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A Genetic Algorithm for Economic Order Quantity with a Short Product Life Cycle

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Abstract. Considering a finite time horizon crossing over multiple stages of a product life cycle, this study presents a genetic algorithm to deal with an economic order quantity model with multiple demand rates under a non-periodic policy. In real, the demand of the product life cycle is a non-linear function but we assumes it as four-segment linear or constant approximations in this work. In addition, a multiple-segment to be combined with linear or constant functions can be approximated a nonlinear function. This study does not focus on this, but it provides a genetic algorithm to deal with this proposed inventory problems. The particular of this research is that we develop a proposed replenishment scheme by the differentiating equation of the total cost with respect to replenishment time. Then, calculate the total cost of the proposed scheme as the fitness function to evaluate the populations. In this paper, an explicit procedure to obtain an approximating solution is provided and numerical examples to illustrate the proposed model are shown as well.

1. Introduction

The collaborative planning, forecasting and replenishment (CPFR) is an aim to assist and support joint practice among entities for supply chain integration. Under scenario of the CPFR, a suppler would request a buyer to build up a medium- to long-term proposal replenishment schedule for their collaborative forecast and inventory policy. Therefore, the planner of the supplier has to map out the interval and demand of each stage and set up an intermediate-range distribution requirement plan according to the buyer’s forecast. This planning horizon is perhaps across over multiple stages on product life cycle. Due to technology advancement and innovation, product life cycle are becoming shorter and shorter especially electronic consumer products recently. For this phenomenon of product life cycle, most previous studies have assumed that the demand pattern is one linear function on production or inventory problems. Nevertheless, the previous researches consider only one linear function over the planning horizon, which is not able to deal with the above situation with different demand trend and multiple stages. One primitive solution following previous one linear function algorithm is to sum each separated stage with the single-piece linear model; but it is neither practical nor effective. To acquire a better solution and release a strict assumption that is no inventory to be held at the end of each stage, this study proposes a genetic algorithm for the economic order quantity problem when the trend of demand is a piecewise linear function.

To consider an increasing demand on the product life cycle or bloom season, Resh et al. [1] were the first to introduce the classical lot-size model with deterministic and time-proportional demand rate. Donaldson [2] first proposed an analytic approach for replenishment problem with a linear (increasing) trend in demand. Based on the above contribution, Henery [3] put forward a recursive procedure for determining the optimal replenishment schedule under the condition of a specified replenishment lots.

Hariga [4] developed an iterative algorithm to derive replenishment schedule for both increasing and decreasing trend in demand. Rau and Ouyang [5] investigated how to model an economic order quantity with a piecewise linear trend in demand. Hill [6] first introduced a general, time-varying, continuous, deterministic demand pattern through a complete product life cycle that broke the demand pattern up into stages and approximated the pattern for each stage by a cubic polynomial.

Genetic algorithms (GAs) are one of most common search methodologies to mimic the process of natural selection and natural genetics for optimization and search problems. The genetic algorithm was first introduced by Holland [7]. Many contributions have widely applied to solve issues for operations and supply chain management, such as inventory control, facility layout, line balancing, production scheduling, and logistics distribution etc.. For characteristics of the chromosome in genetic algorithms, the formulation is ideally suited for using GAs, so Khouja et al. [8] proposed a genetic algorithm to handle the economic lot size problem (ELSP) with discrete demand. Genetic algorithms only need a computable objective function with no requirements of mathematical theory proof such as convexity. Consequently, Gaafar [9] selected genetic algorithms for the deterministic time-varying lot sizing problem with batch ordering and backorders. Recently, Bera et al [10] studied GAs applying to a realistic inventory model with continuous demand under finite horizon. Considering deteriorating, inflation, budget constraints, shortages and finite planning time horizon, Ouyang [11] and Jana et al [12] also selected GAs for inventory models with continuous demand.

2. Assumptions and Notation

2.1. Assumptions

According to characteristics of product life cycle stages, the demand starts to increase gradually in the introduction stage. Next, the demand increase rapidly during the growth stage. Then, the demand becomes to steady over the maturity stage. Finally, the demand ends to decrease fast under the decline stage. In real, the demand of the product life cycle is a non-linear function but we treat it as four-segment linear or constant approximations in here. The demand pattern of this proposed product life cycle is shown in Figure 1. Except the above assumptions, we still have some relative inventory restrictions in our study as follows:

- A finite foreseeable time horizon is considered.
- The demand of each stage is a constant, linearly increasing or decreasing function.
- The ending point of the first stage is the starting point of the second stage and time is continuous.
- A single item is considered.
- Lead-time or shortage is not considered.
- Demand is always greater than zero at the end of the time horizon.
- No stock is held at the beginning and the end of the time horizon.

