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# The Role of Pressure Anisotropy in Cosmic-Ray Hydrodynamics

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## Abstract

The mean free path of cosmic rays in diffuse interstellar and intracluster gas is determined primarily by pitch angle scattering from hydromagnetic waves with wavelength of order the cosmic-ray gyroradius. In the theory of cosmic-ray self confinement, the waves are generated by instabilities driven by the cosmic rays themselves. The dominant instability is due to bulk motion, or streaming, of the cosmic rays, parallel to the background magnetic field $\boldsymbol{B}$, and transfers cosmic-ray momentum and energy to the thermal gas as well as confining the cosmic rays. Classical arguments and recent numerical simulations show that self confinement due to the streaming instability breaks down unless the cosmic-ray pressure and thermal gas density gradients parallel to $\boldsymbol{B}$ are aligned, a condition that is unlikely to always be satisfied We investigate an alternative mechanism for cosmic-ray self confinement and heating of thermal gas based on pressure anisotropy instability. Although pressure anisotropy is demonstrably less effective than streaming instability as a self-confinement and heating mechanism on global scales, it may be important on mesoscales, particularly near sites of cosmic-ray injection.

*Unified Astronomy Thesaurus concepts:* Cosmic rays (329); Galactic winds (572); Interstellar medium (847); Plasma astrophysics (1261)

## 1. Introduction

Cosmic rays provide a window into high energy processes throughout the universe, significantly affect interstellar and intracluster gas dynamics and energy balance, and are agents of star formation and black hole feedback. All these aspects of cosmic-ray astrophysics depend on how cosmic rays propagate through the ambient magnetic field $\boldsymbol{B}$.

The near isotropy and long confinement times of Galactic cosmic rays imply that their propagation is largely diffusive (see Grenier et al. 2015 for a recent review). While diffusion through space can be produced by propagation along randomly wandering magnetic field lines $\boldsymbol{B}$, diffusion parallel to $\boldsymbol{B}$, and to a lesser extent perpendicular to $\boldsymbol{B}$, is thought to be primarily due to scattering by magnetic fluctuations on scales of order the cosmic-ray gyroradius.

There are two theories for the origin of these fluctuations. In the self-confinement theory, the fluctuations are hydromagnetic waves that have been amplified by kinetic instabilities driven by cosmic-ray momentum space anisotropy (Wentzel 1968; Kulsrud & Pearce 1969). The unstable feature, or free energy source for the instability, is generally bulk drift, or streaming, which arises naturally, e.g, from the presence of discrete cosmic-ray sources and global Galactic gradients. In the extrinsic turbulence theory, the fluctuations are also hydromagnetic waves, but are driven by a mechanism such as a turbulent cascade that is independent of cosmic rays.

In both theories, momentum is transferred between the cosmic rays and the thermal gas through what can be described in the limit of short scattering mean free path as a pressure gradient force $-\boldsymbol{\nabla}P_c$. In self-confinement theory, the thermal gas is also heated collisionlessly by damping the waves at the rate the cosmic rays excite them; for the streaming instability, this works out to be $|\boldsymbol{v}_A \cdot \boldsymbol{\nabla}P_c|$. In the extrinsic turbulence theory, there is no heating, provided the fluctuations have no preferred direction of propagation. Here, $P_c$ and $\boldsymbol{v}_A$ are the cosmic-ray pressure and Alfvén velocity in the plasma component $\boldsymbol{B}/\sqrt{4\pi\rho_i}$, with $\rho_i$ the plasma mass density. Both theories lend themselves to fluid descriptions of cosmic-ray interactions with thermal gas ("cosmic-ray hydrodynamics"), and can be smoothly bridged when both extrinsic and self-generated waves are present (Zweibel 2017). Because both theories are based on scattering, they include spatial diffusion, but in many cases it is weak compared to advection by the thermal gas or Alfvénic streaming relative to it.

