Paper

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A fan-type variable capacitor rotation angle sensor for university physics experiments

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Abstract

In this article, we examine the rotation angle dependence of a hand-made fan-type variable capacitor for use in physics experiments in university classes. We made two circular bakelite plates, and the two terminal plates of the capacitor were 90° fan-type copper disks. The capacitance varied linearly with the rotation angle. The standard deviation from the linear fit was 1.4%, which was probably generated by the fact that the capacitor plates were not perfectly circular. Furthermore, we made a rotation angle sensing system using an AC high-pass filter. We selected the AC frequencies of 1 kHz and 100 Hz, which are easy to use in physics experiments. These experimental results also showed a linear relationship.

1. Introduction

The physics of capacitors is an important topic in electromagnetism and electronics [1]. The capacitor plays a crucial role in the circuits of many electronic devices. One of the most famous uses of the capacitor is in FM tuners (NAT 02, NAIM Co. Ltd, UK) and transmitters. The circuit that receives radio waves is an inductor and a variable capacitor connected in parallel. In the modern world, integrated circuits using capacitors, inductors, and resistors are in the devices people use every day: mobile PCs, tablets, smart phones, radio, and TV, to name a few. Therefore, it is important that students understand the principles behind these circuits. Another use of these circuits is in rotation angle sensors. Many researchers have proposed rotation detection sensors [2–5]. Fulmek *et al* made a capacitive angle and angular rate sensor [2], in which two fan-shaped rotors were used as a capacitive relative angle sensor. Two rotors with asymmetrically arranged blades and a center angle of 60° were mounted symmetrically on two concentric shafts. Ye *et al* proposed the cylindrical structure of electrodes in a multi-plane electrical capacitance tomography (ECT) sensor for obtaining high-quality capacitance measurements [3]. Li *et al* proposed a capacitive angular-position sensor [4]. The capacitor was made by three electrode circular disks in an axis-symmetrical configuration. A 45° screen electrode was sandwitched by a common electrode disk and a 15° segmented electrode. A positional sensor, which can measure rotational and lateral displacements, was invented by Duden [5], which can determine the position of an object to within a few nanometers. He mentioned that this sensor provides information on both angular displacement and 2D lateral displacement of moving objects, and can be used in devices requiring highly precise measurement of movement, such as electron microscopes and optical systems.

These devices can measure high-precision angle and displacement. However it is thought that the physical principle is slightly difficult for an abecedarian. Therefore we suggest a physical experiment for the electromagnetism education that made two pieces of disks and simple fan type capacitors using the copper foil.

In this article, we propose a basic experiment of a rotation angle sensor by means of a fan-type variable capacitor for the purpose of understanding the physics of capacitors.

2. Experimental setup

We made circular bakelite plates 0.100 m in diameter, which means 5.00×10^{-2} m radius, and 1.20×10^{-3} m thick, as shown in figure 1, by means of a circular cutter. Figure 2 shows the photo of the capacitor of the verge to actually measure took from a bottom slant. The terminal plates of the capacitor were 90° fan-type copper seats.

Figure 3 shows the schematic of the fan-type capacitor. We used a Cu adhesion seat (CU-35C, 3M Co., Ltd) for the terminal plates of the capacitor. An adhesive is painted in one side of the seat. The seat was cut in a fan-type and put the seat on the bakelite plate like a figure 3. The cutaway view of the capacitor was also shown in figure 3. As for the upper plate, the copper adhesion sheet was stuck on the top surface of the bakelite, and the plate was rotatable. As for the lower plate, the copper adhesion sheet was stuck on the under surface of the bakelite of the lower plate, and was fixed to the center pipe made of bakelite. The plate of both contacts, and the gap between the live and the neutral copper electrode plates was 2.40 mm. We also displayed the schematic of the upper and lower plates from a bottom slant, as shown in figure 4. In this figure, the upper plate and the lower plate were described to be separated to explain the structures of the plates clearly. A red and a white wire were solderd to the copper seat of upper plate, and that of lower plate, respectively. The copper seats of upper plate and lower plate were connected to the live terminal and normal terminal of the AC generator, respectively, via red and white wires.

