

An elaborated diffusion mathematic model of radiative transfer in an extinction medium

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Abstract. The computational method of diffusion approach, used sometimes for simulating the radiative heat transfer, differs in simplicity of its algorithm, but can not take account of radiation anisotropy. The reasons of its insufficient accuracy have been revealed; it is shown that inadequate boundary conditions produce unpredictable errors. Ways for eliminating shortcomings of the diffusion method have been developed. The accurate diffusion model has been theoretically elaborated for radiative heat transfer in dust-laden gases. To eliminate singularity of temperature curves on the surface of confining walls, the enclosed extinction medium was hypothetically considered as an unbounded one. The radiation intensity has been expanded in a series and integrated over the spherical solid angle, what has allowed to obtain more precise differential equation for the resulting radiation flux both in the unbounded and enclosed extinction medium. As a result, shortcomings peculiar to the previous method of diffusion approach have been eliminated, and correct Neumann and Robin boundary conditions have been formulated. As a result, the elaborated diffusion model got the increased accuracy in simulating radiative heat transfer, and concurrently it offers an advantage of simplicity and universality of the diffusion approach method.

Key words: radiative heat transfer, selective gases, scattering dust particles, boundary conditions, diffusion mathematic model.

1. Introduction

The efficiency of many high-temperature technological plants can be improved with the help of numerical researching the radiative heat transfer in dust-laden gases. To ensure adequate simulation results, it is necessary to admit that triatomic gases emit and absorb the radiant energy selectively, generally in lines of the spectrum [1], while particles of the technological dust or ash usually form the gray extinction medium, which can scatter rays. An elongated scattering indicatrix can be approximately replaced by an isotropic one [2], considering that exactly forward scattering is equivalent to no scattering at all [3].

This assumption allows to write down the conservation differential equation for the radiation intensity I_l in any ray direction:

$$\frac{\partial I_l}{\partial l} = \alpha_p I_o - k I_l + \beta \bar{I}, \quad (1)$$



where I_0 is the total intensity of equilibrium radiation, $\text{W}/(\text{m}^2 \cdot \text{sr})$; α_p is the mean Planck absorption coefficient, characterizing the common emitting capacity of the medium, m^{-1} ; k is its local extinction coefficient, m^{-1} ,

$$k = \alpha + \beta;$$

α is the local absorption coefficient of the medium, m^{-1} ; β is the isotropic-scattering coefficient of dust particles, m^{-1} ; l is a distance along the ray, m ; \bar{I} is a local radiation intensity, averaged in the spherical solid angle, $\text{W}/(\text{m}^2 \cdot \text{sr})$,

$$\bar{I} \equiv \frac{1}{4\pi} \int_{(4\pi)} I_l d\omega = \frac{\sigma}{\pi} T_{\text{rad}}^4;$$

σ is the Stefan–Boltzmann constant, $\text{W}/(\text{m}^2 \cdot \text{K}^4)$; T_{rad} is the radiant temperature [4], K,

$$T_{\text{rad}}^4 \equiv \frac{1}{4\sigma} \int_{(4\pi)} I_l d\omega; \quad (2)$$

ω is the solid angle, sr, defined by means of the plain polar angle θ and azimuthal one ψ ,

$$d\omega = \sin\theta d\theta d\psi.$$

Integrating the equation (1) over spherical solid angle gives the conservation differential equation for radiation flux:

$$\text{div} \mathbf{q}_{\text{rad}} = 4\sigma (\alpha_p T^4 - \alpha T_{\text{rad}}^4), \quad (3)$$

where \mathbf{q}_{rad} is a vector of the resulting radiation flux, W/m^2 ; T is the thermodynamic temperature, K.

In scientific publications, much attention is given at present time to problems of adequate absorption coefficients of selective gases [5, 6]. Not less important is an aim to elaborate some easy algorithm for solving differential equations of the radiative transfer [7].

