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# How the individuals' memory affects the evolution of prisoners' dilemma game in the two-layer network

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**Abstract** – In general, multilayer networks are often a significantly more apt description of real-life systems than isolated or single networks. In this paper, we explore the effect of memory on the evolution of prisoner's dilemma (PD) game by constructing different kinds of two-layer networks. The results show that the heterogeneous network structure is conducive to promoting individuals to adopt cooperative behaviors. However, as the lure income $T$ increases, the individuals who take cooperative behavior in the entire system gradually decrease. Further research shows that if no more than one layer of network presents large heterogeneity, then the less the individuals are affected by historical gains, the better the cooperation will be among individuals. By contrast, if both layers of networks are less heterogeneous, the greater the impact of historical returns on individuals, the easier it is for cooperation between individuals. Furthermore, if individuals change their strategies mainly by imitating the strategies of their neighbors, then it is beneficial to promote cooperation among individuals in the entire system. However, it is not conducive to cooperation among individuals if individuals change their own strategy mainly through strategies of their counterparts. The final result indicates that, if at least one layer is a heterogeneous network structure, the cooperation between individuals in the entire system will be blocked when the length of the individual's historical memory is too long.



**Introduction.** – Understanding of human behavior has always been the focus of sociology, psychology and economics, and how to analyze human behavior quantitatively is a very important research topic in science today. We can not only improve human understanding of their own behavior, but also improve their understanding of the evolution of social systems through the study of human behavior [1–4]. Although it is still a huge challenge to understand the cooperative phenomenon of complex systems in different disciplines for individuals [5,6], the game theory and complex network methods provide some effective ways to effectively study the phenomenon of individual cooperation [7–14].

The study of complex networks is part of the study of complexity theory. As a powerful tool for studying complex science and complex systems, complex networks provide a new perspective for studying evolutionary games [15–17]. Abramson *et al.* firstly studied the prisoner's dilemma game model in a small-world network, the individual in the model adopts a deterministic strategy to update the rules. After each round, the individual adopts the strategy of the most profitable individual among his neighbors [18]. Afterwards, models based on pairwise interactions such as prisoner's dilemma games and snowdrift game models in complex networks, and models based on group interactions such as public goods games, have received widespread attention as emergence of research collaborations and evolutionary paradigm [19–22]. Moreover, scholars have studied the effects of changes in link weights between individuals, individual visibility, limited resources, and other factors on the evolution of cooperation among individuals [23–26]. Overall, however, the study of game evolution on complex networks mainly focuses on the influence of factors such as network topology, individual selection mechanisms, and external environment interference on game evolution dynamics.

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For example, Kleinebergd studied the impact of metric clusters on the evolutionary game dynamics in scale-free networks [27]. The results of the research by Wu *et al.* show that if more profit is given to those nodes with large degrees, it can effectively promote cooperation among individuals in the entire network [28].

In the study of information diffusion and contact dissemination, many scholars have proposed different network studies for its inherent characteristics [29–33]. These different network topologies can better characterize the complexity of human activities. The complex inter-individual communication network is not only varying, but also multi-layered, with clustering and community phenomena [34–39]. However, most of the current researches on evolutionary games focus on single-layer networks [40]. In addition, individuals' strategies change are affected by not only neighboring nodes directly connected to them, but also their historical gains. Based on the discussion above, we construct different two-layer networks to study the impact of memory on the evolution of the PD game. The results demonstrate that the heterogeneous network structure is conducive to promoting individuals to adopt cooperative behavior. As the lure income $T$ increases, the individuals' density who take cooperative behavior in the entire system gradually decreases. In addition, if there is a layer of network that is more heterogeneous under the two-layer network structure, then the less the individual is affected by historical gains, the better the cooperation will be among individuals. However, if both layers of networks are less heterogeneous, the greater the impact of historical returns on individuals is, the more beneficial it is for cooperation among individuals. Furthermore, if individuals change their strategies mainly by imitating the strategies of their neighbors, then it is beneficial to promote cooperation among individuals in the entire system. However, it is a disadvantage for cooperation among individuals if individuals change their own strategy mainly through by strategies of their counterparts. The final result indicates that, if at least one layer in the double-layer network structure is a heterogeneous network structure when the length of the individual's historical memory is too long, the cooperation between individuals in the entire system will be blocked.