The calculated capacitance of the capacitor from its dimensions for $\theta = 0°$ (the lower electrode plate completely overlaps the upper electrode plate), and the electrical permittivity of the bakelite ($\varepsilon = 4.5\varepsilon_0$) [6] was 65 pF. The angle dependence of the capacitance of the fan-type capacitor was measured using an LCR meter, as shown in figure 5. Figure 6 shows the measurement system, and figure 7 shows a schematic of the measurement system. The AC input voltage was produced by the generator. The output signal was filtered (by a band pass filter) and amplified by 20 dB (10 times) using an multifunction filter. The frequency of the AC voltage was measured by an frequency counter. The output voltage V_2 was measured by a digital volt meter (DVM). The input voltage V_1 was 1.50 V AC. V_1 and V_2 represent input voltage and output voltage, respectively. We selected the frequencies of the AC signal f as 1.0 k Hz and 100 Hz. The data were imported into a laptop PC via GPIB cables. The software program used to analyse the data was a BASIC/98 system.

3. Results and discussion

Figure 8 shows the rotation angle dependence of the capacitance C of the capacitor. The data indicates that the capacitance was linearly proportional to the rotation angle. The standard deviation from the linear fit was 1.4%, which was probably due to the fact that the capacitor plates were not perfectly circular.

The measurements of the capacitance C of the capacitor using the LCR meter was 80 pF for $\theta = 0°$ (the lower electrode plate completely overlaps the upper electrode plate) and 20 pF for $\theta = 90°$ (the lower electrode plate does not overlap with the upper electrode plate at all). The capacitance for $\theta = 90°$ was not zero. This was due to the capacitance of the electrode plate around the central axis. There, the excess capacitance was 20 pF. As mentioned above, the calculated capacitance was 65 pF, which was comparable to the experimental result of $C = (80–20) = 60$ pF for $\theta = 0°$, when the excess capacitance was subtracted.

Next, we changed the voltage signal sent to the capacitor. Capacitors are inpenetrable to DC current, but they do pass AC current. Accordingly, the electrical conductivity changes with the frequency of the current or the capacitance of the capacitor. In order to make use of this characteristic, we installed a high pass filter in the electric circuit in order to change the voltage signal sent to the capacitor.

Figure 9 shows the schematic of the high pass filter. We considered the frequency dependence at 50 pF, which was the median value of the capacitance. We selected frequencies at 1 kHz or lower than 1 kHz. It is conceivable that the change in the conductivity is large for low frequencies. For the high pass filter, the ratio V_2/V_1 can be expressed as

$$\frac{V_2}{V_1} = \frac{1}{\sqrt{1+\left(\frac{\omega_0}{\omega}\right)^2}}. \quad (1)$$

In equation (1), $\omega_0 = \frac{1}{CR}$ and $\omega = 2\pi f$, where C (in V) represents the capacitance of the capacitor, R (in Ω) represents the resistance of the shunt resister between the hot and cold lines, and f (in Hz) represents the frequency of the AC signal. Accordingly, equation (1) can be rewritten in terms of C, R, and f as

$$\frac{V_2}{V_1} = \frac{1}{\sqrt{1+\left(\frac{1}{2\pi RfC}\right)^2}}. \quad (2)$$

If $2\pi RfC \ll 1$, V_2/V_1 can be written as

$$\frac{V_2}{V_1} = 2\pi RfC. \quad (3)$$

Equation (3) indicates that when $C \ll \frac{1}{2\pi Rf}$, V_2/V_1 is proportional to the capacitance C:

$$\frac{V_2}{V_1} \propto C \quad (4)$$

By using equation (2), we chose the frequency f and resistance R with the software Igor Pro Ver. 6 [7]. First, we selected 1 kHz as the frequency of the AC signal. This frequency is easy to generate in conventional AC generators used for physics experiments. The resistance R was calculated as 3.18 MΩ at $C = 50$ pF by using $\omega_0 = \frac{1}{CR}$ in equation (1). The frequency dependence of V_2/V_1 at $C = 50$ pF calculated using equation (2) is shown in figure 10. For $R = 3.18$ MΩ, V_2/V_1 decreased significantly with decreasing frequency around 1 kHz. For a comparison, the result for $R = 100$ kΩ is also plotted in figure 10. The V_2/V_1 ratio decreased significantly with decreasing frequency around 30 kHz.

