



Propagation of Alfvén Waves in the Expanding Solar Wind with the Fast–Slow Stream Interaction

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Abstract

We carry out two-dimensional magnetohydrodynamic simulations of an ensemble of Alfvénic fluctuations propagating in a structured, expanding solar wind including the presence of fast and slow solar wind streams. Using an appropriate expanding box model, the simulations incorporate the effects of fast–slow stream shear and compression and rarefaction self-consistently. We investigate the radial and longitudinal evolution of the cross helicity, the total and residual energies and the power spectra of outward and inward Alfvénic fluctuations. The stream interaction is found to strongly affect the radial evolution of Alfvénic turbulence. The total energy in the Alfvén waves is depleted within the velocity shear regions, accompanied by the decrease of the normalized cross helicity. The presence of stream compression facilitates this process. Residual energy fluctuates around zero due to the correlation and de-correlation between the inward/outward waves but no net growth or decrease of the residual energy is observed. The radial power spectra of the inward/outward Alfvén waves show significant longitudinal variations. Kolmogorov-like spectra are developed only inside the fast and slow streams and when both the compression and shear are present. On the other hand, the spectra along the longitudinal direction show clear Kolmogorov-like inertial ranges in all cases.

Unified Astronomy Thesaurus concepts: [Solar wind \(1534\)](#); [Interplanetary turbulence \(830\)](#); [Alfvén waves \(23\)](#); [Corotating streams \(314\)](#)

1. Introduction

Turbulence is one of the most important phenomena in space. Inside the heliosphere, it is believed to be fundamental to various physical processes such as the heating and acceleration of the solar corona and wind and the acceleration and propagation of energetic particles, etc. Thus, understanding solar wind turbulence has been one of the most crucial tasks in heliospheric physics and astrophysics as a whole. In addition, as direct measurements of solar wind accumulate, the wind itself serves as a laboratory for the study of the fundamental physics of plasma turbulence.

The study of fluctuations in the solar wind began in the 1960s, when Coleman (1968), using Mariner 2 data, created the first frequency spectra of magnetic field energy and showed that they were power-law compatible with the well-known Kolmogorov power-law spectrum. Coleman therefore proposed that this turbulence was created by the differential flow in fast and slow solar wind streams. Belcher & Davis (1971), by analyzing Mariner 5 data, showed however that much of the fluctuations comprising the turbulence, especially those in the fast streams, have correlations between velocity and the magnetic field consistent with large amplitude Alfvén waves propagating away from the Sun, and should therefore be generated somewhere close to the Sun and propagate outward. A sequence of theoretical works on the propagation of Alfvén waves were developed, starting from linear models, both WKB theory (e.g., Alazraki & Couturier 1971; Belcher 1971; Hollweg 1974) and non-WKB theory (e.g., Heinemann & Olbert 1980). However, whether the Alfvén waves are generated near the Sun or evolve dynamically in the solar wind was still a problem. In fact one major progress in the theory of incompressible magnetohydrodynamic (MHD) turbulence, namely, dynamical alignment (e.g.,

Dobrowolny et al. 1980), was developed to understand the dominance of outwardly propagating Alfvénic fluctuations in the solar wind as a result of an ongoing nonlinear cascade.

In the 1980s, with the *Helios* data, it was found that the energy spectra of the turbulence steepen toward the well-known Kolmogorov’s 5/3 law or Iroshnikov–Kraichnan’s 3/2 law (Iroshnikov 1964; Kraichnan 1965), indicating nonlinear processes in the evolution of the fluctuations (Bavassano et al. 1982). Meanwhile, MHD turbulence transport models were developed. For example, Tu et al. (1984) established a WKB-like Alfvénic turbulence model that reproduces the steepening of the energy spectrum successfully. However, in this model the energy of inward propagating Alfvénic fluctuations, required for a nonlinear cascade, is an input parameter rather than the self-consistent outcome of the calculation. Thus, understanding the generation and the propagation of the inward propagating Alfvén waves is crucial. Roberts et al. (1987a, 1987b) analyzed the *Voyager* and *Helios* data and concluded that the dominance of the outward propagation weakens as the heliocentric distance increases. They also proposed that the outward propagating fluctuations are generated near the Sun but the sheared streams in the solar wind accounts for the generation of the inward waves. Grappin et al. (1990) and Marsch & Tu (1990) studied the radial evolution and the properties of Alfvénic turbulence spectra and their dependence on various solar wind parameters by analyzing the *Helios* data by means of Elsässer variables. Roberts et al. (1992) carried out 2D incompressible MHD simulations with large-scale velocity shear and isotropic Alfvénic fluctuations. Their results showed that the velocity shear layer can produce a nonlinear cascade toward smaller scale fluctuations and the normalized cross helicity, i.e., the relative abundance of the outward propagating Alfvén waves, is decreased systematically at all wave numbers

by the shear, while the kinetic and magnetic energies remain approximately equipartitioned (see also Goldstein et al. 1989). In addition to the sheared streams, the inhomogeneity of the large-scale solar wind structures due to the expansion of the solar wind leads to reflection of the outward propagating Alfvén waves and couples the outward and the inward waves linearly, which may account for the decrease of the “Alfvénicity” (e.g., Velli et al. 1991; Velli 1993).

Although analytical modeling of MHD turbulence has been developed significantly since the 1980s (e.g., Zhou & Matthaeus 1990; Zank et al. 1996, 2012), limitations still exist in the models. First, various approximations must be made to close the moment equations. Whether the closures are physically correct is important and is often debatable. Second, all of the existing models deal with incompressible or nearly incompressible turbulence while compressible effects may be important in the solar wind, especially in the fast–slow stream interaction regions. Third, the models are based on the two-scale separation method and thus the large-scale structure is not evolved self-consistently. In addition, the source terms generating the turbulence, e.g., the terms related to the velocity shear and the compressional effects, are only phenomenologically derived. Considering the drawbacks of the models, it is good to adopt the direct numerical simulations as a method to study the turbulence as the simulation solves the physical system self-consistently and can be fully compressible. However, vast computational capacity is required in order to fully simulate the turbulence evolution inside the heliosphere due to the huge separation of spatial scales. Compromised methods were developed, e.g., the expanding box model (EBM; Grappin & Velli 1996; Tenerani & Velli 2017), which by tracking a box co-moving with the radial mean flow, neglecting the high-order curvature terms, allows one to simulate the nonlinear evolution of the waves and turbulence and the stream structures with the expansion effect taken into consideration.

In this work, we carry out 2D MHD simulations based on the EBM to study the propagation of Alfvén waves and the evolution of the turbulence in the inner heliosphere. Especially, we focus on the effects of the evolving fast–slow stream interaction present in the simulations. The simulation parameters are chosen to be close to the real solar wind conditions. We inspect the radial evolution and the longitudinal variation of some parameters that are important in the study of MHD turbulence, i.e., the energy in the Elsässer variables, the normalized cross helicity, and the normalized residual energy. We show that all of the parameters are significantly affected by the velocity shear and the compression between the streams. We also investigate the power spectra of the Elsässer variables. The paper is organized as follows. In Section 2, we describe the numerical method that is used in this study and the setup of the simulations. In Section 3, we present the simulation results. In Section 4, we conclude and discuss prospective future works.

2. Numerical Method

In this section we describe the numerical method, i.e., the corotating EBM, in Sections 2.1 and 2.2 and then present the initial setup and the choice of parameters in Section 2.3.

2.1. EBM in the Conservation Form

The derivation of the EBM based on the convective form of the MHD equation has been well described in previous papers (e.g., Grappin et al. 1993; Grappin & Velli 1996). The idea is to break the velocity \mathbf{U} into two parts: the radial mean flow and the velocity in the frame of the mean flow:

$$\mathbf{U} = U_0 \hat{e}_r + \mathbf{u}, \quad (1)$$

where U_0 is the constant radial speed. The simulation domain is a thin box (small radial extent) co-moving with the mean flow and the (normalized) expanding coordinate system $(\tilde{x}, \tilde{y}, \tilde{z})$ transforms from the inertial coordinates by

$$\tilde{x} = x - R(t), \tilde{y} = \frac{R_0}{R(t)}y, \tilde{z} = \frac{R_0}{R(t)}z, \quad (2)$$

where $R(t) = R_0 + U_0 t$ such that the derivatives are

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \tilde{x}}, \frac{\partial}{\partial y} = \frac{R_0}{R(t)} \frac{\partial}{\partial \tilde{y}}, \frac{\partial}{\partial z} = \frac{R_0}{R(t)} \frac{\partial}{\partial \tilde{z}}. \quad (3)$$

Inside the expanding box, the mean flow can be written, by neglecting the high-order curvature terms, as

$$U_0 \hat{e}_r \approx U_0 \left[\hat{e}_x + \frac{y}{R(t)} \hat{e}_y + \frac{z}{R(t)} \hat{e}_z \right]. \quad (4)$$

Plugging Equation (4) into the MHD equation gives the EBM equation set

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) - \frac{2}{\tau} \rho \quad (5a)$$

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} = & -\mathbf{u} \cdot \nabla \mathbf{u} - \frac{1}{\rho} \nabla \left(p + \frac{1}{2} B^2 \right) + \frac{1}{\rho} \mathbf{B} \cdot \nabla \mathbf{B} \\ & - \frac{1}{\tau} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{u} \end{aligned} \quad (5b)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \frac{1}{\tau} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{B} \quad (5c)$$

$$\frac{\partial p}{\partial t} = -\mathbf{u} \cdot \nabla p - \kappa (\nabla \cdot \mathbf{u}) p - \frac{2\kappa}{\tau} p, \quad (5d)$$

where κ is the adiabatic index and $\tau = R(t)/U_0$ is the “expansion time.” Equation (5) is very similar to the normal MHD equation set except for (1) the velocity field is in the reference frame of the radial mean flow. (2) New terms with the expansion time τ are introduced by the radial mean flow and they represent the expansion effect. A more detailed discussion of the EBM properties can be found in Grappin & Velli (1996).