A main problem ensues from that there is not any distinct differential equation for interrelating the vector of the resulting radiation flux \mathbf{q}_{rad} with radiation parameters. Therefore it is usually found numerically by means of angular integrating projections of the radiation intensity:

$$q_{\text{rad}} = \int_{(4\pi)} I_l \cos\theta d\omega = \int_0^{2\pi} d\psi \int_0^\pi I_l \cos\theta \sin\theta d\theta, \quad (4)$$

such an algorithm requires the radiation intensity be computed previously at many points along numerous rays by solving the equation of radiative heat transfer (1).

Different simulating methods have been applied for this purpose. There are methods of discrete transfer [8], discrete ordinates [9], finite volumes [10], finite elements [11], natural elements [12], and a zonal one [13]. Despite their variety, all these algorithms display excess complicacy and too high time of numerical solving tasks.

A specific place is held by the method of spherical harmonics [14], which higher orders of approximation allow for spatial anisotropy of the radiation intensity with the help of numerical solving a sequence of differential equations. Comparison of various numerical models of heat transfer in a high-temperature flame [7] showed that the approach P-3 of this method yields results of high precision at smaller expense of the computer time. The shortest computation resides in the approach P-1 of the method of spherical harmonics. However, it does not consider the anisotropy of radiation, what leads to some raised error of computational results.

The approach P-1 essentially coincides with the method of diffusion approach, which has been considered in details in [15]. Its advantage is an easy compatibility with simulating algorithms of the

convective heat transfer. Therefore, it would be worthwhile to adapt the method of diffusion approach to the anisotropic radiative transfer in the extinction medium. The aim of this report is to represent a more perfect diffusion method for numeric solving the radiative heat-transfer equations in dust-laden gases.

2. Methods

An obvious source of anisotropic radiation intensity is the surface of the walls confining a volume of dusty gases. To exclude it temporarily from consideration, a concept of an unbounded medium has to be introduced. This hypothetical medium ought to have the same radiation properties and temperature, as the dust-laden gases in the confined volume. The hypothetical absence of the confining walls simplifies substantiating equations of radiative heat transfer, since the temperature curve in the unbounded medium can be presented by continuous smooth functions without any singularity in points that coincide with the surface of the confining walls.

An expression for the radiation intensity in the unbounded medium follows from the equation (1):

$$I_l = \frac{1}{k} \left(\alpha_p I_o + \beta \bar{I} - \frac{\partial I_l}{\partial l} \right).$$

It is significant to mention that such a representing form of the radiation intensity has sense only in respect to the energy of spectrum that would be absorbed in the extinction medium. As a consequence, it requires some new modes of computing the absorption coefficient for its using in the radiative heat transfer equation.

This mathematical form can be repeatedly differentiated along length of the ray, supposing that lumped radiation sources are absent there, and the extinction coefficient would be constant. Higher derivatives in right-hand sides of each of obtained equalities have to be replaced with expressions, represented by their next equalities. As a result, the radiation intensity will be expanded in a series:

$$I_l = \frac{\alpha_p}{k} \left(I_o - \frac{\partial I_o}{k \partial l} + \frac{\partial^2 I_o}{k^2 \partial l^2} - \frac{\partial^3 I_o}{k^3 \partial l^3} + \frac{\partial^4 I_o}{k^4 \partial l^4} - \frac{\partial^5 I_o}{k^5 \partial l^5} + \dots \right) + \frac{\beta}{k} \left(\bar{I} - \frac{\partial \bar{I}}{k \partial l} + \frac{\partial^2 \bar{I}}{k^2 \partial l^2} - \frac{\partial^3 \bar{I}}{k^3 \partial l^3} + \frac{\partial^4 \bar{I}}{k^4 \partial l^4} - \frac{\partial^5 \bar{I}}{k^5 \partial l^5} + \dots \right). \quad (5)$$

