Massive MIMO Detection Algorithms Based on MMSE-SIC, ZF-MIC, Neumann Series Expansion, Gauss-Seidel, and Jacobi Method

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Abstract. The massive multiple-input multiple-output (MIMO) systems is an important technology in the fifth generation of mobile communication. To get the result of a MIMO system require some algorithm to approximate the precise result as the computation complexity is too large. There are several methods that have been advanced in the fitting, like zero forcing (ZF) method or minimum mean-square error (MMSE) method. However, in massive MIMO system, these methods require further simplify because of the increasing complexity of matrix inversion. In the papers, many methods were presented to get the approximation matrix: like MMSE-SIC, ZF-MIC, Gauss-Seidel, Jacobi and Neumann series expansion. The Jacobi's iterative method and Newton's iterative method both use iteration to approach the MMSE estimation. Their BER performance can outperform current methods and require less computational complexity. Almost every method can get a nearly ideal fit when the number of users or antennas is large enough. But when the number is small, the approximation will not be very accurate.

1. Introduction

The MIMO technology can multiply the wireless communication system's several times without increasing the bandwidth and emitting frequency. The Massive MIMO system is a key technology in realizing the fifth generation of mobile communication and is now being researched and studied worldwide because of its potential in improving the speed of transmitting data and also the stability of it. The benefits of using the massive MIMO system is profitable both economically and environmentally. However, due to the greatly increase of the number of antennas at the BS (base station), the system model may need some change. Now, research of how to apply the traditional MIMO's detection method to the new massive MIMO system is a focus.

Many adapted methods are presented by researchers worldwide. In paper [1], the writers used MMSE-SIC and ZF-SIC algorithm, and they found out that these two methods' results are very close to the ones from ZF and MMSE method. And they further conclude that even the basic MRC and ZF method can provide good result if the antennas at the base stations greatly outnumber the users. Paper [2] give a method based on relaxation iterative, it's better than the methods using polynomial expansion like Neumann series expansion because it can reach more optimal result with same degree of computational complexity. Besides, the method is also better than the Richardson method which is also based on iteration. Paper [3] use Gauss-Seidel model, which is also an iterative method, the result is similar like the relaxation iterative method. Paper [4] use Jacobi iterative method. To get the MMSE estimation, in spite that the Jacobi method use iteration to simplify the matrix inversion, the result is equivalent to that using the Neumann Series Expansion method. Compared with the Gauss-Seidel method, this method has slow convergence rate. However, by providing the initial estimation which is close to the exact MMSE solution, we can speed up the convergence. In paper [5], the authors use Newton's iterative method to simulate and find approximate matrix inverse. The inverse they get has low latency. Besides, they find out that by making an approximate matrix inverse, the complexity can be saved twofold because of iterations. For the result part, the SD scheme they applied outperforms the conventional method while giving the same error performance. Paper [6] show us a method based on Neumann series expansion, the author admit that only with the diagonally dominant condition, the NS-based matrix inversion approximation can achieve satisfying accuracy with quick convergence. Or else, it can't guarantee quick convergence.

By comparing all the methods above, we can find that most method use iteration to get the inverse matrix, and its performance is the best with a rather low complexity and quick convergence. There are several iterative methods, each have its specialty. Other method, such as MMSE-SIC and ZF-SIC, can also reach optimal result under some conditions; but the polynomial expansion method is not that good compare with the iterative method.

2. Iterative method

For this part, we will mainly focus on some methods which primarily deal with the matrix multiplication and initialization for iterative methods. Here we will take Jacobi method [4] and Gauss-Seidel [3] method as two examples. For the Jacobi model, we consider a system of B base station antennas serving K users, and we assume $\mathrm{B} >> \mathrm{K}$ for massive MIMO system.

