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The Fretting Contact of Coated Bodies. Part I – Contact Parameters

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Abstract. In case of a mismatch between the elastic properties of the contacting bodies, the solution of the fretting contact can only be achieved numerically due to the connection between the contact pressure and the shear tractions. The numerical treatment of the contact processes employs an iterative strategy, and therefore requires the computation of the displacement response of the elastic body, assumed as an elastic half-space, subjected to general loadings. Unlike the case of homogeneous bodies with known Green's functions, a closed-form expression of the response of layered semi-infinite solids to unit point loads has only been attained in the frequency domain. The latter formulas are used in this paper to assess the displacement due to arbitrary normal and shear surface tractions, thus empowering the application of a trial-and-error approach in finding the parameters of a fretting contact: the contact area, the slip and the stick regions, and the distributions of pressure and shear tractions. The contact parameters are assessed in a nested loop strategy, involving three levels of iterations. The inner level, based on the conjugate gradient method for linear systems of equations, finds the pressure when solving the contact problem along the normal direction, disconnected from any tangential effects, or the shear tractions when considering the contact equations in the tangential direction, disconnected from the pressure influence. The middle level stabilizes the pressure with respect to the shear tractions, thus assuring that the global instantaneous contact solution is achieved. The outer level manages the reproduction of the loading history in a fretting loop, by load incrementation. The proposed strategy proves itself as a robust tool for the prediction of the fretting contact process involving coatings.

1. Introduction

Modern engineering applications, including bearings, gears, dental crowns, hip prostheses, hard disks or electronic parts, involve functionally graded materials and/or coatings that prolong the service life of critical components by preventing surface damage. In many contact scenarios, the service conditions involve oscillatory relative movement of small amplitudes, also known as fretting. The contact stresses developed in the layered body under contact load are of chief importance for predicting the service life of the contacting element. Giving the complexity of the fretting phenomenon, closed-form solutions are available for very selective cases, and numerical techniques such as the finite element analysis (FEM) or the semi-analytical methods (SAM) present themselves as

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attractive alternatives for calculating the fretting contact stresses. Whereas the advantage of FEM resides in its generality, SAM brings excellent computational efficiency, allowing [1] the calculation of a 3D contact scenario with the computational resources needed for a 2D FEM simulation.

Gallego, Nélias and Jacq [2] first used a fully grown SAM capable of repeated resolution of the contact problem in the normal direction, while accounting for the change in conformity due to wear. The model was later refined by Gallego and Nélias [3], who advanced a numerical model for the fretting wear either under partial or gross slip conditions, assisted by efficient three-dimensional algorithms for both the normal and the tangential contact problem. Chen and Wang [4] presented a three-dimensional numerical model for the simulation of the contacts of elastically dissimilar materials based on the Boussinesq–Cerruti integral equations. An elastic contact model for three-dimensional layered or coated materials under coupled normal and tangential loads, with consideration of partial slip effects, was further developed by these authors [5]. Gallego, Nélias and Deyber [6] advanced a general algorithm for fretting contact problems with application to fretting modes I, II and II, in which the coupling between the normal and the tangential contact problems can or not be taken into account. A numerical analysis of the partial slip contact under a tangential force and a twisting moment was also performed [7].

The load path dependence is a distinctive feature of the fretting contact, which was addressed in more detail by Spinu and Glovnea [8], and by Spinu and Frunza [9], who studied the hysteretic behaviour of partial slip elastic contacts undergoing a fretting loop.

The fretting contact of layered materials is less covered in the literature giving the additional difficulty in expressing the elastic response of the contacting bodies. In this paper, a technique for the computation of displacement and stresses in bilayered half-spaces is combined with an iterative solver for the fretting contact. The resulting numerical analysis technique is both fast and accurate, and can be used with various contact configurations. The relation between the contact processes in the normal and in the tangential directions, often neglected, are accounted for in this contact study.

2. Contact model review and solution

In the absence of friction, a point contact under normal load has a contact area given by the Hertz theory. The mutual contact pressure established between the contacting surfaces yields normal compression but also shear displacements. However, if the contact interface can sustain friction, the shape and size of the contact area are affected only if the elastic constant of the contacting materials are different. The latter case is assumed in the present work, focused on the indentation of a coated flat by a rigid sphere.

