A Letters Journal Exploring the Frontiers of Physics

January 2020

EPL, **129** (2020) 28001
doi: 10.1209/0295-5075/129/28001

www.epljournal.org

# Power-law return-volatility cross-correlations of Bitcoin

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received 22 August 2019; accepted in final form 28 January 2020
published online 11 February 2020

PACS 89.65.Gh – Economics; econophysics, financial markets, business and management
PACS 05.45.Tp – Time series analysis

**Abstract** – This paper investigates the return-volatility asymmetry of Bitcoin. We find that the cross-correlations between return and volatility (squared return) are mostly insignificant on a daily level. In the high-frequency region, we find that a power-law appears in negative cross-correlation between returns and future volatilities, which suggests that the cross-correlation is long-ranged. We also calculate a cross-correlation between returns and the power of absolute returns, and we find that the strength of the cross-correlations depends on the value of the power.



**Introduction.** – It has long been known that return and volatility are negatively correlated, and early studies [1,2] attempted to explain the return-volatility asymmetry as a leverage effect: a drop in the value of a stock increases finance leverage or debt-to-equity ratio, which makes the stock riskier and increases the volatility. The other promising explanation for the return-volatility asymmetry is the volatility feedback effect discussed in [3,4]: if volatility is priced, an anticipated increase in volatility raises the required return, leading to an immediate stock price decline. Although the two effects suggest the same negative correlations, the causality is different [5].

Comparing the two effects empirically, Baekaert *et al.* [5] and Wu [6] argue that the dominant determinant is the volatility feedback effect. However, the studies using GARCH-type models [7–9] suggest that volatility increases more after negative returns than positive ones, which favors the leverage effect.

To discuss the full temporal structure of return-volatility asymmetry, using squared returns as a proxy of volatility, Bouchaud *et al.* [10] calculate the return-volatility correlation function and find that returns and future volatilities are negatively correlated. On the other hand, reverse correlations, *i.e.*, correlations between future returns and volatilities are found to be negligible. The results are fitted to an exponential function, and it is concluded that the correlations are short-ranged. In addition, the decay times[1] are estimated to be about 10 (50) days for stock indices (individual stocks).

[1]The correlations are fitted with an exponential function of $\alpha\exp(-t/\tau)$, and the decay time is defined by $\tau$.

While, for most developed markets, negative correlations between returns and future volatilities are found, an interesting phenomenon is observed in Chinese markets. Qiu *et al.* [11] calculate the return-volatility correlation function for equities in the Chinese market and find that returns and future volatilities are "positively" correlated, which is called the anti-leverage effect. Further studies [12,13] also support the anti-leverage effect in the Chinese market.

In this study, we focus on the return-volatility asymmetry of the Bitcoin market. Since the first proposal of cryptocurrency in 2008 [14], the Bitcoin system, based on a peer-to-peer network and blockchain technology, developed quickly, and Bitcoin has become widely recognized as a payment medium. In recent years, a large body of literature has investigated various aspects of Bitcoin, *e.g.*, hedging capabilities [15], inefficiency [16–19], multifractality [20], extreme price fluctuations [21], liquidity and efficiency [22,23], transaction activity [24], complexity synchronization [25], long memory [26], and so forth.

Although the return-volatility asymmetry of Bitcoin has been investigated using various models, such as asymmetric GARCH-type and stochastic volatility, it seems that a consistent picture of the return-volatility asymmetry of Bitcoin has not yet been obtained. For instance, while Bouoiyour *et al.* [27] observe a volatility asymmetry that reacts to negative news rather than positive, Katsiampa [28] and Baur *et al.* [29] find an inverted volatility asymmetry that reacts to positive news rather than negative. Moreover, several studies [20,30,31] find no evidence of a leverage effect in Bitcoin prices.