Both theories have been implemented in models of galactic winds and star formation feedback (Breitschwerdt et al. 1991; Everett et al. 2008; Uhlig et al. 2012; Agertz et al. 2013; Booth et al. 2013; Salem & Bryan 2014; Girichidis et al. 2016; Ruszkowski et al. 2017; Farber et al. 2018; Mao & Ostriker 2018; Bustard et al. 2019; Chan et al. 2019; Hopkins et al. 2020). These works show that the mass flux, momentum flux, thermal structure, and even the existence of galactic winds are sensitive to the model of cosmic-ray transport, as is the degree to which cosmic-ray feedback suppresses star formation. For example, assuming that cosmic rays are advected with the thermal gas but neither stream nor diffuse suppresses wind launching in Milky Way–like disks (Uhlig et al. 2012), but lowers the star formation rate more than models with cosmic-ray streaming, which are more effective in launching winds but less effective in suppressing star formation (Ruszkowski et al. 2017). With observational constraints on cosmic-ray transport in star-forming galaxies now emerging from models of their $\gamma$-ray emission (Chan et al. 2019), it is more important than ever to understand all the physical processes in play.

The streaming instability can be excited when the bulk drift speed $v_D$ is super-Alfvenic. The growth rate increases with $v_D$, and cosmic rays can be considered self confined if the $v_D$ required to overcome damping is not too much greater than $v_A$. While instability growth and damping rates depend only on local conditions that can be evaluated from point to point, self confinement also depends on the global structure of the system due to a "bottleneck effect," which was first hypothesized by Skilling (1971) and first demonstrated in numerical simulations

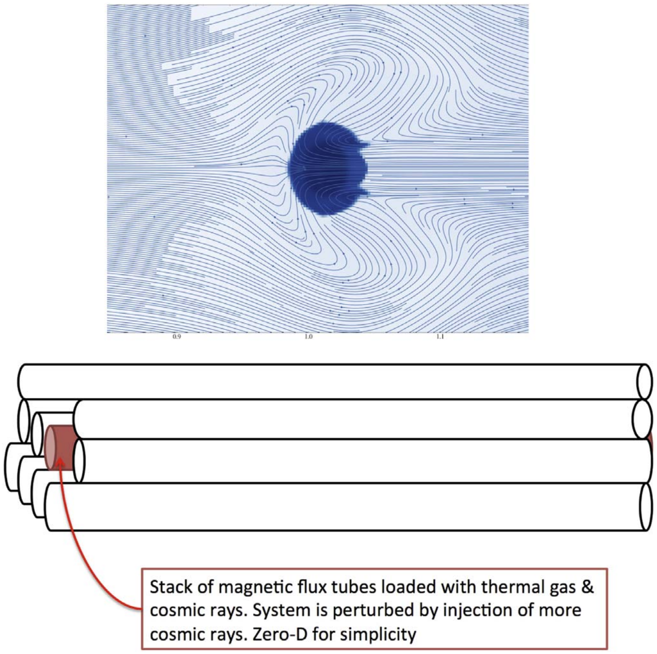


**Figure 1.** Top: the blue curves, from an unpublished simulation by J. Wiener, trace magnetic field lines that have been distorted by the buildup of cosmic-ray pressure between a source of cosmic rays at the left boundary and a dense cloud, shown in darker blue, in which the Alfvén speed is lower than in the surrounding medium. This is a bottleneck of the kind described in Section 1 and in Wiener et al. (2019). In the absence of scattering, the cosmic-ray pressure becomes anisotropic due to the changes in the volume and length of the magnetic flux tubes. Bottom: the simplified model on which the calculations in this paper are based. Cosmic rays are injected uniformly onto the thin magnetic flux tube shown in red, which expands at the magnetoacoustic speed to maintain pressure equilibrium with its surroundings. Spatial gradients (other than the kinetic scales associated with growing hydromagnetic waves) do not enter the model, which we therefore describe as Zero-D.

by Wiener et al. (2017). The drift anisotropy is associated with a spatial gradient in cosmic-ray pressure $P_{\rm c}$ along the background magnetic field $\boldsymbol{B}$ such that the drift is down the gradient and the unstable waves propagate in the same direction as the drift. It can then be shown that $P_{\rm c}$ varies along a magnetic flux tube in proportion to $\rho_i^{\gamma_{\rm c}/2}$, where $\gamma_{\rm c} \sim 4/3$ is the cosmic-ray adiabatic index. If $\rho_i$ increases in the direction of cosmic-ray streaming, this relation predicts that $P_{\rm c}$ increases as well. This implies that waves going in the *opposite* direction should be unstable. Under these irreconcilable conditions, the cosmic-ray pressure gradient flattens, there are no waves, and cosmic rays do not exchange energy or parallel momentum with the ambient medium.

