

Radiation properties of an oscillating atom in the presence of external fields

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Abstract

In the first part of the present work, the correction to the photon emission rate of an oscillating two-level atom in the presence of an electromagnetic quantum vacuum field has been investigated for two different configurations: (i) an atom is trapped in the vicinity of a perfect conductor and (ii) an atom is trapped between two perfect conductors. In the second part, the correction to the decay rate of an initially excited oscillating two-level atom due to the presence of a perfectly conducting surface is found.

Keywords: photon emission rate, two-level atom, quantum vacuum, decay rate, oscillating atom

(Some figures may appear in colour only in the online journal)

1. Introduction

Quantum vacuum fluctuations are a ubiquitous result of quantizing classical field theories. One of the most important classical field theories with extensive applications in science and technology is Maxwell's electromagnetic field theory. Electromagnetic quantum vacuum fluctuations are a direct result of quantizing the electromagnetic field theory so that its manifestation can be approved from experimental observations such as the Casimir effect [1–10], Lamb shift of atomic transitions [11–14] and spontaneous emission of an initially excited atom [14–20]. In addition to quantum vacuum fluctuations, there may be some external fields. Then, the fluctuating field satisfies predefined boundary conditions at the location of the external fields. For example, in the case of electromagnetic vacuum fluctuations, the presence of metallic or dielectric materials leads to a nonhomogeneous electromagnetic energy density due to boundary conditions causing Casimir forces among the material fields [21, 22]. Also, the spectroscopic measurements of the atoms show that the transition frequencies of an atom in the presence of boundary conditions differ from their free space value [23]. Therefore, the radiation properties of atoms will change when they are placed in front of a metallic or dielectric surface [13, 14]. On other hand, the dynamical interaction between light and moving atoms in the realm of non-relativistic atom optics has been studied extensively in related fields such

as laser cooling and trapping and quantum optomechanics [24, 25]. A neutral atom can interact with the electric component of the electromagnetic field through its electric dipole moment. In addition, a moving electric dipole carries a magnetic dipole moment that can interact with the magnetic component of the electromagnetic field. Therefore, a moving atom can couple to both electric and magnetic components of the electromagnetic field. Consequently, the moving atoms or equivalently dipoles, exhibit phenomena such as Röntgen quantum phase shift [26, 27]. Barton and Calegoracos investigated the spontaneous emission of atoms moving in a classically assigned trajectory [28]. Also, Muller studied the role of the vacuum fluctuations in spontaneous excitation of a uniformly accelerated atom in its ground state [29]. This process corresponds to the famous Unruh effect [30]. In general, the Unruh effect expresses that the vacuum state is different for inertial and accelerated observers. The vacuum state for the inertial observer means no particle state, but the accelerated observer detects a thermal bath of particles at temperature T . In [31], the non-relativistic oscillation of an atom in its ground state has been investigated in free space. In the present article, we have generalized the problem studied in [13, 31] to the case where there are boundary conditions due to the presence of metallic surfaces that modify the Green's tensor of the fluctuating field. In the absence of the boundary conditions, the results are in agreement with those reported in [13, 31].

The main motivation for the present work is: (i) in order to have more control on the photon emission rate of an

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oscillating two-level atom, some boundary conditions such as the presence of conductors have been considered as a generalization of the results reported in [31] and (ii) from the experimental point of view, holding an atom at an exact distance from a surface is impossible due to Heisenberg uncertainty relations; the best we can do is to trap the atom at an average distance from a surface. Therefore, the decay rate of an initially excited two-level atom needs to be corrected due to the oscillation of the atomic center of mass.

The paper is organized as follows. In section 2, the basic formulation is presented and the correction to the photon emission rate of an oscillating two-level atom in the presence of an electromagnetic quantum vacuum field has been investigated in two different cases: (i) an atom is trapped in the vicinity of a perfect conductor and (ii) an atom is trapped between two perfect conductors. In section 3, the spontaneous decay rate of an initially excited atom oscillating in the vicinity of a perfect conductor has been investigated. Finally, we conclude in section 4.

2. The basic formalism

In this section, we introduce very briefly the basic material and notations that will be used in the following subsections. Let us consider a two-level atom described by the ground state $|g\rangle$ and excited state $|e\rangle$ with energies E_g and E_e , respectively. The internal frequency is defined by $(E_e - E_g)/\hbar = \omega_0$ and $\mathbf{r}(t)$ denotes the center of mass motion of the moving atom with corresponding velocity $\mathbf{v}(t) = \dot{\mathbf{r}}(t)$. Then, the interaction Hamiltonian in dipole approximation in CGS units can be written as:

$$H_{int} = -\mathbf{d} \cdot \left[\hat{\mathbf{E}}(\mathbf{r}(t)) + \frac{\mathbf{v}(t)}{c} \times \hat{\mathbf{B}}(\mathbf{r}(t)) \right], \quad (1)$$

where electric and magnetic field operators are in the interaction picture and are evaluated at the position of the atom. Let the atom be initially in its ground state, then the single-photon emission rate can be calculated from the time-dependent perturbation theory as:

$$\Gamma = \frac{1}{T} \frac{1}{\hbar^2} \left| \int_0^T \langle 1_{\mathbf{q},\lambda}, e | H_{int}(t) | 0, g \rangle dt \right|^2, \quad (2)$$

where T is the duration of the interaction. In equation (2), by $|0, g\rangle$ we mean that the atom is in its ground state $|g\rangle$ and the field state is the vacuum state $|0\rangle$. Similarly, $|e, 1_{\mathbf{q},\lambda}\rangle$ means that the atom is in its excited state and the field state is a single-particle state described by a photon with wavenumber \mathbf{q} and polarization λ .

The electromagnetic field operators can be decomposed into positive $\hat{\mathbf{E}}^+(\hat{\mathbf{B}}^+)$ and negative $\hat{\mathbf{E}}^-(\hat{\mathbf{B}}^-)$ frequency parts as:

$$\begin{aligned} \hat{\mathbf{E}}(\mathbf{r}, t) &= \frac{1}{\sqrt{2\pi}} \int_0^\infty d\omega [\hat{\mathbf{E}}^+(\mathbf{r}, \omega) e^{-i\omega t} + \hat{\mathbf{E}}^-(\mathbf{r}, \omega) e^{i\omega t}], \\ \hat{\mathbf{B}}(\mathbf{r}, t) &= \frac{1}{\sqrt{2\pi}} \int_0^\infty d\omega [\hat{\mathbf{B}}^+(\mathbf{r}, \omega) e^{-i\omega t} + \hat{\mathbf{B}}^-(\mathbf{r}, \omega) e^{i\omega t}], \end{aligned} \quad (3)$$

where the negative and positive frequency parts involve the photonic creation and annihilation operators, respectively. The dipolar matrix elements in the interaction picture can be written as $\hat{d}_i(t) = \hat{d}_i(0) e^{-i\omega_0 t}$. For a spherically symmetric atomic state, the matrix elements of the electric dipole satisfy:

$$\overline{\langle e | \hat{d}_i(0) | g \rangle \langle g | \hat{d}_j(0) | e \rangle} = \delta_{ij} |\langle e | \mathbf{d}(0) | g \rangle|^2 / 3. \quad (4)$$

Using equations (1)–(4), we find that the single-photon emission rate has contributions from quadratic terms in electric and magnetic fields given by

$$\Gamma = \Gamma^{EB} + \Gamma^{EE} + \Gamma^{BB}, \quad (5)$$

where

$$\begin{aligned} \Gamma^{EB} &= \frac{1}{T} \frac{|\langle e | \mathbf{d}(0) | g \rangle|^2}{6\pi\hbar^2} \int_0^\infty d\omega \int_0^\infty d\omega' \delta_{il} \\ &\times \int_0^T dt \int_0^T dt' e^{-i\omega_0(t-t')} e^{-i\omega t + i\omega' t'} \\ &\times \left[\left\langle 0 | \hat{E}_i^+(\mathbf{r}(t), \omega) \left(\frac{\mathbf{v}(t')}{c} \times \hat{\mathbf{B}}^-(\mathbf{r}(t'), \omega') \right) | 0 \right\rangle \right. \\ &\left. + \left\langle 0 | \left(\frac{\mathbf{v}(t)}{c} \times \hat{\mathbf{B}}^+(\mathbf{r}(t), \omega) \right) \hat{E}_l^-(\mathbf{r}(t'), \omega') | 0 \right\rangle \right], \end{aligned} \quad (6)$$

$$\begin{aligned} \Gamma^{EE} &= \frac{1}{T} \frac{|\langle e | \mathbf{d}(0) | g \rangle|^2}{6\pi\hbar^2} \int_0^\infty d\omega \int_0^\infty d\omega' \delta_{il} \\ &\times \int_0^T dt \int_0^T dt' e^{-i\omega_0(t-t')} e^{-i\omega t + i\omega' t'} \\ &\times \langle 0 | \hat{E}_i^+(\mathbf{r}(t), \omega) \hat{E}_l^-(\mathbf{r}(t'), \omega') | 0 \rangle, \end{aligned} \quad (7)$$

