

Optimization of Lucy-Richardson Algorithm Using Modified Tikhonov Regularization for Image Deblurring

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Abstract. In this paper, we propose the method to optimize the iteration number of the Lucy-Richardson algorithm for image deblurring. This technique based on the modified Tikhonov regularization which composed of 2 parts which are designed for measuring the image similarity and noise enhancement due to the deblurring process. The regularization parameter will be used to control the desired deblurred image. Several sizes of the Gaussian blur kernel are applied for generating the degraded image in the simulation experiment. The Peak Signal to Noise Ratio (PSNR) metric is used to measure the deblurring performance. The results show that this method can be used to estimate the optimal iteration number and it also gives the PSNR value higher than the default Lucy-Richardson method and regularized filter all sizes of the experimental blur kernel. Moreover, it also tolerance to the deviated blur kernel especially it smaller than exact blur kernel.

1. Introduction

When the imaging system captured a scene, the acquired image will always be degraded due to both intrinsic and extrinsic physical properties such as detector, optic instruments, electronic system, camera motion, including the environmental atmosphere. Generally, the image formation model can be expressed as the following equation.

$$g(x, y) = p(x, y) \otimes f(x, y) + n(x, y) \quad (1)$$

where $g(x, y)$ is the degraded image, $p(x, y)$ is the blur kernel which also known as the point spread function (PSF), $f(x, y)$ is the original image (scene), $n(x, y)$ is an additive noise and \otimes denotes the convolution operator. Lucy-Richardson (LR) iterative algorithm is one popular technique in the image deblurring community. It was developed by Richardson [1] and Lucy [2]. It was derived from a statistical point of view as it converges to the maximum-likelihood solution under the condition that the image data corresponding a Poisson distribution [3]. The restored image can be defined as

$$\hat{f}^{(k+1)} = \hat{f}^{(k)} \left[p^* \otimes \frac{g}{p \otimes \hat{f}^{(k)}} \right] \quad (2)$$



where $\hat{f}^{(k+1)}$ is a restored image in the present step, $\hat{f}^{(k)}$ is a restored image in the previous step and p^* is a reversal of p along each of its dimensions[4]. The main disadvantages of this algorithm are image boundary artifacts and lack of information about the optimal number of the algorithm iterations [5]. To overcome this problem, the technique to estimate the optimal iteration number of the LR method is proposed. This technique is based on the modified Tikhonov regularization. The paper is organized as follows. The related works are summarized in section 2. The proposed algorithm is described in section 3 and the experimental results are demonstrated in section 4. Finally, the conclusion is given.

2. Related works

2.1. Lucy-Richardson algorithm

The LR method has become popular in the fields of astronomy. After that, several researchers developed and modified the method that based on LR method to restore the degraded image in many fields such as remote sensing [6,7,8], medical [9,10], computer vision [4,11], etc.

2.2. Tikhonov regularization

In the optimization framework, the Tikhonov regularization is one of the most common ways to deal with ill-conditioned problems [12]. The general form of this technique can be expressed as

$$\min_x \psi(y, x) + \alpha \Phi(x) \quad (3)$$

where $\psi(x, y)$ is a function that measures how much a given candidate estimate x deviates from explaining the data y , $\Phi(x)$ is a regularization function [13] and α is the regularization parameter. Many researchers applied this regularization for deblurring the degraded image [14,15].

3. Proposed method

3.1. Modified Tikhonov regularization

The Tikhonov regularization in (3) is modified to control the iteration convergence which can be written as the following equation.

$$\min_f \psi(\hat{f}, g) + \alpha \Phi(\hat{f}, g) \quad (4)$$

The first term is employed to measure the similarity of the deblurred image compared with the original image in the degradation viewpoint. That is $\psi(g, \hat{f}) = \frac{1}{2} \|g - p \otimes \hat{f}\|_2^2$. The second term, we design it to measure the noise enhancement due to the restoration process [16]. The noise enhancement is inversely proportional to the Signal to Noise Ratio (SNR). If we let g be the signal and $\hat{f} - g$ be the noise, we can formulate the second term as $\Phi(g, \hat{f}) = \left\| \frac{\hat{f} - g}{g} \right\|_2^2$. Therefore, the modified Tikhonov regularization can be formulated as follows.

$$\min_f \frac{1}{2} \|g - p \otimes \hat{f}\|_2^2 + \alpha \left\| \frac{\hat{f} - g}{g} \right\|_2^2 \quad (5)$$

3.2. Image deblurring algorithm

In this section, the algorithm for image deblurring is described as followed.

