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Tooth contact analysis of spur and helical gears with crowned profile for pinion

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Abstract. The paper presents tooth contact analysis (TCA) of modified gears using semi-analytical methods (SAMs). The semi-analytical methods use Love's analytical formulation for Boussinesq equation of linear half-space theory and Hartnett approach for numerical formulation. The linear contact between two teeth has a finite length and during meshing process the edge effect appears, with a harmful influence on the gear reliability. To eliminate this, some tooth profile modifications are needed. To turn a line contact into a point contact, the crowned lead profile surface is considered. The ways used to obtain the matrices of separation are discussed for both spur and helical cylindrical gears. The large pressures' systems of equations were solved using the method of conjugate gradients and fast Fourier transform. For validation, in cases of hertzian geometries the SAM results have been compared with results obtained analytically, while for non-hertzian geometries, the SAM results have been compared with FEM results as presented in the specific literature. The developed SAM proved to be an accurate, robust and very fast tool for TCA of spur and helical cylindrical gear.

1. Introduction

The mechanics of concentrated contact is a fundamental aspect regarding gear design. Some aspects with a profound impact on gear reliability are derived from the postulates of concentrated contact, Johnson [1]. There exist various factors that affect gear reliability. Some authors, Qin and Guan [2] tried to emphasize how contact fatigue can be predicted. An unwanted influence upon gear reliability is the edge effect that manifests as a high gradient increase of contact pressures at the edge zones, Cretu [3]. In order to provide accurate values of the contact pressures. Pedrero et al. [4] proposed a combination between the Hertz equation and the non-uniform model of load distribution along the contact line. Casanova, [5], established that there are interdependences between the values of the face load factor and the gear width, the position of the gear on the shaft, the shaft's length and the ratio between the pitch radius and the shaft radius.

It has been found that the usage of SAM for tooth contact analysis is getting attention among recent researches. This happens because the SAM computing time is at least two orders of magnitude shorter than the necessary time when FEM is used, Guilbault and Najjari [6,7]. Casanova [8] presented various algorithms and computerized ways dedicated to perform tooth contact analysis. The results obtained with SAM were compared with FEA results or with analytical results, provided by AGMA [9]. In Pop et al.[10] it is exemplified a spur cylindrical gear case where contact pressures calculation was accomplished using semi-analytical methods (SAM). Proper lead modifications able to change the pure linear contact into a point contact are discussed and exemplified. In Litvin and Fuentes [11] and Feng, [12] different ways of determination of instantaneous contact ellipse for crown profiled teeth were proposed. Another study with regard TCA of spur gears using SAM or FEM is presented in Ye, Tsai [13], for different gear types, with different geometry modifications. In the current paper, the tooth flank geometry was modelled using the methodology presented in Litvin and Fuentes, [14]. The usage of SAM

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is also presented in Cretu et al. [15], taking into account different profile modifications for spur gear pairs. In this paper the needed modifications of the flank longitudinal profile have been done according to ISO 6336 [16].

2. Tooth contact analysis (TCA) using semi – analytical methods (TCA)

2.1. The non – Hertz method

When two bodies are in a line contact and a normal force is applied, these two bodies suffer deformations. The shape of the real contact area and the pressures distribution on that area are unknown. These two surfaces have the same tangent and the same normal in the contact point or line.
On the tangent plane of the two surfaces that are in contact, it is considered a rectangular virtual contact area chosen so it can cover the real contact area, figure 1.

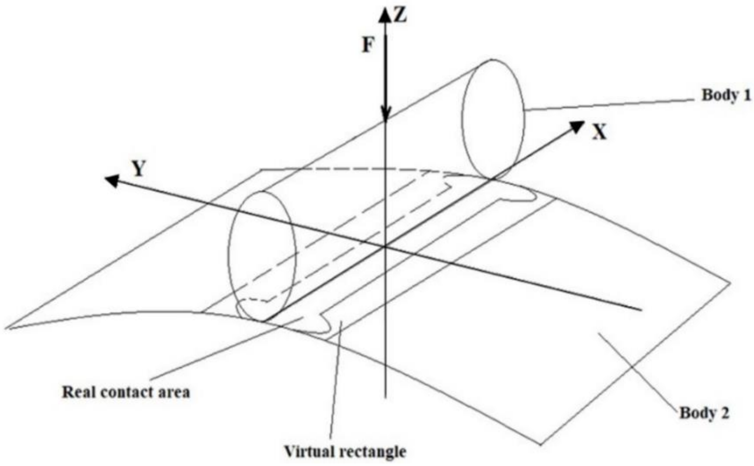

Figure 1. Two bodies in contact.

