Study on Regional Division of Air Traffic Management in Mainland China in the Future

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Abstract: With the problems of poor coordination and frequent flight delays among major air traffic control regions in mainland China becoming increasingly prominent, a series of air traffic control reform measures are imperative. In order to meet the development need of the transition from a large civil aviation country to power one in the future, it is necessary to divide the air traffic management regions reasonably in advance. Therefore, based on the flights between the airports from 2011 to 2017, this paper combines the grey prediction algorithm with the gravity model and considers the impact of high-speed rail on the flights of civil aviation, then constructs the grey gravity prediction model. According to the forecast result of the future airport flow by the grey gravity prediction model, the flight plans of each airport are generated by using the equal proportion enlargement method. On the basis of the flight plans of each airport, the flight situation in mainland China at a certain time is simulated. According to the simulated aircraft longitude and latitude data, the optimal clustering results are obtained by using the fuzzy C-means clustering algorithm and clustering validity evaluation index. Finally, the clustering results are processed by clustering boundary recognition algorithm, the future air traffic management regional division results in mainland China are obtained. The experimental results show that the prediction accuracy of the grey gravity prediction model considering the influence of high-speed rail is better than the traditional grey prediction algorithm, and the original seven major air traffic control zones in mainland China are re-divided into three major air traffic control zones: North China, South Central China and Northwest China, which can provide reference for future air traffic control reform and development.

1.Introduction

The development of air traffic management in China is very serious. Even though the construction of air traffic control system has been strengthened, the development is still not optimistic. First, flight delays are always difficult to control effectively, and the poor coordination of flight time slot jump is also the main problem in air traffic management. At present, the air traffic control regions in the mainland of China are divided into seven regions. It is difficult to achieve timely and efficient coordination among these regions, and to meet the needs of civil aviation development in the mainland of China in the future. In order to solve the above problems, starting from the re-planning of air traffic zoning in mainland China, this paper considers a method of re-zoning air traffic management zones in China by clustering simulation operation based on the flight volume of each airport pair in the future.

At present, the research of traffic subarea division is mainly concentrated in the field of ground traffic, while the research of air traffic subarea division at macro level is still blank. With the rapid development of economic globalization and urbanization, ground traffic congestion has become one of the most important concerns of governments in many developed regions. Traffic subarea division is considered by many scholars as an effective way to solve this dilemma. In order to solve the problem of urban traffic congestion, Li Chungui et al. and Haifeng Guo et al. set up their own evaluation index to divide the ground traffic subarea. Dawen Xia et al.[1] In order to get rid of the restriction of traffic signal intersection as research unit, collect the trajectory data of taxis in Beijing, use a parallel clustering algorithm to process the trajectory data of taxis, and take the clustering results as the basis of traffic subarea division in Beijing. In the field of ground transportation, only Dawen Xia's traffic subarea division method, which clustered trajectory data, can break the limitation of existing division. Based on the above analysis, in order to break the traditional rules of regional division of air traffic management according to national boundaries, and try to establish a new regional division model of global air traffic management, global air traffic subarea division must be based on aircraft trajectory data, so as to provide a reference for the future development of global air traffic control integration.

Therefore, based on the flight volume data between the airports from 2011 to 2017, this paper combines the grey prediction algorithm with the gravity model and considers the impact of high-speed rail on the flight volume of civil aviation, and constructs the grey gravity prediction model. According to the forecast result of the future airport flight volume based on the grey gravity prediction model, the flight plans of each airport are generated by using the equal proportion enlargement method. According to the flight plans of each airport, the flight situation in mainland China at a certain time is simulated. According to the simulated aircraft longitude and latitude data, the optimal clustering results are obtained by using the fuzzy C-means clustering algorithm and clustering validity evaluation index. Finally, the clustering results are processed by clustering boundary recognition algorithm, and the future air traffic management regional division results in mainland China are obtained.

