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Study of temperature fields in a ball bearings bodies under boundary conditions of the third kind

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Abstract. The reliability of the rolling bearing body directly affects the continuity and duration of operation of industrial equipment. Ball bearings are often used in centrifugal pumps of thermal power plants and water supply systems. Overheating and subsequent failure of such bearings lead to failure of pumping equipment. To avoid such breakdowns, it is necessary to study the thermal processes inside the rolling elements, which can lead to their deformation and destruction. In this paper, the problems of non-stationary heat transfer inside rolling bodies (spherical bodies) are considered. An analytical solution of the problem in dimensionless values of temperatures and coordinates is obtained on the basis of Fourier and Bubnov-Galerkin methods under boundary conditions of the third kind. The solution of such a problem requires the use of an extensive mathematical apparatus, so special software tools such as Mathcad and MatLab were used in the work. So solving the problem analytically in dimensionless quantities is convenient because it allows you to scale the results to all such objects without reference to a specific rolling body. As a result, the solution is obtained in the form of a power polynomial that does not contain special functions.

1. Introduction

Ball bearings are widely used in modern industry (from agriculture to aerospace) and in the energy industry. The reliability of most technological equipment depends on it condition. Ball bearings are everywhere used in centrifugal pumps at thermal power plants, as well as in step-up pumping stations of heat supply and water supply systems. The analysis of actual works showed [1-5] that the most part of breakdowns of the pumping equipment is connected with failure of bearings. For example, [1] bearing defects at Nile pumping stations resulted in an 18% reduction in pump efficiency.

Therefore, the study of the influence of various factors (external and internal) on the characteristics of centrifugal pump bearings is of particular interest [6]. In the course of its operation, the rolling bearing body is heated because of friction forces, especially when there is insufficient or complete absence of lubricants. This can cause deformation of the rolling body, jamming and destruction of the bearing. Many studies [7-9] of thermal processes in bearings have shown a negative effect of overheating of the rolling body on the operation of the bearing and all equipment (centrifugal pump).

There are several works [10-16], which are aimed at studying the thermal processes in the rolling bodies of bearings. However, the majority of researchers consider questions of thermal interaction of rolling bodies with bearing lubrication, without paying due attention to internal thermal processes of a rolling body. These processes can cause structural damage inside the bearings. Thus, the actual problem is to study the processes of thermal conductivity inside the rolling bearings bodies (spherical bodies). The results of this study will allow to better analyze the work of centrifugal pump bearings at

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thermal stations and can be used in the system of diagnostics and automatic control of pumping equipment of the station. Moreover, the results of the study should be scaled to bearings of different sizes under different external conditions (ambient temperature), so the task must be solved in dimensionless values of temperatures and coordinates.

Statement of the problem: the boundary value problem of thermal conductivity for a sphere under symmetric boundary conditions of the third kind will have the following form:

$$\frac{\partial t(x,\tau)}{\partial \tau} = a\left(\frac{\partial^2 t(x,\tau)}{\partial x^2} + \frac{2}{x}\frac{\partial t(x,\tau)}{\partial x}\right) \qquad (\tau > 0; \quad 0 \le x \le r); \tag{1}$$

$$t(x,0) = t_0; \tag{2}$$

$$\partial t(0,\tau)/\partial x = 0; \tag{3}$$

$$-\lambda\frac{\partial t(r,\tau)}{\partial x} = \alpha(t(r,\tau) - t_{\text{ср}}), \tag{4}$$

where t – temperature; x – coordinate; τ – time; t_0 – initial temperature; $t_{\text{ст}}$ – wall temperature; a – thermal diffusivity; r – radius of a sphere.

Using dimensionless parameters, we reduce the problem (1) – (4) to a dimensionless form

$$\frac{\partial \Theta(\xi,\text{Fo})}{\partial \text{Fo}} = \frac{\partial^2 \Theta(\xi,\text{Fo})}{\partial \xi^2} + \frac{2}{\xi}\frac{\partial \Theta(\xi,\text{Fo})}{\partial \xi} \qquad (\text{Fo} > 0; \quad 0 < \xi < 1); \tag{5}$$

$$\Theta(\xi,0) = 1; \tag{6}$$

$$\frac{\partial \Theta(0,\text{Fo})}{\partial \xi} = 0; \tag{7}$$

$$\frac{\partial \Theta(1,\text{Fo})}{\partial \xi} + \text{Bi}\,\Theta(1,\text{Fo}) = 0, \tag{8}$$

where $\Theta = (t - t_{\text{ср}})/(t_0 - t_{\text{ср}})$ – dimensionless temperature; $\xi = x/r$ – dimensionless coordinate; $\text{Fo} = (a\tau)/r^2$ – Fourier criterion (dimensionless time).