The arrangement of this letter is as follows: firstly, we construct a prisoner's dilemma game model in a two-layer network, analyze the dynamic behavior of game evolution, and discuss several game dynamic behaviors with typical parameters and networks. Then we use the ER random network and the BA scale-free network, respectively, to construct different two-layer game networks, and numerically analyze the effects of different parameters on evolutionary games. Finally, the conclusions of this letter are given.

**The PD game in a two-layer network by considering individuals' memory.** – We consider the evolutionary game model of the prisoner's dilemma on a two-layer network containing layer $A$ and layer $B$: each individual $i$ in layer $A$ has a symmetric individual $i$ in layer $B$. Behaviors between individuals in layer $A$ and layer $B$ will affect each other. For a pair of game nodes in layer $A$ or layer $B$, if both sides of the game players adopt a cooperative strategy, then the returns of both sides are $R$; if both sides of game players adopt a non-cooperator strategy, then the returns of both sides are $P$; if one side adopts a cooperative strategy and the other side adopts a non-cooperative strategy, then the benefit of the counterpart is $S$, and the benefit of the non-counterpart is $T$ (lure income). For simplicity, this letter studies the weak evolutionary game model, that is, $S = P = 0$, $R = 1$ (see fig. 1).

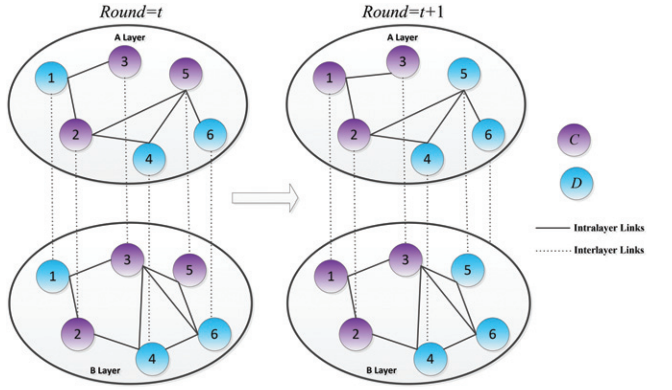


Fig. 1: Schematic diagram of evolutionary games in a two-layer network with 6 nodes in each layer. Each individual $i$ in layer $A$ has a symmetric individual $i$ in layer $B$. The behaviors of individuals in the two-layer network will affect each other.

We propose the following prisoner's dilemma game with individuals' memory. After $t$ game round, the gains of individual $i$ in the layer $A$ and layer $B$ network is calculated as follows:

$$\begin{aligned} g_{ij}^{A(B)}(t) = & \frac{1}{4}(1+s_i^{A(B)})(1+s_j^{A(B)})R \\ & +\frac{1}{4}(1+s_i^{A(B)})(1-s_j^{A(B)})S \\ & +\frac{1}{4}(1-s_i^{A(B)})(1+s_j^{A(B)})T \\ & +\frac{1}{4}(1-s_i^{A(B)})(1-s_j^{A(B)})P, \end{aligned} \quad (1)$$

$s_i^{A(B)}$ is the state of individual $i$ at layer $A$ (layer $B$), $s_i^{A(B)} = 1$ means cooperation, $s_i^{A(B)} = -1$ means non-cooperation. Then, the total returns of individual $i$ from all his neighbors in the layer $A$ (layer $B$) network is given as follows:

$$g_i^{A(B)}(t) = \sum_{j\in\partial i} g_{ij}^{A(B)}(t), \quad (2)$$

where $\partial i$ is the set of neighbors of player $i$ in the layer $A$ (layer $B$) network.