Equality (5) can be written more compactly as follows:

$$I_l = \frac{1}{k} \left[(\alpha_p I_o + \beta \bar{I}) + \sum_{n=1}^{\infty} (-1)^n \frac{\partial^n (\alpha_p I_o + \beta \bar{I})}{k^n \partial l^n} \right].$$

Hereinafter, a simplified one-dimensional consideration of spatial transformations will be applied, what, however, would not depreciate generality of the achieved mathematic results. Thus, it is assumed that a distance l along any ray and coordinate y in direction of the radiation-flux vector are interrelated by the ratio:

$$dl = dy / \cos \theta.$$

The series (5) is convergent under certain conditions and, consequently, it admits mathematical transformations. For example, its integrating over the solid angle in accordance with formulae (2) and (4) gives series of the radiant temperature and the resulting radiation-flux density in the unbounded medium:

$$T_{\text{rad}}^4 = \frac{1}{k} \left[(\alpha_p T^4 + \beta T_{\text{rad}}^4) + \frac{\pi}{\sigma} \sum_{n=1}^{\infty} \frac{\partial^{2n} (\alpha_p I_o + \beta \bar{I})}{(2n+1) k^{2n} \partial l^{2n}} \right], \quad (6)$$

$$q_{\text{rad}}^{\infty} = -\frac{4\sigma}{k} \left[\frac{\partial (\alpha_p T^4 + \beta T_{\text{rad}}^4)}{3k \partial y} + \frac{\pi}{\sigma} \sum_{n=1}^{\infty} \frac{\partial^{2n+1} (\alpha_p I_o + \beta \bar{I})}{(2n+3) k^{2n+1} \partial l^{2n+1}} \right]. \quad (7)$$

The series (6) can be written in the following form:

$$\alpha_p T^4 + \beta T_{\text{rad}}^4 = k T_{\text{rad}}^4 - \frac{\pi}{\sigma} \sum_{n=1}^{\infty} \frac{\partial^{2n} (\alpha_p I_o + \beta \bar{I})}{(2n+1) k^{2n} \partial l^{2n}}.$$

If the right-side part of this expression would be introduced under the sign of the first derivative in the series (7), then a more accurate expression will be obtained for the resulting radiation-flux density in the unbounded extinction medium:

$$q_{\text{rad}}^{\infty} = -\frac{4\sigma}{3k} \frac{\partial T_{\text{rad}}^4}{\partial y} - 4\pi \frac{\alpha_p}{k} \left(\frac{4}{45} \frac{\partial^3 I_o}{k^3 \partial y^3} + \frac{8}{105} \frac{\partial^5 I_o}{k^5 \partial y^5} + \frac{4}{63} \frac{\partial^7 I_o}{k^7 \partial y^7} + \dots \right) - \\ - 4\pi \frac{\beta}{k} \left(\frac{4}{45} \frac{\partial^3 \bar{I}}{k^3 \partial y^3} + \frac{8}{105} \frac{\partial^5 \bar{I}}{k^5 \partial y^5} + \frac{4}{63} \frac{\partial^7 \bar{I}}{k^7 \partial y^7} + \dots \right). \quad (8)$$

It can be produced in the follow generalized form:

$$q_{\text{rad}}^{\infty} = -\frac{4\sigma}{k} \left[\frac{\partial (\alpha_p T^4 + \beta T_{\text{rad}}^4)}{3k \partial y} - \frac{\pi}{\sigma} \sum_{n=1}^{\infty} \frac{4n}{3(2n+1)(2n+3)} \frac{\partial^{2n+1} (\alpha_p I_o + \beta \bar{I})}{k^{2n+1} \partial l^{2n+1}} \right].$$