$$\begin{aligned} \Gamma^{BB} &= \frac{1}{T} \frac{|\langle e | \mathbf{d}(0) | g \rangle|^2}{6\pi\hbar^2} \int_0^\infty d\omega \int_0^\infty d\omega' \delta_{il} \\ &\times \int_0^T dt \int_0^T dt' e^{-i\omega_0(t-t')} e^{-i\omega t + i\omega' t'} \\ &\times \left[\left\langle 0 | \left(\frac{\mathbf{v}(t)}{c} \times \hat{\mathbf{B}}^+(\mathbf{r}(t), \omega) \right)_i \right. \right. \\ &\left. \left. \times \left(\frac{\mathbf{v}(t')}{c} \times \hat{\mathbf{B}}^-(\mathbf{r}(t'), \omega') \right)_l | 0 \right\rangle \right], \end{aligned} \quad (8)$$

and the repeated subscript indices ($i, j, l = x, y, z$) are summed over. Equation (6) can be rewritten as:

$$\begin{aligned} \Gamma^{EB} &= -\frac{1}{T} \frac{|\langle e | \mathbf{d}(0) | g \rangle|^2}{6\pi\hbar^2 c} \int_0^\infty d\omega \int_0^\infty d\omega' \\ &\times \int_0^T dt \int_0^T dt' e^{-i\omega_0(t-t')} e^{-i\omega t + i\omega' t'} \\ &\times [\langle 0 | \hat{A}_i^+(\mathbf{r}(t), \omega) \hat{A}_l^-(\mathbf{r}(t'), \omega') | 0 \rangle \\ &\times \left(\omega' \left(\frac{\mathbf{v}(t)}{c} \cdot \mathbf{q} \right) + \omega \left(\frac{\mathbf{v}(t')}{c} \cdot \mathbf{q}' \right) \right)], \end{aligned} \quad (9)$$

where we made use of the following identities:

$$\begin{aligned}\hat{E}_i^+(\mathbf{r}, \omega) &= (i\omega/c)\hat{A}_i^+(\mathbf{r}, \omega), \\ \hat{\mathbf{B}}^+(\mathbf{r}, \omega) &= \nabla \times \hat{\mathbf{A}}^+(\mathbf{r}, \omega), \\ \hat{E}_i^-(\mathbf{r}, \omega) &= (-i\omega/c)\hat{A}_i^-(\mathbf{r}, \omega), \\ \hat{\mathbf{B}}^-(\mathbf{r}, \omega) &= \nabla \times \hat{\mathbf{A}}^-(\mathbf{r}, \omega).\end{aligned}\quad (10)$$

The fluctuation-dissipation relation connects the correlation functions of the vector potential components to the imaginary part of the electromagnetic Green's tensor as [32]:

$$\begin{aligned}\langle 0|\hat{A}_i^+(\mathbf{r}, \omega)\hat{A}_j^-(\mathbf{r}', \omega')|0\rangle \\ = 2\hbar \operatorname{Im}[G_{ij}](\mathbf{r}, \mathbf{r}', \omega) \delta(\omega - \omega').\end{aligned}\quad (11)$$

The electromagnetic Green's tensor in a medium described by the dielectric function $\varepsilon(\mathbf{r}, \omega)$ satisfies the equation:

$$\begin{aligned}\left(q^2 \varepsilon(\mathbf{r}, \omega) \delta_{mi} - \frac{\partial^2}{\partial x_m \partial x_i} + \delta_{mi} \nabla^2\right) G_{ij}(\mathbf{r}, \mathbf{r}', \omega) \\ = -4\pi \delta_{mj} \delta(\mathbf{r} - \mathbf{r}'),\end{aligned}\quad (12)$$

where $q^2 = \omega^2/c^2$. Therefore, the contribution Γ^{EB} can be rewritten in terms of the Green's tensor as:

$$\begin{aligned}\Gamma^{EB} &= \frac{1}{T} \frac{|\langle e|\mathbf{d}(0)|g\rangle|^2}{3\pi\hbar c} \int_0^\infty d\omega \omega \\ &\times \int_0^T dt \int_0^T dt' e^{-i(\omega+\omega_0)(t-t')} \\ &\times \left[-\frac{\mathbf{v}(t)}{c} \cdot \mathbf{q} - \frac{\mathbf{v}(t')}{c} \cdot \mathbf{q} \right] \operatorname{Im}[G_{ii}(\mathbf{r}(t), \mathbf{r}(t'), \omega)].\end{aligned}\quad (13)$$

Similarly, we have:

$$\begin{aligned}\Gamma^{EE} &= \frac{1}{T} \frac{|\langle e|\mathbf{d}(0)|g\rangle|^2}{6\pi\hbar^2 c^2} \int_0^\infty d\omega \omega \\ &\times \int_0^\infty d\omega' \omega' \int_0^T dt \int_0^T dt' e^{-i(\omega_0)(t-t')} \\ &\times [e^{-i\omega t + i\omega' t'} \langle 0|\hat{A}_i^+(\mathbf{r}(t), \omega)\hat{A}_i^-(\mathbf{r}(t'), \omega')|0\rangle], \\ &= \frac{1}{T} \frac{|\langle e|\mathbf{d}(0)|g\rangle|^2}{3\pi\hbar c^2} \int_0^\infty d\omega \omega^2 \\ &\times \int_0^T dt \int_0^T dt' e^{-i(\omega+\omega_0)(t-t')} \\ &\times \operatorname{Im}[G_{ii}(\mathbf{r}(t), \mathbf{r}(t'), \omega)].\end{aligned}\quad (14)$$

$$\begin{aligned}\Gamma^{BB} &= \frac{1}{T} \frac{|\langle e|\mathbf{d}(0)|g\rangle|^2}{6\pi\hbar^2} \int_0^\infty d\omega \int_0^\infty d\omega' \delta_{il} \\ &\times \int_0^T dt \int_0^T dt' e^{-i\omega_0(t-t')} \\ &\times \left\{ e^{-i\omega t + i\omega' t'} \left(\langle 0 \left| \left[\frac{\mathbf{v}(t)}{c} \times (\mathbf{q} \times \hat{\mathbf{A}}^+(\mathbf{r}(t), \omega)) \right]_i \right. \right. \right. \\ &\times \left. \left. \left[\frac{\mathbf{v}(t')}{c} \times (\mathbf{q}' \times \hat{\mathbf{A}}^-(\mathbf{r}(t'), \omega')) \right]_l \right| 0 \right) \right\} \\ &= \frac{1}{T} \frac{|\langle e|\mathbf{d}(0)|g\rangle|^2}{3\pi\hbar c^2} \int_0^T dt \int_0^T dt' \int_0^\infty d\omega e^{-i(\omega+\omega_0)(t-t')} \\ &\times [v(t)v(t')q^2\delta_{jk} - (\mathbf{q} \times \mathbf{v}(t))_j(\mathbf{q} \times \mathbf{v}(t'))_k] \\ &\times \operatorname{Im}[G_{jk}(\mathbf{r}(t), \mathbf{r}(t'), \omega)].\end{aligned}\quad (15)$$

Recently, the population of one-photon states resulting from the non-relativistic oscillatory motion of an atom initially prepared in its ground state and interacting with the quantum vacuum in free space was investigated in [31]. In the following subsections, we generalize this problem to the case where there are external material fields such as the presence of perfect conductors.

2.1. Oscillating atom in the vicinity of a perfect conductor

In this section, we will find the single-photon emission rate from the non-relativistic oscillatory motion of a two-level atom in front of a perfect conductor. The atom moves along the z -axis, which is perpendicular to the conductor surface. In this case, the dielectric function is defined by:

$$\varepsilon(z, \omega) = \begin{cases} \infty, & z \leq 0 \\ 1, & z > 0 \end{cases} \quad (16)$$

and the electromagnetic Green's tensor can be obtained easily following the method applied in [33] (appendix A). The location of the trapped atom along the z -axis at time t is given by $\mathbf{r}(t) = z(t)\hat{k} = [b + a \cos(\omega_{cm}t)]\hat{k}$ (ω_{cm} is the frequency of oscillation) and implicitly we have assumed that $b > a$, see figure 1. We have:

$$\begin{aligned}|\mathbf{r}(t) - \mathbf{r}(t')| &= |z(t) - z(t')|, \\ z(t) + z(t') &= 2b + a[\cos(\omega_{cm}t) + \cos(\omega_{cm}t')], \\ z(t) - z(t') &= a[\cos(\omega_{cm}t) - \cos(\omega_{cm}t')].\end{aligned}\quad (17)$$

