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Variation Method Overview of Image Segmentation

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Abstract. With the research and development of image segmentation methods for several decades, the various theories and methods have been applied to the image segmentation. Among the many methods of image segmentation, the variation method is applied widely, because its model process is easier, the expansibility of the method is better and the implementation process is simple. On the basics of many achievements of variation method research of image segmentation recently, the variation method of image segmentation is divided into the model based on the boundary and the field according to various structural driving force based on energy functional. In this paper, we analyze these models based on these two methods and discuss their advantages and disadvantages. Relevant innovation algorithms and breakthrough progresses recently have been researched. Some development directions of image segmentation variation method have been given. The application field and development of variation image segmentation still have great research value.

1. Introduction

The thought of variation method emerged as the fastest linear problem proposed by John Bernoull at first. Later, in 1744, Euler who is a mathematician gave the universal solution of the problem at the first. The publishing of his works, “Variation Theory”, symbolized the birth of the variation method in the new branch of mathematics. Then, a large number of scientists made the theoretical contribution in this field. They were widely applied to physics, dynamics, optimum control and so on.

Since 1980s, the variation method has been successfully applied to the field of image segmentation, and gradually become into a powerful image segmentation technology[1-4]. The greatest advantages of variation method of image segmentation lie in that the curve can keep continuity and smoothness in the process of evolution. Therefore, it can realize the continuous, closed and smooth representation of target boundary which is the ability of boundary extraction that the conventional segmentation method has not realized.

With the development for decades, abundant segmentation models have been proposed based on variation method. The segmentation algorithms based on energy functional can be divided into two categories: parametric active contour model, geometric active contour model. Parametric active contour model can be controlled to realize by minimizing energy functional. Level set algorithm is the representation of the geometric active contour model [5].

The research of variation method of image segmentation promotes interdisciplinary and fusion and puts the complete and unified theoretical framework constructed forward in the field of image segmentation so as to solve many theoretical and practical application problems existed in image segmentation. At the same time, intensive study of image segmentation promotes the development of variation method and proposes the new problem and challenge for the development. Thus, the research

of variation method of image segmentation has not only tremendous practical value, but also very importantly theoretical and scientific meaning.

Note that there are many variation models and methods of image segmentation. The typical variation methods of image segmentation are outlined on the basis of synthesizing various methods. In addition, many important works must be not involved in the paper and please comprehend.

The advantages and disadvantages of the various models and relative algorithms based on boundary and fields of variation method of image segmentation are introduced in detail in the second part in the paper.

The development direction of variation method of image segmentation in the future are concluded and expected in the third part in the paper.

2. Research Status of Variation Method of Image Segmentation

2.1. Variation Method of Image Segmentation Based on Boundary

The models based on boundary rely on the image gradient function to control the speed of movement of the curve. Where the image gradient is smaller such as in the flat area of the image, the evolution speeds of the contour curve are faster. However, where the image gradient is bigger such as in the edge of the image object, the evolution speeds of the contour curve are slower and even stop. The boundary extraction of the target object is realized.

2.1.1. Parameter Active Contour Theory. In 1988, **Kass**, **Witkin** and **Terzopolous** proposed a very different new method of segmenting the images, which is called snake model [6]. The model defines an energy functional containing external and internal energy. The defined energy functional carried out iterative operation to obtain the minimal value of the function. The minimal value points constitute final curve segmented in the function. But the model is much more sensitive to the original position of the active curves. The various result curves can be obtained by different curve initialization. Even the correct segmentation can't be obtained. The constructed energy functional is not strict convex function. Thus, it is prone to fall into local minimal value of the function during the function evolution and the image can't be segmented correctly. The function can't make topical changes of the curves in the process of the curve iteration.

For these disadvantages, **Cohen** put forth an improved algorithm called Balloon Snake [7-8]. The algorithm can keep the internal energy items of the energy function unchanged, but it can change the external energy items by adding expansion related items to the external energy items. The algorithm can effectively avoid falling into local minimum value which is disadvantage. Xu. et. al. [9] introduced the concept of gradient vector flow into original active contour model and used it to the external force from the curve evolution movement. The method can improve the capture range of Snake-active contour model.

Because the resolution complexity of Snake-active contour model is very high, **Aimini** et. al. [10] put forth to use dynamic programming algorithm in the process of solving the active contour model and **Williams** et. al. [11] used greedy algorithm based on the above. The two methods can apply the algorithm and theory of software solution to the solution of active contour model. They are extremely outstanding innovation.

2.1.2. Geodesic Active Contour Method. **Caselles** et. al. put forth the geometric active contour model based on mean curvature flow [12]. **Caselles & Kimmel** et. al. put forth geodesic active contour method [13]. Level set method can be added to the contour model. During data process, the level set function can be used to evolve the curve. The 2-dimentional active curve can be replaced by higher dimension level set function. These methods can make evolution curve change with topological structure.

