A Symmetric Successive Overrelaxation (SSOR) based Gauss-Seidel Massive MIMO Detection Algorithm

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Abstract. With an increasing number of antennas and users, the complexity increases dramatically, in order to simplify the process. In this paper, We propose existed algorithms discussing their principles and their advantages as well as the comparison between each method and then we provide the possible improvement in theory changing steps in iteration process to avoid the matrix inversion, by introducing a pre-conditioned gradient (PCG) method and further improved Jacobi method and using Neumann-series terms to estimate an approximate matrix inversion and the Gauss-Seidel (GS) method with a diagonal-approximate initial solution to the method and biconjugate gradient stabilized (BICGSTAB) method and above all of the methods are based on the minimum mean square error (MMSE) detector including the application in multi-user MIMO system. Another approach is from a signal detector based on symmetric successive over relaxation (SSOR) method without matrix inversion and Neumann series based on the zero forcing signal detector. The third algorithm uses a modified version of the conjugate gradient least square (CGLS). These methods are mainly replacing the inversion process into iteration process to reduce the complexity of the algorithm not only replacing the stages but optimizing the speed from setting the initial solution as well as the improvement using the Neumann-series to replace the iteration part. Finally, we proposed a symmetric successive overrelaxation (SSOR) based Gauss-Seidel for massive MIMO detection.

1. Introduction

In order to increase the quality of communication, MIMO aims at equipping with various of antennas on the two sides which can fully use the resources without adding the peripheral configuration. The advantages of the MIMO system are to increase the capacity of the channel and reliance of the channel. as the number of antennas used increases, the complexity of MIMO technology is greatly increased, which limits the number of antennas used and does not fully exploit the advantages of MIMO technology. At present, how to reduce the algorithm complexity and implementation complexity of MIMO technology on the basis of ensuring certain system performance has become a huge challenge facing the industry.

In this paper, we introduce several methods which is relate to improve the speed of the MIMO system and the corresponding efficiency. There are many methods introducing to improve the efficiency of the MIMO system. First, by using the incomplete Cholesky (IC) factorization pre-conditioner which can transform the system in a much more convenient way for increasing the convergence rate by setting the parameter η and setting parameters to zero is also for reducing the complexity which also varies with low complexity and especially , for configuration of 128*32 antenna [1]. Second, when the MIMO system can be solved in linear equations, which I provide the initial estimation using the iteration method to avoid the matrix inversion for a large-scale MIMO system. Based on the Gauss-Seidel iteration structure which increase the range of the reducing complexity compared with the Neumann series approximation, I have proposed the symmetric successive overrelaxation (SSOR) scheme, where different weights are set in the iteration structure which can accelerate the convergence rate which in general, utilizing the GS method for signal detection in uplink large-scale MIMO system [2]. Third, by proposing a low-complexity near-optimal signal detection algorithm avoiding the complicated matrix inversion including two steps. First, designing the algorithm based on MMSE detection algorithm and biconjugate gradient stabilized method (BICGSTAB) by proving the special property of the MMSE filtering matrix that is diagonally dominant [3] and then proving the convergence of the algorithm to ensure the feasibility in practice. Finally, by introducing VLSI architecture for the based soft-output data detection algorithm which is specific for the medium BS-to-user antenna ratio which is about reducing the 30% of the hardware complexity (in terms of the area-delay product). While also we provide the improvement performance of the existed algorithm which have divided into three parts regarding the size of the MIMO system, from the MMSE algorithm to SSOR algorithm and the CGLS algorithm which matches with the size of the MIMO system respectively and make the comparison with the different methods in many aspects [4]. Finally, we proposed a symmetric successive overrelaxation (SSOR) based Gauss-Seidel for massive MIMO detection.

The rest of the paper is organized as follows, Section II describes the system model. Section III specifies the different methods. The Gauss-Seidel based SSOR method is shown in Section IV. Finally, the summary of the whole paper is provided in Section V.

