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Astronomy and the new SI

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Abstract

In 2019 the International System of units (SI) conceptually re-invented itself. This was necessary because quantum-electronic devices had become so precise that the old SI could no longer calibrate them. The new system defines values of fundamental constants (including c, h, k, e but not G) and allows units to be realized from the defined constants through any applicable equation of physics. In this new and more abstract SI, units can take on new guises—for example, the kilogram is at present best implemented as a derived electrical unit. Relevant to astronomy, however, is that several formerly non-SI units, such as electron-volts, light-seconds, and what we may call "gravity seconds" GM/c^3, can now be interpreted not as themselves units, but as shorthand for volts and seconds being used with particular equations of physics. Moreover, the classical astronomical units have exact and rather convenient equivalents in the new SI: zero AB magnitude amounts to $\simeq 5 \times 10^{10}$ photons $\mathrm{m^{-2}\,s^{-1}}$ per logarithmic frequency or wavelength interval, 1 au $\simeq$ 500 lt-s, 1 pc $\simeq 10^8$ lt-s, while a solar mass $\simeq$5 gravity-seconds. As a result, the unit conversions ubiquitous in astrophysics can now be eliminated, without introducing other problems, as the old-style SI would have done. We review a variety of astrophysical processes illustrating the simplifications possible with the new-style SI, with special attention to gravitational dynamics, where care is needed to avoid propagating the uncertainty in G. Well-known systems (GPS satellites, GW170817, and the M87 black hole) are used as examples wherever possible.

Key words: astrophysical processes

1. Introduction

While the whole point of units is to stay at fixed values, definitions of units do in practice change from time to time, in response to scientific developments. From 1990 the SI (Système International d'unités) faced a rebellion of electrical units. The development of Josephson junctions beginning in the 1960s provided voltage as precisely $h/(2e)$ times a frequency. The discovery of the quantum Hall effect in 1980 provided a standard resistance e^2/h. From these two processes a new conventional volt V_{90} and conventional ohm Ω_{90} emerged, which were more precise than the SI-prescribed standards involving force between current-carrying wires. Not only that, the derived electrical unit

$$V_{90}^2\ \Omega_{90}^{-1}\ \mathrm{m^{-2}\ s^3} \tag{1}$$

was equal to a kilogram but more precise than the SI kilogram. The presence of an alternative standard that was more precise threatened to make the SI irrelevant. Faced with this crisis, the SI began a long process of reinventing itself (see Taylor & Mohr 2001) leading to the reforms of 2018/2019. In the new SI (BIPM 2019) only the second is specified by a specific physical process (a spectral line in Cs). Other units are defined implicitly, as whatever makes the speed of light come out as 299,792,458 $\mathrm{m\,s^{-1}}$, Planck's constant come out as $6.626070040 \times 10^{-34}$ J s, and so on. (Table 1 in the Appendix summarizes.) Any equation of physics may be used for realization of units. Thus, if one wishes to interpret the kilogram as a derived electrical unit, there is no longer a conflict with the SI. A nice toy example where this happens is the LEGO watt balance (Chao et al. 2015) which measures mass as electrical watts times $\mathrm{s^3\,m^{-2}}$.

Astronomers cannot be said to have rebelled against the SI, because they never really joined. In the 19th century astronomers set up units standardized from the sky rather than in the laboratory. Ancient stellar magnitudes were formalized as a logarithmic scale for brightness (see Jones 1968; Schaefer 2013, for this history). The Sun became the prototype mass. Most interesting was the astronomical unit of length (for the history, see Sagitov 1970). Anticipating the explicit-constants style of the new SI, the au was defined such that the formal angular velocity $\sqrt{GM_\odot/\mathrm{au}^3}$ equals exactly 0.01720209895 radians per day (which is close to 2π per sidereal year).

Modern practice has continued with the classical units of length, mass, and brightness, but calibrated them against the SI.

1. The astronomical unit of length is now defined as 1 au = 149,597,870,700 m. A parsec is the distance at which 1 au subtends one arc-second, hence now also defined as a fixed (albeit irrational) number of meters. This calibration of astronomical distances to the SI, retiring the 19th-century definition of the au, was adopted quite recently, in 2012.[1]

[1] Resolution B2 *on the re-definition of the astronomical unit of length*, adopted at the IAU General Assembly (2012).

2. The Sun's mass times the gravitational constant (known as the solar mass parameter) is nowadays stated in SI units.[2] It is measured as

$$GM_{\odot} = 1.3271244 \times 10^{20}\,\mathrm{m^3\,s^{-2}} \tag{2}$$

in Newtonian dynamics. Two more significant digits are available if general-relativistic time dilation is included, which we will discuss later (Section 3.4).

3. The AB-magnitude scale (Oke & Gunn 1983) sets a spectral flux density of a jansky (that is, $10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$) as AB = 8.90. This style is not unique to astronomy—in the moment-magnitude scale in seismology (Hanks & Kanamori 1979) a zero-magnitude earthquake corresponds to a gigajoule (or rather $10^{9.05}$ J), and two magnitudes correspond to a factor of a thousand in energy.

Along with these SI-calibrated classical units, astronomers commonly use several named units that are SI units times a simple multiplicative factor. Some of them are universally understood (hours, minutes, also degrees, arc-minutes and arc-seconds) and recognized in Table 8 of the SI brochure as useful alongside SI units. Others (ergs, Gauss, and Å) are legacies of pre-SI conventions.

But there remains one important unsolved problem in calibrating conventional astronomical units against the SI, and again it has to do with the kilogram. In the laboratory, mass is a measure of inertia, but in astrophysics, mass is observable from the gravitational field it produces. Whether the mass is the Sun, or an asteroid less than 100 km across and $\sim 10^{-12} M_{\odot}$ (e.g., Goffin 2014), the astrophysical observable is not M but GM. Unfortunately, G is known only to 10^{-4} (Li et al. 2018), and while there are some creative new ideas for measuring G (Christensen-Dalsgaard et al. 2005; Rosi et al. 2014), there is no near-term prospect of more than four significant digits. The error propagates to any astrophysical mass expressed in kilograms, and in particular to $M_{\odot} = 1.988(2) \times 10^{30}$ kg. This much uncertainty in the mass of the Sun would be fatal for precision applications like spacecraft dynamics. Hence, kilograms cannot be used to express gravitating masses in precision applications. With the kilogram unusable, it is not surprising that astronomers have little appetite for changing any of their conventions to SI units.

