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Axion excitation by intense laser fields in a plasma

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Abstract

We study the excitation of axions by intense laser pulses propagating in a plasma. We assume that the pulses propagate along the direction of a static magnetic field. We examine two different configurations. One is that of a long pulse, with a duration Δt much longer than the electron plasma period, $\Delta t \gg 1/\omega_p$. In this case, the axion field couples with the electron plasma waves (or plasmons) which are excited by the laser pulse, due to a modulational instability. The dispersion relation of the coupled axion-plasmon modes, the unstable regimes and the instability growth rates are established. The other configuration is that of a very short laser pulse, with a duration of the order of (or shorter than) the electron plasma period. In this case, the modulational instability is absent, but a laser wakefield can be excited. The latter consists of both electron density and axion field perturbations. The amplitude of this wakefield is described with a simple one-dimensional model. The two configurations (long and short pulse) can eventually be used to create axions in the laboratory and we suggest that laser plasma experiments could shed some light on the existence of these hypothetical dark matter particles.

Keywords: axions, intense lasers, wakefields, modulational instabilities, plasmas

(Some figures may appear in colour only in the online journal)

1. Introduction

Dark matter is a key ingredient in physical cosmology and one of the mysteries of modern science [1–3]. Among several proposed candidates (generically referred as WIMPs and WISPs, standing for weakly interaction massive and slim particles, respectively), axions or axion-like particles (ALPs) are the most plausible constituents of dark matter [4]. The axion was proposed as a hypothetical particle that could solve the fundamental problem of charge-parity (CP) invariance in QCD [5–11]. The proof of existence of this particle has been actively searched for more than one decade, so far without success, using laboratory experiments and astrophysical observations [12]. This research activity relies on the existence of a weak axion-photon coupling, and has led to solar observation campaigns [13, 14], light shining through a wall (LSW) schemes [15, 16] and other experimental arrangements [17–19].

All these experiments use static magnetic fields, in order to excite the axion-photon coupling. The axions could then decay into photons, providing a signature of their existence. In recent years, it became clear that static magnetic fields could be replaced by varying fields as those of a laser, and that intense laser pulses could be used to excite or detect axions [20–24]. Another approach uses static magnetic fields, but in a plasma medium. The signature of axions could then be found in the dispersion relation of plasma waves [25], which shows a distortion due to the excitation of the axion-polariton mode in the region of high phase velocities. The case of axion excitation by laser wakefields in unmagnetised plasmas was also considered in [26].

Here we discuss the possible excitation of axion-polaritons, using intense laser pulses in a magnetised plasma. We study two different configurations. First, we consider long laser pulses, with a duration much longer than the plasma period $\Delta t \gg 1/\omega_p$, where ω_p is the electron plasma frequency. These long pulses excite large amplitude electrostatic

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oscillations as a result of a modulational instability which, subsequently, induce large amplitude perturbations of the axion field. The dispersion relations of the resulting axion-polariton modes, the unstable regimes and the instability growth rates will be established. Second, short laser pulses, with a duration of the order of (or shorter than) the plasma period will be considered. Here, despite the absence of a modulational instability, a laser wakefield is excited, which acquires a finite axionic component. The amplitude of the two coupled plasma and axion wakefields will be described by a one-dimensional model.

Our work shows that resonant regimes of axion excitation by intense laser pulses can take place in a magnetised plasma. Given the large uncertainty associated with the axion mass and its coupling constant, the mechanisms described here could help clarify the dark matter problem by providing a new experimental approach based on intense laser-plasma interactions. In particular, axions are expected to couple to plasmon oscillations excited by intense laser pulses. The ultimate goal is the discovery of this hypothetical dark matter particle. Furthermore, in the astrophysical context, the physical mechanisms discussed here could eventually imply that local sources of dark matter exist in the Universe, specially in the vicinity of dense astronomical objects, where intense beams of electromagnetic radiation, dense plasmas and strong magnetic fields can simultaneously be present.

2. Basic equations

We consider an axion field φ in the presence of an intense laser pulse which propagates in a plasma along the direction of a static magnetic field $\mathbf{B}_0$. This magnetic geometry differs considerably from that used in the current axion detection experiments and LSW schemes, as illustrated in figure 1. The *axion helioscope*, first proposed by Sikivie [27], was aimed to detect the axion flux originated from the Sun, using a static magnetic field perpendicular to the direction of propagation of the incoming axions, by converting them into x-ray photons via the inverse Primakov effect. The usual LSW configurations use a cw laser beam propagating perpendicularly to a static magnetic field in vacuum.

In contrast, the present configuration assumes that the laser pulse propagates along the direction of a static field $\mathbf{B}_0$, not in vacuum but in a plasma. This means that the laser field is always perpendicular to $\mathbf{B}_0$, and cannot directly be coupled to the axion field, as explained below. Coupling is established by the electrostatic field associated with the plasma oscillations, and not directly by the laser field itself. Experiments on laser-plasma interactions show that these two electric fields (the transverse field associated with the laser, and the electrostatic field associated with the electron plasma waves excited by the laser) can be of comparable magnitude [28].

