}$ which can generate masses for the up and down quarks. In this case, the matrices which couple the left-handed quarks u_L and d_L to those in the mass bases are$U_{uL} = U_{dL} = 1$, and the CKM quark mixing matrix is equal to the identity matrix. However, as we know, the quark mixing matrix is given in equation (2) which is a little different from the unit matrix, and it adheres to the following approximate pattern[5]:

$$|V_{\mathrm{CKM}}| \sim \begin{pmatrix} 0.97446 & 0.224\,52 & 0 \\ 0.22438 & 0.973\,59 & 0 \\ 0 & 0 & 0.999\,11 \end{pmatrix}. \tag{4}$$

To obtain the quark mixing form in equation (4) we propose an additional $SU(2)_L$ doublet H'' put in $\underline{1}''$ under D_4 coupling to $\bar{q}_{\beta L}u_{\beta R}$ and $\bar{q}_{\beta L}d_{\beta R}$, respectively, contributing to the elements (12) and (21) of the up and down quark mass matrices M_u and M_d. The Yukawa interactions

[5] This is a good approximation for the realistic quark mixing matrix, which implies that the mixings among the quarks are small.

of the model are[6]:

$$\begin{aligned}
-\mathcal{L}_Y &= h_1\bar{\psi}_{1L}Hl_{1R} + h_2(\bar{\psi}_{\alpha L}H)_{\underline{2}}l_{\alpha R} + h_3(\bar{\psi}_{\alpha L}H^{'})_{\underline{2}}l_{\alpha R} \\
&+\frac{x_1}{2}(\bar{\psi}_{1L}\tilde{H})_{\underline{1}}\nu_{1R} + \frac{x_2}{2}(\bar{\psi}_{\alpha L}\tilde{H})_{\underline{2}}\nu_{\alpha R} + \frac{x_3}{2}(\bar{\psi}_{\alpha L}\tilde{H}^{'})_{\underline{2}}\nu_{\alpha R} \\
&+\frac{y_1}{2}(\bar{\nu}^c_{1R}\nu_{1R})_{\underline{1}}\varphi + \frac{y_2}{2}(\bar{\nu}^c_{\alpha R}\nu_{\alpha R})_{\underline{1}}\varphi + \frac{y_3}{2}(\bar{\nu}^c_{\alpha R}\nu_{1R} + \bar{\nu}^c_{1R}\nu_{\alpha R})_{\underline{2}}\phi \\
&+h_{1u}(\bar{Q}_{1L}\tilde{H})_{\underline{1}}u_{1R} + h_{2u}(\bar{Q}_{\alpha L}\tilde{H})_{\underline{2}}u_{\alpha R} + h_{3u}(\bar{Q}_{\alpha L}\widetilde{H^{'}})_{\underline{2}}u_{\alpha R} \\
&+h_{4u}(\bar{Q}_{\alpha L}u_{1R} + \bar{Q}_{1L}u_{\alpha R})_{\underline{2}}\widetilde{H^{''}} + h_{1d}(\bar{Q}_{1L}H)_{\underline{1}}d_{1R} + h_{2d}(\bar{Q}_{\alpha L}H)_{\underline{2}}d_{\alpha R} \\
&+h_{3d}(\bar{Q}_{\alpha L}H^{'})_{\underline{2}}d_{\alpha R} + h_{4d}(\bar{Q}_{\alpha L}d_{1R} + \bar{Q}_{1L}d_{\alpha R})_{\underline{2}}H^{''} + \text{h.c.}
\end{aligned} \quad . \tag{5}$$

It is noted that the following interactions, $\bar{\psi}_{iL}H''l_{jR}$ and $\bar{\psi}_{iL}\bar{H}''\nu_{jR}$ $(i, j = 1, 2, 3)$, are forbidden because of, at least, the Z_4 symmetry violation thus they are not included in equation (5).

Since there are many Yukawa couplings in Higgs potential and it is easy to arrange a suitable Higgs potential [112, 116]. In order to generate the remarkable fermion mixing pattern, from the potential minimization conditions (see appendix B1), we choose the VEVs of scalar fields as follows:

$$\begin{aligned}
&\langle H\rangle = (0 \quad v_H)^T, \quad \langle H'\rangle = (0 \quad v_{H'})^T, \quad \langle H''\rangle = (\langle H''_1\rangle, \quad 0), \\
&\langle H''_1\rangle = (0 \quad v_{H''})^T, \quad \langle\varphi\rangle = v_\varphi, \quad \langle\phi\rangle = (\langle\phi_1\rangle, \quad \langle\phi_1\rangle), \quad \langle\phi_1\rangle = v_\phi.
\end{aligned} \tag{6}$$

We note that, the VEVs of H and φ conserve D_4 symmetry while that of H' breaks this symmetry down to Z_4 which consists of four elements $\{e, a, a^2, a^3\}$ where a is the $\frac{\pi}{2}$ rotation [115]. The VEV of H'' breaks D_4 symmetry down to Z_2 which consists of two elements $\{e, b\}$ while the VEV of ϕ breaks this symmetry down to Z_2 which consists of two elements $\{e, a^3b\}$ where b is the reflection [115]. Furthermore, $SU(2)_L$ singlet φ breaks the $U(1)_{B-L}$ symmetry and $SU(2)_L$ doublets H breaks the $SU(2)_L \times U(1)_Y$ symmetry down to $U(1)_{em}$ [32–35].

After symmetry breaking, Dirac and right-handed neutrinos get masses, where Dirac neutrino mass $M_D \sim v_H, v_{H'}$ and right-handed neutrino mass $M_R \sim v_\phi, v_\varphi$, thus, $B-L$ gauge symmetry can provide a natural framework for the seesaw mechanism. The scale of $B-L$ symmetry breaking was discussed in detail in [32–35].

3. Lepton masses and mixings

From the lepton Yukawa terms given by equation (5), we rewrite the Yukawa interactions in the charged lepton sector:

$$\begin{aligned}
-\mathcal{L}_{\text{clep}} &= h_1\bar{\psi}_{1L}Hl_{1R} + h_2(\bar{\psi}_{2L}Hl_{2R} + \bar{\psi}_{3L}Hl_{3R}) \\
&+h_3(\bar{\psi}_{2L}H^{'}l_{2R} - \bar{\psi}_{3L}H^{'}l_{3R}) + \text{h.c.}
\end{aligned} \quad . \tag{7}$$

In the charged-lepton sector, D_4 is spontaneously broken down to Z_4 by the VEV of H', there exist no lepton flavor changing interactions thus the lepton flavor changing processes are

[6] $\widetilde{H}, \widetilde{H'}$ and $\widetilde{H''}$ are, respectively, the complex conjugate fields of H, H' and H'', i.e. $\widetilde{H} = i\sigma_2 H^* = (H_2^{0*} \; -H_1^-)^T \sim [1, 2, -1/2, 0, \underline{1}, 1]$, $\widetilde{H'} \sim [1, 2, -1/2, 0, \underline{1}', 1]$ and $\widetilde{H''} \sim [1, 2, -1/2, 0, \underline{2}, -1]$.

suppressed. With the help of (6), the Lagrangian mass term of the charged leptons can be written in the form:

$$-\mathcal{L}^{\text{mass}}_{\text{clep}} = (\bar{l}_{1L}, \bar{l}_{2L}, \bar{l}_{3L})M_l(l_{1R}, l_{2R}, l_{3R})^T + \text{h.c.}, \tag{8}$$

where

$$M_l = \text{diag}(h_1 v_H,\ h_2 v_H + h_3 v_{H'},\ h_2 v_H - h_3 v_{H'}) \equiv \text{diag}(m_e,\ m_\mu\ m_\tau), \tag{9}$$

which has the diagonal form. Thus, the diagonalization matrices are therefore $U_{lL} = U_{lR} = 1$ and the lepton mixing matrix depends on only that of the neutrinos. The masses of muon and tau are explicitly separated by scalar H' resulting from the breaking $D_4 \to Z_4$ and this is why we introduce H' in accompanying with H.