2.Analysis method

2.1 Grey gravity prediction model

The grey gravity prediction model is based on the combination of grey prediction model and gravity model, and the ratio of the shortest flight time to the minimum travel time of high-speed railway is added as the traffic impedance factor. Due to the limitation of data conditions, it is impossible to predict the flight volume by using neural network and other prediction algorithms which need a large number of data samples to do training set because they only have the data of each airport from 2011 to 2017.The advantages of grey forecasting include that it does not need a lot of data and is suitable for medium and long-term forecasting, but it lacks dynamic characteristics[2]. Gravity model can deal with the dynamic situation of airport-to-airport forecasting by self-defining traffic impedance function[3]. Based on this, this paper preliminarily explores the impact of high-speed rail on future civil aviation flights. The grey gravity prediction model is defined as follows:

$$flightnum_{ij} = t_i TF_i l_j LF_j f\left(c_{ij}\right) \tag{1}$$

Where $flightnum_{ij}$ indicates flight volume that the take-off airport is i, the landing airport is j, t_i is the take-off volume parameter of airport i, TF_i is the take-off volume of airport i, l_j is the landing volume parameter , and LF_j is the landing volume. $f\left(c_{ij}\right)$ is traffic impedance function of the takeoff airport i and the landing airport j ,which is defined here as:

$$f\left(c_{ij}\right) = \left(\frac{flight_{ij}}{train_{ij}}\right)^{-\gamma} \tag{2}$$

Among them, $flight_{ij}$ indicates the shortest flight time of the take-off airport i and the landing airport j, $train_{ij}$ is the shortest travel time of the high-speed railway, which takes the city where the take-off airport i is located, as the starting point and the city where the landing airport j is the end point. The specific values of t_i and l_j can be generated by cyclic iteration, as follows:

$$t_i^{m+1} = \frac{1}{\sum_j l_j^m LF_j f(c_{ij})} \tag{3}$$

$$l_j^{m+1} = \frac{1}{\sum_i t_i^m TF_i f(c_{ij})} \tag{4}$$

Where t_i^m represents the m-th iteration result of the take-off volume parameter of airport i, and l_j^m represents the m-th iteration result of the landing volume parameter of airport j, and exits the iteration when the following conditions are satisfied:

$$1-\varepsilon < \frac{t_i^{m+1}}{t_i^m} < 1+\varepsilon \tag{5}$$

$$1-\varepsilon < \frac{l_j^{m+1}}{l_j^m} < 1+\varepsilon \tag{6}$$

TF_i and LF_j represent the take-off volume of airport i and the landing volume of airport j calculated based on the grey prediction model, which can be expressed as:

$$TF_i^{(1)}(k+1) = \left[TF_i^{(1)}(1) - \frac{b}{a} \right] e^{-ak} + \frac{b}{a} \tag{7}$$

$$LF_j^{(1)}(k+1) = \left[LF_j^{(1)}(1) - \frac{b}{a} \right] e^{-ak} + \frac{b}{a} \tag{8}$$

Where k represents the number of data samples, $TF_i^{(1)}(k+1)$ and $LF_j^{(1)}(k+1)$ represent the grey prediction results, and the values of a and b are obtained by the operation of the original dataset Y and one-accumulation dataset U. Take TF_i as an example, it can be defined as:

$$Y = \left\{ TF_i^{(0)}(1), TF_i^{(0)}(2), ..., TF_i^{(0)}(k) \right\} \tag{9}$$

$$U = \left\{ TF_i^{(1)}(1), TF_i^{(1)}(2), ..., TF_i^{(1)}(k) \right\} \tag{10}$$

$$TF_i^{(1)}(k) = TF_i^{(0)}(1) + TF_i^{(0)}(2) + ... + TF_i^{(0)}(k) \tag{11}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = (U^T U)^{-1} U^T Y \tag{12}$$

2.2 Generation of future flight plans and operation simulation

2.2.1 Generation of future flight plans. Using the proportional magnification method to generate a flight plan for a future day, the specific steps are as follows:

Step1: First, statistics are made on the flight plans of the same day in the current year to obtain the flight volume and flight time of each airport pair.

Step2: Calculates the proportion of flights per airport pair on that day to the total number of flights at the airport for the whole year.

Step3: Uses the proportion obtained in Step2 to multiply the predicted future airport pair flight volume to obtain the airport pair flight volume in the future.

Step4: Compare the future flight volume with the current flight volume, and if it is reduced, delete the corresponding flight time. If it is increased, the flight is added after the existing flight time in ten time slots.

