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Solitary pulses of a conformable nonlinear differential equation governing wave propagation in low-pass electrical transmission line

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Abstract

In this work, we investigate soliton solutions for the conformable nonlinear differential equation governing wave-propagation in low-pass electrical transmission lines. Adopting two integration techniques, we construct dark and bright solitary waves, jacobian elliptic function solutions and trigonometric solutions. The obtained results are relevant and will probably help to carry data and codify them in telecommunication.

Keywords: solitary waves, conformable derivative, wave-propagation

(Some figures may appear in colour only in the online journal)

1. Introduction

Nowadays, investigation of exact solutions of the nonlinear differential equation advance beyond measure. Moreover, a conformable derivative nonlinear ordinary differential equation have become very important do to their application in communication system. So, the problem of obtaining an exact solution of the conformable nonlinear equation attracts a lot of attentions. However, several works have been done in optical fibers, fluid mechanics, biology, economy just to name a few [1–5]. Thus, various mathematical methods have been developed to solve them such as F-expansion method, the generalize Kudryashov method, $\exp(-\psi(\xi))$-expansion method, the (G'/G^2)-expansion method, the trial expansion method and so on [6–19]. We aim, in this work to apply the auxiliary equation method and Sinh–Gordon method to ease the investigation of the exact solutions to the conformable derivative nonlinear differential equation governing slowly modulated wave propagating in low-pass electrical transmission line, which consist of a number of LC (inductance-capacitance) connected and the dissipative effect is neglected [5]. The rest of details of the derivation of (1) are given by [5].

$$D_{tt}^{2\alpha}u(x,t) - vD_{tt}^{2\alpha}u^2(x,t) + \beta D_{tt}^{2\alpha}u^3(x,t) - \delta D_{xx}^{2\alpha}u(x,t) - \frac{\delta^4}{12}D_{xxxx}^{4\alpha}u(x,t) = 0,\ 0 < \alpha \leqslant 1. \quad (1)$$

However, $u(x, t)$ is the voltage in transmission lines and v, δ and β are constants, while the variable x is the propagation distance and t represents the slow time.

The rest, of our work is organized a follows: section 2 summarize the conformable derivative theorem. The glimpse of the methods apply are given in sections 3 and 4 applies the methods for obtaining soliton solutions to the proposed model, follows by some graphical representation. The last section concludes the work.

2. The conformable derivative

Let us gives the short definition of the conformable derivative of order $\alpha\epsilon(0,\,1)$ as follows:

$$\frac{\mathrm{d}^{\alpha}h(t)}{\mathrm{d}t^{\alpha}}=\lim_{\varsigma\to+\infty}\frac{h(t+\varsigma t^{1-\alpha})-h(t)}{\varsigma},\quad f:(0,\,\infty)\to\mathbb{R}. \tag{2}$$

Theorem 1. *Let $\alpha\epsilon(0,\,1]$ and $f=f(t),\,h=h(t)$ be α-conformable differentiable at $t<0$. Hence,*

- $D_t^{\alpha}(af+bh)=aD_t^{\alpha}f+bD_t^{\alpha}h,$ *and* $a,\,b\epsilon\mathbb{R}.$
- $D_t^{\alpha}(t^{\beta})=\beta t^{\beta-\alpha}$ *and* $\beta\epsilon\mathbb{R}.$
- $D_t^{\alpha}(fh)=hD_t^{\alpha}(f)+fD_t^{\alpha}(h)$.
- $D_t^{\alpha}\left(\frac{f}{h}\right)=.\frac{hD_t^{\alpha}(f)-fD_t^{\alpha}(h)}{g^2}.$

3. Glimpse of the methods

3.1. The auxiliary equation method

We surmise the general conformable derivative nonlinear PDE as follows

$$P(u,\,D_t^{\alpha}u,\,D_x^{\alpha}u,\,D_y^{\alpha}u,\,D_t^{2\alpha}u,\,D_t^{\alpha}D_x^{\beta}u,\,\ldots)=0,\quad 0<\alpha,\,\beta<1, \tag{3}$$

and $u(x,\,t)$ is an unknown function, while P is a polynomial of u and its partial fractional derivatives. The next following steps describe the traveling wave solution obtained by auxiliary equation method.

Step 1: We first used the following fractional complex transformation, and suppose $u(x,\,t)=U(\xi)$,

$$\xi=\frac{k_1}{\Gamma(1+\alpha)}t^{\alpha}+\frac{k_2}{\Gamma(1+\alpha)}x^{\alpha}, \tag{4}$$

where $k_1,\,k_2$ are constants to be determined later, and $k_1,\,k_2\neq0$. By using the above transformation, it is obtained the ordinary differential equation of the form

$$G(U,\,U',\,U'',\,U''',\,\ldots)=0, \tag{5}$$

Step 2: Surmise that the exact solutions of ODE (5) can be expressed [20].

$$U(\xi)=A_0+\sum_{i=1}^{n}A_i[g(\xi)]^i, \tag{6}$$

and $g(\xi)$ satisfies the following auxiliary equation.

$$g^2{}_{\xi}=2(C_0+C_1g+C_2g^2+C_3g^3+C_4g^4), \tag{7}$$

$$g_{\xi\xi}=C_1+2C_2g+3C_3g^2+4C_4g^3, \tag{8}$$

with $g_{\xi}=\frac{\partial g}{\partial\xi}$, $C_i(i=(1,\,2,\,3,\,4))$, $A_0,\,A_i,\,i=(1,\,2,\,\ldots..,\,N)$, are constants to be determined later, and $A_i\neq0$.

Step 3: Under the terms of the method, it is suppose that solution of (6) can be written in the following form

$$U(\xi)=A_0+A_1g(\xi)+A_2g(\xi)^2+A_3g(\xi)^3+\ldots\ldots A_Ng(\xi)^N, \tag{9}$$

where A_0, A_1, A_3, A_4 and A_N are constant to be determined later.

Step 3: The value of integer N, is obtained by using the balance principle between the nonlinear terms and the highest order derivative come in the obtained ordinary differential equation. More precisely, if the degree of $U(\xi)$ is $\deg[U(\xi)]=N$, then the degree of the other terms will be written as follows

$$\deg\left[\frac{\mathrm{d}^qU(\xi)}{\mathrm{d}\xi^q}\right]=N+q, \tag{10a}$$

$$\deg\left[(U(\xi))^p\left(\frac{\mathrm{d}^qU(\xi)}{\mathrm{d}\xi^q}\right)^s\right]=Np+s(N+q). \tag{10b}$$

Step 4: Substituting (9), (8) and (7) into (5) provides a polynomial $g(\xi)$ of ξ. Next, collecting all the coefficient $g(\xi)^i$, $(i=0,\,1,\,2,\,\ldots\ldots..N)$ yield a system of algebraic equation. Solving this system, we describe the variable coefficients of $A_0,\,A_i,\,i=(1,\,2,\,\ldots..,\,N)$ then the solution to (3) can be obtained in terms of $g(\xi)$.

Step 5: To obtain the exact solutions to (3), the following solutions of (7) and (8) are used.

case 1: for $C_2>0$ and $C_4<0$, $C_0=C_1=C_3=0$.

$$g(\xi)=\sqrt{\frac{-C_2}{C_4}}\,\mathrm{sech}(\sqrt{2C_2}\,\xi), \tag{11}$$

case 2: for $C_2<0$ and $C_4>0$, $C1=C3=0$, $C_0=C_2^2/4C_4$.

