

# The voltages and polarity of an inductor

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## Abstract

One may be perplexed about the voltage across an inductor, since sometimes it is  $-LdI/dt$ , but more commonly it is  $LdI/dt$ . We explain why this is so, and point out that in fact, each of them is fine to be used if under its own sign convention (listed in table 2 in the last section of this paper), and the latter, mostly used in circuit equations, is for the purpose of intentionally grouping  $L$ ,  $C$  and  $R$  as passive components (consuming electric potential energy), enabling circuit analyses more conveniently and results more understandable. As well, we re-examine a closely related subject matter: the determination of the polarities in an a.c. circuit.

## 1. Introduction

Inductors work on the principle of electromagnetic induction. The self-induced (or called back) e.m.f. across an inductor of inductance  $L$  is

$$\varepsilon = -L \frac{dI}{dt}, \quad (1)$$

where  $I$  is the current through it. The negative sign is a manifestation of Lenz's law. Very normally, one may think that the voltage across an inductor appearing wherever in the texts is the self-induced e.m.f.  $\varepsilon$ , and hence equation (1) has to be employed. However, this is not always the case. Indeed, in circuits, it always refers to the no-negative-sign form, i.e.  $V_L = LdI/dt$ . The choice, whether or not there is a negative sign, purely depends on one basic consideration: treating the inductor as an active or a passive component (section 3). Therefore, we start off with revisiting the distinction between the electromotive force (the

voltage of an active component) and the potential difference (the voltage of a passive component) in the next section. Whichever the voltage form is, the actual polarity of an inductor can be determined from a sign convention of its own (sections 4 and 5). The inductor discussed here is assumed to be pure, that is, it has no internal resistance. Besides inductors, most of the concepts discussed in this paper are applicable to other components, especially the capacitors.

## 2. E.m.f. and P.d.

In general, a voltage can mean an *electromotive force* (e.m.f.), or a *potential difference* (p.d.).

An e.m.f. source, e.g. an ideal battery, of one volt is defined as one joule of electric potential energy (e.p.e.) converted from other form is gained by one coulomb of positive charge passing through the source, implying that the current-leaving terminal of the source has a more positive electric potential with respect to its current-entering terminal [1]. In other words, whenever stating a positive e.m.f. of a source, it is understood that the current-leaving terminal of the source is the positive polarity (figure 1(a)).



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What is a p.d.? In electrostatics, a p.d. between any two specified points is the difference of the electric potentials at these two points, but in current electricity, the p.d. across a passive component, e.g. a resistor, must be the electric potential at its current-entering terminal minus the electric potential at its current-leaving terminal. Hence, whenever stating a positive p.d. across a component, it is understood that the current-*entering* terminal of the component is the positive polarity (figure 1(b)). A passive component has a p.d. of one volt implies that one joule of e.p.e. is dissipated when one coulomb of positive charge passes through it [2].

Nowadays, it seems that the *voltage drop* (or known as potential drop) tends to be more popularly used than the p.d., especially on the web. According to Wikipedia, the voltage drop is defined as the decrease of electric potential along the path of a current flowing in an electric circuit [3]. Obviously, it is synonymous with the circuits' p.d., which however has been used for decades. If a reader is used to using the voltage drop or the potential drop, just 'replaces' the p.d. in the rest of this paper with it.

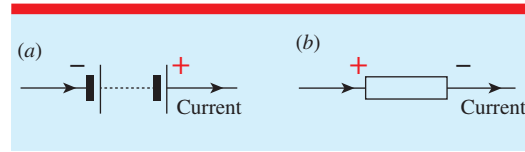
### 3. Inductor is dual-status

Although an inductor produces electric potential energy (e.p.e) from magnetic energy, it transforms e.p.e. back to magnetic energy as well. An inductor creates and consumes e.p.e. in half-and-half of the time in an a.c. cycle, so it is, at least a semi-passive component, nevertheless qualifying for using the term 'p.d.'. Hence, an inductor can legitimately be treated as an active (e.p.e. generating) or a passive (e.p.e. consuming) component [4]. The voltage of the former is the e.m.f.  $\varepsilon$ , while that of the latter is the p.d.  $V_L$ .

