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A new simple model for the α-decay

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Abstract

We analytically find a new simple α-decay formula for a modified harmonic oscillator with the spherical Coulomb potential by employing Wentzel–Kramers–Brillouin approximation in terms of taking into account the Bohr–Sommerfeld quantization condition. We systematically investigate the favored α-decay half-lives of $52 \leqslant Z \leqslant 107$ parent nuclei. Our results are in good agreement with the experimental data and other models. We emphasize the importance of shape of the nuclear potential in the examination of α-decay half-lives of the parent nuclei.

Keywords: α-decay, half-life, WKB method

(Some figures may appear in colour only in the online journal)

1. Introduction

The first correlation between the half-life $T_{1/2}$ and the decay energy Q of the emitted α particle was empirically found by Geiger and Nuttall in 1911 [1]. The origin of this relation was understood after quantum mechanics was developed. In 1928, this relation was explained within the framework of the quantum tunneling through the Coulomb barrier by Gamow [2], and independently by Gurney and Condon [3]. The famous decay law of Geiger and Nuttall depending on α-decay energy Q is $\log T_{1/2} = A + BQ^{-1/2}$ and the decay constants A as well as B are obtained by fitting experimental data. In the Gamow model, the constants A and B are related to the parameters of the Coulomb potential and mass of the interacting particles. The extended version of the Geiger and Nuttall decay law was given by Viola–Seaborg [4]. The Viola–Seaborg relation is $\log T_{1/2} = A + BQ^{-1/2} + \log F$. Here Q, A, B and F are the α-decay energy, coefficients dependent on atomic number and hindrance factor presented to unpaired nucleons in the nucleus, respectively [4]. The parameter in the Viola–Seaborg relation was readjusted for new experimental data of the nuclei by Sobiczewski *et al* [5].

Since the nuclear decay is one of the main sources of our knowledge about the nuclear structure, the experimental and theoretical studies on the determination of decay half-life are still one of the most current issues in the nuclear physics. After Geiger–Nuttall, the discovery of the new empirical or semi-empirical formulas to determine the decay half-life of the nuclei is still up to date over the last forty years. Some of the interesting articles are the followings: Poenaru *et al* [6, 7] develop the analytical super asymmetric fission model (ASAFM). In ASAFM, the action integral is defined from the overlapping region (inner part of the barrier) to the separated fragments region and obtained the analytical formula called the universal curve of cluster radioactivity (UNIV). A nearly universal straight line for the α-decay half-life is found as a function of $Z^{0.6}/\sqrt{Q}$ in [8]. The analytical formula depending on atomic mass number A, atomic number Z and cluster decay energy Q for decay half-life is suggested by Royer [9] and a detailed examination is given in [10] as well as the modified form in [11, 12]. Royer also extends the generalized liquid drop model (GLDM) to include the volume, surface, a proximity energy term and Coulomb energies as well as numerically reproduce the α-decay half-life of the parent nuclei by Wentzel–Kramers–Brillouin (WKB) method [13]. In [14], the α-decay half-life of super-heavy nuclei is examined by extending the GLDM with 1977 nuclear proximity potential. The empirical expressions depending on the angular momentum of the α-particle for the even–odd, odd–even and odd–odd nuclei are proposed in [15] and extended version is studied in [16]. Another angular momentum depended empirical relations are given in [17, 18]. Horoi suggests a truncated Coulomb potential from inner turning point to touching radius and taking the point Coulomb potential from touching radius to outer turning point and finds an analytical cluster decay formula [19]. The decay formula of Gamow is modified by using some approximation and linearization depending on the cluster decay energy, atomic number and atomic mass of the interaction nuclei [20]. In [21, 22], a decay formula called universal decay law (UDL) based on the R-matrix description of the cluster decay is obtained by using the Coulomb–Hankel function with some approximation. The standard deviations of α-decay half-lives are calculated with the UNIV and compared with UDL [21, 22] for a total of 534 α-emitters [23]. Delion [24] investigates the α-decay half-life formula by using the outgoing spherical Coulomb–Hankel function. The last term of obtained formula is modified by using the ground state of a shifted harmonic oscillator [24]. The universal decay formula studies based on the WKB penetration probability or the R-matrix description of the cluster decay are to proceed with some approximations [20, 25–31].

In literature, it is seen that the universal decay formulas obtained empirically or semi-empirically depend only on Coulomb potential parameters and atomic mass. Moreover, the universal decay formulas are linearized by using some approximation with the atomic mass number, atomic number and angular momentum quantum number. However, if we want to understand the dynamic nature of the parent nucleus, we need to take into account not only the Coulomb barrier, but also the effective potential between the daughter nucleus and the alpha cluster. In order to study the dynamic structure of the parent nuclei, the effective potential could be obtained by the inversion from the universal decay formula to the effective potential within the framework of the WKB penetration probability or the R-matrix description of the cluster decay. In this paper, our aim is to find the α-decay half-life formula depending on the parameters of the effective potential by employing WKB method within the framework of the Bohr–Sommerfeld quantization condition for the effective potential including the spherical Coulomb potential and modified harmonic oscillator.