After straightforward calculations, we find from equations (13), (14) and (15) (appendix A):

$$\begin{aligned}\Gamma^{EB} &= \frac{1}{6} \Gamma_0 \frac{v_{\max}^2}{c^2} \left(\frac{\omega_{cm}}{\omega_0} - 1 \right)^3 \left(\frac{\omega_0}{\omega_{cm}} - 1 \right), \\ \Gamma^{EE} &= \frac{1}{4} \Gamma_0 \frac{v_{\max}^2}{c^2} \left(\frac{\omega_{cm}}{\omega_0} - 1 \right)^3 \left(1 - \frac{\omega_0}{\omega_{cm}} \right)^2 \\ &\times \left[\frac{1}{3} + \sin(2b q_0) \left(\frac{3}{4} \frac{1}{b^5 q_0^5} - \frac{3}{2} \frac{1}{b^3 q_0^3} + \frac{1}{2} \frac{1}{b q_0} \right) \right. \\ &\left. + \cos(2b q_0) \left(\frac{-3}{2} \frac{1}{b^4 q_0^4} + \frac{1}{b^2 q_0^2} \right) \right], \\ \Gamma^{BB} &= \frac{1}{4} \Gamma_0 \frac{v_{\max}^2}{c^2} \left(\frac{\omega_{cm}}{\omega_0} - 1 \right)^3 \\ &\times \left[\frac{2}{3} + \cos(2b q_0) \left(\frac{-3}{4} \frac{1}{b^2 q_0^2} + \frac{3}{4} \frac{1}{b^4 q_0^4} \right) \right. \\ &\left. + \sin(2b q_0) \left(\frac{7}{8} \frac{1}{b^3 q_0^3} - \frac{1}{2} \frac{1}{b q_0} - \frac{3}{8} \frac{1}{b^5 q_0^5} \right) \right],\end{aligned}\quad (18)$$

where we have assumed $0 \leq q_0 = (\omega_{cm} - \omega_0)/c$, $v_{\max} = \omega_{cm}a$, and Γ_0 is the free space emission rate

$$\Gamma_0 = \frac{4 |\langle e|\mathbf{d}(0)|g\rangle|^2 \omega_0^3}{3\hbar c^3}. \quad (19)$$

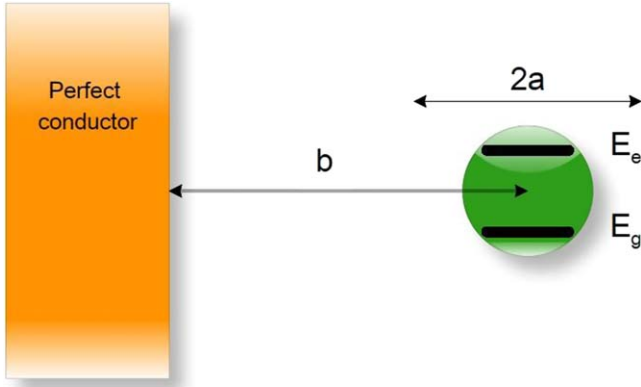


Figure 1. Two-level atom oscillating in non-relativistic regime with frequency ω_{cm} in the vicinity of an ideal conducting plate. Parameter b denotes the distance from the center of oscillation: $z(t) = b + a \cos(\omega_{cm}t)$, ($b > a$).

Therefore,

$$\Gamma = \Gamma_1 \left\{ 1 + 3 \frac{\left(1 - \frac{\omega_0}{\omega_{cm}}\right)^2}{1 + \left(\frac{\omega_0}{\omega_{cm}}\right)^2} \times \left[\sin(2b q_0) \left(\frac{3}{4} \frac{1}{b^5 q_0^5} - \frac{3}{2} \frac{1}{b^3 q_0^3} + \frac{1}{2} \frac{1}{b q_0} \right) + \cos(2b q_0) \left(\frac{-3}{2} \frac{1}{b^4 q_0^4} + \frac{1}{b^2 q_0^2} \right) \right] + \frac{3}{1 + \left(\frac{\omega_0}{\omega_{cm}}\right)^2} \times \left[\sin(2b q_0) \left(\frac{-3}{8} \frac{1}{b^5 q_0^5} + \frac{7}{8} \frac{1}{b^3 q_0^3} + \frac{-1}{2} \frac{1}{b q_0} \right) + \cos(2b q_0) \left(\frac{3}{4} \frac{1}{b^4 q_0^4} + \frac{-3}{4} \frac{1}{b^2 q_0^2} \right) \right] \right\}. \quad (20)$$

When the atom is far enough from the plate, the radiation rate tends to the radiation rate in free space Γ_1 [31]:

$$\Gamma_1 = \frac{1}{12} \Gamma_0 \frac{v_{\max}^2}{c^2} \left(1 + \left(\frac{\omega_0}{\omega_{cm}} \right)^2 \right) \left(\frac{\omega_{cm}}{\omega_0} - 1 \right)^3, \quad (\omega_{cm} \geq \omega_0). \quad (21)$$

Note that the result reported in [31] slightly differs from equation (21) due to a minor typographical error². In figure 2, the scaled emission rate Γ/Γ_1 of a non-relativistic oscillating two-level atom located in front of a perfectly conducting plate has been depicted in terms of the scaled distance b/b_0 ($b_0 = c/2\omega_{cm}$) for $\omega_{cm} = 1.1\omega_0$ and $\omega_{cm} = 3\omega_0$. By increasing the distance between the atom and the plate, the reflected field by the plate becomes weaker. Therefore, the emission rate tends to the free space emission rate, as expected. But in the vicinity of the plate, the plate effect becomes substantial. Here, we have not considered a real plate where plasmonic effects are

² By taking integrals over angular variables in equation (4) in [31], one recovers the same equation, equation (21) in the present work.

dominant at $b \rightarrow 0$ [14]. For $\omega_{cm}/\omega_0 = 3$ and within limit $b \rightarrow 0$, the emission rate tends to $\Gamma = \frac{13}{25}\Gamma_1$. Due to the constructive and destructive interference of the vacuum field in the location of the atom, a damped oscillatory behavior of the photon emission rate around the free space value Γ_1 occurs with the maximum and minimum values appearing at regular distances from the conductor.

2.2. The emission rate of an oscillating atom between perfectly conducting parallel plates

In order to have more control on the quantum dynamics of the oscillating two-level atom, we consider the geometry depicted in figure 3.

$$\varepsilon(z, \omega) = \begin{cases} \infty, & z \leq 0 \\ 1, & d > z > 0 \\ \infty, & z \geq d \end{cases}. \quad (22)$$

Following calculations similar to the previous section, we will find (appendix B):

$$\begin{aligned} \Gamma^{EB} &= \frac{1}{6} \Gamma_0 \frac{v_{\max}^2}{c^2} \left(\frac{\omega_{cm}}{\omega_0} - 1 \right)^3 \left(\frac{\omega_0}{\omega_{cm}} - 1 \right), \\ \Gamma^{EE} &= \frac{\langle e | \mathbf{d}(0) | g \rangle^2}{3\hbar} \frac{v_{\max}^2}{c^2} \frac{(\omega_{cm} - \omega_0)^2}{\omega_{cm}^2} \\ &\quad \times \text{Im} \left[\int_0^{q_0} du (u^2 \cot(u d) + \frac{u^4}{q_0^2} (\cot(u d) \cos(2u b) + \sin(2u b))) \right], \\ \Gamma^{BB} &= \frac{\langle e | \mathbf{d}(0) | g \rangle^2}{6\hbar} \frac{v_{\max}^2}{c^2} \text{Im} \left[\int_0^{q_0} du u (q_0^2 + u^2) \right. \\ &\quad \times \left. \left(\cot(u d) - \frac{u^2}{q_0^2} (\cos(2u b) \cot(u d) + \sin(2u b)) \right) \right], \end{aligned} \quad (23)$$

where $u = \sqrt{q_0^2 - q_{\parallel}^2}$. The emission rate is:

$$\begin{aligned} \Gamma &= \Gamma_1 \left\{ 1 - \frac{2}{1 + \left(\frac{\omega_0}{\omega_{cm}}\right)^2} - \frac{\left(1 - \frac{\omega_0}{\omega_{cm}}\right)^2}{1 + \left(\frac{\omega_0}{\omega_{cm}}\right)^2} \right. \\ &\quad + \frac{3}{2 \left(1 + \left(\frac{\omega_0}{\omega_{cm}}\right)^2\right)} \text{Im} \left[\int_0^{q_0} du u \frac{(q_0^2 + u^2)}{q_0^3} \right. \\ &\quad \times \left. \left(\cot(u d) - \frac{u^2}{q_0^2} (\cos(2b u) \cot(u d) + \sin(2b u)) \right) \right] \\ &\quad + \frac{3 \left(1 - \frac{\omega_0}{\omega_{cm}}\right)^2}{1 + \left(\frac{\omega_0}{\omega_{cm}}\right)^2} \text{Im} \left[\int_0^{q_0} du u \frac{u^2}{q_0^3} \right. \\ &\quad \times \left. \left(\cot(u d) + \frac{u^2}{q_0^2} (\cos(2b u) \cot(u d) + \sin(2b u)) \right) \right] \left. \right\}. \end{aligned} \quad (24)$$

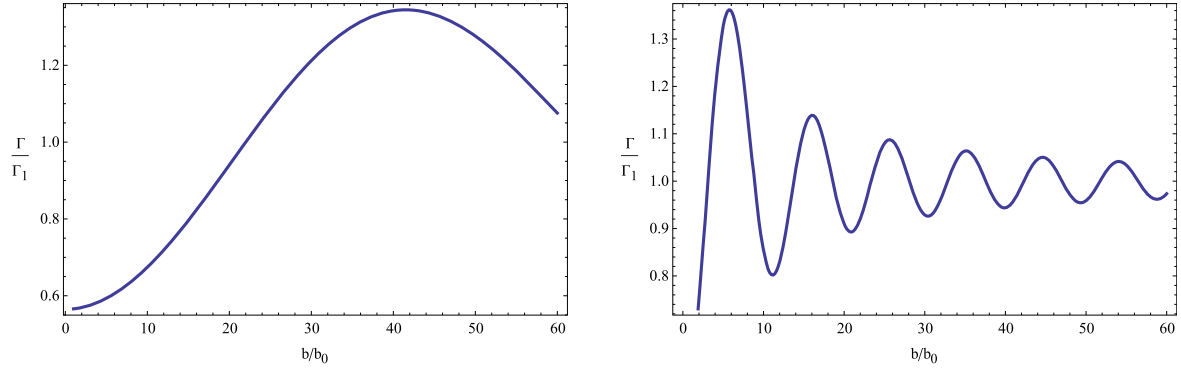


Figure 2. Scaled emission rate Γ/Γ_1 of a non-relativistic oscillating two-level atom located in front of a perfectly conducting plate in terms of the scaled distance b/b_0 , ($b_0 = c/2 \omega_{cm}$) for $\omega_{cm} = 1.1 \omega_0$ (left) and $\omega_{cm} = 3 \omega_0$ (right).