For the conservation form of the MHD equation, care must be taken with the expansion terms. The expansion terms for the density and magnetic field equations remain unchanged as in Equation (5) but not for the momentum and energy equations. Take the momentum equation as an example. Because the left-hand side of the momentum equation can be written as

$$\begin{aligned} \frac{\partial(\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = & \left[\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) \right] \\ & + \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) \right] \mathbf{u}, \end{aligned} \quad (6)$$

the expansion term thus consists of the part that comes from the velocity equation (Equation 5(b)) and that from the density equation (Equation 5(a)):

$$E_m = -\frac{1}{\tau} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rho \mathbf{u} - \frac{2}{\tau} \rho \mathbf{u} = -\frac{1}{\tau} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \rho \mathbf{u}. \quad (7)$$

Similarly, one can show that the expansion term of the energy equation

$$\frac{\partial e}{\partial t} = -\nabla \cdot \left[\left(e + p + \frac{1}{2} B^2 \right) \mathbf{u} - (\mathbf{u} \cdot \mathbf{B}) \mathbf{B} \right], \quad (8)$$

where $e = \frac{p}{\kappa - 1} + \frac{1}{2} \rho u^2 + \frac{1}{2} B^2$ is

$$E_e = -2 \frac{\kappa}{\kappa - 1} \frac{p}{\tau} - \frac{\rho}{\tau} (u_x^2 + 2u_y^2 + 2u_z^2) - \frac{1}{\tau} (2B_x^2 + B_y^2 + B_z^2). \quad (9)$$

In summary, the EBM equation set in conservation form is

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) - \frac{2}{\tau} \rho \quad (10a)$$

$$\begin{aligned} \frac{\partial (\rho \mathbf{u})}{\partial t} = & -\nabla \cdot \left[\rho \mathbf{u} \mathbf{u} + \left(p + \frac{1}{2} B^2 \right) \mathbf{I} - \mathbf{B} \mathbf{B} \right] \\ & - \frac{1}{\tau} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \rho \mathbf{u} \end{aligned} \quad (10b)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \frac{1}{\tau} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{B} \quad (10c)$$

$$\begin{aligned} \frac{\partial e}{\partial t} = & -\nabla \cdot \left[\left(e + p + \frac{1}{2} B^2 \right) \mathbf{u} - (\mathbf{u} \cdot \mathbf{B}) \mathbf{B} \right] \\ & - \frac{1}{\tau} \left[\frac{2\kappa}{\kappa - 1} p + \rho (u_x^2 + 2u_y^2 + 2u_z^2) \right. \\ & \left. + (2B_x^2 + B_y^2 + B_z^2) \right], \end{aligned} \quad (10d)$$

with

$$e = \frac{p}{\kappa - 1} + \frac{1}{2} \rho u^2 + \frac{1}{2} B^2. \quad (11)$$

2.2. Corotating Expanding Box

As explained by Grappin & Velli (1996), in order to simulate the compression between fast and slow streams, we need to rotate the expanding box coordinates by a small angle α such that the new coordinate system \mathbf{x}' is

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix}. \quad (12)$$

The angle α is constant and is the initial inclination of the interface between the fast and slow streams with respect to the radial direction. The initial condition for the stream structure is

$$\mathbf{u}_0 = u_0(y') \hat{e}_{\tilde{x}}, \quad \rho_0 = \rho_0(y'), \quad (13)$$

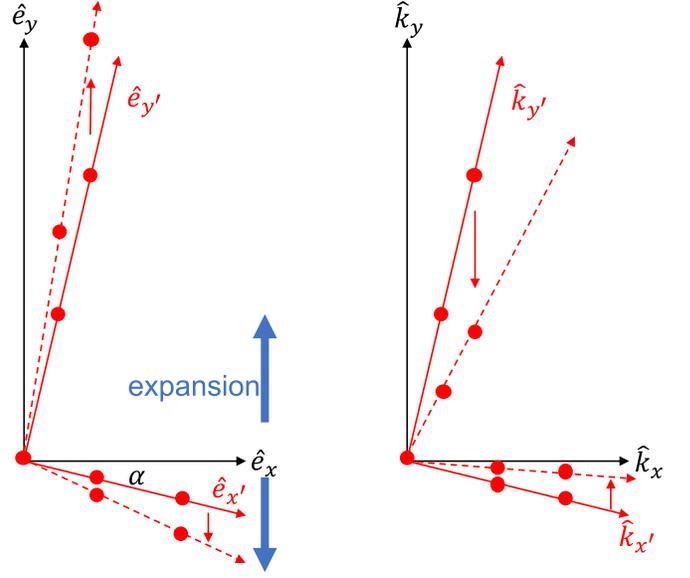


Figure 1. Illustration of the deformation of the corotating expanding box coordinates \mathbf{x}' (left) and the wave vector \mathbf{k}' (right). Black axes are the normal expanding box coordinates (wave vectors) with \hat{e}_x (\hat{k}_x) to be radial. The solid red axes represent the initial state of the corotating coordinates \mathbf{x}' (wave vectors \mathbf{k}'), which is orthogonal. The red dots represent mesh grid points in the simulation domain. The dashed red lines are axes $\hat{e}_{x'}$ ($\hat{k}_{x'}$) and $\hat{e}_{y'}$ ($\hat{k}_{y'}$) after the simulation starts.

i.e., the velocity is along the radial direction but varies with y' instead of y so that compression is induced. The temperature of the stream $T_0 = T_0(y')$ is such that $p_0 = \rho_0 T_0$ is uniform.

We should point out that, although the coordinates \mathbf{x}' are orthogonal at the beginning, they do not remain orthogonal as the box expands unless $\alpha = 0$, as illustrated in the left panel of Figure 1. The black axes show the normal expanding box coordinates with \hat{e}_x aligned with the radial direction and \hat{e}_y along the azimuthal (φ) direction. The solid red axes represent the initial state of the corotating expanding box coordinates \mathbf{x}' , an orthogonal coordinate system rotated by an angle α with respect to the radial direction. The red dots represent a few mesh points in the simulation domain. Due to the expansion along the φ direction, both the $\hat{e}_{x'}$ and $\hat{e}_{y'}$ turn away from the radial axis, as shown by the dashed red axes. That is to say, the angle between $\hat{e}_{x'}$ and $\hat{e}_{y'}$ becomes larger than $\pi/2$ after the simulation starts. A positive aspect of this frame is that if we set the initial magnetic field to be aligned with $\hat{e}_{x'}$

$$\mathbf{B}_0 = B_0(y') \hat{e}_{x'}, \quad (14)$$

it will remain aligned with $\hat{e}_{x'}$ for all time. Thus, in all the simulations we set up \mathbf{B}_0 like in Equation (14) and we call $\hat{e}_{x'}$ the parallel direction hereinafter. Note that, although the axes in real space are turned away from the radial direction, the wave vectors are actually turned toward the radial direction (right panel of Figure 1) due to the increase of the grid spacing in y .

The code operates mainly in the Fourier space ($k_{x'}, k_{y'}$). A third-order Runge–Kutta method is used for time integration. Vectors remain defined in the $(\hat{e}_{\tilde{x}}, \hat{e}_{\tilde{y}}, \hat{e}_{\tilde{z}})$ directions although the mesh grid is on (x', y') . At each time step, fluxes are calculated in real space first and then Fourier transformed. Time advance is done in Fourier space and we need the following projection in order to transform the derivatives on

(x', y') to the derivatives on (x, y) :

$$\begin{aligned} k_x &= k_{x'} \cos \alpha + k_{y'} \sin \alpha \\ k_y &= \frac{R_0}{R(t)} (-k_{x'} \sin \alpha + k_{y'} \cos \alpha), \end{aligned} \quad (15)$$

$\nabla \cdot \mathbf{B} = 0$ is automatically preserved by this algorithm. Because we are interested in the evolution of turbulence, rather than heating or plasma thermodynamics, we apply a smooth numerical filter to all fields to ensure proper de-aliasing rather than explicit viscosity or resistivity. The filter is defined in Fourier space:

$$\hat{\mathbf{f}}(k_{x'}, k_{y'}) = \mathbf{f}(k_{x'}, k_{y'}) \times T(k_{x'}) \times T(k_{y'}), \quad (16)$$

where \mathbf{f} is the field before filtering and $\hat{\mathbf{f}}$ is the field after filtering. The function $T(k)$ is the same as the fourth-order filter of the compact finite difference scheme (Equations (C.2.2) and (C.2.4) of Lele 1992) with constraints $\beta = d = 0$

$$T(k) = \frac{a + b \cos(w) + c \cos(2w)}{1 + 2\lambda \cos(w)}, \quad (17)$$

where $w = 2\pi k \Delta \in [-\pi, \pi]$ is the normalized wave number (Δ is the grid spacing) and $a = (5 + 6\lambda)/8$, $b = (1 + 2\lambda)/2$, $c = -(1 - 2\lambda)/8$ with λ to be a free parameter in the range $[-0.5, 0.5]$ (refer to Figure 19 of Lele 1992 for the shape of $T(k)$). $\lambda = 0.5$ corresponds to no filtering at all. In our simulations we set $\lambda = 0.45$ such that the numerical stability is ensured without too much numerical dissipation.

2.3. Initial Setup and Parameters

The initial condition consists of the large-scale stream structure and the Alfvén waves. As mentioned in Section 2.2, the stream structure is of the form

$$\mathbf{u}_0 = u_0(y') \hat{e}_x, \quad \rho_0 = \rho_0(y'), \quad \mathbf{B}_0 = B_0(y') \hat{e}_{x'}, \quad (18)$$

with double-tanh profiles for u_0 and ρ_0 :

$$u_0(y') = \begin{cases} \frac{1}{2} \left[(u_f + u_s) + (u_f - u_s) \tanh \left(\frac{y' - \frac{1}{4} L_{y'}}{a} \right) \right], & y' < \frac{L_{y'}}{2} \\ \frac{1}{2} \left[(u_f + u_s) - (u_f - u_s) \tanh \left(\frac{y' - \frac{3}{4} L_{y'}}{a} \right) \right], & y' \geq \frac{L_{y'}}{2} \end{cases} \quad (19a)$$

$$\rho_0(y') = \begin{cases} \frac{1}{2} \left[(\rho_f + \rho_s) + (\rho_f - \rho_s) \tanh \left(\frac{y' - \frac{1}{4} L_{y'}}{a} \right) \right], & y' < \frac{L_{y'}}{2} \\ \frac{1}{2} \left[(\rho_f + \rho_s) - (\rho_f - \rho_s) \tanh \left(\frac{y' - \frac{3}{4} L_{y'}}{a} \right) \right], & y' \geq \frac{L_{y'}}{2} \end{cases} \quad (19b)$$

and a uniform magnetic field

$$B_0(y') = B_0 \quad (20)$$

in all the simulations. The width of the shear region is $a = 0.075 L_{y'}$ with $L_{y'}$ to be the size of the simulation domain along $\hat{e}_{y'}$. u_s , u_f , ρ_s , ρ_f are the speeds and densities for the slow

and fast streams, respectively. For all the runs, the initial location of the simulation domain is

$$R_0 = 30R_s = 0.14 \text{ au}, \quad (21)$$

where R_s is the solar radius and the size of the domain is

$$L_{x'} \times L_{y'} = 10R_s \times \pi R_0, \quad (22)$$

i.e., the domain is a half circle in the ecliptic plane. The initial spiral angle α , if not zero, is set to be

$$\alpha = 0.142 \quad (23)$$

so that at 1 au the spiral angle is around $\pi/4$, in accordance with the observation. The strength of the magnetic field is $B_0 = 250 \text{ nT}$ so that at 1 au $B_r \approx B_\varphi \approx 5 \text{ nT}$. The densities of the slow and fast streams are $n_s = 360 \text{ cm}^{-3}$ and $n_f = 140 \text{ cm}^{-3}$. The speeds of the slow and fast streams are $u_s = 340 \text{ km s}^{-1}$ and $u_f = 700 \text{ km s}^{-1}$ and the mean radial speed is $U_0 = 464 \text{ km s}^{-1}$. The thermal pressure is $p_0 = 5 \text{ nPa}$ so that the temperatures of the slow and fast streams are $T_s = 1.0 \times 10^6 \text{ K}$ and $T_f = 2.6 \times 10^6 \text{ K}$. The adiabatic index is $\kappa = 3/2$ instead of $\kappa = 5/3$ to prevent the plasma from cooling down too fast. Note that the radial decay of the temperature due to expansion obeys $T \propto R^{-2(\kappa-1)}$ so that with $\kappa = 3/2$ the temperatures of the slow and fast streams at 1 au are $T_s = 1.4 \times 10^5 \text{ K}$ and $T_f = 3.6 \times 10^5 \text{ K}$. The normalization units are $\bar{B} = 250 \text{ nT}$, $\bar{n} = 200 \text{ cm}^{-3}$ and $\bar{L} = R_s$, which lead to the unit speed $\bar{U} = \bar{B} / \sqrt{\mu_0 m_i \bar{n}} = 385.6 \text{ km s}^{-1}$ and unit pressure $\bar{p} = \bar{n} m_i \bar{U}^2 = 49.7 \text{ nPa}$, where m_i is the proton mass.