But actually, an inductor is a combination of an active and a passive component, how can the e.m.f.  $\varepsilon$  or the p.d.  $V_L$  itself work singly to reflect the alternate shifts between these two statuses? Simply, this is achieved through its sign (or the sign of its power in a.c. circuits), as summarised in table 1:

One form of Kirchhoff's voltage law is  $\Sigma \text{e.m.f.s} = \Sigma \text{p.d.s}$  in any loops of a circuit. Therefore, in an  $RLC$  series a.c. circuit, we have, if the inductor is treated as an active component,

$$\xi + \varepsilon = V_C + V_R, \quad (2)$$



**Figure 1.** (a) The current-leaving terminal of a battery is positive. (b) The current-entering terminal of a resistor is positive.

where  $\xi$  is the source e.m.f.,  $\varepsilon$  is the self-induced e.m.f. due to  $L$ ,  $V_C$  and  $V_R$  are the p.d.s across the capacitor and resistor, respectively. After the self-induced e.m.f.  $\varepsilon$  is moved from the left-hand side to the right-hand side, the above expression becomes

$$\xi = V_L + V_C + V_R, \quad (3)$$

where

$$V_L = -\varepsilon \quad (4)$$

is the p.d. across the inductor. From equations (1) and (4), we have

$$V_L = L \frac{dI}{dt}. \quad (5)$$

Equation (2) is rewritten as equation (3), marking a change of the status of the inductor.

Under what circumstances should the self-induced e.m.f.  $\varepsilon$  (equation (1)), or the p.d.  $V_L$  (equation (5)) be used? Simply put, if the action of the electromagnetic induction is what we focus on, most probably the self-induced e.m.f.  $\varepsilon$  should be used, but in circuits, quite surely the p.d.  $V_L$  is the right one.

In d.c. and a.c. circuits, an inductor is usually connected in series or parallel with other passive components, such as resistors and/or capacitors. As to *compare their individual effects conveniently*, the meaning of the 'voltage' used in each of them need to be the same. The only option is that their 'voltages' all mean p.d.s, because it is nonsense to say 'e.m.f. of a resistor'.

Countless conveniences will be brought about if treating  $L$ ,  $C$  and  $R$  are all passive components, including: in a series resonance,  $V_L$  and  $V_C$  are added up to zero, symbolising the effects of  $L$  and  $C$  cancel each other out; if  $L$  is not pure, its external p.d. is the sum of its internal  $V_L$  and the p.d. due to its internal resistance; the same manipulation and understanding of their individual and combined phasor diagrams; the same

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interpretation of their voltage signs (see figure 4). Just think that how odd or counter-intuitive all these would become if they are re-expressed in terms of the self-induced e.m.f.  $\varepsilon$  (do not forget  $\varepsilon = -V_L$ ).

### 4. Determination of polarity—D.C.

If we put aside the consideration of which one is more convenient, both the e.m.f.  $\varepsilon$  and the p.d.  $V_L$  can be used to determine the actual polarity of an inductor.

There are two steps. First, if  $\varepsilon$  ( $V_L$ ) is used, assigning the current-leaving (-entering) terminal as the positive polarity defines the sign convention for the positive  $\varepsilon$  ( $V_L$ ), as shown in figure 2. Secondly, the actual polarity at a particular instant is determined by referring the then sign of  $\varepsilon$  ( $V_L$ ) to its own sign convention.

Without loss of generality, we consider a *time-increasing* d.c.  $I$  which flows in the direction as shown in figure 2. Hence, we find  $\varepsilon = -LdI/dt < 0$ , indicating that the then polarity is opposite to that defined from positive  $\varepsilon$ , therefore  $P$  is positive and  $Q$  is negative. Absolutely, the same result will be got if the p.d.  $V_L$  is used instead. In this case,  $V_L = LdI/dt > 0$ , therefore the polarity is the same as that defined from positive  $V_L$ . The answer can be easily checked by applying Lenz's law.