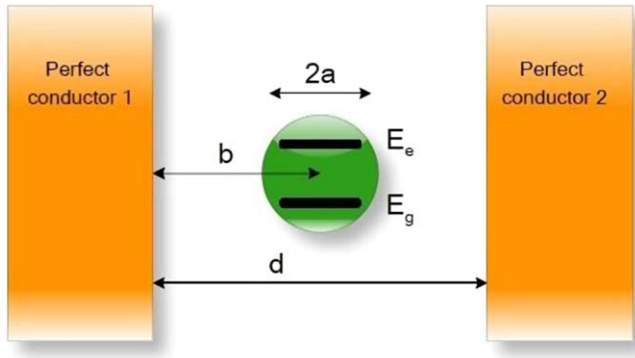


Figure 3. Two-level atom oscillating in non-relativistic regime with frequency ω_{cm} between ideal conducting plates. Parameter b denotes the distance from the center of oscillation: $z(t) = b + a \cos(\omega_{cm}t)$, ($d > b > a$).

As a consistency check, it can be easily seen that for the case where one of the plates goes to infinity ($d \rightarrow \infty$), equation (20) is reproduced, as expected.

In figure 4, the scaled emission rate Γ/Γ_1 of a non-relativistic oscillating two-level atom located between perfect parallel conductors ($b = d/2$) has been depicted in terms of the scaled distance d/d_0 , ($d_0 = c/\omega_{cm}$) for $\omega_{cm} = 1.1 \omega_0$ and $\omega_{cm} = 3 \omega_0$. By increasing the distance between the plates, the emission rate tends to the free space result as expected. Again, due to the constructive and destructive interference of the vacuum field at the location of the atom, a damped oscillatory behavior of the photon emission rate around the free space value Γ_1 occurs with the maximum and minimum values appearing at regular distances from the conductors.

3. The decay rate of an oscillating atom

In this section, we calculate the spontaneous decay rate of an initially excited two-level atom due to coupling to the vacuum field in the vicinity of a perfect conductor. The atom oscillates in z direction perpendicular to the conductor surface with frequency Ω . The position of the center of mass of the atom is considered as:

$$z = b + a \cos(\Omega t), \quad (25)$$

with the velocity

$$v = -v_{\max} \sin(\Omega t), \quad (26)$$

where $v_{\max} = a \Omega$. We consider a fully quantum model and assumed that the atom is initially in its excited state denoted by $|e\rangle$ and the fluctuating field is in its vacuum state $|0\rangle$. In order to obtain a real description of atom-field interaction within the laboratory frame, we add the Röntgen term to the interaction Hamiltonian:

$$H_{int}(t) = -\mathbf{d} \cdot \left(\hat{\mathbf{E}}(\mathbf{r}(t)) + \frac{\mathbf{v}(t)}{c} \times \hat{\mathbf{B}}(\mathbf{r}(t)) \right), \quad (27)$$

where the atomic dipole operator is defined by [34]:

$$\mathbf{d}(0) = \langle g | \mathbf{d}(0) | e \rangle (|g\rangle \langle e| + |e\rangle \langle g|). \quad (28)$$

The electric and magnetic field operators in the presence of an ideal conductor plate can be obtained as [34]:

$$\begin{aligned} \hat{\mathbf{E}}(\mathbf{r}, t) &= \sum_{\mathbf{k}, \lambda} -\sqrt{2\pi\hbar\omega_{\mathbf{k}}} \mathbf{f}(\mathbf{r})_{\mathbf{k}, \lambda} \hat{a}_{\mathbf{k}, \lambda}(t) + H.c \\ \hat{\mathbf{B}}(\mathbf{r}, t) &= \sum_{\mathbf{k}, \lambda} i c \sqrt{\frac{2\pi\hbar}{\omega_{\mathbf{k}}}} (\nabla \times \mathbf{f}(\mathbf{r})_{\mathbf{k}, \lambda}) \hat{a}_{\mathbf{k}, \lambda}(t) + H.c, \end{aligned} \quad (29)$$

where $\hat{a}_{\mathbf{k}, \lambda}$ annihilates a photon with wavenumber \mathbf{k} and polarization λ , and $\hat{a}_{\mathbf{k}, \lambda}^\dagger$ creates such a photon. The function $\mathbf{f}_{\mathbf{k}, \lambda}$ is the spatial profile for the two polarizations $\lambda = TE, TM$ given by:

$$\begin{aligned} \mathbf{f}_{\mathbf{k}, TE} &= \sqrt{\frac{2}{V}} (\hat{\mathbf{k}}_{\parallel} \times \hat{\mathbf{z}} \sin k_z z) e^{ik_{\parallel} r}, \\ \mathbf{f}_{\mathbf{k}, TM} &= \sqrt{\frac{2}{V}} \left(\hat{\mathbf{k}}_{\parallel} \frac{k_z}{k} \sin k_z z + i \hat{\mathbf{z}} \frac{k_{\parallel}}{k} \cos k_z z \right) e^{ik_{\parallel} r}, \end{aligned} \quad (30)$$

respectively. The polarization $TE(TM)$ means that the electric (magnetic) field is parallel to the plate. In equations (30), k_{\parallel} and k_z refer to the parallel and perpendicular components of the wave vector \mathbf{k} with respect to the conductor surface, respectively. The normalization or quantization volume is denoted by V . The decay rate of an initially excited atom can

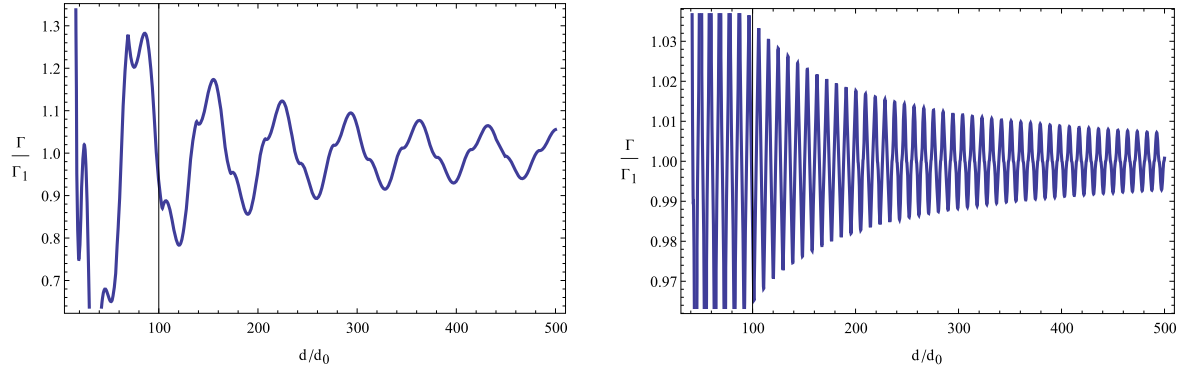


Figure 4. Scaled emission rate Γ/Γ_1 of a non-relativistic oscillating two-level atom located between two perfectly conducting plates ($b = d/2$) in terms of the scaled distance d/d_0 , ($d_0 = c/\omega_{cm}$) for $\omega_{cm} = 1.1 \omega_0$ (left) and $\omega_{cm} = 3 \omega_0$ (right).

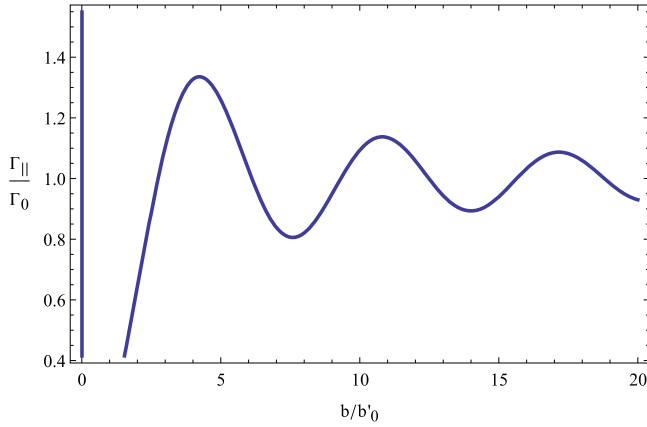


Figure 5. Dimensionless decay rate of an initially excited oscillating atom between conducting plates in terms of the dimensionless variable b/b'_0 , ($b'_0 = c/2 \omega_0$) for $v_{\max} = 343 \text{ m s}^{-1}$ and parallel polarization.