We add circularly polarized Alfvénic wave bands on top of the stream structure:

$$\begin{aligned} \mathbf{b}_{1,o} &= \delta b \sum_{N=1}^{N_{\max}} \frac{1}{\sqrt{N}} \left[\cos \left(\frac{2\pi N}{L_{x'}} x' + \phi_{N,o} \right) \hat{e}_{y'} \right. \\ &\quad \left. + \sin \left(\frac{2\pi N}{L_{x'}} x' + \phi_{N,o} \right) \hat{e}_z \right], \quad \mathbf{u}_{1,o} = -\frac{\mathbf{b}_{1,o}}{\sqrt{\rho_0(y')}} \end{aligned} \quad (24a)$$

$$\begin{aligned} \mathbf{b}_{1,i} &= r_{io} \times \delta b \sum_{N=1}^{N_{\max}} \frac{1}{\sqrt{N}} \left[\cos \left(\frac{2\pi N}{L_{x'}} x' + \phi_{N,i} \right) \hat{e}_{y'} \right. \\ &\quad \left. + \sin \left(\frac{2\pi N}{L_{x'}} x' + \phi_{N,i} \right) \hat{e}_z \right], \quad \mathbf{u}_{1,i} = \frac{\mathbf{b}_{1,i}}{\sqrt{\rho_0(y')}}. \end{aligned} \quad (24b)$$

Here δb is the amplitude of the magnetic perturbation of the outward wave, r_{io} is the ratio between the amplitudes of the inward and outward waves, $\phi_{N,o}$ and $\phi_{N,i}$ are the random phases of the mode N of the outward and inward waves. The slope of the power spectrum of the wave band is -1 . In order to make sure $\nabla \cdot \mathbf{b}_1 = 0$, \mathbf{b}_1 is invariant along y' and \mathbf{u}_1 varies with y' due to the nonuniform density. This leads to the inhomogeneity of the Alfvén wave energy along the y' direction: the wave energy is larger in the fast stream than the slow stream. Five 2D runs are carried out and they are listed in Table 1. By choosing the parameter δb , the total energies in the waves are invariant among the runs. We fix $N_{\max} = 16$ in all the simulations. The maximum simulation time is $t = 200$, corresponding to a radial distance $R = 270.9R_s = 1.26 \text{ au}$. The resolution is $n_{x'} \times n_{y'} = 2048 \times 4096$. In addition, we also make a 1D run ($L_{y'} = \pi R_0$

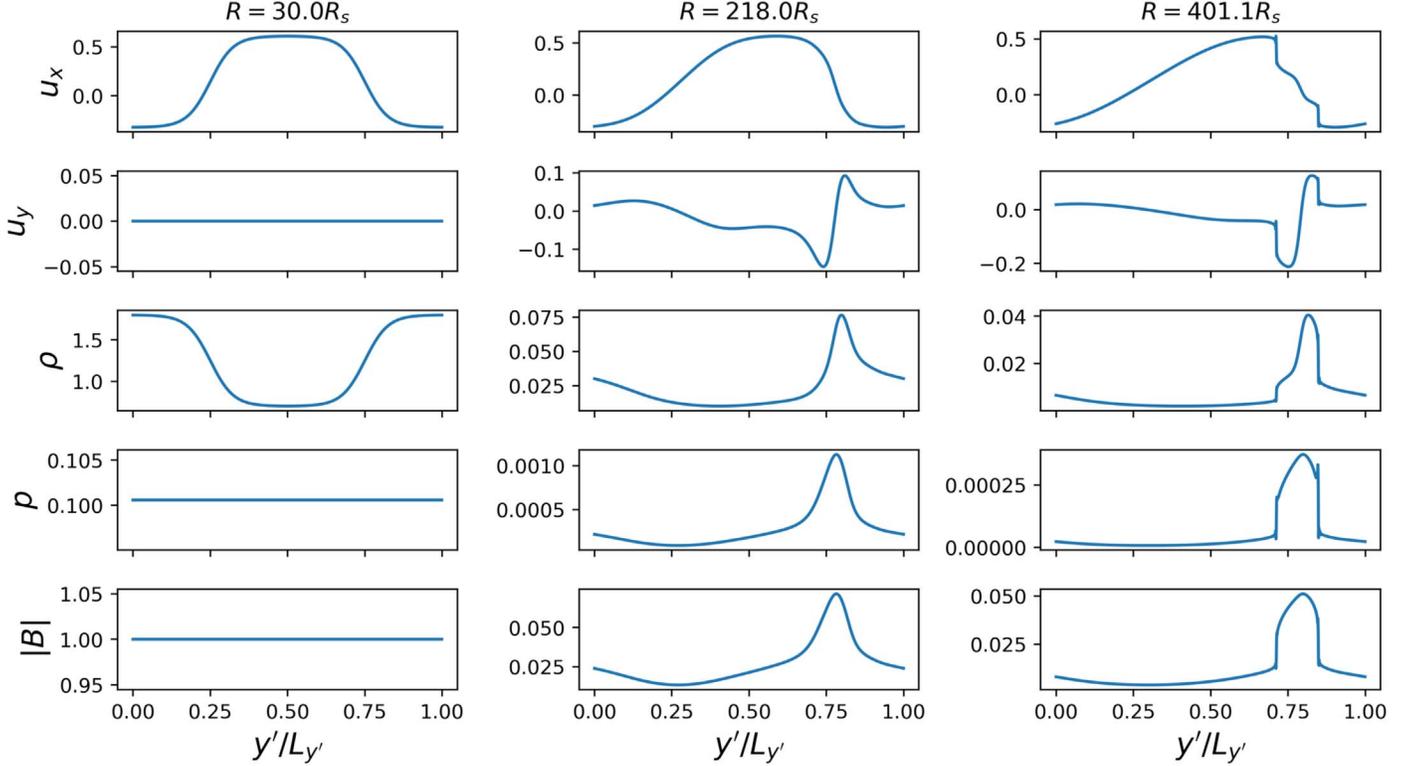


Figure 2. 1D simulation of the large-scale stream structure. From top to bottom, the rows are longitudinal (y') profiles of the radial velocity, the azimuthal velocity, the density, the pressure, and the magnitude of magnetic field, respectively. From left to right the columns are snapshots at $R = 30.0R_s$, $218.0R_s$, and $401.1R_s$.

Table 1
Parameters of the 2D Runs

Run	Expansion	Corotation	r_{io}	δb
A0	N	N	0.2	0.2
A	Y	N	0.2	0.2
B	Y	Y	0.2	0.2
C	Y	Y	1.0	0.144
D	Y	Y	5.0	0.04

Note. Here r_{io} is the ratio between the amplitude of the inward wave band and the amplitude of the outward wave band. δb is the amplitude of the outward wave band and the five runs have the same total wave energies. If expansion is present, the radial mean speed $U_0 = 1.2$. If corotation is present, the initial spiral angle $\alpha = 0.142$.

and $n_{y'} = 1024$) without adding waves to show the evolution of the stream structure up to $R = 400R_s$.

3. Results

3.1. 1D Run without Waves

In this section we show a 1D test simulation of the stream structure without adding any waves. This run serves as a test of the code. For convenience, we refer to $y'/L_{y'}$ as the normalized “longitude” hereinafter although $\hat{e}_{y'}$ is not exactly along the azimuthal direction \hat{e}_y . Figure 2 shows the radial evolution of the longitudinal profiles of the radial velocity u_x , the azimuthal velocity u_y , the density ρ , the pressure p , and the magnitude of the magnetic field $|B|$ (from top to bottom rows). The left, middle, and right columns are snapshots at $R = 30.0R_s$,

$218.0R_s$, and $401.1R_s$, respectively. At around 1 au (middle column), a clear compression region already forms. The flows are deflected away from the interface between fast and slow streams. The density, pressure, and magnetic field peak around the compression region. Further out, a forward-backward shock pair, which bounds the compression region, forms as shown in the right column. The results, shown in Figure 2, are consistent with the results of Grappin & Velli (1996) and may be benchmarked against their Figure 3.

3.2. Diagnostics of the Alfvénic Turbulence

Before presenting the results of the 2D simulations, we first introduce the diagnostics adopted for the analysis of the simulation data.

The analysis is mainly based on the perturbed Elsässer variables z_{out} and z_{in} . The procedure to calculate them is described as follows. We first calculate the x' averaged, i.e., the background, magnetic, and velocity fields:

$$\begin{aligned} \mathbf{B}_0(y') &= \frac{1}{L_{x'}} \int_0^{L_{x'}} \mathbf{B}(x', y') dx', \\ \mathbf{u}_0(y') &= \frac{1}{L_{x'}} \int_0^{L_{x'}} \mathbf{u}(x', y') dx' \end{aligned} \quad (25)$$

and then the perturbed magnetic and velocity fields:

$$\begin{aligned} \mathbf{b}_1(x', y') &= \mathbf{B}(x', y') - \mathbf{B}_0(y'), \\ \mathbf{u}_1(x', y') &= \mathbf{u}(x', y') - \mathbf{u}_0(y'). \end{aligned} \quad (26)$$