In next section, we introduce the model including the α clustering in parent nuclei within the framework of the Bohr–Sommerfeld quantization condition by employing the WKB method. In section 3, we give the results and discussion. Then, we conclude the results in section 4.

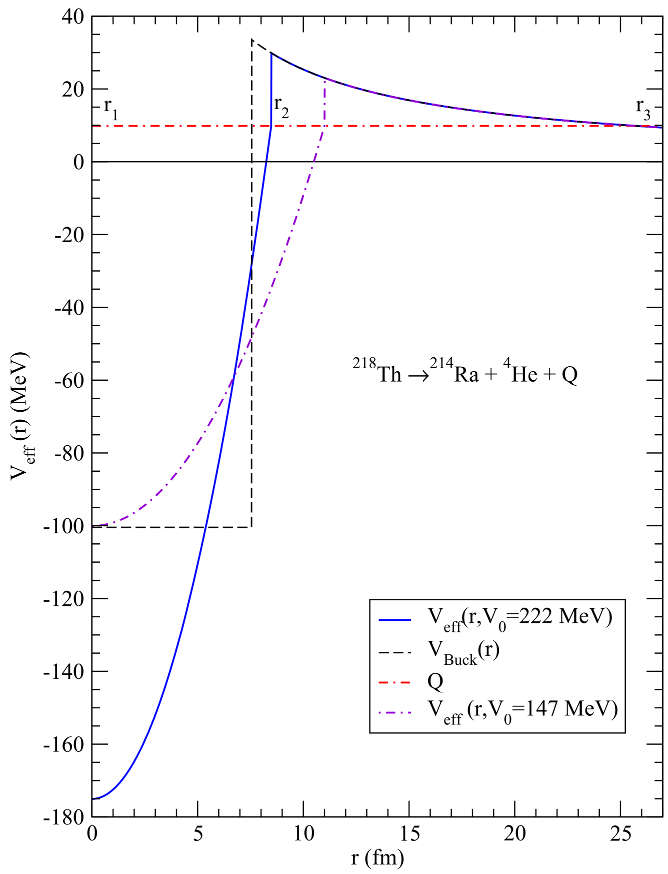

Figure 1. The effective potentials in terms of the different potential depths are shown as a function of the nuclear radius for the ^{218}Th $\rightarrow$ ^{214} Ra + α system with α-decay energy $Q = 9.848$ MeV. Dashed line shows the effective potential of Buck *et al* [32].

2. The model

One of the first successful applications of the quantum mechanics is the use of quantum tunneling through a potential barrier by Gamow [2], independently by Gurney and Condon [3] to explain the α-decay. With the idea of tunneling of the α particle through the Coulomb barrier, Gamow obtained a simple analytical formula similar to the empirical formula proposed by Geiger and Nuttall [1] for the decay half-life of the α particle.

Buck *et al* [32, 33] has proposed a simple cluster model so that the α particle is already preformed in the parent nucleus and revolves around it. The interacting potential between the α cluster and core (daughter nucleus) purposed by Buck *et al* [32, 33] has the square-well plus surface charge Coulomb form for the inside of the parent nucleus and a point charge Coulomb potential for the outside in figure 1. The model potential of Buck *et al* [32, 33] is as follows

$$V(r) = \begin{cases} -V_N + \dfrac{C}{R}, & r < R \\ \dfrac{C}{r}, & r > R, \end{cases} \tag{1}$$

where V_N, R and $C = 2(Z - 2)e^2$ are the potential depth and radius as well as the product of charges, respectively. This potential model has two important features. First one, an analytical relationship between the experimental observables such as the excitation energy E^*, half-lives $T_{1/2}$, etc and potential parameters can be obtained. In this way, it can be easily discussed how the experimental observables depend on nuclear potential parameters. Secondly, there is one degree of freedom in the potential which is depth of the nuclear potential. Decreasing the degree of freedom reduces the uncertainty in the nuclear potential [34]. On the other hand, since the nucleus has a diffusive structure towards the surface, the square-well potential is not suitable in describing the α cluster and core interactions. The model of Buck *et al* [32, 33] has also a simplification for the spherical Coulomb potential which is considered a constant inside of the nucleus. However, the Coulomb potential has r-dependent form inside of the nucleus for the spherical nuclei.

In this paper, we propose a modified harmonic oscillator potential to systematically examine the α-decay half-life of the parent nuclei as follows

$$V_N(r) = -V_0 + V_1 r^2, \tag{2}$$

where V_0 and V_1 are depth and diffusivity parameters of the nuclear potential. This potential model is very important from a physical point of view. While we expand an arbitrary potential $V(r)$ at the equilibrium distance, we can only take the constant plus harmonic oscillator terms for the low vibration limit [35]. Therefore, we can analytically determine the approximate form of realistic nuclear potential in the low vibration limit. Moreover, the modified harmonic oscillator has a diffused form towards the surface of the nuclei and has an analytical solution with the spherical Coulomb potential.