2.2.2 Aircraft position simulation. Since the result of the aircraft position simulation is to determine the specific position of the aircraft at a specific time in the future, it is necessary to first determine the specific time t of the prediction. According to the predicted departure time dep_{ij} and the shortest flight time $flight_{ij}$ from airport i to airport j, the flights of $dep_{ij} \le t$ and $dep_{ij} + flight_{ij} \ge t$ are selected. The location of a specific flight can be expressed as:

$$flight_{lat} = \begin{cases} lat_i + \dfrac{t - dep_{ij}}{flight_{ij}} \times \left(lat_j - lat_i\right), lat_j \ge lat_i \\ lat_i - \dfrac{t - dep_{ij}}{flight_{ij}} \times \left(lat_i - lat_j\right), lat_j < lat_i \end{cases} \tag{13}$$

$$flight_{lon} = \begin{cases} lon_i + \dfrac{t - dep_{ij}}{flight_{ij}} \times \left(lon_j - lon_i\right), lon_j \ge lon_i \\ lon_i - \dfrac{t - dep_{ij}}{flight_{ij}} \times \left(lon_i - lon_j\right), lon_j < lon_i \end{cases} \tag{14}$$

Where $flight_{lat}$ and $flight_{lon}$ represent the latitude and longitude of the flight, lon_i and lat_i represent the longitude and latitude of the departure airport, lon_j and lat_j represent the longitude and latitude of the landing airport, respectively.

2.3 Fuzzy C-means clustering algorithm

In essence, the problem of air traffic subarea division can be regarded as a problem of dividing airspace into several categories through certain rules. Clustering algorithm determines the fuzzy partition and clustering results of data sets by solving optimization problems. Under the background of air traffic area partition, it can use fuzzy clustering algorithm to solve air traffic area partition according to the characteristics of aircraft data sets. As the most widely used soft partitioning algorithm, Fuzzy C-Means algorithm can get rid of the hard partitioning "either-or" clustering situation by introducing the concept of membership degree and has good robustness to data amplification or reduction[4]. In the air traffic subarea division problem, there are obvious differences in the number of aircraft at different times. For example, the operation data in the daytime peak period is significantly more than that in the night non-peak period. In view of this situation, the robustness of FCM algorithm can accurately reflect the situation of clustering centers.

Assuming that n sample points in sample space $\boldsymbol{X} = \{x_1, x_2, ..., x_n\}$ are divided into c categories, the cluster centers of c categories are assumed to be $\boldsymbol{V} = \{v_1, v_2, ..., v_n\}$, $\boldsymbol{U}$ is the membership matrix, and J is the objective function, it can be expressed as:

$$J(\boldsymbol{U}, \boldsymbol{V}) = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^{\mathrm{m}} \left\| x_j - v_i \right\|^2 \tag{15}$$

Among them: u_{ij} denotes the membership degree of the j data sample in the i class, m denotes the fuzzy weighted index, x_j denotes the j data sample, and v_i denotes the i clustering center. In addition, u_{ij} needs to satisfy the following constraints:

$$\sum_{i=1}^{c} u_{ij} = 1 \tag{16}$$

Under the condition that the limit value of objective function J is calculated by Lagrange multiplier method, the mutual deduction between u_{ij} and v_i can be carried out, thus constituting the basis of continuous iteration of FCM algorithm.

When applying FCM algorithm, the distance between data points and clustering centers has an important influence on the objective function of the algorithm. Because the data set is the longitude and latitude of the aircraft, the distance it represents is the distance from each aircraft in the objective world to the cluster center (a point in reality). By analyzing the clustering results, the boundary location of air traffic subarea division and the central location of traffic management can be defined. In addition, because the experimental data used in this paper are real and anomalous longitude and latitude data of aircraft, it can make up for the shortcoming that FCM algorithm is sensitive to noise data[5].

2.4 Cluster validity index

Clustering analysis, as an unsupervised classification process, mainly depends on the user's experience and related domain knowledge in the selection of the number of clusters, which has great subjectivity. Therefore, it is necessary to use some index to verify the validity of clustering results.

Xie and Beni[6] put forward the effectiveness index based on the ratio of "compactness" to "separation", which has become one of the most representative indicators of effectiveness. Since then, many scholars have made continuous innovations on this basis. For data sets with good segregation, the proposed validity indicators can find the optimal number of clusters, but due to the non-uniform distribution of data and overlap between classes, the existing validity indicators can not effectively find the ideal number of clusters. The reason is that these indicators do not consider the overlap between classes, and data overlap between classes is one of the reasons for the misclassification. Considering that all the existing validity indicators do not involve the calculation of overlap between classes, this paper uses a new validity index[7] based on the full fusion of intra-class compactness and inter-class overlap.