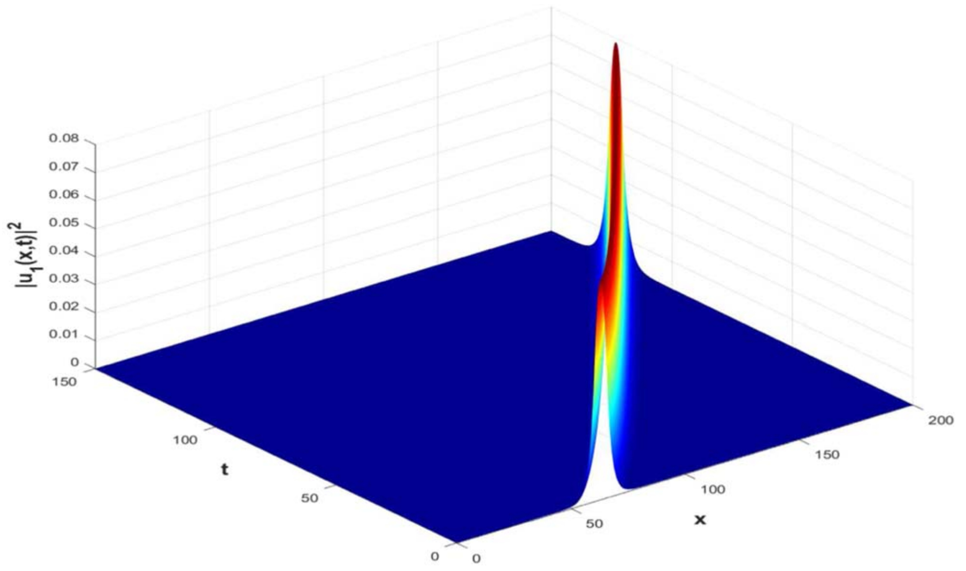
$$g(\xi)=\sqrt{\frac{-C_2}{C_4}}\,\tanh(\sqrt{-C_2}\,\xi), \tag{12}$$

case 3: for $C_2>0$ and $C_4>0$, $C_0=C_1=0$.

$$g(\xi)=\frac{C_2\,\mathrm{sech}^2(\sqrt{-C_2}\,\xi)}{2\sqrt{C_2C_4}\,\tanh\left(\sqrt{2C_2}\,\frac{\xi}{2}\right)}, \tag{13}$$

case 4: for $C_2>0$ and $C_2{}^2-4C_2C_4>0$, $C_0=C_1=0$.

$$g(\xi)=\frac{2C_2\,\mathrm{sech}^2(\sqrt{2C_2}\,\xi)}{\sqrt{C_2{}^2-4C_2C_4}-C_3\,\mathrm{sech}(\sqrt{2C_2}\,\xi)}, \tag{14}$$



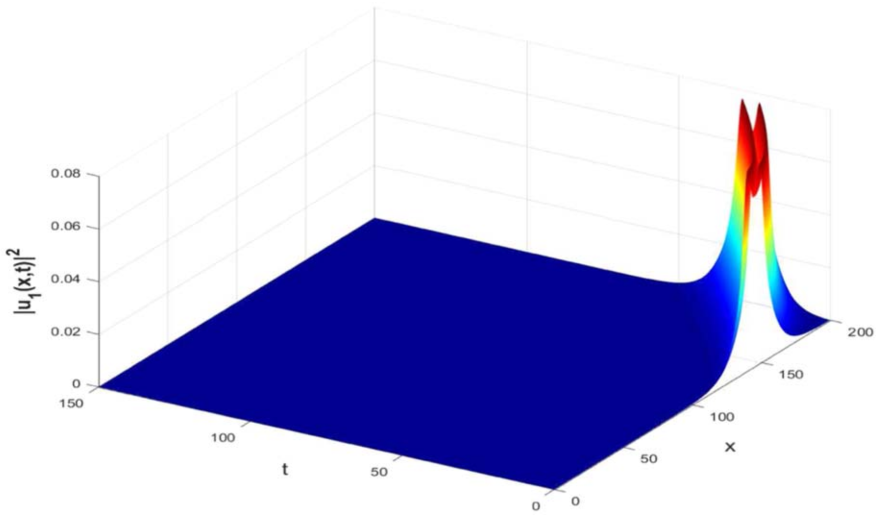


Figure 1. Spatiotemporal plot of bright solitons $u_1(x, t)$ at $\alpha = 0.9$ and $\alpha = 0.75$ respectively.

case 5: for $C_2 > 0$, $C_0 = C_1 = 0$.

$$g(\xi) = \frac{2C_2 C_3 \operatorname{sech}^2\left(\sqrt{2C_2 \frac{\xi}{2}}\right)}{C_2 C_4\left(1 - \tanh\left(\sqrt{2C_2 \frac{\xi}{2}}\right)\right)^2}, \tag{15}$$

where C_0, C_1, C_2, C_3 and C_4 are arbitrary constants. Therefore, using (10–114) and (9), the exact solutions to (3) can be obtained.

3.2. The Sinh–Gordon method

For the Sinh–Gordon equation, the following preliminaries are adopted [3]

$$\frac{\partial^2 u}{\partial x \partial t} = \alpha \sinh u, \tag{16}$$

where α is a constant. By using the traveling-wave transformation (4) and then adopted $u(x, t) = U(\xi)$, gives the ordinary differential equation

$$\frac{\mathrm{d}^2 U}{\mathrm{d}\xi^2} = \frac{\alpha}{(k_1 + k_2)} \sinh U \tag{17}$$

k_1, k_2 are constants to be determined. Integrating (17) once gives

$$\left(\frac{\mathrm{d}}{\mathrm{d}\xi}\frac{1}{2}U\right)^2 = \frac{\alpha}{(k_1 + k_2)} \sinh^2\left(\frac{1}{2}U\right) + c. \tag{18}$$

with integration constant c. To surmise $c = 0$, $\frac{\alpha}{(k_1 + k_2)} = 1$ and $\frac{1}{2}U = w$, so, it is obtained

$$\frac{\mathrm{d}w(\xi)}{\mathrm{d}\xi} = \sinh w(\xi). \tag{19}$$

Thereafter, with (19), it can be obtained soliton solutions to the nonlinear partial differential equation (3), and we note that (19) is a special case of (17) or (18).

