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The magnetic field of a three-phase power line with an ordinary and triangular arrangement of wires

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Abstract. Power lines are sources of magnetic fields, which requires attention to ensure safety for people and to consider electromagnetic compatibility. In this article the nondimensional form expressions for calculation of magnetic field strength of a three-phase line with an ordinary and triangular arrangement of wires were obtained what makes it possible to calculate the necessary distance to the core of the line if the maximum permissible level of magnetic field strength is provided. The approximate expression of magnetic field strength at large distance from the line was considered. It is valid with the measurement accuracy less than 1%, which shows the proportion between magnetic field strength and distances between wires and inverse proportion of square distance to the wire core. It was determined that the triangular arrangement of three-phase line wires has lower magnetic field strength than the arrangement of wires in row due to more compact arrangement of wires.

1. Introduction

High current cables of a power system are a source of electromagnetic field strength [1-6]. It requires calculations to estimate electromagnetic safety and electromagnetic compatibility in system designing [7-9].

Electromagnetic fields of 0.4-10 kV cables can usually be neglected, but industrial frequency magnetic fields can exceed the critical value, especially in case of lines with an ordinary and triangular arrangement of wires.

A magnetic field of one wire or two wires single-phase line is calculated and can be written in expressions [10-12].

For a three-phase line, vectors of a magnetic field are not equal in amplitude (due to different distances to the point of observation), shifted in angle in space (due to the mismatch of directions of vectors) and shifted in phase. It makes calculations of a magnetic field more difficult. Approximate values and graphics methods are usually used, which leads to a large measurement error [11-15].

This paper considers universal nondimensional expressions for magnetic field strength of a three-phase line. There are three cases of wire arrangement: horizontal row, vertical row and triangular.

2. Materials and Methods

In figure 1 three-phase line with arrangement of wires in horizontal row is shown. Coordinate axes are taken along the line connecting wires instead of the case where axes are connected with position vectors and changed with shift of the observation point.

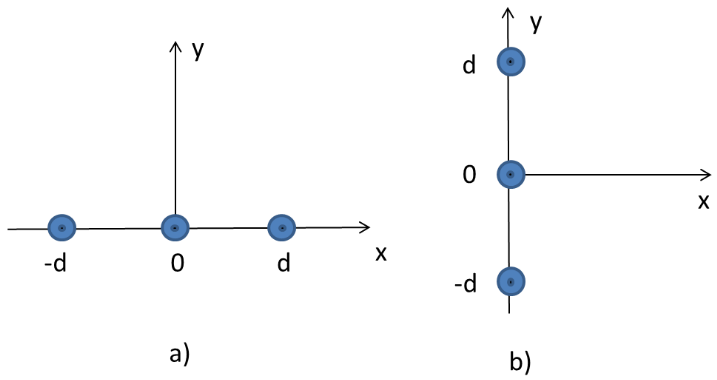

Figure 1. Three-phase line with arrangement of wires in row:
a) horizontal arrangement; b) vertical arrangement.

Figure 2 shows the location of magnetic vectors of each of the wires of figure 1(option a). Vectors of magnetic field act along lines H_A, H_B, H_C, which are perpendicular to position vectors of the corresponding wire core.

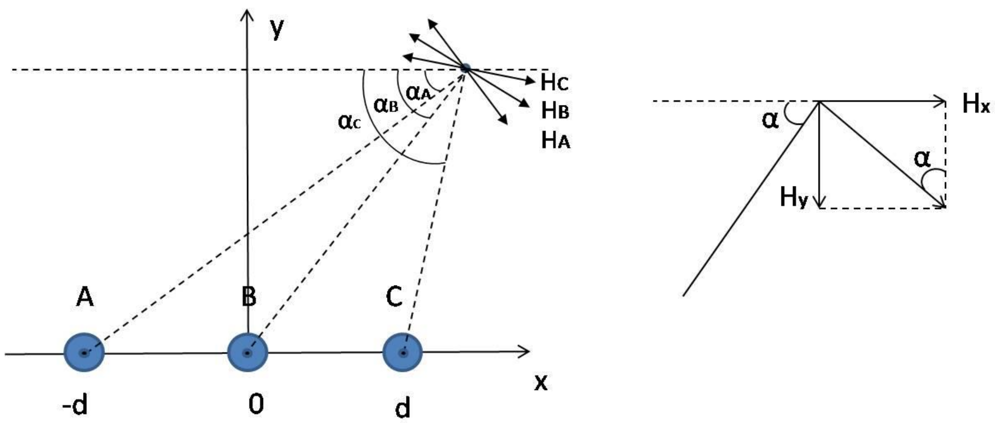

Figure 2. Scheme of location of magnetic field strength vectors for horizontal arrangement Magnetic field strength of a straight wire with current I(t) is expressed as.