Note that for the individual with the memory, the fitness of the individual $i$ in the layer $A$ (layer $B$) can be calculated as follows:

$$f_i^{A(B)}(t) = \frac{1}{H+1}\sum_{\tau=t-H}^{t} e^{-\alpha(t-\tau)} g_i^{A(B)}(\tau), \quad (3)$$

where $H \in N$ is the memory length of historical return, and $\alpha \geq 0$ is the memory decay factor of historical return. In the case of $\alpha = 0$, it means that the historical income has a great influence on individual's behavior change. On the contrary, if $\alpha \gg 0$, it means that historical gains have a small impact on individual's behavior change.

For individual $i$ in layer $A$ (layer $B$), there are the following two ways to change his own strategy after each round of game.

1) The probability that node $i$ selects a neighbor node $j$ in the same layer and imitates its strategy is $\beta$. In layer $A$ (layer $B$), individual $i$ will randomly select a neighbor node $j$ to compare the returns after a round of game, the probability of strategy to be changed for individual $i$ is as follows:

$$P(s_i^{A(B)} \to s_j^{A(B)}) = \frac{1}{1+\exp\left(-\frac{f_j^{A(B)}-f_i^{A(B)}}{\kappa}\right)}, \quad (4)$$

$f_i^A(f_i^B)$ is the fitness of the individual $i$ in the layer $A$ (layer $B$) and an optional neighbor node $j$ in layer $A$ (layer $B$), $\kappa$ is the impact of environmental noise.

2) For a node $i$, the probability of adopting the strategy from his counterpart is $1-\beta$. Based on the mean-field theory, the density $\rho_A^C$ of the collaborators in layer $A$ changes in a homogeneous network is as follows:

$$\rho_A^C(t+1) = \rho_A^C(t) + \beta[\rho_A^{D\to C} - \rho_A^{C\to D}] + (1-\beta)\pi^{B\to A}, \quad (5)$$

$\rho_A^C(t)$ is the density of cooperator at time $t$ in layer $A$, $\rho_A^{D\to C}$ is the density transfer from non-cooperative individual ($D$) to cooperative individual ($C$) in layer $A$; $\pi^{B\to A}$ denotes how the individuals in layer $A$ are influenced from their symmetrical individuals in layer $B$. Note that the formula of $\pi^{B\to A}$ represents the general form of individual behavior's interaction between two layer networks. For different impact modes, specific impact forms can be set.

Based on the returns of cooperative and non-cooperative individuals, the expressions for the conversion rate $\rho_A^{D\to C}$ between non-cooperatives and cooperatives and the conversion rate $\rho_A^{C\to D}$ between cooperatives and non-cooperatives are as follows:

$$\begin{aligned}\rho_A^{D\to C} &= (1-\rho_A^C)\rho_A^C P(D\to C)\\ &= (1-\rho_A^C)\rho_A^C \tfrac{1}{1+e^{-\frac{f_C-f_D}{\kappa}}}\end{aligned} \quad (6)$$

and

$$\begin{aligned}\rho_A^{C\to D} &= \rho_A^C(1-\rho_A^C)P(C\to D)\\ &= \rho_A^C(1-\rho_A^C)\tfrac{1}{1+e^{-\frac{f_D-f_C}{\kappa}}}.\end{aligned} \quad (7)$$

If we consider layer $A$ (layer $B$) as the homogeneous network, and the average degree of the network is $\langle k\rangle$, we can get the average returns of collaborator $C$ and non-collaborator $D$ in layer $A$ (layer $B$) as $f_C = \langle k\rangle\rho^C$ and $f_D = \langle k\rangle\rho^C T$, respectively. Then we can obtain

$$\rho_A^{D\to C} - \rho_A^{C\to D} = (1-\rho_A^C)\rho_A^C \tanh\frac{(1-T)\langle k\rangle\rho_A^C}{2\kappa}. \quad (8)$$

In addition, we can analyze the changes of the collaborator density in layer $B$ in the same way.