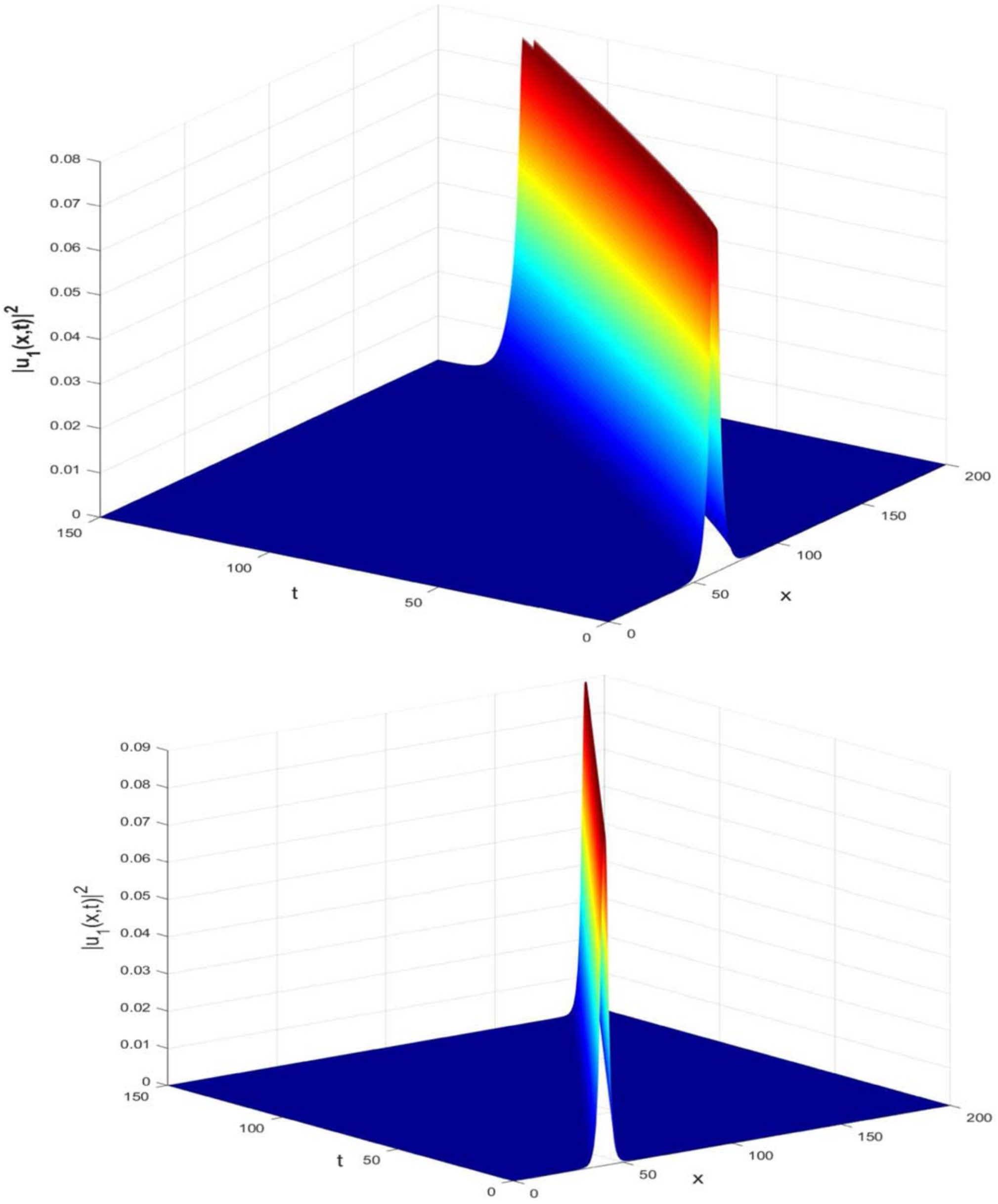


Figure 2. Spatiotemporal plot of bright $u_1(x, t)$ at $\alpha = 1$ and $\alpha = 0.58$ respectively.

To build relevant jacobian elliptic function solution to (3), the following form of (19) can be adopted

$$\frac{d^2 w(\xi)}{d^2\xi} = \frac{1}{2}\sinh 2w(\xi). \tag{20}$$

Considering $\phi = 2w$, it is obtained from (20) the following expression.

$$\left(\frac{dw(\xi)}{d\xi}\right)^2 = \sinh^2 w(\xi) + c. \tag{21}$$

where c is an integration constant. However, from (21), it is recovered the following solutions

$$\sinh[w(\xi)] = cs(\xi; m), \tag{22a}$$

$$\cosh[w(\xi)] = ns(\xi; m), \tag{22b}$$

Here m is the modulus of Jacobian elliptic function solutions [4]. Substitute (22a) and (22b) into (20), the integration constant c satisfies

$$c = 1 - m^2, \tag{23}$$

and will be used in the following results. To apply the method, the following steps will be follows

Step 1: We suppose that (5) admits the following solutions

$$U(\xi) = U(w(\xi)) = A_0 + \sum_{i=1}^{N}\cosh^{i-1} w[A_i \sinh w + B_i \cosh w], \tag{24}$$

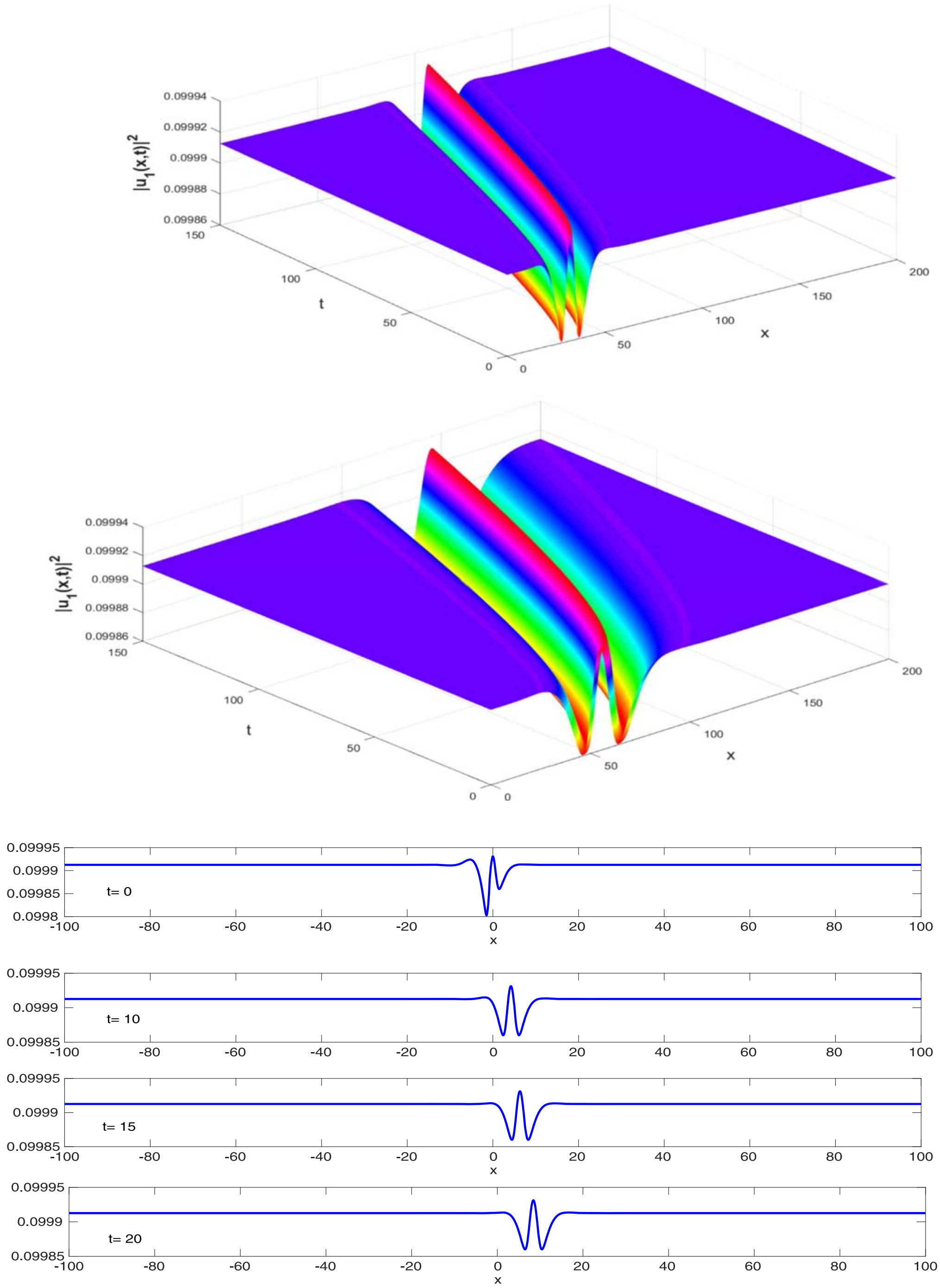


Figure 3. Spatiotemporal plot and the corresponding profile of W-shaped bright solitons $u_1(x, t)$ at $\alpha = 0.75$ and $\alpha = 0.6$ respectively.

where $w = w(\xi)$ satisfies (20) or (21) and $A_i(i = 1, 2, 3, \dots N)$, $B_i(i = 1, 2, 3, \dots, N)$ are constants to be determined later.

Step 2: The degree N is obtained by balancing the highest degree linear term and nonlinear term in (5).

Step 3: Substitute (24) along with (19) and (21) into (5) and the hyperbolic polynomial for w is recovered.

Step 4: Thereafter, set to zero the coefficients of $\sinh^j w \cosh^j w$ $(i = 0, 1; j = 0, 1, 2, \dots, N)$, to obtain a set of algebraic equations with the parameters $A_i(i = 1, 2, 3, \dots N)$, $B_i(i = 1, 2, 3, \dots, N)$.