Next, combining expressions of $m_{e,\mu,\tau}$ in equation (9) with the experimental values for masses of the charged leptons at the weak scale given in [60]: $m_e \simeq 0.510\,99$ MeV, $m_\mu \simeq 105.658\,37$ MeV, $m_\tau \simeq 1776.86$ MeV, we get[7] $|h_1| \sim 5.1 \times 10^{-6}$, $|h_2| \sim 9.4 \times 10^{-3}$, $|h_3| \sim 8.4 \times 10^{-3}$, i.e. the hierarchy of Yukawa couplings are required to obtain the hierarchy of charged-lepton masses ($|h_1| \ll |h_2| \simeq |h_3|$).

The neutrino masses arise from the couplings of $\bar{\psi}_{iL}\nu_{jR}$ and $\bar{\nu}^c_{iR}\nu_{jR}$ $(i, j = 1, 2, 3)$ to scalars, where $\bar{\psi}_{1L}\nu_{1R}$, $\bar{\psi}_{1L}\nu_{\alpha R}$ and $\bar{\psi}_{\alpha L}\nu_{\alpha R}$ $(\alpha = 2, 3)$ transform as $SU(2)_L$ doublets and $\underline{1}$, $\underline{2}$ and $(\underline{1}, \underline{1}', \underline{1}'', \underline{1}''')$ under D_4 symmetry, respectively, while $\bar{\nu}^c_{1R}\nu_{1R}$, $\bar{\nu}^c_{1R}\nu_{\alpha R}$ and $\bar{\nu}^c_{\alpha R}\nu_{\alpha R}$ transform as $SU(2)_L$ singlets and $\underline{1}$, $\underline{2}$ and $(\underline{1}, \underline{1}', \underline{1}'', \underline{1}''')$ under D_4, respectively. For the $SU(2)_L$ scalar doublets H and H', the following interactions $\bar{\psi}_{iL}\widetilde{H}\nu_{jR}$ and $\bar{\psi}_{iL}\widetilde{H'}\nu_{jR}$ are available, however, these terms only contribute to the Dirac mass matrix M_D. To generate the Majorana mass terms, we propose two new $SU(2)_L$ singlets (φ, ϕ), respectively, transform as $\underline{1}$ and $\underline{2}$ under D_4 as given in table 3.

From equation (5), the Yukawa Lagrangian in neutrino sector reads:

$$\begin{aligned} -\mathcal{L}_\nu \ =\ & \frac{x_1}{2}\bar{\psi}_{1L}\tilde{H}\nu_{1R} + \frac{x_2}{2}(\bar{\psi}_{2L}\tilde{H}\nu_{2R} + \bar{\psi}_{3L}\tilde{H}\nu_{3R}) \\ & + \frac{x_3}{2}(\bar{\psi}_{2L}\tilde{H}'\nu_{2R} - \bar{\psi}_{3L}\tilde{H}'\nu_{3R}) + \frac{y_1}{2}\bar{\nu}^c_{1R}\varphi\nu_{1R} + \frac{y_2}{2}(\bar{\nu}^c_{2R}\nu_{2R} + \bar{\nu}^c_{3R}\nu_{3R})\varphi \\ & + \frac{y_3}{2}[(\bar{\nu}^c_{2R}\nu_{1R} + \bar{\nu}^c_{1R}\nu_{2R})\phi_1 + (\bar{\nu}^c_{3R}\nu_{1R} + \bar{\nu}^c_{1R}\nu_{3R})\phi_2] + \text{h.c.} \end{aligned} \tag{10}$$

With the VEVs given in equation (6), i.e. D_4 is broken to Z_2, we get the Dirac neutrino mass matrix (M_D) and the right-handed Majorana neutrino mass matrix (M_R) as follows

$$M_D = \text{diag}(a_D, b_D - c_D, b_D + c_D), \quad M_R = \begin{pmatrix} a_R & c_R & c_R \\ c_R & b_R & 0 \\ c_R & 0 & b_R \end{pmatrix}, \tag{11}$$

where

$$\begin{aligned} & a_D = x_1 v_H, \quad b_D = x_2 v_H, \quad c_D = x_3 v_{H'}, \\ & a_R = y_1 v_\varphi, \quad b_R = y_2 v_\varphi, \quad c_R = y_3 v_\phi. \end{aligned} \tag{12}$$

We introduced the additional symmetries $U(1)_{B-L} \times Z_4 \times D_4$ to prevent some Yukawa interactions thus giving rise to the predictive textures for the lepton and quark sectors. For instance, under $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \times D_4$ symmetry, the coupling $\bar{\psi}_{\alpha L} l_{\alpha R}$

[7] We use $v_H \sim v_{H'} \sim 100$ GeV for their scale.

transform as $(\mathbf{1}, \mathbf{2}, 1/2, 0, \underline{1} + \underline{1}' + \underline{1}'' + \underline{1}''')$. For the known scalars $H, H', H'', \varphi, \phi$, $(\overline{\psi}_{\alpha L} l_{\alpha R})H''$ is forbidden by the D_4 and Z_4 symmetries, $(\overline{\psi}_{\alpha L} l_{\alpha R})\varphi$ is prevented by the $B-L$, Z_4 and $SU(2)_L$ symmetries, and $(\overline{\psi}_{\alpha L} l_{\alpha R})\phi$ is prevented by the $B-L$, D_4, Z_4 and $SU(2)_L$ symmetries. Consequently, in the charged-lepton sector, there are only three terms invariant under all symmetries $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \times D_4$, which corresponds to $(\bar{\psi}_{1L} l_{1R})H$, $(\bar{\psi}_{\alpha L} l_{\alpha R})_{\underline{1}} H$ and $(\bar{\psi}_{\alpha L} l_{\alpha R})_{\underline{1}'} H'$ that provide a simple form of M_l as indicated by equation (9). The situation is similar for the remaining couplings that generate the desired mass matrices given in equations (11) and (40).

