Application of Mesh-Less Algorithms in Image Processing

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Abstract. Mesh-less method is a discrete governing equation which does not need to generate grids in numerical calculation and constructs interpolation functions according to some arbitrarily distributed coordinate points. This method can easily simulate flow fields of various complex shapes. In this paper, mesh-less method is applied to phase unwrapping in image processing of InSAR. This method can forbid error accumulate to the whole image in the sense of least squares. The precision of this method are can showed by real data.

1. Introduction

Mesh-less method [1-2] is a numerical calculating method that developed in recent year and has become research hotspot in many fields. In this thesis, we employ the mesh-less method to deal with phase unwrapping in the least-squares sense. Interferometric synthetic aperture radar (InSAR) [3]can get the generation of digital elevation model (DEM). The height is related to the absolute phase that can be get from two registered SAR(synthetic aperture radar) pictures of the same scene. However, measured phase has been restricted in $[-\pi,\pi]$ because it is calculated through an inverse trigonometric function (arctan or arccos). To obtain the unwrapped phase from wrapped phase, phase unwrapping process will be introduced for InSAR.

Two basic approaches have been introduced to deal with phase unwrapping. The first method is local approach, which connect the residues and select the integration path, then avoid crossing turbulent regions to forbid error accumulate to whole image. The another method is a overall method, these methods is on a minimum norm theory [4-6], This method obtains the optimal solution by minimizing the cost function. The approach is so robust and direct, but is not to deal with the error that created by residues directly, which lead to the unwrapped phase inaccurate comparatively and computational inefficiency.

2. Mesh-less method

Mesh-less method is a numerical calculation method that the approximate solution is not rely on grid but only need to know the node information. The problem field can be indicated by arbitrary node distribution. In problem field, field variable of arbitrary point can be expressed approximately by the point in local support field. The generation of the mesh-less basis function is a key step in mesh-less method, which can adapt the approximate value of arbitrary field node in field function. Point interpolation method (PIM) is one method that can create mesh-less basis function, assume one of the calculation point $U(x,y)$ in problem domain is:

$$U(x,y)=\sum_{i=1}^{m} p_i(x,y)a_i=\{p_1(x,y)p_2(x,y)\cdots p_m(x,y)\}\begin{Bmatrix} a_1 \\ a_2 \\ \cdots \\ a_n \end{Bmatrix}=Qa \tag{1}$$

Wherein Q denotes basis function; a denotes the coefficient of the $p_i(x,y)$, m denotes the basis function' number, $a=\{a_1a_2a_3\cdots a_m\}^T$ is unknown coefficient vector. In order to obtain the coefficient a, calculation point x need to be generated in local support domain, which have included n field node defined by Eq.(2):

$$\left.\begin{aligned} u_1 &= \sum_{i=1}^{m} a_i p(x_1,y_1)=a_1+a_2x_1+\ldots+a_m p_m(x_1,y_1) \\ u_2 &= \sum_{i=1}^{m} a_i p(x_2,y_2)=a_1+a_2x_2+\ldots+a_m p_m(x_2,y_2) \\ &\cdots \\ u_n &= \sum_{i=1}^{m} a_i p(x_i,y_i)=a_1+a_2x_i+\ldots+a_m p_m(x_i,y_i) \end{aligned}\right\} \tag{2}$$

Eq.(2) can be simplified to the following matrix form in Eq.(3):

$$\text{Us=Wa} \tag{3}$$

Wherein Us= is the node function vector.

$$W=\begin{bmatrix} 1 & x_1 & y_1 & x_1y_1 & \cdots & p_m(x_1,y_1) \\ 1 & x_2 & y_2 & x_2y_2 & \cdots & p_m(x_2,y_2) \\ 1 & x_3 & y_3 & x_3y_3 & \cdots & p_m(x_3,y_3) \\ 1 & x_4 & y_4 & x_4y_4 & \cdots & p_m(x_4,y_4) \\ & \cdots & & & & \\ 1 & x_n & y_n & x_ny_n & \cdots & p_m(x_n,y_n) \end{bmatrix} \tag{4}$$