Although the ideal segmentation results can be solved by these methods for high contrast, and a lot of different areas can be segmented simultaneously, the satisfied segmentation results can't be

obtained by these models and algorithms when the noise in the image is stronger, and the objects can't be extracted accurately by these algorithms for the uneven gray image when the contrast between the target and the background is lower.

For the problem of geodesic active contour method, **Paragois** et. al [14] put forth geometric active contour model based on gradient vector flow which applied the driving force constructed by gradient vector flow of the image. The model increased the convergence rate of evolution curve and was not sensitive to primary contour position [15]. A new inertial force field is proposed in the references [16-17]. Combined gradient vector flow with inertial force, the evolution curve can converge on deep depression area in these methods. In addition, applying the methods to choose the primary contour position is robust.

2.2. Image Segmentation Variation Based on Region

The image region division is realized by using description of the global and local statistical information of the image region based on the region model so as to extract target boundary profile and to segment fuzzy edge and discontinuous target in the image. Meanwhile, it has noise immunity. Topology changes of the curve can be handled effectively based on geometric active contour model so as to make up the disadvantages that topology changes can't be made by parameter active contour model. The evolution of the geodesic active contour model moves only along a certain direction. The final image segmentation results depend on the initialization location of the contour curve.

2.2.1. Mumford-Shah Model. Mumford-Shah model [18] is the most classical image variation segmentation model based on the region. The problem of image segmentation is turned into optimization problem. The image segmentation is opened up new areas. But the research and application of the model is very difficult. Because the model involves two different dimensions and unknown variables, it is difficult to solve them simultaneously. On the other hand, the energy functional isn't optimized based on classical Euler-Lagrange method, because it hasn't differentiability in Sobolev spaces. Moreover, the model involves an unknown and discontinuous free boundary set. It is difficult to be appropriately discretized in mathematical theory. Generally, a set of regular functional need be defined in Sobolev spaces to approach Mumford-Shah functional. But numerical implementation based on the method is very complex and amount of computation is huge which is difficult to be applied to the practice.

2.2.2. Geometric Active Contour. The energy functional of the designated problem and the edge line of split target in the image are defined respectively by the image information itself and the continuous contour curve of the geometric active contour. Euler-Langrange function is inferred in the light of variation method to reach the corresponding evolution function. The level set function is introduced to obtain the evolutionary partial differential equation of the energy function relevant to level set function. Finally, the evolutionary partial differential equation is obtained using numerical treatment to find the optimal target contour.

Two phase piecewise constant model proposed by **Chan & Vese** [18] is a simplified form of Mumford-Shah model [19]. Assume that the target and background of the image to be segmented is two different regions constituted of gray values respectively. Thus, the curve evolution is guided by the internal and external gray average of the evolution curve. The CV model has ideal segmentation results for edge without gradient meaning and image with weak edge and isn't subjected to noise to some extent. The curve evolution is guided by global gray information fully used of the image based on CV model which can avoid gradient information influence directly on image segmentation effect. The image edge can be segmented accurately by the model. But it is sensitive to the initial contour of the model. The algorithm complexity is higher and the segmentation efficiency is lower [20].

Since Chan-Vese model emerged, a lot of research and improvement work has made the model apply widely to poly-phase image segmentation, gray uneven image segmentation, texture image segmentation and medical image segmentation. **Zhao** et.al. [21] adopted the thought which used N

level sets to express N regions and put forth multi-phase segmentation of regional competition. **Vese** et.al. [22] put forth Poly-phase image segmentation model based on the thought of using N level sets to express 2N regions and extension of Chan-Vese model. But, the number of level set function is increased as segmentation regions rise based on the model which can result in increase of computational complexity of the algorithm. For solving the problem, **Lie** et.al. [23] put forth the segmentation method of poly-phase level set using a level set to express 2N regions so as to degrade greatly space storage rate and computational complexity.

For gray uneven image segmentation, **Tsai** et.al. [24] put forth piecewise smooth model which can segment gray uneven image well. But the numerical implementation is very complex and amount of computation is very huge based on the model. So, the practical application of the model is limited. Later, **Li** et.al. [25-26] used local region formation to fit gray scale of approximate image and then to segment gray uneven image. For relative piecewise smooth model of the model, computational efficiency is increased dramatically.

For texture image segmentation, at first **Sandberg** et.al. [27] mixed multidimensional characteristics extracted by filter into vector valued active contour model to get better segmentation effect of the texture image. Later, **Wang** et.al. [28] put forth a uniform tensor active contour model combined with image multiple features. Many active contour models can be uniformed by the model and better segmentation results can be solved by the texture images. In reference [29], the difference between each pixel and adjacent pixel of the image can be considered, and pixel nonuniformity factors can be built up which can be combined with active contour model. More complexly natural image can be segmented using the method such as gray uneven image and texture image.