The effective neutrino mass matrix obtained through the type-I seesaw mechanism as follows:

$$M_{\rm eff} = -M_D^T M_R^{-1} M_D = \begin{pmatrix} A_0 & B_0 & B_0 \\ B_0 & C_0 - D_0 & D_0 \\ B_0 & D_0 & C_0 - D_0 \end{pmatrix} + \begin{pmatrix} 0 & a_1 & -a_1 \\ a_1 & b_1 + b_2 & c_1 \\ -a_1 & c_1 & -b_1 + b_2 \end{pmatrix}, \tag{13}$$

where

$$\begin{aligned} A_0 &= \frac{a_D^2 b_R}{-a_R b_R + 2c_R^2}, \quad B_0 = \frac{a_D b_D c_R}{a_R b_R - 2c_R^2}, \\ C_0 &= \frac{a_R b_D^2}{-a_R b_R + 2c_R^2}, \quad D_0 = \frac{b_D^2 c_R^2}{-b_R(a_R b_R - 2c_R^2)}, \end{aligned} \tag{14}$$

$$\begin{aligned} a_1 &= \frac{a_D c_D c_R}{-a_R b_R + 2c_R^2}, \quad b_1 = \frac{2b_D c_D(a_R b_R - c_R^2)}{b_R(a_R b_R - 2c_R^2)}, \\ b_2 &= \frac{c_D^2(-a_R b_R + c_R^2)}{b_R(a_R b_R - 2c_R^2)}, \ c_1 = \frac{c_D^2 c_R^2}{a_R b_R^2 - 2b_R c_R^2}. \end{aligned} \tag{15}$$

We note that A_0, B_0, C_0 and D_0 given in equation (14) accommodated in the first matrix of equation (13) due to the contribution of one $SU(2)_L$ scalar doublet (H) and two $SU(2)_L$ scalar singlets φ, ϕ. The last matrix in equation (13) is deviation from the contribution of the $SU(2)_L$ scalar doublet H' only. If there is no contribution of H', the deviations a_1, $b_{1,2}$, c_1 will vanish and the mass matrix $M_{\rm eff}$ in (13) reduces to its first term which generates a nearly tri-bimaximal mixing. Thus, second term in (13), which is proportional to the contribution of H', will take the role for a small deviation of θ_{13} and being responsible for the CP violating phase in the lepton sector.

The first matrix in equation (13) has three eigenvalues,

$$m_{1,2} = \frac{1}{2}\left(A_0 + C_0 \pm \sqrt{8B_0^2 + (A_0 - C_0)^2}\right), \ m_3 = C_0 - 2D_0, \tag{16}$$

and the corresponding lepton mixing matrix takes the form:

$$U_0 = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\frac{\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \tag{17}$$

where

$$\theta = \arccos\left(\frac{\sqrt{2}}{\sqrt{K^2 + 2}}\right), \ K = \frac{2B_0}{C_0 - m_1} = \frac{2B_0}{m_2 - A_0}, \tag{18}$$

with A_0, B_0, C_0 and $m_{1,2}$ are defined in equations (14) and (16), respectively.

The matrix U_0 in equation (17) implies $\theta_{13}=0$, $\theta_{23}=\pi/4$ and $\theta_{12}=\theta$ which was rule out by the recent data[8], however, the contribution of the second term in equation (13) will improve this situation.

At the first order of perturbation theory, the second matrix in equation (13) contributes to both eigenvalues and eigenvectors. In this case, the neutrino masses are given as:

$$\lambda_1 = m_1 + (b_2 + c_1)\sin^2\theta,\quad \lambda_2 = m_2 + (b_2 + c_1)\cos^2\theta,\quad \lambda_3 = m_3 + b_2 - c_1, \tag{19}$$

with b_2, c_1 and $m_{1,2,3}$ are given in equations (15) and (16), respectively. The corresponding lepton mixing matrix becomes:

$$U = U_0 + \delta U, \tag{20}$$

where U_0 is defined by (17), and δU has the following entries:

$$\begin{aligned}
(\delta U)_{11} &= \frac{(b_2 + c_1)\sin^2\theta\cos\theta}{m_2 - m_1},\quad (\delta U)_{12} = \frac{(b_2 + c_1)\sin\theta\cos^2\theta}{m_1 - m_2},\\
(\delta U)_{13} &= \frac{(b_1\cos\theta + \sqrt{2}a_1\sin\theta)\sin\theta}{m_3 - m_2} + \frac{(-\sqrt{2}a_1\cos\theta + b_1\sin\theta)\cos\theta}{m_1 - m_3},\\
(\delta U)_{21} &= \frac{1}{\sqrt{2}}\left(\frac{(b_2 + c_1)\cos^2\theta\sin\theta}{m_2 - m_1} + \frac{\sqrt{2}a_1\cos\theta - b_1\sin\theta}{m_1 - m_3}\right),\\
(\delta U)_{22} &= \frac{1}{\sqrt{2}}\left(\frac{(b_2 + c_1)\cos\theta\sin^2\theta}{m_2 - m_1} + \frac{\sqrt{2}a_1\sin\theta + b_1\cos\theta}{m_2 - m_3}\right),\\
(\delta U)_{23} &= \frac{1}{\sqrt{2}}\left(\frac{(\sqrt{2}a_1\sin\theta + b_1\cos\theta)\cos\theta}{m_3 - m_2} + \frac{(\sqrt{2}a_1\cos\theta - b_1\sin\theta)\sin\theta}{m_1 - m_3}\right),\\
(\delta U)_{31} &= \frac{1}{\sqrt{2}}\left(\frac{(b_2 + c_1)\cos^2\theta\sin\theta}{m_2 - m_1} + \frac{-\sqrt{2}a_1\cos\theta + b_1\sin\theta}{m_1 - m_3}\right),\\
(\delta U)_{32} &= \frac{1}{\sqrt{2}}\left(\frac{(b_2 + c_1)\cos\theta\sin^2\theta}{m_2 - m_1} + \frac{\sqrt{2}a_1\sin\theta + b_1\cos\theta}{m_3 - m_2}\right),\\
(\delta U)_{33} &= (\delta U)_{23}.
\end{aligned} \tag{21}$$

In the three-neutrino framework, the lepton mixing matrix (U_{PMNS}) can be parameterized as [60]

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\mathrm{e}^{-\mathrm{i}\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & c_{23}c_{13} \end{pmatrix} \times P, \tag{22}$$

where δ is the Dirac CP violating phase and $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ with θ_{12}, θ_{23} and θ_{13} being the solar angle, atmospheric angle and the reactor angle respectively. P contains two Majorana phases (α_{21}, α_{31}) which play no role in neutrino oscillations, $P = \mathrm{diag}(1, \mathrm{e}^{\mathrm{i}\frac{\alpha_{21}}{2}}, \mathrm{e}^{\mathrm{i}\frac{\alpha_{31}}{2}})$.