Step 5: Solving the obtained algebraic equations, derived the doubly periodic solutions of the ODE (5).

4. Application of the methods

4.1. On solving the conformable derivative nonlinear differential equation governing wave propagation in low-pass electrical transmission lines with auxiliary equations method

In this section we will apply the auxiliary equation method to construct exact solutions of the conformable derivative nonlinear differential equation governing wave propagation in low-pass electrical transmission lines [4].

To achieve the nonlinear ordinary differential equation of (1), the traveling wave transformation (4) is used. Thereafter,

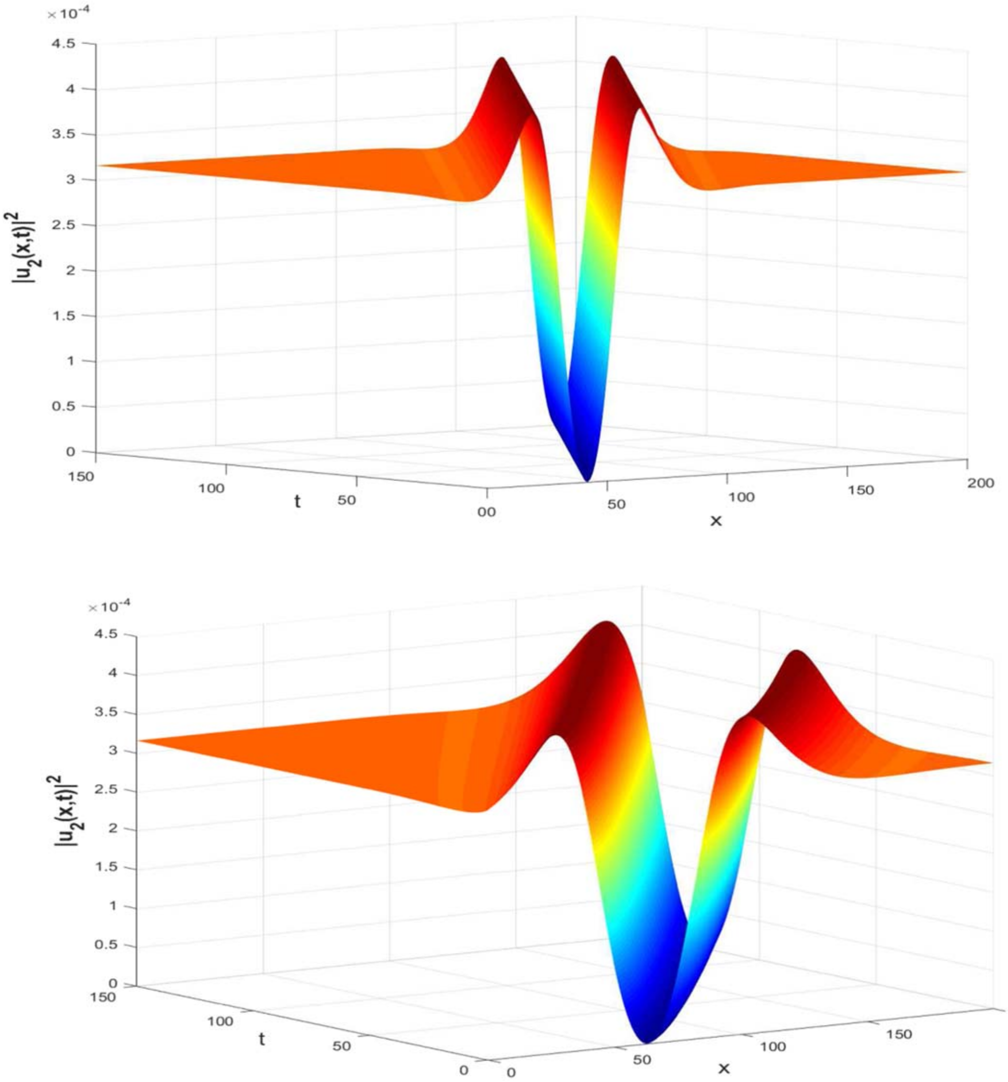


Figure 4. Spatiotemporal plot dark solitons $|u_2(x,\,t)|^2$ at $\alpha = 1$ and at $\alpha = 0.9$ respectively.

the nonlinear differential equation is obtained as follows

$$(k_1^2 - \delta k_2^2)U(\xi) - k_1^2 v U^2(\xi) + \beta k_1^2 U^3(\xi) - \frac{\delta^4}{12}k_2^2 U''(\xi) = 0. \tag{25}$$

Using the balance principle between $U^3(\xi)$ and U'', gives $N = 1$. Hence, we surmise that (6) can take the following expression.

$$U(\xi) = A_0 + A_1(g(\xi)), \tag{26}$$

Substitute (26) and (8) into (25), it is obtained the polynomial algebraic equation in terms of $(g(\xi))^j$, $(j = 0, \pm 1, \pm 2, \pm 3, \ldots)$ for each case.

Collect all the coefficient of the derive polynomials and setting them to zero the set of algebraic equations is obtained. Thereafter, by aid of the MAPLE, it is obtained the following results for each case, **R1**: $A_0 = \frac{3}{2}\frac{(k_1^2 - \delta k^2{}_2)}{k_1^2 v}$, $A_1 = A_1$, $k_1 = k_1$, $k_2 = k_2$, $\beta = \frac{2}{9}\frac{k_1^2 v^2}{(k_1^2 - \delta k_2^2)}$, $C_2 = \frac{3(-k_1^2 + \delta k_2^2)A_1}{\delta^4 k_2^2}$, $C_4 = -\frac{2}{3}\frac{A_1^3 k_1^4 v^2}{\delta^4 k_2^2(-k_1^2 + \delta k_2^2)}$

$$u_1(x,\,t) = A_0 + A_1\sqrt{\frac{-C_2}{C_4}}\,\mathrm{sech}\left(\sqrt{2C_2}\left(\frac{k_1}{\Gamma(1+\alpha)}t^{\alpha} + \frac{k_2}{\Gamma(1+\alpha)}x^{\alpha}\right)\right), \tag{27}$$

R2: $A_0 = -\frac{3}{2}\frac{(k_1^2 - \delta k^2{}_2)}{k_1^2 v}$, $A_1 = A_1$, $k_1 = k_1$, $k_2 = k_2$, $\beta = -\frac{2}{9}\frac{k_1^2 v^2}{-k_1^2 + \delta k_2^2}$, $C_2 = \frac{3(-k_1^2 + \delta k_2^2)A_1}{\delta^4 k_2^2}$, $C_4 = -\frac{2}{3}\frac{A_1^3 k_1^4 v^2}{\delta^4 k_2^2(-k_1^2 + \delta k_2^2)}$ Thus, for $C_2 < 0$ and $C_4 > 0$, and $C_0 = C_1 = C_3 = 0$, dark solitary waves

$$u_2(x,\,t) = A_0 + A_1\sqrt{\frac{-C_2}{C_4}}\,\tanh\left(\sqrt{-C_2}\left(\frac{k_1}{\Gamma(1+\alpha)}t^{\alpha} + \frac{k_2}{\Gamma(1+\alpha)}x^{\alpha}\right)\right), \tag{28}$$