The Coulomb potential due to a point charge $Z_\alpha e$ interacting with a charge $Z_d e$ distributed over a uniformly sphere with radius R_C is taken as [36]

$$V_C(r) = \begin{cases} \dfrac{Z_\alpha Z_d e^2}{2R_C}\left(3 - \dfrac{r^2}{R_C^2}\right), & r \leqslant r_2 \\ \dfrac{Z_\alpha Z_d e^2}{r}, & r > r_2 \end{cases} \tag{3}$$

where $R_C = 1.07(A_\alpha^{1/3} + A_d^{1/3})$ [37]. The symbols Z_α, A_α, Z_d and A_d denote the atomic numbers, and atomic mass numbers of and daughter nuclei, respectively. Since we only consider the favored α-decay of the parent nuclei, the angular momentum quantum number L is taken zero in the calculations. The effective potential becomes with the spherical Coulomb potential as follows

$$V_{\text{eff}}(r) = \begin{cases} C_0 - V_0 + (V_1 - C_1)r^2, & r \leqslant r_2 \\ \dfrac{C_2}{r}, & r > r_2, \end{cases} \tag{4}$$

where $C_0 = \frac{3Z_\alpha Z_d e^2}{2R_C}$, $C_1 = \frac{Z_\alpha Z_d e^2}{2R_C^3}$ and $C_2 = Z_\alpha Z_d e^2$. The innermost, inner and outermost classical turning points r_1, r_2 and r_3 can be calculated by $V_{\text{eff}}(r) = Q$ equation. The innermost turning point r_1 is zero. The inner and outermost turning points can be analytically obtained as $r_2 = \sqrt{\frac{Q + V_0 - C_0}{V_1 - C_1}}$ and $r_3 = \frac{C_2}{Q}$. The turning points explicitly depend on the effective potential parameters and α-decay energy Q. We plot the effective potential given by equation (4) and compare the model of Buck *et al* [32, 33] given by equation (1) in figure 1. The advantage of our potential model is both more physical than the model of Buck *et al* [32, 33] and has analytical solution with the spherical Coulomb potential. The weakness of our model is that it has a sharp discontinuity in the region between r_2 and at the top of the barrier. This discontinuity can be removed by adding the inverted harmonic oscillator potential at the region of top of the barrier [38, 39]. Moreover, many parent nuclei have the deformed structures and the deformation effect should be take into account in calculations [40–43]. In order to avoid losing simplicity in our model, we do not consider deformed or more complex potential forms.

In order to reduce the degree of freedom of the effective potential describing the interaction between the daughter nucleus and α cluster, we take into account the cluster model by using the Bohr–Sommerfeld quantization condition which is [33]:

$$\int_0^{r_2} \mathrm{d}r k(r) = (G+1)\frac{\pi}{2}, \tag{5}$$

where $k(r) = \sqrt{\frac{2\mu}{\hbar^2}(Q - V_{\mathrm{eff}}(r))}$ is the wave number in the internal region of the effective potential. Q and μ are the α-decay energy and reduced atomic mass of the daughter and α cluster system, respectively. In calculation, Q and μ are evaluated from [49]. The relation between V_0 and V_1 parameters can be analytically found by using the Bohr–Sommerfeld quantization condition as follows

$$V_1 = C_1 + \frac{\mu}{2\hbar^2}\left(\frac{Q + V_0 - C_0}{1+G}\right)^2, \tag{6}$$

under the integral condition, $Q + V_0 > C_0$ and $V_1 > C_1$. Thus, we have the opportunity to systematically examine the α-decay half-life of parent nuclei with a single potential parameter V_0. The global quantum number G in equation (5) is determined by Wildermuth rule [44].

In semiclassical WKB approximation, the α-decay width Γ is given by [45]

$$\Gamma = PF\frac{\hbar^2}{4\mu}\exp(-2S_{23}), \tag{7}$$

where P, F and S_{23} are the preformation factor of the daughter-α cluster system, the normalization factor and the action integral, respectively. The normalization factor is [45]

$$F\int_0^{r_2} \mathrm{d}r\frac{1}{2k(r)} = 1, \tag{8}$$

and we can analytically obtain

$$F = \frac{4}{\pi}\sqrt{\frac{2\mu}{\hbar^2}(V_1 - C_1)}, \tag{9}$$

with the integral condition, $Q + V_0 > C_0$ and $V_1 > C_1$. The action integral S_{23} is as follows [45]

$$S_{23} = \int_{r_2}^{r_3} \mathrm{d}r\kappa(r), \tag{10}$$

where $\kappa(r) = \sqrt{\frac{2\mu}{\hbar^2}(V_{\mathrm{eff}}(r) - Q)}$ is the wave number in the barrier region of the effective potential. With the integral conditions $Q > 0$ and $C_2/Q > r_2 \geqslant 0$ in equation (10), the action integral can be analytically obtained as,

$$S_{23} = \sqrt{\frac{2\mu}{\hbar^2}} \frac{C_2}{\sqrt{Q}} \left(\arccos\left(\sqrt{\frac{Qr_2}{C_2}}\right) - \sqrt{\frac{Qr_2}{C_2} - \left(\frac{Qr_2}{C_2}\right)^2} \right). \tag{11}$$