According to the initial value problem considered by Wiener et al. (2017), the characteristic timescale for bottleneck formation is of order the Alfvén transit time $D/v_{\rm A}$ between the cosmic-ray source and the point a distance $D$ away where the density gradient reverses. Because the Alfvén transit time must compete with other timescales such as the cosmic-ray source modulation time and the timescales on which density structures form, disperse, and move, bottlenecks may not always reach a steady state. Nevertheless, the possibility of bottlenecks suggests a picture in which cosmic-ray self confinement, and the heating and momentum transfer that accompany it, could be quite intermittent in space and time. Because much of the diffuse gas in the universe is clumpy and astrophysical magnetic fields are usually tangled, this intermittency might be the generic state. Bottlenecks may be a confounding issue, for example, in treatments of cosmic-ray propagation and cosmic-ray heating in clusters of galaxies, where the magnetic geometry is usually assumed to be simple (Loewenstein et al. 1991; Guo & Oh 2008; Wiener et al. 2013; Jacob & Pfrommer 2017a, 2017b; Wiener & Zweibel 2019).

Because cosmic-ray heating and momentum transfer are important in so many astrophysical systems, we must ask whether drift anisotropy is the only path to self confinement. In this paper we develop a complementary theory for cosmic-ray–thermal gas coupling mediated by hydromagnetic waves. The theory is based on an instability similar to the drift instability, but the energy source is pressure anisotropy.

Pressure anisotropy instability is described qualitatively in Kulsrud (2005). It was studied previously by Lazarian & Beresnyak (2006, hereafter LB06), and by Yan & Lazarian (2011), who investigated it as a mechanism for cosmic-ray self confinement and an energy sink for interstellar turbulence, and was recently studied numerically by Lebiga et al. (2018). We instead consider a case in which the instability is driven by an input of cosmic-ray energy itself. This is the most direct conceptual analog of heating due to streaming down the cosmic-ray pressure gradient.

The main outcome of our work is that for global or galactic scale processes, pressure anisotropy instability is much weaker than drift anisotropy as a mechanism for cosmic-ray self confinement and plasma heating. We find that under otherwise similar conditions, the spatial diffusivity resulting for pressure anisotropy is a factor of order $c/v_{\rm A}$ larger than the diffusivity resulting for drift anisotropy, and the heating rate associated with pressure anisotropy is lower than that due to drift anisotropy by a factor of order $v_{\rm A}/c$. The underlying reason is that when scattering is due to drift anisotropy instability, it balances the tendency of cosmic rays to stream down their pressure gradient at the speed of light. In the case of pressure anisotropy, scattering balances the rate of change of the magnetic field, the characteristic speed of which is typically the magnetoacoustic speed in the thermal gas. Therefore, a level of turbulent fluctuations that is lower by a factor of approximately $v_{\rm A}/c$ is required to maintain marginal stability against pressure anisotropy. The lower energy density in fluctuations translates to a lower level of heating, which is quantified in Section 4.3. Phenomena on smaller spatial scales with a shorter intrinsic timescale can drive anisotropy harder, leading to stronger scattering and possibly a significant level of heating. Thus, pressure anisotropy instability might play an important role in cosmic-ray self confinement near a young supernova remnant that has been an active cosmic-ray source for a few thousand years on scales of several parsecs, but less of a role near a galactic starburst that has been active for tens of megayears on scales of 100 or more parsecs.

In Section 2, we pose the simplest possible problem that brings out the relevant effects: expansion of a magnetic flux tube due to spatially uniform injection of cosmic-ray pressure. In Section 3, we discuss the pressure anisotropy instability. In Section 4, we derive an expression for the heating rate that results from instability due to cosmic-ray driven flux tube expansion, and in Section 5, we discuss the implications of the instability for cosmic-ray self confinement and estimate the spatial diffusion coefficient. In Section 6, we apply the theory to the problem of a bottleneck formed between a galactic halo cloud and a galactic disk (Wiener et al. 2019). Section 7 is a summary and speculation on how drift and pressure anisotropy instability might combine and a conclusion.

## 2. Formulation of the Problem

### 2.1. Macroscopic Dynamics

Here, we develop a simple model in which input of cosmic-ray energy drives expansion of the ambient gas, which weakens the magnetic field and causes cosmic-ray pressure anisotropy. The anisotropy triggers instabilities that scatter the cosmic rays and heat the gas, leading to a relationship between the energy injection rate, the rate of doing work, and the rate of heating. An example of a situation in which these events might occur is shown in Figure 1(a), while the simplified problem we solve is shown in Figure 1(b).