Eq.(3) can be obtained and Eq.(5) will be obtained:

$$a=W^{-1}U_s \tag{5}$$

Take the Eq.(5) substituting into Eq.(1):

$$U(x,y)=QW^{-1}U_s=\sum_{i=1}^{m}\phi_i u_i=P(x,y)U_s \tag{6}$$

Wherein p(x,y) in Eq.(7) denotes shape function vector:

$$q(x,y)=PW^{-1}=\{q_1(x,y) \quad q_2(x,y) \quad \cdots \quad q_m(x,y)\} \tag{7}$$

3. Image processing by mesh-less method

To solve the phase unwrapping problem of image processing by mesh-less method, a cost function in the sense of least squares can be constructed first. The wrapped phase has been restricted between $[-\pi, \pi]$ in the interferogram S as shown in the Figure. 1.

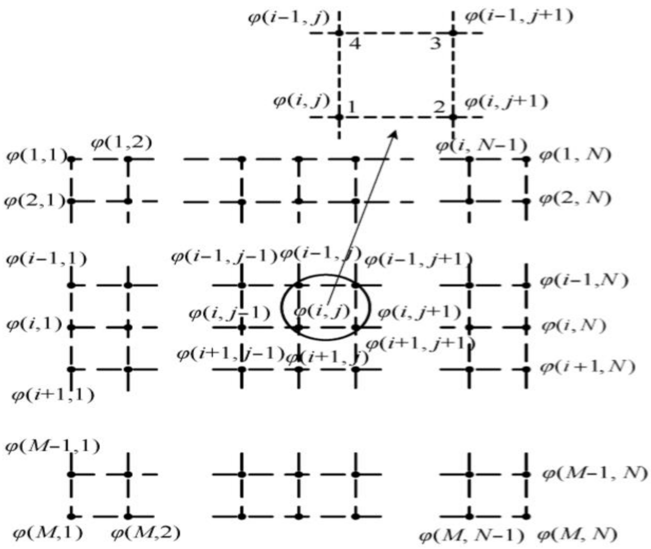

Figure 1. Element and node classification of interferogram S

$$\phi(i,j) = \psi(i,j) + n(i,j) \cdot 2\pi, (i,j) \in S \quad (i = 0,1,\cdots,M-1; j = 0,1,\cdots,N-1) \tag{8}$$

Wherein $\phi(i,j)$ denotes unwrapped phase, $\psi(i,j)$ denotes wrapped phase, the integer number $n(i,j)$ of 2π phase must be added or subtracted to each phase value that can realize phase unwrapping.
The wrapped phase $\psi(i,j)$ is be given, and it can get the unwrapped phase $\phi(i,j)$ at the same grid locations, the $\phi(i,j)$ need to base the phase gradient of the $\psi(i,j)$ in the least-squares sense. The difference of the wrapped phase $\nabla\psi$ and the unwrapped phase $\nabla\phi$ are obtained by Eq. (9):

$$L(\phi) = \|\nabla\phi - \nabla\psi\|^2 = \iint_s \nabla\phi \cdot \nabla\phi dxdy + \iint_s \nabla\psi \cdot \nabla\psi dxdy + 2\iint_s \nabla\phi \cdot \nabla\psi dxdy \tag{9}$$

Wherein the x and y superscripts refer to the column and row of the pixels. As the $\psi(i,j)$ and the $\iint_s \nabla\psi\nabla\psi dxdy$ at the right-hand side of Eq.(9) have been known, so the Eq.(9) can obtain the minimum by minimizing $\iint_s \nabla\phi\nabla\phi dxdy - 2\iint_s \nabla\phi\nabla\psi dxdy$.