**Numerical simulation and results.** – For different kinds of complex network structures and general parameters, numerical simulation methods are used to analyze the evolutionary dynamic behaviors of the PD game. We mainly focus on three different types of two-layer networks to study the PD game. 1) Both layer $A$ and layer $B$ are the ER random network with $n = 1000$ nodes in each layer, the average degree of each network is $\langle k\rangle = 6$; 2) Both layer $A$ and layer $B$ are the BA scale-free network with $n = 1000$ nodes in each layer, the degree distribution is $P(a) \sim a^{-\gamma}$, $\gamma = 2.1$, and the average degree of each network is $\langle k\rangle = 6$; 3) Layer $A$ is the ER random network with $n = 1000$ nodes, layer $B$ is the BA scale-free network with $n = 1000$ nodes, the degree distribution is $P(a) \sim a^{-\gamma}$, $\gamma = 2.1$, and the average degree of each network is $\langle k\rangle = 6$. At the beginning of the game, the probability that each individual in the game layer is cooperator $C$ or non-cooperator $D$ is the same. In order to eliminate the influence of random factors on the results, each of the following simulation results is the average of 100 independent experiments. For convenience, we let $\rho_A^C(t)$ and $\rho_B^C(t)$ denote the density of collaborators in layer $A$ and layer $B$ at $t$ game round, respectively, $\rho_A^C(\infty)$ and $\rho_B^C(\infty)$ denote the density of collaborators in $A$ and $B$ layers at steady state, respectively. In addition, we let $\rho^C(t) = \frac{1}{2n}[\sum_{i=1}^n \delta(s_i^A(t),1) + \sum_{i=1}^n \delta(s_i^B(t),1)]$ and $\rho^C(\infty) = \frac{1}{2n}[\sum_{i=1}^n \delta(s_i^A(\infty),1) + \sum_{i=1}^n \delta(s_i^B(\infty),1)]$ denote the density of collaborators of the whole system at $t$ game round and at steady state, respectively, where $\delta(x,y)$ is the $\delta$-function, when $x = y$, $\delta(x,y) = 1$, otherwise $\delta(x,y) = 0$.

Figure 2 plots the evolution of collaborators density $\rho^C(t)$ as a function of the number of games in the whole system. Comparing and analyzing fig. 2(a) and fig. 2(b), we can find that, when both layer networks are ER random networks, the individual density of cooperative behavior in the entire system gradually decreases with the increasing number of games. However, when both layer networks are BA scale-free networks, the individuals who take cooperative behavior reach the highest level in the entire system. The results demonstrate that the heterogeneous network structure is beneficial to promoting individuals to adopt the cooperative behaviors. In addition, fig. 2 illustrates that the individual density of cooperative behavior in the entire system gradually decreases as the lure income $T$ increases. The practice of individual cooperative

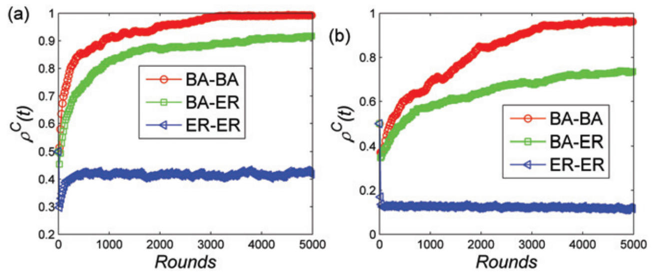


Fig. 2: The evolution diagram of collaborators density $\rho^C(t)$ against the number of games in the whole system, with the two-layer network structure under different circumstances. The parameter settings are as follows: $\alpha = 0.5, \beta = 0.5, H = 1$. (a) The collaborators density $\rho^C(t)$ varies with lure income $T = 1.2$. (b) The collaborators density $\rho^C(t)$ varies with the lure income $T = 1.4$.