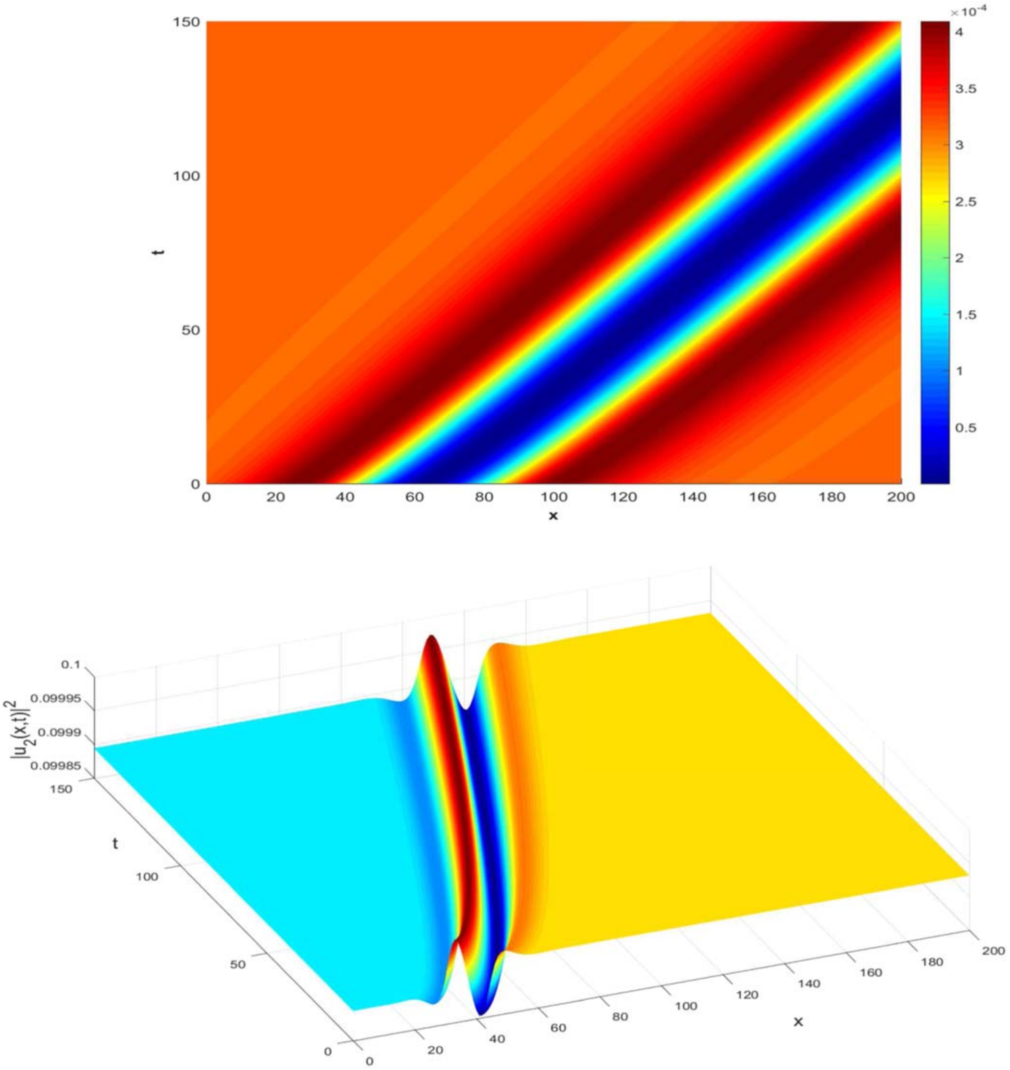


Figure 5. Spatiotemporal contour plot of $|u_2(x, t)|^2$ at $\alpha = 0.9$ and $\alpha = 0.72$.

R3: $A_0 = A_0$, $A_1 = A_1$, $k_1 = k_1$, $k_2 = k_2$, $\beta = \frac{-k_1^2 + \delta k_2^2 + k_1^2 vA_0}{k_1^2 A_0^2}$, $C_2 = \frac{6A_1(-2k_1^2 + k_1^2 vA_0 + 2\delta k_2^2)}{\delta^4 k_2^2}$, $C_3 = \frac{4A_1^2(2k_1^2 vA_0 - 3k_1^2 + 3\delta k_2^2)}{A_0\delta^4 k_2^2}$, $C_4 = \frac{3(-k_1^2 + \delta k_2^2 + k_1^2 vA_0)A_1^3}{A_0^2\delta^4 k_2^2}$.

Hence, for $C_2 > 0$ and $C_4 > 0$, and $C_0 = C_1 = 0$, it is obtained soliton solutions

$$u_3(x, t) = A_0 + A_1\frac{C_2\,\mathrm{sech}^2\left(\sqrt{-C_2}\left(\frac{k_1}{\Gamma(1+\alpha)}t^\alpha + \frac{k_2}{\Gamma(1+\alpha)}x^\alpha\right)\right)}{2\sqrt{C_2C_4}\tanh\left(\sqrt{2C_2}\frac{\left(\frac{k_1}{\Gamma(1+\alpha)}t^\alpha + \frac{k_2}{\Gamma(1+\alpha)}x^\alpha\right)}{2}\right)}, \tag{29}$$

R4: $A_0 = A_0$, $A_1 = A_1$, $k_1 = k_1$, $k_2 = k_2$, $\beta = \frac{-k_1^2 + \delta k_2^2 + k_1^2 vA_0}{k_1^2 A_0^2}$, $C_2 = \frac{6A_1(-2k_1^2 + k_1^2 vA_0 + 2\delta k_2^2)}{\delta^4 k_2^2}$, $C_3 = \frac{4A_1^2(2k_1^2 vA_0 - 3k_1^2 + 3\delta k_2^2)}{A_0\delta^4 k_2^2}$, $C_4 = \frac{3(-k_1^2 + \delta k_2^2 + k_1^2 vA_0)A_1^3}{A_0^2\delta^4 k_2^2}$. Hence, for $C_2 > 0$ and ${C_2}^2 - 4C_2C_4 > 0$, and $C_0 = C_1 = 0$, it is recovered soliton solutions

$$u_4(x, t) = A_0 + A_1\frac{2C_2\,\mathrm{sech}^2\left(\sqrt{2C_2}\left(\frac{k_1}{\Gamma(1+\alpha)}t^\alpha + \frac{k_2}{\Gamma(1+\alpha)}x^\alpha\right)\right)}{\sqrt{{C_2}^2 - 4C_2C_4} - C_3\,\mathrm{sech}\left(\sqrt{2C_2},\left(\frac{k_1}{\Gamma(1+\alpha)}t^\alpha + \frac{k_2}{\Gamma(1+\alpha)}x^\alpha\right)\right)}, \tag{30}$$

R5: $A_0 = A_0$, $A_1 = A_1$, $k_1 = k_1$, $k_2 = k_2$, $\beta = \frac{-k_1^2 + \delta k_2^2 + k_1^2 vA_0}{k_1^2 A_0^2}$, $C_2 = \frac{6A_1(-2k_1^2 + k_1^2 vA_0 + 2\delta k_2^2)}{\delta^4 k_2^2}$, $C_3 = \frac{4A_1^2(2k_1^2 vA_0 - 3k_1^2 + 3\delta k_2^2)}{A_0\delta^4 k_2^2}$, $C_4 = \frac{3(-k_1^2 + \delta k_2^2 + k_1^2 vA_0)A_1^3}{A_0^2\delta^4 k_2^2}$.