- Step 1: Input the degraded image and the associated blur kernel.

- Step 2: Initialization

Set the value of the regularization parameter and initial deblurred image as the degraded image.

Calculate the initial error from optimization function (5) as $e^{(0)} = \frac{1}{2} \|g - p \otimes g\|_2^2$

- Step 3: Iteration

Update the deblurred image using the LR method (2) and calculate the error from optimization function (5) as

$$e^{(k)} = \frac{1}{2} \|g - p \otimes \hat{f}^{(k)}\|_2^2 + \alpha \left\| \frac{(\hat{f}^{(k)} - g)}{g} \right\|_2^2 \quad (6)$$

- Step 4: Optimization condition

The iteration process will be stopped when the error in (6) is more than the previous iteration.

- Step 5: Stop the iteration and output the resulting image which is the deblurred image before the last deblurred image.

For the best benefit, the degraded image and the deblurred image in (6) should be considered only the image area that has not the image boundary artifact.

4. Simulation results and discussions

In this section, the experiments are divided into 2 parts. The first part, the blur kernel used in the deblurring process is same as the blur kernel in the degradation process. The second part, the blur kernel used in the deblurring process will be deviated by the scaling factor to test the tolerance performance from non-exact blur kernel estimation. The regularization parameter was defined as 4 in all experiments.

4.1. Exact blur kernel

The original image will be convolved with several isotropic Gaussian blur kernel. After that, the same blur kernel will be employed in the deblurring process. The visual results will be demonstrated in Figure 1. The graph results of PSNR value versus the sigma of Gaussian blur (σ) when to compare with the default LR method and the regularized filter are shown in Figure 2. Due to the edge artifact of all method, the PSNR will be determined from the area that not including the image boundary artifact. We can see in Figure 1, the PSNR results of our proposed method are better than both the default LR method (iteration number is 10 for the default LR method) and regularized filter. The iteration number will be changeable although the regularization parameter is a fixed value. The visual results in Figure 2 show that the high detail in the deblurred image of our proposed method. However, the image boundary artifact is always presented.

4.2. Non-exact blur kernel

The blur kernel in the deblurring process will be deviated by the scaling factor before applying to each deblurring method. The comparison results of the PSNR value versus the scaling factor where the sigma of the blur kernel in the degradation process as 1.5 are demonstrated in Figure 3. We can see, our proposed method gives the PSNR value higher than others when the scaling factor is less than about 1.1. The maximum PSNR value is located on the scaling factor as 1. That is, the proposed method will provide the best efficiency if the estimated blur kernel is close to the exact blur kernel.