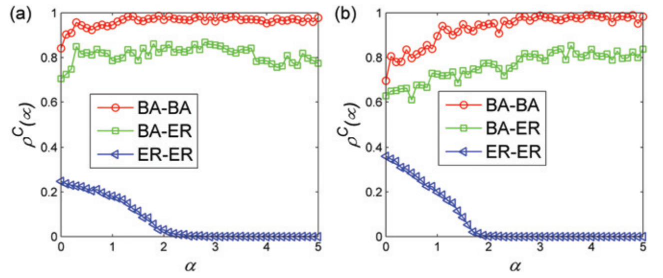


Fig. 4: The evolution of the final cooperators' density $\rho_C^A(\infty)$ and $\rho_C^B(\infty)$ as a function of $\alpha$. The final cooperators' density in the two-layered network gradually decreases as $\alpha$ increases. The parameters are set as: $T = 1.1$, $\beta = 0.5$, $\kappa_1 = \kappa_2 = 0.1$. (a) The collaborator density $\rho^C(\infty)$ varies with $\alpha$, while the memory length $H = 1$. (b) The collaborator density $\rho^C(\rho)$ varies with $\alpha$, while the memory length $H = 3$.

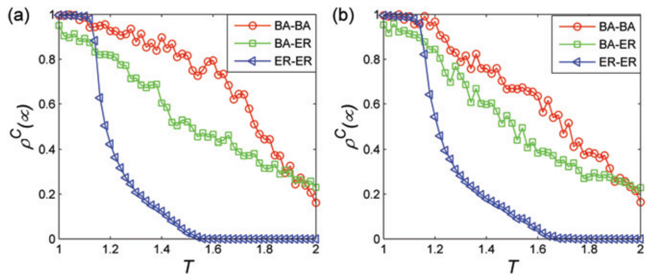


Fig. 3: The evolution diagram of the final collaborators density $\rho^C(\infty)$ against the lure income $T$ in the whole system, with the two-layer network structure under different circumstances. The parameter settings are as follows: $\alpha = 0.5$, $\beta = 0.5$. (a) The collaborator density $\rho^C(\infty)$ varies with memory length $H = 1$. (b) The collaborator density $\rho^C(\infty)$ varies with memory length $H = 3$.

behavior is not only related to the connection method between individuals and other individuals, but also related to the attitude (reward) of the entire social environment to collaborators and betrayers. Under the same network structure, the lure income $T$ affects how much the betrayer individual gains. Therefore, it can be obtained from the simulation results that if the betrayer obtains more benefits, it is not conducive to the diffusion of individual cooperative behavior.

Figure 3 presents the evolution diagram of the final collaborators density $\rho^C(\infty)$ against the lure income $T$ in the whole system under different kinds of two-layer network structures. In general, a larger lure income $T$ will bring higher expected income to the non-cooperative individual $D$. As shown in fig. 3, as the lure income $T$ increases, the final cooperative density $\rho^C(\infty)$ gradually decreases in three different types of two-layer networks. Through comparison analysis, it can be found that, when both the two-layer topologies are scale-free networks, the cooperative density $\rho^C(\infty)$ will remain at a higher level at the beginning as the lure income $T$ increases. However, the value of $\rho^C(\infty)$ will decrease rapidly, once the lure income $T$ increases to a certain threshold. When both the initial two-layer topologies are the ER random network, the final cooperative density $\rho^C(\infty)$ decreases faster as the lure income $T$ increases. Moreover, there will be no cooperative individual in the entire system when the lure income $T \geq 1.6$. However, cooperators and non-cooperators will still coexist in the system if the initial two-layer topologies are the BA networks when the lure income $T \geq 1.6$. Therefore, we can further conclude that heterogeneous networks can effectively promote cooperation among individuals.