2.2. Notation

n	number of replenishment cycles.
n^*	optimal replenishment cycles.
H_1	end time of the introduction stage.
H_2	end time of the growth stage.
H_3	end time of the maturity stage.
H_4	end time of the decline stage.
H	planning horizon.
W	total cost, including set up and holding cost.
$W^*(n)$	optimal total cost for n replenishment cycles.
c_1	set up cost per order.
c_2	holding cost per unit per year.
$f_i(t)$	demand function during the ith stage.
a_1	demand at $t = 0$
b_1	rate of demand change per unit of time during stage i and $b_i = 0$ during maturity stage.

t_i Terminating time of ith replenishment cycles time or starting time of i+1th replenishment cycles period i = 0, 1,…, n-1.

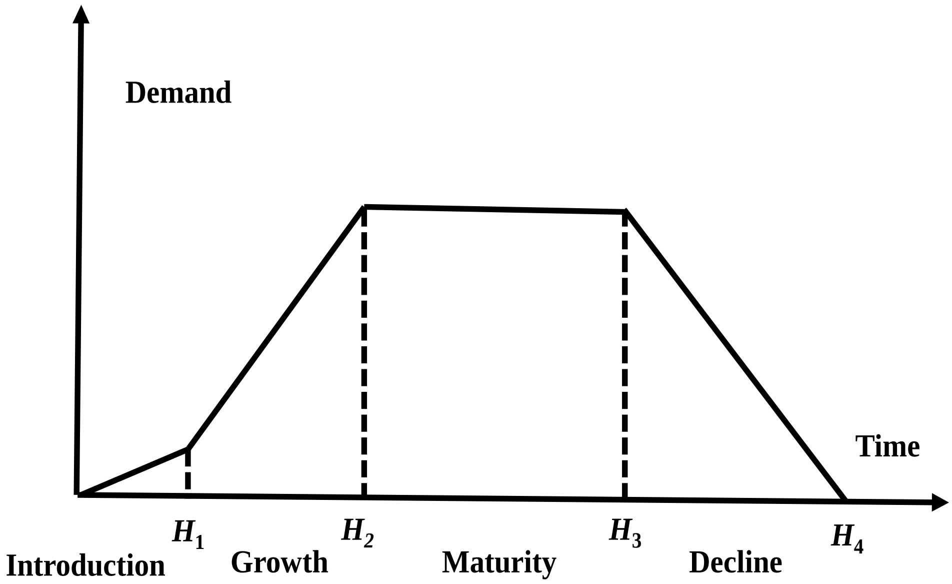

Figure 1. Demand pattern for this proposed product life cycle

3. The Mathematical Development

3.1. Single-Stage Model

The single-stage is a case that only one demand rate over the planning horizon. Suppose a fixed number of n replenishments and the demand function is $f(t) = a_1 + b_1 t$ under the planning horizon H. Therefore, we have the total relevant cost, including the replenishment cost and holding cost as follows:

$$W = nc_1 + c_2 \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} \int_{t}^{t_i} f(u)dudt \tag{1}$$

From equation (1), it is easy to understand that this proposed inventory model is a nonlinear problem, where we have to resolve the replenishment schedule t_i and number of production cycles n over a planning horizon H. Under a s deterministic n, the derivative of equation (1) with respect to t_i, i =1, 2….n-1, is

$$\frac{\partial W}{\partial t_i} = c_2 \left[\left(t_i - t_{i-1} \right) f(t_i) - \int_{t_i}^{t_{i+1}} f(t)dt \right] \tag{2}$$

Hence, W has the stationary point at t_i if equation (2) = 0, which is a necessary condition for obtaining the optimal solution, that is

$$\int_{t_i}^{t_{i+1}} f(t)dt = (t_i - t_{i-1}) f(t_i) \qquad i = 1, 2, \ldots\ldots.. n-1. \tag{3}$$

Solve equation (3) and derive

$$t_{i+1} = (-a + \sqrt{(-2b^2 t_{i-1} t_i + 3b^2 t_i^2 - 2abt_{i-1} + 4abt_i + a^2)}) / b\,. \tag{4}$$

3.2. Two-stage model

The first stage demand function is $f_1(t) = a_1 + b_1 t$ with planning time from 0 to H_1 and the second demand function is $f_2(t) = a_1 + b_1 H_1 + b_2(t - H_1)$ with planning time from H_1 to H, which is the end of the whole planning horizon, as shown in equation (4).

