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Evaluation of inertia effects in planar squeeze flow inside soft, porous layers

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Abstract. The resistance to flow of a fluid squeezed out from a soft and porous structure under compression, generates high load capacity. The process is dependent on permeability, which is variable during compression of the porous layer. This mechanism (named Ex-Poro-Hydrodynamic Lubrication – XPHD) plays an important role in a wide range of tribological applications (biolubrication, shock absorbers, etc.). A literature search reveals that Darcy's law is intensively used for modeling flow through porous media. However, for high porosity materials and relatively high velocity flows that may occur during impact, the inertia effects could be significant. The full squeeze solution of high velocity flow inside deformable porous materials including inertia effects and permeability variation with the level of compression could not be found in the literature. A numerical study for planar squeeze flow of a Newtonian liquid inside a soft porous layer, based on the Darcy flow model including Forchheimer correction, is presented in this paper. The porous material is compressed at high speed between two perfectly rigid, parallel and impermeable discs. Finite differences technique is used to solve the system of equations. Pressure distribution, load and Reynolds pore based number variation during compression are obtained and compared with similar values predicted by simple analytical models based on Darcy law. The results define the limits of Darcy approach in XPHD lubrication.

1. Introduction

Lubrication with fluid film is the main topic of numerous studies with various applications. A particular application based on fluids imbibed inside a highly deformable porous media is emerging and there are studies that prove higher load capacity. Load is generated by the resistance to flow inside the porous matrix with its variable (decreasing) permeability during compression. This mechanism (named Ex-Poro-Hydrodinamic Lubrication – XPHD [1]) shows great potential for squeeze dampers. For squeeze flow inside porous materials, Darcy law is widely used. However, for high velocity flow inertia effects could be important. It was already proven that Darcy law is less accurate when pore based Reynolds number Re_p becomes greater than unity [2]÷[4]. There is no well established limit or transition zone between laminar to turbulent flow regime. According to Zeng & Grigg [5] the transitional flow occurs for Re_p=1÷100, while Boomsma & Poulikakos [6] proposed a Re_p=5.

First proposal for modification of Darcy law by introducing a quadratic term was made by Dupuit [7], and Forchheimer [8] by analogy with Navier-Stokes equations. Presently the equation proposed by Ward [9] and Beavers et al. [10], based on a series of experimental results, is widely used:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{\eta}{\phi}u - \frac{C_f \cdot \rho}{\sqrt{\phi}}u^2 \tag{1}$$

where C_f is a nondimensional Forchheimer coefficient, experimentally determined, ϕ is material permeability, η dynamic viscosity, ρ density. Some studies tried to find a universal value for this coefficient. Ward [9] stated that C_f=0.55 can be used for all permeable materials. Later experimental results reported that C_f is different for various classes of materials. Beavers and Sparrow [11] found C_f=0.078÷1.32 for porous media in the form of latticework of metallic fibers and water. For particles beds, Schwartz and Probstein [12] used C_f=0.26. Other studies tried to find a connection between C_f and porosity. Ergun and Orning [13] proposed for gases that $\frac{C_f}{\sqrt{\phi}} = b''\frac{(1-\varepsilon)}{\varepsilon^3}$ where b'' is a factor of proportionality. According to Irmay [14], for any fluid, the dimensionless coefficient is defined as $C_f=\alpha\cdot(\beta\cdot\varepsilon^3)^{-1/2}$, and later, Bear [3] suggested: α=0.6 and β=180. Thus $C_f=0.045\cdot\varepsilon^{-3/2}$.

Despite a relatively great number of papers dedicated to inertia effects in porous media, no attempt could be found for the case of a highly deformable media with time variable porosity and permeability.

This study is focused on a particular application of XPHD lubrication for damping devices subjected to medium impact speed, when inertia effect can be significant. It is of major interest to evaluate the limits of Darcy flow model when squeezing speed is high. In the first stage, the analysis is performed for constant speed squeeze in a disc-on-plane configuration.

2. Problem formulation and main assumptions

A simple case of squeeze of an imbibed porous material between two circular parallel discs is considered for this study. The upper rigid disc approaches with constant speed V=dh/dt and compresses the material with initial thickness h_0. The porous material has the same radius R as the rigid discs. The model is axisymmetric with radial flow and atmospheric pressure on the outer boundaries. The geometry of the model is presented in figure 1.

The porous material is highly deformable (compression forces generated by the solid structure are neglected), homogeneous and isotropic. A reticulated structure with open pores, saturated with liquid, is considered. The fluid is Newtonian and the flow is laminar, isothermal and isoviscous.