Consider a uniform medium with a background magnetic field $\boldsymbol{B}$, thermal gas pressure $P_{\rm g}$, and cosmic-ray pressure $P_{\rm c}$. This equilibrium system is perturbed by injecting cosmic rays onto a magnetic flux tube, or bundle of field lines, of radius $R$, at the same rate $\Delta\dot{P}_{\rm c}$, everywhere along the flux tube and uniformly over its cross section, ignoring the possibility of instability due to a cross field gradient (Riquelme & Spitkovsky 2010). In a more realistic situation the cosmic-ray source is likely to be localized to a small region along the flux tube and the cosmic rays stream down their gradient along the field lines as well as causing local transverse expansion of the magnetic field. We briefly speculate on the joint effects of parallel gradients and transverse expansion in Section 7; here we focus on the latter.

The flux tube responds to cosmic-ray injection by expanding perpendicular to its major axis. We approximate the speed of expansion by the magnetoacoustic speed $C_{\rm ma}$

$$C_{\rm ma} \equiv \left(\frac{\gamma_{\rm g} P_{\rm g} + \gamma_{\rm c} P_{\rm c} + \gamma_{\rm m} P_{\rm m}}{\rho_{\rm g}}\right)^{1/2}, \quad (1)$$

where $\gamma_{\rm g}$ and $\gamma_p$ are the thermal and cosmic-ray polytropic indices, $P_{\rm m} \equiv B^2/(8\pi)$ is the magnetic pressure, and $\gamma_{\rm m} = 2$ for transverse expansion. In writing Equation (1) we have made several simplifying assumptions. We have replaced the perpendicular cosmic-ray pressure $P_{\rm c\perp}$ by the isotropic pressure $P_{\rm c}$; assuming that scattering by waves keeps the pressure anisotropy near marginal stability, this is accurate to order $v_{\rm A}/c$. We have also assumed that the effects of heating and cooling on the thermal gas can be subsumed into an effective polytropic index $\gamma_{\rm g}$; a simplification that is common in interstellar gas dynamics.

The characteristic expansion timescale for the tube is $\tau_D \sim R/C_{\rm ma}$, while the total pressure changes on a timescale of $\tau_P \sim (P_{\rm g} + P_{\rm c} + P_{\rm m})/\Delta\dot{P}_{\rm c}$. If $\tau_D/\tau_P \ll 1$, the tube maintains pressure equilibrium with its surroundings; this sets an upper limit on the radius $R$. By the same assumptions made in deriving Equation (1),

$$C_{\rm ma}^2 \frac{d\rho_{\rm g}}{dt} = -\Delta\dot{P}_{\rm c}, \quad (2)$$

or because $B/\rho_{\rm g}$ is constant for uniform, transverse expansion

$$\frac{\dot{\rho}_{\rm g}}{\rho_{\rm g}} = \frac{\dot{B}}{B} = -\frac{\Delta\dot{P}_{\rm c}}{\rho_{\rm g} C_{\rm ma}^2} \equiv -\frac{1}{\tau_B}, \quad (3)$$

where we have introduced the magnetic field timescale $\tau_B$ for later use. From the First Law of thermodynamics, the rate $\Delta\dot{W}_{\rm c}$ at which the cosmic rays do work is

$$\Delta\dot{W}_{\rm c} = \frac{P_{\rm c}\Delta\dot{P}_{\rm c}}{\rho_{\rm g} C_{\rm ma}^2} = -P_{\rm c}\frac{\dot{B}}{B} = \frac{P_{\rm c}}{\tau_B}. \quad (4)$$

In Section 4, we derive a rate of heating in terms of the rate of doing work.