The elastic model of surface deformation under the normal load F is defined by the following equations:
- geometric equation of elastic contact:

$$g_{ij} = h_{ij} + w_{ij} - \delta_0 \tag{1}$$

- integral equation of normal displacement of elastic half – space frontier:

$$w_{ij} = \sum_{k=1}^{Nx} \sum_{l=1}^{Ny} (K_{i-k,j-l} \cdot p_{kl}) \tag{2}$$

- static equilibrium equation

$$\Delta x \Delta y \sum_{i=1}^{Nx} \sum_{j=1}^{Ny} p_{ij} = F \tag{3}$$

where for the point with coordinates (*i*, *j*, *0*). The notations h_{ij}, w_{ij}, g_{ij} and δ_0 represent the initial separation, the elastic deformation along the normal direction, the gap resulted and the rigid displacement, respectively. The equations (1) – (3) represents an algebraic system with $(n + 1)$ equations and unknowns: n pressures p_{ij} and the displacement δ_o. The hypotheses regarding the linear elasticity lead to the following restrictions:
- non - adhesion and non -penetration conditions lead to:

$$g_{ij} = 0 \xrightarrow{yields} p_{ij} > 0, \ (i,j) \in A_r \tag{4}$$

$$g_{ij} > 0 \xrightarrow{yields} p_{ij} = 0, \ \ (i,j) \notin A_r \tag{5}$$

- elastic-perfect plastic behaviour of the material:

$$p_{ij} > p_Y => p_{ij} = p_Y \tag{6}$$

More details can be found in [3]. The values of initial separations h_{ij} are determined considering the equations of each contacting surfaces. The high accuracy and very fast convergence speed recommended the Conjugate Gradients Method for solving the above equations [3]. The validation of the accuracy of pressures distribution subroutine was done by comparisons with:
i) pressures distributions provided by Hertz relations [3], when the contact is of hertzian type, and

ii) FEM results obtained by neutral researchers.
In figure 2, it is exemplified the comparison between FEM results obtained by J. de Mul et al.[17] and the result provided by SAM. Very good fit between these approaches and the developed SAM have been obtained.

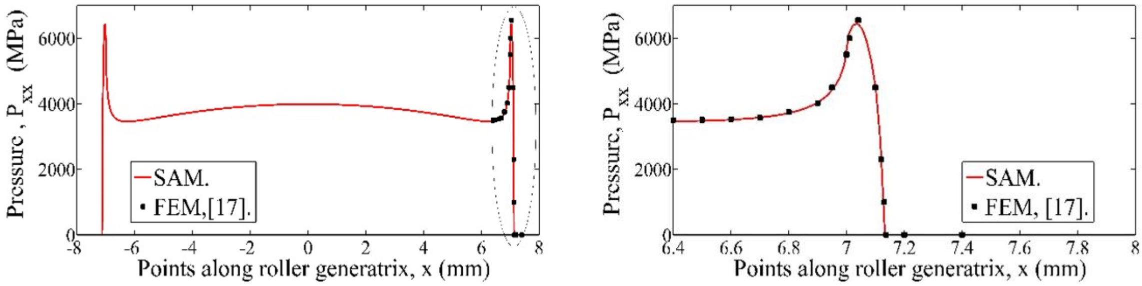

Figure 2. The synchronization between the non-Hertz algorithm results and FE, results [17].

2.2.Matrix of separations for contacts between teeth with crowned profile in helical gears
The equations that define a typical surface of a helical gear tooth flank are as follows:

$$\begin{cases} x_1 = r_{b1}\cos(\theta_1 + \mu_1) + u_1\cos\lambda_{b1}\sin(\theta_1 + \mu_1) \\ y_1 = r_{b1}\sin(\theta_1 + \mu_1) - u_1\cos\lambda_{b1}\cos(\theta_1 + \mu_1) \\ z_1 = -u_1\sin\lambda_{b1} + p_1\theta_1 \end{cases} \tag{7}$$

The meaning of each term is according to Litvin and Fuentes, [14].
To calculate the matrix of separations in such a case, the curve resulted from the intersection between the described surface and the plane $Z = 0$ (red dashed involute curve, figure 3) is rotated with respect to OY'' - axis, figure 3. In this way, a crown profiled pinion is obtained.