From equations (20)–(22) the lepton mixing angles can be defined via the elements of the neutrino mixing matrix:

[8] In the case $\cos\theta = \sqrt{2/3}$ ($\theta \simeq 35.26^\circ$), U_0 becomes an exact tribimaximal form [117–120].

$$t_{12}=\frac{|U_{12}|}{|U_{11}|},\quad t_{23}=\frac{|U_{23}|}{|U_{33}|},\quad s_{13}=U_{13}e^{i\delta}, \tag{23}$$

By combining equations (20), (21) and (23) we get:

$$a_1=\frac{e^{-i\delta}\left[(m_1-m_3)(m_3-m_2)s_{13}+\frac{(m_1-m_2)\cos\theta\sin\theta e^{i\delta}}{1-t_{23}^2}(\alpha-\sqrt{\beta})\right]}{\sqrt{2}\,[m_1-m_3+(m_2-m_1)\cos^2\theta]},$$
$$b_1=\frac{\alpha-\sqrt{\beta}}{t_{23}^2-1},\qquad c_1=-b_2-\frac{(m_1-m_2)(\sin\theta-t_{12}\cos\theta)}{\cos\theta[t_{12}+\cos\theta(\sin\theta-t_{12}\cos\theta)]}, \tag{24}$$

where

$$\alpha=-(m_1\sin^2\theta+m_2\cos^2\theta-m_3)(1+t_{23}^2)+(m_1-m_2)\sin\theta\cos\theta s_{13}(t_{23}^2-1)\cos\delta,$$
$$\beta=2[2(m_3-m_1\sin^2\theta-m_2\cos^2\theta)^2+(m_1-m_2)^2\cos^2\theta\sin^2\theta s_{13}^2]t_{23}^2$$
$$-(m_1-m_2)^2\cos^2\theta\sin^2\theta s_{13}^2(1+t_{23}^4-(t_{23}^2-1)^2\cos^2\delta. \tag{25}$$

We found that two elements U_{11} and U_{12} depend only on two parameters θ, θ_{12}, namely $U_{11}=1/(\cos\theta+t_{12}\sin\theta)$, $U_{12}=t_{12}/(\cos\theta+t_{12}\sin\theta)$ with $t_{12}=0.670$ is the best-fit value from the global analysis given in [59]. Furthermore, at 3σ confidence level [59], $U_{11}\in(0.797, 0.842)$. Thus we get $\cos\theta=0.831$ ($\theta=33.8°$) and $U_{12}=0.557$.

Although the global analysis in [121] shows a hint in favor of the NH over the inverted one at more than 3σ and the global analysis in [59] based on LBL+ reactor data obtain a preference for NH at about 2σ, however, the neutrino mass spectrum is currently unknown and it can be NH ($m_1<m_2<m_3$) or IH ($m_3<m_1<m_2$) depending on the sign of Δm_{31}^2 (Δm_{32}^2) [60]. In this work, two types of the neutrino mass spectrum can be found in which the model parameters are in good agreement with the global analysis in [59] for both normal and inverted hierarchy (IH).

3.1. Normal spectrum

By taking the best-fit values for neutrino mass squared splittings, leptonic mixing angles and the Dirac CP violating phase for NH with Super-Kamiokande atmospheric neutrino data as given in table 1, we get a solution[9]:

$$m_1=-1.5\times10^{-6}m_2+1.74\sqrt{\gamma_1},$$
$$b_2=7.5\times10^{-7}m_2-1.3\times10^{-6}\sqrt{\gamma_1}+1.84\sqrt{\gamma_2}-0.5m_3, \tag{26}$$

with

$$\gamma_1=-2.45\times10^{-5}+0.331m_2^2,$$
$$\gamma_2=1.81\times10^{-4}+7.4\times10^{-2}m_2^2-2.66\times10^{-7}m_2\sqrt{\gamma_1}. \tag{27}$$

The sum of neutrino masses in NH, $\sum_N=\sum m_\nu=\sum_{i=1}^3\lambda_i^N$, is defined as:

$$\sum\nolimits_N=m_2+1.74\sqrt{\gamma_1}+3.68\gamma_2, \tag{28}$$

[9] There are four solutions for m_2, b_2 however they give the same value for Δm_{ij}^2 and the same absolute values of $\lambda_{1,2,3}$ thus we only consider in detail this solution.

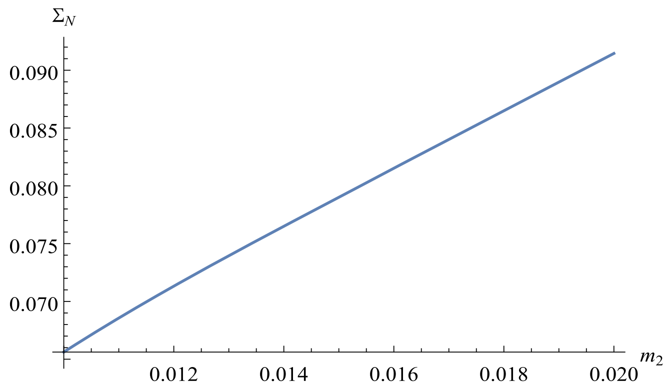

Figure 1. $\sum_N$ as a function of m_2 with $m_2 \in (0.01, 0.02)$ eV in the NH.

which is depicted in figure 1 with[10] $m_2 \in (0.01, 0.02)$ eV. In the case $m_2 = 0.014$ eV, three light neutrino masses are explicitly given as:

$$\lambda_1^N = 1.1 \times 10^{-2}, \text{eV}, \ \lambda_2^N = 1.4 \times 10^{-2}, \text{eV}, \ \lambda_3^N = 5.14 \times 10^{-2}\,\text{eV}, \tag{29}$$

and the sum of three neutrino masses is found to be $\sum_N = 7.65 \times 10^{-2}$ eV. Let us note that at present there exist various different bounds for $\sum m_\nu$. For example, it has the upper limits [122] $\sum m_\nu < 0.152$ eV at 95% C.L in the minimal $\Lambda\text{CDM} + \sum m_\nu$ model, $\sum m_\nu < 0.118$ eV by adding the high-l polarization data [122], $\sum m_\nu < 0.101$ eV is achieved in NPDDE model, $\sum m_\nu < 0.093$ eV in the NPDDE+r model and the most aggressive constraint is $\sum m_\nu < 0.078$ eV in NPDDE+r with the R16 prior while the most recent constraint [123] is $\sum m_\nu < 0.46 \div 1.3$ eV. Thus our model predicts most of updated bounds on sum of neutrino masses in various cosmological scenarios presented in [122, 123].

We found allowed region of m_3 that the elements of lepton mixing matrix U_{2i} and $U_{3i}(i = 1, 2, 3)$ can reach the constraint on the absolute values of the entries of the lepton mixing matrix given in [59] . Indeed, in the case $m_3 = 0.05$ eV, the other model parameters are found in table 4 and the magnitude of the lepton mixing matrix in equation (20) then takes the form:

$$|U^N| = \begin{pmatrix} 0.831 & 0.557 & 0.15 \\ 0.296 & 0.587 & 0.765 \\ 0.499 & 0.59 & 0.649 \end{pmatrix}, \tag{30}$$

which is consistent with the constraint on the absolute values of the entries of the lepton mixing matrix given in [59].

3.2. Inverted spectrum $m_3 < m_1 < m_2$

Similar to the normal spectrum, taking the best-fit values of neutrino oscillation parameters for IH with Super-Kamiokande atmospheric neutrino data as given in table 1, we get a solution[11]:

[10] In NH [59], $\lambda_2 \simeq \sqrt{|\Delta m_{21}^2|} \simeq 0.0086$ eV thus we can assume $m_2 \in (0.01, 0.02)$ eV.

[11] There are four solutions for m_1, b_2 however they give the same value for Δm_{ij}^2 and the same absolute values of $m_{1,2,3}$ thus we only consider in detail the solution in equations (31), (32).

Table 4. The model parameters in the case $m_2 = 0.014$ eV and $m_3 = 0.05$ eV in NH.