2. Method of solution

2.1. Analytical solution

The approximate analytical solution is based on the joint application of the method of separation of variables and orthogonal methods of weighted residuals. Obtaining a high-precision approximate solution of the thermal conductivity problem is achieved by directly satisfying the differential equation of the Sturm-Liouville boundary value problem in a given number of points of the spatial variable [17].

Obtaining a solution is limited only by the possibility of separating the variables in the original differential equation. At the same time, there are practically no restrictions on the form of the differential equation of The Sturm-Liouville boundary value problem obtained after the separation of variables.

The solution of the problem (5-8), following the method of separation of variables, is found in the form

$$\Theta(\xi,\text{Fo}) = \phi(\text{Fo})\psi(\xi). \tag{9}$$

Substituting (9) in (5), we find

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$$\frac{d\phi(Fo)}{dFo}+\nu\phi(Fo)=0; \tag{10}$$

$$\frac{d^2\psi(\xi)}{d\xi^2}+\frac{d\psi(\xi)}{d\xi}\frac{2}{\xi}+\nu\psi(\xi)=0, \tag{11}$$

where ν − some constant.

The solution of equation (10) is known and has the form

$$\phi(\mathrm{Fo})=A\exp(-\nu\mathrm{Fo}), \tag{12}$$

where A – unknown coefficient.

Substituting (9) into (7), (8) we obtain

$$d\psi(0)/d\xi=0; \tag{13}$$

$$\frac{\partial\psi(1)}{\partial\xi}+\mathrm{Bi}\cdot\psi(1)=0. \tag{14}$$

Solving the Sturm-Liouville boundary value problem (11), (13), (14) accepted as

$$\psi(\xi)=B_0+\sum_{i=1}^{r}B_i\xi^{i+1}, \tag{15}$$

where $B_i\ (i=\overline{1,\ r})$ – unknown coefficients. Note that the relation (15) satisfies the boundary condition (13).

The relation (13) allows to enter one more boundary condition

$$\psi(0)=const=1. \tag{16}$$

Substituting (15) in (16), we find B_0 = 1. Demand that the relation (15) satisfies the boundary condition (14) and the equation (11) at the points $\xi=0;1/3;2/3;1$. Substituting (15), limited to five terms of the series, in the ratio (14) and equation (11), with respect to points $\xi=0;1/3;2/3;1$ with respect to unknown coefficients $B_i\quad(i=\overline{1,5})$ we obtain a system of five algebraic linear equations. Then from the solution of this system at Bi = 1 we find the coefficients $B_i\quad(i=\overline{1,5})$:

$$B_1=-\frac{\nu}{6};$$

$$B_2=\frac{13\nu^4-2943\nu^3+179250\nu^2-2906280\nu+6123600}{3c};$$

$$B_3=-\frac{29\nu^4-7782\nu^3+563580\nu^2-9520740\nu+20207880}{3c};$$

$$B_4=\frac{9\nu^4-2601\nu^3+197892\nu^2-3450600\nu+7348320}{c};$$

$$B_5=-\frac{3\nu^4-900\nu^3+69390\nu^2-1229580\nu+2624400}{c};$$

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$$c = 4\nu^3 - 512\nu^2 + 1620\nu - 204120.$$

Find the integral of the weighted residual of equation (11)

$$\int_0^1 \left[\frac{\partial^2}{\partial \xi^2}\left(1+\sum_{i=1}^{r} B_i \xi^{i+1}\right) + \frac{2}{\xi}\frac{\partial}{\partial \xi}\left(1+\sum_{i=1}^{r} B_i \xi^{i+1}\right) + \nu\left(1+\sum_{i=1}^{r} B_i \xi^{i+1}\right)\right] d\xi = 0. \quad (17)$$

Calculating the integrals in (17), taking into account the found values of the coefficients $B_i \quad (i=\overline{1,5})$ with respect to the eigenvalues ν_k, we obtain an algebraic equation of the fifth degree

$$\nu^5 - 1161\nu^4 + 246360\nu^3 - 14911200\nu^2 + 243777600\nu - 514382400 = 0. \quad (18)$$

From the solution of equation (18) we obtain five eigenvalues $\nu_1 = 907.242263$; $\nu_2 = 166.248673$; $\nu_3 = 63.157269$; $\nu_4 = 21.884347$; $\nu_5 = 2.467446$.