Figure 4 shows the evolution diagram of the final collaborators density $\rho^C(\infty)$ as a function of the historical return memory decay factor $\alpha$ in the whole system. The results indicate that, once more than one layer of the network structure is a BA scale-free network, the collaborator density $\rho^C(\infty)$ gradually increases as the historical gain memory decay factor $\alpha$ increases. However, it is interesting to note that when both the initial two-layer network topologies are ER random networks, the density of the collaborator gradually decreases as the parameter $\alpha$ increases. Historical return memory decay factor $\alpha$ reflects the degree to which individuals are affected by historical returns. The smaller the value of $\alpha$, the more individuals are affected by historical returns. In contrast, the larger the value of $\alpha$ is, the less individuals are affected by historical returns. Therefore, it can be concluded from the simulation results that, if the network is more heterogeneous, the less the individuals are affected by historical returns, the better the cooperation will be among individuals. However, if both layers of the network are less heterogeneous, the more individuals are affected by historical returns, the better the cooperation among individuals is.

Figure 5 illustrates the evolution diagram of the final collaborators density $\rho^C(\infty)$ against the optional parameter $\beta$ in the whole system under different two-layer network structures. The results of the simulation indicate that the final collaborators density $\rho^C(\infty)$ in the system increases gradually with the optional parameter $\beta$.

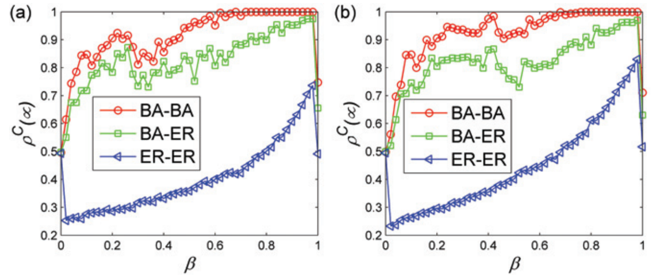


Fig. 5: The final collaborators density $\rho^C(\infty)$ against the optional parameter $\beta$. The parameter settings are as follows: $T = 1.2$, $\alpha = 1$. (a) The collaborator density $\rho^C(\infty)$ varies with $\beta$, while the memory length $H = 1$; (b) The collaborator density $\rho^C(\infty)$ varies with $\beta$, while the memory length $H = 3$.

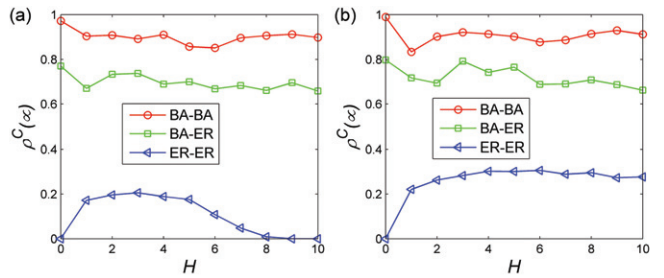


Fig. 6: The final collaborators density $\rho^C(\infty)$ with respect to memory length $H$. The parameter settings are as follows: $T = 1.3$, $\beta = 0.5$. (a) The collaborator density $\rho^C(\infty)$ varies with memory length $H$, while $\alpha = 1$. (b) The collaborator density $\rho^C(\infty)$ varies with memory length $H$, while $\alpha = 0.5$.

This result shows that if individuals change their own strategies mainly by imitating the strategies of their neighbors, it is beneficial to promote cooperation among individuals in the entire system. However, if individuals change their own strategy mainly by the strategies of the counterpart, then it is not conducive to cooperation among individuals.