The axion field is described by a Klein–Gordon equation, which includes a photon-axion coupling term proportional to

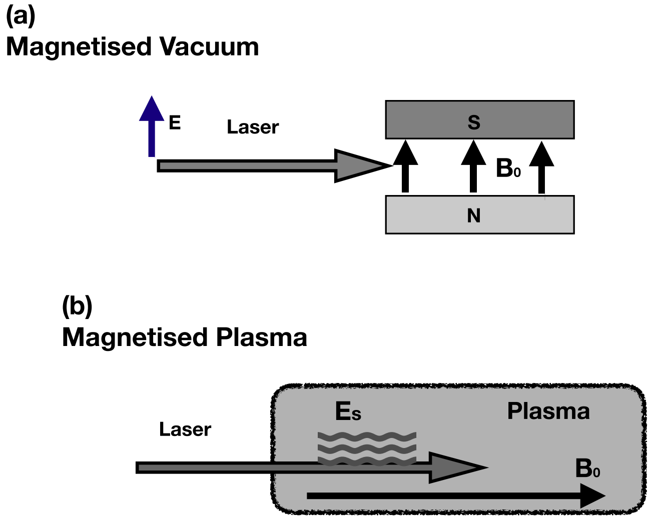

Figure 1. Schematic representation of two different physical configurations: (a)—the LSW scheme, where a cw laser beam propagates in a magnetised vacuum. The laser electric field is parallel to the static magnetic field; (b)—the present scheme, where an intense laser pulse propagates in a magnetized plasma along the direction of a static magnetic field $\mathbf{B}_0$. The laser field is perpendicular to the magnetic field, but the secondary electric field of an electrostatic character and parallel to $\mathbf{B}_0$ establishes a possible coupling with the axion field.

$(\mathbf{E}\cdot\mathbf{B}_0)$, namely

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2 + \Omega_\varphi^2\right)\varphi = 2\epsilon_0\frac{gc^4}{\hbar}(\mathbf{E}\cdot\mathbf{B}_0). \quad (1)$$

Here, $\Omega_\varphi^2 = m_\varphi^2 c^4/\hbar^2$, m_φ is the axion mass, g the coupling constant, and ϵ_0 the vacuum permittivity. The electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$ respectively, are determined by noting that we can define the fields $\mathbf{E}'$ and $\mathbf{B}'$ which obey Maxwell equations in their usual form. In particular, [29]

$$\mathbf{E}' = (\mathbf{E} + cg\varphi\mathbf{B}), \;\; \mathbf{B}' = \left(\mathbf{B} - \frac{g}{c}\varphi\mathbf{E}\right). \quad (2)$$

These new fields therefore satisfy the wave equation

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)\mathbf{E}' - c^2\nabla(\nabla\cdot\mathbf{E}') = \frac{1}{\epsilon_0}\frac{\partial\mathbf{J}}{\partial t}. \quad (3)$$

Here, the current density is $\mathbf{J} = -en_e\mathbf{v}$, where the electron mean density, n_e, and mean velocity, $\mathbf{v}$, can be determined by the electron fluid equations, or in alternative, by an electron kinetic equation.

We assume that two distinct electric fields are present: the electrostatic field $\mathbf{E}_s$ associated with plasma oscillations, and the transverse field $\mathbf{E}_t$ of the laser pulse. Similarly, for the magnetic field, we have the external static field $\mathbf{B}_0$ and the laser field component $\mathbf{B}_t$. This allows us to write: $\mathbf{E} = \mathbf{E}_s + \mathbf{E}_t$, and $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_t$. According to the wave equation (3), the electrostatic field $\mathbf{E}_s$ is described by a modified Gauss's law of the form

$$\nabla\cdot(\mathbf{E}_s + cg\varphi\mathbf{B}_0) = -\frac{e}{\epsilon_0}\tilde{n}, \quad (4)$$

where n_0 is the equilibrium plasma density and $\tilde{n} = (n_e - n_0)$ represents the deviations from equilibrium. On the other hand, the transverse field $\mathbf{E}_t$ is described by

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)[\mathbf{E}_t + cg\varphi(\mathbf{B}_0 + \mathbf{B}_t)] = \frac{1}{\epsilon_0}\frac{\partial \mathbf{J}}{\partial t}. \quad (5)$$

We assume that the laser pulse propagates along the static field $\mathbf{B}_0$, leading to $(\mathbf{E}_t \cdot \mathbf{B}_0) = 0$ together with $(\mathbf{E}_t \cdot \mathbf{B}_t) = 0$. It should be noticed that the laser magnetic field $\mathbf{B}_t$ is always orthogonal to the electrostatic field $\mathbf{E}_s$, and therefore cannot couple to the axion field. In order to describe the intense laser pulse, a dimensionless vector potential $\mathbf{a}$ can be used, defined as

$$\mathbf{E}_t = -\frac{m_e c}{e}\frac{\partial \mathbf{a}}{\partial t}. \quad (6)$$