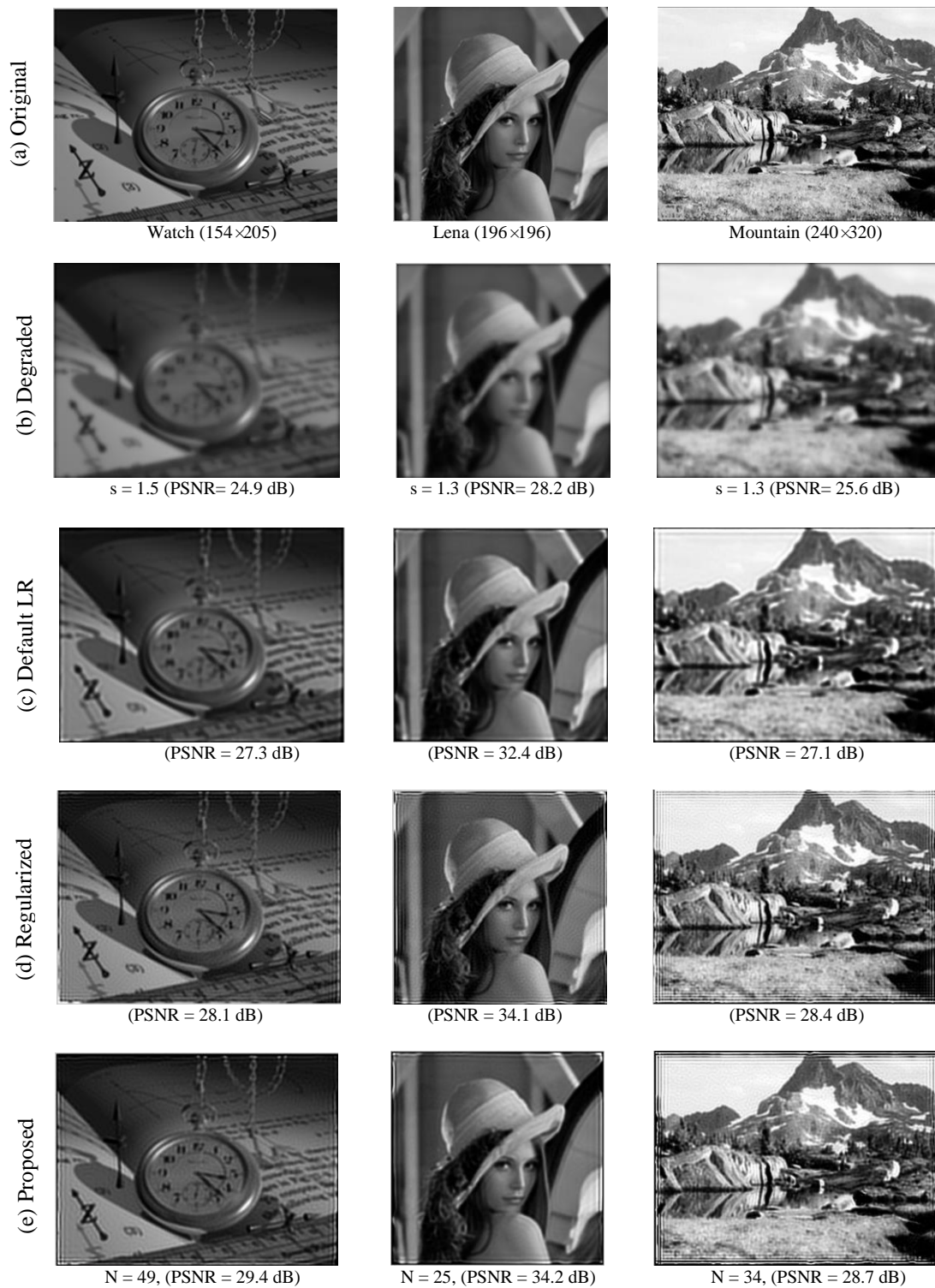


Figure 1. Visual results of the tested image when the sigma of Gaussian blur kernel is 1.3, (a) original image, (b) degraded image, (c) proposed method, (d) default LR algorithm and (e) regularized filter