The relation between the α-decay half-life and decay width is $T_{1/2} = \hbar \frac{\ln 2}{\Gamma}$ [45]. With equation (7) the decay half-life can be formulated by the following equation,

$$T_{1/2} = \frac{\ln 2}{PF} \frac{4\mu}{\hbar} e^{2S_{23}}. \tag{12}$$

Here, the most dominant term in calculation of α-decay half-lives of the parent nuclei is the action integral S_{23} depending on the reduced mass and atomic number of daughter-α system, α-decay energy and r_2 terms. In fact, the action integral is the area intersecting between Coulomb barrier and decay energy from the turning point r_2 to turning point r_3 in figure 1. Since the turning point r_3 is constant for a decay mechanism under examination, the area depends on r_2 turning point. Since the turning point r_2 also depends on the potential parameters V_0 and V_1, the form of the nuclear potential seems to be important in determining the action integral and the α-decay half-life. As a result, the nuclear potential should be taken into account in cluster decay calculations.

Inserting the normalization factor F in equation (9) and the action integral S_{23} in equation (11) into (12) and taking the decimal logarithm, we can analytically obtain decay formula as

$$\log T_{1/2} = A + \frac{B}{\sqrt{Q}}, \tag{13}$$

where A and B are decay parameter as follows:

$$A = \log\left(\frac{\pi\hbar \ln 2}{P} \frac{(1+G)}{(Q+V_0-G)}\right),$$
$$B = 2C_2 \log(e) \sqrt{\frac{2\mu}{\hbar^2}} \left(\arccos\left(\sqrt{\frac{Qr_2}{C_2}}\right) - \sqrt{\frac{Qr_2}{C_2} - \left(\frac{Qr_2}{C_2}\right)^2} \right). \tag{14}$$

It should be noted that the outermost turning point r_3 is generally much larger than the outer turning point r_2, i.e. $r_3 \gg r_2$. Therefore, an approximation to the terms in round brackets in B parameter can be made [20]. By using basic principles of the quantum mechanic, we obtain the traditional Geiger and Nuttall formula [1] for the modified harmonic oscillator in terms of the spherical Coulomb potential. The most important feature of this formula is that it establishes a correlation between the experimental observables and parameters of interaction potential. In this way, we can examine the nuclear structure and reaction dynamic of the parent nuclei [32, 46–48].

3. Results and discussion

Considering the α clustering in the parent nucleus by using Bohr–Sommerfeld quantization condition with the Wildermuth rule, we systematically examine the ground state to ground state α transitions of the parent nuclei in the range of $52 \leqslant Z \leqslant 107$ nuclei which have 136 even–even, 48 even–odd, 49 odd–even, 30 odd–odd and totally 263 isotopes. In the calculation, we have three parameters which are depth of the nuclear potential V_0, global quantum number G and preformation factor P. In the optimization of the parameters V_0, G and P, we firstly use the literature values for G and P parameters in [43, 45] and determine V_0 values for each decaying parent nuclei by using equation (13) and calculate average value as $V_0 = 195.8\,\text{MeV}$ and the root-mean-square (rms) deviation $\delta = 0.58$ for all parent nuclei. By using G and P in [33], we calculate average value of potential depth $V_0 = 223.0\,\text{MeV}$ and rms error $\delta = 0.53$. By changing potential depth we reduce the rms error and we find $V_0 = 221\,\text{MeV}$ as well as $\delta = 0.52$. In order to compare our results with Buck *et al* [33], we recalculate α-decay half-lives of the parent nuclei for the model of Buck *et al* [33] since it is only calculated the α-decay half-lives of even–even nuclei in [33] and the α-decay energy as well as reduced mass of daughter-α cluster are slightly different in their calculations. We obtain the total rms error of Buck *et al* [33] model is $\delta = 0.53$ for all parent nuclei.