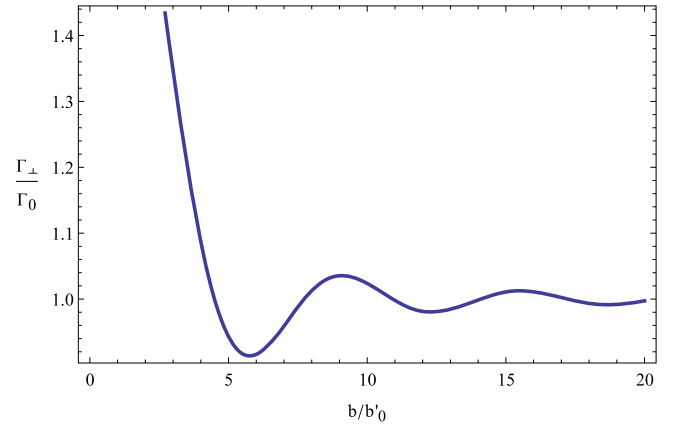


Figure 6. Dimensionless decay rate of an initially excited oscillating atom between conducting plates in terms of the dimensionless variable b/b'_0 , ($b'_0 = c/2 \omega_0$) for $v_{\max} = 343 \text{ m s}^{-1}$ and perpendicular polarization.

be obtained from the time-dependent perturbation theory as:

$$\Gamma_{|e\rangle \rightarrow |g\rangle} = \frac{1}{T} \frac{1}{\hbar^2} \left| \int_0^T \langle 1_{\mathbf{k},\lambda}, g | H_{int}(t) | 0, e \rangle dt \right|^2. \quad (31)$$

Now, by inserting equation (27) into equation (31), and doing straightforward calculations, we find the decay rate of the oscillating atom (appendix C). In the limiting case $a \rightarrow 0$, we recover the results reported in [13]:

$$\begin{aligned} \Gamma_{|e\rangle \rightarrow |g\rangle} = & \frac{4 |d_{ge}|^2 \omega_0^3}{3 \hbar c^3} \left[1 - 3 \left(\frac{\cos(2k_0 b)}{(2k_0 b)^2} - \frac{\sin(2k_0 b)}{(2k_0 b)^3} \right) \right] \\ & + \frac{4 |d_{ge}|^2 \omega_0^3}{3 \hbar c^3} \left[1 - \frac{3}{2} \left(\frac{\sin(2k_0 b)}{2k_0 b} \right. \right. \\ & \left. \left. - \frac{\sin(2k_0 b)}{(2k_0 b)^3} + \frac{\cos(2k_0 b)}{(2k_0 b)^2} \right) \right], \end{aligned} \quad (32)$$

where $k_0 = \omega_0/c$ (ω_0 is the characteristic frequency of the atom). Also, within the limits $a = 0$, $b \rightarrow \infty$, the spontaneous emission rate tends to the vacuum spontaneous decay rate $\Gamma_0 = 4 |e| \langle \mathbf{d}(0) | g \rangle^2 \omega_0^3 / 3 \hbar c^3$, as expected.

In figures 5 and 6, the dimensionless decay rate for parallel and perpendicular polarizations of an initially excited

oscillating atom near a conducting half-space is depicted in terms of the dimensionless variable b/b'_0 , ($b'_0 = c/2 \omega_0$).

4. Conclusions

The corrections to the photon emission rate of an oscillating two-level atom in the presence of electromagnetic quantum vacuum were investigated for two geometries: (i) an atom was trapped in the vicinity of a perfect conductor and (ii) an atom was trapped between two perfect conductors. The presence of conductors caused a damped oscillatory behavior of the photon emission rate around the free space result Γ_1 with maximum and minimum values occurring at regular distances from the conductors. This oscillatory behavior was due to the constructive and destructive interference of the vacuum field by the plates. By increasing the distance between the atom and the plates, the emission rate tended to the free space result. In the vicinity of a perfectly conducting plate and within the limit $b \rightarrow 0$, the emission rate tended to $\Gamma = \frac{13}{25} \Gamma_1$. The spontaneous decay rate of an initially excited atom oscillating in the vicinity of a perfectly conducting plate was investigated. The decay rates had a damped oscillatory

behavior around the free space value Γ_0 with the maximum and minimum values occurring at regular distances from the conductor.

Appendix A

Using the Fourier transform in x - y directions, we define the dimensionally reduced dyadic tensor [33]:

$$\mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = \int \frac{d^2 \mathbf{q}_{\parallel}}{(2\pi)^2} e^{i \mathbf{q}_{\parallel}(\mathbf{r}_{\parallel} - \mathbf{r}'_{\parallel})} \mathbf{g}(z, z', \omega, \mathbf{q}_{\parallel}), \quad (\text{A.1})$$

where $\mathbf{q}_{\parallel} = (q_x, q_y, 0)$ and $\mathbf{r}_{\parallel} = (x, y, 0)$. The diagonal components of \mathbf{g} matrix for conducting plate boundary conditions ($z, z' > 0$) are given by [33]:

$$\begin{aligned} g_{xx}(z(t), z(t'), \omega, \mathbf{q}_{\parallel}) &= 2\pi i \frac{\eta q}{q^2} [e^{i \eta q |z - z'|} - e^{i \eta q (z + z')}], \\ g_{yy}(z(t), z(t'), \omega, \mathbf{q}_{\parallel}) &= 2\pi i \frac{1}{\eta q} [e^{i \eta q |z - z'|} - e^{i \eta q (z + z')}], \\ g_{zz}(z(t), z(t'), \omega, \mathbf{q}_{\parallel}) &= 2\pi i \frac{q_{\parallel}^2}{(\eta q) q^2} [e^{i \eta q |z - z'|} + e^{i \eta q (z + z')}] \\ &\quad - \frac{4\pi}{q^2} \delta(z(t) - z(t')), \end{aligned} \quad (\text{A.2})$$

where $\eta = \sqrt{1 - \frac{q_{\parallel}^2}{q^2}}$. By inserting the coordinates given in equation (17) into (A.2), the components of the reduced Green's tensor up to the second order in $\alpha = qa$, are:

$$\begin{aligned} g_{xx}(z(t), z(t'), \omega, \mathbf{q}_{\parallel}) &\approx 2\pi i \frac{\eta q}{q^2} \left[(1 - e^{2i b \sqrt{q^2 - q_{\parallel}^2}}) - i(\eta \alpha) (n e^{2i b \sqrt{q^2 - q_{\parallel}^2}} - m) \right. \\ &\quad \left. + \frac{1}{2} (\eta \alpha)^2 (n^2 e^{2i b \sqrt{q^2 - q_{\parallel}^2}} - m^2) \right], \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} g_{yy}(z(t), z(t'), \omega, \mathbf{q}_{\parallel}) &\approx 2\pi i \frac{1}{\eta q} \left[(1 - e^{2i b \sqrt{q^2 - q_{\parallel}^2}}) - i(\eta \alpha) (n e^{2i b \sqrt{q^2 - q_{\parallel}^2}} - m) \right. \\ &\quad \left. + \frac{1}{2} (\eta \alpha)^2 (n^2 e^{2i b \sqrt{q^2 - q_{\parallel}^2}} - m^2) \right], \end{aligned} \quad (\text{A.4})$$

and

$$\begin{aligned} g_{zz}(z(t), z(t'), \omega, \mathbf{q}_{\parallel}) &\approx 2\pi i \frac{q_{\parallel}^2}{\eta q^3} \left[(1 + e^{2i b \sqrt{q^2 - q_{\parallel}^2}}) + i(\eta \alpha) (n e^{2i b \sqrt{q^2 - q_{\parallel}^2}} + m) \right. \\ &\quad \left. - \frac{1}{2} (\eta \alpha)^2 (n^2 e^{2i b \sqrt{q^2 - q_{\parallel}^2}} + m^2) \right] \\ &\quad - \frac{4\pi}{q^2} \delta(z(t) - z(t')), \end{aligned} \quad (\text{A.5})$$

where $m = \cos(\omega_{cm} t) - \cos(\omega_{cm} t')$ and $n = \cos(\omega_{cm} t) + \cos(\omega_{cm} t')$.

By inserting equations (A.3), (A.4) and (A.5) into equation (A.1), and using equations (13), (14) and (15), we will find within large-time limit:

$$\begin{aligned} \Gamma^{EB} &= 2 \frac{\|e|\mathbf{d}(0)|g\|^2}{3\hbar} \frac{v_{\max}^2}{c^2} \left(\frac{\omega_0 - \omega_{cm}}{\omega_{cm}} \right) \text{Re} \int_0^{q_0} du u^2, \\ \Gamma^{EE} &= \frac{\|e|\mathbf{d}(0)|g\|^2}{3\hbar} \frac{v_{\max}^2}{c^2} \frac{(\omega_{cm} - \omega_0)^2}{\omega_{cm}^2} \\ &\quad \times \left(\frac{q_0^3}{3} + \text{Re} \int_0^{q_0} du \frac{u^4}{q_0^2} e^{2iub} \right), \\ \Gamma^{BB} &= \frac{\|e|\mathbf{d}(0)|g\|^2}{6\hbar} \frac{v_{\max}^2}{c^2} \\ &\quad \times \text{Re} \left[\int_0^{q_0} du u (q_0^2 + u^2) \left(1 - e^{2iub} \frac{u^2}{q_0^2} \right) \right], \end{aligned} \quad (\text{A.6})$$

where $q^2 = q_z^2 + q_{\parallel}^2$, $q_0 = (\omega_{cm} - \omega_0)/c$, $v_{\max} = \omega_{cm} a$, and $u = \sqrt{q_0^2 - q_{\parallel}^2}$.