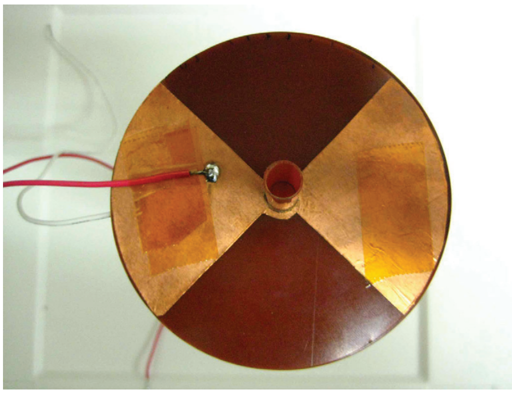

Figure 1. Photo of the top view of the fan-type variable capacitor used in this experiment. The diameter of the capacitor is 0.100 m.

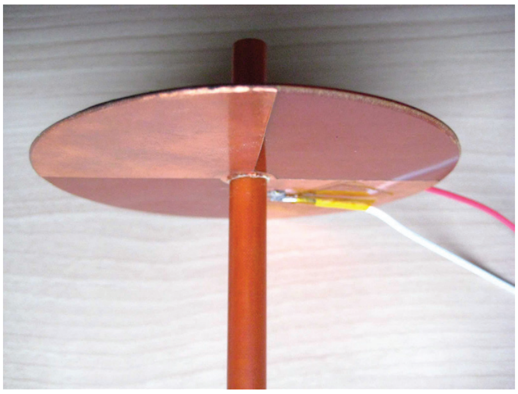

Figure 2. Photo of the capacitor from a bottom slant.

The range of the resistance R and the frequency f can be selected in order to use the rotating capacitor as an angle sensor as indicated by changes in the output voltage V_2.

First, we calculated the dependence of V_2/V_1 on C at $f = 1.0$ kHz and $R = 3.18$ MΩ, as shown in figure 11. The result indicates that V_2/V_1 versus C was not linear. Figure 12 plots the calculated results of the dependence of V_2/V_1 on C at $R = 3.18$ MΩ, with various frequency values. The result at $f \geqslant 200$ Hz deviated from linearity. By contrast, for frequencies below 200 Hz, there was a linear V_2/V_1 versus C relationship for the rotation range of the capacitor. In order to use the capacitor as a rotation angle sensor, it is essential that the relationship between C and V_2/V_1 is linear. Therefore, we used a frequency of 100 Hz in

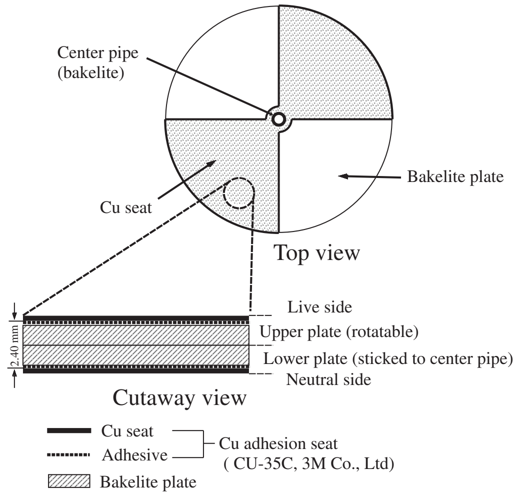

Figure 3. The schematic of the fan-type capacitor.

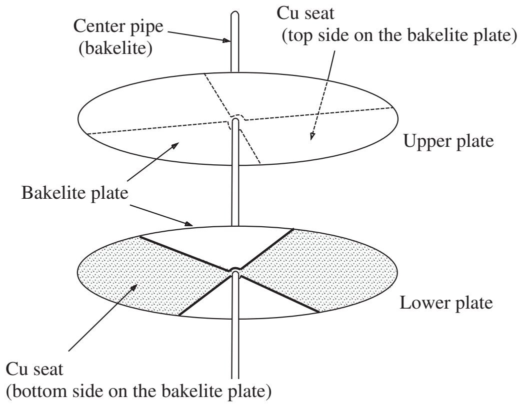

Figure 4. The schematic of the upper and lower plate. Upper plate was rotatable, and lower plate was fixed to the center pipe by glue. In this figure, the upper plate and the lower plate were described to be separated to explain the structures of the plates clearly.