$$f(t) = \begin{cases} a_1 + b_1 t & t \le H_1 \\ a_1 + b_1 H_1 + b_2(t - H_1) & H_1 < t \le H \end{cases} . \tag{5}$$

Suppose the kth replenishment period is across stages one and two and there are two linear demand functions $f_1(t)$ and $f_2(t)$ at the kth replenishment period. Then, the total cost, including the replenishment and inventory holding cost, is given by

$$W = nc_1 + c_2\left(\sum_{i=0}^{k-2}\int_{t_i}^{t_{i+1}}\int_t^{t_i+1} f_1(u)dudt + \int_{t_{k-1}}^{H_1}\int_t^{H_1} f_1(u)dudt + (H_1 - t_{k-1})\int_{H_1}^{t_k} f_1(t)dt \right.$$
$$\left. + \int_{H_1}^{t_{k1}}\int_t^{t_{k1}} f_2(u)dudt + \sum_{i=k1}^{n-1}\int_{t_i}^{t_{i+1}}\int_t^{t_{i+1}} f_2(u)dudt \right) . \tag{6}$$

Similarly, under a deterministic n value, the derivative of equation (6) with respect to t_i, i =1,2….n-1 and then we have a stationary point at t_i as follows:

$$\begin{aligned} &(t_i - t_{i-1})\, f_1(t_i) = \int_{t_i}^{t_{i+1}} f_1(u)du && i = 1,..,k-1 \\ &(t_i - t_{i-1})\, f_1(t_i) = \int_{t_i}^{H_1} f_1(t)dt + \int_{H_1}^{t_{i+1}} f_2(t)dt && i = k-1 \\ &(t_i - t_{i-1})\, f(t_i) = \int_{t_i}^{t_{i+1}} f_2(u)du && i = k+1,..,n-1 \end{aligned} . \tag{7}$$

We solve equation (7) and obtain the following equation

$$\begin{aligned} &t_{i+1} = (-a + \sqrt{(-2b_1^2 t_{i-1}t_i + 3b_1^2 t_i^2 - 2ab_1 t_{i-1} + 4ab_1 t_i + a^2)})/b_1 \\ &\text{when} \quad t_{i+1} \le H_1 \quad \text{and} \quad i = 1,..,k-1 \\ &t_{i+1} = \frac{-a + \sqrt{H_1^2 b_1 b_2 + H_1^2 b_2^2 + 2b_1 b_2 t_{i-1} t_i - 3b_1 b_2 t_i^2 + 4H_1 ab_2 + 2ab_2 t_{i-1} - 4ab_2 t_i + a^2}}{b_2} \\ &\text{when} \quad t_i \le H_1 \quad \text{and} \quad t_{i+1} \ge H_1 \quad \text{and} \quad i = k-1 \\ &t_{i+1} = (-a + \sqrt{(-2b_2^2 t_{i-1}t_i + 3b_2^2 t_i^2 - 2ab_2 t_{i-1} + 4ab_2 t_i + a^2)})/b_2 \\ &\text{when} \quad t_{i+1} \ge H_1 \quad \text{and} \quad i = k+1,..,n-1 \end{aligned} \tag{8}$$

3.3. Multiple-Stage Model

Actually, the two-stage model can be easily extended into multiple-stage models because of deterministic the period of each stage. We are able to follow the procedure for handling two-stage model when we encounter piecewise linear demand.

4. Genetic Algorithm

In this proposed model, the number of replenishment cycles n and replenishment schedule t_i have to be solved, which is a complicated nonlinear problem. Thus, we provide this genetic algorithm with the chromosome of real number type for seeking to an approximately optimal solution. Based on Bellman's principle of optimization [13], once t_1 is found correctly and the remaining replenishment schedule t_2 to t_n can be determined by equation (4) for single-stage and equation (8) for multiple-stage. Thus, we only have to search t_1 and neglect n. Our proposed genetic algorithm to derive an approximate solution is shown in Figure 2 and illustrated as follows:

Chromosome representation:
Due to short life product cycle, we assume a finite planning horizon that is always less than one year. Consequently, randomly select a chromosome consisting of 20-bits for the first replenishment cycle time t_1 under a reasonable range. For example, the terminating time of the first replenishment cycles t_1 = 0.036 is near bit string 00001001001101110101 by binary expression.