$$H(t) = \frac{I(t)}{2\pi r}$$

where r is the distance from the observation point to the wire core. Projections of magnetic field are equal: $H_x(t) = H(t)\cos\alpha$, $H_y(t) = H(t)\sin\ \alpha$.

Then, in our case, according to Figure 2, the projections of the vectors at the point with coordinates xy from the wire core:

$$H_{Ax} = \frac{I_0 sin(\omega t - 120°)}{2\pi(y^2 + (x+d)^2)^{\frac{1}{2}}} \cdot \frac{y}{(y^2 + (x+d)^2)^{\frac{1}{2}}} H_{Ay} = -\frac{I_0 sin(\omega t - 120°)}{2\pi(y^2 + (x+d)^2)^{\frac{1}{2}}} \cdot \frac{(x+d)}{(y^2 + (x+d)^2)^{\frac{1}{2}}}$$

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$$H_{Cx} = \frac{I_0 \sin(\omega t + 120°)}{2\pi(y^2 + (x-d)^2)^{\frac{1}{2}}} \cdot \frac{y}{(y^2 + (x-d)^2)^{\frac{1}{2}}} H_{Cy} = \\ = -\frac{I_0 \sin(\omega t + 120°)}{2\pi(y^2 + (x-d)^2)^{\frac{1}{2}}} \cdot \frac{(x-d)}{(y^2 + (x-d)^2)^{\frac{1}{2}}} \tag{1}$$

where I_0– current amplitude. Projections of the magnetic field are obtained by summing up the projections of individual cores:

$$H_x = H_{Ax} + H_{Bx} + H_{Cx}, \qquad H_y = H_{Ay} + H_{By} + H_{Cy} \tag{2}$$

The square of the effective value of the magnetic field is determined from the expression:

$$H_{RMS}^2 = \frac{1}{2\pi}\int_0^{2\pi} (H_x + H_y)^2 d\omega t \tag{3}$$

Expressions (1) and (2) are summed up, and then they are squared:

$$H_x^2 = \frac{I_0^2}{4\pi^2}\left(\frac{y^2 sin^2(\omega t - 120°)}{(y^2 + (x+d)^2)^2} + \frac{y^2 sin^2(\omega t)}{(y^2 + x^2)^2} + \frac{y^2 sin^2(\omega t + 120°)}{(y^2 + (x-d)^2)^2} + \frac{2y^2 sin(\omega t) sin(\omega t - 120°)}{(y^2 + (x+d)^2)(y^2 + x^2)} \right. \\ \left. + \frac{2y^2 sin(\omega t) sin(\omega t + 120°)}{(y^2 + (x-d)^2)(y^2 + x^2)} + \frac{2y^2 sin(\omega t - 120°) sin(\omega t + 120°)}{(y^2 + (x+d)^2)(y^2 + (x-d)^2)}\right)$$

$$H_y^2 = \frac{I_0^2}{4\pi^2}\left(\frac{(x+d)^2 sin^2(\omega t - 120°)}{(y^2 + (x+d)^2)^2} + \frac{x^2 sin^2(\omega t)}{(y^2 + x^2)^2} + \frac{(x-d)^2 sin^2(\omega t + 120°)}{(y^2 + (x-d)^2)^2} \right. \\ + \frac{2x(x+d) sin(\omega t) sin(\omega t - 120°)}{(y^2 + (x+d)^2)(y^2 + x^2)} + \frac{2x(x-d) sin(\omega t) sin(\omega t + 120°)}{(y^2 + (x-d)^2)(y^2 + x^2)} \\ \left. + \frac{2(x-d)(x+d){I_0}^2 sin(\omega t - 120°) sin(\omega t + 120°)}{(y^2 + (x+d)^2)(y^2 + (x-d)^2)}\right) \tag{4}$$