We denote the base station's received signal as: $\mathrm{Y} = \mathrm{Hx} + \mathrm{z}$, here$\mathrm{H} = [\mathrm{h}_0, \mathrm{h}_1, \cdots, \mathrm{h}_{\mathrm{K}-1}]$ is the channel matrix which denotes the channel response between the k-th user and the base station. The noise vector: $\mathrm{Z} = [\mathrm{z}_0, \mathrm{z}_1, \cdots, \mathrm{z}_{\mathrm{B}-1}]^{\mathrm{T}}$. The typical MMSE estimation can be expressed as $\hat{\mathrm{x}} = (\mathrm{H}^{\mathrm{H}}\mathrm{H} + \sigma_{\mathrm{Z}}^2\mathrm{I}_{\mathrm{K}})^{-1}\mathrm{H}^{\mathrm{H}}\mathrm{y} = \mathrm{W}^{-1}\mathrm{y}^{\mathrm{MF}}$, in which $\mathrm{y}^{\mathrm{MF}} = \mathrm{H}^{\mathrm{H}}\mathrm{y}$ is the matched filter output. For $\mathrm{W} = \mathrm{G} + \sigma_{\mathrm{Z}}^2\mathrm{I}_{\mathrm{K}}$,$\mathrm{G} = \mathrm{H}^{\mathrm{H}}\mathrm{H}$ denotes the Gram matrix. For simplicity, we can rewrite $\hat{\mathrm{x}} = (\mathrm{H}^{\mathrm{H}}\mathrm{H} + \sigma_{\mathrm{Z}}^2\mathrm{I}_{\mathrm{K}})^{-1}\mathrm{H}^{\mathrm{H}}\mathrm{y} = \mathrm{W}^{-1}\mathrm{y}^{\mathrm{MF}}$ as $\mathrm{W}\hat{\mathrm{x}} = \mathrm{y}^{\mathrm{MF}}$. Then we can use the Jacobi iteration method:

$$\hat{\mathrm{x}}^{(\mathrm{i}+1)} = \mathrm{D}^{-1}((\mathrm{D} - \mathrm{W})\hat{\mathrm{x}}^{(\mathrm{i})} + \mathrm{y}^{\mathrm{MF}}) \tag{1}$$

In the equation above, $\mathrm{D} = \mathrm{diag}\{\mathrm{W}\}$. To converge, the equation must satisfy:

$$\lim_{\mathrm{i}\to\infty}(\mathrm{I}_{\mathrm{k}} - \mathrm{D}^{-1}\mathrm{W})^{\mathrm{i}} = 0. \tag{2}$$

In addition, if the initial estimation is given as: $\mathrm{X}^{(0)} = \mathrm{D}^{-1}\mathrm{y}^{\mathrm{MF}}$, than we have:

$$\mathrm{x}^{(\mathrm{i}+1)} = \left(\sum_{\mathrm{l}=0}^{\mathrm{i}+1}\left(-\mathrm{D}^{-1}(\mathrm{W} - \mathrm{D})^{\mathrm{l}}\mathrm{D}^{-1}\right)\right)\mathrm{y}^{\mathrm{MF}} \tag{3}$$

From $\hat{\mathrm{x}}^{(\mathrm{i}+1)} = \mathrm{D}^{-1}((\mathrm{D} - \mathrm{W})\hat{\mathrm{x}}^{(\mathrm{i})} + \mathrm{y}^{\mathrm{MF}})$ and $\mathrm{W} = \mathrm{H}^{\mathrm{H}}\mathrm{H} + \sigma_{\mathrm{Z}}^2\mathrm{I}_{\mathrm{K}}$ we can get

$$\mathrm{WX}^{(\mathrm{i})} = \mathrm{H}^{\mathrm{H}}\mathrm{Hx}^{(\mathrm{i})} + \sigma_{\mathrm{Z}}^2\mathrm{x}^{(\mathrm{i})} \tag{4}$$

And we can alternatively express the iteration as: $\hat{\mathrm{x}}^{(\mathrm{i}+1)} = \hat{\mathrm{x}}^{(\mathrm{i})} + \mathrm{D}^{-1}(\mathrm{y}^{\mathrm{MF}} - \mathrm{H}^{\mathrm{H}}\mathrm{Hx}^{(\mathrm{i})} + \sigma_{\mathrm{Z}}^2\mathrm{x}^{(\mathrm{i})})$.