Parameters	The derived values(eV)
m_1	1.1×10^{-2}
a_1	$(-3.29 + 2.42i)10^{-3}$
b_1	3.21×10^{-3}
b_2	7.25×10^{-4}
c_1	-7.25×10^{-4}

Table 5. Some model parameters in the case $m_2 = 0.0505$ eV and $m_3 = 0.055$ eV in IH.

Parameters	The derived values (eV)
m_1	4.98×10^{-2}
a_1	$(0.726 + 5.22i)10^{-4}$
b_1	3.82×10^{-4}
$b_2 = -c_1$	-2.44×10^{-2}

$$m_1^I = m_1^N,$$
$$b_2 = 7.5 \times 10^{-7} m_2 - 1.3 \times 10^{-6}\sqrt{\gamma_1} + 1.3\sqrt{\gamma_3} - 0.5 m_3, \tag{31}$$

$$\gamma_3 = -3.71 \times 10^{-4} + 0.148 m_2^2 - 5.32 \times 10^{-7} m_2\sqrt{\gamma_1} - 0.5 m_3, \tag{32}$$

with γ_1 is defined in equation (27).

In IH [59], $\lambda_2 \simeq \sqrt{|\Delta m_{32}^2|} = 0.0501$ eV thus we can put $\lambda_2 = 0.0505$ eV and get[12] $m_2 = 0.0505$ eV. The sum of neutrino mass is found to be $\sum_{i=1}^{3}\lambda_i^I = 0.106 - 2.22 \times 10^{-16} m_3 \simeq 0.106$ eV which is well consistent with most of the bonds presented in [122, 123]. In the case $m_2 = 0.055$eV the other parameters are found in table 5 and three physical neutrino masses are explicitly given as $\lambda_1^I = 4.98 \times 10^{-2}$ eV, $\lambda_2^I = 5.05 \times 10^{-2}$ eV, $\lambda_3^I = 6.18 \times 10^{-3}$ eV. The sum of neutrino mass is found to be $\sum_{i=1}^{3}\lambda_i^I = 0.106$ eV which is well consistent with almost upper bounds from cosmology given in [122].

The magnitude of the lepton mixing matrix in equation (20) then takes the form:

$$|U^I| = \begin{pmatrix} 0.831 & 0.557 & 0.15 \\ 0.386 & 0.533 & 0.766 \\ 0.419 & 0.649 & 0.649 \end{pmatrix}, \tag{33}$$

which is consistent with the constraint on the absolute values of the entries of the lepton mixing matrix given in [59].

3.3. Effective neutrino mass parameters

The effective neutrino masses governing the beta decay (m_β) and neutrinoless double beta decay ($\langle m_{ee}\rangle$) [124–128] has the forms

[12] In this model under consideration $\cos\theta \simeq 0.83$, $c_1 = -b_2 = 2.4 \times 10^{-2}$ hence $\lambda_2 = m_2 + 2(c_1 + b_2)\cos^2\theta = m_2$.

$$\langle m_{ee}\rangle = \left|\sum_{i=1}^{3} U_{ei}^2 m_i\right|, \quad m_\beta = \left(\sum_{i=1}^{3} |U_{ei}|^2 m_i^2\right)^{1/2}, \tag{34}$$

where U_{ei} ($i = 1, 2, 3$) is the PMNS leptonic mixing matrix elements and m_i correspond to the masses of three light neutrinos . With the model parameters obtained in sections 3.1 and 3.2, the effective neutrino masses in the beta decay and neutrinoless double beta decay, for the Normal and Inverted neutrino mass orderings, acquire the following values:

$$\langle m_{ee}\rangle = \begin{cases} 1.23 \times 10^{-2}\,\text{eV} & \text{for NH,}\\ 4.88 \times 10^{-2}\,\text{eV} & \text{for IH,}\end{cases} \tag{35}$$

and

$$m_\beta = \begin{cases} 1.42 \times 10^{-2}\,\text{eV} & \text{for NH,}\\ 5.07 \times 10^{-2}\,\text{eV} & \text{for IH.}\end{cases} \tag{36}$$

Furthermore, the Jarlskog invariant which controls the size of CP violation takes the values [60, 129–131]:

$$J_{\text{CP}} = \text{Im}(U_{23}U_{13}^*U_{12}U_{22}^*) = \begin{cases} -2.45 \times 10^{-2}\,\text{eV} & \text{for NH,}\\ -3.26 \times 10^{-2}\,\text{eV} & \text{for IH.}\end{cases} \tag{37}$$

The resulting effective neutrino mass parameters in equations (35) and (36), for both normal and inverted hierarchies, are below all the upper bounds arising from present $0\nu\,\beta\beta$ decay experiments, such as, KamLAND-Zen [132] $\langle m_{ee}\rangle < 0.05 \div 0.16$ eV, GERDA [133] $\langle m_{ee}\rangle < 0.12 \div 0.26$ eV, MAJORANA [134] $\langle m_{ee}\rangle < 0.24 \div 0.53$ eV, EXO [135–137] $\langle m_{ee}\rangle < 0.17 \div 0.49$ eV, CUORE [138, 139] $\langle m_{ee}\rangle < 0.11 \div 0.5$ eV, and they are very well consistent with the meV limit of the effective neutrino mass can be reached by the planning of future experiments [140–142].

4. Quark mass

The Yukawa interactions in quark sectors can be expanded from equation (5):

$$\begin{aligned} -\mathcal{L}_q \;=\; & h_{1u}\bar{Q}_{1L}\tilde{H}u_{1R} + h_{2u}\bar{Q}_{2L}\tilde{H}u_{2R} + h_{3u}\bar{Q}_{2L}\tilde{H}^{'}u_{2R} + h_{2u}\bar{Q}_{3L}\tilde{H}u_{3R}\\ & -h_{3u}\bar{Q}_{3L}\tilde{H}^{'}u_{3R} + h_{4u}(\bar{Q}_{2L}\widetilde{H_1^{''}}u_{1R} + \bar{Q}_{1L}\widetilde{H_1^{''}}u_{2R} + \bar{Q}_{3L}\widetilde{H_2^{''}}u_{1R} + \bar{Q}_{1L}\widetilde{H_2^{''}}u_{3R})\\ & +h_{1d}\bar{Q}_{1L}Hd_{1R} + h_{2d}\bar{Q}_{2L}Hd_{2R} + h_{3d}\bar{Q}_{2L}H^{'}d_{2R} + h_{2d}\bar{Q}_{3L}Hd_{3R}\\ & -h_{3d}\bar{Q}_{3L}H^{'}d_{3R} + h_{4d}(\bar{Q}_{2L}H_1^{''}d_{1R} + \bar{Q}_{1L}H_1^{''}d_{2R} + \bar{Q}_{3L}H_2^{''}d_{1R} + \bar{Q}_{1L}H_2^{''}d_{3R})\\ & +\text{h.c.} \end{aligned} \tag{38}$$