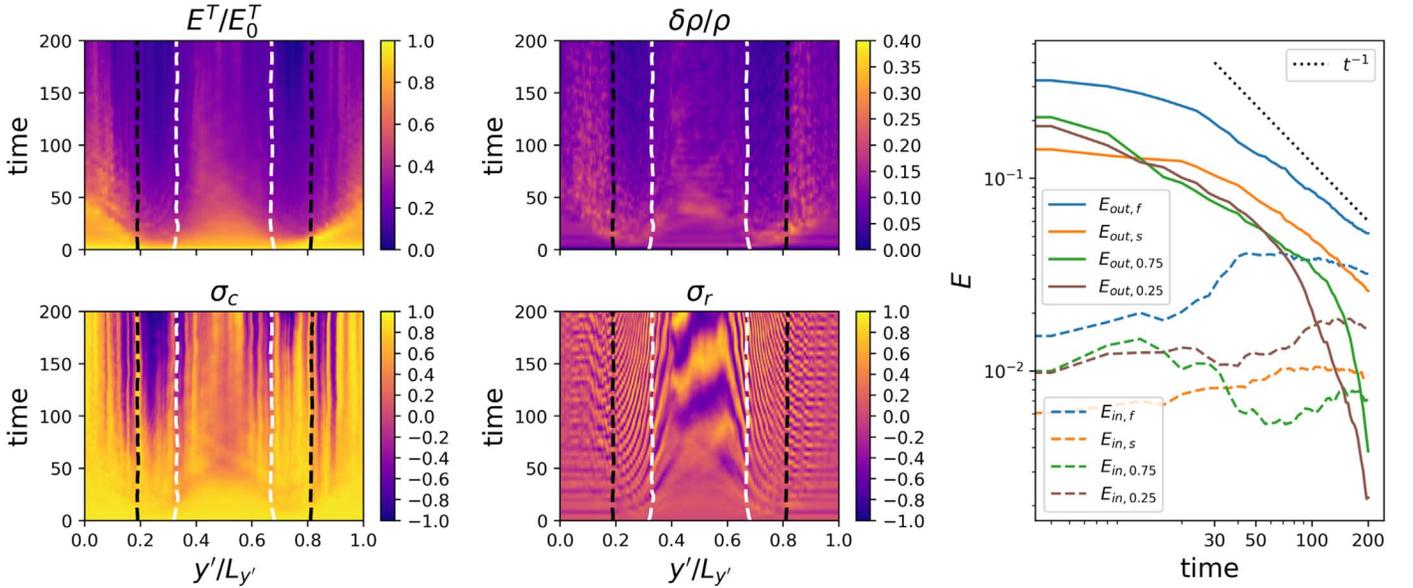


Figure 3. Results of Run A0. The left two columns are the y' - t contours of the total Elsässer energy E^T/E_0^T , where E_0^T is $E^T(t=0)$ (top left), the normalized density perturbation $\delta\rho/\rho$ (top middle), the normalized cross helicity σ_c (bottom left), and the normalized residual energy σ_r (bottom middle). White dashed lines mark the longitudes where the x' -averaged radial speed u_{0x} equals 650 km s^{-1} and the black dashed lines mark the longitudes where the x' -averaged radial speed u_{0x} equals 400 km s^{-1} . The right panel shows the time evolution of the Elsässer energies in log-log scale where solid/dashed curves are the outward/inward waves and blue, orange, green, and brown represent fast stream (“f”), slow stream (“s”), shear region around $y' = 0.75L_{y'}$, and shear region around $y' = 0.25L_{y'}$. The black dotted line is $E \propto t^{-1}$ for reference.

The Elsässer variables are then calculated by

$$\mathbf{z}_{\text{out}} = \mathbf{u}_1 - \text{sign}(B_{0x}) \frac{\mathbf{b}_1}{\sqrt{\rho}}, \quad \mathbf{z}_{\text{in}} = \mathbf{u}_1 + \text{sign}(B_{0x}) \frac{\mathbf{b}_1}{\sqrt{\rho}}, \quad (27)$$

where $\text{sign}(B_{0x})$ is the sign of the radial background magnetic field. Note that the density is not x' averaged but the local density. We further project the Elsässer variables defined by Equation (27) into three directions: the out-of-plane direction \hat{e}_z , the parallel-to- \mathbf{B}_0 direction $\hat{e}_{x'}$, and the in-plane perpendicular-to- \mathbf{B}_0 direction $\hat{e}_\perp = \hat{e}_z \times \hat{e}_{x'}$. In the analysis hereinafter, we only deal with the z -component and the perpendicular component and exclude the parallel component. At a certain time t , various energies as functions of y' are calculated by integrating along the x' direction, e.g., the outward Elsässer energy:

$$E_{\text{out}}(y', t) = \frac{1}{2} \int_{x'} (z_{\text{out},z}^2 + z_{\text{out},\perp}^2). \quad (28)$$

The total energy, the normalized cross helicity, and the normalized residual energy are then calculated by

$$E^T = E_{\text{out}} + E_{\text{in}}, \quad \sigma_c = \frac{E_{\text{out}} - E_{\text{in}}}{E_{\text{out}} + E_{\text{in}}}, \quad \sigma_r = \frac{E_u - E_b}{E_u + E_b}. \quad (29)$$

The kinetic and magnetic energies are those in the perturbations \mathbf{u}_1 and \mathbf{b}_1 and we do not include the parallel component in calculating σ_r . We have verified that including the parallel component in E_u and E_b does not make a significant difference. The normalized density perturbation $\delta\rho/\rho$ is the root mean square

value of ρ along x' divided by the x' -averaged density ρ_0 :

$$\frac{\delta\rho}{\rho}(y', t) = \frac{1}{\rho_0(y', t)} \sqrt{\frac{1}{L_{x'}} \int_{x'} [\rho(x', y', t) - \rho_0(y', t)]^2}. \quad (30)$$

Power spectra of \mathbf{z}_{out} and \mathbf{z}_{in} are calculated along $\hat{e}_{x'}$ and $\hat{e}_{y'}$ by applying the Fourier transform to the z and perpendicular components of them, e.g., $E_{\text{out},z}(k_{x'}, y', t) = |\mathcal{F}_{x'}(z_{\text{out},z})|^2$, where $\mathcal{F}_{x'}$ is the Fourier transform in coordinate x' . When we present the spectra, we further average the spectra along the non-Fourier-transformed coordinates to eliminate the strong oscillations. The details of the averaging procedure of the spectra will be discussed later.

3.3. Run A0: No Corotation, No Expansion, Outward-dominant Waves

In Run A0, the background fields are radial, i.e., there is no compression and rarefaction. Besides, the expansion effect is turned off. The initial condition consists of the outward-dominant Alfvén wave band. The result of Run A0 is shown in Figure 3.

The top-left panel shows the y' - t contour of the total Elsässer energy E^T/E_0^T , where E_0^T is $E^T(t=0)$. The white dashed lines mark y' , where u_{0x} equals 650 km s^{-1} and the black dashed lines mark y' , where u_{0x} equals 400 km s^{-1} (the same in the other three contours). We see that the total Elsässer energy E^T at all longitudes decays with time while in the shear region the energy decays much faster. The right panel of Figure 3 shows the time evolution of the Elsässer energies of the outward wave (solid curves) and inward wave (dashed curves) averaged in different regions bounded by the white and black dashed lines in the contours, i.e., the fast stream (blue), the slow stream (orange), the shear region around $y' = 0.75L_{y'}$ (green), and the

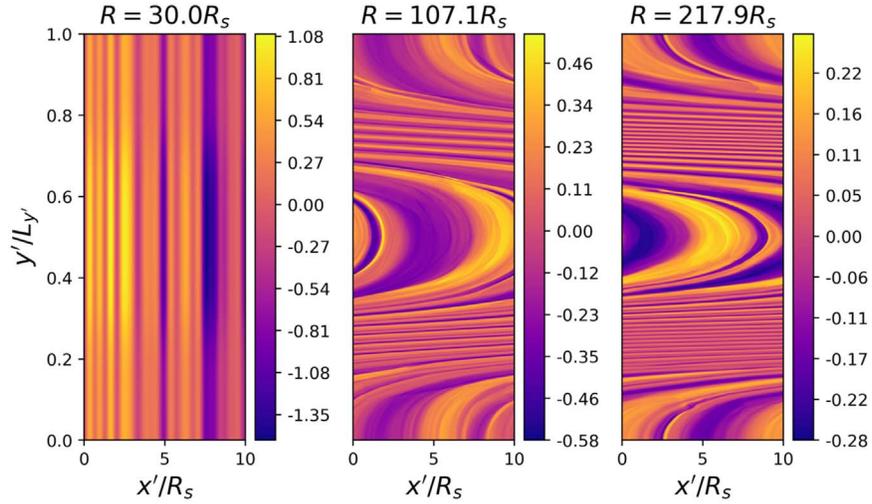


Figure 4. Contours of the z -component of the outward Elsässer variable $z_{\text{out},z}$ at $R = 30.0R_s$, $107.1R_s$, and $217.9R_s$ in Run A.

shear region around $y' = 0.25L_{y'}$ (brown). The black dotted line is $E \propto t^{-1}$ for reference. The evolution of the wave energies inside the fast and slow streams is similar: the outward wave energy decays with time at a rate slower than t^{-1} and the inward wave energy increases with time slightly. Inside the shear regions, the outward wave energy decays slower than t^{-1} first and the decay rate is similar to that of the outward wave inside the fast/slow streams. However, after some time ($t \gtrsim 100$ in the shear region around $y' = 0.75L_{y'}$ and $t \gtrsim 70$ in the shear region around $y' = 0.25L_{y'}$) the wave energy starts to drop very fast. The inward wave energy grows slowly at the beginning, followed by a drop at $t \approx 30$, and then starts to grow again in the two shear regions. Note that in the shear region at $y' \approx 0.75L_{y'}$, the drop of the inward wave energy is stronger than that in the shear region at $y' \approx 0.25L_{y'}$. Here we must point out that the initial configuration, although symmetric in y' , does not evolve symmetrically because the y' gradients of the background fields are of opposite signs, while the initial perturbations along y' (e.g., $u_{1y'}$) do not change sign at the two shear regions. This, for example, will lead to an increase of ρ at one shear region and a decrease of ρ at the other one.

The top-middle panel of Figure 3 shows the y' - t contour of the relative density fluctuation $\delta\rho/\rho$. The value of $\delta\rho/\rho$ remains small ($\lesssim 0.2$) throughout the simulation. The largest density fluctuation $\delta\rho$ is found to be inside the slow stream near the boundaries of the shear regions, as can be seen from the contour. The bottom-left panel displays the y' - t contour of the normalized cross helicity σ_c , which decays with distance in all the flow regions. The decay rate is largest inside the shear region at $y' \approx 0.25L_{y'}$, where σ_c almost reaches -1 at the end of the simulation. This can also be seen from the right column of Figure 3, which shows that the outward Elsässer energy is one order of magnitude smaller than the inward energy in the shear region at $y' \approx 0.25L_{y'}$ at the end of the simulation. A notable phenomenon is the striped structures in the contour of σ_c , showing that σ_c decays much faster within some narrow channels in y' compared to the ambient streams. The evolution of σ_c we find is very similar to that in the incompressible simulation by Roberts et al. (1992) (see their Figure 12). The bottom-middle panel shows the y' - t contour of the normalized

residual energy σ_r . Strong oscillations are observed. On average σ_r is 0, but the instant amplitude can be as large as ≈ 1 . The oscillation of σ_r is strongest inside the shear regions due to the large longitudinal gradient of the relative speed between the counter-propagating waves. A trend of increasing of $|\sigma_r|$ in some regions, e.g., the fast stream and the shear regions, is also seen. This is because of the decrease of $|\sigma_c|$: if $\sigma_c = 0$, the inward and outward waves are of the same amplitude and thus $|\sigma_r|$ will be equal to 1 if the two populations of waves are non-correlated. Actually, we can see that at $y' \approx 0.25L_{y'}$, $|\sigma_r|$ increases at $t \lesssim 150$ and then starts to drop, which is anticorrelated with $|\sigma_c|$.

3.4. Run A: No Corotation, Expansion, Outward-dominant Waves

In this subsection, we present the results of Run A ($\alpha = 0$, $r_{io} = 0.2$ and $\delta b = 0.2$), where the compression between the fast and slow streams is absent but the expansion effect is turned on.

Figure 4 shows the contours of the out-of-plane component of the outward Elsässer variable $z_{\text{out},z}$ at three radial distances: $R = 30.0R_s$, $107.1R_s$, and $217.9R_s$. From Figure 4, it can be clearly seen that the differential radial flow leads to the phase mixing of the Alfvén waves, the wave vector of which is tilted from $\hat{e}_{x'}$ toward $\hat{e}_{y'}$. The strongest phase mixing happens in the regions where the velocity shear is the largest (around $y' = 0.75L_{y'}$ and $y' = 0.25L_{y'}$). The dissipation of waves is observed at these regions since the phase mixing transfers the wave energy to small scales where the numerical dissipation is strong. For other Elsässer variables, i.e., $z_{\text{out},\perp}$, $z_{\text{in},z}$, and $z_{\text{in},\perp}$, a similar evolution is also observed.