The different displacements of dissimilarly elastic materials, induced in each body by the equal (in magnitude) but opposite (in direction) shear tractions, lead to a relative peripheral velocity at the interface, i.e. local or micro-slip, if not opposed by friction. Practically, the contact area of the three-dimensional point contact under normal load consists in a central region of stick enveloped by slip zones towards the edge of the contact. On the stick region the surfaces adhere in the tangential direction, meaning that mating points on the two surfaces stick together after the deformation, and therefore have vanishing relative displacement. As opposed to the slip peripheral zone, where mating points undergo different tangential displacements and consequently slips occurs. As the contact area grows with the normal load increase, points initially laying outside the adhesion region will progressively be enveloped in the stick region.

A subsequently applied tangential force may generate a global sliding motion, if greater than the force of limiting friction (i.e. that for the case when sliding is about to occur), or a tendency to slide. In the latter case, although there is no nominal relative velocity between the bulks of the bodies, local slip occurs at specific regions of the interface. The tangential tractions induced by the tangential force affect the size and shape of the contact area, and also the pressure distribution, only if the two solids have different elastic constants. Otherwise, for similarly elastic materials, given that the normal displacements induced by the shear tractions will be equal, the contacting surfaces will warp conformingly. Consequently, the pressure distribution and the contact area will be given by the initial contact geometry, and will be independent of the tangential force.

Considering the aforementioned interdependencies, it can be concluded that the tangential tractions interact with pressure when the solids have different elastic properties. The practice of contact problems involving homogenous solids usually assumes that pressure and tangential tractions are independent of each other, as the influence of the latter on pressure is small, especially for low frictional regimes. For the contact of coated materials, the aforementioned assumption may be too strong, and therefore it is discarded in the present study. The interdependence between the normal and the tangential tractions is overcome by numerical analysis, in an iterative approach whose convergence is dictated by the stabilization of pressure with respect to the shear tractions.

One distinctive feature of the contact process is that, although the contact is assumed quasi-static and in the frame of linear elasticity, due to the irreversibility of friction, the final stress state will depend upon the loading history rather than solely on the final magnitudes of the normal and tangential loads. Consequently, the surface tractions can only be found by following the complete loading history in incremental steps.

Given the aforementioned contact process features, the model discretization has to be made in both spatial and temporal dimensions (although the latter does not imply that the time parameter is explicitly needed, as long as the loading history is reproduced). Two or three indexes, respectively, may be needed to substitute the continuous coordinates, allowing for positioning on the surface (e.g., i, j in equations (1)-(5)) or in the volume computational domain, respectively, whereas an additional index (e.g., k in equations (1)-(5)) may be needed for the sequence of the loads application. In order to speed up the computation of convolutions resulting from the superposition of effects, techniques derived from the digital signal processing are applied, which require a uniform rectangular mesh and piecewise constant distributions. This state-of-the-art approach is sustained by the analytical solutions existed in the literature for the stress or displacement effect induced in a half-space by a pressure or shear stress excitation uniformly distributed on a rectangular patch located on the half-space boundary. Given the aforementioned solutions, an additional assumption is needed, commonly used in contact mechanics, i.e. the approximation of solids of general profiles with half-spaces. This assumption is reasonable in case of concentrated contacts, when the contact stresses are confined to a limited body domain, and not influenced by the proximity of its boundary. The latter assumption warrants the calculation of displacements using superposition of fundamental solutions derived for the elastic half-space, and also forces contact pressure to act perpendicular to the interface regardless of the deformation it produces. Moreover, the strains developed in the contact process are assumed sufficiently small to fit the framework of the linear theory of elasticity.