Taking into account the neutron shell closure, we optimize global quantum number G and determine $G = 20$ for $N \leqslant 82$, $G = 22$ for $82 < N \leqslant 126$ and $G = 24$ for $N > 126$. We choose the preformation factor to be $P = 1.25$ for even–even nuclei, $P = 1$ for odd-A nuclei and $P = 1$ for odd–odd nuclei. Thus, we calculate average value of potential depth as $V_0 = 223.1\,\text{MeV}$ and $\delta = 0.484$ and search optimum values of potential depth as $V_0 = 222\,\text{MeV}$ and obtain rms error $\delta = 0.482$. In table 1, we calculate the α-decay half-lives of spherical even–even parent nuclei from the ground state to ground state α transitions and compare with the experimental data and theoretical models. In calculation, the α-decay energy and reduced mass of daughter-α cluster system are evaluated from [49]. The first column shows symbols of the parent nuclei and the second column is the α-decay energy. The third and fourth columns denote the decimal logarithm of the experimental and our theoretical α-decay half-lives. The fifth, sixth and seventh columns show logarithmic ratio of the experimental and theoretical α-decay half-lives which are the present model, Buck *et al* [33] model and Denisov *et al* [41] model called unified model for the α-decay and α-capture (UMADAC), respectively. We recalculate the α-decay half-lives of all parent nuclei with the model of Buck *et al* [33] by employing same α-decay energy Q and reduced mass μ of daughter-α cluster system in our calculations in table 1. It is know that there are differences in the use of the α-decay energy Q and reduced mass μ [17]. In table 1, we compare our results with Buck *et al* [33] model and UMADAC model [41] which take into account the quadrupole and hexadecapole deformations of the daughter nuclei. Our theoretical model seems to produce better results than Buck *et al* [33] and UMADAC [41] models for even–even parent nuclei (table 1). Similar comparisons for the α-decay half-lives of the spherical even–odd, odd–even and odd–odd parent nuclei are shown in tables 2 and 3, respectively. In figure 2, we show the decimal logarithm deviations between the experimental and calculated α-decay half-lives for the even–even (e–e), even–odd (e–o), odd–even (o–e) and odd–odd (o–o) parent nuclei versus the atomic number Z of the parent nuclei. The decimal logarithm deviations between the experimental and calculated α-decay half-lives are approximately ± 0.5 which are reproduced to within a factor of three.

For a more quantitative comparison of deviation between the experimental data and theoretical results, we calculate the rms deviation of the decimal logarithmic values by employing the following equation [41]

$$\delta = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\log_{10} T_{i,1/2}^{\text{theo}} - \log_{10} T_{i,1/2}^{\text{exp}})^2}\,, \tag{15}$$

where n indicates number of the parent nuclei. In table 4 we calculate rms errors of the decimal logarithm of the favored α-decay half-lives in the range of $52 \leqslant Z \leqslant 107$ parent nuclei which have 136 even–even, 48 even–odd, odd–even 49, 30 odd–odd and totally 263 isotopes. The first, second and third line show the rms errors of the present study, Buck *et al* [33] and UMADAC [41] models. The other lines shows different models and are taken from

Table 1. Comparison of the experimental and calculated α-decay half-lives of spherical even–even parent nuclei for the ground state to ground state α transitions. Q are evaluated from [49] and the experimental half-lives are taken from [41].

Parent nuclei	Q(MeV) [49]	$\log_{10} T_{1/2}^{\text{exp}}$(s) [41]	$\log_{10} T_{1/2}^{\text{theo}}$(s)	$\log_{10} \frac{T_{1/2}^{\text{exp}}}{T_{1/2}^{\text{theo}}}$	$\log_{10} \frac{T_{1/2}^{\text{exp}}}{T_{1/2}^{\text{Buck}}}$ [33]	$\log_{10} \frac{T_{1/2}^{\text{exp}}}{T_{1/2}^{\text{Denisov}}}$ [41]
${}^{144}_{60}\text{Nd}_{84}$	1.903	22.86	22.87	0.01	0.107	−0.58
${}^{146}_{62}\text{Sm}_{84}$	2.529	15.51	15.21	0.30	0.36	−0.12
${}^{148}_{64}\text{Gd}_{84}$	3.271	9.37	9.05	0.32	0.37	0.01
${}^{150}_{66}\text{Dy}_{84}$	4.351	3.08	2.73	0.35	0.4	0.27
${}^{186}_{82}\text{Pb}_{104}$	6.47	0.68	0.92	−0.24	−0.15	0.28
${}^{188}_{82}\text{Pb}_{106}$	6.109	2.06	2.4	−0.34	-0.24	−0.16
${}^{190}_{82}\text{Pb}_{108}$	5.698	4.25	4.27	−0.02	0.09	0.12
${}^{192}_{82}\text{Pb}_{110}$	5.221	6.57	6.71	−0.14	−0.02	0.34
${}^{194}_{82}\text{Pb}_{112}$	4.738	9.99	9.57	0.42	0.55	0.86
${}^{210}_{82}\text{Pb}_{128}$	3.792	16.57	15.57	1.	1.06	−0.02
${}^{204}_{84}\text{Po}_{120}$	5.485	6.28	6.38	−0.1	0.05	0.01
${}^{206}_{84}\text{Po}_{122}$	5.327	7.14	7.23	−0.09	0.07	0.06
${}^{210}_{84}\text{Po}_{126}$	5.408	7.08	6.81	0.27	0.43	0.8
${}^{212}_{84}\text{Po}_{128}$	8.954	−6.52	−6.86	0.34	0.32	0.15
${}^{214}_{84}\text{Po}_{130}$	7.834	−3.78	−3.92	0.14	0.14	0.06
${}^{216}_{84}\text{Po}_{132}$	6.906	−0.84	−0.9	0.06	0.08	−0.21
${}^{210}_{86}\text{Rn}_{124}$	6.159	3.95	4.12	−0.17	−0.02	0.47
${}^{212}_{86}\text{Rn}_{126}$	6.386	3.16	3.15	0.01	0.16	0.37
${}^{214}_{86}\text{Rn}_{128}$	9.209	−6.57	−6.79	0.22	0.19	0.18
${}^{216}_{86}\text{Rn}_{130}$	8.197	−4.35	−4.22	−0.13	−0.13	−0.32
${}^{218}_{86}\text{Rn}_{132}$	7.263	−1.46	−1.33	−0.13	−0.12	−0.47
${}^{220}_{86}\text{Rn}_{134}$	6.405	1.75	1.9	−0.15	-0.12	−0.35
${}^{222}_{86}\text{Rn}_{136}$	5.59	5.52	5.68	−0.16	-0.12	−0.25
${}^{214}_{88}\text{Ra}_{126}$	7.273	0.39	0.61	−0.22	-0.08	0.19
${}^{216}_{88}\text{Ra}_{128}$	9.525	−6.74	−6.85	0.11	0.09	−0.01
${}^{216}_{90}\text{Th}_{126}$	8.073	−1.57	−1.2	−0.37	−0.24	−0.05
${}^{218}_{90}\text{Th}_{128}$	9.848	−6.96	−6.93	−0.03	−0.06	−0.16