Although better segmentation results can be found based on the variation model above mentioned, global minimum is difficult to be guaranteed so as to result in wrong segmentation results because the energy functional is non-convexity. For non-convexity of Chan-Vese model, **Brown** et.al.[30] put forth a fully convex active contour model. The model can be solved by primal dual algorithm to enormously optimize convergence rate of the algorithm. Different from the above methods, **Lee** et.al.[31] put forth an active contour model with global minimal solution from different perspectives. A displacement function, Heaviside, is introduced to limit level set function to local minimum based on the model so as to get much more satisfactory segmentation results.

For the further improvement of the nonconvex problem, **Cai** et.al.[32] put forth a convex model, called Mumford-Shah. The global minimum of the model is a smooth approximation image of the primary image. The image region extraction can be obtained by K-mean clustering method. The improved method is a convex relaxation method substantially. The thought of solving nonconvexity of Chan-Vese model is changed significantly.

2.3. New method combination of image segmentation variation method

2.3.1. Combination of Variation Method and Wavelet Transform. **Yang Zhiqiao** [33] put forth a new segmentation model combined global energy with local energy in order to fully use the global image information. Firstly, image edge information is extracted according to wavelet transformation characteristics. Secondly, edge detection function is constructed on above basis. Thirdly, edge detection function is introduced into LBF model so as to new LBF model based on wavelet transformation, called WLBF model. The method is not only more efficient, but also more accurate. The satisfactory segmentation results can be obtained during handling the images when gray is obviously uneven and noise is louder.

2.3.2. Application of Variation Method to 3D Image. In medical image segmentation, **Shen** [34] put forth a novel, multitasking and full convolution network structure applied to automatic segmentation of brain tumor which is breakthrough progress for 3D image segmentation realization. Combined with Variation image segmentation, **Li Laquan** [35] who built up PET/CT multimodal tumor segmentation model, put forth a PET/CT multimodal tumor segmentation method based on deep learning network

and variation segmentation model This is unsupervised deep learning segmentation method based on 3D full convolution neural network. Self-learning of deep learning network parameters can be realized without sample label. But the image is only divided into two regions by using the method. For unsupervised deep learning network, more complex image segmentation implementation has not been researched deeply. Moreover, matching PET with CT image acquired by default is precisely registered. However, the image in practice can't been matched precisely and the practicability is bad.

3. Conclusion and Expectation

Variation image segmentation method is mainly based on edge and region respectively. The edge information of image target such as gradient and curvature is used to segment the image by variation image segmentation method based on edge. But the problem of curve topology changes isn't been solved well. The curve evolution is considered in higher dimension space by using level set thought based on variation image segmentation of region. The split and merger of the curve is solved by the method well. Each model has good characteristics and shortcomings.

Multiple models recently have been researched based on image segmentation variation. Here, the future development directions of the application of image segmentation variation are proposed as the following:

(1) 2D and 3D image segmentation. Only 2D image segmentation method has been researched based on image segmentation variation recently. The research of 2D and 3D image segmentation is to be studied.

(2) Model parameter setting. Parameter setting is very important which depends on experiences and properties of specific images. Firstly, the parameter values are given tentatively. Then they are adjusted ceaselessly according to segmentation effect which need take up time.

(3) The integration of model framework. The models in the paper are come down in one continuous line. If these models can be integrated into a big framework, various adaptive models can be obtained by setting some parameters according to different images based on image segmentation variation which is worthy of researching deeply.

(4) Multi-level set image segmentation. The research on single level set has been mainly research trend. However, application of single level set method is limited. Poor segmentation can be solved based on multi-level set image segmentation. The precision of multi-level set image segmentation is much higher than existed methods and the robustness is better. Therefore, application of multi-level set image segmentation method is to be a hot topic.

(5) Existence and convergence of global optimal solution. So far, a number of image segmentation variation models are non-convex. Global optimal solution can't be guaranteed access using optimization algorithm solution. Recently, some researchers have tried to solve the problem and put forth convex variation segmentation model that practicability is not high.

(6) Regularization of evolution curve of level set. Level set regular terms need be introduced to constraint level set function in order to not be prone to noise and weak edge, because noise and weak edge exist in image. So, designing effective regularization method of overcoming high noise and avoiding edge leakage is worthy of researching.

(7) The problem of gray uneven image segmentation. Gray distribution overlap in various image region resulted from uneven gray can greatly reduce accuracy of image segmentation. The method using global image information to handle uneven gray image is to be researched.

(8) SAR image segmentation. Imaging system (SAR) has an ability of working 24 hours a day and large area data acquisition. It has an advantage of side looking. It is pivotal technology combined radar early warning with target recognition and tracking. SAR image has characteristics of high noise and fuzzy edge. At present, the research of SAR image segmentation is still in its nascent stages.

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