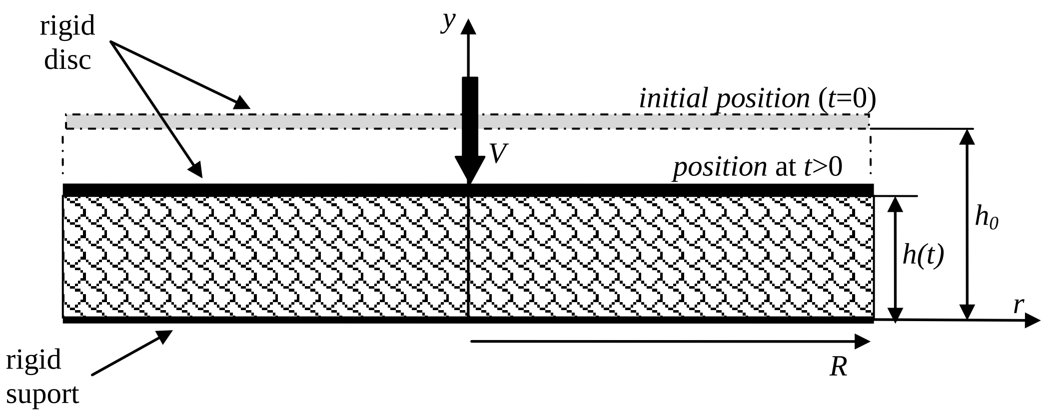

Figure 1. The geometry of the axisymmetric planar squeeze flow

The equation of mass conservation in cylindrical coordinates (r,u) is:

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(ru) + \frac{\mathrm{d}v}{\mathrm{d}y} = 0 \tag{2}$$

Using Kozeny-Carman equation [15] permeability-porosity correlation is:

$$\phi = \frac{D \cdot \varepsilon^3}{(1-\varepsilon)^2} \tag{3}$$

where D is a complex parameter, function of the characteristic size of the porous structure.

The material deformation is produced only in the normal direction of the porous layer and the conservation of the solid fraction is used *h·(1-ε)*=ct. Thus introducing dimensionless layer thickness $\bar{h} = h / h_0$ ecuation (3) becomes:

$$\phi = \frac{D \cdot (\bar{h} - 1 + \varepsilon_0)^3}{\bar{h} \cdot (1-\varepsilon_0)^2} \tag{4}$$

The following dimensionless parameters are introduced: $\bar{y} = y / h_0$, $\bar{v} = v / V$, $\bar{u} = u / V$, $\bar{p} = p \cdot D / (\eta V R)$. Permeability ϕ is also replaced with dimensionless form (eq. (4)).

The dimensionless form Darcy-Forchheimer (D-F) equation (1) in dimensionless form is:

$$\frac{\mathrm{d}\bar{p}}{\mathrm{d}\bar{r}} = -\frac{\bar{h}\left(1-\varepsilon_0\right)^2}{\left(\bar{h} - 1 + \varepsilon_0\right)^3}\bar{u} - \left(C_f \cdot K\right)\frac{1}{\sqrt{\dfrac{\left(\bar{h} - 1 + \varepsilon_0\right)^3}{\bar{h}\left(1-\varepsilon_0\right)^2}}}\bar{u}^2 \tag{5}$$

where $\bar{y} = y / h_0$, $\bar{v} = v / V$, $\bar{u} = u / V$, $\bar{p} = p \cdot D / (\eta V R)$ and $K = \rho V \sqrt{D} / \eta$. If we integrate eq. (2) with respect to radius *r*, while considering the boundary conditions (at *y*=0, *v*=0 and at *y*=*h*, *v*=*V*) we obtain eq. (6) in a dimensionless form:

$$\frac{\mathrm{d}}{\mathrm{d}\bar{r}}(\bar{r}\bar{h}\bar{u}) = -\bar{r}\left(\frac{R}{h_0}\right) \tag{6}$$