Consequently, for $C_2 > 0$, and $C_0 = C_1 = 0$, it is obtained soliton solutions

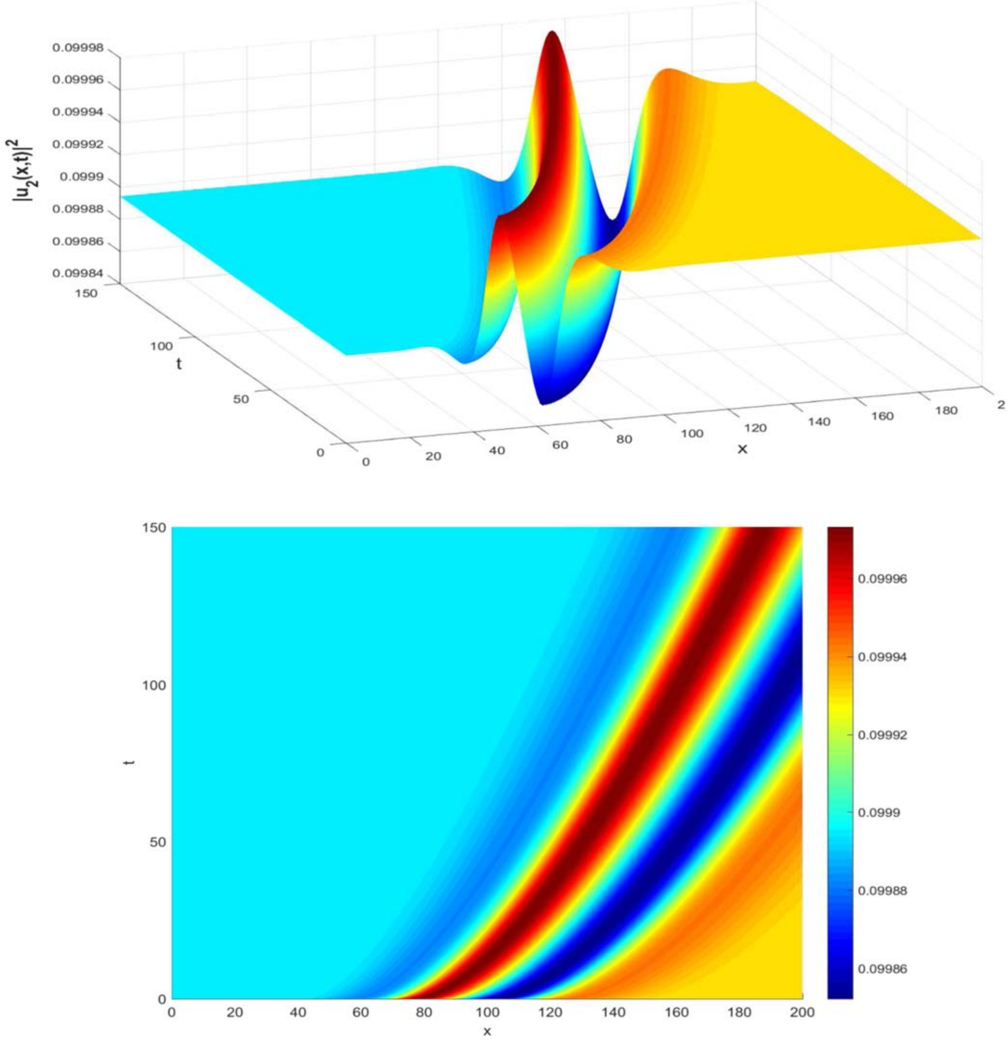


Figure 6. Spatiotemporal plot evolution and contour plot of $|u_2(x,\,t)|^2$ at $\alpha = 0.65$ and $\alpha = 0.55$.

$$u_5(x,\,t) = A_0 + A_1 \frac{2C_2C_3\,\mathrm{sech}^2\left(\sqrt{2C_2}\,\frac{\left(\frac{k_1}{\Gamma(1+\alpha)}t^{\alpha} + \frac{k_2}{\Gamma(1+\alpha)}x^{\alpha}\right)}{2}\right)}{C_2C_4\left(1 - \tanh\left(\sqrt{2C_2}\,\frac{\left(\frac{k_1}{\Gamma(1+\alpha)}t^{\alpha} + \frac{k_2}{\Gamma(1+\alpha)}x^{\alpha}\right)}{2}\right)\right)^2}. \quad (31)$$

4.2. On solving the conformable derivative nonlinear differential equation governing wave propagation in low-pass electrical transmission lines with Sinh–Gordon method

From (25) it is recovered $N = 1$, and (24) gives

$$U(\xi) = A_0 + A_1 \sinh w + B_1 \cosh w. \quad (32)$$

Substitute (32), (23), (21) and (19) into (25), it is obtained a set of algebraic equations.

- $(\cosh(w(\xi)))^3$: $\beta k_1^2 A_1^3 - \frac{1}{6}\delta^4 k_2^2 A_1 + 3\beta k_1^2 B_1^2 A_1 = 0$,
- $(\cosh(w(\xi)))^2$: $3\beta k_1^2 A_0 B_1^2 + 3\beta k_1^2 A_0 A_1^2 = 0$,
- $\sinh(w(\xi))(\cosh(w(\xi)))^2$: $\beta k_1^2 B_1^3 - \frac{1}{6}\delta^4 k_2^2 B_1 + 3\beta k_1^2 B_1 A_1^2 = 0$,
- $\cosh(w(\xi))$: $-\delta k_2^2 A_1 + \frac{1}{6}\delta^4 k_2^2 A_1 - \frac{1}{12}\delta^4 k_2^2 A_1 s + k_1^2 A_1 + 3\beta k_1^2 A_0^2 A_1 - 3\beta k_1^2 B_1^2 A_1 = 0$,
- $\sinh(w(\xi))\cosh(w(\xi))$: $6\beta k_1^2 A_0 A_1 B_1 = 0$,
- $\sinh(w(\xi))$: $k_1^2 B_1 - \delta k_2^2 B_1 - \frac{1}{12}\delta^4 k_2^2 B_1 s + 3\beta k_1^2 A_0^2 B_1 - \beta k_1^2 B_1^3 + \frac{1}{12}\delta^4 k_2^2 B_1 = 0$,
- Constant: $k_1^2 A_0 - k_1^2 v V^2 - \delta k_2^2 A_0 - 3\beta k_1^2 A_0 B_1^2 + \beta k_1^2 A_0^3 = 0$,