Bouri *et al.* [32] investigate return-volatility asymmetry in two periods separated by the price crash of 2013. They find that, while before the crash Bitcoin shows inverted volatility asymmetry, after the crash, and for the whole period, no significant volatility asymmetry is observed. Using the stochastic volatility model, Philip *et al.* [33] find that one day ahead volatility and returns are negatively correlated.

Here, we approach the return-volatility asymmetry of Bitcoin through return-volatility cross-correlations. We calculate a cross-correlation between returns and a power of absolute returns. This is in part motivated by the existence of the Taylor effect [34,35], which suggests that the strength of autocorrelations of a power of absolute returns, $|r|^d$, is dependent on the value of power $d$, and, typically, the maximum autocorrelations are obtained at $d \approx 1$ for stocks [35] and at $d \approx 0.5$ for exchange rates [36]. The Taylor effect is also present for Bitcoin [37]. Thus, we investigate how the cross-correlation of Bitcoin is dependent on the value of power.

This paper is organized as follows. The next section describes the data and methodology. The third section presents the empirical results. Finally, we conclude in the fourth section.

**Data and methodology.** – We use Bitcoin tick data (in dollars) traded on Bitstamp from January 10, 2015 to January 23, 2019 and downloaded from Bitcoincharts[2]. Let $p_{t_i}$; $t_i = i\Delta t$; $i = 1, 2, \ldots, N$ be the time series of Bitcoin prices with sampling period $\Delta t$. We define the return, $R_i$, by the logarithmic price difference, namely,

$$R_{i+1} = \log p_{t_{i+1}} - \log p_{t_i}. \tag{1}$$

In this study, we consider high-frequency returns with $\Delta t = 2$ min, and we also consider daily returns. We further calculate the normalized returns by $r_i = (R_i - \bar{R})/\sigma_R$, where $\bar{R}$ and $\sigma_R$ are the average and standard deviation of $R_i$, respectively. We calculate the cross-correlation, $CC_d(j)$, between returns and the $d$-th power of absolute returns at lag $j$ as

$$CC_d(j) = \frac{E[(r_t - \mu_r)(|r_{t+j}|^d - \mu_{|r|^d})]}{\sigma_r \sigma_{|r|^d}}, \tag{2}$$

where $\mu_r$ and $\mu_{|r|^d}$ are the averages of $r_i$ and $|r_i|^d$, and $\sigma_r$ and $\sigma_{|r|^d}$ are the standard deviations of $r_i$ and $|r_i|^d$, respectively. $E[O_j]$ in eq. (2) stands for the average over $N - j$ values of $O_j$. For $d = 2$, eq. (2) reduces to the usual definition of the return-volatility correlation that uses squared returns as a proxy of volatility [12,13] except the normalization.

We calculate $CC_d(j)$ for $d = 0.1$ to $3.0$ every $0.1$ step. For positive $j$'s, $CC_d(j)$ at $d = 2$ evaluates the relationships between returns and future volatilities. The reverse correlations, *i.e.*, relationships between future returns and volatilities, are obtained for negative $j$'s.

[2]http://api.bitcoincharts.com/v1/csv/.

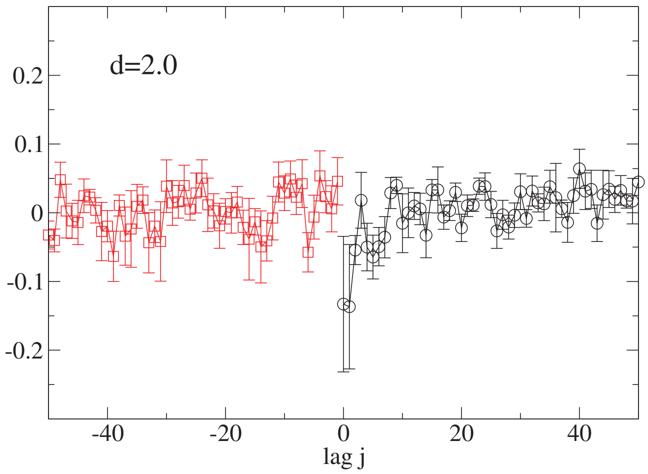


Fig. 1: Cross-correlation $CC_d(j)$ for daily returns as a function of lag $j$ at $d = 2.0$. Error bars of data points represent one-sigma errors calculated by the jackknife method.