The obtained expressions are substituted in (3) and integrated over the period:

$$H_{RMS}^2 = \frac{I_0^2}{8\pi^2}\left(\frac{1}{y^2 + (x+d)^2} + \frac{1}{y^2 + x^2} + \frac{1}{(y^2 + (x-d)^2)^2} + \frac{y^2 + x(x+d)}{(y^2 + (x+d)^2)(y^2 + x^2)} \right. \\ \left. - \frac{y^2 + x(x-d)}{(y^2 + (x-d)^2)(y^2 + x^2)} + \frac{y^2 + (x-d)(x+d)}{(y^2 + (x+d)^2)(y^2 + (x-d)^2)}\right) \tag{5}$$

The following expressions were used during the integration:

$$\overline{sin^2(\omega t)} = \frac{1}{2}$$

$$\overline{sin^2(\omega t \pm 120°)} = \frac{1}{2}$$

$$\overline{sin(\omega t) sin(\omega t \pm 120°)} = \frac{1}{2}\big(cos(\pm 120°) - cos(2\omega t \pm 120°)\big) = -\frac{1}{4}$$

$$\overline{sin(\omega t + 120° \omega t) sin(\omega t - 120°)} = \frac{1}{2}\big(cos(240°) - cos(2\omega t)\big) = -\frac{1}{4}$$

The expression (5) is led to the common denominator:

$$H_{RMS}=\frac{I_{RMS}}{2\pi}\left(\frac{d^2\left[3\left(y^2+x^2\right)+d^2\right]}{\left(y^2+(x-d)^2\right)\left(y^2+(x+d)^2\right)\left(y^2+x^2\right)}\right)^{\frac{1}{2}} \quad (6)$$

The maximum value of magnetic field strength is directly over the vertical line at x=0:

$$H_{RMS}=\frac{I_{RMS}}{2\pi}\frac{\mathrm{d}(3h^2+\mathrm{d}^2)^{\frac{1}{2}}}{\mathrm{h}(\mathrm{h}^2+\mathrm{d}^2)} \quad (7)$$

At $\frac{\mathrm{h}}{\mathrm{d}}\geq 10$ with accuracy better that 1% the approximate expression is applicable

$$H_{RMS}=\frac{I_{RMS}}{2\pi}\frac{\mathrm{d}\sqrt{3}}{\mathrm{h}^2}=0{,}276 I_{RMS}\frac{d}{\mathrm{h}^2} \quad (8)$$

Approximation accuracy (8) referred to (7) is shown in table 1.

Table 1. Approximation accuracy (8) referred to (7).

$\frac{\mathrm{h}}{\mathrm{d}}$	Accuracy (%)
5	3.2
10	0.8
20	0.2

According to the expression (6), the exposure radius to the value $0.5H_{RMSmax}$ is approximately equal to x ≈ h.

2.1. A three-phase line with the vertical arrangement of wires in row

The case of vertical arrangement of wires in row is obtained by shifting the system to 90 and replacing coordinates x and y in the expression (6)

$$H_{RMS}=\frac{I_{RMS}}{2\pi}\left(\frac{\mathrm{d}[3(x^2+y^2)+\mathrm{d}^2]}{(x^2+(y-\mathrm{d})^2)(x^2+(y+\mathrm{d})^2)(x^2+y^2)}\right)^{\frac{1}{2}}$$

The maximum value of magnetic field strength would be above the line x=0. In case y>d

$$H_{RMS}=\frac{\sqrt{3}I_{RMS}d}{2\pi}\frac{\left(y^2+\frac{1}{3}\mathrm{d}^2\right)^{\frac{1}{2}}}{y(y^2-\mathrm{d}^2)}$$

The approximate expression (8) is applied for large distances h. The approximate expression is also applied for any direction from the line and any distance h>>d.

$$H_{RMS}=0{,}276\frac{I_{\mathrm{RMS}}\mathrm{d}}{(h+d)^2}$$

2.2. A three-phase line with the triangular arrangement of wires

A three-phase line with the triangular arrangement of wires is shown in figure 3.