2.4.1 aircraft compactness index. The compactness of data within a class is one of the important criteria and basic conditions for evaluating the validity of fuzzy clustering results. Based on FCM algorithm, this paper uses the compactness index in the validity analysis of fuzzy clustering, which is defined as follows:

$$Comp(c,\boldsymbol{U}) = \frac{1}{S} \times \sum_{i=1}^{n} \delta(\max_{1\le j\le c} u_{ij}) \tag{17}$$

$$\delta(\max_{1\le j\le c} u_{ij}) = \begin{cases} 1, & \max\limits_{1\le j\le c} u_{ij} \ge \alpha \\ \max\limits_{1\le j\le c} u_{ij}, & \beta \le \max\limits_{1\le j\le c} u_{ij} < \alpha \\ 0, & \max\limits_{1\le j\le c} u_{ij} < \beta \end{cases} \tag{18}$$

S is the number of data objects whose maximum membership degree satisfies condition $\max\limits_{1\le j\le c} u_{ij} \ge \beta$; α and β are two constants. When the maximum membership degree of the data object is greater than threshold α , the value of $\delta(\max\limits_{1\le j\le c} u_{ij})$ is 1, indicating that the data object belongs to the corresponding class; When the maximum membership degree is between α and β , the value of $\delta(\max\limits_{1\le j\le c} u_{ij})$ is equal to the maximum membership degree, indicating that the data object is most likely to belong to a certain class; When the maximum membership degree is less than β , it indicates that the data object belongs to a certain class to a lower degree and may be in the overlap area between classes. By calculating the value of $\delta(\max\limits_{1\le j\le c} u_{ij})$, the degree of tightness within the class can be obtained. The larger the $Comp(c,\boldsymbol{U})$ value is, the higher the degree of intra-class compactness of the fuzzy clustering is.

2.4.2 Region Overlap index. For data objects whose maximum membership degree is less than β, the maximum membership degree does not reach the threshold β, so it may be in the overlapping region of multiple class boundaries. In order to find such data points, threshold γ is introduced. If $\left|u_{ip}-u_{iq}\right| \le \gamma$, data object x_i is considered to be in the overlapping region of class p and class q. Overlap index is defined as follows:

$$Overlap(c,\boldsymbol{U}) = \frac{1}{n} \times \frac{2}{c(c-1)} \times \sum_{i=1}^{n} \sum_{\forall 1 \le p,q \le c}^{R} \varphi(\left|u_{ip}-u_{iq}\right|) \tag{19}$$

$$\varphi(\left|u_{ip}-u_{iq}\right|)_{1 \le p,q \le c} = \begin{cases} 1, \max\limits_{1 \le j \le c} u_{ij} < \beta \& \left|u_{ip}-u_{iq}\right| \le \gamma \\ 0 \qquad \text{other} \end{cases} \tag{20}$$

R is the number of matrix elements satisfying both conditions $\max\limits_{1 \le j \le c} u_{ij} < \beta$ and $\left|u_{ip}-u_{iq}\right| \le \gamma$. Setting threshold γ, when the maximum membership degree is less than β, that is, the data object is in the overlapping region of multiple class boundaries, if $\left|u_{ip}-u_{iq}\right| \le \gamma$ is satisfied at the same time, it shows that the data object belongs to the two classes equally, so that the $\varphi(\left|u_{ip}-u_{iq}\right|)_{1 \le p,q \le c}$ value at this time is equal to 1, and the overlapping definition $Overlap(c,\boldsymbol{U})$ is obtained by adding all the above conditions to average. The smaller the $Overlap(c,\boldsymbol{U})$ value, the lower the degree of clustering overlap.

2.4.3 Comprehensive validity index. Based on the above compactness index and overlap index, a comprehensive validity index is proposed. For the calculation of compactness index and overlap index, The value of c is $c = 2,3,...,50$. The maximum values are obtained and normalized respectively:

$$FComp(c,\boldsymbol{U}) = \frac{Comp(c,\boldsymbol{U})}{\max\limits_{2 \le c \le 50} Comp(c,\boldsymbol{U})} \tag{21}$$

$$FOverlap(c,\boldsymbol{U}) = \frac{Overlap(c,\boldsymbol{U})}{\max\limits_{2 \le c \le 50} Overlap(c,\boldsymbol{U})} \tag{22}$$

Combining formula (21) and formula (22), the comprehensive effectiveness index can be obtained:

$$F = FComp(c,\boldsymbol{U}) - FOverlap(c,\boldsymbol{U}) \tag{23}$$

The index shows that the greater the degree of compactness within a class, the greater the value of $FComp(c,\boldsymbol{U})$; the smaller the degree of overlap between classes, the smaller the value of $FOverlap(c,\boldsymbol{U})$. Obviously, the better the clustering results, the larger the F value. Therefore, the number of cluster corresponding to the maximum value of F is the optimal number of clusters.