Substituting (12), (15) in (9), for each eigenvalue we will have partial solutions of the form

$$\Theta_k(\xi, Fo) = A_k \exp(-\nu_k \mathrm{Fo})\left(1+\sum_{i=1}^{r} B_i(\nu_k)\xi^{i+1}\right). \quad (19)$$

Each particular solution of (19) exactly satisfies the boundary conditions (7), (8) and approximately (in the third approximation) – equation (5). However, none of them, including their sum

$$\Theta(\xi, Fo) = \sum_{k=1}^{n}\left[A_k \exp(-\nu_k \mathrm{Fo})\left(1+\sum_{i=1}^{r} B_i(\nu_k)\xi^{i+1}\right)\right], \quad (20)$$

do not satisfy the initial condition (6). To fulfill the initial condition, its residual is made and the orthogonality of the residual to each eigenfunction is required, i.e.

$$\int_0^1 \left\{ \sum_{k=1}^{5}\left[A_k\left(1+\sum_{i=1}^{r} B_i(\nu_k)\xi^{i+1}\right)\right] - 1\right\} \psi_j(\nu_j, \xi) d\xi = 0. \quad (j=1,\,2,\,3;\; r=5) \quad (21)$$

By calculating the integrals in (21) to find $A_k \quad (k=\overline{1,5})$ we obtain a system of five algebraic linear equations. Its decision:

$$A_1 = 0.018464;\; A_2 = -0.108447;\; A_3 = 0.228104;\; A_4 = -0.398377;\; A_5 = 1.265116.$$

The results of calculations by the formula (20) in the fourth approximation are presented in Fig. 1. Their analysis allows us to conclude that in the range of numbers $0{,}01 \le \mathrm{Fo} < \infty$ the difference between the obtained solution and the numerical one does not exceed 1.5%.

To improve the accuracy of the solution, it is necessary to increase the number of members of the series (15). To obtain additional equations in order to determine the unknown coefficients of B_i we will increase the number of points along the ξ, coordinate in which equation (11) should be performed. And, in particular, taking the number of such points equal to 8 (with a step $\Delta\xi = 1/7$, starting from point $\xi = 0$), we obtain 9 equations with respect to unknown coefficients B_i (another equation is added as a result of the boundary condition (14)). After solving the system of these equations and finding the coefficients $B_i \, (i=\overline{1,10})$ the further course of the solution is repeated.

In Figure 1 in addition to the numerical solution and the solution in the 4-th approximation, as represented by decision in the 8th approximation and from the analysis of the temperature distribution can

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be judged to increase the solution accuracy by increasing the number of approximations. In addition, the discrepancy between the eighth approximation and the numerical solution in the range of numbers $0.01 \le \text{Fo} < \infty$ does not exceed 0.5%.

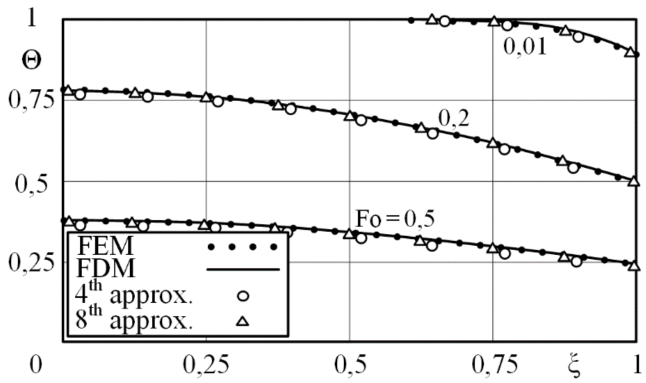

Figure 1. Graphs of temperature changes in the ball.

Figure 2 shows the residual of differential equation (5) in the 4th, 6th and 8th approximations at $\text{Fo} = 0.01$, the analysis of which allows us to conclude that the above method converges.

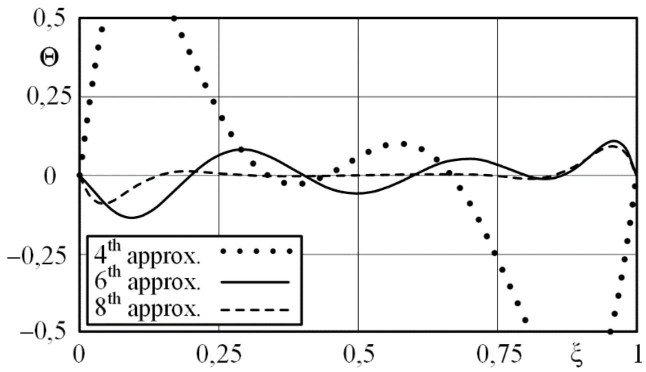

Figure 2. Residual of the differential equation.

2.2. Solution in ANSYS

In the framework of this work, the problem of thermal conductivity for the ball was also solved in the ANSYS. In order to compare the solution obtained in ANSYS and dimensionless analytical solution, the corresponding properties and parameters presented in table 1 were set.