It is also known that, at relativistic intensities, the electron momentum is approximately equal to $\gamma\mathbf{v}/c = \mathbf{a}$, where γ is the electron relativistic factor. This allows to rewrite the wave equation (5) in terms of the dimensionless vector, as

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)\mathbf{a} = c^2\omega_p^2\left(1 + \frac{\tilde{n}}{n_0}\right)\frac{\mathbf{a}}{\gamma}, \quad (7)$$

where $\omega_p = (e^2 n_0/\epsilon_0 m_e)^{1/2}$ is the electron plasma frequency. This completes the discussion of the basic field equations pertinent to our model.

The spectrum of the laser pulse is not monochromatic, and the total field $\mathbf{a}$ can be described as a superposition of modes $\mathbf{a}_k$ associated with different photon wavenumber $\mathbf{k}'$. Because the photon dispersion relation specifies the frequency ω'_k, we can use a single integral to represent the total field: $\mathbf{a} = \int \mathbf{a}_k \exp(-\mathrm{i}\omega'_k t)\mathrm{d}\mathbf{k}'/(2\pi)^3$. It is sometimes useful to write this integral in terms of the photon density N_k, as shown in appendix A.

3. Modulational instability

We first consider the case of long laser pulses, with duration Δt longer than the plasma period, $\Delta t \gg 1/\omega_p$. We consider density perturbations $\tilde{n}$ evolving in space and time as $\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r} - \mathrm{i}\omega t)$, where the wavevector $\mathbf{k}$ and the frequency ω should not be confused with the wavevectors $\mathbf{k}'$ and the frequencies ω' of the laser pulse spectrum.

Using the procedure described in appendix B, we can derive a dispersion relation for the combined axion-plasma oscillations, in the presence of a laser pulse, namely

$$(1 - \chi_\varphi)(1 - \chi_e - \chi_{\mathrm{ph}}) = \frac{\Omega^4}{\omega^4}. \quad (8)$$

Here, χ_φ, χ_e and χ_{ph} represent the axion, electron and laser field (or photon) susceptibilities. The axion coupling factor is $\Omega = \sqrt{\omega_c\omega_g}$, where ω_c is the electron cyclotron frequency and ω_g is the axion coupling frequency, as defined by (see appendix B)

$$\omega_c = \frac{eB_0}{m_e}, \;\; \omega_g^2 = 2g^2c^5\frac{n_{e0}m_e}{\hbar}. \quad (9)$$

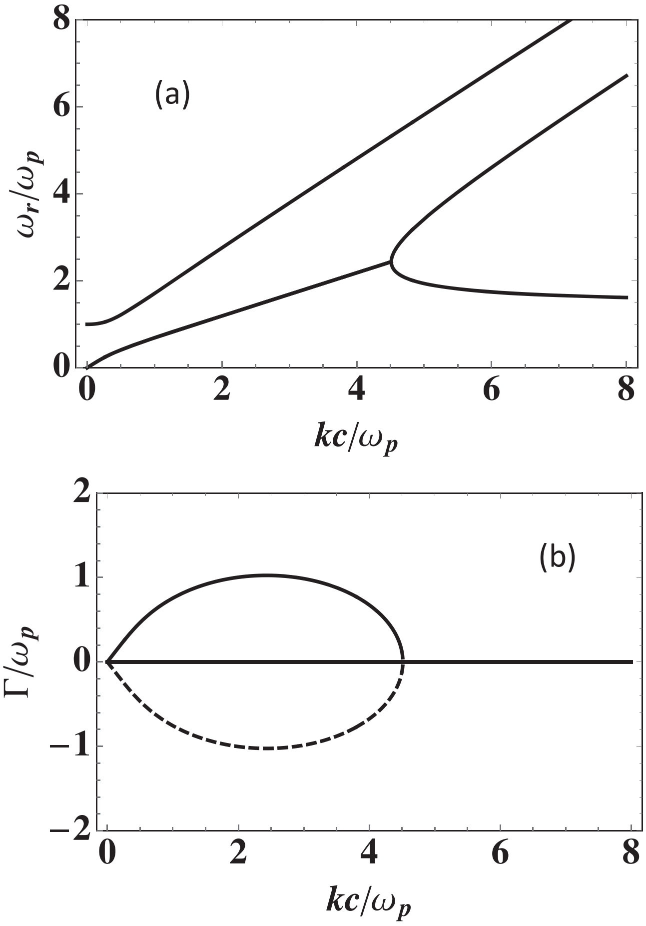

Figure 2. Dispersion relation for the modulation instability created by an intense laser pulse in a magnetized plasma: (a) real part fo the frequency ω_r/ω_p, and (b) instability growth rates Γ/ω_p.