If $\mathrm{X}^{(0)} = \mathrm{D}^{-1}\mathrm{y}^{\mathrm{MF}}$, the initial estimation will be $\mathrm{x}^{(1)} = \mathrm{x}^{(0)} + \mu\mathrm{r}^{(0)}$. Where $\mu = \frac{\left(\mathrm{r}^{(0)}\right)^{\mathrm{H}}\mathrm{r}^{(0)}}{(\mathrm{Wr}^{(0)})^{\mathrm{H}}\mathrm{r}^{(0)}}$ and $\mathrm{r}^{(0)} = \mathrm{y}^{\mathrm{MF}} - \mathrm{Wx}^{(0)}$. Then use the Jacobi iteration, we get:

$$\begin{aligned}\mathrm{x}^{(2)} &= \mathrm{x}^{(1)} + \mathrm{D}^{-1}\left(\mathrm{y}^{\mathrm{MF}} - \mathrm{Wx}^{(1)}\right) \\ &= \mathrm{x}^{(0)} + \mu\mathrm{r}^{(0)} + \mathrm{D}^{-1}\left(\mathrm{r}^{(0)} - \mu\mathrm{Wr}^{(0)}\right)\end{aligned} \tag{5}$$

The computation complexity can be reduced from $O(B \times K^2)$ to $O(B \times K)$ in this method. The computational complexity including the initialization computation is far less than the existing method with the increase of user's number. Besides, for the BER performance when B=128 and K=16 (Figure

1), the proposed approximation leads to closer result of the exact MMSE estimation than existing work. So if a better estimation is used, the number of iterations for Jacobi method is reduced.

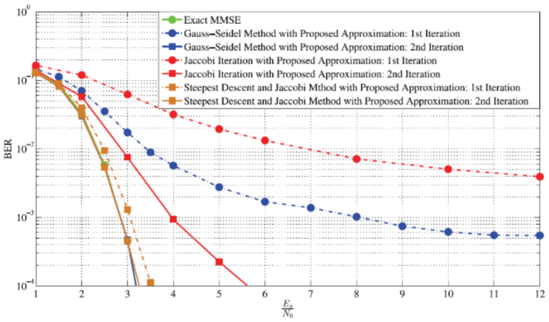

Figure 1. BER performance of Jacobi method [4]

That's a brief description of the Jacobi's method; we will next introduce a similar method, the Gauss-Seidel method [5]. The $N \times 1$ receive signal can be represent as: $Y = Hx + z$, where z represent the noise vector. By the MMSE method, we can get the transmitted signal vector x as: $\hat{x} = (H^H H + \sigma_Z^2 I_K)^{-1} H^H y = W^{-1}\hat{y}$. Where $\hat{y} = H^H y$ and the filtering matrix is W can be represented as $W = G + \sigma_Z^2 I_U$. In the expression of W, we have the gram matrix as $G = H^H H$. Let $E = W^{-1}G$ represent the equivalent channel matrix, and we can get $U = W^{-1}H^H(W^{-1}H^H)^H = W^{-1}GW^{-1}$. Analyzing $Y = Hx + z$ and $\hat{x} = (H^H H + \sigma_Z^2 I_K)^{-1} H^H y = W^{-1}\hat{y}$together, we can represent the MMSE approximated signal vector x as $\hat{x} = Ex + W^{-1}H^H n$. So for the kth user, the approximated transmitting signal is $\hat{x}_k = \mu_k s_k + v_k$. Besides, the channel gain is $\mu_k = E_{kk}$, and the NPI variance is $v_k^2 = \sum_{m \neq k}^{K} |E_{mk}|^2 + U_{kk}\sigma^2$, where E_{mk} and U_{mk} symbol for matrix element in the mth row and kth column.

However, since K is always a relatively large number in the uplink large-scale MIMO systems, the computational complexity for matrix inversion, which is $O(K^3)$ will be large, so we need further operation. W, the MMSE filtering matrix, is Hermitian positive definite, so W can be decomposed as $W = D + L + L^H$. In the expression above, D represents the diagonal component of W. L represents the strictly lower triangular component of W. And L^H represents the strictly upper triangular component of W. Then by applying the GS method, the transmitted signal vector can be written as:

$$x^{(i)} = (D + L)^{-1}\left(\bar{y} - L^H x^{(i-1)}\right), i = 1,2, \ldots \tag{6}$$

Where i is the number of iterations. We then use Diagonal-Approximate method to get the initial solution. Also, with this method, we can achieve a faster convergence rate of GS-method based algorithm.