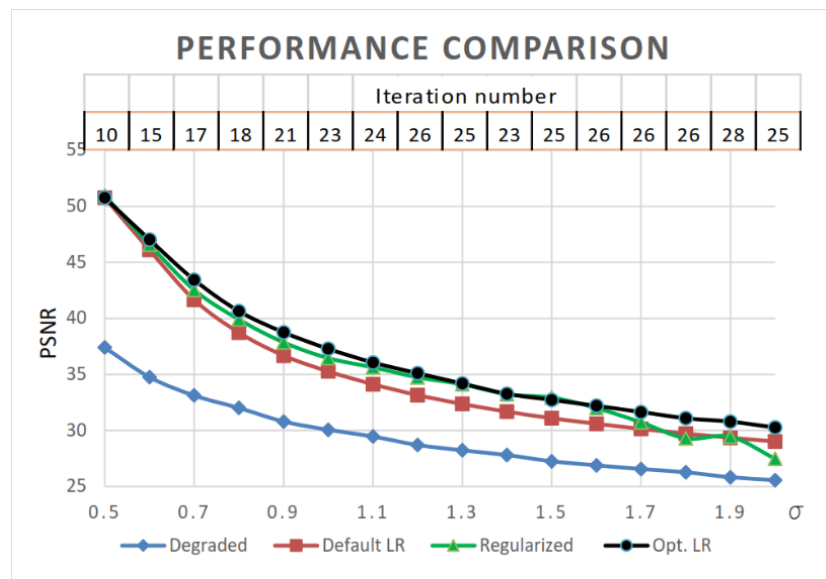


Figure 2. Comparison of the PSNR value versus the sigma of the Gaussian blur kernel for Lena tested image (196×196) with default LR algorithm, regularize filter and proposed method (Opt. LR)

However, the PSNR value of our proposed method is rapidly decreasing when the scaling factor is greater than 1.2, and its PSNR value is lower than the PSNR of the degraded image when the scaling factor is more than 1.3. From this experiment results, we can summarize as followed. Our method can be applied to restore both the degraded image in case of the soft blur and hard blur. The estimated blur kernel that uses in the deblurring process must be estimated close to the real blur kernel in the degraded process for the best of the deblurred image. If the estimated blur kernel is not exact, the size of the tuned blur kernel should be reduced.

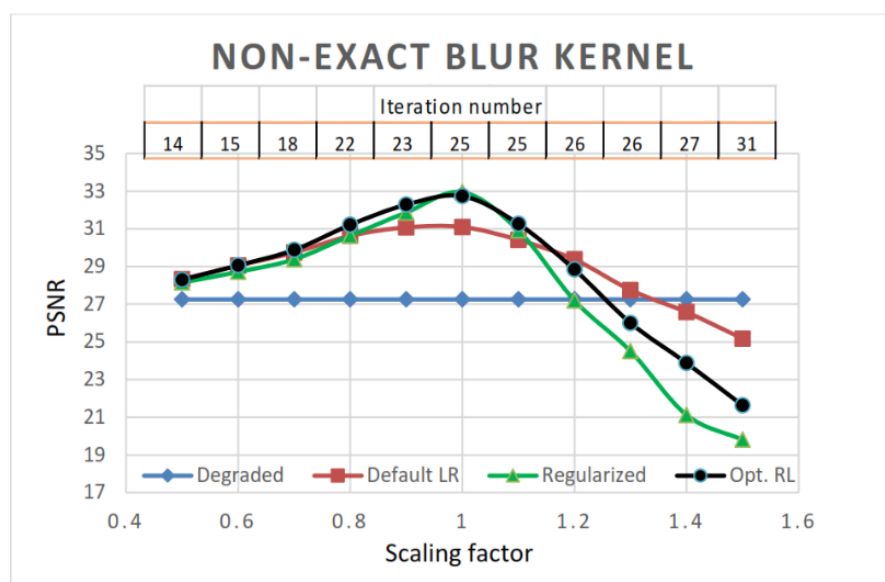


Figure 3. Comparison of the PSNR value versus the scaling factor for the Lena tested image (196×196) with default LR algorithm, regularize filter and proposed method (Opt. LR)

5. Conclusions

The Tikhonov regularization was modified to control the iteration number of the Lucy-Richardson method. This regularization is composed of 2 terms as the similarity in the degradation viewpoint and noise enhancement. The proposed method is performance tested by comparing with the default Lucy-Richardson method and the regularized filter in 2 cases as exact blur kernel and non-exact blur kernel. The results show that the proposed method can be given the PSNR value higher than others all sizes of blur kernel and can tolerate the deviated blur kernel especially it has smaller than exact blur kernel.

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