Appendix B

The diagonal components of \mathbf{g} in the presence of two parallel conducting plates for $0 < z, z' < d$ are given by [33]:

$$\begin{aligned} g_{xx}(z(t), z(t'), \omega, \mathbf{q}_{\parallel}) &= 2\pi i \frac{\eta q}{q^2} \left\{ e^{-i \eta q |z - z'|} + \frac{1}{(1 - e^{-2i \eta q d})} [-e^{-i \eta q (z + z')} \right. \\ &\quad \left. - e^{-2i \eta q d} e^{i \eta q (z + z')} + 2 e^{-2i \eta q d} \cos(\eta q (z - z'))] \right\} \\ &\quad - \frac{4\pi}{q^2} \delta(z(t) - z(t')), \\ g_{yy}(z(t), z(t'), \omega, \mathbf{q}_{\parallel}) &= 2\pi i \frac{1}{\eta q} \left\{ e^{-i \eta q |z - z'|} + \frac{1}{(1 - e^{-2i \eta q d})} [-e^{-i \eta q (z + z')} \right. \\ &\quad \left. - e^{-2i \eta q d} e^{i \eta q (z + z')} + 2 e^{-2i \eta q d} \cos(\eta q (z - z'))] \right\}, \\ g_{zz}(z(t), z(t'), \omega, \mathbf{q}_{\parallel}) &= 2\pi i \frac{q_{\parallel}^2}{(\eta q) q^2} \left\{ e^{-i \eta q |z - z'|} + \frac{1}{(1 - e^{-2i \eta q d})} [e^{-i \eta q (z + z')} \right. \\ &\quad \left. + e^{-2i \eta q d} e^{i \eta q (z + z')} + 2 e^{-2i \eta q d} \cos(\eta q (z - z'))] \right\} \\ &\quad - \frac{4\pi}{q^2} \delta(z(t) - z(t')). \end{aligned} \quad (\text{B.1})$$

By inserting equation (17) into (B.1), and expanding the result up to the second order in $\alpha = qa$, we find:

$$\begin{aligned}
 g_{xx}(z(t), z(t'), \omega, \mathbf{q}_{\parallel}) & \approx 2\pi i \frac{\eta q}{q^2} \frac{e^{-2i b \sqrt{q^2 - q_{\parallel}^2}}}{e^{2i d \sqrt{q^2 - q_{\parallel}^2}} - 1} [(-1 + e^{2i b \sqrt{q^2 - q_{\parallel}^2}}) \\
 & \times (-e^{2i b \sqrt{q^2 - q_{\parallel}^2}} + e^{2i d \sqrt{q^2 - q_{\parallel}^2}}) + i(\eta\alpha)(-n(e^{4i b \sqrt{q^2 - q_{\parallel}^2}} \\
 & - e^{2i d \sqrt{q^2 - q_{\parallel}^2}}) + m e^{2i b \sqrt{q^2 - q_{\parallel}^2}}(e^{2i d \sqrt{q^2 - q_{\parallel}^2}} - 1) \\
 & + \frac{1}{2}(\eta\alpha)^2(n^2(e^{4i b \sqrt{q^2 - q_{\parallel}^2}} + e^{2i d \sqrt{q^2 - q_{\parallel}^2}}) \\
 & - m^2(e^{2i b \sqrt{q^2 - q_{\parallel}^2}} + e^{2i b \sqrt{q^2 - q_{\parallel}^2}} e^{2i d \sqrt{q^2 - q_{\parallel}^2}}))] \\
 & - \frac{4\pi}{q^2} \delta(z(t) - z(t')),
 \end{aligned} \tag{B.2}$$

$$\begin{aligned}
 g_{yy}(z(t), z(t'), \omega, \mathbf{q}_{\parallel}) & \approx 2\pi i \frac{1}{\eta q} \frac{e^{-2i b \sqrt{q^2 - q_{\parallel}^2}}}{e^{2i d \sqrt{q^2 - q_{\parallel}^2}} - 1} [(-1 + e^{2i b \sqrt{q^2 - q_{\parallel}^2}}) \\
 & \times (-e^{2i b \sqrt{q^2 - q_{\parallel}^2}} + e^{2i d \sqrt{q^2 - q_{\parallel}^2}}) + i(\eta\alpha)(-n(e^{4i b \sqrt{q^2 - q_{\parallel}^2}} \\
 & - e^{2i d \sqrt{q^2 - q_{\parallel}^2}}) + m e^{2i b \sqrt{q^2 - q_{\parallel}^2}}(e^{2i d \sqrt{q^2 - q_{\parallel}^2}} - 1) \\
 & + \frac{1}{2}(\eta\alpha)^2(n^2(e^{4i b \sqrt{q^2 - q_{\parallel}^2}} + e^{2i d \sqrt{q^2 - q_{\parallel}^2}}) \\
 & - m^2(e^{2i b \sqrt{q^2 - q_{\parallel}^2}} + e^{2i b \sqrt{q^2 - q_{\parallel}^2}} e^{2i d \sqrt{q^2 - q_{\parallel}^2}}))] ,
 \end{aligned} \tag{B.3}$$

$$\begin{aligned}
 g_{zz}(z(t), z(t'), \omega, \mathbf{q}_{\parallel}) & \approx 2\pi i \frac{q_{\parallel}^2}{\eta q (q^2)} \frac{e^{-2i b \sqrt{q^2 - q_{\parallel}^2}}}{e^{2i d \sqrt{q^2 - q_{\parallel}^2}} - 1} [(e^{2i b \sqrt{q^2 - q_{\parallel}^2}} + 1) \\
 & \times (e^{2i b \sqrt{q^2 - q_{\parallel}^2}} + e^{2i d \sqrt{q^2 - q_{\parallel}^2}}) + i(\eta\alpha)(n(e^{4i b \sqrt{q^2 - q_{\parallel}^2}} \\
 & - e^{2i d \sqrt{q^2 - q_{\parallel}^2}}) + m e^{2i b \sqrt{q^2 - q_{\parallel}^2}}(e^{2i d \sqrt{q^2 - q_{\parallel}^2}} - 1) \\
 & - \frac{1}{2}(\eta\alpha)^2(n^2(e^{4i b \sqrt{q^2 - q_{\parallel}^2}} + e^{2i d \sqrt{q^2 - q_{\parallel}^2}}) \\
 & + m^2(e^{2i b \sqrt{q^2 - q_{\parallel}^2}} + e^{2i b \sqrt{q^2 - q_{\parallel}^2}} e^{2i d \sqrt{q^2 - q_{\parallel}^2}}))] \\
 & - \frac{4\pi}{q^2} \delta(z(t) - z(t')).
 \end{aligned} \tag{B.4}$$

Now, by inserting equations (B.2), (B.3) and (B.4) into equation (A.1), and using equations (13), (14) and (15), we find the following results for the geometry of parallel conductors:

$$\begin{aligned}
 \Gamma^{EB} & = \frac{2 \langle |e|\mathbf{d}(0)|g\rangle^2}{3\hbar} \frac{v_{\max}^2}{c^2} \left(\frac{\omega_0 - \omega_{cm}}{\omega_{cm}} \right) \\
 & \times \text{Re} \int_0^{q_0} dq_{\parallel} (q_0^2 - q_{\parallel}^2),
 \end{aligned} \tag{B.5}$$

$$\begin{aligned}
 \Gamma^{EE} & = \frac{1}{T} \frac{\langle |e|\mathbf{d}(0)|g\rangle^2}{3\pi\hbar c^2} \int_0^\infty d\omega (\omega a)^2 \\
 & \times \int_0^T dt \int_0^T dt' e^{-i(\omega + \omega_0)(t-t')} \\
 & \times \text{Re} \left\{ \int_0^q dq_{\parallel} q_{\parallel} \left(\frac{\sqrt{q^2 - q_{\parallel}^2}}{2 q^2} \right) \frac{e^{-2i\eta q b}}{e^{2i\eta q d} - 1} \right. \\
 & \times [-(q^2 - q_{\parallel}^2) + q^2 + q_{\parallel}^2] \\
 & \times ((e^{2i\eta b q} + e^{2i\eta b q} e^{2i\eta d q})(\cos(\omega_{cm} t) - \cos(\omega_{cm} t')^2) \\
 & + (-q_{\parallel}^2 + q^2 + (q^2 - q_{\parallel}^2))(e^{4i\eta q b} + e^{2i\eta q d}) \\
 & \left. \times (\cos(\omega_{cm} t) + \cos(\omega_{cm} t')^2) \right\},
 \end{aligned} \tag{B.6}$$