The new SI, however, changes the situation. There are no new units in the new SI, nor do any values change significantly, instead the reform is conceptual. In particular, the kilogram is now defined implicitly. This suggests leaving astrophysical masses in kilograms implicit (pending future developments in the measurement of G) and working with the mass parameter in SI units. Of course, in solar system dynamics, this is already done, only $\mathrm{m^3\,s^{-2}}$ seems too un-memorable for general use. But that problem is easily remedied, by using GM/c^2 in meters, or GM/c^3 in seconds. Basically, one could work with the formal Schwarzschild radius in place of the mass.

This paper will develop some ideas like the above. The aim is not some kind of formal compliance with the SI, but to suggest some formulations that (a) have a precise meaning in the new SI, (b) simplify formulas and help understand astrophysical processes better, and (c) would be reasonably easy to convert to. This paper will focus on the au, pc, $M_{\odot}$, and optical magnitudes. If those get converted, units like the Å and Gauss can be trivially replaced, and do not need discussion here. Section 2 discusses SI formulations that are equivalent to the classical astronomical units, but have somewhat different interpretations. Section 3 is devoted to example applications, where we see that replacing the au, pc, solar mass, optical magnitudes with SI equivalents is quite convenient and can provide insight into diverse astrophysical phenomena.

2. Length, Mass, and Brightness Units

Astronomical distances are often stated in light seconds. In topics where general relativity plays a role, GM/c^3 in seconds may appear as a surrogate for mass. AB magnitudes are equivalent to photon flux per logarithmic wavelength interval (see Equation (1) from Tonry et al. 2012). Let us discuss these in turn.

2.1. Light-seconds

The light-second is a common and useful informal unit, and has conveniently round conversions:

$$\begin{aligned} 1\ \mathrm{au} &\longrightarrow 5.0 \times 10^{2}\ \mathrm{s} \\ 1\ \mathrm{pc} &\longrightarrow 1.0 \times 10^{8}\ \mathrm{s}. \end{aligned} \tag{3}$$

Table 2 in the Appendix gives precise conversions.

To use light-seconds more than informally (that is, in formulas and computer programs), we need to decide whether a light second is a length or a time. That is, a light-second could be a synonym for 299,792,458 m, *or* it could be this length divided by c. In this paper, the latter interpretation will be used. That is, a light-second will be taken as a normal second measurable by a clock, just being used to measure a length divided by c.

An analogous situation applies to the electronvolt. The list of useful non-SI units in Table 8 of the SI brochure gives the electronvolt as $1.602176634 \times 10^{-19}$ J. But the electronvolt is also used to measure mass or even frequency. Hence, it is more useful to understand the electronvolt as just a volt, with the "electron" label denoting that, according to context, one is measuring energy divided by the electron charge e, or mass times c^2/e, or frequency times h/e. Formerly, such a multiplication depended on the experimentally determined value of e, but not any more, because in the new SI e is a defined constant.

[2] Resolution B3 *on recommended nominal conversion constants for selected solar and planetary properties,* adopted at the IAU General Assembly (2015).

Later in this paper, we will write several distances in light seconds. For this, let us fix the notation

$$\begin{pmatrix} \bar{a} \\ \bar{R} \\ \bar{D} \end{pmatrix} \equiv \begin{pmatrix} a/c \\ R/c \\ D/c \end{pmatrix} \tag{4}$$

to express lengths as times.

2.2. Gravity Seconds

We could choose GM/c^2 in meters as an easier variant of the mass parameter, but GM/c^3 in seconds blends somewhat better with light-seconds. There is no standard term for measuring

$$\mathscr{M} \equiv \frac{GM}{c^3} \tag{5}$$

in seconds, but it would be useful to have one. Let us use "gravity-second" to denote a second being used in this way. The solar mass in gravity-seconds is

$$\mathscr{M}_\odot \simeq 4.9 \times 10^{-6}\ \mathrm{s} \tag{6}$$

while for the Earth, the value is 15 ps. Again, Table 2 gives precise values. The values of $2\mathscr{M}c$ (3 km for the Sun, 1 cm for the Earth) are more familiar, but the gravity-second values are not difficult to remember. Because of the uncertainty in G, we do not know precisely how many kilograms a gravity-second is, but since the constancy of G has been much better tested than the value has been measured, we do know that a gravity-second is precisely *some* number of kilograms.

One seemingly weird consequence of light-seconds and gravity-seconds is that density will come out in gravity-seconds per cubic light-second, which is frequency squared. To get $\mathrm{kg\ m^{-3}}$ we need to divide by G, as in

$$\frac{\mathscr{M}}{(4\pi/3)\bar{R}^3} \times \frac{1}{G}. \tag{7}$$

In contexts where particle interactions are of interest, density in $\mathrm{eV\ m^{-3}}$ may be more useful. That is easily obtained by a further factor, as in

$$\frac{\mathscr{M}}{(4\pi/3)\bar{R}^3} \times \frac{c^2}{Ge}. \tag{8}$$

For gravitational phenomena, however, density as frequency squared is ideal—recall that the crossing time in a gravitating system depends only on the enclosed density, as manifest in, for example, the destination-independent travel time of 42 minutes on the "gravity-train" through the Earth (Cooper 1966). We can also write the gravitational constant as

$$1/G = (1.075\ \mathrm{hr})^2\ \mathrm{g\ cm^{-3}} \tag{9}$$

explicitly relating density and time squared.

One thing one musn't do with mass in gravity seconds is to take $\mathscr{M}c^2$ to get energy! Instead, to turn gravity-seconds into joules, we have to multiply by

$$\frac{c^5}{G} = 3.628 \times 10^{52}\ \mathrm{W}. \tag{10}$$

This constant, sometimes called the Planck power, is the luminosity scale of merging black holes.