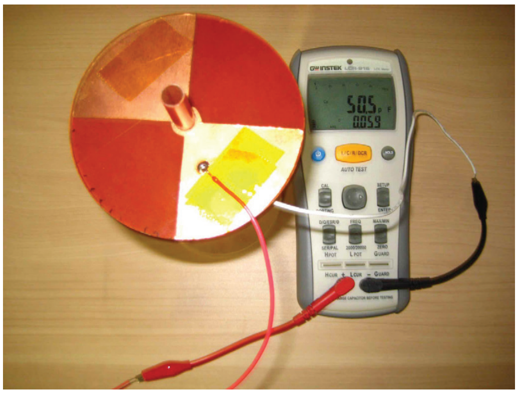

Figure 5. Photo of the fan-type variable capacitor and the LCR meter used in this experiment.

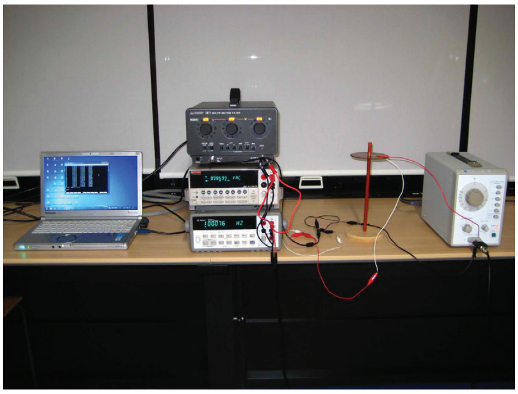

Figure 6. Photo of the measurement system. From right to left: AC generator, fan-type variable capacitor, multifunction filter, DVM, frequency counter, laptop PC.

order to maintain this linearity and also to maximise the output voltage. The experimental result of V_2/V_1 versus C at $R = 3.18$ MΩ and $f = 100$ Hz is shown in figure 13, which indicates linearity for the rotation angle range of the capacitor.

The linearity of V_2/V_1 versus C was maintained even if the resistance was changed without changing frequency. Figure 14 shows the calculated results of the dependence of V_2/V_1 on C at $R = 100$ kΩ and $f = 1.0$ kHz. These calculated results are in excellent accordance with the linear fit to the experimental data, as shown in table 1.

Figure 15 shows the experimental result of V_2/V_1 versus C at $R = 100$ kΩ and $f = 1.0$ kHz. The experimental result showed a linear relation. As shown in table 1, the standard deviation of V_2/V_1 versus C from linearity was 1.22%, which was larger than that of the calculated result.

It is conceivable that this was due to the imprecision of the shape of the capacitor plates. In this study, we made a fan-shaped capacitor by hand, which was necessary to facilitate student learning. However, manual construction inevitably leads to 1%–2% errors.

Now we compare the errors of our meansument with other fan-type capacitive sensors. Mentioned above, Fulmek *et al* made a capacitive rotational angle-measurement sensor [2]. The

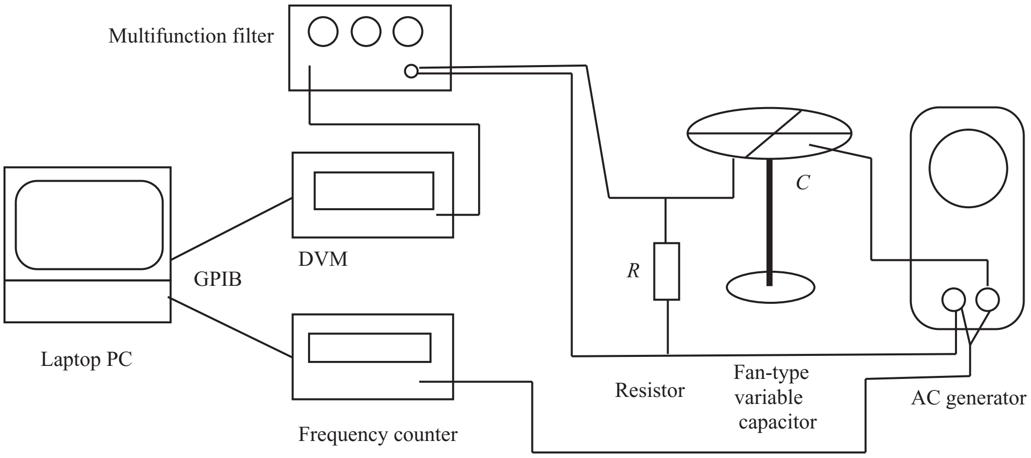

Figure 7. Schematic of the measurement system.