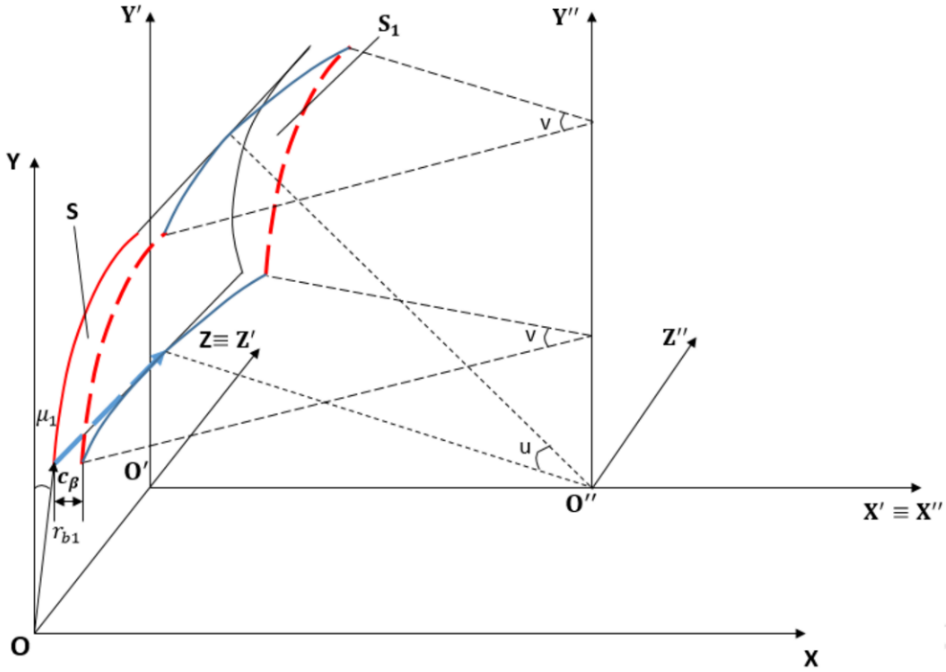

Figure 3. Schematic representation of crowned profile tooth geometry.

Surface S is the surface defined by eq. (7). The continuous red curve on the plane $Z = 0$ is the intersection between surface S and this plane. This curve is then translated along the Z – axis and OX' -

axis and its equations are written with respect to $X''O''Y''Z''$. The surface that is obtained by rotating this curve with respect to Y'' - axis has the following equations:

$$\begin{cases} x'' = \left[r_{b_1}\sin(\theta_1+\mu_1) - p_1\theta_1\cot\lambda_{b_1}\cos(\theta_1+\mu_1) - r_{b_1}\sin\mu_1 - R\right]\cdot\cos v \\ y'' = \cos(\theta_1+\mu_1) + p_1\theta_1\cot\lambda_{b_1}\sin(\theta_1+\mu_1) \\ z'' = \left[r_{b_1}\sin(\theta_1+\mu_1) - p_1\theta_1\cot\lambda_{b_1}\cos(\theta_1+\mu_1) - r_{b_1}\sin\mu_1 - R\right]\cdot\sin v \end{cases} \quad (8)$$

where v is the angle of rotation of the involute curve with respect to OY'' - axis and u is the angle between any point of surface S_1 and OX' - axis, which represents the crown profiled surface. The limits of these two angles can be easily found by calculating the angle between the straight lines that intersect each corner of the flank surface (for v) and the angle between the two straight lines that pass through the top and the bottom of surface, figure 3. The value of crowning height –figure 3 – is taken according to [16]. A tangent plane is defined so that it pass through the maximum point of the corrected surface, corresponding to the value of B/2 on the Z – axis (half pinion width). This surface is meshed in a number of points – using equations (8) – and the matrix of separations is calculated as the distances between each point and the tangent plane. In the above equations, R is the radius of the lowest circle arc that passes through the corrected surface.

3. SAM algorithms results

3.1. TCA of a spur gear pair

The initial data for the current spur gear pair are identical with those presented in [10], under the same conditions. This paper is taken as reference for comparison with the current results, figure 4. To obtain a spur gear pair, in the computerized program, the helix angle takes the value of 0 degrees.