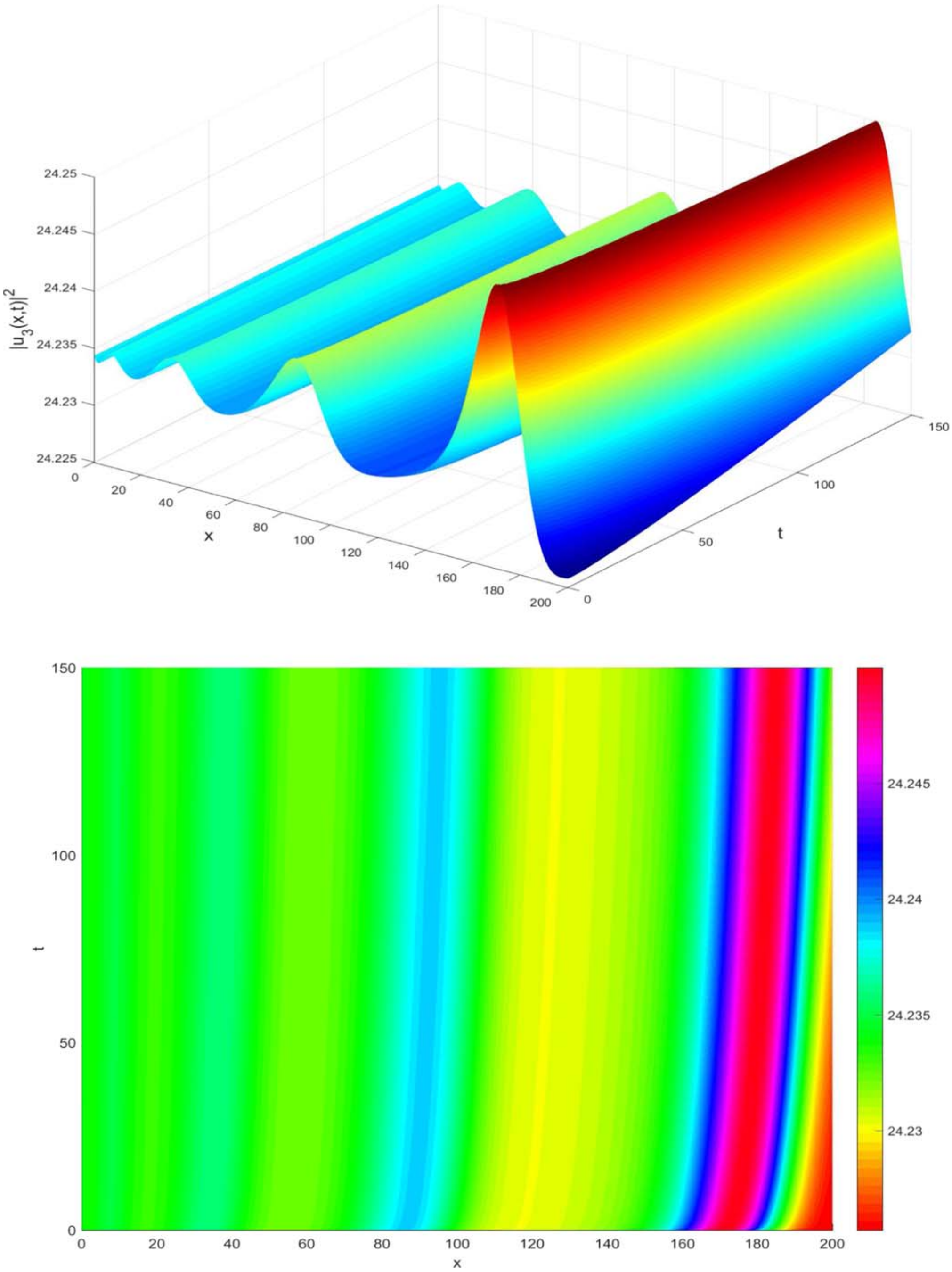


Figure 7. Spatiotemporal plot evolution and contour plot of $|u_3(x,\, t)|^2$ at $\alpha = 0.4$ and $k_2 = -0.34$, $k_1 = 9.4$, $k = 10.2$, $\delta = 5.85$, $A_1 = 0.005$, $v = 0.0025$.

hence it is obtained from the above system of algebraic equations with help of Maple, the following results after setting to zero all the coefficients of $\sinh^j w \cosh^j w$ ($i = 0,\, 1$; $j = 0,\, 1,\, 2,\, \ldots,\, N$). Thus, by taking into account (22*a*) and (22*b*), follows the Jacobian elliptic function solutions to (1).

SET 1: $A_0 = \frac{1}{2}\sqrt{2m^2 + 2}\,B_1$, $A_1 = 0$, $B_1 = B_1$, $k_2 = \sqrt{\frac{6}{\delta(\delta^3 m^2 + \delta^3 + 6)}}\,k_1$, $k_1 = k_1$, $v = \frac{3}{2}\frac{\delta^3\sqrt{2\,m^2 + 2}}{(\delta^3 m^2 + \delta^3 + 6)B_1}$, $\beta = \frac{\delta^3}{(\delta^3 m^2 + \delta^3 + 6)B_1^2}$, $\delta = \delta$.

$$u1b(\xi) = \frac{1}{2}B_1\left[\sqrt{2m^2 + 2} + 2k_1 ns\left(\frac{1}{\Gamma(1 + \alpha)}t^\alpha + \frac{\sqrt{\frac{6}{\delta(\delta^3 m^2 + \delta^3 + 6)}}}{\Gamma(1 + \alpha)}x^\alpha,\, m\right)\right] \tag{33}$$

SET 2: $A_0 = 0, A_1 = -B_1, B_1 = B_1, \quad k_2 = 2\sqrt{-\frac{6}{\delta(2\delta^3m^2-\delta^3-24)}}k_1, \quad k_1 = k_1, v = 0, \quad \beta = \frac{-\delta^3}{(\delta^3m^2-\delta^3-24)B_1^2}$, $\delta = \delta$.

$$u2b(\xi) = -B_1k_1\left[cs\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{2\sqrt{-\frac{6}{\delta(2\delta^3m^2-\delta^3-24)}}}{\Gamma(1+\alpha)}x^\alpha, m\right) - ns\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{2\sqrt{-\frac{6}{\delta(2\delta^3m^2-\delta^3-24)}}}{\Gamma(1+\alpha)}x^\alpha, m\right)\right], \tag{34}$$

SET 3: $A_0 = \sqrt{2m^2-1}B_1, A_1 = B_1, B_1 = B_1, \quad k_2 = 2\sqrt{-\frac{3}{\delta(2\delta^3m^2-\delta^3+12)}}k_1, k_1 = k_1, \quad v = \frac{3}{2}\frac{\delta^3\sqrt{2m^2+2}}{(2\delta^3m^2-\delta^3+12)B_1}, \quad \beta = \frac{1}{2}\frac{\delta^3}{(\delta^3m^2-\delta^3+12)B_1^2}$, $\delta = \delta$.

$$u3b(\xi) = B_1[\sqrt{2m^2-1} + k_1cs\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{2\sqrt{-\frac{3}{\delta(2\delta^3m^2-\delta^3+12)}}}{\Gamma(1+\alpha)}x^\alpha, m\right) + k_1ns\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{2\sqrt{-\frac{3}{\delta(2\delta^3m^2-\delta^3+12)}}}{\Gamma(1+\alpha)}x^\alpha, m\right) \tag{35}$$

SET 4: $A_0 = \sqrt{2m^2-1}B_1, A_1 = -B_1, B_1 = B_1, \quad k_2 = 2\sqrt{-\frac{3}{\delta(2\delta^3m^2-\delta^3+12)}}k_1, k_1 = k_1, \quad v = \frac{3}{2}\frac{\delta^3\sqrt{2m^2+2}}{(2\delta^3m^2-\delta^3+12)B_1}, \quad \beta = \frac{1}{2}\frac{\delta^3}{(\delta^3m^2-\delta^3+12)B_1^2}$, $\delta = \delta$.

$$u4b(\xi) = B_1k_1\left[\frac{\sqrt{2m^2-1}}{k_1} - cs\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{2\sqrt{-\frac{3}{\delta(2\delta^3m^2-\delta^3+12)}}}{\Gamma(1+\alpha)}x^\alpha, m\right) + ns\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{2\sqrt{-\frac{3}{\delta(2\delta^3m^2-\delta^3+12)}}}{\Gamma(1+\alpha)}x^\alpha, m\right)\right]. \tag{36}$$

From (33)–(36), when $m \to 1$ it is obtained solitary waves solutions to (1),

$$u11(\xi) = \frac{1}{2}B_1\left[2(1+k_1)\tanh\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{\sqrt{\frac{6}{\delta(2\delta^3+6)}}}{\Gamma(1+\alpha)}x^\alpha\right)\right], \tag{37}$$

$$u12(\xi) = -B_1k_1\left[csch\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{2\sqrt{-\frac{6}{\delta(\delta^3-24)}}}{\Gamma(1+\alpha)}x^\alpha\right) - \coth\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{2\sqrt{-\frac{6}{\delta(2\delta^3-24)}}}{\Gamma(1+\alpha)}x^\alpha\right)\right], \tag{38}$$

$$u13(\xi) = B_1\left[1 + k_1csch\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{2\sqrt{-\frac{3}{\delta(\delta^3+12)}}}{\Gamma(1+\alpha)}x^\alpha\right) + k_1\coth\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{2\sqrt{-\frac{3}{\delta(\delta^3+12)}}}{\Gamma(1+\alpha)}x^\alpha\right)\right], \tag{39}$$

$$u14(\xi) = -B_1k_1\left[-\frac{1}{k_1} + csch\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{2\sqrt{-\frac{3}{\delta(\delta^3+12)}}}{\Gamma(1+\alpha)}x^\alpha\right) - \coth\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{2\sqrt{-\frac{3}{\delta(\delta^3+12)}}}{\Gamma(1+\alpha)}x^\alpha\right)\right], \tag{40}$$