In the absence of a laser beam, $\chi_{\mathrm{ph}} = 0$, equation (8) simply describes the polariton modes already discussed in our previous work [25]. They are nothing but axion-plasmon coupled oscillations. The presence of a pulse, $\chi_{\mathrm{ph}} \neq 0$, introduces an important qualitative change, as it leads to an instability of the axion-plasmon field. In order to better illustrate this, we explicitly separate the complex frequency ω into its real and imaginary parts, $\omega = \omega_r + \mathrm{i}\Gamma$, with $\Gamma > 0$ quantifying the instability growth rate. These two quantities are plotted in figure 2. The Ω factor is responsible for the excitation of axion oscillations by coupling the axion field to the electrostatic perturbations that results from the modulational instability determined by $(1 - \chi_e - \chi_{\mathrm{ph}}) = 0$.

It is useful to give a simple analytical estimate of the expected growth rates of this axion-plasmon instability in the magnetised plasma. As such, we assume that the real part of the frequency is much larger than the growth rate, $\omega_r \simeq \omega_p \gg |\Gamma|$. We use the dispersion relation (8), and assume a *triple resonance identity*

$$\omega_r = \omega_\varphi = kv_0 = \sqrt{\frac{1}{\gamma_a}(\omega_p^2 + k^2S_e^2)}, \quad (10)$$

where v_0 is the laser group velocity, γ_a is the electron relativistic factor in the laser field, and S_e the electron thermal

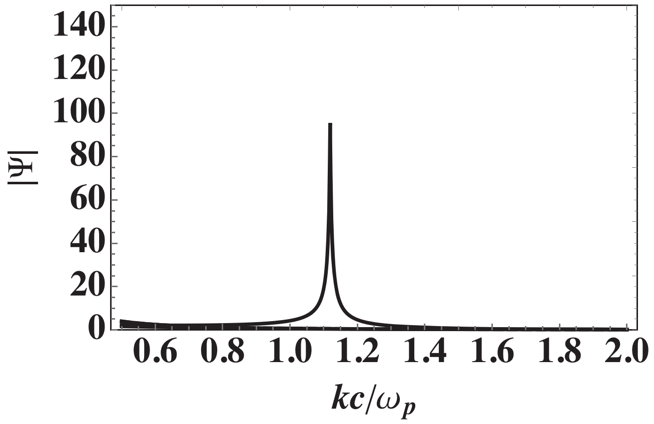

Figure 3. Absolute value of the dimensionless axion field $\Psi = g\varphi(\omega_p^2/\omega_g)^2$, as a function of the normalised wavenumber kc/ω_p, for $\omega_c/\omega_p = 1$ and $\tilde{n}/n_0 = 1$, for the same conditions of figure 1.

velocity (see appendix B for definitions). We can then obtain an equation for the growth rate Γ, of the form

$$\Gamma^3 + \Gamma\frac{\omega_g^2\omega_c^2}{4\omega^2} - \frac{i}{2}\frac{\omega_p^2\nu_{\rm ph}^2}{\gamma_a\omega_r} = 0, \tag{11}$$

where $\nu_{\rm ph} = kc|a_0|/4$ is the photon density parameter, proportional to the normalised laser field amplitude a_0, as derived in appendix B. Noting that the term containing ω_g^2 is very small, the maximum instability growth rate will be given by

$$\Gamma = \frac{\sqrt{3}}{2}\omega_p\left(\frac{\nu_{\rm ph}}{\omega_p}\right)^{2/3}. \tag{12}$$

Formally, this is a typical result for beam-type instabilities, valid under the assumption of the triple identity condition (10), corresponding to $\Gamma \propto a_0^{2/3}$. A similar result could be obtained for the an electron beam with density n_b, where we would get $\Gamma \propto n_b^{1/3}$. Note, however, that not all the unstable wavenumbers are equally relevant to the excitation of axions. This can be shown by looking at the relative importance of $\tilde{n}$ and φ inside the unstable domain. Using equation (B4), we can write the dimensionless relation

$$(g\varphi) = -\mathrm{i}\frac{\omega_c}{kc}\frac{\omega_g^2}{(\omega^2 - \omega_\varphi^2)}\frac{\tilde{n}}{n_0}. \tag{13}$$

This expression shows that the axion field is resonantly excited in a very small frequency band, where ω_r is nearly equal to the characteristic axion frequency ω_φ. This is illustrated in figure 3, where the absolute value of the normalised axion field $\Psi = g\varphi(\omega_p^2/\omega_g^2)$ is represented in the unstable range of wavenumbers k, assuming that the instability saturates at $\tilde{n} \sim n_0$. Note that, despite the increase of the axion field for very large wavelengths, the limit of $k \to 0$ eventually becomes irrelevant due to the finite size of any experimental system.

4. Axion wakefield

The above discussion is valid when the laser pulse duration is much longer than the electron plasma wave period, $\omega_p\Delta t \gg 1$.