The sum of items with higher partial derivatives can be replaced here with a simpler expression that has been got from the following approximate equality:

$$\frac{\partial}{k \partial y} \int I_l \cos^8 \theta d\omega = 4\pi \frac{\alpha_p}{k} \left[\frac{\sigma}{\pi} \frac{\partial T^4}{9k \partial y} + \left(\frac{\partial^3 I_o}{11k^3 \partial y^3} + \frac{\partial^5 I_o}{13k^5 \partial y^5} + \dots \right) \right] + \\ + 4\pi \frac{\beta}{k} \left[\frac{\sigma}{\pi} \frac{\partial T_{\text{rad}}^4}{9k \partial y} + \left(\frac{\partial^3 \bar{I}}{11k^3 \partial y^3} + \frac{\partial^5 \bar{I}}{13k^5 \partial y^5} + \dots \right) \right] \approx \frac{4\sigma}{9} \frac{\partial T_{\text{rad}}^4}{k \partial y}.$$

Indeed, the items with higher derivatives contain in this equality numerical coefficients that differ little from those in corresponding items of the right-hand side of the equation (8). As a final result, an easy and precise enough formula would substitute for the series (7):

$$q_{\text{rad}}^{\infty} \approx -\frac{4\sigma}{3k} \frac{\partial T_{\text{rad}}^4}{\partial y} - \frac{4\sigma}{9k} \left(\frac{\alpha}{k} \frac{\partial T_{\text{rad}}^4}{\partial y} - \frac{\alpha_p}{k} \frac{\partial T^4}{\partial y} \right).$$

It allows for anisotropy of the radiation in the hypothetical unbounded extinction medium.

3. Discussions

Radiative heat transfer in an unbounded extinction medium

To simplify equation forms, it would be advantageous to introduce the notion of the determining temperature T_Σ , which accounts for the summary influence of the radiant and thermodynamic temperatures on transferring the radiant energy in the unbounded extinction medium:

$$T_\Sigma^4 \equiv T_{\text{rad}}^4 + \frac{1}{3k} (\alpha T_{\text{rad}}^4 - \alpha_p T^4). \quad (9)$$

Following this, the specified differential expression of the resulting radiation-flux vector in the unbounded extinction medium takes a gradient form that is similar to the formula of the diffusion-approach method:

$$\mathbf{q}_{\text{rad}}^\infty = -\frac{4\sigma}{3k} \text{grad } T_\Sigma^4. \quad (10)$$

Following this, the specified differential expression of the resulting radiation-flux vector in the unbounded extinction medium takes a gradient form that is similar to the formula of the diffusion-approach method:

$$\text{div} \mathbf{q}_{\text{rad}}^\infty = \frac{4\sigma}{1 + \alpha/(3k)} (\alpha_p T^4 - \alpha T_\Sigma^4). \quad (11)$$

If the gradient expression (10) for the resulting radiation-flux vector would be introduced in the last equality, a specified transport equation for the radiation-heat transfer in the hypothetical unbounded medium can be obtain. It takes the following form in Cartesian coordinates x, y, z :

$$\frac{\partial}{\partial x} \left(\frac{\partial T_\Sigma^4}{k \partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\partial T_\Sigma^4}{k \partial y} \right) + \frac{\partial}{\partial z} \left(\frac{\partial T_\Sigma^4}{k \partial z} \right) + \frac{9}{3 + \alpha/k} (\alpha_p T^4 - \alpha T_\Sigma^4) = 0. \quad (12)$$

This equation differs from the differential equation of the diffusion-approach method both in a functional variable and in a numerical coefficient of its item.

The temperature of the hypothetical unbounded medium is not strictly determined outside the confined gaseous space. Hence, it makes sense to accept a physically reliable assumption that the temperature curves have inflection points that coincide with the surface of enclosing walls. According to this postulate, all the derivatives of even orders are equal to zero at these points.

At the flexion points of the temperature curve, the series (6) establishes the following relation between the radiant and thermodynamic temperatures:

$$T_{\text{rad.bn}}^4 = \frac{1}{k} (\alpha_p T_w^4 + \beta T_{\text{rad.bn}}^4),$$

where T_w is the wall thermodynamic temperature; $T_{\text{rad.bn}}$ is a radiant temperature at points of the unbounded extinction medium that coincide with the wall surface.