The contact model [10,11] for the dry, frictional contact, under normal and tangential load, the latter generating a partial slip regime, is restated here for clarity and completeness. The equations are reported to a Cartezian coordinate system with the origin in the initial point of contact, and with the x_1-axis conveniently aligned with the direction of the tangential force, without losing generality. The contacting bodies are compressed by a steady normal load W, giving birth to a contact area $A(k)$, while an oscillating tangential force $\mathbf{T}(T_1, T_2 = 0)$ is subsequently applied, having an amplitude insufficient to cause the sliding of the two surfaces. The static force equilibrium equates the applied normal and tangential loads to the contact pressure p and the shear tractions $\mathbf{q}(q_1, q_2)$, respectively:

$$W(k) = \Delta \sum_{(i,j)\in A(k)} p(i,j,k); \quad T_n(k) = \Delta \sum_{(i,j)\in A(k)} q_n(i,j,k), n = 1,2, \tag{1}$$

where Δ is the elementary cell area. On the computational domain P, the displacement equation in the normal direction results from comparison of contact geometry before and after the deformation, whereas that in the tangential direction from consideration of subsequent load increments:

$$u_3(i,j,k) = h(i,j,k) - hi(i,j) + \delta_3(k), (i,j) \in P\,; \tag{2}$$

$$\begin{bmatrix} u_1(i,j,k)-u_1(i,j,k-1) \\ u_2(i,j,k)-u_2(i,j,k-1) \end{bmatrix} = \begin{bmatrix} s_1(i,j,k)-s_1(i,j,k-1) \\ s_2(i,j,k)-s_2(i,j,k-1) \end{bmatrix} + \begin{bmatrix} \delta_1(k)-\delta_1(k-1) \\ \delta_2(k)-\delta_2(k-1) \end{bmatrix}, (i,j)\in A(k). \quad (3)$$

In equation (2), h is the separation between the two surfaces, whereas hi denotes the sum of initial (i.e., in unloaded state) digitized body profiles, u_3 the composite (i.e., relative) displacement in the normal direction, and δ_3 the rigid-body approach, measured as the approach of body points distant to the contact zone, measured along the normal direction. Equation (3) relates the increments of the tangential displacements u_i with the those of the tangential rigid displacements δ_i and with those of slip s_i, $i=1,2$. As δ_i are position-independent, it follows from the latter equation that surface points with vanishing slip (i.e., in stick) undergo the same tangential displacement.

The contact problem solution is difficult to achieve because: (1) the division of the computational domain into the contact and non-contact zones is not known in advance, such as that of the contact area into slip and stick regions, and (2) these separations keep changing during the loading history due to the interaction between pressure and the tangential traction. In the numerical model, these divisions are expressed by the boundary complementarity conditions, and are found by trial-and-error.

The boundary conditions express the complementarity between the contact and the non-contact regions, as well as between the slips and stick zones:

$$\begin{cases} p(i,j,k)>0 \quad \text{and} \quad h(i,j,k)=0, \quad (i,j)\in A(k); \\ p(i,j,k)=0 \quad \text{and} \quad h(i,j,k)>0, \quad (i,j)\in P-A(k); \end{cases} \quad (4)$$

$$\begin{cases} \|\mathbf{q}(i,j,k)\| \le \mu p(i,j,k) \quad \text{and} \quad \|\mathbf{s}(i,j,k)-\mathbf{s}(i,j,k-1)\|=0, (i,j)\in S(k); \\ \|\mathbf{q}(i,j,k)\| = \mu p(i,j,k) \quad \text{and} \quad \|\mathbf{s}(i,j,k)-\mathbf{s}(i,j,k-1)\|>0, (i,j)\in A(k)-S(k). \end{cases} \quad (5)$$

Equation (4) implies that tensile normal tractions cannot be sustained, which is reasonable for contact process involving metallic solids with inherent microtopography that inhibit the contribution of the adhesion force in the normal direction. The relationship between pressure and the tangential tractions is assumed according to the Amontons'a law of sliding friction, as shown in equation (5). The frictional coefficient μ, determined by the materials properties and by the physical state of the interface, is assumed uniform over the whole contact region and constant during the loading history, although this is not a limitation imposed by the numerical scheme, which can handle mapped friction data with no additional algorithm modifications.

