PAPER

Phys. Educ. **55** (2020) 035001 (4pp) iopscience.org/ped

Rolling sphere along an inclined multi-slotted track

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Abstract

In this work we study the motion of a small metal ball along an inclined multi-slotted track. Specifically, the effect of the slot width on the ball speed at the lower part of the track is analyzed in detailed both analytically and experimentally. Experimental results are in satisfactory agreement with the theoretical model.

S Supplementary material for this article is available online

1. Introduction

Usually in fundamental physics lab courses the study of the motion of a rolling sphere along an inclined plane is performed by releasing a sphere from some height and letting it move along an incline plane under the assumption that there is only one contact point between the rolling sphere and the surface. In this sort of experience it is usual to measure the ball speed at the base of the incline as a function of the height from which the ball was released, by keeping the inclination angle of the incline constant, and on the basis of the results obtained, the mechanical energy conservation is verified and an analysis of the transformation of gravitational potential energy into kinetic and rotational energy is performed. In this paper, we want to go one step further by studying the motion of the rolling sphere along a slotted track. Specifically, we study the effect of the slot width on the ball speed at the base of the track. To do this, we propose a mechanical design which is relatively easy to implement at any workshop. The proposed design not only allow us to analyse and visualize the process of energy transformation in a more attractive, amazing and compelling manner but has the advantage that students see the interaction between theory and experiment that is so characteristic of scientific methodology in physics.

2. Theoretical background

It is well known that for a sphere rolling along an inclined plane from a height H, see figure 1, its speed at the base of the incline is given by equation (1). This equation [1] is valid under the assumption that the sphere rolls without slipping and that it starts from rest

$$v = \sqrt{\frac{10}{7} gH}. \tag{1}$$

However, when the sphere rolls along a slotted track made in the inclined plane, equation (1) is no longer valid since the sphere in this case is in contact with two points of the slot, see figure 2. From the geometry, the effective radius r of the sphere is given by equation (2)

$$r = \sqrt{R^2 - \frac{D^2}{4}} = R\sqrt{1 - \left(\frac{D}{2R}\right)^2} \tag{2}$$

where equation (2) is valid for $0 \leqslant D \leqslant 2R$. To find the speed of the sphere at the base of the incline we use the law of mechanical energy conservation [2]. Being the mechanical energy at the top and bottom of the incline equal, we can write,

$$mgH = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \tag{3}$$

this equation simply states the conversion of gravitational potential energy of the sphere at the top into translational and rotational energy at the bottom, where $I = \frac{2}{5}mR^2$ is the moment of inertia of a uniform sphere of mass m and radius R about an axis through its centre. If the sphere is rolling without slipping along the slot we have the condition $v = \omega r$. From equations (2) and (3) and the last two expressions, the speed for the sphere is given by equation (4).

$$v = \sqrt{\frac{10}{7}gH} \times \sqrt{\frac{1-\left(\frac{D}{2R}\right)^2}{1-\frac{5}{7}\left(\frac{D}{2R}\right)^2}}. \tag{4}$$

Notice that equation (4) simply states how the speed of the sphere at the base of the incline is strongly dependent on the slot width, D. On the other hand, it is clear that if $D \to 0$ then the value for v given by equation (1) is recovered and that if $D \to 2R$ then $v \to 0$. Now let us write the Newton's second law of motion for the rolling sphere. For the translational part we have

$$mg\sin\theta - 2f = ma \tag{5}$$

and for the rotational part

$$2fr = I\alpha = \frac{2}{5}mR^2\alpha \tag{6}$$

where f and α represent the frictional force and the the angular acceleration. From equations (2), (5), (6) and the rolling condition $a = \alpha r$ we get

$$f = \frac{mg\sin\theta}{\left(7-5(\frac{D}{2R})^2\right)} \tag{7}$$

$$a = \frac{5}{7}g\sin\theta\frac{\left(1-(\frac{D}{2R})^2\right)}{\left(1-\frac{5}{7}(\frac{D}{2R})^2\right)}. \tag{8}$$

It is interesting to see from equations (7) and (8) that as $D \to 2R$, the frictional force f grows monotonically and the acceleration a is reduced to zero. Therefore, it is expected that as the sphere rolls in a slot where $D \to 2R$ then it will move more slowly and spin faster. On the other hand, if $D = 0$, the value for the acceleration is $a = \frac{5}{7}g\sin\theta$, which corresponds to known result for a sphere that rolls without slipping along an incline.