This relation can be reduced to explicit form by means of identical transformations:

$$\alpha T_{\text{rad.bn}}^4 = \alpha_p T_w^4.$$

Finally, boundary conditions of the differential equation (12) would be found with the help of the formula (9) for determining temperature:

$$\alpha T_{\Sigma.bn}^4 = \alpha_p T_w^4, \quad (13)$$

were $T_{\Sigma, \text{bn}}$ is a determining temperature at points of the unbounded extinction medium that coincide with the wall surface.

Radiative heat transfer in the enclosed medium

A difference between the effective radiation flux, propagating from confining walls, and the hypothetical radiation flux outside of the unbounded extinction medium forms an anisotropic additional radiation flux. Separate calculating absorption of the additional radiation would not only complicate the computing algorithm, but also would introduce new errors into the computer simulation. It is more appropriate to find conditions allowing to apply the equations (9) – (12) to the summary radiation transferred. With the defining temperature in enclosed space denoted as T_s , they receive following forms:

$$T_s^4 \equiv T_{\text{rad}}^4 + \frac{1}{3k} (\alpha T_{\text{rad}}^4 - \alpha_p T^4), \quad (14)$$

$$\mathbf{q}_{\text{rad}} \approx -\frac{4\sigma}{3k} \text{grad } T_s^4, \quad (15)$$

$$\text{div} \mathbf{q}_{\text{rad}} = \frac{4\sigma}{1 + \alpha/(3k)} (\alpha_p T^4 - \alpha T_s^4), \quad (16)$$

$$\frac{\partial}{\partial x} \left(\frac{\partial T_s^4}{k \partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\partial T_s^4}{k \partial y} \right) + \frac{\partial}{\partial z} \left(\frac{\partial T_s^4}{k \partial z} \right) + \frac{9}{3 + \alpha/k} (\alpha_p T^4 - \alpha T_s^4) = 0. \quad (17)$$

Despite formal identity, expression (14) differs in values of its radiant temperature T_{rad} from the analogue (9). Also, the differential equations (12) and (17) might produce different results being solved with distinct boundary conditions.

The radiation fluxes that are required for stating the boundary conditions on surface of the walls are depicted in Fig. 1. It is accepted here that the effective radiation of walls is diffusive, i.e. it is emitted and reflected with equal radiation intensity in all the directions within the hemispherical solid angle.

When the confined dust-laden gaseous volume has a fairly large optical thickness, the additional radiation propagated from the opposite wall is expected to be absorbed completely by the medium. Thus, it may be assumed, that the incident radiation flux on the enclosing walls practically equals in this case to such of a flux produced by the hypothetical unbounded medium with the same temperature field. This radiation flux can be determined by integrating the series (5) of the radiation intensity, multiplied by $\cos \theta$, over the hemispherical solid angle. Since the even temperature derivatives at the bounds of the domain are supposed to be equal to zero, the surface density of the incident radiation flux would be represented as follows:

$$E_{\text{inc}} = \frac{\alpha_p}{\alpha} \sigma T_w^4 - 2 \frac{\pi}{k} \sum_{n=0}^{\infty} \left[\frac{\partial^{2n+1} (\alpha_p I_o + \beta \bar{I})}{(2n+3) k^{2n+1} \partial l^{2n+1}} \right]_{\text{bn}}$$

where subscript "bn" denotes derivatives of the radiation intensity found at the points of the unbounded medium that coincide with the wall surface.

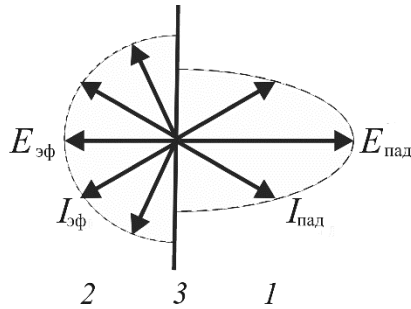


Figure 1. A scheme of radiation fluxes on surface of the walls: 1 – incident radiation 2 – effective radiation; 3 – wall surface.