2. System Model

By considering the MIMO system in a base station with M antennae which serve N user terminals(usually we assume that M is much larger than N). we can express the symbol y$\in \mathbb{C}^{M}$ as :

$$Y= Hx+n \tag{1}$$

where H=$[h_1, h_2, h_3, \dots, h_N] \in \mathbb{C}^{M*N}$, x=$[x_1, x_2, x_3, \dots, x_N]^{\mathrm{T}}$ is the transmitted symbol vector. The MMSE equalizer was used in [4], which we calculate the $\hat{x}$ of the transmitted symbol as follows

$$\hat{x} = (H^H H + \sigma^2 I)^{-1} H^H y = A^{-1}\hat{y} \tag{2}$$

Which we derive the equations into

$$\hat{y} = H^H y \tag{3}$$

$$A=(H^H H + \sigma^2 I) \tag{4}$$

3. Introduction of Different Methods

3.1. A Proposed the PCG method

In this chapter, we will discuss the advantages of the PCG method compared with the traditional CG method while encountering the increasing number of user terminals. First, we conventional CG method, can be generated by

$$\hat{x}^{(i)} = \hat{x}^{(i-1)} \frac{(\hat{y}^{(i-1)} * \hat{y}^{(i-1)})}{(A * \hat{p}^{(i-1)} * \hat{p}^{(i-1)})} \times \hat{p}^{(i-1)} \tag{5}$$

$$\hat{y}^{(i)} = \hat{y}^{(i-1)} + \frac{(\hat{y}^{(i-1)} * \hat{y}^{(i-1)})}{(A * \hat{p}^{(i-1)} * \hat{p}^{(i-1)})} \times \mathrm{A} \times \hat{p}^{(i-1)} \tag{6}$$

$$\hat{P}^{(i)} = \hat{y}^{(i)} + \frac{(\hat{y}^{(i)} * \hat{y}^{(i)})}{(\hat{y}^{(i-1)} * y^{(i-1)})} \times \hat{p}^{(i-1)} \tag{7}$$

However, lack of robustness is considered as the disadvantages of iteration methods such as CG method. Therefore, I propose to set a pre conditioner under CG frame with a matrix M which can be written as

$$M^{-1}\mathrm{A}\hat{x} = M^{-1}\hat{y} \tag{8}$$

Therefore, by adding $z^{(j)} = (LL^H)^{-1}\widehat{y^{(J)}}$ into (8), each iteration steps which should easy to be solved and construct and apply. while the most popular pre-conditioned method is the incomplete Cholesky (IC) factorization method which construct the $L^H L$ to approach the matrix A in the system model and then we can generate as

$$(L^H\mathrm{L})^{-1}A \approx \mathrm{I} \tag{9}$$

where I is an identify matrix and the main idea of IC factorization is to construct a set U which include all the points which equal to zero while in the matrix A.

$$\mathrm{U}= \{(\mathrm{i},\mathrm{j})|A_{(i,j)} = 0\} \tag{10}$$

However , in real cases the off-diagonal points are not zero and one of the reasons are the ratio of M/N is affecting the result and naturally by introducing the ratio M/N into consideration which we derive the formula with a newly set parameter which

$$\mathrm{U}= \{(\mathrm{i},\mathrm{j})|A_{(i,j)} \leq \eta\} \tag{11}$$

where

$$\eta = \epsilon(1 - \mathrm{N/M})A_{(i,i)} \tag{12}$$

where ϵ is decided by the performance and complexity. For uplink large-scale MIMO system, the channel matrix H is column full-rank and column asymptotically orthogonal [1], which ensures that the MMSE filtering matrix A is Hermitian positive definite. The GS method can iteratively solve the system model with low complexity where we can divide the matrix A into three different parts.

3.2. Gauss-Seidel Method

For uplink large-scale MIMO system, the channel matrix H have two characters which are column full-rank and column asymptotically orthogonal [2], which ensures that the MMSE filtering matrix A is Hermitian positive definite. The GS method can iteratively solve the system model with low complexity where we can divide the matrix A into three different parts:

$$A = D + L + L^H \tag{13}$$

Thus we derive the transmitted signal vector s as follows:

$$s^{(i)} = (\mathrm{D} + \mathrm{L})^{-1}(\hat{y} - L^H s^{(i-1)}),\ \mathrm{i=1,2..} \tag{14}$$