Figure 6 indicates the evolution diagram of the final collaborators density $\rho^C(\infty)$ as a function of memory length $H$. Comparing and analyzing fig. 6(a) and fig. 6(b), it can be found that, if both layers of the network structure are BA scale-free network, or one layer is a BA scale-free network and the other layer is an ER random network, the collaborators density $\rho^C(\infty)$ in the final system gradually decreases with the increasing of the historical memory length $H$. However, if both layers of the network structure are ER random networks, the collaborator density $\rho^C(\infty)$ gradually increases in the final system at the beginning, and then gradually decreases when the individual memory decay factor $\alpha$ is large. Moreover, the collaborator density $\rho^C(\infty)$ gradually increases in the final system, when the individual memory decay factor $\alpha$ is small. Comparing fig. 6(a) and fig. 6(b), we can get that when the length of the individual's memory of historical returns is too long, then the cooperation among individuals in the entire system will be blocked if at least one layer network is a heterogeneous structure.

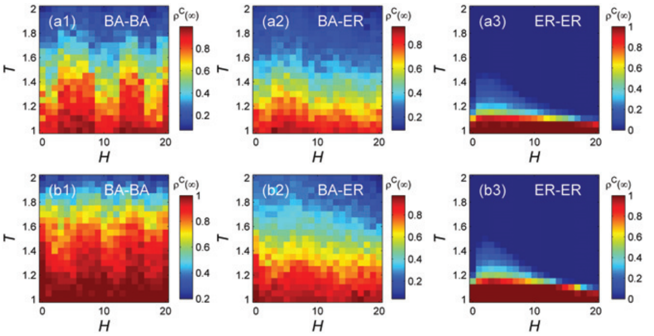


Fig. 7: The evolution of the final collaborators density $\rho^C(\infty)$ with memory length $H$ and lure income $T$. The parameter settings are as follows: $\alpha = 1$. (a1)–(a3) With the different two-layer network structure, the collaborator density $\rho^C(\infty)$ varies with $H$ and $T$, while $\beta = 0.3$. (b1)–(b2) With the different two–layer network structure, the collaborator density $\rho^C(\infty)$ varies with $H$ and $T$, while $\beta = 0.7$.

Figure 7 shows the evolution diagram of the final collaborators density $\rho^C(\infty)$ against memory length $H$ and lure income $T$ in the whole system. With the increase of the lure income $T$, the final partner density $\rho^C(\infty)$ gradually decreases in the three different types of two-layer networks. Comparing fig. 7(a1) to fig. 7(b3), it can be observed that when the length of the individual's memory of historical returns is too long, then the cooperation among individuals in the entire system will be blocked if at least one layer in the two-layer network structure is a heterogeneous network structure. If both layers of the network structures are ER random networks, the collaborator density $\rho^C(\infty)$ gradually increases in the final system at the beginning, and then gradually decreases when the individual memory decay factor $\alpha$ is large. Therefore, if both the two-layer network structures are the ER random networks, there will be an optimal memory decay factor $H$, which maximizes the cooperation of individuals.

**Conclusions.** – In general, the multilayer networks are often a significantly more apt description of real-life systems than isolated or single networks. In this paper, we analyze the evolution of the PD game in a two-layered network, by considering the effect of memory. The results of the research indicate that the heterogeneous network structure is conducive to promoting individuals to adopt cooperative behaviors. As the lure income $T$ increases, the individual density of cooperative behavior in the entire system gradually decreases. In addition, if there is a layer of network that is more heterogeneous, then the less the individual is affected by historical gains, the better the cooperation will be among individuals. However, if both layers of the networks are less heterogeneous, the greater the impact of historical returns on individuals is, the more beneficial it is for cooperation among individuals.

Furthermore, if individuals change their strategies mainly by imitating the strategies of their neighbors, then it is beneficial to promote cooperation among individuals in the entire system. However, it is not conducive to cooperation among individuals if individuals change their own strategy mainly through strategies of their counterparts. The final result shows that, if at least one layer in the two-layer network structure is heterogeneous, when the length of the individual's historical memory is too long, the cooperation between individuals in the entire system will be blocked. How to rationally quantify and study individual behavioral strategies has always been an unsolved and important problem with the complex inter-individual communication. Our research attempts to provide a feasible and reasonable solution for game research in multilayer networks to reveal the interaction relationship among individuals.

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