2.3. Photon Flux versus Magnitude

The usual physical measure of brightness in astronomy is the spectral flux density f_ν. The relevant SI unit of $\mathrm{W\ m^{-2}\ Hz^{-1}}$ is extremely large, because 1 Hz is a very small spectral range to pack $1\ \mathrm{W\ m^{-2}}$, and the convention in astronomy is to use a jansky, defined as 10^{-26} of the SI unit. At optical wavelengths, detectors in use nowadays measure the photon flux in some band

$$\int W_\nu \frac{f_\nu}{h\nu}\, d\nu \tag{11}$$

where W_ν is the throughput of the filter being used. Many filters with calibrated transmissions are in use (Bessell 2005).

Since h is a defined constant in the new SI, let us define

$$\phi_\nu \equiv \frac{f_\nu}{h} \tag{12}$$

which has units of $\mathrm{counts\ m^{-2}\ s^{-1}}$. If we then change the frequency scale to logarithmic, the photon flux (11) becomes

$$\int W_\nu\, \phi_\nu\, d(\ln\nu). \tag{13}$$

Applying the condition from Oke & Gunn (1983) that $h\phi_\nu = 1\ \mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}} = 10^{23}\ \mathrm{Jy}$ corresponds to $\mathrm{AB} = -48.60$ gives

$$h\phi_\nu = 10^{-22.44-\mathrm{AB}/2.5}\ \mathrm{J\ m^{-2}} \tag{14}$$

relating ϕ_ν to AB. Dividing by h we have

$$\phi_\nu = 10^{-\mathrm{AB}/2.5} \times 5 \times 10^{10}\ \mathrm{m^{-2}\ s^{-1}} \tag{15}$$

and the precise coefficient is given in Table 2. Colors are simply the ratio of photon fluxes $10^{\Delta\mathrm{AB}/2.5}$.

To get the photon flux over a bandpass, we need to take the integral (13). If the spectral density is fairly flat and the throughput is ≈ 1, the result will be $\sim\phi_\nu \ln(\nu_2/\nu_1)$ where ν_2 and ν_1 are the edges of the band. If ratio of those is $\simeq 1.2$ (which is typical of optical bands) the photon flux over a band comes to roughly $10^{-\mathrm{AB}/2.5} \times 1 \times 10^{10}\ \mathrm{m^{-2}\ s^{-1}}$. In other words, zero AB magnitude in a typical optical band gives about 10^{10} photons $\mathrm{m^{-2}\ s^{-1}}$.

The photon flux ϕ_ν is clearly much more intuitive that the magnitude scale. A logarithmic frequency scale also brings some advantages. First, it does not matter whether the scale is frequency or wavelength or energy: $\phi_\nu = \phi_\lambda = \phi_E$. Second, redshift is simply a shift along the spectral scale and does not change the shape of ϕ_ν.

We remark in passing that the SI has three special units relating to brightness: the candela, the lumen, and the lux.

These units, however, are designed to quantify the physiological effect of light. The SI specifies that a watt of monochromatic green light at 540 THz is worth 683 lumens, thus defining a lumen. (A lux is a lumen per square meter, while a candela is a lumen per steradian.) Light at other visible frequencies has fewer lumens per watt, the exact number being given by a model throughput function for human vision, known as the luminous efficacy. Household LED lights deliver about 100 lumens of white light per watt of electrical power.

3. Examples

The preceding sections drew attention to the characteristic style of the new SI, wherein constants are defined explicitly and units are defined implicitly thereby, and suggested that the classical astronomical units for length, mass, and brightness could be replaced by SI units while leaving meters, kilograms, and watts per hertz implicit. We will now discuss examples from different topics in astrophysics, illustrating how expressing distance in light-seconds, mass in gravity-seconds, and brightness as ϕ_ν or ϕ_λ can be useful in describing diverse astrophysical processes.

The basic strategy is to work in the SI, while using the freedom given by the new SI to use any equation of physics to change variables or introduce new quantities. Light-seconds, gravity-seconds, and logarithmic spectral scales are particular instances of this strategy—a time variable may be used to represent a length divided by c, and so on. There is no requirement to use light-seconds for all lengths, or gravity-seconds for all masses—we can use whatever formulation is most convenient, while keeping to the principle that all dimensional quantities are unambiguously in SI units. Classical astronomical units will be converted to SI units at the input stage, and sometimes classical units will be re-introduced at the output stage. What we want to avoid is mixing unit systems inside a derivation or a computation. Fundamental constants relating different SI units (see Table 1) will be used whenever needed or useful. The exception is G, which we will use explicitly only where it is unavoidable, because of its large uncertainty.

For the purposes of this paper, it is not necessary to consider the most general or most sophisticated form of each process. The approximate conversions Equations (3), (6), and (15) will be useful, as will some common spherical-cow idealizations.

3.1. The Sun and the CMB

Let us compare the nearest and furthest sources of light in astronomy. For this purpose, we approximate the solar surface by a blackbody at $T_\odot = 5800$ K, and the cosmic microwave background by an ideal Planckian spectrum with $T_{\rm cmb} = 2.725$ K. The former approximation is comparatively drastic, but that does not matter for this example.

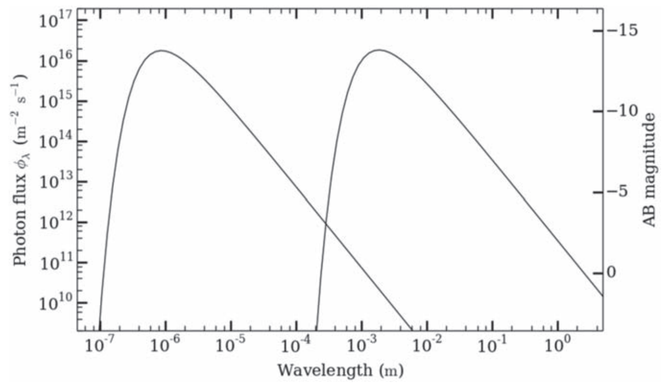

Figure 1. Approximate photon spectrum of the Sun, if seen from about 450 au, and the all-sky spectrum from the CMB, shown as photons m^{-2} s^{-1} and AB magnitudes.

The photon flux per logarithmic spectral interval has a simple interpretation for thermal sources. A blackbody has

$$\phi_\lambda = \frac{c}{\lambda^3} \times \frac{2\pi}{e^{hc/\lambda kT} - 1} \tag{16}$$

at its surface. For a spherical blackbody of radius r_1 observed from distance r_2, ϕ_λ is reduced by a factor $(r_1/r_2)^2$. The ϕ_λ spectrum peaks at $5.100\,\mathrm{mm} \times T^{-1}$ K. At this peak value, the second factor in Equation (16) is of order unity (actually 0.40). This makes it useful to imagine the c/λ^3 as a packed column of ball-like photons whose diameter is the peak wavelength, moving at the speed of light.