Table 1. Properties and parameters.

Property	Value	Unit of measurement
Density ρ	1	kg/m^3
Specific heat C	1	J/kg·°C
Thermal conductivity λ	1	W/m·°C
Ambient temperature t_{cp}	0	°C
Initial temperature t_0	1	°C
Heat transfer coefficient α	1	W/m^2°C
The radius of the ball r	1	m

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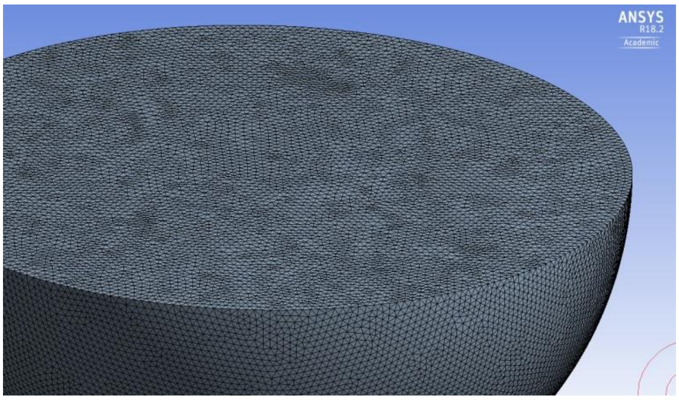

Figure 3. The original geometry.

The solution obtained as a result of simulation of the studied process in ANSYS allows to estimate the body temperature at any point and at any time, as well as to derive graphical and tabular temperature distributions. Figure 4 shows the temperature distribution at $\mathrm{Fo} = 0{,}01$. The obtained data clearly illustrate the process of thermal conductivity in the direction from the center of the ball to the outside. Also, the inverse dependence of the heat conduction process of the ball will have the same form.

Table 2 shows the specific temperature values imported from the ANSYS solution at different points of the globe at different times, and Figure 1 they can be compared with numerical and analytical solution.

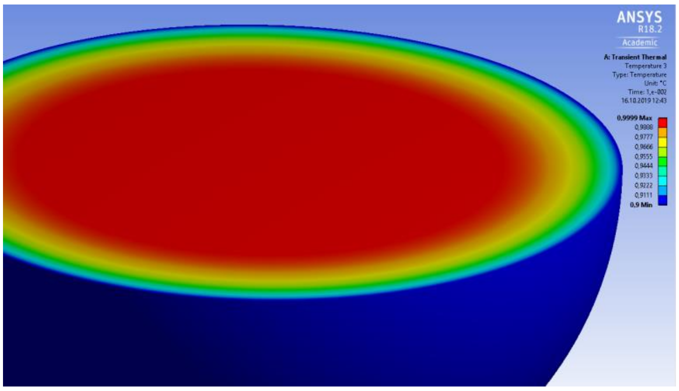

Figure 4. Temperature distribution.

3. Conclusion

As a result of joint application of a method of separation of variables and orthogonal methods of the weighted residuals the approximate analytical solution allowing to estimate thermal processes in a sphere (bearing) has been received. The obtained dependences can be presented in a simple and logical form, which will allow to estimate the influence of thermal conductivity processes on the rolling bodies of bearings in different temperature conditions of the environment, also these mathematical dependences can be easily converted into a program code that can be used in automation systems, diagnostics systems and control systems of the centrifugal pump.

A variant of the solution of this problem in the ANSYS program was also described, the results of which correspond to the analytical solution. However, the solution of such tasks in the program ANSYS require the installation of this software and, in addition, there is a need for high computing power of the PC.

Table 2. Temperature values imported from the ANSYS.

ξ	Fo 0.01	0.2	0.5
0	0.99990	0.78271	0.41581
0.05	0.99990	0.78209	0.41538
0.10	0.99989	0.78023	0.41412
0.15	0.99987	0.77712	0.41202
0.20	0.99983	0.77275	0.40909
0.25	0.99978	0.76710	0.40534
0.30	0.99969	0.76015	0.40079
0.35	0.99957	0.75188	0.39544
0.40	0.99938	0.74226	0.38931
0.45	0.99909	0.73128	0.38243
0.50	0.99865	0.71890	0.37481
0.55	0.99798	0.70511	0.36649
0.60	0.99694	0.68990	0.35750
0.65	0.99535	0.67326	0.34785
0.70	0.99289	0.65521	0.33760
0.75	0.98906	0.63576	0.32676
0.80	0.98309	0.61495	0.31539
0.85	0.97375	0.59284	0.30352
0.90	0.95913	0.56950	0.29120
0.95	0.93615	0.54502	0.27846
1	0.90001	0.51952	0.26536

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