With the VEV alignments of H, H', H'' as given in equation (6), the mass Lagrangian of quarks reads

$$\begin{aligned}
-\mathcal{L}_q^{\mathrm{mass}} &= h_{1u}v_H^*\bar{u}_{1L}u_{1R} + h_{4u}v_{H''}^*(\bar{u}_{2L}u_{1R} + \bar{u}_{1L}u_{2R})\\
&\quad +(h_{2u}v_H^* + h_{3u}v_{H'}^*)\bar{u}_{2L}u_{2R} + (h_{2u}v_H^* - h_{3u}v_{H'}^*)\bar{u}_{3L}u_{3R}\\
&\quad +h_{1d}v_H\bar{d}_{1L}d_{1R} + h_{4d}v_{H''}(\bar{d}_{2L}d_{1R} + \bar{d}_{1L}d_{2R})\\
&\quad +(h_{2d}v_H + h_{3d}v_{H'})\bar{d}_{2L}d_{2R} + (h_{2d}v_H - h_{3d}v_{H'})\bar{d}_{3L}d_{3R} + \mathrm{h.c.}\\
&\equiv (\bar{u}_{1L},\, \bar{u}_{2L},\, \bar{u}_{3L})M_u(u_{1R},\, u_{2R},\, u_{3R})^T + (\bar{d}_{1L},\, \bar{d}_{2L},\, \bar{d}_{3L})M_d(d_{1R},\, d_{2R},\, d_{3R})^T\\
&\quad +\mathrm{h.c.},
\end{aligned} \tag{39}$$

where the up-and down-quark mass matrices are

$$M_u = \begin{pmatrix} h_{1u}v_H^* & h_{4u}v_{H''}^* & 0\\ h_{4u}v_{H''}^* & h_{2u}v_H^* + h_{3u}v_{H\prime}^* & 0\\ 0 & 0 & h_{2u}v_H^* - h_{3u}v_{H\prime}^*\end{pmatrix} \equiv \begin{pmatrix} a_{1u} & a_{4u} & 0\\ a_{4u} & a_{2u} + a_{3u} & 0\\ 0 & 0 & a_{2u} - a_{3u}\end{pmatrix},$$
$$M_d = \begin{pmatrix} h_{1d}v_H & h_{4d}v_{H''} & 0\\ h_{4d}v_{H''} & h_{2d}v + h_{3d}v_{H\prime} & 0\\ 0 & 0 & h_{2d}v_H - h_{3d}v_{H\prime}\end{pmatrix} \equiv \begin{pmatrix} a_{1d} & a_{4d} & 0\\ a_{4d} & a_{2d} + a_{3d} & 0\\ 0 & 0 & a_{2d} - a_{3d}\end{pmatrix}. \tag{40}$$

The matrices $M_{u,d}$ in equation (40) are, respectively, diagonalized as

$$\begin{aligned}
U_{uL}^{+}M_u U_{uR} &= \mathrm{diag}(m_u,\ m_c,\ m_t),\\
U_{dL}^{+}M_d U_{dR} &= \mathrm{diag}(m_d,\ m_s,\ m_b),
\end{aligned} \tag{41}$$

where

$$\begin{aligned}
m_{u,c} &= \Lambda_{1u} \mp \Lambda_{2u},\ \ m_t = a_{2u} - a_{3u},\\
m_{d,s} &= \Lambda_{1d} \mp \Lambda_{2d},\ \ m_b = a_{2d} - a_{3d},
\end{aligned} \tag{42}$$

$$U_{(u,d)L} = U_{(u,d)R} = \begin{pmatrix} \cos\theta_{(u,d)} & -\sin\theta_{(u,d)} & 0\\ \sin\theta_{(u,d)} & \cos\theta_{(u,d)} & 0\\ 0 & 0 & 1\end{pmatrix}, \tag{43}$$

with

$$\sin\theta_{(u,d)} = \frac{1}{\sqrt{K_{(u,d)}^2 + 1}},\ K_{(u,d)} = \frac{a_{1(u,d)} - a_{2(u,d)} - a_{3(u,d)} - \Lambda_{2(u,d)}}{2a_{4(u,d)}}, \tag{44}$$

and

$$\begin{aligned}
\Lambda_{1(u,d)} &= a_{1(u,d)} + a_{2(u,d)} + a_{3(u,d)},\\
\Lambda_{2(u,d)} &= [(a_{2(u,d)} + a_{3(u,d)} - a_{1(u,d)})^2 + 4a_{4(u,d)}^2]^{\frac{1}{2}}.
\end{aligned} \tag{45}$$

The quark mixing matrix is defined as

$$U_{\mathrm{CKM}} = U_L^{d\dagger}U_L^u = \begin{pmatrix} \cos\theta_u\cos\theta_d + \sin\theta_u\sin\theta_d & \cos\theta_u\sin\theta_d - \sin\theta_u\cos\theta_d & 0\\ \sin\theta_u\cos\theta_d - \cos\theta_u\sin\theta_d & \cos\theta_u\cos\theta_d + \sin\theta_u\sin & 0\\ 0 & 0 & 1\end{pmatrix}. \tag{46}$$

This is a good approximation for the realistic quark mixing matrix, which implies that the mixings among the quarks are very small [60]. With the best-fit results of $|V_{\mathrm{CKM}}|$ [60], $(U_{\mathrm{CKM}})_{11} = 0.974\,46$ and $(U_{\mathrm{CKM}})_{12} = 0.224\,52$, from equation (46) we get

$$\sin\theta_u = 0.899\ (\theta_u \simeq 64.1^\circ),\ \sin\theta_d = 0.974\ (\theta_d \simeq 77.1^\circ). \tag{47}$$

From equations (42), (44) and (45) we have a solution[13]:

$$\begin{aligned}
&a_{1u} = m_u + (mc - m_u)\sin^2\theta_u,\ a_{2u} = \frac{1}{2}[m_c + m_t + (m_u - m_c)\sin^2\theta_u],\\
&a_{3u} = \frac{1}{2}[m_c - m_t + (m_u - m_c)\sin^2\theta_u],\ a_{4u} = \mathrm{i}(m_c - m_u)\sin\theta_u\sqrt{\sin^2\theta_u - 1},\\
&a_{1d} = m_d + (m_s - m_d)\sin^2\theta_u,\ a_{2d} = \frac{1}{2}[m_b + m_s + (m_d - m_s)\sin^2\theta_d],\\
&a_{3d} = \frac{1}{2}[-m_b + m_s + (m_d - m_s)\sin^2\theta_d],\ a_{4d} = \mathrm{i}(m_d - m_s)\sin^2\theta_d\sqrt{\sin^2\theta_d - 1}.
\end{aligned} \tag{48}$$

By substituting the best-fit values of m_u, m_d, m_c, m_s, m_t, m_b in equation (3) and obtained values of $\sin\theta_{u,d}$ in equation (47) into equation (48), we get:

$$\begin{aligned}
&a_{1u} = 1.03 \times 10^9\,\mathrm{eV},\ a_{2u} = 8.66 \times 10^{10}\,\mathrm{eV},\\
&a_{3u} = -8.63 \times 10^{10}\,\mathrm{eV},\ a_{4u} = 4.99 \times 10^8\,\mathrm{eV},\\
&a_{1d} = 8.85 \times 10^7\,\mathrm{eV},\ a_{2d} = 2.09 \times 10^9\,\mathrm{eV},\\
&a_{3d} = -2.09 \times 10^9\,\mathrm{eV},\ a_{4d} = 1.93 \times 10^7\,\mathrm{eV},
\end{aligned} \tag{49}$$

i.e. $\frac{h_{1u}}{h_{1d}} \simeq 11$, $\frac{h_{2u}}{h_{2d}} \simeq \frac{h_{3u}}{h_{3d}} \simeq 41$, $\frac{h_{4u}}{h_{4d}} \simeq 25$, and if $v_{H''} \sim v_{H'} \sim v_H$, the Yukawa coupling hierarchies are $|h_{1(u,d)}| \simeq |h^{4(u,d)}| < |h_{2(u,d)}| \simeq |h_{3(u,d)}|$.