### 2.2. Microscale Response

As the flux tube expands, the cosmic-ray momentum distribution becomes anisotropic with respect to $\boldsymbol{B}$. If the magnetic field changes slowly relative to the cosmic-ray gyration frequency and there is no scattering, the particle motion is adiabatic and two orbital properties, the magnetic moment $p^2(1-\mu^2)/B$ and longitudinal action $p\mu B/n$, which for the situation here is just $p\mu$, can be treated as constant. Here, $p$ is the magnitude of the momentum and $\mu \equiv \boldsymbol{p}\cdot\boldsymbol{B}/pB$ is the cosine of the pitch angle. The evolution of the cosmic-ray phase space distribution function $f(\boldsymbol{p}, t)$ is given by

$$\frac{\partial f}{\partial t} + \frac{dp}{dt}\frac{\partial f}{\partial p} + \frac{d\mu}{dt}\frac{\partial f}{\partial \mu} = \Delta\dot{f}(p, t), \quad (5)$$

or assuming adiabatic motion,

$$\frac{\partial f}{\partial t} + \frac{\dot{B}}{B}\frac{(1-\mu^2)}{2}\left(p\frac{\partial f}{\partial p} - \mu\frac{\partial f}{\partial \mu}\right) = \Delta\dot{f}(p, t), \quad (6)$$

where $\Delta\dot{f}(p, t)$ is the phase space counterpart of $\Delta\dot{P}_{\rm c}$ and we assume particles are injected isotropically (see Lichko et al. 2017 for a more general version of Equation (6) that includes shearing as well as compression). In Equation (6), anisotropy arises through the term multiplying $\dot{B}/B$ in the sense that if $\dot{B}/B \equiv 0$ and $f$ is initially isotropic, it will remain isotropic. We therefore refer to this term as the driver of anisotropy in Equation (9). It should be borne in mind, however, that in the problem considered here it is $\Delta\dot{f}(p, t)$, or $\Delta\dot{P}_{\rm c}$, which causes $\dot{B}/B$ to be nonzero in the first place, so in an energetic sense, $\Delta\dot{P}_{\rm c}$ is the driver.

The evolution of $f$ can be followed more easily if we expand it as a Legendre series in $\mu$

$$f(p, \mu, t) = \Sigma_{l=0}^{\infty} f_l(p, t) P_l(\mu), \quad (7)$$

where

$$f_l(p, t) \equiv \frac{2l+1}{2}\int_{-1}^{1} f(p, \mu, t) P_l(\mu) d\mu. \quad (8)$$

It can be shown from well-known relations between Legendre functions that the solution of Equation (6) contains all harmonics of even order $l = 2n$, even if $f$ is initially isotropic. However, we show in Section 3 that the magnitude of the anisotropy, which we estimate as $|f_2/f_0|$, is capped at a value of order $v_{\rm A}/c$. Therefore, the primary driving term is isotropic, and we approximate Equation (6) by

$$\frac{\partial f}{\partial t} + \frac{\dot{B}}{3B}(P_0(\mu) - P_2(\mu)) p\frac{\partial f_0(p, t)}{\partial p} = \Delta\dot{f}(p, t). \quad (9)$$

According to Equation (9), if $f$ is initially isotropic, it generates $P_2$ anisotropy but no higher orders. In our problem $\dot{B}/B < 0$, so assuming $df_0/dp < 0$, the anisotropy is positive.

The physical significance of $f_2$ becomes apparent if we compute the pressure anisotropy

$$\Delta P_c \equiv P_{c\|} - P_{c\perp} \equiv \int fpv\left(\mu^2 - \frac{1-\mu^2}{2}\right)p^2 dp d\mu$$
$$= \int fpvP_2(\mu)p^2 dpd\mu = \frac{2}{5}\int f_2 pvp^2 dp. \quad (10)$$

Equation (10) shows that $f_2$ is a direct measure of pressure anisotropy. Strictly speaking, it is $P_{c\perp}$ that drives the expansion of the flux tube, but for the small anisotropy expected here, this is a correction that we can ignore.

## 3. Pressure Anisotropy Instability

We are interested in instabilities of hydromagnetic waves that are driven by gyroresonant particles. A particle gyroresonates with a circularly polarized wave of frequency $\omega$ and wavenumber $k_\|$ parallel to $\boldsymbol{B}$ if its parallel velocity $v_\|$ and relativistic gyrofrequency $\Omega = \Omega_0/\gamma$ (where $\Omega_0$ is the nonrelativistic gyrofrequency and $\gamma$ is the Lorentz factor) satisfy the condition that the wave frequency Doppler shifted to the particle frame matches the cyclotron frequency

$$\omega - k_\| v\mu \pm \Omega = 0. \quad (11)$$

Here, the $\pm$ signs refer to right and left circular polarization, respectively. We assume throughout the paper that the cosmic rays and interstellar ions are both protons, but the analysis can be generalized to other ion species.