From the photon spectrum (16) it is clear that the photon flux will be $\propto T^3$. Thus, at the solar surface, the Sun has a photon flux $(T_\odot/T_{\rm cmb})^3 \simeq 10^{10}$ times that of the CMB. From a distance of $(T_\odot/T_{\rm cmb})^{3/2} R_\odot \simeq 450$ au the photon fluxes become equal. Figure 1 illustrates. The corresponding peaks are at 1.871 mm (or 160.2 GHz) for the CMB and 880 nm for the Sun. The latter may seem wrong, because the solar spectrum is well-known to peak in the middle of the visible range—but that is only because the quantity usually plotted for the solar spectrum is the energy against wavelength, or $hc\lambda^{-2}\phi_\lambda$. The AB magnitude of the Sun is indeed brightest in filters near 900 nm (see e.g., Willmer 2018).

3.2. The Oort and Hubble Parameters

Classical astronomical units often appear mixed with SI units. Thus, we may have an expression of the type

$$\frac{\partial v_i}{\partial x_j} \tag{17}$$

where v_i is a velocity field (mean velocity at location x_i), with v_i measured in km s^{-1} and x_i in pc. Dimensionally, such expressions are inverse times, and it is useful to remember

$$1\ \mathrm{m\,s^{-1}\,pc^{-1}} = 1\ \mathrm{km\,s^{-1}\,kpc^{-1}} \approx (1\ \mathrm{Gyr})^{-1} \tag{18}$$

while more digits are given in Table 2. The divergence (trace of Equation (17)) of the velocity field of galaxies is the Hubble constant. The (generalized) Oort constants are analogous quantities for the stellar velocities in the solar neighborhood. Of course, none of these are really constants, just values at our location or time, so "parameter" may also be used. Conversions of the type $100\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \simeq (9.78\,\mathrm{Gyr})^{-1}$ are often used in both contexts.

The classical Oort parameters A and B, defined as

$$2A = \frac{v_\phi}{r} - \frac{\partial v_\phi}{\partial r}$$
$$-2B = \frac{v_\phi}{r} + \frac{\partial v_\phi}{\partial r} \tag{19}$$

in cylindrical coordinates, describe the differential rotation of the Galaxy. Solid-body rotation would give $A = 0$, whereas a flat rotation curve gives $A + B = 0$. Recent measurements (e.g., Li et al. 2019) give $A - B \simeq 28\,\mathrm{m\,s^{-1}\,pc^{-1}}$ and all the other components are an order of magnitude smaller. Expressing this in SI units as $A - B \simeq 9 \times 10^{-16}\,\mathrm{s^{-1}}$ does not seem much of an improvement. If we note, however, that $A - B$ is an angular velocity, then $A - B = v_\phi/r \simeq 2\pi/(220\,\mathrm{Myr})$ is simple.

In the case of the Hubble parameter, it is easy to work with the Hubble time

$$H_0^{-1} = (4.4 \pm 0.2) \times 10^{17}\,\mathrm{s} = (14 \pm 0.5)\,\mathrm{Gyr} \tag{20}$$

which, as well as being the reciprocal expansion rate, is also approximately the time since the big bang. This already applied in the old SI.[3] The new SI, however, encourages further useful formulations. Recall that $3/(8\pi G) \times H_0^2$ is the critical density of the universe. The result in $\mathrm{kg\,m^{-3}}$ is not very intuitive, but with the help of the SI constants we can also write

$$\frac{3}{8\pi}\frac{H_0^2}{G} \times \frac{c^2}{e} \simeq 5\,\mathrm{GeV\,m^{-3}} \tag{21}$$

or roughly 5 atomic mass units per cubic meter. (Of course, only a fraction $\Omega_b \simeq 0.05$ of this will be in baryonic matter.) If the Hubble time is expressed in seconds, no astro-specific unit-conversion factors are needed, just constants in SI units. Furthermore, the density expression $3/(8\pi) \times H_0^2$ in gravity-seconds per cubic light-seconds can also be of use. Dividing by $\mathscr{M}_\odot$ gives the density in solar masses per cubic light second. We can then calculate a notional distance

$$\left(\frac{3}{8\pi}\frac{\Omega_b H_0^2}{\mathscr{M}_\odot}\right)^{-1/3} \sim c \times 5 \times 10^{10}\,\mathrm{s} \tag{22}$$

meaning that if all the baryonic matter in the universe were in Sun-like stars, these would be on average roughly 500 pc apart.

[3] For example, Sandage (1962) in his prediction of redshift drift (which has yet to be observed, but see Lazkoz et al. 2018) gives $H_0^{-1} = 13$ Gyr as an illustrative value, and does not bother with $\mathrm{km\,s^{-1}\,Mpc^{-1}}$ at all.

3.3. The Milky Way and Andromeda

We saw above that classical astronomical units can leave us having to disentangle a mixture of pc, $\mathrm{km\,s^{-1}}$, and $M_\odot$, with G in SI or cgs units thrown in for good measure. Another example where this happens is the Local Group timing argument, in which the Galaxy and M31 appear as an unusual but very interesting binary system.

The basic observational facts are that the two galaxies are some 800 kpc apart and are approaching each other at roughly $120\,\mathrm{km\,s^{-1}}$. Using this information, one can estimate the combined mass of these two galaxies by computing how much mass would be needed to have countered the expansion of the universe in the Local Group in a Hubble time. The idea goes back to Kahn & Woltjer (1959) and has been developed further by many researchers (e.g., Banik & Zhao 2016). The inferred mass provides a simple and robust estimate of the dark matter in the Local Group.

Converting the distance to light seconds, we have

$$\bar{R}_{\mathrm{M31}} \simeq 8 \times 10^{13}\,\mathrm{s} \qquad d\bar{R}_{\mathrm{M31}}/dt \simeq -4 \times 10^{-4} \tag{23}$$

using the notation from Equation (4) to denote distance in light-seconds. Dividing distance by speed gives a formal time of 2×10^{17} s, which is of the same order as the Hubble time. This suggests modeling the system as a radial two-body orbit that started to move out at the big bang, and has since turned around and is now approaching a collision. To do so, let us introduce some notation for binaries in general.