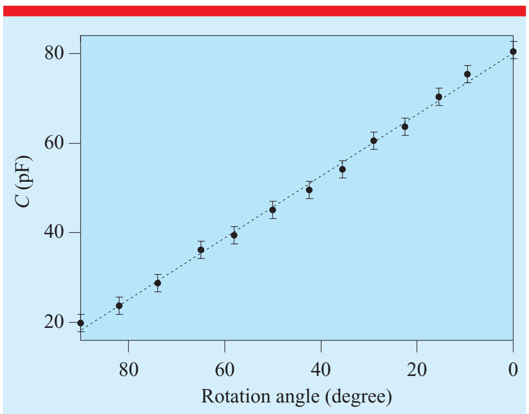

Figure 8. Rotation angle dependence of the capacitance C of the capacitor. The dotted straight line indicates the linear fit.

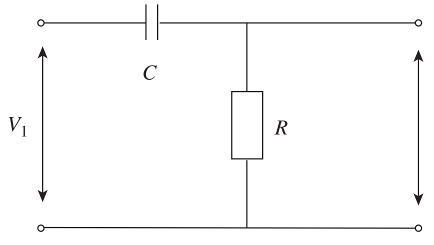

Figure 9. Schematic of the high pass filter. V_1 and V_2 denote input voltage and output voltage of the capacitor, respectively.

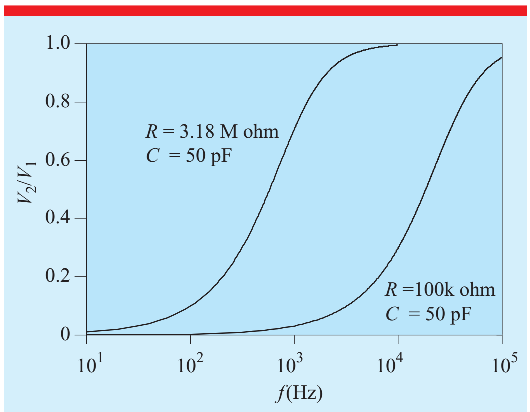

Figure 10. Frequency dependence of V_2/V_1 at $C = 50$ pF.

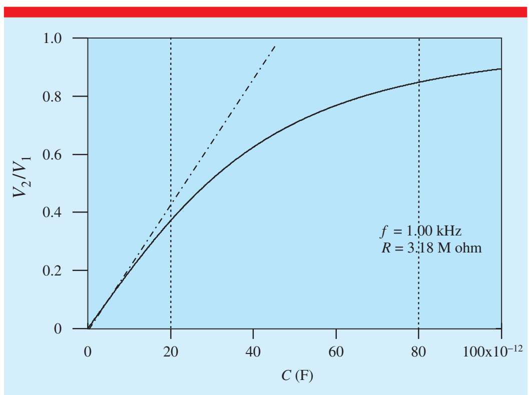

Figure 11. The calculated dependence of V_2/V_1 on C for $f = 1.0$ kHz and $R = 3.18$ MΩ. The vertical dotted lines indicate the rotation range between $\theta = 0°$ and 90°. The linear fit around the origin is shown by the dot–dashed line.

sensor was configurated by the transmitter plate with 16 segments, two mirror-symmetrical rotors, and receiving electrode. The result of the error of the relative angle versus angle position was 0.32% (with the range of +0.12% and −0.25%).

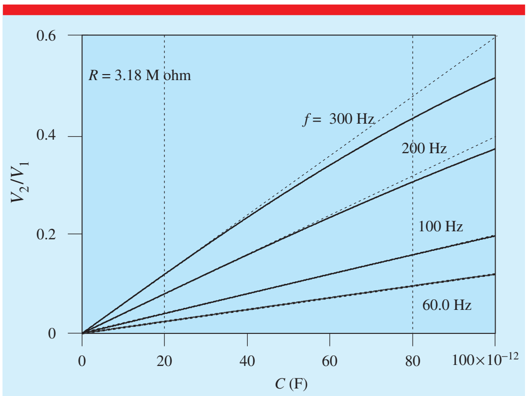

Figure 12. Dependence of V_2/V_1 on C for $R = 3.18$ MΩ and $f = 60.0$, 100, and 300 Hz. The linear fits around the origin are represented by the dotted lines.