Because $|\omega/k_\| v| \sim v_A/c$, we can drop $\omega$ in evaluating the resonance condition, unless $|\mu| \ll 1$. The resonant $\mu$, $\mu_r$, is then

$$\mu_r = \pm\frac{\Omega}{k_\| v} = \pm\frac{m_i\Omega_0}{k_\| p} \equiv \pm\frac{p_1}{p}, \quad (12)$$

where $p_1 \equiv m_i\Omega_0/k_\|$ is the minimum momentum that can resonate with a wave of parallel wavenumber $k_\|$.

There are three types of hydromagnetic waves: the fast and slow magnetosonic waves, and the shear (or intermediate) Alfvén wave. Kulsrud & Pearce (1969) showed that waves propagating parallel to $\boldsymbol{B}$ grow faster than oblique waves, so we assume parallel propagation here. In this case, one of the magnetosonic waves becomes a pure acoustic wave, which does not perturb the magnetic field and for which there are no gyroresonant effects, and the other magnetosonic wave becomes degenerate with the Alfvén wave, with both being purely transverse and both satisfying the dispersion relation $\omega = kv_A$. In this fully degenerate case, both linearly and circularly polarized waves are valid normal modes, but degeneracy holds only in the limit $\omega/\Omega_{ci} \to 0$. When higher order terms in $\omega/\Omega_{ci} \to 0$ are included, the normal modes are circularly polarized waves, with the left- and right-hand waves becoming ion cyclotron and whistler waves at higher frequencies, respectively.

The gyroresonant cosmic-ray pressure anisotropy instability also breaks the frequency degeneracy of left and right circularly polarized parallel propagating hydromagnetic waves (the gyroresonant cosmic-ray streaming instability, however, is independent of polarization, so degeneracy remains). The growth rate of parallel propagating, circularly polarized waves, is

$$\Gamma_c = \frac{\pi^2 q^2}{2}\frac{v_A^2}{c^2}\int p^2 dpd\mu v(1-\mu^2)\delta(\omega - kv\mu \pm \Omega)A(f, \omega, k) \quad (13)$$

where we have suppressed the subscript on $k$ because we now take $k = k_\|$. The anisotropy functional $A$ is

$$A[f, \omega, k] \equiv \frac{\partial f}{\partial p} + \left(\frac{kv}{\omega} - \mu\right)\frac{1}{p}\frac{\partial f}{\partial \mu}. \quad (14)$$

From here on we will drop the $\mu$ term multiplying $\partial f/\partial\mu$ relative to $kv/\omega \sim c/v_A$ in $A$.

Integrating Equation (13) over $\mu$, using Equation (12), and rearranging the prefactor gives

$$\Gamma_c = \frac{\pi}{8}\frac{\Omega_0}{n_i}\int_{p_1} p_1 dp(p^2 - p_1^2)A[f, \omega, k]. \quad (15)$$

Instability requires $A > 0$ in at least some part of phase space. Because $\partial f/\partial p < 0$ for typical cosmic-ray distributions, the source of instability must be in the anisotropy term. However, due to the factor of $kv/\omega \sim c/v_A$ multiplying $\partial f/\partial\mu$, the anisotropic part of $f$ need only be of order $v_A/c$ relative to the isotropic part for the wave to be unstable.

The classical streaming instability occurs for drift anisotropy of the form

$$f(p, \mu, t) = f_0(p, t) - \alpha p\frac{\partial f_0(p, t)}{\partial p}P_1(\mu). \quad (16)$$

In order to carry out the stability analysis we will make the standard assumption that the time dependence of $f$ is so slow that it can be dropped relative to the instability frequency and growth rate, i.e., we will approximate $f(p, \mu, t)$ by $f(p, \mu)$.