We can rewrite the Eq.(2) as follows:

$$L(\phi) = \iint_s \nabla\phi \cdot \nabla\phi dxdy - 2\iint_s \nabla\phi \cdot \nabla\psi dxdy \tag{10}$$

To obtain the phase value of the pixel (i, j) ,its neighbors $(i-1, j)$, $(i, j+1)$ and $(i-1, j+1)$ can be divided into the local support domain as shown in Figure1, For ease to calculate, pixel (i, j) and its neighbors $(i-1, j)$, $(i, j+1)$ and $(i-1, j+1)$ is similarly identified with a couple of integers(node index):1,2,3,4. Then Eq.(11) can be obtained based on Eq.(6):

$$\phi^e(x,y) = P^e(x,y) \cdot \phi^e \tag{11}$$

$$\nabla \varphi_x^e(x, y) = P^e(x, y) \cdot \nabla \varphi_x^e \tag{12}$$

$$\nabla \varphi_y^e(x, y) = P^e(x, y) \cdot \nabla \varphi_y^e \tag{13}$$

Wherein

$$\phi^e = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}, \nabla\varphi_x^e = \begin{pmatrix} \nabla\varphi_{x1} \\ \nabla\varphi_{x2} \\ \nabla\varphi_{x3} \\ \nabla\varphi_{x4} \end{pmatrix}, \quad \nabla\varphi_y^e = \begin{pmatrix} \nabla\varphi_{y1} \\ \nabla\varphi_{y2} \\ \nabla\varphi_{y3} \\ \nabla\varphi_{y4} \end{pmatrix} \tag{14}$$

To implement Eq.(10) by using mesh-less method, we must divide the whole domain into elements. Take $L^e(\phi^e)$ as the sub-functional in this local support domain, Eq.(15) can be obtained as follows:

$$L^e(\phi^e) = \phi^e\left(\iint_{S_e} \nabla p^e \bullet \nabla p^e dxdy\right)\phi^e + 2\phi^e\left(\iint_{S_e} p^e \bullet \nabla p^e dxdy\right)_x \nabla\varphi_x^e + 2\phi^e\left(\iint_{S_e} p^e \bullet \nabla p dxdy\right)_y \nabla\varphi_y^e \tag{15}$$

To minimize the cost function, the first derivatives of the ϕ^e must yield zero in $\frac{\partial L^e}{\partial \phi^e} = 0$:

$$\frac{\partial L^e}{\partial \phi^e} = \left[\frac{\partial L^e}{\partial \phi_1^e}, \frac{\partial L^e}{\partial \phi_2^e}, \frac{\partial L^e}{\partial \phi_3^e}, \frac{\partial L^e}{\partial \phi_4^e}\right] \tag{16}$$

Wherein $\varphi^e = [\varphi_1^e \varphi_2^e \varphi_3^e \varphi_4^e]$ denotes column vector that need to unwrap in sequence in local support domain. Eq.(16) can be expressed by mathematical derivation:

$$\begin{vmatrix} 4 & -1 & -2 & -1 \\ -1 & 4 & -1 & -2 \\ -2 & -1 & 4 & -1 \\ -1 & -2 & -1 & 4 \end{vmatrix} \bullet \begin{bmatrix} \phi_1^e & \phi_2^e & \phi_3^e & \phi_4^e \end{bmatrix}^{\mathrm{T}} = \begin{vmatrix} 2 & 1 & 2 & -1 \\ -2 & -1 & 1 & 2 \\ -1 & -2 & -1 & -2 \\ 1 & 2 & -2 & -1 \end{vmatrix} \begin{bmatrix} \varphi_{x1} & \varphi_{x3} & \varphi_{y2} & \varphi_{y4} \end{bmatrix} \tag{17}$$