For uplink large-scale MIMO systems, the channel matrix H will be close to orthogonal when $N >> K$. Therefore we can get: $\frac{h_m^H h_k}{N} \to 0, m \neq k,\ \ m, k = 1,2, \ldots, K$, in which h_m represents the mth column of H. The matrix W^{-1} is also diagonally dominant, so when the ratio of N/K increases, the non-diagonal matrix W^{-1} and the diagonal one D^{-1} become increasingly similar. We can set the initial solution of $x^{(i)} = (D + L)^{-1}\left(\bar{y} - L^H x^{(i-1)}\right), i = 1,2, \ldots$ as:

$$x^{(0)} = D^{-1}\hat{y}. \tag{7}$$

By this estimation, we are able to get a faster convergence rate. By comparing, we find that for any number of iterations, the proposed GS-based algorithm can simplify the computational complexity from the level of $O(K^3)$ to $O(K^2)$. There is a series of conclusion we can get from simulation. First, if the number of iterations is relatively large (i>=3), the performance is satisfying. Besides, the convergence rate can be accelerated by using the approximation. Figure 2 is the BER performance comparison between the proposed GS-based algorithm and the conventional Neumann Series-based algorithm, When B is128, and K is 16. As the Figure 2 shows, the Gauss-Seidel based algorithm is clearly better if the number of iterations are the same.

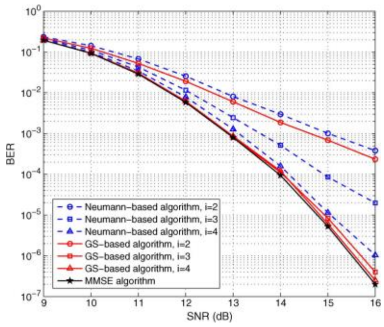

Figure 2. BER performance of GS method [3]

In addition, when considering about the BER performance relationship with the number of antennas at base station (B) with users K = 16 is considered. The proposed GS-based method is near optimal compared with the ML algorithm if B>>K, while the Neumann-based algorithm suffers non-negligible performance loss.

By comparing the two similar methods, we find that both the two methods use diagonal matrix D of the MMSE filtering matrix W to simplify the computation complexity. The main difference is that GS method further simplify W as D because W is diagonal dominant if N is relatively large, and the paper prove that this replacement has little effect to the result. And the Jacobi method use D in iteration part to simplify the computation work, and it is equivalent to Neumann series expansion. Both algorithms can reach a better result than existing work. They are more close to the exact MMSE BER performance. The GS method can reduce the computational complexity from $O(K^3)$ to $O(K^2)$, and the Jacobi method can reduce the complexity from $O(B \times K^2)$ to $O(B \times K)$. From Figure 1 and Figure 2, we can clearly see that the steepest descent and Jacobi method with proposed approximation: 2nd iteration and Gauss-Seidel method with proposed approximation: 2nd iteration can get near optimal result as exact MMSE get.

3. Decomposing method

The Neumann series method [6] is based on the Zero-Forcing method. However, in massive MIMO systems, to support a large number of users, the Zero-Forcing method requires much larger dimensions of matrix inversion. Hence the Neumann Series was applied to the approximation of matrix inversion. It's more suitable to massive MIMO system and it also has unique advantages in hardware implementation. Zero-Forcing's precoding and detection is able to reach wonderful performance very close to both the downlink's and the uplink's channel capacity. So for massive MIMO systems, the ZF method has always been regarded as one of most the practical method. We take M for the number of antennas, and B for the number of users. In the article, the authors stressed to solve three main problems in the NS method for computing the matrix inversion. First, they gave a M/K ratio requirement that gives high convergence rate. Besides, because the K × K matrix should be diagonally dominant, they derived another high probability M/K ratio. Finally, they give the specific approximation error analysis for the Neumann Series based matrix inversion approximation in hardware implementation.