Consider two masses, $\mathscr{M}_1$, $\mathscr{M}_2$ in gravity seconds, in a two-body orbit, with

$$\mathscr{M} \equiv \mathscr{M}_1 + \mathscr{M}_2 \qquad \eta \equiv \frac{\mathscr{M}_1\mathscr{M}_2}{\mathscr{M}^2} \tag{24}$$

being the total mass and the symmetric mass ratio. Let $\bar{a}$ be the orbital semimajor axis in light-seconds, and let us use the expression

$$\bar{a} = \frac{\mathscr{M}}{\beta^2} \tag{25}$$

to define a dimensionless constant β. For a circular orbit β is clearly the orbital speed in light units. For general bound orbits, β can be worked out using the virial theorem as the orbit-averaged rms speed

$$\beta = \frac{\sqrt{\langle v^2 \rangle}}{c} \tag{26}$$

in light units. The orbital period can be expressed as

$$P = 2\pi\frac{\mathscr{M}}{\beta^3} \tag{27}$$

thus relating mass in gravity-seconds to an observable time. Expressing $\bar{a}$ in Equation (25) as a time is not simply formal either—in pulsar binaries (see e.g., Lorimer & Kramer 2012) the light-crossing time is the observable size of the orbit (because no resolved image of the system is observed), and is known as the Roemer time delay after the 17th-century measurement of the light-travel time across the solar system. The Earth's orbit provides a nice illustration of Equations (25) and (27). Since $\mathscr{M}_\odot = 5 \times 10^{-6}$ s and $\bar{a} = 500$ s we have $\beta = 10^{-4}$ (or 30 km s^{-1}). For the orbital period we recover the well-known mnemonic that a year is $\pi \times 10^7$ s.

Turning now to the Galaxy and Andromeda, let us consider these as being on a radial two-body orbit. Such a system has a well-known solution, which in our notation is

$$\begin{aligned} t &= \mathscr{M}\beta^{-3}(\psi - \sin\psi) \\ \bar{R}_{\rm M31} &= \mathscr{M}\beta^{-2}(1 - \cos\psi) \\ d\bar{R}_{\rm M31}/dt &= \beta\sin\psi/(1 - \cos\psi) \end{aligned} \tag{28}$$

with a formal independent variable ψ serving to give the time dependence implicitly. To find the current value of ψ, we consider the dimensionless product

$$\frac{t}{\bar{R}_{\rm M31}} \times \frac{d\bar{R}_{\rm M31}}{dt} = \frac{\sin\psi\,(\psi - \sin\psi)}{(1 - \cos\psi)^2} \tag{29}$$

and in it we put $t = 4 \times 10^{17}$ s (about a Hubble time) and the distance and velocity values from Equation (23) The value of the expression (29) comes to -2. A numerical solution for ψ yields $\psi \simeq 4.2$. With ψ determined, it is easy to solve for β and $\mathscr{M}$. The result is $\mathscr{M} = 2.5 \times 10^7$ s. Recalling the conversion (6) gives a mass of $5 \times 10^{12} M_\odot$, which is more than an order of magnitude above the stellar mass in the Local Group.

In conventional astronomical units, the above steps would be basically the same. But because we converted the input to SI at the start, and converted the inferred mass from gravity-seconds to $M_\odot$ at the very end, the calculation was much easier. A computer is useful when solving for ψ, but the rest of the arithmetic is trivial.

3.4. Time Dilation in Orbiting Clocks

When the SI was first instituted in 1960, the second was defined from astronomy, as a fraction of a mean solar day. The Caesium-clock standard changed that a few years later. Still, astronomy has not entirely ceded time measurement to atomic physics, because some applications require time dilation to be taken into account. The TCB (temps coordonnée barycentrique) is defined as the time kept by clocks moving with the solar system barycenter but outside all gravitational fields (Brumberg & Groten 2001).

It is well known that global navigation satellites have to correct for relativistic time dilation. A detailed treatment can be found in Ashby (2003) but a simple estimate makes a nice illustration of gravity-seconds, as follows. Let us consider a two-body orbital system as in Section 3.3 above, except that $\eta \to 0$ since the satellite mass is negligible, and $\mathscr{M}$ is simply the Earth's mass. Let us write $\bar{R}$ for the radius of the Earth in light seconds. A clock at the surface of the Earth runs slower than TCB by a fraction $\mathscr{M}/\bar{R}$, neglecting the spin of the Earth. From weak-field relativity, a clock in a circular orbit of radius $\bar{a}$ (in light-seconds) runs slower than TCB by $\frac{3}{2}\mathscr{M}/\bar{a}$. The difference of these two time dilations is observable. Multiplying by the orbital period (27) and rearranging, we have a delay of

$$2\pi\left(\frac{3}{2} - \frac{\bar{a}}{\bar{R}}\right)\frac{\mathscr{M}}{\beta} \tag{30}$$

per orbit. We can estimate its value from easily remembered quantities, as follows: (i) the circumference of the Earth is 40,000 km, which gives $\bar{R}$, (ii) using $g/c = \mathscr{M}/\bar{R}^2$ and putting $g \simeq 9.8$ m s^{-2} gives $\mathscr{M} = 15$ ps (Table 2 gives the precise value.) (iii) equating $2\pi\mathscr{M}/\beta^3$ to the orbital period of 12 hr for GPS satellites gives β, (iv) using $\beta^2 = \mathscr{M}/\bar{a}$ gives $\bar{a}$. Doing the arithmetic, we find a delay of 38.5 microsec per day.

GPS clocks are also in orbit around the Sun. Hence, though they tick faster than terrestrial clocks, satellite clocks run slower than a notional TCB clock. To find the time delay over one orbit (that is, one year), we use the first term in Equation (30) and substitute the solar mass and the Earth's orbital speed. The latter, we have already seen, is yet another conveniently round number $\beta = 10^{-4}$. The result is 0.5 s per year. The solar mass parameter (see Equation (2)) is $GM_\odot = 1.32712442099(10) \times 10^{20}$ m^3 s^{-2} in TCB but measurably different with respect to terrestrial time (Luzum et al. 2011).