$$\begin{aligned}
 \Gamma^{BB} & = \frac{1}{T} \frac{\langle |e|\mathbf{d}(0)|g\rangle^2}{6\pi\hbar} \frac{v_{\max}^2}{c^2} \\
 & \times \text{Re} \left\{ \int_0^\infty d\omega \int_0^T dt \int_0^T dt' e^{-i(\omega + \omega_0)(t-t')} \right. \\
 & \times \sin(\omega_{cm} t) \sin(\omega_{cm} t') \int_0^q dq_{\parallel} \frac{q_{\parallel}}{\sqrt{q^2 - q_{\parallel}^2}} \frac{2q^2 - q_{\parallel}^2}{q^2} \\
 & \times \left[(q_{\parallel}^2) \left(\left(\frac{1 + e^{-2i\eta b q}}{e^{2i\eta d q} - 1} \right) (e^{2i\eta d q} + e^{2i\eta b q}) \right) \right. \\
 & \left. \left. + (2q^2 - q_{\parallel}^2) \left(\left(\frac{1 - e^{-2i\eta b q}}{e^{2i\eta d q} - 1} \right) (e^{2i\eta d q} - e^{2i\eta b q}) \right) \right] \right\},
 \end{aligned} \tag{B.7}$$

which lead to equations (23) after straightforward simplifications. The integrals in equations (23) can be calculated leading to the following results:

$$\Gamma^{EB} = \frac{1}{6} \Gamma_0 \frac{v_{\max}^2}{c^2} \left(\frac{\omega_{cm}}{\omega_0} - 1 \right)^3 \left(\frac{\omega_0}{\omega_{cm}} - 1 \right), \tag{B.8}$$

$$\begin{aligned}
 \Gamma^{EE} & = \frac{1}{4} \Gamma_0 \frac{v_{\max}^2}{c^2} \left(\frac{\omega_{cm}}{\omega_0} - 1 \right)^3 \left(1 - \frac{\omega_0}{\omega_{cm}} \right)^2 \\
 & \times \frac{1}{12 b^5 d^5 q_0^5} [6 b q_0 d \cos(2b q_0) \\
 & \times (d^4(-3 + 2b^2 q_0^2) \\
 & + b^4 \left(-2d^2 q_0^2 \left[\Phi \left(e^{2id q_0}, 2, \frac{-b}{d} \right) + \Phi \left(e^{2id q_0}, 2, \frac{b}{d} \right) \right] \right. \\
 & + 3 \left[\Phi(e^{2id q_0}, 4, \frac{-b}{d}) + \Phi(e^{2id q_0}, 4, \frac{b}{d}) \right] \left. \right) \\
 & + 4b^5 d^3 q_0^3 (d q_0 + 3 \text{Arg}(1 - e^{-2id q_0}) \\
 & + 3 \text{Li}_2(e^{-2id q_0}) \\
 & - 3 \sin(2b q_0) \left(6 b^5 d^2 q_0^2 \left[\Phi \left(e^{2id q_0}, 3, \frac{-b}{d} \right) \right. \right. \\
 & \left. \left. - \Phi \left(e^{2id q_0}, 3, \frac{b}{d} \right) \right] \right) \\
 & + 3b^5 \left[\Phi \left(e^{2id q_0}, 5, \frac{b}{d} \right) - \Phi \left(e^{2id q_0}, 5, \frac{-b}{d} \right) \right] \\
 & + d^5(-3 + 6b^2 q_0^2 - 2b^4 q_0^4 \\
 & + 2b^4 q_0^4 \left[{}_2F_1 \left(1, \frac{-b}{d}, 1 - \frac{b}{d}, e^{2id q_0} \right) \right. \\
 & \left. \left. + {}_2F_1 \left(1, \frac{b}{d}, 1 + \frac{b}{d}, e^{2id q_0} \right) \right] \right] \left. \right],
 \end{aligned} \tag{B.9}$$

and

$$\begin{aligned}
 \Gamma^{BB} = & \frac{1}{8} \Gamma_0 \frac{v_{\max}^2}{c^2} \left(\frac{\omega_{cm}}{\omega_0} - 1 \right)^3 \frac{1}{4 b^5 d^5 q_0^5} \\
 & \times [6 b q_0 d \cos(2b q_0) (d^4 (1 - b^2 q_0^2) \\
 & + b^4 \left(d^2 q_0^2 \left[\Phi \left(e^{2id q_0}, 2, \frac{-b}{d} \right) + \Phi \left(e^{2id q_0}, 2, \frac{b}{d} \right) \right] \right. \\
 & \left. - \left[\Phi \left(e^{2id q_0}, 4, \frac{-b}{d} \right) + \Phi \left(e^{2id q_0}, 4, \frac{b}{d} \right) \right] \right) \\
 & + \frac{16}{3} b^5 d^3 q_0^3 (d q_0 (d q_0 + 3 \operatorname{Arg} (1 - e^{-2id q_0}) \\
 & + 3 \operatorname{Li}_2(e^{-2id q_0}) \\
 & + \sin(2b q_0) (d^5 (-3 + 7b^2 q_0^2 - 4 b^4 q_0^4) \\
 & + b^4 \left(7 b d^2 q_0^2 \left[\Phi \left(e^{2id q_0}, 3, \frac{-b}{d} \right) \right. \right. \\
 & \left. \left. - \Phi \left(e^{2id q_0}, 3, \frac{b}{d} \right) \right] + 3b \left[\Phi \left(e^{2id q_0}, 5, \frac{b}{d} \right) \right. \right. \\
 & \left. \left. - \Phi \left(e^{2id q_0}, 5, \frac{-b}{d} \right) \right] \right) \\
 & + 4 d^5 q_0^4 \left[{}_2F_1 \left(1, \frac{-b}{d}, 1 - \frac{b}{d}, e^{2id q_0} \right) \right. \\
 & \left. \left. + {}_2F_1 \left(1, \frac{b}{d}, 1 + \frac{b}{d}, e^{2id q_0} \right) \right] \right] \right]. \quad (B.10)
 \end{aligned}$$

Appendix C

By inserting equation (27) into (31), we find:

$$\begin{aligned}
 \Gamma_{|e\rangle \rightarrow |g\rangle} = & \sum_{k, \lambda} \int_0^T \int_0^T dt dt' e^{i(\omega_k - \omega_0)(t-t')} \\
 & \times \{ 2\pi \hbar \omega_k [\mathbf{d}_{ge} \cdot \mathbf{f}_{k, \lambda}^*(\mathbf{r}(t))] \\
 & \times [\mathbf{d}_{ge} \cdot \mathbf{f}_{k, \lambda}(\mathbf{r}(t'))] \\
 & + \frac{2\pi \hbar}{\omega_k} v_{\max}^2 [\mathbf{d}_{ge} \cdot (\hat{\mathbf{z}} \times [\mathbf{k} \times \mathbf{f}_{k, \lambda}^*(\mathbf{r}(t))]) \\
 & \times [\mathbf{d}_{ge} \cdot (\hat{\mathbf{z}} \times [\mathbf{k} \times \mathbf{f}_{k, \lambda}(\mathbf{r}(t'))])] \sin(\Omega t) \sin(\Omega t') \\
 & + 2\pi \hbar v_{\max} [\mathbf{d}_{ge} \cdot (\hat{\mathbf{z}} \times [\mathbf{k} \times \mathbf{f}_{k, \lambda}^*(\mathbf{r}(t))]) \\
 & \times [\mathbf{d}_{ge} \cdot \mathbf{f}_{k, \lambda}(\mathbf{r}(t'))] \sin(\Omega t) \\
 & + 2\pi \hbar v_{\max} [\mathbf{d}_{ge} \cdot (\hat{\mathbf{z}} \times [\mathbf{k} \times \mathbf{f}_{k, \lambda}(\mathbf{r}(t'))]) \\
 & \times [\mathbf{d}_{ge} \cdot \mathbf{f}_{k, \lambda}^*(\mathbf{r}(t))] \sin(\Omega t') \}, \quad (C.1)
 \end{aligned}$$

where ω_0 is the internal frequency of the atom and $\mathbf{d}_{ge} = \langle g | \mathbf{d} | e \rangle$. By inserting equation (30) into equation (C.1) and expanding the result up to the second order in $k_z a \ll 1$,

we obtain:

$$\begin{aligned}
 \Gamma_{|e\rangle \rightarrow |g\rangle}^{EE} = & \sum_{\mathbf{k}} \frac{4\pi \omega_{\mathbf{k}}}{V} \frac{1}{T \hbar} \int_0^T dt \int_0^T dt' e^{i(\omega_{\mathbf{k}} - \omega_0)(t-t')} \\
 & \times \left\{ |d_{ge}|^2 \left[\frac{k_{\parallel}^2}{k^2} (1 + \cos 2k_z b) \right. \right. \\
 & - \frac{k_{\parallel}^2}{k^2} (\alpha' n' \sin(2k_z b) + \frac{k_{\parallel}^2}{k^2} \left(\frac{-(\alpha' m')^2}{2} \right) \\
 & + \frac{k_{\parallel}^2}{k^2} \cos(2k_z b) \left(\frac{-(\alpha' n')^2}{2} \right) \left. \right] \\
 & + \frac{|d_{ge}|^2}{2} \left[\left(1 + \frac{k_z^2}{k^2} \right) (1 - \cos(2k_z b)) \right. \\
 & + \left(1 + \frac{k_z^2}{k^2} \right) (\alpha' n' \sin(2k_z b)) \\
 & + \left(1 + \frac{k_z^2}{k^2} \right) \left(\frac{-(\alpha' m')^2}{2} \right) \\
 & \left. \left. - \left(1 + \frac{k_z^2}{k^2} \right) \cos(2k_z b) \left(\frac{-(\alpha' n')^2}{2} \right) \right] \right\}, \quad (C.2)
 \end{aligned}$$