This equality would be presented more obviously in a wide form:

$$E_{\text{inc}} = \frac{\alpha_p}{\alpha} \sigma T_w^4 - 2\pi \frac{\alpha_p}{k} \left[\frac{1}{3k} \left(\frac{\partial I_o}{\partial y} \right)_{\text{bn}} + \frac{1}{5k^3} \left(\frac{\partial^3 I_o}{\partial y^3} \right)_{\text{bn}} + \frac{1}{7k^5} \left(\frac{\partial^5 I_o}{\partial y^5} \right)_{\text{bn}} + \dots \right] -$$

$$- 2\pi \frac{\beta}{k} \left[\frac{1}{3k} \left(\frac{\partial \bar{I}}{\partial y} \right)_{\text{bn}} + \frac{1}{5k^3} \left(\frac{\partial^3 \bar{I}}{\partial y^3} \right)_{\text{bn}} + \frac{1}{7k^5} \left(\frac{\partial^5 \bar{I}}{\partial y^5} \right)_{\text{bn}} + \dots \right].$$

From here, in view of the series (7), a simple expression will be obtained for the surface density of the incident radiation flux on the walls that enclose the extinction medium:

$$E_{\text{inc}} \approx \frac{\alpha_p}{\alpha} \sigma T_w^4 + \frac{1}{2} q_{\text{rad.w}}^{\infty} \quad (18)$$

The surface density of the effective flux of radiation emitted and reflected by the confining walls can be determined with the well-known formula [4]:

$$E_{\text{eff}} = \frac{\alpha_p}{\alpha} \sigma T_w^4 + \left(\frac{1}{\varepsilon_w} - 1 \right) q_{\text{rad.w}}, \quad (19)$$

where ε_w is the emissivity of the wall surface; $q_{\text{rad.w}}$ is the density of the resulting radiation flux from the extinction medium to the wall. The ratio of absorption coefficients had been introduced in formula (18), in order to match more accurately the radiation fluxes of the walls and those of a volume of the extinction medium in accordance with the equality (13).

The difference between the incident (18) and effective (19) radiation fluxes gives a formula that connects on the wall surface the resulting radiation fluxes in confined and unbounded extinction media:

$$q_{\text{rad.w}} = q_{\text{rad.w}}^{\infty} \varepsilon_w / 2. \quad (20)$$

Boundary conditions

In the diffusion approach method, the boundary conditions on the wall surface are inherently approximate. Besides, the incident radiation is assumed, as a rule, to be diffusive. Such a rough stating leads to unpredictable mistakes in numerical simulations.

In the elaborated diffusion model, Neumann more accurate boundary conditions for the differential equation (17) directly follow from a ratio (20) in view of the gradient expressions (10) and (15):

$$\left(\partial T_s^4 / \partial y \right)_w = \frac{\varepsilon_w}{2} \left(\partial T_{\Sigma}^4 / \partial y \right)_w. \quad (21)$$

The right-side part of this equality has to be computed in each of iteration after solving the differential equation (12) of radiative heat transfer in the unbounded medium.

Robin boundary conditions can be derived from the balance equation that interrelates the radiant temperature and surface fluxes of radiation [16]:

$$4\sigma T_{\text{rad.w}}^4 = m_{\text{eff}} E_{\text{eff}} + m_{\text{inc}} E_{\text{inc}},$$

where $T_{\text{rad.w}}$ is the radiant temperature, calculated on the wall surface; m_{eff} and m_{inc} are ratios of hemispherical integrals that determine volumetric and surface radiation fluxes on the wall. It can be assumed $m_{\text{eff}}=2$ for the diffused radiation of wall.