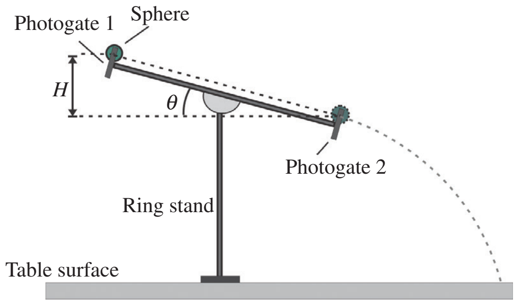

Figure 1. A sphere rolling along an incline.

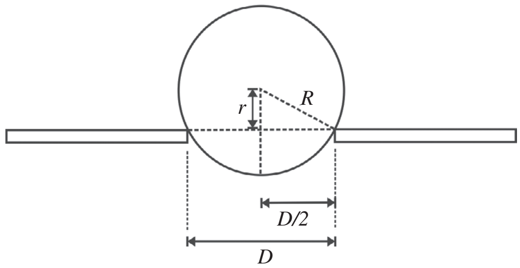

Figure 2. Front view of a sphere resting on a slotted track.

3. Experimental procedure

To obtain the dependence of the sphere speed at the base of the incline on the slot width, a multi-slotted plane was designed and built. A picture of it is shown in figure 3 and it consists of a aluminium sheet of dimensions $31\frac{1}{2}'' \times 5\frac{11}{12}'' \times \frac{3}{8}''$ possessing 8 rectangular slots of $\frac{15}{64}''$, $\frac{9}{32}''$, $\frac{10}{32}''$, $\frac{23}{64}''$, $\frac{25}{64}''$, $\frac{7}{16}''$, $\frac{15}{32}''$ and $\frac{33}{64}''$ in width, respectively. We had the slots cut by a water jet cutting local company. A water jet cutter is an industrial tool that uses a very high-pressure jet of water, or a mixture of water and an abrasive substance. With this tool is possible to cut a wide variety of materials including metals and its advantage over a laser cutting machine is that the piece to be cut is not exposed to high temperatures and therefore causing a potentially permanent deformation due to overheating. Despite the cuts made by the machine being fine, it was necessary to install a narrow alignment sheet to make sure the width of each slot was uniform along its length. The resulting apparatus provided a sturdy multi-track of uniform width and smoothness. The service charge turned out to be less than 100 dollars. The design of the multi-slotted plane was created in CorelDraw which is a graphic design software that allows us to create vector files. The design was saved as a dxf file, which is the format required by cutting machines.

In this experience we used eight identical steel balls of 13.5 mm $\approx \frac{17}{32}''$ in diameter, the inclination angle of the plane was 18°. Each sphere was allowed to roll, from rest, a distance $L = 76$ cm down each slot and two photogates were used; the first one was placed at the top of the incline and the second at the bottom running in pulse/gate mode and in a synchronized manner [3]. In a synchronized manner means that the first time the photogate at the top is blocked the time is set to zero and starts running and as the second photogate is blocked the corresponding timer system [4] measures not only the time it takes for the sphere to go from top to bottom but also the time interval that the IR beam remains blocked by the rolling sphere. Knowing this time interval and the diameter of the sphere is therefore possible to find the sphere speed at the base of the incline for each slot.

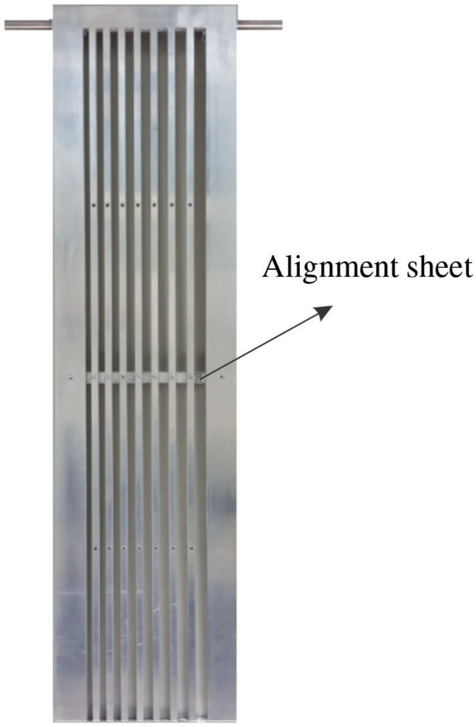

Figure 3. Multi-slotted aluminium sheet. The alignment sheet makes sure parallelism between the slots.