To solve the contact model, it is required to find the discrete distributions of surface tractions transmitted between the two solids at their surface of contact. To this end, the contact model (1)-(5) is first divided into two submodels for the normal and the tangential direction. Given the fact that calculation of displacement in any direction require knowledge of all contact tractions, the submodels will not be independent. However, an algorithm for the solution of each model with known, yet arbitrary output from the other, is readily available. The scheme was originally proposed for the frictionless contact or rough bodies by Polonsky and Keer [11], and was subsequently adapted [12] to the submodel in the tangential direction. An outer loop [8] is required to stabilize the inputs of the two submodels, i.e. the tangential tractions with respect to the contact pressure. The latter algorithm strategy applies in full to the contact of coated bodies under the assumption that the elastic displacements are accurately computed. An algorithm for the computation of the elastic response of coated bodies to general loading is detailed in the companion paper. From the point of view of the latter contact solver strategy, it is important to note that the method to solve each of the two submodels neither requires nor calculates the rigid-body displacements δ_i, $i=1,2$. In other words, only the relative values of displacements are needed for the contact process solution, which warrants a simplified approach in displacement calculation, as detailed in the companion paper.

3. Results and discussions

For benchmarking purposes, two simpler scenarios with well-known solutions were replicated with the newly proposed computer program. Firstly, the pressure profiles in the spherical indentation of a coated flat by a rigid indenter were obtained for the frictionless contact. The Young modulus of the coating E_1 was varied with respect to that of the substrate, E_2, which was kept constant. The Hertz contact parameters a_H and p_H, calculated with the substrate elastic parameters, were used as normalizers for the radial coordinate and the contact pressure, respectively. The coating thickness was fixed and equal to the Hertz radius. The semi-elliptical Hertz pressure profile was also added for reference. These results depicted in figure 1 match well the pressure distributions obtained by O'Sullivan and King [13] using a different method, and validate the contact scheme for the normal contact problem, as well as the procedure for the calculation of normal displacements in coated bodies.

Secondly, simulation of a fretting loop for homogenous materials allows validation of the contact solvers in the normal and in the tangential directions. The reference closed-form relations were deduced in [10], based on the solutions of the partial slip contact obtained independently by Cattaneo [14] and by Mindlin [15]. The corresponding curves are displayed with continuous grey lines in figure 2, which shows the distributions of shear tractions at specific moments from the loading history, in a fretting loop of amplitude $T_{\lim} = 0.9\mu W$, in the time interval τ between the first and the third null of the tangential force. In this case, the pressure is independent of the shear tractions and of the loading history, and obeys the Hertz theory.

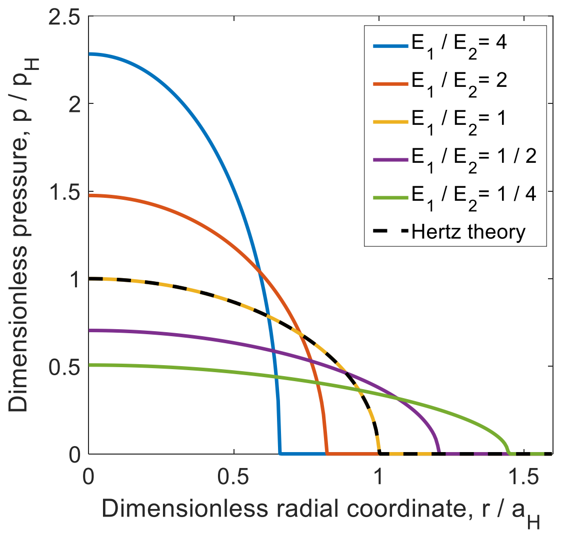

Figure 1. Influence of elastic moduli ratio on the pressure profiles.

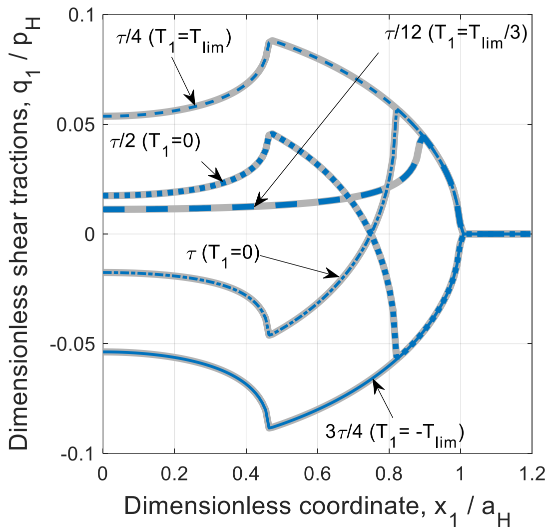

Figure 2. Shear tractions q_1/p_H in a fretting loop, $\mu = 0.1$.