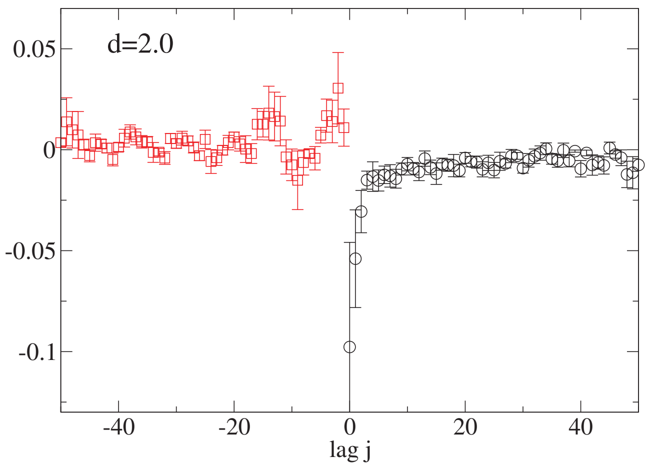


Fig. 2: Cross-correlation $CC_d(j)$ for high-frequency returns as a function of lag $j$ at $d = 2.0$. Error bars of data points represent one-sigma errors calculated by the jackknife method.

**Empirical results.** – First, in fig. 1, we show the cross-correlation, $CC_d(j)$, of the daily returns for $d = 2.0$. The cross-correlations are mostly consistent with zero for both positive and negative lags, $j$, except for $j = 0$ and $1$, at which negative correlations are observed. For other $d$'s, similar results are obtained. Thus, at the daily level, the cross-correlations are mostly insignificant, except for contemporaneous and small, positive lags.

Next, in fig. 2, we show the cross-correlation, $CC_d(j)$, calculated with 2 min, high-frequency returns for $d = 2.0$. For positive $j$'s, we find negative cross-correlations lasting from small to large lags, which is consistent with the results observed for developed markets [10,38]. For negative $j$'s, we observe positive, but smaller, cross-correlations at several small lags. For larger (negative) lags, the