3. Numerical solution

There is no analytical solution possible for these equations and consequently, a numerical approach based on finite differences method is proposed. Because there is no flow on $\bar{y}$ direction, the domain is split only in radial direction in small elements. Pressure and radial flow velocity is assumed to be constant across the thickness of the porous layer. An explicit first-order FD scheme with forward derivatives is used for the discretization of the eq. (5) and (6). A one-dimensional grid (along $\bar{r}$) based on *i*=1...*N*+1 equally-spaced nodes is used and leads to the following set of discrete equations:

$$\bar{p}_i = \bar{p}_{i+1} + \left(f \cdot \bar{u}_i + C_f K \frac{1}{\sqrt{1/f}} \bar{u}_i^2 \right) \Big/ \Delta\bar{r} \tag{7}$$

$$\bar{u}_i = \left(\bar{u}_{i-1} \cdot \bar{r}_{i-1} + \frac{\Delta\bar{r}}{\bar{h}_k} \cdot \frac{\bar{r}_i + \bar{r}_{i-1}}{2} \cdot \frac{R}{h_0} \right) \Big/ \bar{r}_i \tag{8}$$

where $f = \dfrac{h_k(1-\varepsilon_0)^2}{(h_k - 1 + \varepsilon_0)^3}$, $\bar{p}_i$ and $\bar{u}_i$ refer to pressure and flow velocity, respectively, on each node and *k* is the subscript that counts for time step.

Gauss-Seidel iterative method without relaxation is used. At each time step pressure distribution is integrated and load calculated. The solution convergence is checked by: the relative variation in force $\bar{F}$ between two iterations must be inferior to 10^{-9}. Grid independence solution was satisfied for N >1000.

4. Results and discussion

The present analysis is focused on a particular application presented in [16]. A high porosity material (knitted fabric made of polyester yarns) and glycerin are considered for impact squeeze. First, an analysis of the values of *K* for various conditions reveals a large interval, spanning from 10^{-3} for low fluid velocity and glycerin to 10^2 for high fluid velocity and water. For a squeeze speed of 1 m/s the coefficient *K*=0.015 was found. A second less viscous fluid (e.g. synthetic oil), was considered which, for same speed, yields to a coefficient *K*=0.07. The algorithm allows determination of the pressure and flow velocity distribution with radius when the porous imbibed material is compressed and the fluid is squeezed out including the inertia influence. Dimensionless pressure distribution on the radius of the porous disc during compression for different values of $\bar{h}$ is presented in figure 2. According to continuity equation, variation of dimensionless flow velocity in radial direction is linear and reaches the maximum value when the pressure gradient is maximum at the outer boundary of the porous disk.

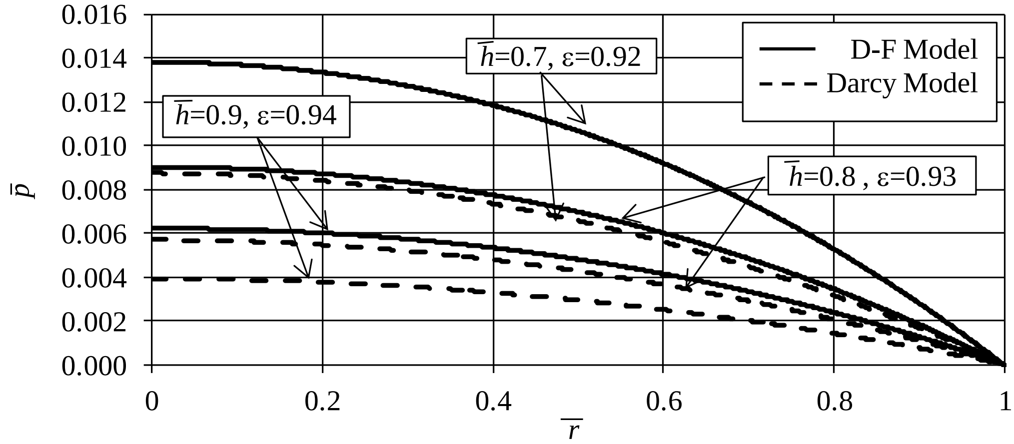

Figure 2. Variation of dimensionless pressure during compression for different values of dimensionless thickness $\bar{h}$ (ε_0=0.95, K=0.015)