2.5 Clustering Boundary Recognition

When using clustering method to partition the global air traffic area, it is very important to recognize the clustering boundary. After processing the longitude and latitude of aircraft with clustering algorithm only, the classification of the longitude and latitude data of aircraft can be realized, and the result is only the scattered data points after classification. Therefore, in order to realize the division of air traffic area, it is necessary to recognize the boundary of scattered data points after clustering. In this paper, the method of establishing coordinate system for clustering center is used to determine the clustering

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boundary. The specific steps are as follows:

Step 1: As shown in Figure 1, take each cluster center point as the origin in turn, establish a Cartesian coordinate system, and divide all regions into n parts.

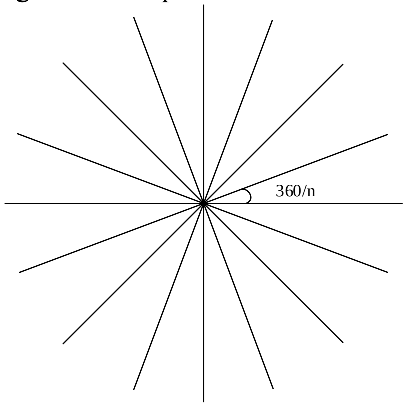

Figure 1. Cartesian coordinate system with cluster center as origin

Step 2: Calculate the distance from all points belonging to the same class to the clustering center, and select the points with the greatest distance from the clustering center from n partitioned regions to form a set of farthest points, like point P in Figure 2.

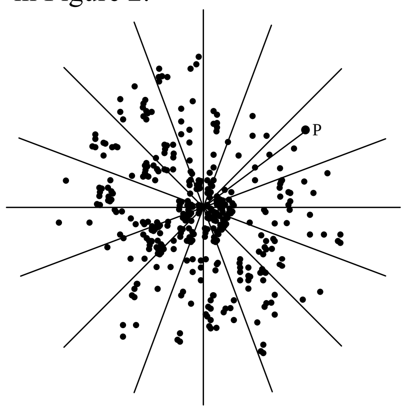

Figure 2. Farthest point recognition of cluster centers

Step 3: Remove some points from the set of farthest points and connect most of the remaining points to form convex hulls as shown in Figure 3 to establish clustering boundaries.

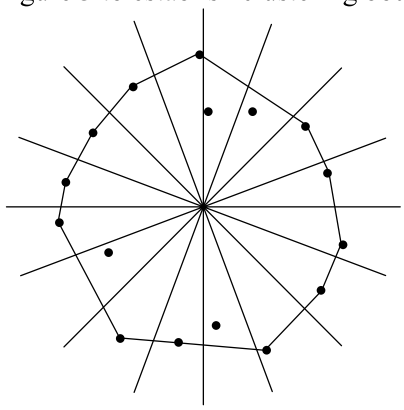

Figure 3. Convex hull formed by farthest point connection

3. Experiments and Results Analysis

3.1 Grey gravity prediction results

In this paper, a total of 21946512 data of take-off and landing flights from 2011 to 2017 in mainland China are selected, and the above flights are classified as statistical takeoff and landing volume of 257 airports as the input of grey gravity prediction model. At the same time, the minimum flight time and the minimum operating time of high-speed rail are calculated according to the statistics of 66049 cities (if there are no flights or no high-speed rail between the two cities, it is recorded as 0). The ratio of the shortest flight time to the minimum travel time of high-speed railway is taken as the traffic impedance function. First of all, the take-off and landing volume of 257 airports are predicted, and the prediction effect is evaluated by the calculated post-check ratio. Taking the top 10 airports in mainland China as an example, the posterior error is shown in Figure 4.