Some other issues such as the constraints from available collider, the predictions for Higgs doublet spectrum and the renormalization group evolution, etc, have been studied in [58, 143–147] so we will not discuss further here. The model predictions sustain under renormalization group evolution because only one Higgs scalar doublet is allowed to have Yukawa interactions with a given type of right-handed fermion [148] and the model could potentially preserve perturbativity and vacuum stability all the way up to the Planck scale [149].

5. Conclusions

We have proposed a renormalizable $B - L$ standard model (SM) extension based on D_4 symmetry which accommodates fermion mass and mixing parameters with CP violation. Both normal and inverted neutrino mass ordering as well as the smallness of the active neutrino masses are generated through a type I seesaw mechanism. The obtained physical parameters are consistent with the global fit of neutrino oscillation in [59] while the quark masses are in good agreement with the recent experimental data [60]. The model also predicts an effective neutrino mass parameter of $\langle m_{ee}\rangle = 1.24 \times 10^{-2}$ eV for normal hierarchy and $\langle m_{ee}\rangle = 4.88 \times 10^{-2}$ eV for IH which are all consistent with the recent experimental limits on neutrinoless double beta decay.

[13] The system of equations has four solutions but they have the same absolute values of $m_{u,d,c,s,t,b}$ so we only consider in detail the solution in equation (48).

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Appendix A. D_4 group and Clebsch–Gordan coefficients

D_4 is the symmetry group of a square. It has eight elements divided into five conjugacy classes, with $\underline{1}$, $\underline{1}'$, $\underline{1}''$, $\underline{1}'''$ and $\underline{2}$ as its five irreducible representations.

In real basis, in which the two-dimensional representation $\underline{2}$ of D_4 is real, $2^*(1^*, 2^*) = 2(1^*, 2^*)$, the Clebsch–Gordan coefficients of D_4 are:

$$\begin{aligned} &\underline{1}(x_1)\otimes\underline{1}(y_1)=\underline{1}(x_1y_1), &&\underline{1}'(x_1)\otimes\underline{1}'(y_1)=\underline{1}(x_1y_1),\\ &\underline{1}''(x_1)\otimes\underline{1}''(y_1)=\underline{1}(x_1y_1), &&\underline{1}'''(x_1)\otimes\underline{1}'''(y_1)=\underline{1}(x_1y_1), \end{aligned}\tag{A1}$$

$$\begin{aligned} &\underline{1}(x_1)\otimes\underline{1}'(y_1)=\underline{1}'(x_1y_1), &&\underline{1}(x_1)\otimes\underline{1}''(y_1)=\underline{1}''(x_1y_1),\\ &\underline{1}(x_1)\otimes\underline{1}'''(y_1)=\underline{1}'''(x_1y_1), &&\underline{1}'(x_1)\otimes\underline{1}''(y_1)=\underline{1}'''(x_1y_1),\\ &\underline{1}''(x_1)\otimes\underline{1}'''(y_1)=\underline{1}'(x_1y_1), &&\underline{1}'''(x_1)\otimes\underline{1}'(y_1)=\underline{1}''(x_1y_1), \end{aligned}\tag{A2}$$

$$\underline{1}(x_1)\otimes\underline{2}\begin{pmatrix}y_1\\y_2\end{pmatrix}=\underline{2}\begin{pmatrix}x_1y_1\\x_1y_2\end{pmatrix},\quad \underline{1}'(x_1)\otimes\underline{2}\begin{pmatrix}y_1\\y_2\end{pmatrix}=\underline{2}\begin{pmatrix}x_1y_1\\-x_1y_2\end{pmatrix},\tag{A3}$$

$$\underline{1}''(x_1)\otimes\underline{2}\begin{pmatrix}y_1\\y_2\end{pmatrix}=\underline{2}\begin{pmatrix}x_1y_2\\x_1y_1\end{pmatrix},\quad \underline{1}'''(x_1)\otimes\underline{2}\begin{pmatrix}y_1\\y_2\end{pmatrix}=\underline{2}\begin{pmatrix}-x_1y_2\\x_1y_1\end{pmatrix},\tag{A4}$$

$$\begin{aligned}\underline{2}\begin{pmatrix}x_1\\x_2\end{pmatrix}\otimes\underline{2}\begin{pmatrix}y_1\\y_2\end{pmatrix}= &\ \underline{1}(x_1y_1+x_2y_2)\oplus\underline{1}'(x_1y_1-x_2y_2)\\ &\oplus\underline{1}''(x_1y_2+x_2y_1)\oplus\underline{1}'''(x_1y_2-x_2y_1).\end{aligned}\tag{A5}$$

The conjugation representations of D_4 are given by

$$\underline{2}^*(1^*, 2^*) = \underline{2}(1^*, 2^*),\tag{A6}$$

$$\underline{1}^*(1^*)=\underline{1}(1^*),\ \ \underline{1}'^*(1^*)=\underline{1}'(1^*),\ \ \underline{1}''^*(1^*)=\underline{1}''(1^*),\ \ \underline{1}'''^*(1^*)=\underline{1}'''(1^*),\tag{A7}$$

where, for example, $\underline{2}^*(1^*, 2^*)$ denotes some $\underline{2}^*$ multiplet of the form $(x_1^*, x_2^*) \sim \underline{2}^*$.

Appendix B. Higgs potential scalar

The general renormalized potential which is invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \times D_4 \times Z_4$ symmetry takes the form[14]:

$$\begin{aligned}V_{\text{total}} = &\ V(H) + V(H') + V(H'') + V(\varphi) + V(\phi)\\ &+ V(H, H') + V(H, H'') + V(H, \varphi) + V(H, \phi) + V(H', H'')\\ &+ V(H', \varphi) + V(H', \phi) + V(H'', \varphi) + V(H'', \phi) + V(\varphi, \phi),\end{aligned}\tag{B1}$$

[14] Here, we denote $V(X \to X_1, Y \to Y_1, \cdots) \equiv V(X, Y, \cdots)|_{\{X=X_1, Y=Y_1, \cdots\}}$.