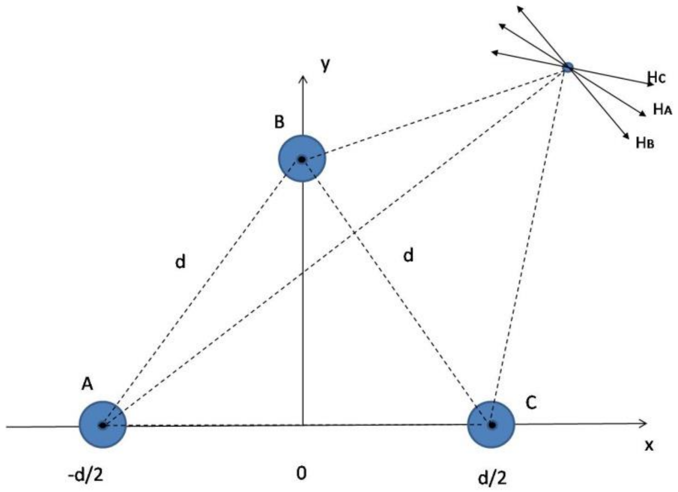

Figure 3. Three-phase line with arrangement of wires in triangle.

The origin of axes is placed in the middle of the lower side of an equilateral triangle. The distance between the wires is "d". We have obtained expressions for effective magnetic field strength by applying the same methodology as we did for the line with the arrangement of wires in row.

$$H_{RMS} = \frac{I_{RMS}\mathrm{d}}{2\pi}\left(\frac{\frac{3}{2}(x^2+y^2)-\frac{\sqrt{3}}{2}y\mathrm{d}+\frac{5}{8}\mathrm{d}^2}{\left(x^2+(y-\frac{\sqrt{3}}{2}\mathrm{d})^2\right)\left(\left(x-\frac{\mathrm{d}}{2}\right)^2+y^2\right)\left(\left(x+\frac{\mathrm{d}}{2}\right)^2+y^2\right)}\right)^{\frac{1}{2}} \tag{9}$$

Figure 6 shows graphs of magnetic field strength which were calculated by using the expression (9) for the different ratio of h/d. In this case, h is also counted from the top of the triangle.

2.3. Special cases are considered for this method of laying the wire core

Case of x=0:

$$H_{RMS} = \frac{I_{RMS}\mathrm{d}\left(\frac{3}{2}y^2-\frac{\sqrt{3}}{2}y\mathrm{d}+\frac{5}{8}\mathrm{d}^2\right)^{\frac{1}{2}}}{2\pi(y-\frac{\sqrt{3}}{2}\mathrm{d})\left(y^2+\frac{\mathrm{d}^2}{4}\right)} = \frac{0{,}195\frac{\mathrm{d}}{l}\left(1+\frac{1}{2}\left(\frac{\mathrm{d}}{l}\right)^2\right)^{\frac{1}{2}}}{l\left(1-\frac{1}{\sqrt{3}}\frac{\mathrm{d}}{\mathrm{l}}\right)^2\left(1-\sqrt{3}\frac{\mathrm{d}}{l}+\frac{1}{3}\left(\frac{\mathrm{d}}{l}\right)^2\right)} \tag{10}$$

where
$$l = \left(x^2+\left(y-\frac{\sqrt{3}}{6}x\right)^2\right)^{\frac{1}{2}}$$

distance from the centre point of the line to the observation point.

Case of y=0, x>d/2:

$$H_{RMS} = \frac{I_{RMS}\mathrm{d}}{2\pi\left(x^2-\frac{\mathrm{d}^2}{4}\right)}\left(\frac{\frac{3}{2}\mathrm{x}^2+\frac{5}{8}\mathrm{d}^2}{x^2+\frac{3}{4}\mathrm{d}^2}\right)^{\frac{1}{2}}$$

The appoximate expression is applicable for large distances h/d:

$$H_{RMS} \approx 0.195\frac{I_{RMS}d}{h^2} \tag{11}$$

For a two-wire single-phase line we have the following expression:

$$H_{RMS} = \frac{I_{RMS}d}{2\pi\left(\left(y^2+\left(x+\frac{d}{2}\right)^2\right)\left(y^2-\left(x-\frac{d}{2}\right)^2\right)\right)^{\frac{1}{2}}} \tag{12}$$

If d<<h we have

$$H_{RMS} = \frac{I_{RMS}d}{2\pi h^2} = 0.159\frac{I_{RMS}d}{(h+0{,}86d)^2}$$

According to the graphs, the triangular arrangement has lower magnetic field strength due to more compact arrangement of wires.