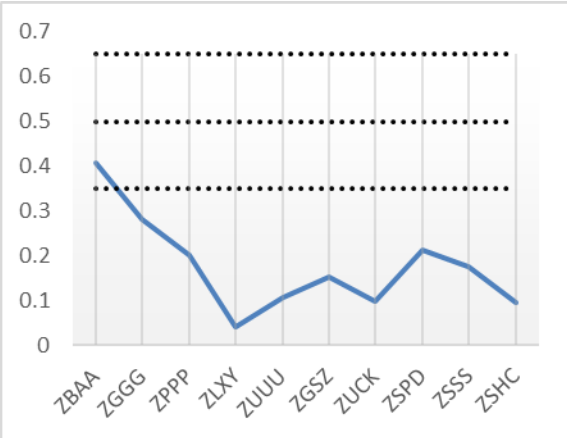

Figure 4. The top ten take-off volumes airport in mainland China as an example of posterior error

When the posterior error is less than 0.35, it is considered that the prediction result of grey prediction is excellent and the smaller the posterior error is, the better the prediction effect is. As can be seen from Figure 4, except for the posterior error of Beijing Capital Airport, the posterior error of the other nine airports is less than 0.35. Among the statistical grey prediction results of take-off and landing volume of 257 airports, the posterior error below 0.35 is as high as 93.7%.

The predicted value of airport take-off and landing volume is replaced by the gravity model with the ratio of the shortest flight time to the minimum travel time of high-speed railway as the traffic impedance function. The results can be better than the direct use of grey prediction to predict the number of flights between airport pairs. Taking the top 10 airport pairs in 2017 as an example, the comparison effect is shown in Figure 5.

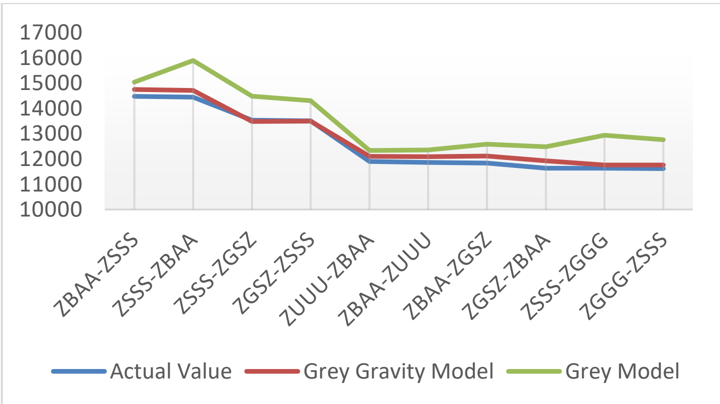

Figure 5. Comparison of the forecasting results of the top ten airports in 2017

According to Figure 5, the prediction deviation of the grey gravity model is smaller than that of the traditional grey prediction model. In the prediction of the top 10 airport pairs in 2017, the average deviation is only 1.41%. The grey gravity model is used to predict the flight volume of the above 10 airports in 2025, the Figure 6 can be obtained.

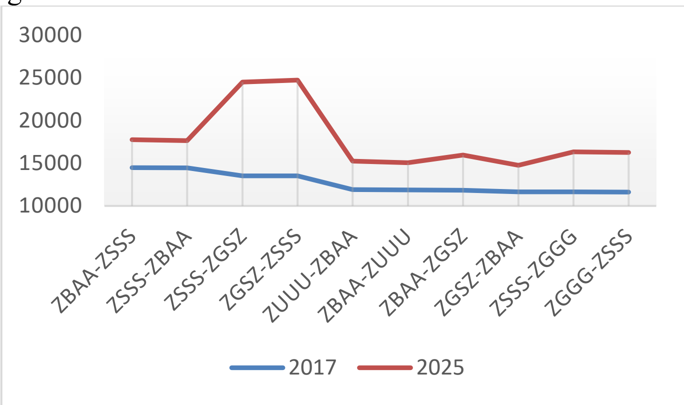

Figure 6. The number of flights at the top ten airports in 2025

As can be seen from Figure.6, the number of flights between the top ten airport pairs has increased to varying degrees in 2025, especially between Shenzhen and Shanghai due to the small ratio of flight time to high-speed rail travel time. It is the airport pair with the largest number of round trip flights in mainland China.