Like the analysis above, we consider a massive MIMO system with M antennas serving K users at the base station. Then, the M × K uplink channel matrix is represented by $\mathrm{H} = [\mathrm{h}_{\mathrm{mk}}]$, and h_{mk} represents coefficient in the mth row and the kth column, where the range is $\mathrm{m} = 1, \ldots, \mathrm{M}$, and $\mathrm{k} = 1, \ldots, \mathrm{K}$. The channel matrix H is also the same for downlink as the uplink, due to the channel reciprocity under the situation we considered. For the ZF method, we also need to calculate the pseudo-inverse of matrix H:

$$\mathrm{H}^{\dagger} = (\mathrm{H}^{\mathrm{H}}\mathrm{H})^{-1}(\mathrm{H}^{\mathrm{H}}) \tag{8}$$

If we take $G = H^H H$, in massive MIMO systems, the major computational complexity lies in the matrix inversion of G, which is a the $K \times K$ matrix. Though K is smaller than M, it is also many times larger than the number of users in conventional MIMO systems. So, it is too costly to compute G^{-1} . The NS method is used to realize the matrix inversion approximation:

$$G_N^{-1} \approx \sum_{n=0}^{N-1}(I_K - \Theta G)^n \Theta \tag{9}$$

N denotes the total terms' number in Neumann series, and Θ represent a K × K diagonal matrix. $\lim_{n\to\infty}(I_K - \Theta G)^n \to 0_K$ has to be satisfied in order to let the formula above work,. The matrix G is a complex central Wishart matrix. We take $\alpha = M/K$, we can get that when K and M grows, the largest eigenvalue of G converge to:

$$\lambda_{max}(G) \to M(1 + \frac{1}{\sqrt{a}})^2 \tag{10}$$

And correspondingly, for the smallest eigenvalue of G:

$$\lambda_{min}(G) \to M(1 - \frac{1}{\sqrt{a}})^2 \tag{11}$$

Therefore, if we choose Θ as:

$$\Theta = \frac{\alpha}{M\,(1+\alpha)} I_K = \frac{1}{M+K} I_K \tag{12}$$

We can get:

$$\lambda_{max}(\Theta G) \to 1 + \frac{2\sqrt{a}}{1+\sqrt{a}} \tag{13}$$

$$\lambda_{min}(\Theta G) \to 1 - \frac{2\sqrt{a}}{1+\sqrt{a}} \tag{14}$$

when $a \geq 1, \frac{2\sqrt{a}}{1+\sqrt{a}} \leq 1$, which means $\lim_{n\to\infty}(I_K - \Theta G)^n \to 0_K$ can surely be satisfied.

Figure 3 shows the performance comparison when M=128 and different K.

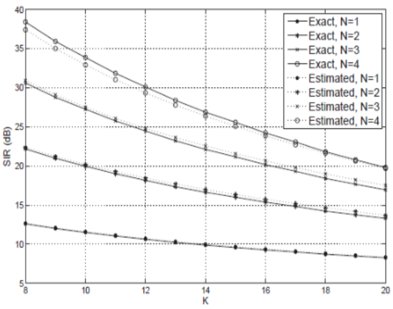

Figure 3. BER performance of NS method [6]

Usually, in massive MIMO systems, a is very large, like $a > 10$ can satisfied the convergence condition. So, the convergence of NS-based matrix inversion approximation can be ensured. Therefore for massive MIMO systems, the NS-method is still practical. Besides by increasing the value of N, its accuracy can be improved. Although Neumann-Series is usable in matrix inversion approximation, there are many constaints and condition for the method. In other words, there are clearly some shortcomings in this method, we'll discuss them later.

There are some other decomposing method than the NS method, such as MMSE-MIC and ZF-MIC. The model is the same as above. First, by analyzing the model of the system, the author proved that we can neglect the small signal decay and only need to consider the large-scale decay. The base station receive M × 1 vector,$y = \sqrt{p_n}Gx + n$, where p_n is every user's average transmitting power. X symbols for a vector of K users' transmitting signals at the same time. And n symbols for the white Gaussian noise. We have $n \in CN(0, I_M)$, and G is the $M \times N$ channel matrix vector: $G = HD^{1/2}$. We use g_{mk} to represent the element of G, which means the transmitting coefficient between the kth user and the mth antenna. H is a $M \times N$ matrix, h_{mk} stands for the Small-scale fading coefficient. D is an $K \times K$ diagonal matrix, the element in it is b_k and $\sqrt{b_k}$ is the Large-scale fading coefficient. Their relationship is: $g_{mk} = h_{mk}\sqrt{\beta_k}$. When K>>M, we have: $\frac{G^H G}{M} = D^{1/2}\frac{H^H H}{M}D^{1/2} \approx D$. in this case, we can ignore the Small-scale fading coefficient, only consider the Large-scale fading coefficient.