It can be shown from the Lorentz invariance of $f$ that the distribution function given Equation (16) is isotropic in a frame moving with speed $\alpha c$, up to factors of order $\alpha^2$. Assuming the particles are ultrarelativistic ($v \sim c$), the parameter $\alpha$ is directly related to the bulk drift, or streaming velocity by

$$v_D \equiv \frac{1}{n_c}\int v\mu f(p, \mu)p^2 dpd\mu = \alpha c. \quad (17)$$

Substituting Equation (16) into (14) and using Equation (17) gives

$$A[f, \omega, k] \equiv A_d[f, \omega, k] = \frac{df_0}{dp}\left(1 - \frac{v_D}{v_A}\right), \quad (18)$$

where we have taken $\omega/k = v_A$. Then, substituting Equation (16) into (15) gives $\Gamma_{cd}$, the growth rate of the streaming instability as

$$\Gamma_{cd} = \frac{\pi}{8}\Omega_0\mathcal{C}\frac{n_c(>p_1)}{n_i}\left(\frac{v_D}{v_A} - 1\right), \quad (19)$$

where $\mathcal{C}$ is a spectrum-dependent constant of order unity and $n_c(>p_1)$ is the number density of cosmic rays with momentum $p > p_1$, which are the only cosmic rays that can resonate with the wave.

In the case of pure pressure anisotropy considered in Section 2, there are no gradients along the magnetic field, so the series (7) contains only terms with even $l$. Therefore, $\partial f/\partial\mu$ is an odd function of $\mu$. If $\partial f/\partial\mu > 0$ ($<0$), only waves propagating in the positive (negative) direction can be unstable. But if particles of some particular $\mu_r$ resonate with a wave of one polarization, particles with $\mu = -\mu_r$ resonate with a wave of the opposite polarization. Therefore, if a circularly polarized wave is unstable, the wave with the opposite circular polarization propagating in the opposite direction is also unstable, and has the same growth rate. This implies that cosmic rays confined by pressure anisotropy instability would be advected by the thermal gas, rather than streaming relative to it at the Alfvén speed.

We make the *ansatz*

$$f(p, \mu) = f_0(p) - \zeta p \frac{df_0}{dp} P_2(\mu) \tag{20}$$

(Equation (42) provides some support for this form of $f_2$). Using Equation (10), the parameter $\zeta$ is related to the pressure anisotropy by

$$\zeta = \frac{5}{12} \frac{\Delta P_c}{P_c}, \tag{21}$$

where $P_c \equiv (2P_{c\perp} + P_{c\|})/3$ is the scalar pressure; for the expanding flux tube considered here, $\zeta > 0$. Substituting Equation (20) into (14) and approximating $\partial f/\partial p$ by $\partial f_0/\partial p$ on the grounds that $|f_2/f_0| \ll 1$ gives

$$A_{\rm pa}[f, \omega, k] = \frac{\partial f_0}{\partial p}\left(1 - \frac{3\zeta k v \mu}{\omega}\right), \tag{22}$$

which is to be evaluated at $\mu_r$. From Equation (12) and using $\zeta > 0$), we see that if the polarization is chosen such that the resonant particles are traveling in the same direction as the wave, the wave can be unstable, while the opposite sense of circular polarization is always damped. That is, right circularly polarized waves propagating in the positive direction and left circular polarized waves propagating in the negative direction are the only possible unstable ones. (Similar arguments show that in a compressing flux tube with $\zeta < 0$, the instabilities are positive propagation direction/left circular polarization and negative propagation direction/right circular polarization.) These results are consistent with the arguments given in Kulsrud (2005).

In the important case of a power law cosmic-ray distribution $f_0(p) \propto p^{-a}$, Equations (15) and (21) yield for the growth rate for pressure anisotropy instability

$$\Gamma_{cpa} = \frac{\pi}{8} \mathcal{C} \Omega_0 \frac{n_c(>p_1)}{n_i}\left[\frac{5a(a-2)}{4(a^2-1)} \frac{c}{v_A} \frac{\Delta P_c}{P_c} - 1\right], \tag{23}$$

with $\mathcal{C} = (a-3)/(a-2)$. In the strong anisotropy limit, which corresponds to dropping the "1" in the square bracket on the right-hand side of Equation (23), our expression for the instability growth rate agrees with the strong anisotropy limit given in LB06. For the local interstellar cosmic-ray spectrum with $a \sim 4.7$, $\mathcal{C} = 0.63$, and the threshold anisotropy for instability is $\zeta > 0.55$, or $\Delta P_c/P_c \sim 1.3 v_A/c$.

## 4. Calculation of the Heating Rate

Our tool for calculating the rate at which cosmic rays heat the thermal plasma will be the Fokker–Planck equation, which is the Vlasov equation plus scattering terms. As is usual in quasilinear diffusion theory, we assume the waves are small amplitude and have random phases. We consider only resonant wave–particle interactions with parallel propagating Alfvén waves. We begin with a brief review of heating when the waves are generated by drift anisotropy and then calculate heating when the waves are generated by pressure anisotropy, which is the main contribution of this paper.