If to take into consideration formulae (17) and (18) for densities of incident and effective radiation fluxes, the preceding equality would be reduced to an expression that contains only coefficient m in its last item:

$$4\sigma T_{\text{rad.w}}^4 = 4 \frac{\alpha_{\text{p}}}{\alpha} \sigma T_{\text{w}}^4 + (1 - \varepsilon_{\text{w}}) q_{\text{rad.w}}^{\infty} + \frac{m}{2} q_{\text{rad.w}}^{\infty}. \quad (22)$$

This coefficient is defined here as follows:

$$m = \left[\int_{(2\pi)} I_l d\omega - 2 \frac{\sigma}{k} (\alpha_{\text{p}} T_{\text{w}}^4 + \beta T_{\text{rad.bn}}^4) \right] / \left(\frac{1}{2} q_{\text{rad.w}}^{\infty} \right).$$

The denominator of this ratio corresponds to the series (7). Its numerator ought to be found by means of integrating termwise the series (5) and admitting the even derivatives to be equal zero:

$$\begin{aligned} & \int_{(2\pi)} I_l d\omega - 2 \frac{\sigma}{k} (\alpha_{\text{p}} T_{\text{w}}^4 + \beta T_{\text{rad.bn}}^4) = \\ & -2\pi \frac{\alpha_{\text{p}}}{k} \left[\frac{1}{2k} \left(\frac{\partial I_0}{\partial y} \right)_{\text{bn}} + \frac{1}{3k^3} \left(\frac{\partial^3 I_0}{\partial y^3} \right)_{\text{bn}} + \frac{1}{5k^5} \left(\frac{\partial^5 I_0}{\partial y^5} \right)_{\text{bn}} + \dots \right] - \\ & -2\pi \frac{\beta}{k} \left[\frac{1}{2k} \left(\frac{\partial \bar{I}}{\partial y} \right)_{\text{bn}} + \frac{1}{3k^3} \left(\frac{\partial^3 \bar{I}}{\partial y^3} \right)_{\text{bn}} + \frac{1}{5k^5} \left(\frac{\partial^5 \bar{I}}{\partial y^5} \right)_{\text{bn}} + \dots \right] \end{aligned}$$

Comparing numerical coefficients before the first derivatives in both numerator and denominator gives us a rough estimation: $m \approx 3/2$. Now replacing the determining temperature $T_{\text{s.w}}$ in the equation (22) with the radiant one in accordance with formula (14) results in a computational expression for the Robin boundary conditions:

$$\sigma T_{\text{s.w}}^4 \approx \sigma T_{\Sigma.\text{bn}}^4 + \frac{3k + \alpha}{12k} (1.75 - \varepsilon_{\text{w}}) q_{\text{rad.w}}^{\infty}. \quad (23)$$

This expression includes the resulting radiation-flux density in the unbounded medium. Thus, in order to set correctly the boundary conditions for the differential equation (17), one has to solve previously the differential equation (12) with the boundary condition (13).

Owing to such an algorithm, the precision of the whole numerical method rises [16]. At some distance from walls, where the dust-particle absorption nullifies the wall additional radiation over the total spectrum, the formula (15) becomes accurate. If, in addition, the resulting radiation flux had been determined by means of the relation (20) with acceptable accuracy, some potential faults of the gradient formula (15) might be revealed only restrictedly in a narrow field beside the walls [17], and they would not be capable to yield any essential inadequacy to computational results.

4. Conclusion

The accurate diffusion model has been theoretically elaborated for radiative heat transfer in dust-laden gases. To eliminate singularity of temperature curves on the surface of confining walls, the enclosed extinction medium was hypothetically considered as an unbounded one. The radiation intensity has been expanded in a series and integrated over the spherical solid angle, what has allowed to obtain more precise differential equation for the resulting radiation flux both in the unbounded and enclosed extinction medium. As a result, shortcomings peculiar to the previous method of diffusion approach have been eliminated, and correct Neumann and Robin boundary conditions have been formulated.

The elaborated diffusion model has an easy algorithm, minimum requirements to computer resources and small time of its numerical realization.

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