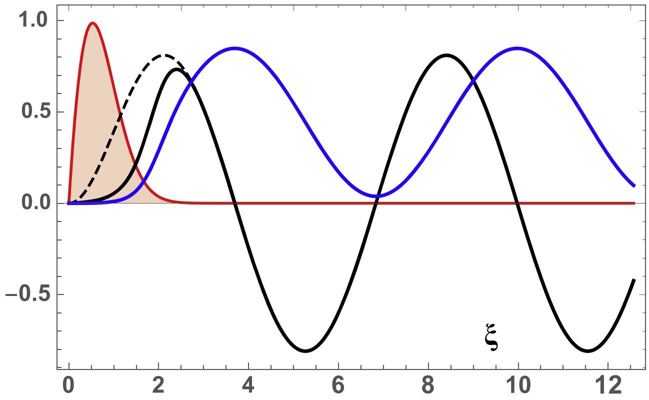

Figure 4. Plasma and axion wakes, assuming high laser intensities, such that $a_0 \simeq \gamma_a = 10$: The pulse shape, $I_0(\xi)/I_0(0)$, is represented in shaded red; the electron plasma density wake, $\tilde{n}(\xi)/n(0)$, is represented in black; the same curve, but for a nonrelativistic pulse with $\gamma_a = 1$, is represented in dashed black; (4) the normalised axion wake, $\Psi = g\varphi(\omega_p^2/\omega_g^2)$, is represented in blue.

We now consider the opposite case of short laser pulses, such that $\omega_p\Delta t \leqslant 1$. The perturbations created by these short pulses form a kind of combined axion-plasmon wakefield, similar to the wake created by a moving boat in the surface of a lake. We will use a simple 1D model to describe the process, as explained in appendix C. The transverse dimensions, that we are ignoring, could be important to describe electron betatron oscillations, but have little effect on the axion-plasmon coupling, because of the radial symmetry of the wake.

We begin the analysis by considering the standard laser wakefield problem, where the axions are ignored (see, for instance, [30]). In terms of the co-moving variable $\xi = z - v_0 t$, the plasma density perturbations associated with the laser wake are determined by

$$\tilde{n}(\xi) = \frac{1}{2}n_0\int_\infty^\xi \frac{1}{\gamma_a^2}\frac{\partial I_0}{\partial\xi}\cos[k(\xi - \xi')]\mathrm{d}\xi', \tag{14}$$

where we have used the laser pulse intensity $I_0 = |a_0|^2$. The plasmon wavenumber k is nearly equal to $\omega_p/\gamma_a c$, and its exact definition is given in appendix C. The coupling term Ω is now responsible for the excitation of the axion wake given by

$$\varphi(\xi) = -2e\frac{g\hbar B_0}{m_\varphi^2}\int_\infty^\xi \tilde{n}(\xi')\mathrm{d}x'. \tag{15}$$

These wakefield solutions are illustrated in figure 4. They show that, in addition to the usual electron plasma oscillations, a wake of axions is now present. Wakefield losses and nonlinearities were neglected, but can easily be incorporated. In the above solutions we have assumed that the relativistic factor γ_a is constant. This is a good approximation in the wakefield region, even for arbitrary intensities (see figure 4). Note that, it is the electrostatic perturbations resulting from the modulational instability that are responsible for the excitation of the axion wake. The latter can, in principle, feedback onto the plasmon modes in a self-consistent manner. However, this effect is very small [25] and we explicitly ignore it here. This also means that indirect detection of axions via their influence on the electrostatic oscillations is extremely difficult. We propose instead the detection of these newly created axions by a separate apparatus (similar to the shining through wall detectors). We have previously proposed the use of relativistic electron beams in a *plasma shining through a wall* (PSW) experiment, with sensitivities that can compete with the usual *light shining through a wall* (LSW) experimental setup [31]. Here we have explored an alternative plasma model, where the relativistic electron beams are replaced by intense laser pulses. The resulting electric fields are comparable. Therefore, the numerical estimates of [31] remain valid in the present context. In recent years, laser-plasma experiments have shown that we can create density perturbations of $\tilde{n} \sim 10^{18}\,\mathrm{cm}^{-3}$, using gas jets together with laser intensities of $I_0 \sim 10^{18}\,\mathrm{Watt\,cm}^{-2}$ [28]. For axion studies, we should work at much lower densities, in order to be at resonance with the expected axion mass, which seems achievable.

5. Conclusions

We have studied the active production of axions in a plasma, using intense laser pulses propagating in the direction of a static magnetic field $\mathbf{B}_0$. Two laser pulse regimes were considered. The long pulse regime, where modulational instabilities take place, and the short pulse regime, where axion-plasmon wakefields can be excited. In particular, we were able to show that axion-plasmon coupled oscillations, first discussed in [25], can become unstable due to modulations of the laser field amplitude and the resulting plasma density perturbations. We have derived new dispersion relations, and established the respective growth rates. The usual dispersion relations of laser-plasma systems are modified by the axion field which inherits the typical modulational instability that occurs in the former. The laser field itself cannot directly couple to axions, because of the transverse polarisation, but coupling is established through the longitudinal plasmon field.