Linear algorithm cannot reach test accuracy requirement in traditional MIMO system, but for large scale MIMO system, the algorithm like MMSE and ZF can improve their performance with the increase in antennas. For non-linear iteration algorithms, like MMSE-MIC and ZF-MIC, are also suitable. The author gives the computation expression of MRC, ZF, MMSE, ZF-MIC and MMSE-MIC. For a M ◊K linear detecting matrix A, the recipient signal is:

$$r = A^H y = \sqrt{p_u} A^H G x + A^H n \tag{15}$$

Let r_k and x_k be the kth element of the vector r and x:

$$\begin{aligned} r &= \sqrt{p_u} A^H G x + A^H n \\ &= \sqrt{p_u} a_k^H g_k x_k + \sqrt{p_u} \textstyle\sum_{i=1, i \neq k}^{k} a_k^H g_i x_i + a_k^H n \end{aligned} \tag{16}$$

In the equation above, the first term represent the expected signal, and the second one represent the interfere with other users, the last term represent noise.

For the traditional linear signal detection methods like MRC, ZF, and MMSE, the linear detecting matrix are:

(1) MRC: $A = G$

(2) ZF: $A = G\left(G^H G\right)^{-1}$

(3) MMSE: $A = G\left(G^H G + (1/p_M) I_k\right)^{-1}$

For non-linear iteration algorithms, we denote W as the linear weight matrix:

(1) MMSE: $W = A^H = (G^H G + \frac{1}{p_u} I)^{-1} G^H$

(2) ZF: $W = A^H = G$

Denote w_k as the kth vector of W, the noise-signal ratio, then:

(1) MMSE: $SNR_k = \frac{p_u |w_i g_i|^2}{p_u \sum_{i \neq k} |w_i g_i| + \sigma_k^2 ||w_k||^2} \sim \frac{1}{||w_k||^2}$

(2) ZF: $SNR_k = \frac{p_u}{\sigma_k^2 ||w_k||^2} \sim \frac{1}{||w_k||^2}$

The BER performance when the number of antennas M is a variable and the number of users K=10 is shown in Figure 4. We can see that in large scale MIMO system, there are no great difference between ZF, MMSE, ZF-SIC, MMSE-MIC. These four algorithms can have its error rate lower than 10^{-4} at M>60. Also, the MRC can reach it when M> 200. Besides, if the number of antennas is far great than users, then MMSE-SIC and ZF-SIC methods don't outperformance the traditional MMSE and ZF method. But when the number of antennas and users are close, the MMSE-SIC and ZF-SIC methods can have much development.

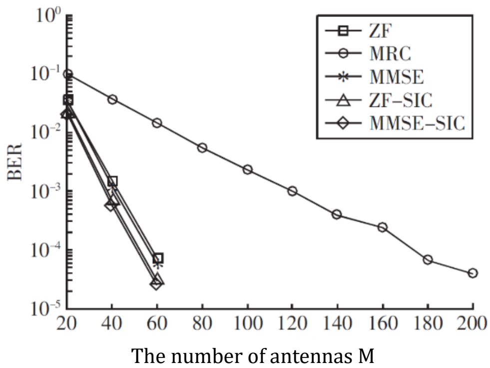

Figure 4. BER performance of MMSE-SIC and ZF-SIC [5]

The two methods discussed in the decompose method part both have some obvious drawbacks. The NS method requires many conditions to make sure it can converge quickly. If the requirements are not met, it may not convergent at all, which means the method is infeasible. As for the MMSE-SIC and ZF-MIC methods, they didn't show many advantages in massive MIMO systems comparing with the traditional MMSE and ZF methods. Compared with the iterative methods which can both reduce the computational complexity and reach better performance, no wonder more scholars favor the iterative methods and the papers of iterative method is far more.

4. Conclusion

In conclusion, both the iterative methods and the decomposing methods can simplify the progress of matrix inversion approximation. However, by comparison, we can acknowledge that the iterative methods generally outperform the decomposing methods. They can be applied to more situations because they suffer less restraints. Besides, the simulation results of the iterative methods are better than the ones of the decomposing method under same level of computational complexity. Hence, the researchers around the globe mainly focus on developing the iterative methods.

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