$$\begin{aligned}
 \Gamma_{|e\rangle \rightarrow |g\rangle}^{BB} = & \sum_{\mathbf{k}} \frac{2\pi}{\omega_{\mathbf{k}} V} v_{\max}^2 \frac{1}{T \hbar} \\
 & \times \int_0^T dt \int_0^T dt' e^{i(\omega_{\mathbf{k}} - \omega_0)(t-t')} \sin(\Omega t) \sin(\Omega t') \\
 & \times \left\{ |d_{ge}|^2 \left[\frac{k_{\parallel}^4 + k_z^4 + k^2 k_z^2}{k^2} \right. \right. \\
 & \left. \left. + \frac{\cos 2k_z b}{k^2} (k_{\parallel}^4 - k_z^4 - k^2 k_z^2) \right] \right\}, \quad (C.3)
 \end{aligned}$$

$$\begin{aligned}
 \Gamma_{|e\rangle \rightarrow |g\rangle}^{EB} = & \sum_{\mathbf{k}} \frac{2\pi}{V} \frac{1}{T \hbar} v_{\max} \int_0^T dt \int_0^T dt' e^{i(\omega_{\mathbf{k}} - \omega_0)(t-t')} |d_{ge}|^2 \\
 & \times \left\{ [(\sin(\Omega t') - \sin(\Omega t)) (\alpha' n')] \right. \\
 & \left. + (\sin(\Omega t') + \sin(\Omega t)) (\alpha' m') \frac{ik_{\parallel}^2 k_z}{k^2} \cos(2k_z b) \right\}, \quad (C.4)
 \end{aligned}$$

where

$$\begin{aligned}
 \alpha' &= k_z a & n' &= \cos(\Omega t) + \cos(\Omega t') \\
 m' &= \cos(\Omega t) - \cos(\Omega t'). \quad (C.5)
 \end{aligned}$$

Now, integrals over time variables t, t' can be calculated similar to (A.7). The summation over the modes can be approximated by integrals according to $\sum_{\mathbf{k}} \rightarrow \frac{V}{(2\pi)^3} \int d^3 \mathbf{k}$, and by changing the integration variable as $u = \sqrt{k^2 - k_{\parallel}^2}$, we obtain:

$$\begin{aligned}
\Gamma_{|e\rangle \rightarrow |g\rangle} &= \Gamma_{|e\rangle \rightarrow |g\rangle}^{EB} + \Gamma_{|e\rangle \rightarrow |g\rangle}^{EE} + \Gamma_{|e\rangle \rightarrow |g\rangle}^{BB} \\
&= \int_0^{\omega_0/c} du \left\{ \frac{2 |d_{ge}|^2}{\hbar} \left(\frac{\omega_0^2}{c^2} - u^2 \right) (1 + \cos(2u b)) \right. \\
&\quad \left. + \frac{|d_{ge}|^2}{\hbar} \left(\frac{\omega_0^2}{c^2} + u^2 \right) (1 - \cos(2u b)) \right\} \\
&\quad + \sum_{i=1,2} \int_0^{q_i} du \frac{|d_{ge}|^2}{\hbar} \frac{a^2}{2} (1 - \cos(2u b)) u^2 (q_i^2 - u^2) \\
&\quad - \int_0^{\omega_0/c} du \frac{|d_{ge}|^2}{\hbar} a^2 (1 + \cos(2u b)) u^2 \left(\frac{\omega_0^2}{c^2} - u^2 \right) \\
&\quad + \sum_{i=1,2} \int_0^{q_i} du \frac{|d_{ge}|^2}{\hbar} \frac{a^2}{4} (1 + \cos(2u b)) u^2 (q_i^2 + u^2) \\
&\quad - \int_0^{\omega_0/c} du \frac{|d_{ge}|^2}{\hbar} \frac{a^2}{2} (1 - \cos(2u b)) u^2 \left(\frac{\omega_0^2}{c^2} + u^2 \right) \\
&\quad + \sum_{i=1,2} \int_0^{q_i} du \frac{|d_{ge}|^2}{\hbar} \frac{v_{\max}^2}{4 c^2} \frac{1}{q_i^2} (((q_i^2 - u^2)^2 + u^4 + q_i^2 u^2) \\
&\quad + \cos(2u b) ((q_i^2 - u^2)^2 - u^4 - q_i^2 u^2)) \\
&\quad + \int_0^{q_1} du \frac{|d_{ge}|^2}{\hbar} \frac{v_{\max}}{2 c} \frac{a}{q_1} u^2 (q_1^2 - u^2) (\cos(2u b) + 1) \\
&\quad - \int_0^{q_2} du \frac{|d_{ge}|^2}{\hbar} \frac{v_{\max}}{2 c} \frac{a}{q_2} u^2 (q_2^2 - u^2) (\cos(2u b) + 1), \tag{C.6}
\end{aligned}$$

where $q_1 = (\omega_0 + \Omega)/c$ and $q_2 = (\omega_0 - \Omega)/c$. From equation (C.6), we have:

$$\begin{aligned}
\frac{\Gamma_{\perp}}{\Gamma_0} &= \left[1 - 3 \left(\frac{\cos(x)}{x^2} - \frac{\sin(x)}{x^3} \right) + \frac{v_{\max}^2}{5 c^2 x^5} (\xi^2) \right. \\
&\quad \times \left(\frac{1}{4} \Delta^5 x^5 + 15(\Delta)x \left(-3 + \frac{1}{4} x^2 \Delta^2 \right) \cos(\Delta x) \right. \\
&\quad + 15 \left(3 - \frac{5}{4} x^2 (\Delta)^2 \right) \sin(x\Delta) + \frac{1}{4} \sigma^5 x^5 \\
&\quad + 15 \sigma x \left(-3 + \frac{1}{4} x^2 \sigma^2 \right) \cos(\sigma x) \\
&\quad + 15 \left(3 - \frac{5}{4} x^2 (\sigma)^2 \right) \sin(x\sigma) \\
&\quad - \frac{1}{2} x^5 + 30x \left(-3 + \frac{1}{4} x^2 \right) \cos(x) \\
&\quad \left. \left. - 30 \left(-3 + \frac{5}{4} x^2 \right) \sin(x) \right) \right], \tag{C.7}
\end{aligned}$$

where $\Omega = \frac{1}{\xi} \omega_0$, $z/z_0 = x = 2 k_0 b$, $\sigma = 1 + \frac{1}{\xi}$ and $\Delta = 1 - \frac{1}{\xi}$. Also,

$$\begin{aligned}
\frac{\Gamma_{\parallel}}{\Gamma_0} &= \left\{ 1 - \frac{3}{2} \left(\frac{\sin(x)}{x} - \frac{\sin(x)}{(x)^3} + \frac{\cos(x)}{(x)^2} \right) \right. \\
&\quad + \frac{6v_{\max}^2}{c^2 x^5} \left[\xi^2 \left[-\frac{1}{30} x^5 \right. \right. \\
&\quad + \frac{1}{2} \left(\cos(x) (-3x + \frac{3}{4} x^3) + \sin(x) (3 - x^2) \right) \\
&\quad + \frac{1}{60} \left((x\Delta)^5 + 15(x\Delta) \cos(x\Delta) \left(-3 + \frac{3}{4} (x\Delta)^2 \right) \right. \\
&\quad + 15 \left(3 - \frac{7}{4} (x\Delta)^2 + \frac{1}{4} (x\Delta)^4 \right) \sin(x\Delta) + (\sigma x)^5 \\
&\quad + 15(\sigma x) \cos(\sigma x) \left(-3 + \frac{3}{4} (\sigma x)^2 \right) \\
&\quad + 15 \left(3 - \frac{7}{4} (\sigma x)^2 + \frac{1}{4} (\sigma x)^4 \right) \sin(\sigma x) \left. \right] \left. \right] \\
&\quad + \frac{1}{\Delta^2} \left(\frac{1}{30} (x\Delta)^5 + \frac{1}{16} (x\Delta)^2 [-3(x\Delta) \cos(x\Delta) \right. \\
&\quad + (3 - (x\Delta)^2) \sin(x\Delta)] + \frac{1}{\sigma^2} \left(\frac{1}{30} (\sigma x)^5 \right. \\
&\quad + \frac{1}{16} (\sigma x)^2 [-3(\sigma x) \cos(\sigma x) + (3 - (\sigma x)^2) \sin(\sigma x)] \left. \right) \\
&\quad - \frac{\xi}{30\Delta} \left(\frac{1}{4} (\Delta x)^5 - 15(\Delta x) \left(-3 + \frac{1}{4} (\Delta x)^2 \right) \cos(\Delta x) \right. \\
&\quad - 15 \left(3 - \frac{5}{4} (\Delta x)^2 \right) \sin(\Delta x) + \frac{\xi}{30\sigma} \left(\frac{1}{4} (\sigma x)^5 \right. \\
&\quad - 15(\sigma x) \left(-3 + \frac{1}{4} (\sigma x)^2 \right) \cos(\sigma x) \\
&\quad \left. \left. - 15 \left(3 - \frac{5}{4} (\sigma x)^2 \right) \sin(\sigma x) \right) \right] \left. \right\}. \tag{C.8}
\end{aligned}$$

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