Furthermore, in the short pulse regime, the electrostatic wakes are slightly modified by the presence of axions. New wakefield solutions were established, which show the existence of an axion wake produced by the laser pulse. Direct detection methods, using electric probes or optical imaging seem unrealistic. The situation changes when we consider the potential use of these laser-plasma interaction processes in the frame of a *shining through wall* experiment. Here, plasma configurations could be relevant to produce axions in the laboratory. Local creation of axions by plasma instabilities and wakes could considerably increase the chances to detect axions with shining through wall detectors (see [31] for a detailed numerical analysis). The design of new experiments using intense lasers and electron beams in a plasma will be examined in a future paper.

Finally, in the astrophysical context, we can easily imagine that intense radiation bursts, emitted by magnetars and active galactic nuclei along locally intense magnetic fields, would allow for the local production of axions through the two processes described here.

Appendix A. Photon number density

Each component $\mathbf{a}_k$ of the vector potential $\mathbf{a} = \int \mathbf{a}_k \exp(-\mathrm{i}\omega'_k t)\mathrm{d}\mathbf{k}'/(2\pi)^3$, can be represented in terms of the photon number density N_k. This quantity is defined by

$$N_k = \frac{W_k}{\hbar\omega'},\quad W_k = \frac{\epsilon_0}{4}\left(\frac{m_e c}{e}\right)^2 \omega' |a_k|^2, \tag{A1}$$

where $W_k \propto |a_k|^2$ is the spectral energy density. In geometric optics, the evolution equation for N_k, takes the form of a Vlasov equation of the form [30]

$$\frac{\mathrm{d}}{\mathrm{d}t}N_k = 0,\quad \frac{\mathrm{d}}{\mathrm{d}t} \equiv \left(\frac{\partial}{\partial t} + \mathbf{v}' \cdot \nabla + \mathbf{F}' \cdot \frac{\partial}{\partial \mathbf{k}'}\right), \tag{A2}$$

where $\mathbf{v}'$ is the group velocity of photon modes with wavevector $\mathbf{k}'$ and frequency $\omega' = \sqrt{k'^2 c^2 + \omega_p^2 (n_e/n_0)}$, and $\mathbf{F}' = -\partial\omega'/\partial\mathbf{r}$ is the force acting on the photons.

Appendix B. Dispersion relation

The electron fluid equations, with a ponderomotive force due to the laser field is

$$\begin{aligned}
&\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{v} = 0,\\
&\frac{\partial}{\partial t}(\gamma_a \mathbf{v}) = -\frac{e}{m_e}(\mathbf{E}_s + \mathbf{v} \times \mathbf{B}_0)\\
&\qquad - \frac{S_e^2}{n_0}\nabla n - \frac{c^2}{2}\frac{\nabla |a|^2}{\gamma_a},
\end{aligned} \tag{B1}$$

where $S_e = \sqrt{T_e/m_e}$ is the electron thermal velocity, and $\gamma_a = \sqrt{1 + |a_0|^2}$ the electron relativistic factor in the laser field. The quantity $|a_0|^2 = \int |a_k|^2 \mathrm{d}\mathbf{k}'/(2\pi)^3$ slowly varies in space and time, and is proportional to the total photon density $N_t = \int N(\mathbf{k}')\mathrm{d}\mathbf{k}'/(2\pi)^3$.

We linearize these equations with respect to the perturbations $\tilde{n} = n_e - n_0$. Noting that $|a|^2 \propto \int N(\mathbf{k}')\mathrm{d}\mathbf{k}'$, and assuming a quasi-1D model with an electrostatic field parallel to the static magnetic field, we get

$$\begin{aligned}
\left(\frac{\partial^2}{\partial t^2} - \frac{S_e^2}{\gamma_a}\nabla^2 + \frac{\omega_p^2}{\gamma_a}\right)\tilde{n} = &-\frac{e n_0}{m_e \gamma_a} c g \mathbf{B}_0 \cdot \nabla\varphi\\
&+ \frac{2\hbar}{m_e}\frac{\omega_p^2}{\gamma_a^2 \omega_0}\nabla^2 \int \tilde{N}(\mathbf{k}')\mathrm{d}\mathbf{k}',
\end{aligned} \tag{B2}$$

where ω_0 is the central laser pulse frequency. Here, the axion field is determined by equation (1), with $\mathbf{E} = \mathbf{E}_s$ and the photon density perturbations $\tilde{N}(\mathbf{k}')$ are described by the linearised kinetic equation (A2). which can be stated as [30]

$$\left(\frac{\partial}{\partial t} + \mathbf{v}' \cdot \nabla\right)\tilde{N}(\mathbf{k}') = -\mathbf{F}' \cdot \frac{\partial}{\partial \mathbf{k}'}N_0(\mathbf{k}'), \tag{B3}$$