from (33)–(36), when $m \to 0$ it is obtained jacobian elliptic function solutions and trigonometric function solutions to (1),

$$u15(\xi) = \frac{1}{2}B_1\left[\sqrt{2} + 2k_1csc\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{\sqrt{\frac{6}{\delta(2\delta^3+6)}}}{\Gamma(1+\alpha)}x^\alpha\right)\right],$$

$$u16(\xi) = -B_1k_1\left[\cot\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{2\sqrt{-\frac{6}{\delta(\delta^3-24)}}}{\Gamma(1+\alpha)}x^\alpha\right) - csc\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{2\sqrt{-\frac{6}{\delta(2\delta^3-24)}}}{\Gamma(1+\alpha)}x^\alpha\right)\right], \tag{41}$$

$$u17(\xi) = B_1\left[1 + k_1csc\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{2\sqrt{-\frac{3}{\delta(\delta^3+12)}}}{\Gamma(1+\alpha)}x^\alpha\right) + k_1\cot\left(\frac{1}{\Gamma(1+\alpha)}t^\alpha + \frac{2\sqrt{-\frac{3}{\delta(\delta^3+12)}}}{\Gamma(1+\alpha)}x^\alpha\right)\right] \tag{42}$$

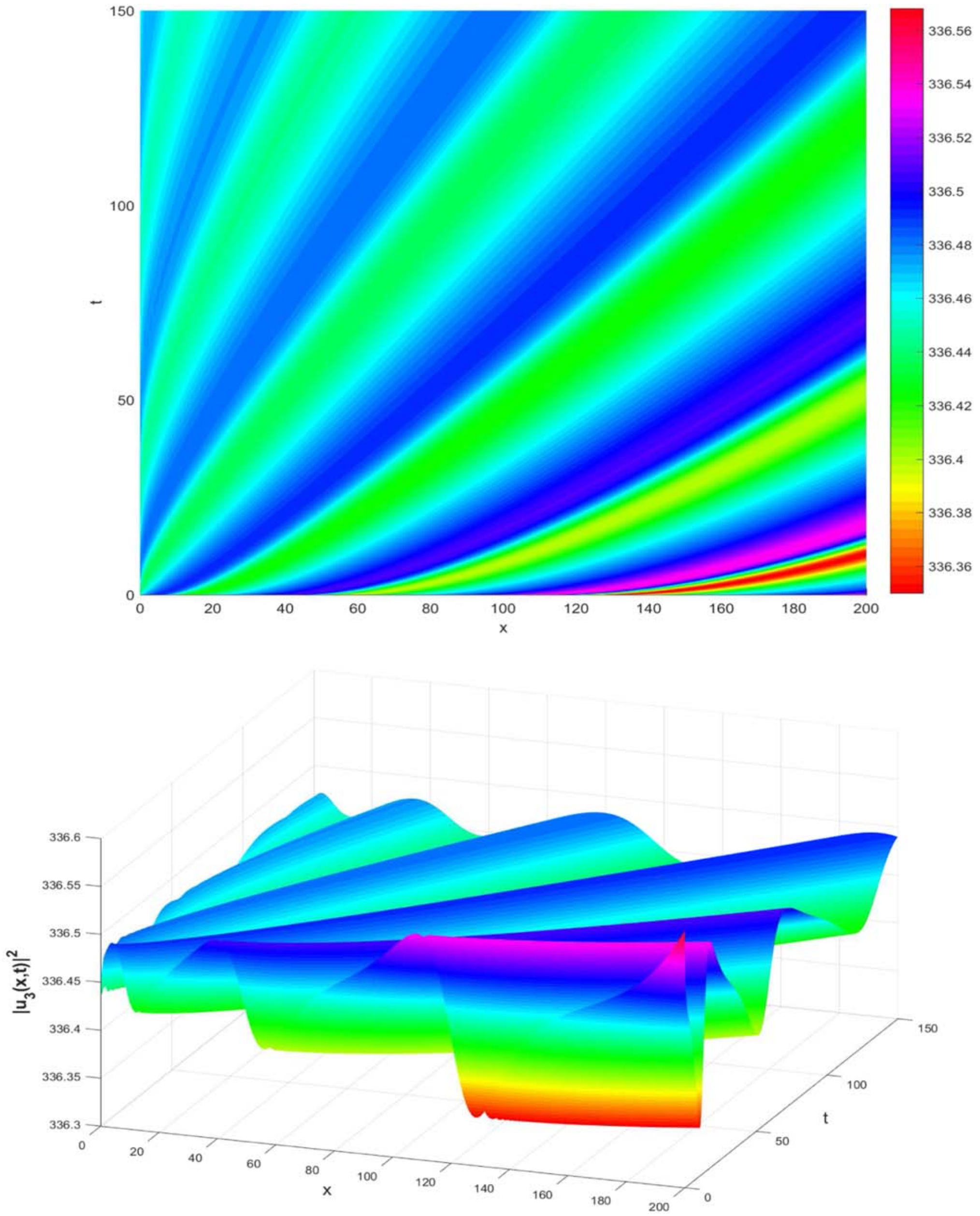


Figure 8. Spatiotemporal plot evolution and contour plot of $|u_3(x,\, t)|^2$ at $\alpha = 0.53$ and $k_2 = -k_1$, $k = 10.02$, $\delta = 5.85$, $A_1 = 0.005$, $v = 0.0025$.

$$u18(\xi) = -k_1 B_1\left[-\frac{1}{k_1} - csc\left(\frac{1}{\Gamma(1+\alpha)}t^{\alpha} + \frac{2\sqrt{-\frac{3}{\delta(\delta^3-12)}}}{\Gamma(1+\alpha)}x^{\alpha}\right) + \cot\left(\frac{1}{\Gamma(1+\alpha)}t^{\alpha} + \frac{2\sqrt{-\frac{3}{\delta(\delta^3-12)}}}{\Gamma(1+\alpha)}x^{\alpha}\right)\right].$$

Figures 1 and 2 are spatiotemporal plot evolution of $|u_1(x,\, t)|^2$ at $k_2 = -k_1$, $k = 100.67$, $\delta = 0.9$, $A_1 = 0.05$, $v = 0.25$ and figure 3 is plot evolution of $|u_1(x,\, t)|^2$ at $k_2 = -k_1$, $k = 100.67$, $\delta = 0.8$, $A_1 = 0.05$, $v = 0.25$. Figure 4–6 are spatiotemporal plot evolution and contour plot evolution of $|u_2(x,\, t)|^2$ at $k_2 = -k_1$, $k = 100.67$, $\delta = 0.8$, $A_1 = 0.0005$, $v = 0.25$. While figure 3 is W-shape bright soliton of $|u_1(x,\, t)|^2$ at $k_2 = -k_1$, $k = 10.67$, $\delta = 1.9$, $A_1 = 0.005$, $v = 0.25$. Figures 5–7, show the spatiotemporal plot of dark solitons $|u_2(x,\, t)|^2$. The obtained results are more general than those obtained by [3, 5]. We also observed that the fractional parameter α has an impact on the width and on the amplitude of the obtained above bright and dark solitons solutions (see figures 1, 5–8). Without doubt, we can predict that the derivative order changes the shape of the traveling-wave in the electrical transmission lines. Furthermore, the memory effect of fractional derivative can have application in electrical transmission line.

5. Conclusion and remarks

This paper studies the conformable derivative nonlinear differential equation governing wave propagation in electrical transmission lines. To obtain the ordinary differential equation, the fraction complex temporal hypothesis is adopted. By means of the auxiliary equation method and the Sinh–Gordon method, it is obtained dark, bright solitons compare to the results obtained by [1–3, 5]. Moreover, Some relevant results have been obtained by using the Sinh–Gordon method. As, it is well known that solitons have a great importance in telecommunication system, the results obtained will probably help to carry information and increase the bit-rate of data.

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