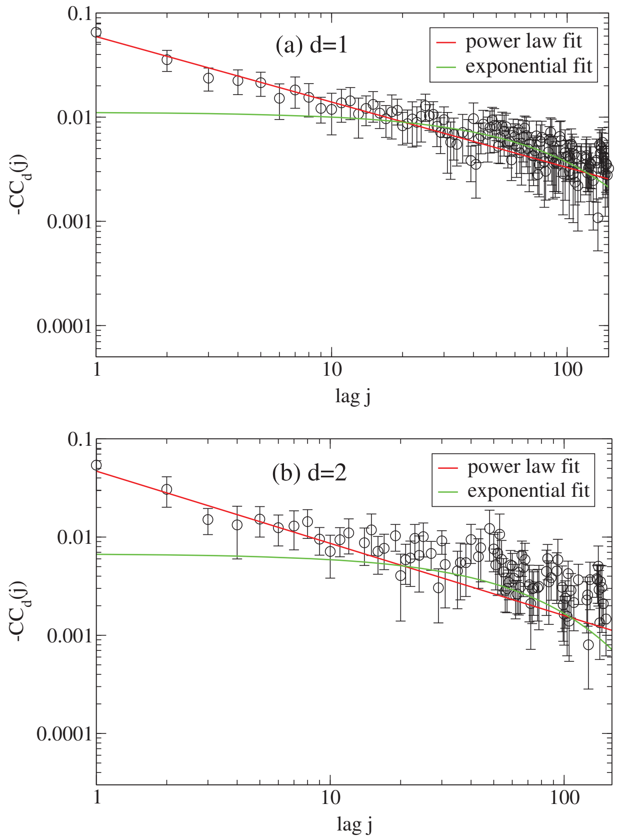


Fig. 3: Cross-correlation $CC_d(j)$ at $d = 1$ and 2 in log-log scale. (a) $d = 1$ and (b) $d = 2$. Error bars of data points represent one-sigma errors calculated by the jackknife method. For better visibility, noisy data that are consistent with zero within 1.5 sigma errors are omitted from the pictures. The red (green) solid curve represents the power-law (exponential) fitting to the data. The reduced chi square: (a) 0.796 (power law) and 1.09 (exponential), (b) 1.13 (power law) and 1.33 (exponential).

cross-correlations are consistent with zero. For the contemporaneous correlations, *i.e.*, $j = 0$, we observe negative cross-correlations.

To examine the scaling properties of the cross-correlations at positive lags[3], we plot negative values of the results, *i.e.*, $-CC_d(j)$ in fig. 3 in log-log scale.

We fit the cross-correlations with the power-law function of $\kappa j^{-\gamma}$ and the exponential function of $\alpha \exp(-j/\tau)$ in a range of $j = [1, 200]$, where $\kappa$, $\gamma$, $\alpha$, and $\tau$ are fitting parameters. The fitting results of the power-law (exponential) function are depicted by the red (green) curve in fig. 3. We find that the cross-correlations are better described by the power-law function than by the exponential function. In particular, we recognize that the exponential function does not adequately describe the data points of cross-correlations at small lags. This finding is different from the results of previous studies that observe

[3]Since the cross-correlations at negative lags quickly become consistent with zero at very small lags, we only consider those at positive lags.

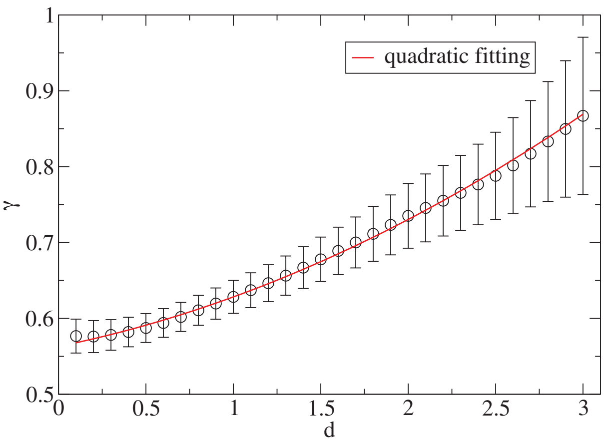


Fig. 4: Fitting results of $\gamma$ as a function of $d$, where $\gamma$ is a parameter of the power-law function of $\kappa j^{-\gamma}$. Error bars of the results represent the asymptotic standard errors of the parameter. The solid curve represents the quadratic fitting to $\gamma$.

Table 1: Results of fitting to a quadratic function, $\gamma(d) = \alpha d^2 + \beta d + \rho$. The values in parentheses represent the asymptotic standard errors of the fitting parameters.

| | $\alpha$ | $\beta$ | $\rho$ |
|---|---|---|---|
| Bitcoin | 0.0184(13) | 0.0470(35) | 0.5630(18) |

an exponential behavior in the cross-correlation [10,11,13]. The exponential behavior in the cross-correlation indicates that the cross-correlation quickly disappears as the lag increases, *i.e.*, the correlation is short-ranged. On the other hand, the power-law behavior[4] that we observe indicates that the cross-correlation decreases slowly with the lag, *i.e.*, the correlation is long-ranged.