Figure 5 displays the y' - R contours of the total Elsässer energy E^T/E_0^T compensated by R/R_0 , where E_0^T is E^T at $t = 0$ (top left), the relative density fluctuation $\delta\rho/\rho$ (top right), the normalized cross helicity σ_c (bottom left), and the normalized residual energy σ_r (bottom right), similar to Figure 3 but note that the y -axis is now radial distance instead of time. The white dashed lines mark $y'(R)$ where the x' -averaged radial speed u_{0x} equals 650 km s^{-1} and the black dashed lines mark $y'(R)$, where u_{0x} equals 400 km s^{-1} . The decay of E^T is in general faster than $1/R$, the WKB prediction of the Alfvén waves in the spherical

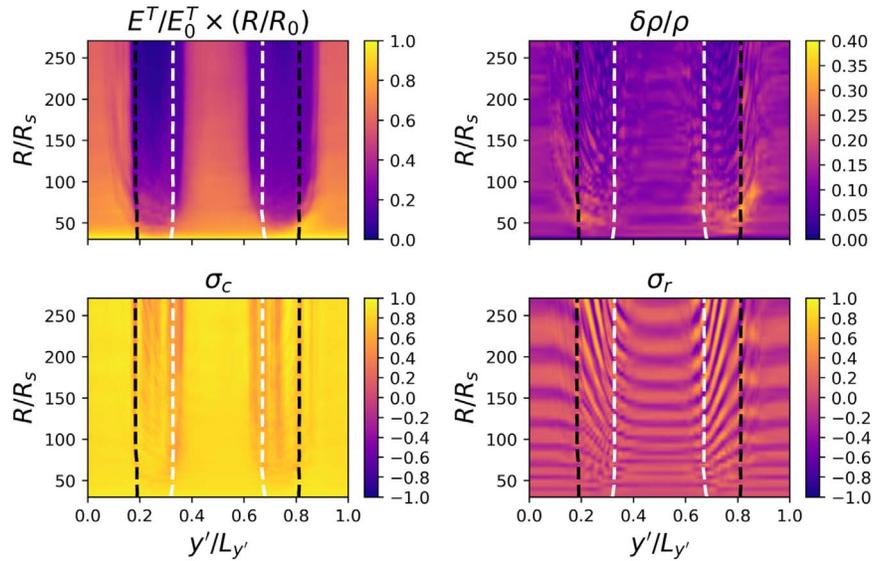


Figure 5. y' - R contours of the total Elsässer energy E^T/E_0^T compensated by R/R_0 , where E_0^T is $E^T(t=0)$ (top left), the normalized density perturbation $\delta\rho/\rho$ (top right), the normalized cross helicity σ_c (bottom left), and the normalized residual energy σ_r (bottom right) for Run A. White dashed lines mark the longitudes where the x' -averaged radial speed u_{0x} equals 650 km s^{-1} and black dashed lines mark the longitudes where the x' -averaged radial speed u_{0x} equals 400 km s^{-1} .

geometry (Belcher 1971). Similar to Run A0, it clearly shows a longitudinal variation: inside the fast and slow streams, the decay is slower than in the shear regions. The relative density fluctuation $\delta\rho/\rho$ is smaller than 0.2 most of the time and it is smaller inside the shear regions compared with the fast and slow streams. It is also observed that some density structures are generated near the boundaries between the shear regions and the slow stream and propagate along the y' direction. The most significant one starts at $R \approx 60R_s$ and $y' \approx 0.8L_{y'}$, with amplitude $\delta\rho/\rho \approx 0.35$. Note that in Run A0 we also observe that the density fluctuation is largest near the boundary between the shear region and the slow stream.

It is known from the observations that the normalized cross helicity decreases with radial distance (e.g., Roberts et al. 1987a, 1987b). The possible mechanisms for the decrease include the generation of inward Alfvén waves due to the velocity shears and the faster decay of outward Alfvén waves with distance compared with the inward waves (Bruno & Bavassano 1991). In Run A0 we already see that the velocity shear leads to the drop of σ_c . From Figure 5, we confirm that σ_c drops with radial distance inside the shear regions, especially near the boundaries of the fast stream. It decreases to values of around 0.7–0.8 within $100R_s$ and then decreases slowly to around 0.5–0.6 until the end of the simulation $R = 270.9R_s$. In the fast and slow streams, σ_c remains almost constant around the initial value 0.92. Compared with Run A0, the contour of σ_c is quite smooth and no stripe-like structures are formed, indicating that the expansion effect slows down the evolution of the wave energies. Last, we look at the residual energy shown in the bottom-right panel. Similar to Run A0, the normalized residual energy fluctuates around 0 and no systematic growth of σ_r is observed. However, the oscillation of σ_r is much weaker in Run A than in Run A0 because the expansion reduces the Alfvén speed so that the relative speed between the outward and inward waves goes down with radial distance.

Figure 6 shows the power spectra of the Elsässer variables along the parallel direction $\hat{e}_{x'}$ (in this run it is aligned with

the radial direction) at (a) $R = 107.1R_s \approx 0.5 \text{ au}$ and (b) $R = 217.9R_s \approx 1 \text{ au}$. Again we divide the domain into four regions: the fast stream, the slow stream, and the two shear regions (in Runs B–D they are the compression/rarefaction regions). The spectra in different regions are displayed in four subplots at each time. The shear region plotted in the top row is the one at $y' \approx 0.75L_{y'}$. The spectra are averaged in y' inside each region and are multiplied by $k_x^{5/3}$. The blue and orange solid curves are the z and \perp components of the outward Alfvén waves and the dashed curves are of the inward waves. Inside the shear regions, the wave energies are strongly damped and inertial ranges are not observed in the spectra and as the radial distance increases, the spectra are eroded rapidly. In the fast and slow streams, the spectra behave similarly and are more stable compared with the shear regions. Especially, z_{out} shows clear three-segment spectra: the large scales with $k_x R_s \lesssim 1$, the intermediate scales with $1 \lesssim k_x R_s \lesssim 20\text{--}30$, and the small scales that are dominated by the numerical dissipation. At 0.5 au, the large scales show slopes close to $-5/3$ in both the fast and slow streams, while at 1 au the large-scale part of the z_{out} spectrum in the fast stream is eroded by the intermediate-scale part and steepening of the spectrum is observed.

Figure 7 shows the power spectra of the Elsässer variables along the $\hat{e}_{y'}$ direction at four radial distances: $R = 30.0R_s$, $107.1R_s$, $217.9R_s$, and $270.9R_s$. The wave number $k_{y'}$ is defined by the normalized y' , i.e., $y'/L_{y'}$, so that $k_{y'} \in [0, n_{y'}/2]$. The blue and orange lines are the z and \perp components of the outward Alfvén wave and the dashed lines are those of the inward Alfvén wave. The spectra are averaged in x' and multiplied by $k_y^{5/3}$. At $R = 30.0R_s$, i.e., the initial state, the curves for z -components are covered by those of \perp -components as the initial wave band is circularly polarized. A Kolmogorov-like inertial range that spans about one decade forms at 0.5 au for all the wave components. It is maintained throughout the simulation for $E_{\text{out},\perp}$ and $E_{\text{in},\perp}$. But for $E_{\text{in},z}$ the inertial range shortens with radial distance and for $E_{\text{out},z}$ the inertial range becomes shallower than $k_y^{-5/3}$ at 1 au. This

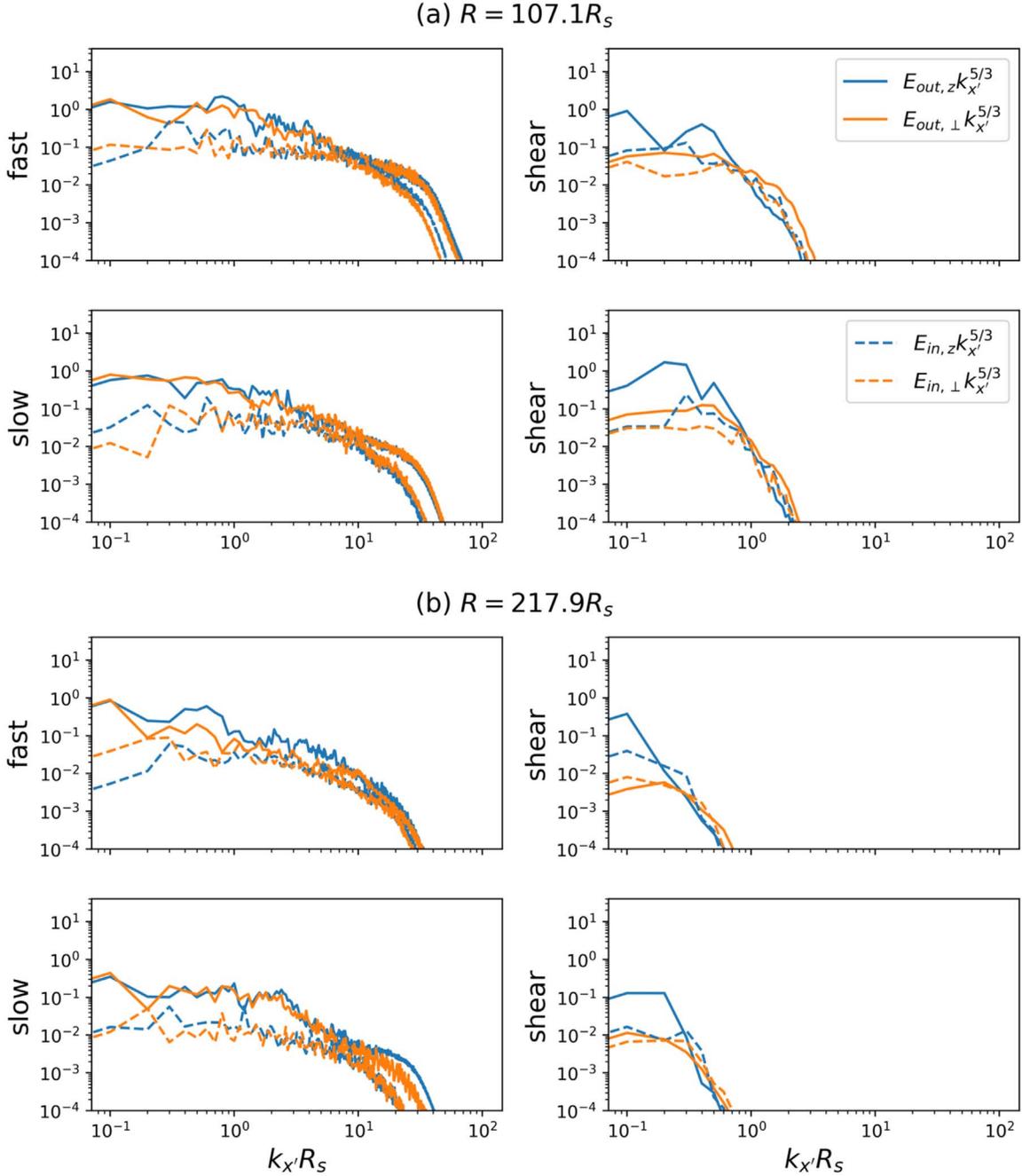


Figure 6. Power spectra of the Elsässer variables in Run A at (a) $R = 107.1R_S$ and (b) $R = 217.9R_S$, averaged in different regions: fast stream (top left), slow stream (bottom left), shear region at $y' = 0.75L_{y'}$ (top right), and shear region at $y' = 0.25L_{y'}$ (bottom right). The spectra are calculated along the parallel direction \hat{e}_x . Blue and orange solid lines are the z -component and in-plane perpendicular-to- \mathbf{B}_0 component of the outward Alfvén wave. Blue and orange dashed lines are the two components of the inward Alfvén wave. The spectra are multiplied by $k_x'^{5/3}$.

asymmetry between the \perp -component and the z -component is due to the uniformity in the z -direction, which rules out the nonlinear interaction between the waves along \hat{e}_z .