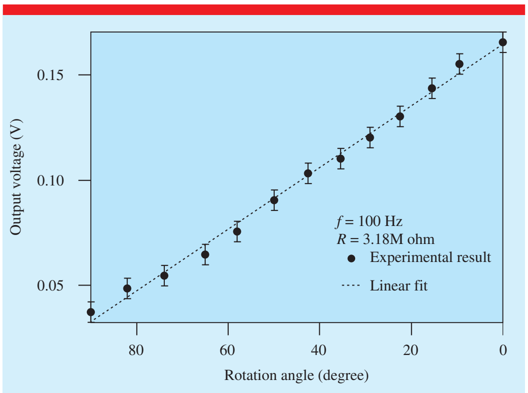

Figure 13. Experimental result of V_2/V_1 versus C at $R = 3.18$ MΩ and $f = 100$ Hz.

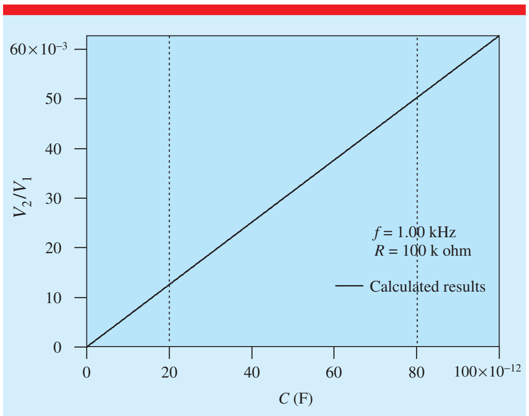

Figure 14. Dependence of V_2/V_1 on C at $R = 100$ kΩ and $f = 1.0$ kHz.

Table 1. The standard deviation of V_2/V_1 versus C from the linearity.

Condition	Deviation (%)
Calculated 3.18 MΩ, 200 Hz	0.092
Calculated 3.18 MΩ, 100 Hz	0.024
Calculated 3.18 MΩ, 60.0 Hz	0.0089
Experimental 3.18 MΩ, 100 Hz	2.20
Calculated 100 kΩ, 1.00 kHz	<0.001
Experimental 100 kΩ, 1.00 kHz	1.48

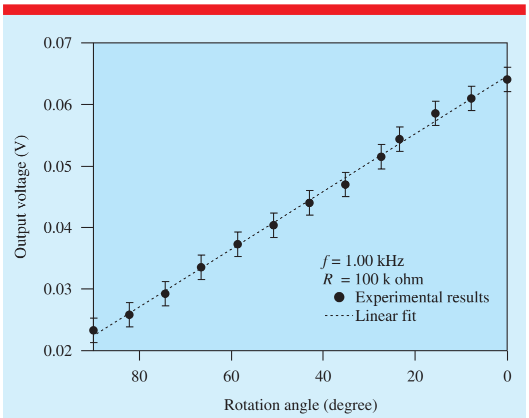

Figure 15. Experimental result of V_2/V_1 versus C at $R = 100$ kΩ and $f = 1.00$ kHz.

Concerned about the capacitive angular-position sensors by Li *et al* [4], the calculations was experimentally verified with a nonlinearity of less than $\pm 17''$ and $\pm 50''$ over measurement ranges of 15° and 90°, respectively. These errors indicates ±0.03% and ±0.015%, respectively. The standard deviation of the sensor obtained by the measurements in this article, shown in table 1, was between 1.4% and 2.2%. The error of the results of the measurements was ±0.7% and ±1.1%, respectively, when the standard deviation was converted into an error. The error in our handmade sensor was more than five-times larger than that of other high-resolution sensors. However, we think that it is a device with enough experimental value for physics education.

4. Conclusion

We examined the rotation angle dependence of a hand-made fan-type variable capacitor, which consisted of bakelite plates and copper foil, for use in physics experiments in university classes. The capacitance varied linearly with the rotation angle. Furthermore, we made a rotation angle sensing system using an AC high-pass filter. These experimental results also showed a linear relation. In this study, the AC generator and the measuring instruments used commercial devices, however it becomes the electronic learning by creating an AC generator by themselves. They can realize a reasonable cost by using a tester. We compared it with other sensors and considered about the usefulness of this device. The error in our hand-made sensor was more than five-times larger than that of other high-resolution sensors. However, it was supposed that it is a device with enough experimental value for physics education. In this study, we made a fan-shaped capacitor by hand, which was necessary to facilitate student learning. In achieving an education effect, it is important to make a device by hand by the physical experiment.

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