In fig. 4, we plot the results of $\gamma$ as a function of $d$ and find that $\gamma$ increases with $d$. We fit the results to a quadratic function, $\gamma(d) = \alpha d^2 + \beta d + \rho$, where $\alpha$, $\beta$, and $\rho$ are fitting parameters; the fitting results are listed in table 1. From the fitting results, we recognize that for $d \to 0$, the power $\gamma$ seems to approach the value around 0.56. To investigate the strength of the cross-correlations, we plot $\kappa$ as a function of $d$ in fig. 5. More precisely, $\kappa$ represents the strength of the cross-correlations at lag $j = 1$. We find that $\kappa$ is a convex function and that the maximum strength is obtained around $d \approx 1.4$. Thus, the correlation $CC_d(1)$ at $d \approx 1.4$ gives a stronger correlation than the traditional cross-correlation defined at $d = 2$.

**Conclusion.** – At the daily level, cross-correlations are mostly insignificant for Bitcoin. By examining

[4]More precisely, the power-law exponent, $\gamma$, should be $\gamma < 1$ for a long-range behavior (*e.g.*, [39]). In addition, the summation of the correlations diverges for $\gamma < 1$. As seen in fig. 4, the condition of $\gamma < 1$ is satisfied, at least for $d < 3$.

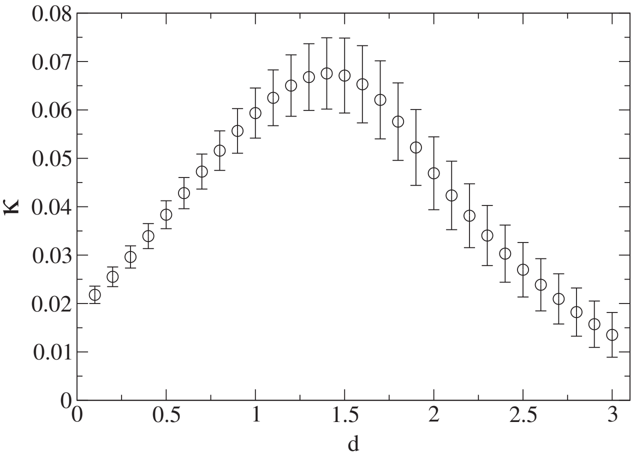


Fig. 5: Fitting results of $\kappa$ as a function of $d$, where $\kappa$ is a parameter of the power-law function of $\kappa j^{-\gamma}$. $\kappa$ corresponds to the strength of a cross-correlation at lag $j = 1$. The error bars of the results represent the asymptotic standard errors of the parameter.

high-frequency Bitcoin returns, we find that returns and future volatilities are negatively correlated and the cross-correlations between returns and future volatilities show a power-law behavior. We calculate cross-correlations between returns and the $d$-th power of absolute returns and find that the maximum cross-correlation is obtained at $d \approx 1.4$. Thus, we were able to obtain clear evidence on the cross-correlation by choosing other values of $d$, rather than the traditional value of $d = 2$.

Our findings on cross-correlations suggest that, in modeling asset time series, we should more seriously consider models that produce power law behavior in the cross-correlations.

For example, ref. [40] proposes a fractional random walk model combined with a simple auto-regressive conditional heteroskedastic model, denoted as FRWARCH, and finds that the FRWARCH model exhibits a power law in the cross-correlations.

There exist universal properties, such as volatility clustering and no autocorrelations in returns, that appear across various assets. These properties are called the stylized facts (*e.g.*, [41]). The existence of stylized facts suggests that the price formation is governed by certain common dynamics. If Bitcoin has a different property in the cross-correlation from other assets, there could be a different type of dynamics in Bitcoin. To come to a definite conclusion about whether the power-law behavior only appears in Bitcoin, it would be desirable to examine other assets in detail.

* * *

Numerical calculations for this work were carried out at the Yukawa Institute Computer Facility and at the facilities of the Institute of Statistical Mathematics. This work was supported by JSPS KAKENHI (Grant No. JP18K01556) and the ISM Cooperative Research Program (2018-ISMCRP-0006).

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