3.5. Run B: Corotation, Expansion, Outward-dominant Waves

In this section we present the results of Run B ($\alpha = 0.142$, $r_{io} = 0.2$, and $\delta b = 0.2$). This run has the most realistic setup: expansion, velocity shear, and compression/rarefaction are all present and the initial perturbations are outward-dominant Alfvén waves.

Figure 8 is a plot similar to that in Figure 5 for Run B. From top left to bottom right are the corrected total Elsässer energy $E^T/E_0^T \times (R/R_0)$, the normalized density fluctuation, the normalized cross helicity, and the normalized residual energy. The white and black dashed lines mark $u_{0x} = 650 \text{ km s}^{-1}$ and $u_{0x} = 400 \text{ km s}^{-1}$, respectively. Similar to Run A, the total energy decays faster than the WKB prediction R^{-1} . However, in the fast and slow streams, the radial decay of E^T is significantly faster in Run B than in Run A. Besides, in Run B, beyond $R \approx 200R_S$, a narrow band inside the compression region forms at $y' \approx 0.75L_{y'}$, where the wave energy is much

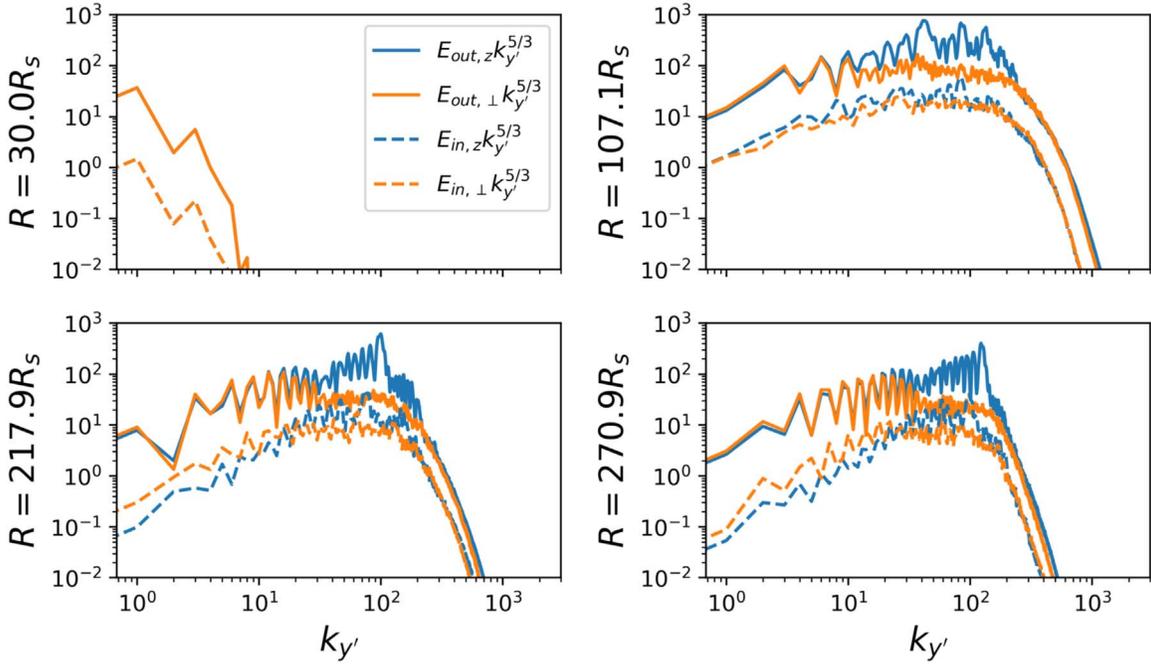


Figure 7. Power spectra of the Elsässer variables calculated along \hat{e}_y' and averaged in x' in Run A. From top left to bottom right the power spectra are at $R = 30.0R_s$, $107.1R_s$, $217.9R_s$, and $270.9R_s$ respectively. The blue and orange solid lines represent the z -component and in-plane perpendicular-to- \mathbf{B}_0 component of the outward Alfvén wave. Blue and orange dashed lines are the two components of the inward Alfvén wave. The spectra are multiplied by $k_y'^{5/3}$.

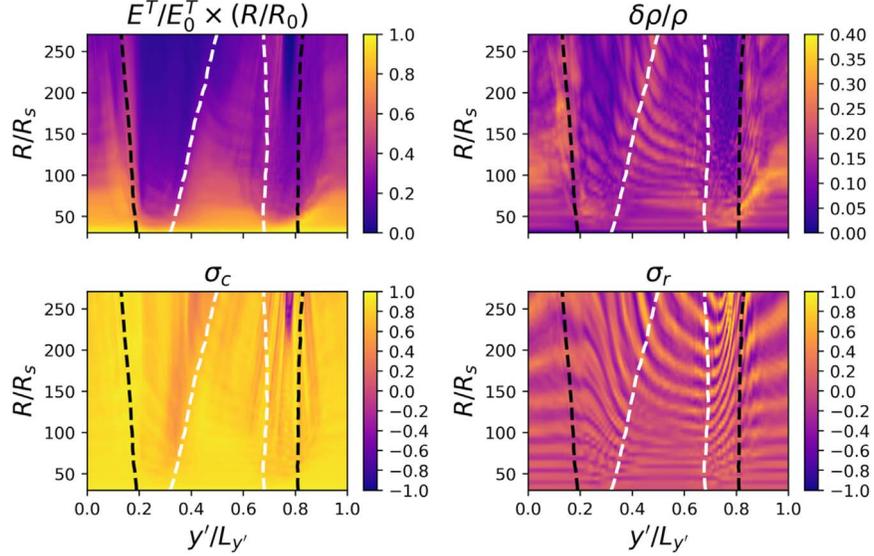


Figure 8. y' - R contours of the total Elsässer energy E^T/E_0^T compensated by R/R_0 , where E_0^T is $E^T(t=0)$ (top left), the normalized density perturbation $\delta\rho/\rho$ (top right), the normalized cross helicity σ_c (bottom left), and the normalized residual energy σ_r (bottom right) for Run B. White dashed lines mark the longitudes where the x' -averaged radial speed u_{0x} equals 650 km s^{-1} and black dashed lines mark the longitudes where the x' -averaged radial speed u_{0x} equals 400 km s^{-1} .

more damped compared with the shear regions in Run A. This might be due to the fact that the compression between the fast and slow streams steepens the velocity profile, enlarging the velocity shear. $\delta\rho/\rho$ and σ_r do not show significant differences between Run A and Run B. Similar to Run A, the decrease of σ_c is more significant in the compression and rarefaction regions than inside the fast and slow streams. In the rarefaction region, mainly in the trailing edge of the fast stream, σ_c drops to around 0.6 very soon at $R \approx 80R_s$ and remains around this

value until the end of the simulation. In the compression region, however, σ_c remains relatively large (>0.5) for a long time followed by a fast drop beyond $R \approx 1 \text{ au}$ and reaches around -0.7 at the end of the simulation $R = 270.9R_s$. The drop of σ_c coincides with the drop of E^T in the compression region (see the top-left panel). In the fast and slow streams, σ_c decreases with distance more slowly, from the initial value of 0.92 to ~ 0.7 – 0.8 at 1 au. Note that in Run A, σ_c remains almost constant around the initial value 0.92 inside the fast and slow

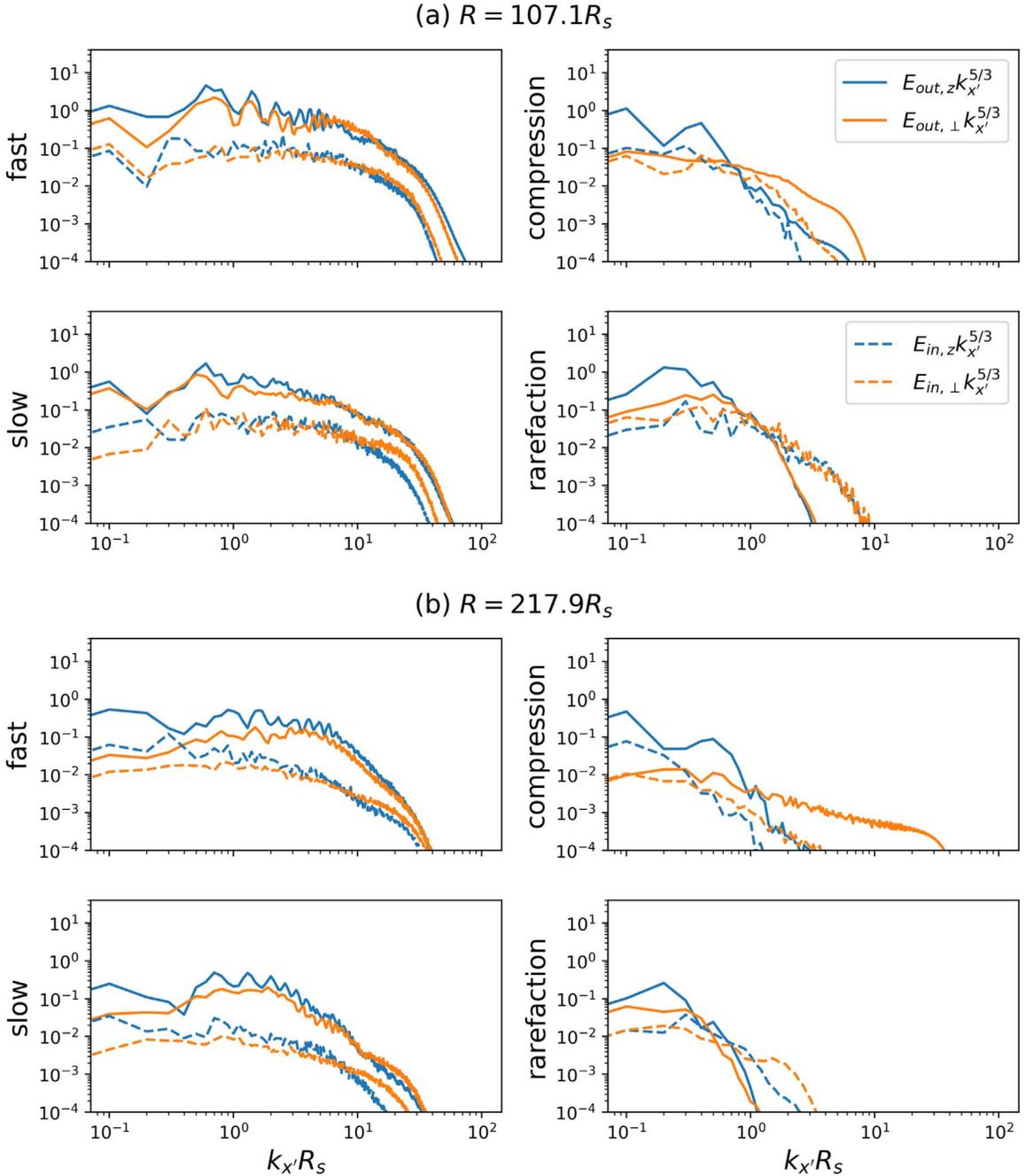


Figure 9. Power spectra of the Elsässer variables in Run B at (a) $R = 107.1R_S$ and (b) $R = 217.9R_S$ averaged in different regions: fast stream (top left), slow stream (bottom left), compression region (top right), and rarefaction region (bottom right). The spectra are calculated along the parallel direction $\hat{e}_{x'}$. The blue and orange solid lines represent the z -component and in-plane perpendicular-to- \mathbf{B}_0 component of the outward Alfvén wave. The blue and orange dashed lines represent the two components of the inward Alfvén wave. The spectra are multiplied by $k_{x'}^{5/3}$.

streams, i.e., the velocity shear only reduces the normalized cross helicity locally in the shear regions. Thus, the compression between the fast and slow streams might play an important role in the radial evolution of σ_c . It not only speeds up the drop of σ_c in the compression region but also speeds up the decrease of σ_c inside the fast and slow streams by steepening the velocity profile at all longitudes.