The simulation of a fretting loop of amplitude $T_{\lim} = 0.75\mu W$, with $\mu = 0.1$, is performed for different elastic moduli ratios between the coating and the substrate. The loading history is reproduced with 400 loading increments, from which the first 100 are reserved for the initial normal indentation. In this stage, although no tangential force is applied, self-equilibrating shear tractions arise due to the mismatch in the elastic properties between the contacting bodies. The indenter is assumed rigid, while the ratio of the elastic moduli ratio between the coating and the substrate is varied. Whereas the contact pressure is not greatly affected, the distributions of shear tractions vary significantly from the elastically similar case presented in figure 2. More so, the maximum tangential force that the contact can withstand before going into full sliding is significantly less than the theoretical one [14,15], $T_{\lim} = \mu W$. The predicted profiles of dimensionless shear tractions are depicted in figure 3.

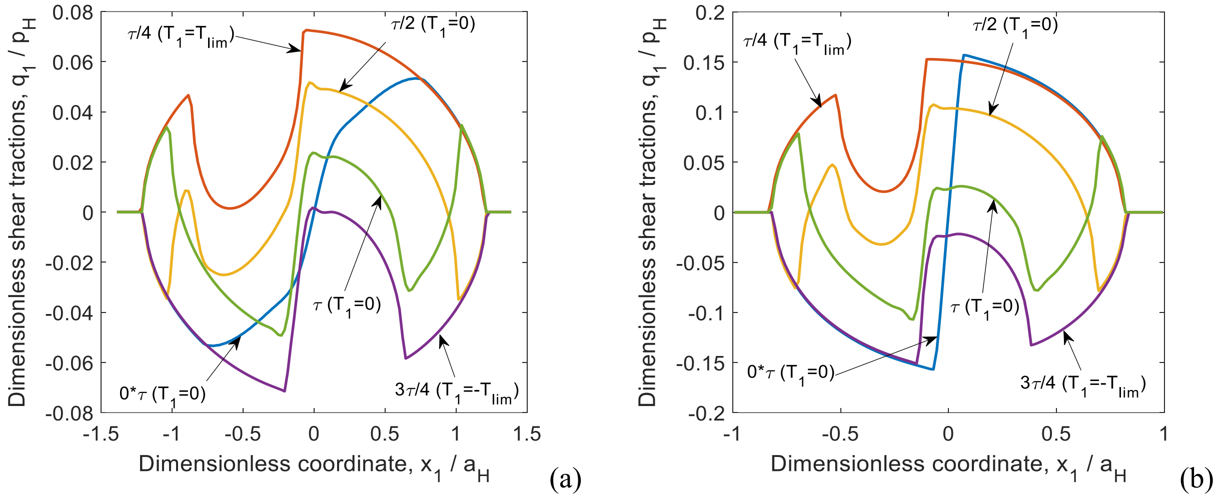

Figure 3. Shear tractions in the indentation of a coated half-space: (a) $E_1/E_2 = 1/2$; (b) $E_1/E_2 = 2$.

4. Conclusions

The lack of analytical solutions for the model of the fretting contact encourage the use of numerical methods that employ trial-and-error strategies to derive the contact area and the slip-stick boundary, as well as their evolution with the loading history. The accurate reproduction of the latter is an important prerequisite for the precise estimation of the contact normal and shear tractions.

A fretting contact solver originally designed for elastic homogenous bodies was successfully adapted to the contact of layered bodies with the assistance of a numerical procedure for the calculation of displacement in multi-layered mediums. Only the relative displacement fields are required, which avoids the evaluation of the frequency response functions in the origin of the frequency domain.

The three-level iterative strategy converged in all situations involving various elastic moduli ratios between the coating and the substrate. Well known literature results were faithfully reproduced, giving confidence in the method ability to advance the understanding of the fretting contact of coated bodies. The knowledge of contact tractions will allow the calculation of stress fields developing in the layered mediums.

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