Eq. (17) can be simplified to the following matrix form in Eq. (18):

$$K\phi^e = b^e \tag{18}$$

Wherein K denotes coefficients of the matrix; ϕ_{x1} and ϕ_{x3} is the gradient of pixel 1 and 3 in x irection respectively; ϕ_{y2} and ϕ_{y4} is the gradient of pixel 2 and 4 in y direction. To unwrap the phase of each pixel in the whole problem domain, the whole functional $L(\phi\ )$ need to be expressed by the sum of the sub-functional:

$$L(\phi) = \sum_{e=1}^{(N-1)(M-1)} L^e(\phi^e) \tag{19}$$

Wherein (N-1) and (M-1) are number in the problem domain. The first Green's identity will be applied to the Eq.(19), then Eq.(20) and Eq.(21) can be obtained:

$$\frac{\partial L}{\partial \phi} = \sum_{e=1}^{N} \frac{\partial L^e}{\partial \phi^e} = \sum_{e=1}^{N} (K^e \phi^e - b^e) = 0 \tag{20}$$

$$K\phi = b \tag{21}$$

Wherein b is known term and K is a block tri-diagonal matrix. then ϕ can be obtained via the iterative solution.

4. Experimental Results

The mesh-less method was used on a real wrapped phase picture that obtain from an X-band SAR Pictures. Figure 2 is the wrapped phase, Figure.3 is the quality map.

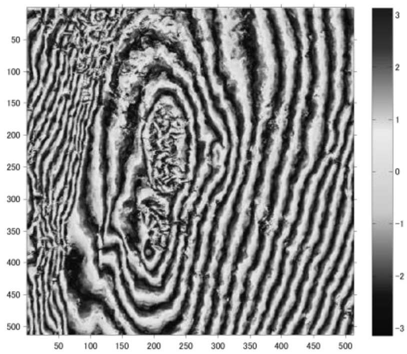

Figure 2. The wrapped phase

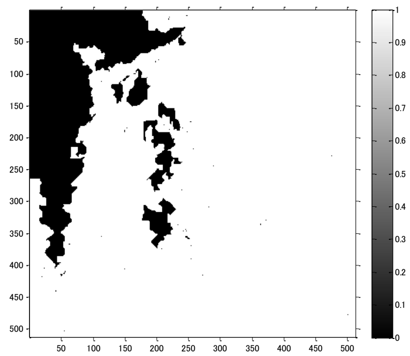

Figure 3. Quality map

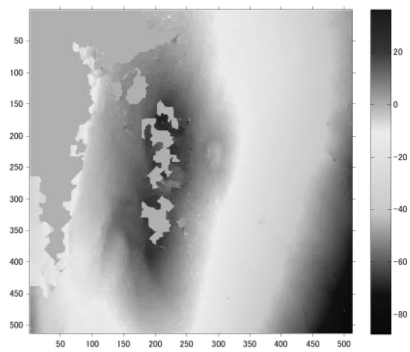

Figure 4. Unwrapped phase by branch-cut

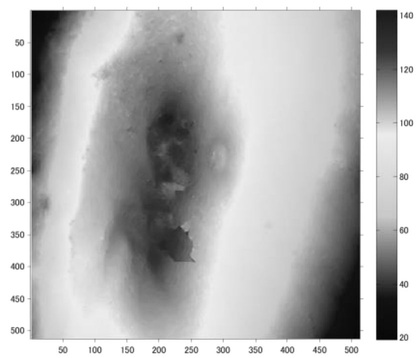

Figure 5. Unwrapped phase obtained by mesh-less

Figure.4 is unwrapped phase used by branch-cut, Figure.5 is unwrapped phase used by the mesh-less method. The mesh-less method has estimated the unwrapped surface solution in a very efficient way such that its rewrapped result best matches the wrapped phase.

5. Conclusion

Mesh-less method has been introduced to phase unwrapping of InSAR in this paper. It is so convenient and efficient implementation of the standard least-squares method, the method can avoid the error propagate to the whole image by dividing the local support domain, and can obtain the better phase unwrapping result than least-square method. Presented experiments on real Mt. Etna datademonstrate the precision and efficiency of the procedure.

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