Figure 9 shows the $k_{x'}^{5/3}$ -corrected power spectra of z_{out} and z_{in} in fast stream, slow stream, the compression region, and the rarefaction region at $R = 107.1R_S$ and $R = 217.9R_S$. By comparing Figures 9 and 6, several differences are observed.

First, inside the compression and rarefaction regions (shear regions in Run A), the Elsässer energies are damped in both runs but in Run B the damping is weaker than Run A. Especially, in Run B the wave energy decays with $k_{x'}$ much slower, indicating that the compression and rarefaction transfer energy from large scales to small scales effectively. Second, in Run B, we also observe an asymmetry between the compression and rarefaction regions: at high- $k_{x'}$ ranges ($k_{x'} R_S \gtrsim 1$), the inward wave energy dominates in the rarefaction region while in the compression region the outward wave energy dominates. Third, inside the fast and slow streams, the evolution of the

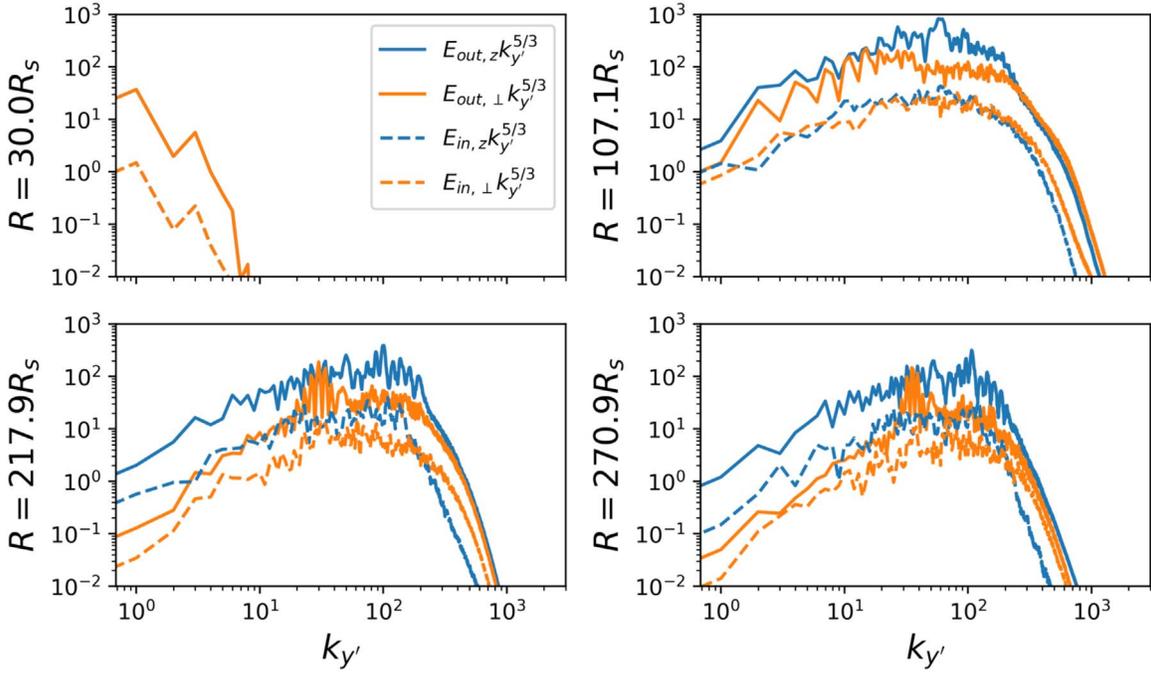


Figure 10. Power spectra of the Elsässer variables calculated along \hat{e}_y' and averaged in x' in Run B. From top left to bottom right the power spectra are at $R = 30.0R_s$, $107.1R_s$, $217.9R_s$, and $270.9R_s$ respectively. The blue and orange solid lines represent the z -component and in-plane perpendicular-to- B_0 component of the outward Alfvén wave. The blue and orange dashed lines represent the two components of the inward Alfvén wave. The spectra are multiplied by $k_y'^{5/3}$.

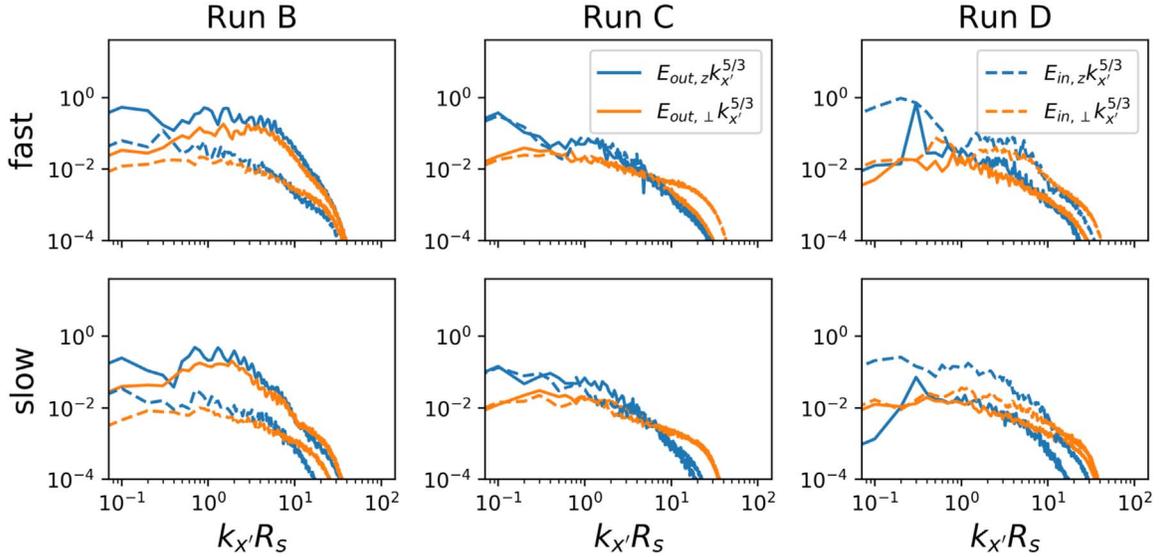


Figure 11. Parallel power spectra multiplied by $k_x'^{5/3}$ of Elsässer variables inside the fast (top row) and slow streams (bottom row) at $217.9R_s$ for Run B (left column), C (middle column), and D (right column). The blue and orange solid lines represent the z -component and in-plane perpendicular component of the outward Alfvén wave. The blue and orange dashed lines represent the two components of the inward Alfvén wave.

spectra is different in Run B compared with Run A. At $R = 107.1R_s$, the inward waves show $-5/3$ spectra over a substantial range of k_x' but the outward waves show spectra steeper than $k_x'^{-5/3}$. During the evolution toward 1 au, the spectra of z_{in} steepen while the spectra of z_{out} develop a Kolmogorov-like inertial range as seen in plot (b) of Figure 9. The span of the inertial range in the fast stream is larger than that in the slow stream.

Figure 10 is the x' -averaged y' -spectra of z_{out} and z_{in} corrected by $k_y'^{5/3}$ in Run B. At $R = 107.1R_s$ the Kolmogorov-type inertial

range is well established for both outward and inward waves. Different from Run A, the shape of the spectra is only slightly changed throughout the simulation in this run.

3.6. Run C and Run D

Run C and Run D have both corotation and expansion turned on, similar to Run B, but have $r_{io} = 1$ and $r_{io} = 5$, respectively. They are carried out to show how the inward and outward waves evolve differently when their amplitudes change.

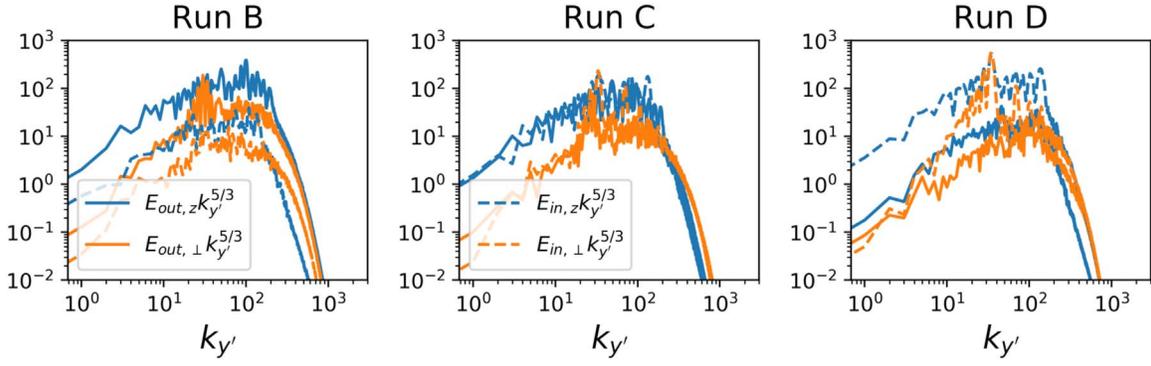


Figure 12. Power spectra of Elsässer variables calculated along \hat{e}_y averaged in x' at $217.9R_s$ for Run B (left), C (middle), and D (right). Blue and orange solid lines are the z -component and in-plane perpendicular component of the outward Alfvén wave. The blue and orange dashed lines represent the two components of the inward Alfvén wave. The spectra are multiplied by $k_y^{5/3}$.

Figure 11 compares the $k_x^{5/3}$ -corrected parallel power spectra of the Elsässer variables inside the fast and slow streams at $217.9R_s$ for Runs B–D. For Run C and Run D, the spectra inside the slow stream show inertial ranges steeper than $k_x^{-5/3}$. Inside the fast stream, Run D shows a short Kolmogorov-like range at $k_x R_s \sim 1\text{--}4$ for E_{in} , while Run C shows a shorter one in $E_{out,z}$ and $E_{in,z}$. Note that in Run B, clear Kolmogorov-like inertial ranges are observed in E_{out} spectra inside both fast and slow streams. In other words, in order to get Kolmogorov-like parallel spectra, the outward-dominant initial condition is preferred to the balanced and the inward-dominant ones. However, as shown in Figure 12, the k_y spectra at $217.9R_s$ are similar for Runs B, C, and D as clear $k_y^{-5/3}$ inertial ranges are observed in all three runs.