In a.c. circuits, the basic idea is the same, but the periodic reversals of the current direction makes the analysis and result a little bit different.

### 5. Determination of polarity—A.C.

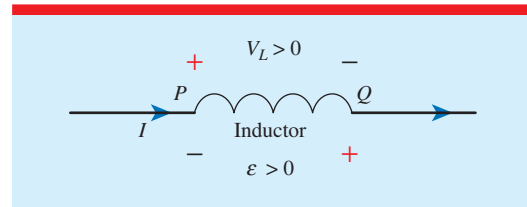
A problem is raised about the  $RLC$  series a.c. circuit shown in figure 3: without directly utilising Lenz's law, how can the polarity of the inductor at any instant be figured out? The method basing on  $V_L$  is explained as follows.

Since side E is earthed, the positive polarity of the a.c. source is predefined at side S, from where the positive current should leave. Thus, the direction of the current  $I$  is defined: when  $I > 0$ ,  $I$  flows clockwise around the circuit, and when  $I < 0$ ,  $I$  flows anticlockwise.

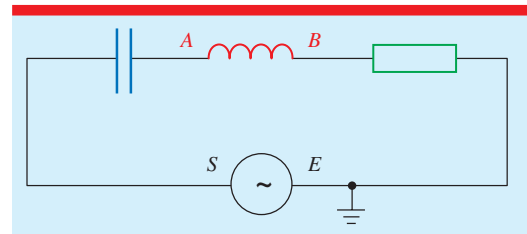
Next, we consider the power of the inductor,  $P = IV_L$ . When the power is positive,  $I$  and  $V_L$  must be both positive or negative. Whatever the case, a positive power of a p.d. means a consumption of

**Table 1.** The signs of  $\varepsilon$  and  $V_L$  correspond to different statuses of the inductor (for a positive current).

	Generating e.p.e.	Consuming e.p.e.
$\varepsilon$	$\varepsilon > 0$	$\varepsilon < 0$
$V_L$	$V_L < 0$	$V_L > 0$



**Figure 2.** The polarity sign convention depends on either the induced e.m.f.  $\varepsilon$  or p.d.  $V_L$  is used.



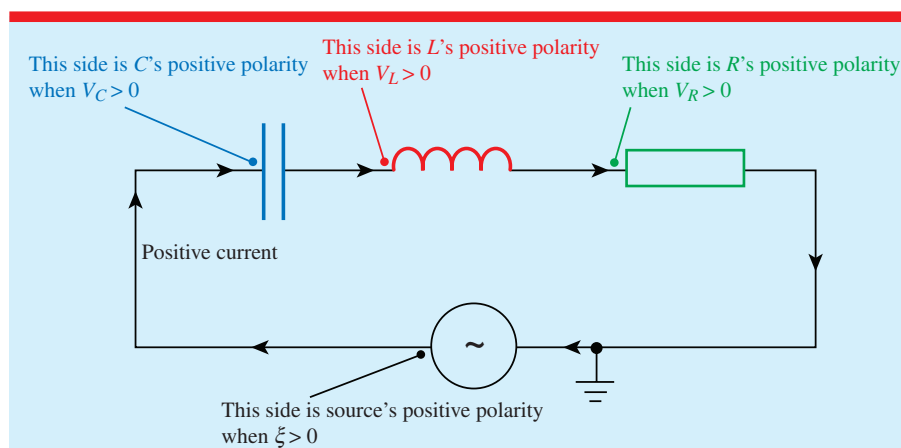
**Figure 3.** An  $RLC$  series a.c. circuit is shown with side E is earthed.

electric potential energy; in other words, the current must flow from a higher potential to a lower potential when crossing the inductor. Hence, in the case of both  $I$  and  $V_L$  are positive (negative),  $I$  flows clockwise (anticlockwise) and side A (B), from where the current enters the inductor then, is the positive polarity. Accordingly, the polarity of  $L$  is determined: when  $V_L > 0$ , the polarity is (A + , B -), and when  $V_L < 0$ , the polarity is (A - , B +). Simply put, *the sign of  $V_L$  gives exactly the sign of polarity at the positive-current-entering terminal*. Here, the positive current enters  $L$  from side A. We can reach the same result by considering a negative power of the inductor. Readers are advised to check this out and work out the case for the self-induced e.m.f.  $\varepsilon$  themselves. All the sign conventions for  $\varepsilon$  and  $V_L$  are listed at once in table 2 in the next section.