Fitness function:
With a trial t_1, we can obtain t_2 by equation (4) for single-stage and equation (8) for multiple-stage. Repeatedly solve t_3, t_4,….., t_{n-1}, t_n until $t_{n-1} < H$ and $t_n \geq H$. Then, let $t_n = H$ and compute total cost $W(n, \{t_i\})$ and let $t_{n-1} = H$ and compute total cost $W(n-1, \{t_i\})$ from equation (3). Finally, compare with the costs of these two scheme and select the lower one. Therefore, the fitness function can be expressed as follows:

$$\begin{cases} W\left(n,\{t_i\}\right) & \text{if} \quad W\left(n,\{t_i\}\right) < W\left(n-1,\{t_i\}\right) \\ W\left(n-1,\{t_i\}\right) & \text{if} \quad W\left(n,\{t_i\}\right) > W\left(n-1,\{t_i\}\right) \end{cases} \tag{9}$$

GA operators:
GA operators create a new generation from parent chromosomes. There are three standard GA operators are used, namely, reproduction, mutation and crossover.

Searching direction:
This GA searching direction is controlled by the self-adjustment rate of operators based on survivor off-springs' rates that are determined after each generation in order to the next search.

Input parameters:
Parameters of genetic algorithm, including population size, probability of mutation, probability of crossover, generation size, initial rate of crossover operation, initial rate of mutation operation, initial rate of reproduction operation and stop condition, are reasonably provided.

Output: The local optimal replenishment schedule and the total cost.

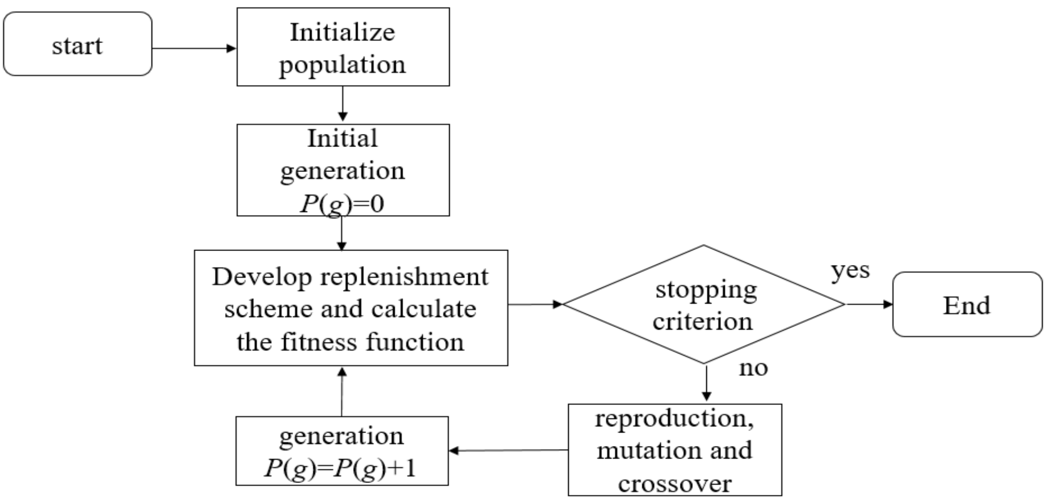

Figure 2. Genetic algorithm for this proposed model

5. Numerical Examples

A VB6.0 program was written to solve the following examples. In order to illustrate our model, we offer two examples as follows:

Under c_1 = 4.5 and c_2=1, suppose a short product life cycle item that $H_1 = 0.2$, $H_2 = 0.5$, $H_3 = 0.8$, and H_4 =1.0. The demand functions are $f_1(t) = 6000$, $f_2(t) = 1200+9000t$, $f_3(t) = 3900$ and $f_4(t) = 3900-19500t$.

Example 1: the planning horizon is from 0 to 0.4 and Example 2: the planning horizon is from 0.6 to 1.0. Applied to the proposed solution procedures, replenishment schedule for these two examples are shown in Table I. The total costs for these two examples are 67.1572 and 89.6796.

Table 1. Replenishment schedules for numerical examples

Example / *i*	1	2	3	4
1	0.1423	0.2444	0.3273	0.4
2	0.6779	0.755	0.8372	1.0

6. Conclusions

For technology advancement, the product life cycle for 3C items are becoming very short. Many industries may encounter that their planning horizon pass through different stages of the product life cycle. In real, this inventory model is a very complex nonlinear problem that is difficult to provide mathematical theory proof such as convexity. Therefore, this study develop a genetic algorithm for relaxing the previous researches with a single-stage linear trend in demand. The particular of our proposed algorithm is that we select the total cost's differentiate equation to build up a proposed replenishment scheme. Then, compute the total cost of this replenishment schedule as the fitness function to evaluate the populations for solving this inventory problem. In terms of the future research, people are able to deliberate our proposed method applying to genetic algorithm for other unconstrained nonlinear problems especially in more complex inventory model with relative issues under a production system.

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