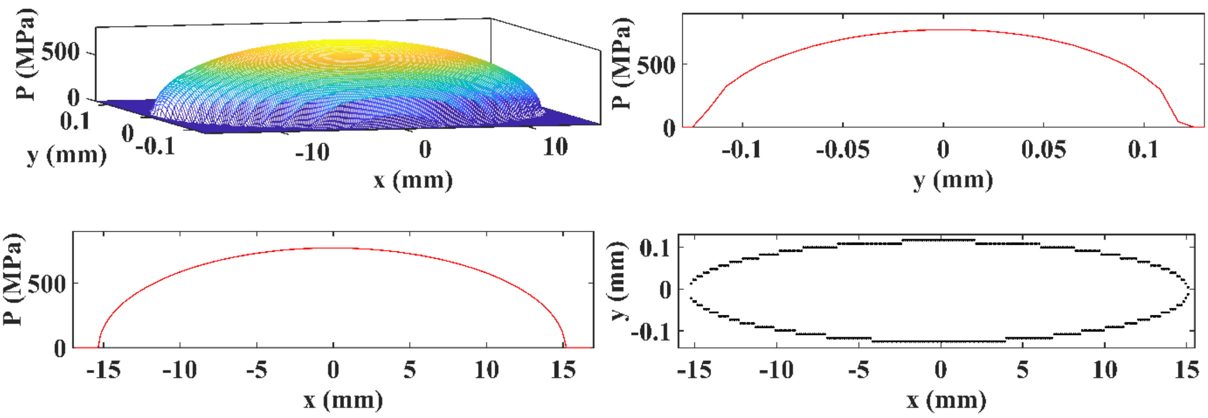

Figure 4. The 3D, transversal and longitudinal pressures distribution and contact area for crowned teeth of a spur gear pair

The maximum pressure value in this case is 772.2 MPa, while in [10] the maximum value is 748 MPa. The maximum error between these two approaches is 3.13%, which is a very good correlation. In [10], the matrix of separations has a different method of determination.

3.2. TCA of a helical gear pair

As long as the validation of the pressures determination subroutine was accomplished, according to figure 1, and the accuracy of the matrix of separations calculation procedure for crown profiled teeth was proven in the previous case (spur gear pair), the following results are provided for a right hand helical gear pair without validation. The initial data, in this case, are given in table 1. The most important aspect with respect these results are the median pressures and the fact that the edge pressures, with two zones of peak pressures, have been avoided.

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Table 1. Initial data for the current helical gear pair

Data	Values
No. of teeth – z_1, z_2	34, 57
Helix angle – β	15^o
Module – m_n	3 mm
Pressure angle – α	20^o
Gear width – B	25 mm
Crowning height – C_β	5 µm
Point action radius – r_{x_1}	Meshing pole

The results are presented in figure 5.

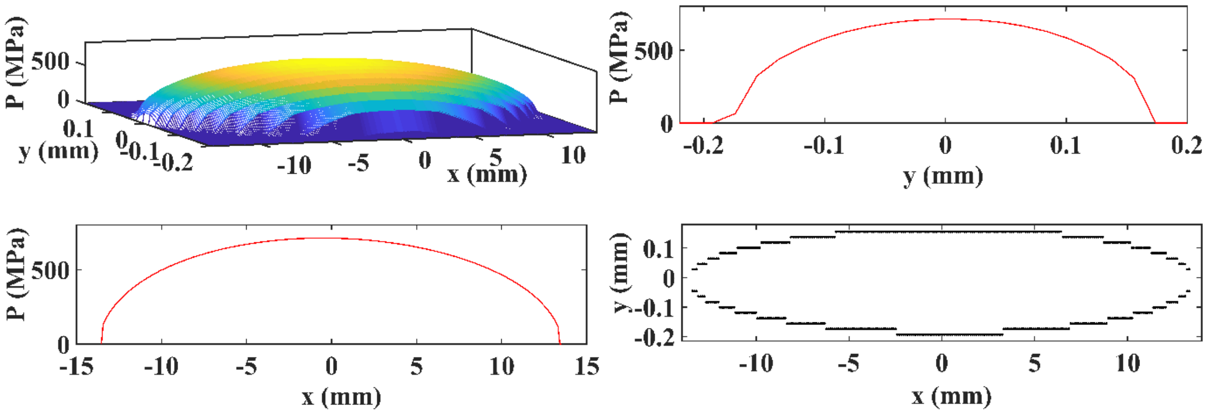

Figure 5. The 3D, transversal, and longitudinal pressures distribution and contact area for crowned teeth of a helical gear pair, helix angle $\beta = 15^0$.

The maximum pressure value is 712 MPa, in this case.

4. Conclusions

Usually, the cylindrical gears have teeth with a slightly modified longitudinal lead profile in order to transform the initial line contact into a point contact, so that the edge effect can be avoided.

To carry out case studies including teeth contact analyses, the finite element method proved to be time consuming.

A semi-analytical method has been developed to obtain the pressures distributions for concentrated contacts developed during meshing processes of spur and helical cylindrical gears.

To obtain the coefficients for matrix of separations at the contact between teeth helical gears, an original way was presented and exemplified for pressures distributions in teeth contacts of helical gears.

If the contact was of hertzian type, the developed SAM would be validated by comparisons of its results with pressures distributions provided by Hertz relations. For non-hertzian type of contacts, the FEM results obtained by neutral researchers were used for comparations and validation.

The main advantages in using semi-analytical methods consist in much lower computation time, accuracy, robustness and a good compliance. These qualities recommend SAM to perform fast TCA of spur and helical cylindrical gears.

5. References

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