[41]. The present model produces better results than other models in calculation the α-decay half-lives of the parent nuclei in table 4.

In figure 3, we plot the decimal logarithm α-decay half-lives of Os, Hg, At and U nuclei as a function of $Q^{-1/2}$ by using the α-decay formula in equation (13) and compare the experimental α-decay half-lives of parent nuclei. There is clearly a linear correlation between the decimal logarithm half-lives and the parameter $Q^{-1/2}$. The effect of neutron shell closure is obviously seen for At isotopes in figure 3.

4. Conclusion

We systematically examine the α-decay half-lives of 263 parent nuclei, which have 136 even–even, 48 even–odd, 49 odd–even, 30 odd–odd and totally 263 isotopes from the ground

Table 2. Same with table 1 but for spherical even–odd and odd–even parent nuclei.

Parent nuclei	Q (MeV) [49]	$\log_{10} T_{1/2}^{\exp}$(s) [41]	$\log_{10} T_{1/2}^{\rm theo}$(s)	$\log_{10} \frac{T_{1/2}^{\exp}}{T_{1/2}^{\rm theo}}$	$\log_{10} \frac{T_{1/2}^{\exp}}{T_{1/2}^{\rm Buck}}$ [33]	$\log_{10} \frac{T_{1/2}^{\exp}}{T_{1/2}^{\rm Denisov}}$ [41]
$^{145}_{61}$Pm$_{84}$	2.323	17.3	17.17	0.13	0.29	−0.62
$^{147}_{63}$Eu$_{84}$	2.991	10.98	11.01	−0.03	0.12	−0.54
$^{207}_{84}$Po$_{123}$	5.215	8.	7.95	0.05	0.31	0.03
$^{213}_{84}$Po$_{129}$	8.536	−5.38	−5.74	0.36	0.44	−0.3
$^{215}_{84}$Po$_{131}$	7.526	−2.75	−2.89	0.14	0.24	−0.56
$^{209}_{85}$At$_{124}$	5.757	5.68	5.62	0.06	0.31	−0.03
$^{211}_{85}$At$_{126}$	5.982	4.79	4.56	0.23	0.48	0.2
$^{213}_{85}$At$_{128}$	9.255	−6.9	−7.13	0.23	0.3	−0.11
$^{215}_{85}$At$_{130}$	8.178	−4.	−4.43	0.43	0.52	−0.06
$^{215}_{86}$Rn$_{129}$	8.839	−5.64	−5.81	0.17	0.25	−0.53
$^{217}_{86}$Rn$_{131}$	7.888	−3.27	−3.23	−0.04	0.05	−0.84
$^{213}_{87}$Fr$_{126}$	6.905	1.54	1.62	−0.08	0.15	−0.08
$^{215}_{87}$Fr$_{128}$	9.54	−7.07	−7.11	0.04	0.11	−0.47
$^{215}_{89}$Ac$_{126}$	7.746	−0.77	−0.47	−0.3	−0.07	−0.24
$^{217}_{89}$Ac$_{128}$	9.832	−7.16	−7.11	−0.05	0.01	−0.39
$^{219}_{89}$Ac$_{130}$	8.827	−4.93	−4.74	−0.19	−0.11	−0.62
$^{217}_{91}$Pa$_{126}$	8.489	−2.45	−1.95	−0.5	−0.28	−0.43
$^{219}_{91}$Pa$_{128}$	10.081	−7.28	−7.02	−0.26	−0.19	−0.68

Table 3. Same with table 1 but for spherical odd–odd parent nuclei.