where $N_0(\mathbf{k}')$ is the unperturbed photon density. Let us now assume evolving as $(\tilde{n}, \tilde{N}(\mathbf{k}'), \varphi) \propto \exp(\mathrm{i}kz - \mathrm{i}\omega t)$. From equation (1), we obtain

$$(1 - \chi_\varphi)\varphi = -2\mathrm{i}\frac{egc^4}{\hbar^2 k\omega^2}B_0\tilde{n}, \tag{B4}$$

where we introduced the *axion susceptibility*, $\chi_\varphi = \omega_\varphi^2/\omega^2$. The new frequency ω_φ is determined by

$$\omega_\varphi^2 = k^2c^2 + \frac{\tilde{m}_\varphi^2 c^4}{\hbar^2}, \tag{B5}$$

where $\tilde{m}_\varphi$ is the effective axion mass

$$\tilde{m}_\varphi^2 = m_\varphi^2 + \frac{\hbar^2}{c^4}\frac{\Omega^4}{\omega_p^2}, \quad \Omega^4 = \omega_c^2\omega_g^2. \tag{B6}$$

For convenience, we also use the electron cyclotron frequency ω_c, and the axion coupling frequency ω_g, defined by

$$\omega_c = \frac{eB_0}{m_e}, \quad \omega_g^2 = 2g^2c^5\frac{n_{e0}m_e}{\hbar}. \tag{B7}$$

A relation between the perturbed quantities $\tilde{N}(\mathbf{k}')$ and $\tilde{n}$, can be established by using

$$\mathbf{v}' = \frac{\partial\omega'}{\partial\mathbf{k}'} = \frac{c^2\mathbf{k}'}{\omega'}, \quad \mathbf{F}' = -\frac{\partial\omega'}{\partial\mathbf{r}} = -\mathrm{i}k\frac{\omega_{p0}^2}{2\omega'\gamma_a}\frac{\tilde{n}}{n_0}. \tag{B8}$$

This leads to

$$\tilde{N}_k = -\frac{\mathrm{i}\mathbf{k}\cdot(\partial N_{k0}/\partial\mathbf{k}')}{(\omega - \mathbf{v}'\cdot k)}\frac{\omega_{p0}^2}{2\omega'}\frac{\tilde{n}'}{n_0}. \tag{B9}$$

Replacing this in equation (B3), integrating by parts and assuming $N_{k0} = N_0\delta(\mathbf{k}' - \mathbf{k}'_0)$, with $k_0' = \omega_0/c$, we obtain

$$(1 - \chi_e - \chi_{\mathrm{ph}})\tilde{n} = \mathrm{i}\frac{\omega_c n_0}{\omega^2}gck\ \varphi, \tag{B10}$$

where the electron and photon susceptibilities are

$$\chi_e = \frac{1}{\gamma_a\omega^2}(\omega_{p0}^2 + k^2S_e^2), \quad \chi_{\mathrm{ph}} = \frac{\omega_{p0}^2}{\gamma_a\omega^2}\frac{\nu_{\mathrm{ph}}^2}{(\omega - kv_0)^2}, \tag{B11}$$

where $v_0 = v'(\mathbf{k}' = \mathbf{k}'_0)$. We have also used the relative photon density parameter ν_{ph}, defined as

$$\nu_{\mathrm{ph}}^2 = \frac{k^2c^2}{4}|a_0|^2 = \frac{\hbar k^2c^2}{m_en_0}\frac{\omega_{p0}^2}{\omega'}N_0. \tag{B12}$$

From here, an effective electron beam density, $n_{\mathrm{eff}} = n_0\nu_{\mathrm{ph}}^2/K^2c^2$, can be defined.

Appendix C. Wakefield equations

We replace equation (B2) by

$$\left(\frac{\partial^2}{\partial t^2} - \frac{S_e^2}{\gamma_a}\frac{\partial^2}{\partial z^2} + \frac{\omega_{p0}^2}{\gamma_a}\right)\tilde{n} = -\frac{en_0}{m_e\gamma_a}cg\mathbf{B}_0\cdot\frac{\partial\varphi}{\partial z} + \frac{c^2}{2}\frac{n_0}{\gamma_a^2}\frac{\partial^2}{\partial z^2}I_0(z - v_0t), \tag{C1}$$

where we have $\gamma_a = \sqrt{1 + I_0}$. For the axion field equation, we can replace equation (1), by

$$\frac{\partial}{\partial z}\left(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial z^2} + \Omega_\varphi^2\right)\varphi = -2\epsilon_0\frac{gc^4}{\hbar}(\mathbf{E}\cdot\mathbf{B}_0), \tag{C2}$$

These equations describe the electron density and axion field, $\tilde{n}$ and φ, produced by the short laser pulse.