For eccentric orbits, the time dilation is more complicated in detail, but of the same order. A good example is the star S2/S0-2, which orbits the black hole at the center of the Milky Way in a highly eccentric orbit. The spectral features of the star amount to a natural clock, and its orbital time dilation has recently been measured (Abuter et al. 2018; Do et al. 2019). Further observable times of the form $\mathscr{M}\beta^n$ (with positive n) can emerge from higher-order relativistic effects (Angélil & Saha 2014) but have not been measured yet.

3.5. Shapiro and Refsdal Delays

As well as the various $\mathscr{M}\beta^{-n}$ in seconds, there is a measurable time that is simply $\mathscr{M}$ times a numerical factor. As one might guess from the absence of β, it involves light.

A light ray, that on its way between source and observer has flown past a mass $\mathscr{M}$, experiences a time delay

$$-2\mathscr{M}\ln(1 - \cos\theta) \tag{31}$$

where θ is the angle on the observer's sky between the mass and the incoming ray. The logarithm in (31) will be negative, assuming θ is not too large, and hence the whole expression will be positive. This delay is well known in ranging experiments in the solar system, and in pulsar timing, and is known as the Shapiro delay after the prediction by Shapiro (1964).

Another manifestation of the same phenomenon, predicted by Refsdal (1964) by very different arguments, appears in gravitational lensing. In the regime of lensing, θ is small. Taking the small-θ limit of (31) gives $-2\mathscr{M}(2\ln\theta - \ln 2)$. But at small θ, the deflection $\Delta\theta$ of the light ray is important, as it contributes a further time delay. The time delay depends on θ and $\Delta\theta$ as

$$t(\theta) = \bar{D}\,\Delta\theta^2 - 4\mathscr{M}\ln\theta \qquad (32)$$

where $\bar{D}$ is an effective distance (in light-seconds) depending on the distances to the mass and the light source. Light then follows Fermat's principle and chooses θ and $\Delta\theta$ so as to make the total light travel time extremal (e.g., Blandford & Narayan 1986). There can be more than one extremal light path, giving multiple images with different light travel times. A good example is Supernova Refsdal observed to appear at displaced locations with time delays (Kelly et al. 2016). In general, a gravitational lens will not be a single mass but an extended mass distribution. Accordingly, the last term in Equation (32) has to be replaced by an integral over the mass density. The observable time delay between different lensed images then depends on details of the how the mass is distributed (see e.g., Mohammed et al. 2015).

If a mass is at a cosmological distance (say at redshift z), the Shapiro or Refsdal delay as measured by an observer will be time-dilated by $(1 + z)$, just like any other time interval at that redshift. That is, mass in gravity-seconds gets redshifted.

3.6. Gravitational-wave Inspiral

In general relativity it is common to put $G = c = 1$, leaving units and dimensions implicit (geometrized units—see Appendix F in Wald 1984). Light seconds and gravity seconds are similar, but with explicit variable changes to keep track of units and dimensions, and using these we can express gravitational-wave inspiral very concisely, while also keeping the comparison with observations simple.

Let us again consider a binary with parameters $\mathscr{M}$, η, and β. For this system let us write

$$\omega = \beta^3/\mathscr{M} \qquad (33)$$

for the angular orbital frequency, and

$$\mathscr{E} = -\tfrac{1}{2}\eta\mathscr{M}\beta^2 \qquad (34)$$

for the orbital energy in gravity-seconds.

Such a binary will produce gravitational waves of angular frequency 2ω, and the strain components at distance $\bar{D}$ will be

$$h_{\rm GW} \sim \frac{\mathscr{E}}{\bar{D}} \qquad (35)$$

with numerical coefficients of order unity, depending on orientation. It may be convenient to think of the strain as a kind of gravitational potential whose source is not the mass but the orbital energy. If we further associate the wave with an energy density $\propto\omega^2 h_{\rm GW}^2$ (see Mathur et al. 2017) and moving with a speed of light, it follows that energy will be emitted at a rate $\propto\omega^2\mathscr{E}^2$. Writing the proportionality constant as κ, we have

$$\frac{d\mathscr{E}}{dt} = -\kappa\,\omega^2\mathscr{E}^2. \qquad (36)$$

This paragraph is not a derivation, just a plausibility argument. Nevertheless, the expression (36) with $\kappa = 128/5$ turns out to be the correct leading-order relativistic result for circular binaries (Peters & Mathews 1963). Eccentric orbits and higher order modify the numerical factor, but do not change the units needed. Notice that the power output (36) is in gravity-seconds per second. To convert to watts, we need to multiply by c^5/G (as noted at Equation (10)).

Low-mass gravitationally radiating binaries can be just as luminous as supermassive binaries, but the latter take longer. To see this, we eliminate $\mathscr{E}$ and ω from the formula (36) and rearrange to get

$$\frac{d\beta}{dt} = \tfrac{1}{4}\kappa\eta \times \frac{\beta^9}{\mathscr{M}} \qquad (37)$$

giving the increasing speed of the inspiralling system. Integrating to get the time left before $\beta = 1$ (when the system would merge), we get

$$T_{\rm insp} = \frac{1}{2\kappa\eta}\,\frac{\mathscr{M}}{\beta^8} \qquad (38)$$

for the inspiral time. Here we have yet another timescale of the form $\mathscr{M}/\beta^n$.

The parameters $\mathscr{M}$, η, and β are, in general, not directly observable for inspiralling binaries. The observables are the two components (polarizations) of the strain, the wave frequency 2ω, and its derivative, known as the chirp. In terms of there we can write

$$\frac{1}{\omega}\frac{d\omega}{dt} = \tfrac{3}{8}T_{\rm insp}^{-1} = \tfrac{3}{2}\kappa\omega^2\mathscr{E}. \qquad (39)$$

Since the left expression is observable, the other two expressions become measurable. That $T_{\rm insp}^{-1}$ can be inferred from the frequency and chirp is not surprising. That $\mathscr{E}$ is measurable is remarkable, and has important consequences. Writing $\mathscr{E}$ as $\frac{1}{2}\eta\mathscr{M}^{5/3}\omega^{2/3}$, we see that the combination $\eta^{3/5}\mathscr{M}$ is also measurable. It is known as the chirp mass. Still more interesting is that gravitational-wave binaries can serve as "standard sirens" enabling distance measurements, as first noted by Schutz (1986). To see how, recall from Equation (35) that $\mathscr{E}$ relates strain and distance. The coefficients in the equation can be determined from the measurable polarization of the gravitational wave. This makes $\bar{D}$ measurable too, and from it H_0 as well if the redshift is measured.