It means a lower value than a three phase line at the same distance between wire cores.

Using expressions (8) and (12), it is possible to calculate the necessary distance to the centre of the line if the maximum permissible level of magnetic field strength 50 Hz [1-3] is provided.

3. Results

Figure 4 shows graphs of distribution of magnetic field strength for a three-phase line in horizontal arrangement at distance *h*.

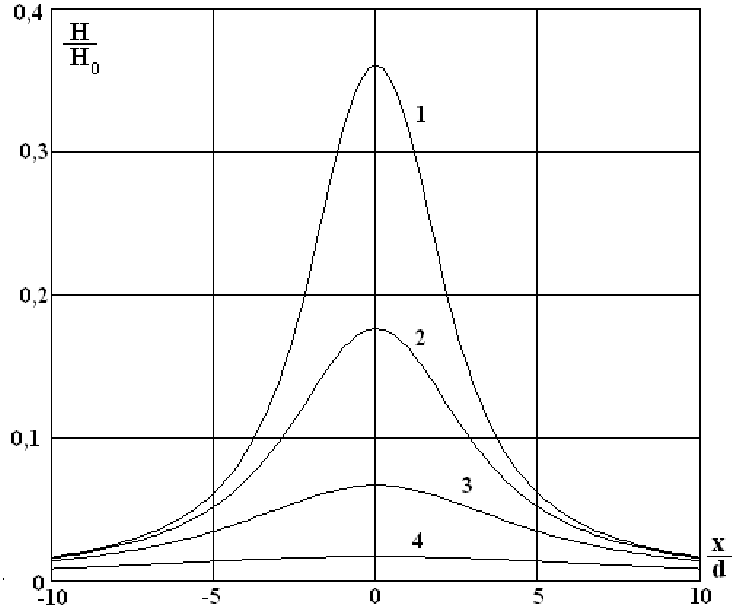

Figure 4. Distribution of nondimensional root mean square value of magnetic field strength in horizontal direction for cases of different distance from horizontal arrangement of row wires $H_0 = \frac{I_{RMS}}{2\pi d}$,

$$1-\frac{h}{d}=10, 2-\frac{h}{d}=5, 3-\frac{h}{d}=3, 4-\frac{h}{d}=2.$$

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Figure 5 shows the distribution of magnetic field strength for different h/d. The distance h in this case is calculated from the upper wire.

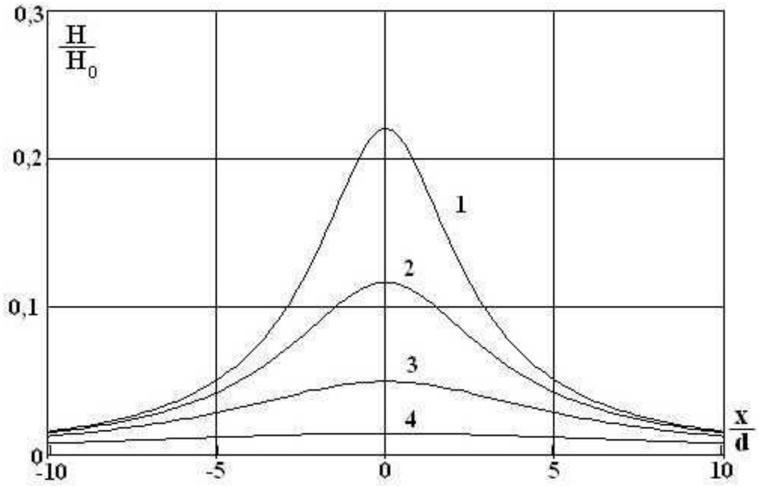

Figure 5. Distribution of nondimensional root mean square value of magnetic field strength in horizontal direction for different distances from the upper wire $H_0 = \frac{I_{RMS}}{2\pi d}$,

$$1 - \frac{h}{d} = 10, 2 - \frac{h}{d} = 5, 3 - \frac{h}{d} = 3, 4 - \frac{h}{d} = 2.$$

Figure 6 shows graphs of magnetic field strength which were calculated by using the expression (9) for the different ratio of h/d. In this case, h is also counted from the top of the triangle.