3.2 Simulation and traffic partition results

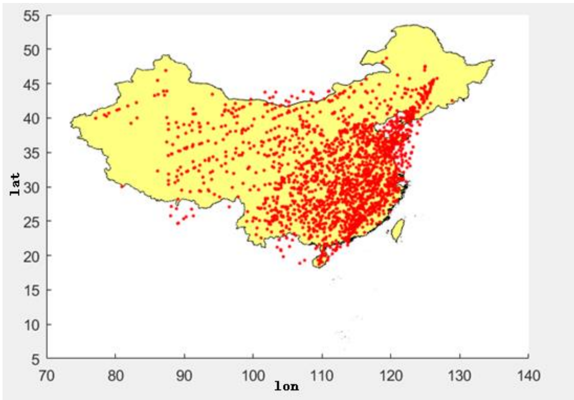

Figure 7. flight position simulation

Flights in mainland China at noon on October 2, 2025 were simulated. As shown in Figure 7, there were 2111 flight points. As can be seen from the figure, flights in central and southern China were particularly intensive in 2025, while those in northwest China were relatively sparse.

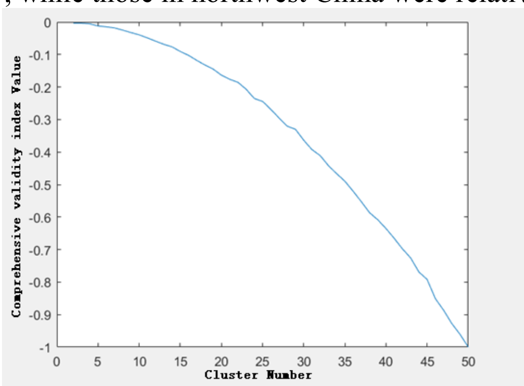

Figure 8. Evaluation of clustering results

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It can be seen from Figure 8. that the clustering number and the traffic partition clustering evaluation index value show an approximately negative correlation relationship, that is, the larger the clustering number, the smaller the clustering evaluation index value. The clustering evaluation index takes into account both the tightness of flights within classes and the overlap of traffic regions between classes. Although the increase of the number of clusters can strengthen the compactness within classes, it will also lead to the geometric growth of overlap between classes. This results in this phenomenon of approximate negative correlation.

Table 1. Comprehensive validity index under different cluster numbers

Cluster number	2	3	4	5
October 2td	-0.0057	-0.0039	-0.0049	-0.0122

As can be seen from Figure 8, the maximum value must be among the first five categories. From the specific calculation results given in Table 1, it can be seen that at 12 o'clock on October 2, 2025, when the number of clusters is 3, the cluster evaluation index value is the largest. This shows that according to the characteristics of the aircraft longitude and latitude data at that time, it is suitable to divide the air traffic system in mainland China into three categories. Taking the optimal clustering number as the input parameter, the FCM algorithm is implemented, and the clustering boundary is identified, and the result of dividing the air traffic region of mainland China into Northwest China, Central South and North China as shown in Figure 9. is obtained.

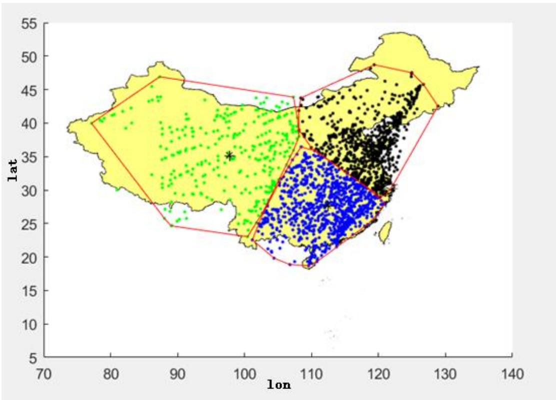

Figure 9. Results of air traffic division in mainland China

4. Conclusion

Based on grey prediction model and gravity model, this paper also considers the impact of high-speed rail on civil aviation flight volume, and constructs a grey gravity prediction model.Experiments show that the grey gravity model has better forecasting effect than the traditional grey forecasting model in forecasting airport traffic volume in the future. According to the prediction results, the longitude and latitude data of aircraft at specific time in the future are obtained by simulation. The optimal number of regional clustering is obtained by using the longitude and latitude data of aircraft and air traffic clustering evaluation index. Then the FCM clustering algorithm and clustering boundary recognition algorithm are used to identify the air traffic area in the mainland of China. The region is divided into three major air traffic areas: North China, Central South and Northwest China. According to the method proposed in this paper, the regions of air traffic management in mainland China can be divided, which can provide reference for the construction and development of air traffic management organization, operation mechanism, rules and procedures, system tools in mainland China.

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