We then inspect the radial evolution of E_{out} and E_{in} inside different regions for Runs A–D and the results are shown in Figure 13. The energies are corrected by R/R_0 and the plot is log–log scale. Solid and dashed curves are E_{out} and E_{in} , respectively. Colors represent different regions as shown in the legend and described in the caption where the subscripts “ f ,” “ s ,” “ c ,” and “ r ” represent the fast stream, slow stream, compression region, and rarefaction region, respectively. For Run C, we multiply the energies in the four regions by different factors, as shown in the plot, in order to separate the overlapped curves and make the plot more readable. The energies are calculated by averaging $\frac{1}{2}|z^\pm|^2$ over different regions at each time. We first compare Run A and Run B. These two runs are both outward dominant but Run A lacks the compression and rarefaction between streams. Compared with Run B, in Run A E_{out} decays much slower inside the fast and slow streams but faster inside the shear regions, i.e., the compression effect speeds up the dissipation of the outward waves in the regions without large velocity gradients but it slows down the dissipation inside the regions with large velocity gradients. The evolution of E_{in} inside the fast and slow streams does not show significant differences between Run A and Run B and approximately follow the R^{-1} WKB prediction. But in the shear regions E_{in} , similar to E_{out} , decays faster in Run A than Run B. Then we compare Run C with Run B. In Run C the initial condition consists of balanced outward/inward waves instead of outward-dominant waves. By comparing the blue and orange curves in panel Run C, we see that the evolution of E_{out} and E_{in} is very similar to each other inside the fast and slow streams and the decay rates are similar to those of E_{out} in Run B. Inside the rarefaction region, the inward waves decay

much slower than the outward waves. Compared to Run B, E_{in} shows a slower decay rate, while E_{out} has a similar decay rate. In the compression region, both E_{in} and E_{out} show similar evolution as in Run B: a decay followed by a plateau or even an increase. Last, we inspect Run D where the initial condition is an inward-dominant wave band, inverse to Run B. Inside the fast stream and the slow stream, E_{in} in Run D evolves similarly with E_{out} in Run B. E_{out} grows at the beginning and then decays with R , similar to E_{in} in Run B, but its growth and decay are stronger. Consistent with Run C, this result shows that when the wave amplitude is large enough, its radial evolution inside the fast and slow streams is not affected by the direction of the propagation. It is likely that there is some mechanism that generates/depletes small-amplitude waves and it works differently for outward and inward waves. In the compression region, E_{in} in Run D evolves similar with E_{out} in Run B and E_{out} in Run D decreases to a smaller level compared with E_{in} in Run B, although both of them reach a plateau beyond $R \approx 10^2 R_s$. In the rarefaction region, E_{in} in Run D has a decay rate similar to that in Run C, i.e., slower than that in Run B. On the other hand, E_{out} in Runs B, C, and D show a very close decay rate beyond $R \approx 10^2 R_s$, indicating that the decay of E_{out} in the rarefaction region is not affected by the wave amplitude significantly.

To summarize the above paragraph, we list the major findings from Figure 13 below: (1) the radial decrease of the wave amplitude is faster than the WKB prediction when the amplitude is large but gets closer to the WKB prediction when the amplitude is small, especially inside the fast and slow streams where the velocity shear is small. (2) The compression between fast and slow streams speeds up the dissipation of the waves inside the fast and slow streams but slows down the dissipation inside the compression and rarefaction regions. (3) Inside the fast and slow streams, the outward and inward waves do not show significant differences: the radial evolution of their energies are controlled mainly by their amplitudes instead of the propagation directions. (4) In the compression region, the outward wave decays faster than the inward wave but both of them decay slower as the radial distance increases. (5) Inside the rarefaction region, the outward and inward waves show strong asymmetry. The radial decay of the outward wave is in general faster than the inward wave and is not affected by the wave amplitude significantly. The decay of the inward wave energy, on the contrary, is modulated by the wave amplitude: the larger the wave amplitude is, the slower E_{in} decreases with distance.

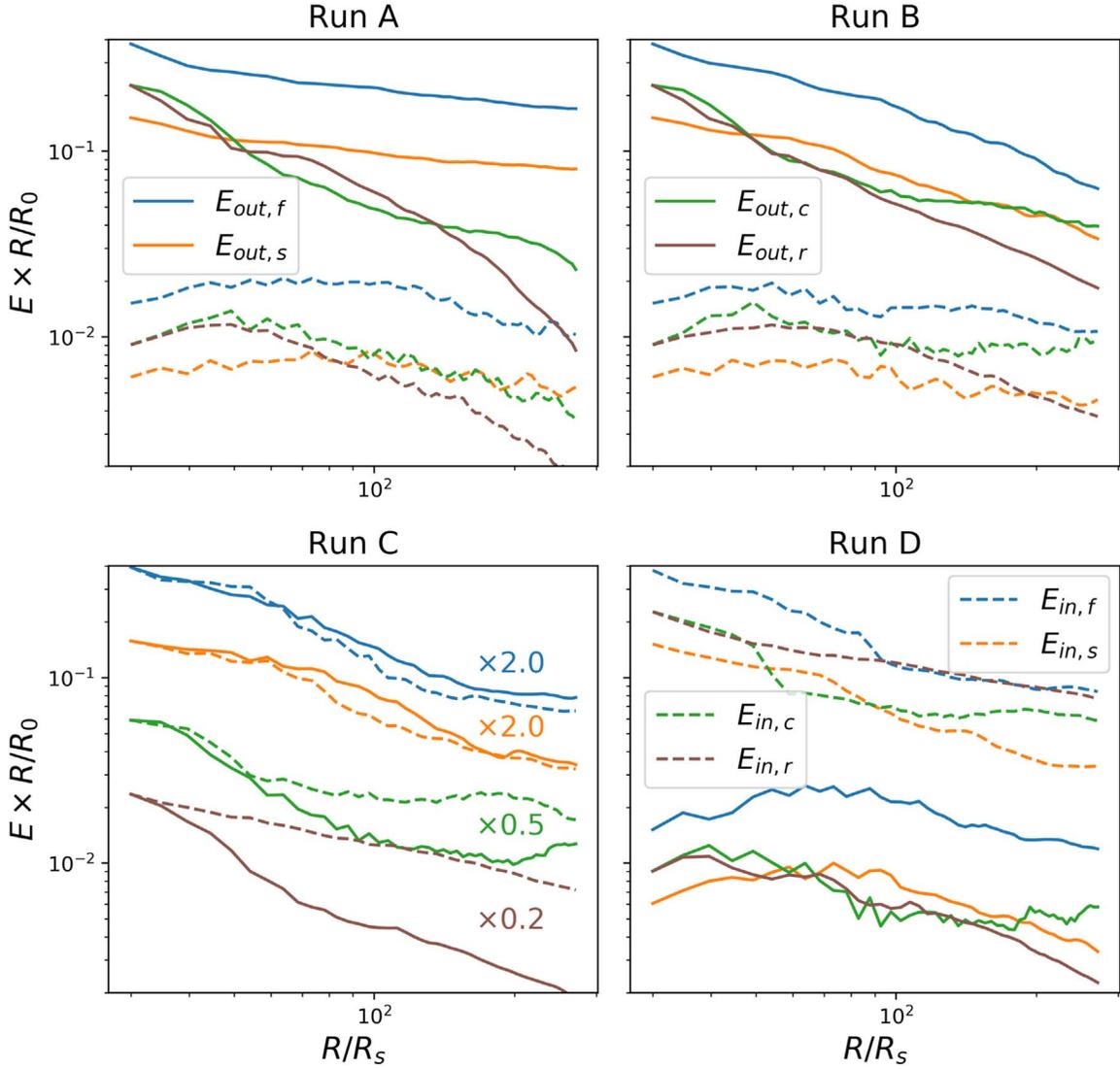


Figure 13. Radial evolution of the Elsässer energies E_{out} (solid curves) and E_{in} (dashed curves) in different regions. The energies are corrected by R/R_0 . From the top-left to bottom-right panels are Runs A–D, respectively. The plot is in log–log scale. Blue, orange, green, and brown represent fast stream (“f”), slow stream (“s”), compression region (“c”), and rarefaction region (“r”). For Run A the compression and the rarefaction regions are the two shear regions around $y' = 0.75L_y$ and $y' = 0.25L_y$, respectively. For Run C we multiply the energies in the four regions by different factors as shown in the plot in order to separate the overlapped curves.

4. Conclusion

In this paper, we use the corotating EBM (2D version) to simulate the propagation of Alfvén waves and turbulence in solar wind. The large-scale stream interactions, including shear, compression, and rarefaction, are evolved self-consistently. The simulation parameters are chosen to be close to the realistic solar wind. We find that the Elsässer energies are depleted in the strong-shear regions due to phase mixing, accompanied by a decrease of the normalized cross helicity. This process is greatly enhanced by the compression between fast and slow streams. The generated density fluctuations are overall small ($\delta\rho/\rho \lesssim 0.2$ on average) and there are high-density structures born near the boundaries between the shear regions and the slow streams. The normalized residual energy fluctuates around zero due to the correlation and de-correlation between the outward and inward waves but no net growth or decrease of it is observed, contrary to the solar wind observations, which show a prevailing excess of magnetic energy (e.g., Roberts et al. 1987b; Chen et al. 2013). The parallel spectra of the

Elässer variables show Kolmogorov-like inertial ranges only inside the fast and slow streams and when the shear and compression between streams are present. Besides, the outward-dominant wave band is preferred. Otherwise, the parallel spectra are in general steeper than $k^{-5/3}$. On the other hand, the perpendicular, or more precisely the quasi-longitudinal, spectra show Kolmogorov-like inertial ranges in all the runs carried out, no matter whether the compression between streams is present and how the initial wave band is configured. Since the Alfvénic fluctuations are in the perpendicular-to- \mathbf{B}_0 direction, it is reasonable to expect that the spectra are more developed in this direction (plane). The radial evolution of the Elsässer energies shows significant longitudinal dependence, symmetry/asymmetry between the outward and inward waves and the wave-amplitude dependence.

In the present paper we have allowed for a large-scale stream structure, but we have not included the corresponding sector structure with heliospheric current sheet. The heliospheric current sheet is known to be embedded inside the slow wind, at

least at solar minimum (Smith 2001), and the corresponding magnetic shears might modify the conclusions reached above. Current sheets themselves might evolve dynamically and interact with a turbulence in a nontrivial way, as in the region where the magnetic field changes sign, the magnetic field velocity field correlation for outwardly propagating Alfvénic fluctuations must also change sign, leaving a region with little correlation and probably a strong magnetic excess in between. Furthermore, fully three-dimensional simulations need to be carried out for a more realistic solar wind configuration. Third, since the Parker Solar Probe will provide observations at locations from $R \sim 10R_s$ to $R \lesssim 200R_s$, it is good to use the data to study the radial evolution of the Alfvénic turbulence in the inner heliosphere in the future.

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