4. Results

Figure 4 shows the dependence of the sphere at the base of the incline on the slot width along with the theoretical prediction of equation (4). The first point corresponds to the movement of the sphere along the surface of the incline, i.e. slot width $D = 0$. The theoretical speed is given by equation (1), i.e. $v = \sqrt{\frac{10}{7}gH} = \sqrt{\frac{10}{7}gL\sin(\theta)} = \sqrt{\frac{10}{7}(9.8)(0.76)\sin(18^o)} = 1.8$ m s^{-1} and the experimental value was 1.9 m s^{-1}. For the rolling of the sphere along the other slots we can see that its speed tends to decrease as the width of the slot becomes larger as predicted by the theoretical model. And above all, the effect of the slot width on the sphere speed is observed experimentally in a very striking fashion. In particular, for the case of the sphere rolling along the widest slot, when it touches the table surface its rotational energy is quickly converted in translational and rotational kinetic energy.

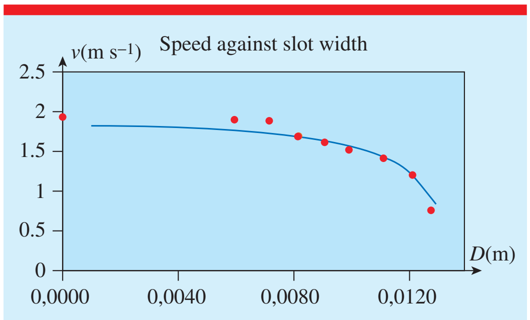

Figure 4. Solid dots: experimental data and continuous curve: theoretical curve given by equation (4).

For the sake of completeness, measurements for the sphere acceleration as a function of the slot width were carried out and the results are shown in figure 5. The data are plotted against the theoretical equation (8). The sphere acceleration was calculated by measuring the time T spent for it to travel the distance $L = 76$ cm, i.e. the distance between the two photogates. From these two quantities, the acceleration is given by the expression $a = \frac{2L}{T^2}$.

From the figure is clear that the sphere acceleration tends to become zero as $D \to 2R$ which is consistent with the theoretical model.

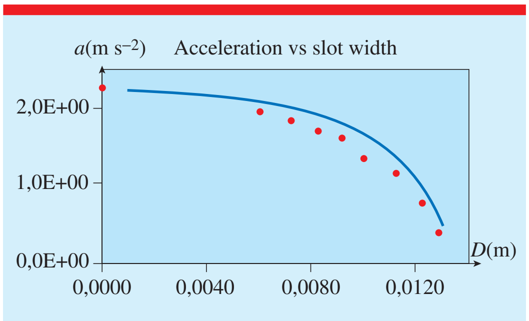

Figure 5. Solid dots: experimental data and continuous curve: theoretical curve given by equation (8).

An issue that is worth mentioning is the impossibility to determine precisely the vertical distance descended for the centre of mass of the sphere for its movement along each slot. In the model this distance is assumed to be constant, but achieving this in practice was a difficult task; the difficulty entailed in this procedure probably caused most of the experimental error. In this experiment, despite the fact that no error handling in data was rigorously applied each measurement made was repeated several times and the average was considered.

5. Conclusions

We have implemented a design of a multi-slotted incline that allows us to study the motion of a rolling sphere along slots of different widths. This approach has the advantage that students can easily visualize the impact that the slot width has on the speed of the sphere and therefore how the process of conversion of gravitational potential energy into translational and rotational energy occurs. This method gives satisfactory results, which together with the simplicity of the design, makes this experiment a good candidate for a general physics practice or demonstration.

Acknowledgments

The authors would like to thank laboratory assistants Yuri Angel, Francisco Diaz and Juan Gonzalez for their assistance in the development and implementation of the multi-slotted incline. We would like to thank the Escuela Colombiana de Ingeniería, Julio Garavito for supporting this project.

Received 18 December 2019, in final form 15 January 2020
Accepted for publication 21 January 2020
https://doi.org/10.1088/1361-6552/ab6e02

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