Parent nuclei	Q(MeV) [49]	$\log_{10} T_{1/2}^{\exp}$(s) [41]	$\log_{10} T_{1/2}^{\rm theo}$(s)	$\log_{10} \frac{T_{1/2}^{\exp}}{T_{1/2}^{\rm theo}}$	$\log_{10} \frac{T_{1/2}^{\exp}}{T_{1/2}^{\rm Buck}}$ [33]	$\log_{10} \frac{T_{1/2}^{\exp}}{T_{1/2}^{\rm Denisov}}$ [41]
$^{208}_{85}$At$_{123}$	5.751	6.04	5.64	0.4	0.65	0.24
$^{214}_{85}$At$_{129}$	8.988	−6.25	−6.51	0.26	0.33	−0.26
$^{216}_{87}$Fr$_{129}$	9.174	−6.15	−6.28	0.13	0.2	−0.27
$^{218}_{87}$Fr$_{131}$	8.013	−2.97	−3.22	0.25	0.35	−0.26
$^{218}_{89}$Ac$_{129}$	9.376	−5.97	−6.09	0.12	0.19	−0.33
$^{218}_{91}$Pa$_{127}$	9.815	−3.76	−6.45	2.69	2.75	1.87

state to ground state α transitions by using WKB method with taking into account the Bohr–Sommerfeld quantization condition in terms of the Wildermuth rule. We propose the modified harmonic oscillator potential which is more convenient than the model of Buck *et al* [33] and has analytical solution with the spherical Coulomb potential in order to explain the decay mechanism of the daughter-α interaction systems. We reduce the degree of freedom to one (V_0) in the effective potential by employing the Bohr–Sommerfeld quantization condition and find the analytical formula for the α-decay half-life. We compare our result with the experimental α-decay half-lives, Buck *et al* [33], UMADAC [41], and other empirical or semi-empirical models [9, 10, 50, 51]. Although we do not take the deformation of the parent nuclei into consideration, our model produces better results than UMADAC model [41]. We

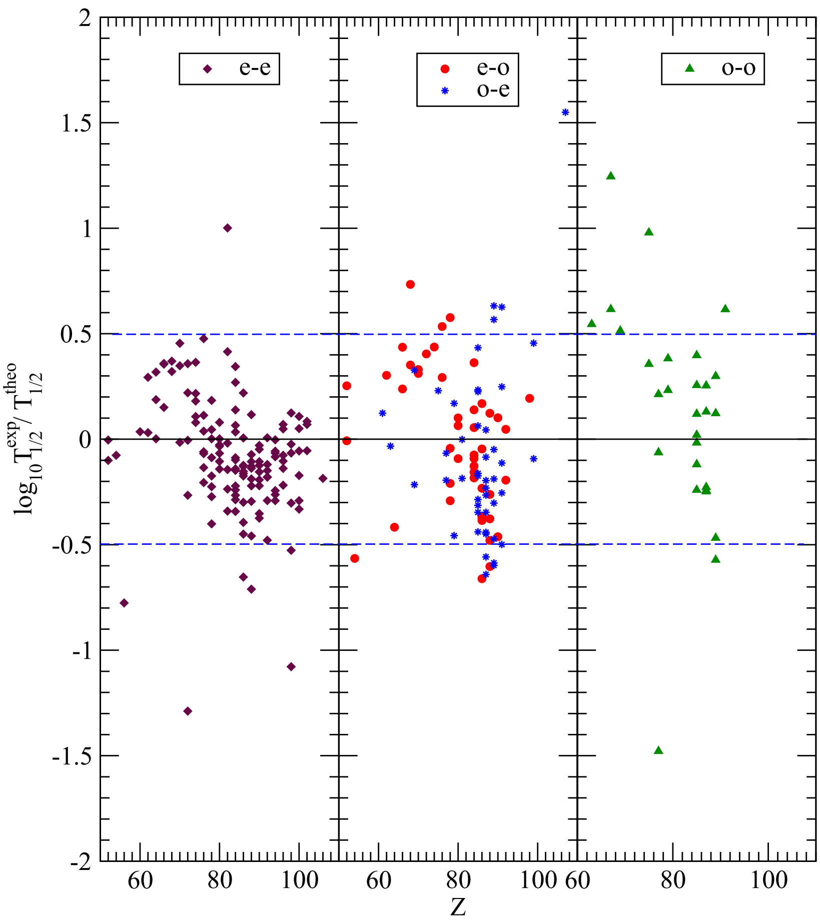

Figure 2. The decimal logarithm deviations between the calculated and experimental α-decay half-lives versus the atomic number of parent nuclei Z in the range of $52 \leqslant Z \leqslant 107$ for the even–even (e–e), even–odd (e–o), odd–even (o–e) and odd–odd (o–o) nuclei.