Note that this problem cannot be solved without a prior definition of the positive current direction. If the positive current direction is really unknown, the result will become an ambiguous

**Table 2.** The polarity of  $L$  is determined from the sign of  $\varepsilon$  or  $V_L$ . For  $\varepsilon < 0$  and  $V_L < 0$ , the polarities are opposite to the corresponding ones.

	d.c. circuits	a.c. circuits
$\varepsilon > 0$	Positive polarity: current-leaving terminal	Positive polarity: positive-current-leaving terminal
$V_L > 0$	Positive polarity: current-entering terminal	Positive polarity: positive-current-entering terminal



**Figure 4.** Since the a.c. source is an active component, its positive-current-leaving side is the positive default polarity. But  $L$ ,  $R$  and  $C$  are passive ones, so to each of them the positive default polarity is at its positive-current-entering side. The symbol  $\xi$  stands for the source e.m.f.

case. Technically, one output of an a.c. source has to be earthed.

Besides  $V_L$ , the signs of  $V_C$  and  $V_R$  are interpreted likewise (one of the pros of treating  $L$ ,  $R$  and  $C$  as the same kind of component), as shown in figure 4.

Figure 4 serves as a visual summing-up of the relationships between the voltage signs and their corresponding polarities in a series a.c. circuit. In fact, such relationships are applicable to other a.c. circuits.

## 6. Discussion and conclusions

It is well known that a current flows from  $(-)$  to  $(+)$  when crossing a d.c. source and from  $(+)$  to  $(-)$  when crossing a resistor. An inductor possesses this *duality*, because it can be used as a source of e.m.f.  $\varepsilon = -LdI/dt$  or a passive component of p.d.  $V_L = LdI/dt$ . From a practical viewpoint, the difference between  $\varepsilon$  and  $V_L$  is nothing more than their opposite default polarities. All their sign conventions are tabulated in table 2.

Nonetheless, there seems to be a rule: *in electric circuits, an inductor is always treated as a passive component*, having exactly the same status as a resistor. Other than qualitative discussions, the self-induced e.m.f. of an inductor is seldom used in formulating circuit equations. In an a.c. circuit, we say ' $V_L$  leads  $I$  by  $90^\circ$ ', rather than ' $\varepsilon$  lags  $I$  by  $90^\circ$ '; in an  $RL$  d.c. circuit, the voltage  $V_L = V_0 \exp(-Rt/L)$  is, with no exception, a p.d., and many other examples. The reason for this inclination is explained in section 3. This is certainly a point worthy to be emphasised to students for avoiding any miscalculations, since 'inductor', 'induced e.m.f.' and 'Lenz's law (the negative sign)' are all too closely related, often they will come to mind together.

Actually, a phase difference between any two a.c. quantities is meaningful only after the meanings of their own signs are explicitly known. So, in our opinion, a diagram like our figure 4 is an indispensable tool for teaching a.c. circuits, because it tells us what the signs of the a.c. voltages truly mean in the simplest way. Moreover, it

is unlikely that one can understand a more complicated system which involves inductances, e.g. an ideal transformer, without a good grasp of the inductor [5].

By the way, the voltage across a capacitor is less confusing, because from the very beginning, the voltage  $V = Q/C$  has been defined and used as the p.d. across a capacitor. Therefore, there is little doubt that a capacitor is a passive component, but indeed, like an inductor, a capacitor can be treated as an e.m.f. source [6].

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