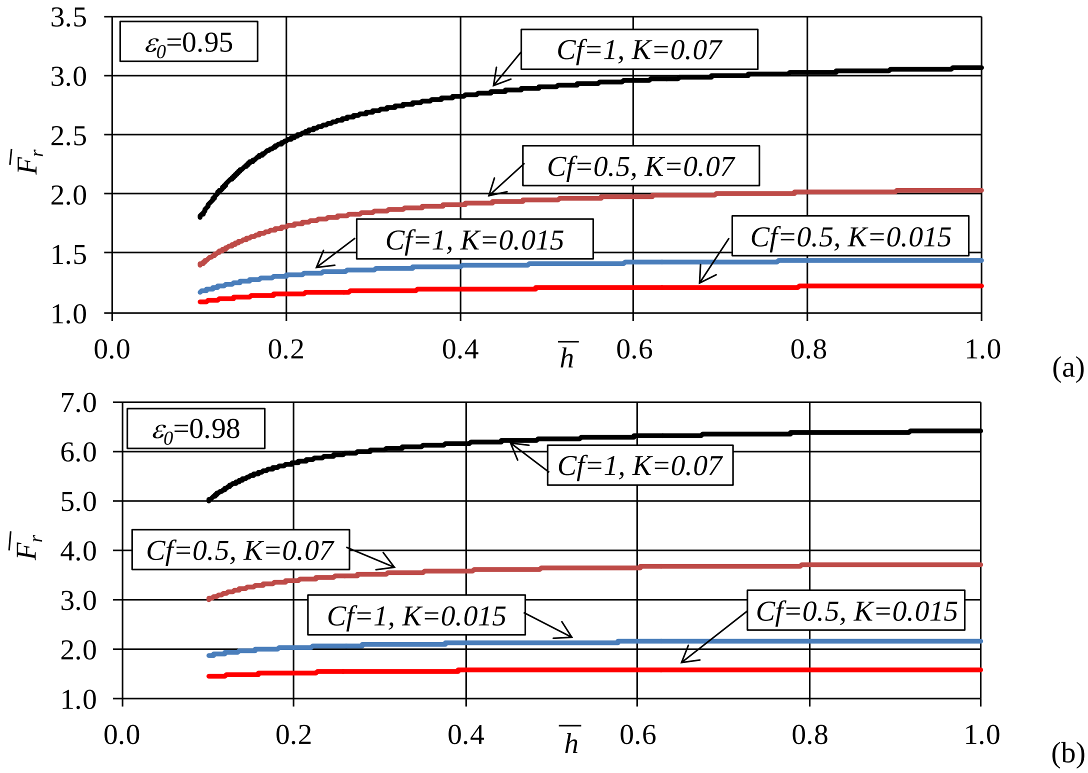

Figure 3. Variation of relative load $\bar{F}_r$ for different values of C_f , K and initial porosity a) ε_0=0.95, b) ε_0=0.98

Further, by numerical integration the load $\bar{F}$ can be determinate. To better understand the influence of inertia effects, relative load $\bar{F}_r = \bar{F} / \bar{F}(C_f = 0)$ was calculated by dividing the load obtained using the proposed model for different values of C_f and K to the load obtained for C_f=0 when no inertia effects are considered. Figure 3 shows the variation of relative load for two different initial porosities: a) ε_0=0.95, b) ε_0=0.98. One can see that for low material thickness inertia effects decrease. The drop is accentuated for low thickness due to the variation of porosity in accordance with conservation of solid phase assumption. The load also increases with increasing C_f. If a less viscous fluid when K is increasing to 0.07, the inertia effects increase and also the estimated load. Further, to validate the numerical model, the variation of load $\bar{F}$ was compared at limit, when Forchheimer coefficient C_f=0, with the analytical solution of the Darcy flow model proposed by Radu [17] for disc on plane squeeze. The difference is very small and can be neglected.

An analysis of pore based Reynolds number variation $Re_p=\rho u \phi^{1/2}/\eta$ is presented in figure 4 for the maximum value of the flow velocity at the outer boundary of the domain. For a low squeeze speed of 1 m/s and K=0.015 the Re_p exceeds the laminar flow limit. When considering the same fluid (K=0.04), if the speed increases, the Re_p increases as well. Furthermore, when a less viscous fluid is used, Re_p exceeds the limit 10. This is in accordance with the assumption that for high squeeze flow inertia must be taken into account. The variation of Rep on the radius is presented in figure 5. It can be seen that the variation is linear from centre, as no radial flow occurs, gradually increasing towards the exterior of the disc where the pressure gradient is maximum and also the flow velocity.