And then due to the iteration steps, and we generally set the initial solution as the zero factor which is usually far away from the right solution. in other words , we can express the value of the initial solution as the important part in the whole process. Here, we provide a diagonal-approximate initial solution to the GS-based algorithm to achieve a better performance. For large-scale MIMO system, the channel matrix H is asymptotically orthogonal when M>>N and then we have $\frac{h_m^H h_k}{M}$is approaching 0 and m is not equal to k where m and k is ranging from 1 to k. where h_m denotes the mth column vector corresponding to the channel matrix H and generally we can observe that the domination of the inversion of matrix A is increasing with the ratio of M/N. which inspires us to use D^{-1} to approximate w^{-1} which then can be chosen as

$$s^{(0)} = D^{-1}\hat{y} \tag{15}$$

Since the expected error should be smaller and the initial solution would be closer compared with the conventional methods. The goal of achieving the faster convergence rate can be done.

3.3. SSOR Method

In this part, by introducing a symmetric successive overrelaxation (SSOR) based signal detector for massive MIMO system without computing the matrix inversion. And the first part of the algorithm is to divide the Hermitian positive matrix A as A=D+L+L^H which is identical to the Gauss-Seidel method and the next part is to compute the first half iteration which is identical to the successive overrelaxation iteration

$$(\mathrm{D}+\omega \mathrm{L})\hat{s}^{(i+1/2)}=(1-\omega)\mathrm{D}\hat{s}^{i}-\omega L^{(i)}+\omega\hat{y} \tag{16}$$

And the third step is adding the 1/2 order as the SOR iteration.

$$(\mathrm{D}+\omega \mathrm{L})\hat{s}^{(i+1)}=(1-\omega)\mathrm{D}\hat{s}^{i+1/2}-\omega L^{(i+1/2)}+\omega\hat{y} \tag{17}$$

The $\hat{s}^{(0)}$ is generally regarded as the zero vector, and the ω is the relaxation parameter, which is very important in the convergence rate. While ω is ranging from 0 to 2, SSOR-based signal detector is convergent at any initial conditions which can be regarded as the improved method which is symmetric which means we can accelerate the convergence rate and the another advantage is that in terms of SSOR method , where the convergence rate is not sensitive which means by using simple approximation would not influence the convergence rate sharply.

And the next part, we focus on the quantifying the relaxation parameter which used as

$$\omega^{opt}=\frac{2}{1+\sqrt{2(1-\rho B_J)}} \tag{18}$$

Where

$$B_J=D^{-1}\mathrm{AL} \tag{19}$$

And D is converging to a fixed value where

$$D^{-1} \approx \frac{1}{v}\mathrm{L} \tag{20}$$

A is a central Wischart matrix, when the ratio M/N is large and the largest eigenvalue can be approximated by

$$\lambda_1=\mathrm{N}(1+\sqrt{\frac{M}{N}})^2 \tag{21}$$

$$\omega^{opt}=\frac{2}{1+\sqrt{2(1-a)}} \tag{22}$$

$$\mathrm{a}=(1+\sqrt{\frac{M}{N}})^2-1 \tag{23}$$

which means that ω^{opt} is only decided by the ratio of M/N. Because ω is not very sensitive which means we can achieve a good performance.

3.4. Biconjugate gradient stabilized BICGSTAB Method

The unique property of the MMSSE filtering matrix shows the appropriateness of the algorithm because it is diagonally dominant with the decreasing number of N. the purpose of this method is to reach a balance between low computing complexity and good detecting accuracy.

First we choose an initial value of the s_0 and by setting the initial value of the equation and we can get the relevant parameters variant. [4]

$$r^0=\hat{y}-\mathrm{A}s^0 \tag{24}$$

Next step, we set the value is generated as:

$$\hat{r}=r^0 \tag{25}$$

Next step is to initial all the relevant variables which will be used in the following steps