Regarding redshifts, it is important to note that a redshift z dilates t in the observer frame by $1 + z$. As a result, observed frequencies are slowed down, and the inferred chirp mass is dilated, by the same factor. The speed β is not affected. To make the strain in Equation (39) come out right, we need to dilate $\bar{D}$ by $1 + z$ as well (i.e., use the luminosity distance). Again, as was the case in lensing, we see mass in gravity-seconds at cosmological distances getting redshifted.

GW170817 (Abbott et al. 2017) was a particularly interesting gravitational-wave source, providing a wealth of observations in addition to the inspiral, and taken together these enabled a reconstruction of the system as two neutron stars at $\simeq$40 Mpc, which corresponds to

$$\mathscr{M} \simeq 1.5 \times 10^{-5}\,\mathrm{s} \qquad \eta \simeq \tfrac{1}{4} \qquad \bar{D} \simeq 4 \times 10^{15}\,\mathrm{s}.$$

The level of strain at merger would be $\mathscr{M}/\bar{D} \sim 10^{-21}$. If we assume $\beta = 0.12$ at the start of the detected event, Equation (33) gives an initial orbital frequency of $\simeq$20 Hz (wave frequency of $\simeq$40 Hz), while Equation (38) implies $T_{\rm insp} \simeq 30$ s, reproducing the observed values.

Galactic binary pulsars are younger systems like GW170817. The gravitational-wave inspiral of these systems has been known since the first measurement by Taylor et al. (1979), but detecting the gravitational-wave strain from these low-frequency sources is not feasible yet. Interestingly, the $h_{\rm GW}$ from Galactic binary pulsars is comparable to that from GW170817. To see this, we can simply scale

$$\beta \rightarrow 10^{-2}\beta, \qquad \bar{D} \rightarrow 10^{-4}\bar{D}$$

which leaves $h_{\rm GW}$ the same. The distance changes to 4 kpc, a 20 Hz orbital frequency ($\propto\beta^3$) changes to a half-day orbital period, and a 30 s inspiral time ($\propto\beta^{-8}$) changes to 10^{10} yr. These values are typical of Galactic binary pulsars (see Lorimer 2008).

3.7. Eddington Luminosity and the M87 Black Hole

In the preceding examples, matter only contributed a gravitational field, and hence mass always appeared multiplied by the gravitational constant, which we were able to absorb inside $\mathscr{M}$, thus eliminating the uncertainty in G. If matter contributes in other ways too (such as producing gas pressure), mass will appear as both M and GM. This will make the uncertainty in G unavoidable—and also offer a way to measure G, as in Christensen-Dalsgaard et al. (2005).

An exceptional but important situation, in that mass appears only as GM even though non-gravitational processes are involved, is Eddington luminosity. In this one has a spherical mass M of ionized gas, and some energy source which gives it a luminosity L, and the radiation pressure from the latter balances the self-gravity. At any radius r inside the sphere, we have

$$\frac{GMm_p}{r^2} = \frac{L/c}{4\pi r^2} \times \frac{2}{3\pi}\left(\frac{\alpha h}{m_e c}\right)^2 \tag{40}$$

where m_p and m_e are the masses of the proton and electron, and α is the fine-structure constant. On the left of this equation we have the weight of an ion, and on the right we have the outward momentum flux times the Thomson cross-section. Radiation pressure acts on the electrons, but the force is transmitted electrostatically to the ions. Expressing the particle masses as equivalent frequencies $\nu_e = m_e c^2/h$, $\nu_p = m_p c^2/h$ and rearranging, we get the Eddington luminosity

$$L = 6\pi^2\, h\nu_p\, (\nu_e/\alpha)^2 \mathscr{M} \tag{41}$$

written with mass in gravity-seconds.

Although originally developed for massive stars, the Eddington luminosity is nowadays also often applied to accreting black holes to estimate the maximum possible luminosity. Accretion by black holes is not a spherical process, so applying the formula to black holes gives a rough estimate at best, but is nonetheless interesting. Let us accordingly approximate an accreting black hole as a blackbody sphere at temperature T. It will radiate at $2\pi^5 c/15 \times (kT)^4/(hc)^3$ per unit area. For the radius of the sphere, we take the radius of the innermost stable orbit, which is $6\mathscr{M}$ (or $6\mathscr{M}c$ in length units) for a non-spinning black hole. Equating the total luminosity to the Eddington luminosity we can define an effective temperature

$$T = \frac{h}{2\pi k}\left(\frac{5\nu_e^2\nu_p}{\alpha^2\mathscr{M}}\right)^{1/4} \tag{42}$$

for the accreting system. The formula for this "Eddington temperature" seems strangely reminiscent of the much much colder Hawking temperature $h/(4k\mathscr{M})$.

Akiyama et al. (2019) present an interferometric image of the silhouette of the supermassive black hole in M87 at a resolution of 20 μas or 10^{-10} rad. The resolution is as expected for mm-wavelengths with a baseline of $\sim$$10^4$ km. The distance to M87 being $\simeq$20 Mpc, the resolved size comes to 2×10^{-3} pc, which is like 2×10^5 s or two light days. The inferred mass is about an order of magnitude smaller than this scale: $6 \times 10^9 M_\odot$ or 3×10^4 gravity-seconds. Plugging the mass in the formula (42) gives $T \simeq 8 \times 10^4$ K. Taken as an upper limit, this value is very reasonable, since a continuum peak around 100 nm, corresponding to an effective temperature of $T \approx 3 \times 10^4$ K, is typical of quasars (Francis et al. 1991; Vanden Berk et al. 2001). The M87 black hole system itself would have a much lower effective temperature, because it is accreting only weakly now. The measured brightness temperature at mm wavelengths is, however, much higher. This tells us that the mm radiation cannot be thermal and must be predominantly reprocessed.