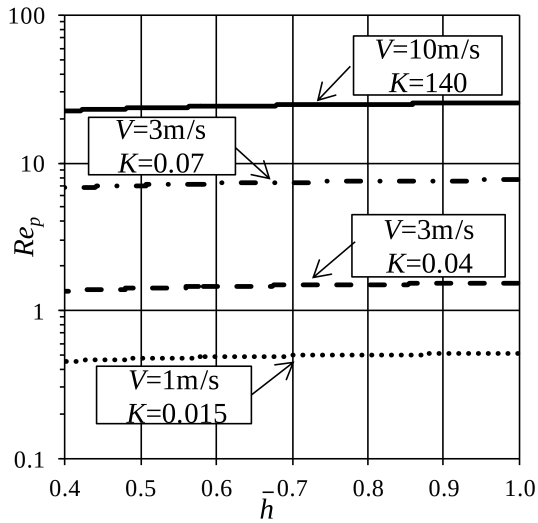

Figure 4. Variation of Re_p on compression for highest radial flow velocity u_{max}

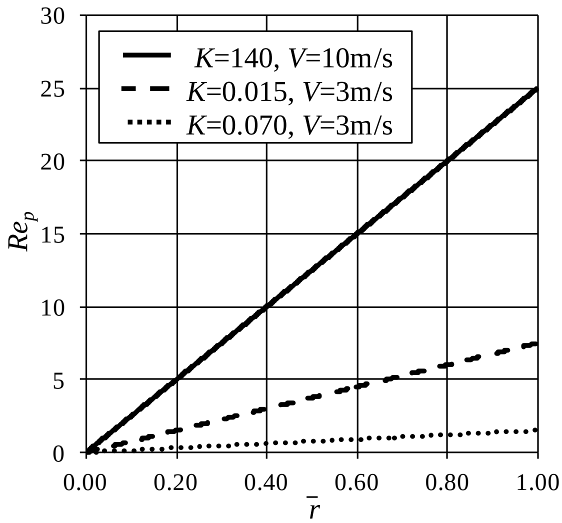

Figure 5. Variation of Re_p on the radius of the disc for $\bar{h}$ =0.8

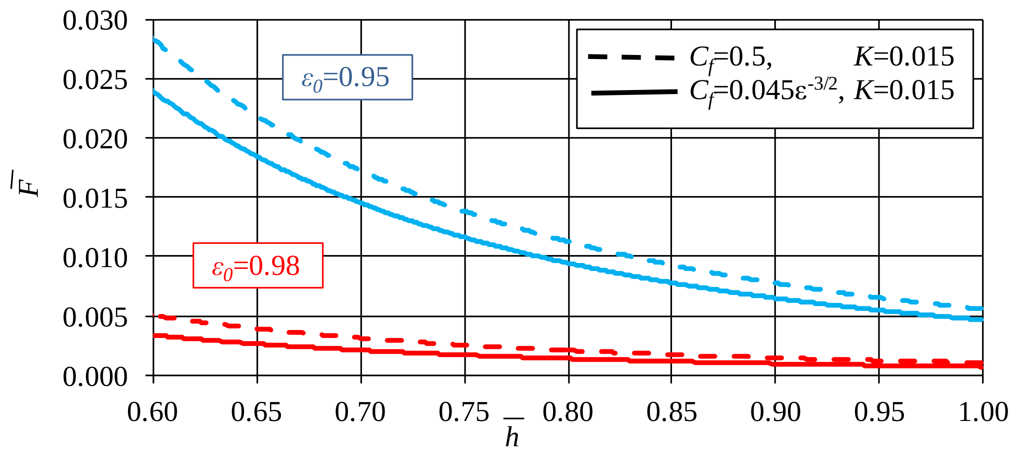

Figure 6. Load variation during squeeze

The effect of Forchheimer coefficient C_f when is variable with porosity according to definition of Irmay [13] was studied in parallel with the case of constant C_f (figure 6). One can see that when C_f is variable the predicted force is lower and the difference becomes higher when the material is more compressed. Two cases for different initial porosities ε_0=0.95 and ε_0=0.98 are used.

5. Conclusions

An evaluation of the inertia effects on force generated during high constant velocity squeezing out of a liquid imbibed in a highly porous and soft layer has been performed. The analysis has been performed numerically using finite differences method.

Pressure distribution was determined and influence of inertia effects on load was evaluated for two values of Forchheimer coefficient. The dimensionless results show that for relatively low squeeze speed inertia effects could be important especially for high porous materials and low viscosity fluids. It is also evident from these results that at high rates of compression (near the end of squeeze process) the inertia contribution vanishes.

An improved numerical solution should consider the dependence of friction factor in Forchheimmer term with Reynolds number.

Due to variable porosity specific to XPHD applications, a variable Forchheimer coefficient with porosity can be more appropriate to predict load. Further theoretical work appears necessary in order to evaluate inertia effects in the case of impact squeeze where fluid velocity decline with time due to damping effect.

6. References

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