where:

$$\begin{aligned}
V(H) &= \mu_H^2 H^\dagger H + \lambda^H (H^\dagger H)^2, \quad V(H') = V(H \to H'),\\
V(H'') &= \mu_{H''}^2 (H''^\dagger H'')_{\underline{1}} + \lambda_1^{H''} (H''^\dagger H'')_{\underline{1}} (H''^\dagger H'')_{\underline{1}} + \lambda_2^{H''} (H''^\dagger H'')_{\underline{1}'} (H''^\dagger H'')_{\underline{1}'}\\
&\quad + \lambda_3^{H''} (H''^\dagger H'')_{\underline{1}''} (H''^\dagger H'')_{\underline{1}''} + \lambda_4^{H''} (H''^\dagger H'')_{\underline{1}'''} (H''^\dagger H'')_{\underline{1}'''},\\
V(\varphi) &= V(H \to \varphi), \quad V(\phi) = V(H'' \to \phi),\\
V(H, H') &= \lambda_1^{HH'} (H^\dagger H)_{\underline{1}} (H'^\dagger H')_{\underline{1}} + \lambda_2^{HH'} (H^\dagger H')_{\underline{1}'} (H'^\dagger H)_{\underline{1}'},\\
V(H, H'') &= \lambda_1^{HH''} (H^\dagger H)_{\underline{1}} (H''^\dagger H'')_{\underline{1}} + \lambda_2^{HH''} (H^\dagger H'')_{\underline{2}} (H''^\dagger H)_{\underline{2}},\\
V(H, \varphi) &= \lambda_1^{H\varphi} (H^\dagger H)_{\underline{1}} (\varphi^\dagger \varphi)_{\underline{1}} + \lambda_2^{HH'} (H^\dagger \varphi)_{\underline{1}} (\varphi^\dagger H)_{\underline{1}},\\
V(H, \phi) &= V(H, H'' \to \phi), \quad V(H', H'') = V(H \to H', H''),\\
V(H', \varphi) &= \lambda_1^{H'\varphi} (H'^\dagger H')_{\underline{1}} (\varphi^\dagger \varphi)_{\underline{1}} + \lambda_2^{H'\varphi} (H'^\dagger \varphi)_{\underline{1}'} (\varphi^\dagger H')_{\underline{1}'},\\
V(H', \phi) &= V(H', H'' \to \phi), \quad V(H'', \varphi) = V(\varphi \to H, H''),\\
V(H'', \phi) &= \lambda_1^{H''\phi} (H''^\dagger H'')_{\underline{1}} (\phi^\dagger \phi)_{\underline{1}} + \lambda_2^{H''\phi} (H''^\dagger H'')_{\underline{1}'} (\phi^\dagger \phi)_{\underline{1}'}\\
&\quad + \lambda_3^{H''\phi} (H''^\dagger H'')_{\underline{1}''} (\phi^\dagger \phi)_{\underline{1}''} + \lambda_4^{H''\phi} (H''^\dagger H'')_{\underline{1}'''} (\phi^\dagger \phi)_{\underline{1}'''}\\
&\quad + \lambda_5^{H''\phi} (H''^\dagger \phi)_{\underline{1}} (\phi^\dagger H'')_{\underline{1}} + \lambda_6^{H''\phi} (H''^\dagger \phi)_{\underline{1}'} (\phi^\dagger H'')_{\underline{1}'}\\
&\quad + \lambda_7^{H''\phi} (H''^\dagger \phi)_{\underline{1}''} (\phi^\dagger H'')_{\underline{1}''} + \lambda_8^{H''\phi} (H''^\dagger \phi)_{\underline{1}'''} (\phi^\dagger H'')_{\underline{1}'''},\\
V(\varphi, \phi) &= V(H \to \varphi, \phi). \qquad \text{(B2)}
\end{aligned}$$

The scalars H, H', H'', φ and ϕ with the VEVs in equation (6) is a solution from the minimization conditions of V_{total}. To see this, in the system of minimization equations, let us put $v_{H_1''} = v_H''$, $v_{H_2''} = 0$, $v_{\phi_1} = v_{\phi_2} = v_\phi$ and $v_H^* = v_H$, $v'^*_H = v'_H$, $v''^*_H = v''_H$, $v_\varphi^* = v_\varphi$, $v_\phi^* = v_\phi$, which reduces to

$$\begin{aligned}
&\mu_H^2 + 2\lambda^H v_H^2 + (\lambda_1^{HH'} + \lambda_2^{HH'}) v'^2_H + 2(\lambda_1^{HH''} + \lambda_2^{HH''}) v''^2_H + (\lambda_1^{H\varphi} + \lambda_2^{H\varphi}) v_\varphi^2\\
&\quad + 2(\lambda_1^{H\phi} + \lambda_2^{H\phi}) v_\phi^2 = 0, \qquad \text{(B3)}
\end{aligned}$$

$$\begin{aligned}
&\mu_{H'}^2 + 2\lambda^{H'} v'^2_H + (\lambda_1^{HH'} + \lambda_2^{HH'}) v_H^2 + 2(\lambda_1^{H'H''} + \lambda_2^{H'H''}) v''^2_H + (\lambda_1^{H'\varphi} + \lambda_2^{H'\varphi}) v_\varphi^2\\
&\quad + 2(\lambda_1^{H'\phi} + \lambda_2^{H'\phi}) v_\phi^2 = 0, \qquad \text{(B4)}
\end{aligned}$$

$$\begin{aligned}
&\mu_{H''}^2 + (\lambda_1^{HH''} + \lambda_2^{HH''}) v_H^2 + (\lambda_1^{H'H''} + \lambda_2^{H'H''}) v'^2_H + 2(\lambda_1^{H''} + \lambda_2^{H''}) v''^2_H\\
&\quad + 2(\lambda_1^{H''\varphi} + \lambda_2^{H''\varphi}) v_\varphi^2 + (2\lambda_1^{H''\phi} + \lambda_5^{H''\phi} + \lambda_6^{H''\phi} + \lambda_7^{H''\phi} - \lambda_8^{H''\phi}) v_\phi^2 = 0, \qquad \text{(B5)}
\end{aligned}$$

$$\begin{aligned}
&\mu_\varphi^2 + 2\lambda^\varphi v_\varphi^2 + (\lambda_1^{H\varphi} + \lambda_2^{H\varphi}) v_H^2 + (\lambda_1^{H'\varphi} + \lambda_2^{H'\varphi}) v'^2_H + 2(\lambda_1^{H''\varphi} + \lambda_2^{H''\varphi}) v''^2_H\\
&\quad + 2(\lambda_1^{\varphi\phi} + \lambda_2^{\varphi\phi}) v_\phi^2 = 0, \qquad \text{(B6)}
\end{aligned}$$

$$\begin{aligned}
&\mu_\phi^2 + (\lambda_1^{HH'} + \lambda_2^{HH'}) v_H^2 + 2\lambda^{H'} v'^2_H + (\lambda_1^{H'H''} + \lambda_2^{H'H''}) v''^2_H + (\lambda_1^{H'\varphi} + \lambda_2^{H'\varphi}) v_\varphi^2\\
&\quad + 2(\lambda_1^{H'\phi} + \lambda_2^{H'\phi}) v_\phi^2 = 0. \qquad \text{(B7)}
\end{aligned}$$

The system of equations from (B3) to (B7) always gives the solution $(v, v', v'', v_\varphi, v_\phi)$ as given in equation (6). It is also noted that this aligned is only one solution to have the desirable results. In this type of the model, the breaking of the $U(1)_{B-L}$ symmetry at a scale larger than the electroweak scale, i.e. $v_H, v_{H'}, v_{H''} \ll v_\phi, v_\varphi$. After the electroweak symmetry breaking there exist the mixing between the remaining degrees of freedom from $SU(2)_L$ singlets with the ones coming from the $SU(2)_L$ doublets which give rise to a number of neutral, charged and pseudoscalar Higgs bosons (one of which will correspond to the SM Higgs) and some of the scalars will be heavier than the others which has been analyzed in [150].

ORCID iDs

V V Vien https://orcid.org/0000-0002-2460-7013

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