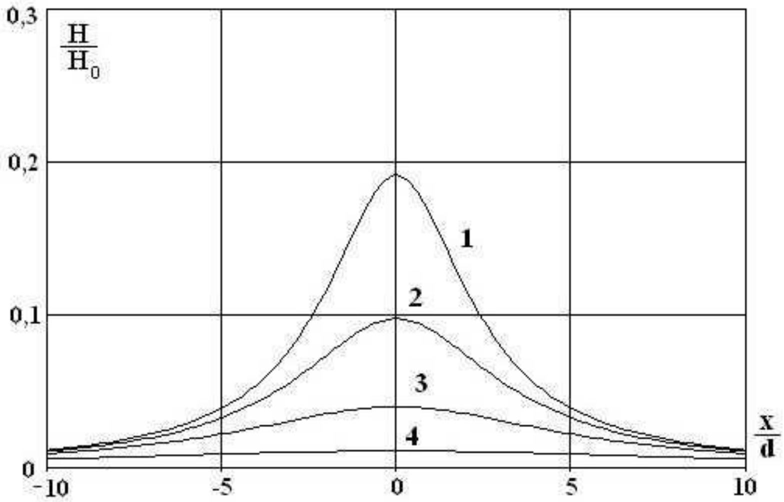

Figure 6. Distribution of nondimensional root mean square value of magnetic field strength in horizontal direction for different distances from the upper wire $H_0 = \frac{I_{RMS}}{2\pi d}$

Figure 7 shows graphs of distribution of magnetic field strength in horizontal plane for 3 options of underground arrangement of wires and h/d=10. (h is counted from the ground level to upper wires). It is less than in case of the arrangement of wires in row.

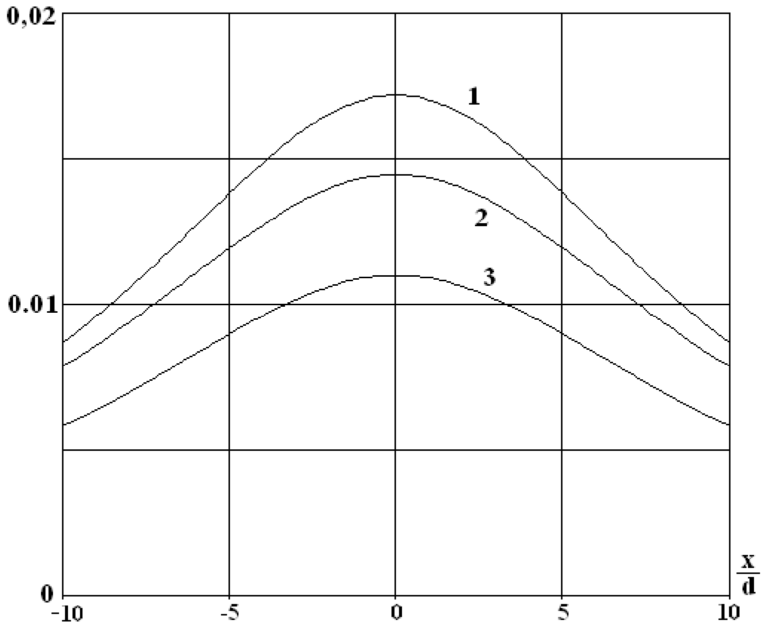

Figure 7. Distribution of nondimensional root mean square value of magnetic field strength in horizontal direction for different arrangement of wires: 1 – horizontal arrangement, 2 – vertical arrangement, 3 – triangular arrangement. $H_0 = \frac{I_{RMS}}{2\pi h}$.

4. Discussion
According to the graphs, the triangular arrangement has lower magnetic field strength due to more compact arrangement of wires.

It means a lower value than a three phase line at the same distance between wire cores.

Using expressions (8) and (12), it is possible to calculate the necessary distance to the centre of the line if the maximum permissible level of magnetic field strength 50 Hz [1-3] is provided.

5. Conclusion
1. Accurate expressions for calculation of magnetic field strength of three-phase line were obtained.
2. Approximations (8,11,12) are valid for d<<h with a measurement accuracy less than 1%, which show the proportion between magnetic field strength and distances between wires (d) and inverse proportion of square distance to the wire core.

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