Table 4. Rms errors of the decimal logarithm for different models. In table, first and second lines show rms errors of the decimal logarithm of the favored α-decay half-lives in the range of $52 \leqslant Z \leqslant 107$ parent nuclei which have 136 even–even, 48 even–odd, 49 odd–even, 30 odd–odd and totally 263 nuclides. Other lines are taken from [41].

Total	Even–even	Even–odd	Odd–even	Odd–odd	References
0.4818	0.2896	0.5060	0.5952	0.8397	This study
0.5335	0.3009	0.5781	0.6960	0.8981	[33]
0.6248	0.3088	0.7816	0.7621	0.7546	[41]
1.3813	1.2928	1.4300	1.5607	1.2751	[50]
1.1130	0.3837	1.4762	1.3688	1.3340	[9]
1.1285	0.3712	1.5425	1.3541	1.3307	[51]
1.0185	0.5165	1.1611	1.3348	1.2568	[10]

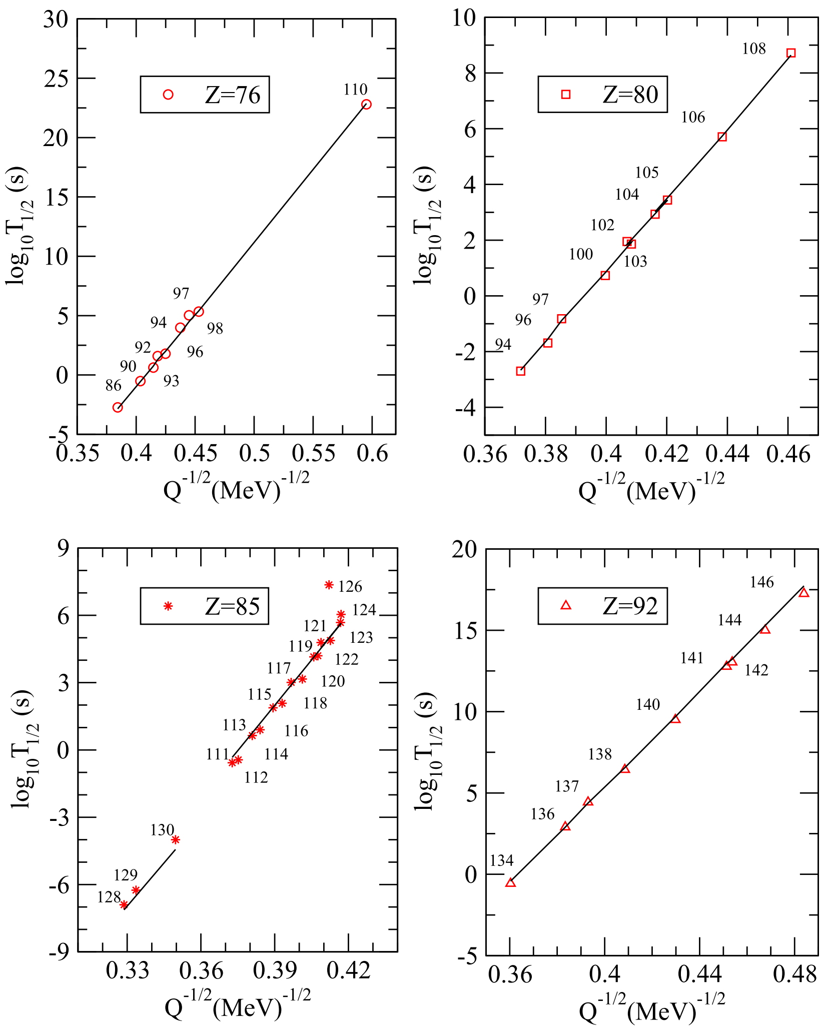

Figure 3. The linear relations between the decimal logarithm α-decay half-lives and α-decay energy $Q^{-1/2}$ for Os, Hg, At and U isotopes. The solid lines show the theoretical results. Circle, square, star and triangle show the experimental α-decay half-lives. Numbers on experimental data show neutron numbers N related Isotopes.

have also achieved better results than Buck *et al* [33] and other empirical or semi-empirical models [9, 10, 33, 50, 51] in explaining the α-decay half-lives of the favored nuclei. In calculations of the α-decay half-life, we observe that the nuclear potential should be taken into account since shape of the nuclear potential changes the turning point r_2 and action integral as well as the decay half-life of the parent nuclei. Since it can be analytically established a correlation between the experimental data and potential parameters in our model, we might have more information about nuclear structure and reaction dynamic of the parent nuclei [32, 46–48]. We only consider the first three terms of the Taylor series of the realistic nuclear potential in our nuclear potential model. A more suitable α-decay half-life formula could be obtained analytically, taking into account more terms in Taylor series of the realistic nuclear potential.

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