$$p_0 = v_0=0 \tag{26}$$

$$\alpha = \rho_0 = \omega_0 = 1 \tag{27}$$

The final step is to iterate and we propose the i-times iteration

$$p_i=\widehat{r_0}r_{i-1} \tag{28}$$

$$\beta = (\frac{\rho_i}{\rho_{i-1}})(\frac{\alpha}{\omega_{i-1}}) \tag{29}$$

$$p_i = r_{i-1} + \beta(p_{i-1} - \omega_{i-1}v_{i-1}) \tag{30}$$

$$v_i=\mathrm{A}p_i \tag{31}$$

$$\alpha=\frac{p_i}{\widehat{r_0}r_{i-1}} \tag{32}$$

$$\mathrm{h}=s_{i-1}+ \alpha p_i \tag{33}$$

$$\mathrm{x}=r_{i-1} - \alpha v_i \tag{34}$$

$$\mathrm{t}=\mathrm{Ax} \tag{35}$$

$$\omega_i=\frac{tx}{tt} \tag{36}$$

$$s_i=\mathrm{h}+\omega_i x \tag{37}$$

We repeat the process until it satisfies the accuracy of the requirement, the advantages of the algorithm is that most of the cases, it can achieve the faster and smoother convergence. This algorithm is designed for the large number of base stations compared with the rather small number of users.

3.5. Jacobi Method

The linear detection in massive MIMO system can be solved in linear equations without the matrix inversion and the process contain three main process and first is to set to the initial value and second is to follow the iterative structure and thus once it get the satisfied simulation result . However, it requires a lot of iteration steps and the previous iterative method has been given two important aspects needed to be considered the initial estimation and the convergence rate. The Jacobi method is proposed as follows:

While the system model is provided $M^{-1}\mathrm{A}\hat{x} = M^{-1}\hat{y}$

While we have

$$\mathrm{A}\hat{x}=\mathrm{y} \tag{38}$$

And we can use the iterative methods to solve this linear problems by using the Jacobi method [5]

$$\widehat{x^{k+1}}=D^{-1}((\mathrm{D}\text{-}\mathrm{A})\,\widehat{x^k}+\mathrm{y}) \tag{39}$$

where D is the diagonal matrix of A and the limit of the method is approaching zero while we also set the initial value to

$$x^{(0)}=D^{-1}\mathrm{y} \tag{40}$$

Except 0 which is much more accurate compared with the previous methods. Finally, we have the expression generated as:

$$x^{(k+1)}=\left(\textstyle\sum_0^{k+1}(-D^{-1}(W - D))^l D^{-1}\right)\mathrm{y} \tag{41}$$

To discuss the Jacobi method in detail where we have the two main steps which first is to consider the matrix vector product and at the beginning of this paper where we have shown the system model which we can find that matrix A is known as the many parts which has already been calculated in total and the next step is to set the initial value which by using the method shown above we can have a more precise result compared with the initial value of zero.

And the result of the method is by replacing the matrix inversion with the vector product, we achieve to degrade the performance loss in the process .

4. SSOR method based on the Gauss-Seidel initialization

A symmetric successive overrelaxation (SSOR) based signal detector for massive MIMO system is a very effective way of iteration without the matrix inversion but it is likely to suffer from the inaccuracy of the initialization because normally we have set the initial value to zero which do not always work. Therefore, based on the Gauss-Seidel method by setting the initial value which approaches the exact value and the first half iteration is computed by me which is identical to the successive overrelaxation iteration

$$s^{(0)} = D^{-1}\hat{y} \tag{42}$$

$$(\mathrm{D}+\omega\mathrm{L})\hat{s}^{(i+1/2)}=(1-\omega)\mathrm{D}\hat{s}^{i}-\omega L^{(i)}+\omega\hat{y} \tag{43}$$

$$(\mathrm{D}+\omega\mathrm{L})\hat{s}^{(i+1)}=(1-\omega)\mathrm{D}\hat{s}^{i+1/2}-\omega L^{(i+1/2)}+\omega\hat{y} \tag{44}$$

Which SSOR method has been discussed before, we can obtain the final formula which is

$$\omega^{opt}=\frac{2}{1+\sqrt{2(1-\rho B_J)}} \tag{45}$$

$$\mathrm{a}=(1+\sqrt{\frac{M}{N}})^2-1 \tag{46}$$

Because ω is not very sensitive, and we also set the accurate initial value which can achieve a better performance.

5. Conclusion

In this paper, we have discussed the general system model which has been widely used in the MIMO system and we have discussed five methods which all focus on improving the convergence rate. By introducing the iterative methods we also have focused on the reduction of the complexity of the algorithm in the simulation process.

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