4. Discussion

The recent reforms of the SI have made the formerly unexciting subject of units scientifically novel. Astronomers, however, have always been unwilling to adopt SI units.

The persistence of classical and other pre-SI units in astrophysics actually has an interesting scientific reason, namely the difficulty of calibrating astronomical observables against the laboratory standards on which the SI and its predecessors are based. The calibration problems are mostly solved now, but one very important problem remains: the uncertainty in G, which makes the kilogram unusable in some precision applications. Without the kilogram, any proposal to change to SI units becomes a non-starter. At most, one sees arguments for a partial change to SI units (see Dodd 2011). And so it is that every new research student in astronomy, having mastered basic physics with SI units, is confronted with magnitudes, parsecs and solar masses, as well as pre-SI decimal metric units like Å, ergs, and Gauss. Expressions mixing different unit systems are especially painful, and make it difficult to catch errors.

The new SI, by giving physical constants the central role, encourages reformulation of observables by inserting factors of c, h, and so on. With this freedom, the classical units of length, brightness, and mass can be usefully replaced by SI units having different dimensions.

1. The au and pc are easily replaceable by light-seconds, and both conversions are close to round numbers (500 and 10^8 s).
2. AB magnitude is equivalent to the photon flux per logarithmic spectral interval, which is much easier to understand and work with. For typical optical bands, zero magnitude $\simeq 10^{10}$ photons $\mathrm{m}^{-2}\,\mathrm{s}^{-1}$.
3. Measuring astronomical mass in gravity-seconds may seem contrived at first, but it is really a simple variant of the mass parameter used in solar system dynamics, and can help gain new insight into diverse astrophysical processes. Especially nice is how some very different observable timescales in orbital systems are set by the mass in gravity-seconds and the dimensionless speed: the classical Roemer delay is $\sim\mathscr{M}/\beta^2$ and the orbital period is $2\pi\mathscr{M}/\beta^3$, in general relativity the time dilation per orbit is $\sim\mathscr{M}/\beta$, the gravitational-wave inspiral time is $\sim\mathscr{M}/\beta^8$ and the Shapiro and Refsdal delays are $\sim\mathscr{M}$.

The light-second and gravity-second are not to be considered as new (and therefore non-SI) units with dimensions of length and mass. They are simply the second being used to measure a length times c^{-1}, or measure a mass times G/c^3.

An indirect benefit of the classical astronomical units is that working astronomers are quite used to converting between different unit systems. Theoretical calculations of idealized systems may be done in geometrized units or even Planckian units (e.g., Saha & Taylor 2018), and compute-intensive work often favors internal units to improve numerical performance. All of these require unit conversion at input and output stages, but provided unit conversion is a minor overhead, it does not cause problems. Hence, SI replacements for the classical astronomical units can simply be incrementally introduced by early adopters, without requiring any formal policy changes.

All that said, which astronomer does not love their parsecs and magnitudes? Moreover, there are parameters that have a parsec inside their definitions: absolute magnitude and the conventional normalization of the cosmological power spectrum σ_8. Now, there is an interesting social phenomenon that sometimes the word for an archaic unit survives, but changes its meaning to a round number of the new unit. For example, contemporary German usage has rounded up a "Pfund" from a pound to 500 g, while in South Asia a "tola" has been rounded down to 10 g. One can imagine the same for the classical astronomical units.

1. A rounded solar-mass unit as $\mathscr{M}_\odot = 5 \times 10^{-6}$ s has already been used in this paper.
2. Rounded parsecs of exactly 10^8 lt-s (which is just over π lt-yr) would be 3% smaller than parsecs. For σ_8 in cosmology, the power-spectrum would get averaged over a volume about 10% smaller, and the change in that average may be insignificant. A rounded au of 500 lt-s would also be useful.
3. Zero magnitude could be conveniently rounded to 10^{10} photons $\mathrm{m}^{-2}\,\mathrm{s}^{-1}$, with reference to a broad band whose width is 20% of its median.

Rounding the classical astronomical units in this way would be harmless for most applications.

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Appendix
Constants and Conversion Factors

This Appendix summarizes the various constants and conversion factors used in this paper. Table 1 consists of physical constants relevant to astronomy, while Table 2 has conversion factors from the classical astronomical units.

The first four constants in Table 1 have defined values in the new SI. The others are experimentally determined and hence have uncertainties. Neither is a complete set, but simply the subset important in astrophysics. Taking advantage of equivalences in the new SI, electric charge is written in joules per volt, and particle masses in e/c^2 times volts. It is worth mentioning that the vacuum permeability μ_0 is no longer a defined constant; instead, permeability and permittivity are given by

$$c\mu_0 = \frac{1}{c\epsilon_0} = 2\alpha\frac{h}{e^2} \tag{43}$$

the latter constant being the vacuum impedance $\simeq 377\Omega$.

Table 2 expresses the classical astronomical units in terms of SI units. Note that in each case, some simple change of variable is involved. The first four numerical factors are actually exact numbers (that is, derived from defined constants) but have been

Table 1
Physical Constants in the New SI

c	299,792,458 m s^{-1}
h	$6.62607015 \times 10^{-34}$ J s
e	$1.602176634 \times 10^{-19}$ J V^{-1}
k	1.380649×10^{-23} J K^{-1}
G	$6.674\,3(2) \times 10^{-11}$ kg^{-1} m^3 s^{-2}
$1/\alpha$	137.03599908(2)
m_e	$e/c^2 \times 0.5109989500(2)$ MeV
m_p	$e/c^2 \times 0.9382720882(3)$ GeV

Table 2
Classical Astronomical Units in SI Terms

AB = 0	5.4795384×10^{10} m^{-2} s^{-1}
au	$c \times 499.00478$ s
pc	$c \times 1.0292713 \times 10^8$ s
m s^{-1} pc^{-1}	$3.2407793 \times 10^{-17}$ s^{-1}
	(0.9777922 Gyr)$^{-1}$
$M_\odot$	$c^3/G \times 4.9254909 \times 10^{-6}$ s
$M_\oplus$	$c^3/G \times 1.4793661 \times 10^{-11}$ s

rounded to eight digits in the table. The mass values are measured quantities whose current uncertainties are smaller than the eight digits given here. As noted in the main text, all the numerical values are close to some round number and hence easy to remember approximately.

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