We use a transformation from (z, t) to $(\xi = z - v_0t, \tau = t)$, and take the quasi-static limit, $\partial/\partial\tau \simeq 0$. Neglecting thermal effects $(S_e^2 \ll c^2)$, we get

$$\left(\frac{\partial^2}{\partial\xi^2} + \frac{\omega_p^2}{\gamma_a(\xi)v_0^2}\right)\tilde{n} = -\frac{n_0\omega_c}{\gamma_a(\xi)}\frac{cg}{v_0^2}\frac{\partial\varphi}{\partial\xi} + \frac{c^2}{2v_0^2}\frac{n_0}{\gamma_a^2(\xi)}\frac{\partial^2 I_0}{\partial\xi^2}, \tag{C3}$$

and

$$\frac{\partial\varphi}{\partial\xi} = -2e\frac{g\hbar B_0}{m_\varphi^2}\tilde{n}. \tag{C4}$$

From here, we can easily derive a wakefield equation in closed form

$$\left[\frac{\partial^2}{\partial\xi^2} + k^2(\xi)\right]\tilde{n} = \frac{n_0}{2\gamma_a^2(\xi)}\frac{\partial^2 I_0}{\partial\xi^2}, \tag{C5}$$

with the wavenumber

$$k(\xi) = \sqrt{\frac{\omega_p^2}{\gamma_a(\xi)v_0^2} - k_\varphi^2(\xi)}, \tag{C6}$$

The axion correction to the usual plasmon wavenumber is

$$k_\varphi^2(\xi) = \frac{\omega_c^2\omega_g^2}{\gamma_a(\xi)}\frac{\hbar^2}{m_\varphi^2c^6} \simeq \frac{\omega_c^2\omega_g^2}{\Omega_\varphi^2c^2}. \tag{C7}$$

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References

[1] Bertone G and Hooper D 2018 *Rev. Mod. Phys.* **90** 45002
[2] Profumo S 2017 *An Introduction to Particle Dark Matter* (Singapore: World Scientific)
[3] Sanders R H 2010 *The Dark Matter Problem* (Cambridge: Cambridge University Press)

[4] Kim J E and Carosi G 2010 *Rev. Mod. Phys.* **82** 557
[5] Peccei R D and Quinn H R 1977 *Phys. Rev. Lett.* **38** 1440
[6] Weinberg S 1978 *Phys. Rev. Lett.* **40** 223
[7] Wilczek F 1978 *Phys. Rev. Lett.* **40** 271
[8] Kim J E 1979 *Phys. Rev. Lett.* **43** 103
[9] Shifman M, Vainshtein A and Zakharov V 1980 *Nucl. Phys.* B **166** 493
[10] Dine M, Fischler W and Srednicki M 1981 *Phys. Lett.* B **104** 199
[11] Zhitnitsky A 1980 *Sov. J. Nucl. Phys.* **31** 260 Yad. Fiz. (1980), Inspire, 31, 497
[12] Abel C *et al* 2017 *Phys. Rev. X* **7** 041034
[13] Arik M *et al* 2014 *Phys. Rev. Lett.* **112** 091302
[14] CAST Collaboration 2017 *Nat. Phys.* **13** 584
[15] Balou R *et al* 2015 *Phys. Rev.* **92** 092002 D
[16] Ehret K *et al* 2010 *Phys. Lett.* B **689** 149
[17] Asztalos S J *et al* 2010 *Phys. Rev. Lett.* **104** 041301
[18] Mei H H, Ni W T, Chen S J and Pan S S 2010 *Mod. Phys. Lett.* A **25** 983 A
[19] Cameron R *et al* 1993 *Phys. Rev.* D **47** 3707
[20] Mendonça J T 2007 *Europhys. Lett.* **79** 21001
[21] Döbrich B and Gies H 2010 *J. High Energy Phys.* **10** 1–27
[22] Di Piazza A, Müller C, Hatsagortsyan K Z and Keitel C H 2012 *Rev. Mod. Phys.* **84** 1177
[23] Villalba-Chávez S, Podszus T and Müller C 2017 *Phys. Lett.* B **769** 233
[24] Dillon B M and King B 2018 *Eur. Phys. J.* C **78** 775
[25] Terças H, Mendonça J T and Rodrigues J D 2018 *Phys. Rev. Lett.* **120** 181803
[26] Burton D A and Noble A 2018 *New J. Phys.* **20** 033022
[27] Sikivie P 1983 *Phys. Rev. Lett.* **51** 1415
[28] Bingham R, Mendonça J T and Shukla P K 2004 *Plasma Phys. Control. Fusion* **46** R1
[29] Visinelli L 2013 *Mod. Phys. Lett.* A **28** 1350162
[30] Mendonça J T 2001 *Theory of Photon Acceleration* (Bristol: IOP Publishing)
[31] Mendonça J T, Rodrigues J D and Terças H 2019 Axion production in unstable magnetised plasmas arXiv:1901.05910v2