### *4.1. Heating Due to Drift Anisotropy*

We assume $f$ has a spatial gradient along the background magnetic field $\boldsymbol{B} = \hat{s}B$ such that the cosmic rays drift toward positive $s$. The Fokker–Planck equation is

$$\frac{\partial f}{\partial t} + v\mu \frac{\partial f}{\partial s} = \frac{\partial F_\mu}{\partial \mu} + \frac{1}{p^2} \frac{\partial}{\partial p}(p^2 F_p), \tag{24}$$

where $F_\mu$ and $F_p$ are the components of the diffusive flux, which can be written in terms of components of the momentum space diffusion tensor $\boldsymbol{D}$ as

$$F_\mu = D_{\mu\mu} \frac{\partial f}{\partial \mu} + D_{\mu p} \frac{\partial f}{\partial p}, \tag{25}$$

$$F_p = D_{p\mu} \frac{\partial f}{\partial \mu} + D_{pp} \frac{\partial f}{\partial p}. \tag{26}$$

In the case of drift anisotropy, only waves propagating toward positive $s$ are present, but with both signs of circular polarization (which we assume have equal intensity). The components of $\boldsymbol{D}$ are (Schlickeiser 1989)

$$D_{\mu\mu} = \frac{\nu(1-\mu^2)}{2}, \tag{27}$$

$$D_{\mu p} = D_{p\mu} = \frac{p v_A}{v} D_{\mu\mu}, \tag{28}$$

$$D_{pp} = \left(\frac{p v_A}{v}\right)^2 D_{\mu\mu}, \tag{29}$$

where $\nu$ is the pitch angle scattering frequency, which is related to the spectral magnetic energy density of resonant waves $W_k$ by

$$\nu(p, \mu) = \Omega \frac{8\pi k W_k}{B^2}. \tag{30}$$

Equations (27)–(29) are valid for small amplitude, parallel propagating Alfvén waves of arbitrary polarization traveling parallel or antiparallel to $\boldsymbol{B}$.

Substituting Equations (27) and (28) into (25) gives

$$F_\mu = \frac{\nu(1-\mu^2)}{2} \frac{p v_A}{v} A(f, k v_A, k), \tag{31}$$

where $A$, the functional introduced in Equation (14), is fundamental to the criterion for instabiity (Equation (13)). Likewise,

$$F_p = \frac{\nu(1-\mu^2)}{2}\left(\frac{p v_A}{v}\right)^2 A(f, k v_A, k). \tag{32}$$

The implications of the close relationship between momentum space diffusion and wave excitation (or damping) will shortly become clear.

In some early studies of self confinement by drift anisotropy, such as Kulsrud & Pearce (1969) and Skilling (1971, 1975), the Fokker–Planck equation is solved in a frame moving with the waves, in which case the cosmic-ray scattering is purely elastic, with $D_{\mu\mu}$ the only nonzero component of the diffusion tensor. Because we are interested in energy exchange between the cosmic rays and the background, and because pressure anisotropy excites waves propagating in both directions, we work in the rest frame of the thermal plasma.

We solve Equation (24) under the assumption that the scattering mean free path $\lambda \equiv v/\nu$ is short compared to the gradient length scale $L_c \equiv |f_c/(\partial f_c/\partial s)|$, and the cosmic-ray streaming anisotropy is $\mathcal{O}(v_A/c) \ll 1$. We then keep only the second term on the left-hand side of Equation (24), and only the first term on the right-hand side. Integrating once with respect to $\mu$ and using Equations (27) and (28) gives a relationship between the spatial gradient of $f$, the scattering frequency $\nu$, and $pv_{AA}/v$

$$\frac{pv_A}{v}\left(\frac{\partial f}{\partial p} + \frac{v}{v_A p}\frac{\partial f}{\partial \mu}\right) = -\frac{v}{\nu}\frac{\partial f}{\partial s}. \tag{33}$$

If we replace $\partial/\partial s$ by a gradient length scale $L^{-1}$, we see that the ratio of the anisotropic to the isotropic part of $f$ is of order $c/(\nu L)$.

Next, we derive an energy equation from Equation (24) by multiplying by particle energy $\epsilon$ and integrating over phase space. The result is

$$\frac{\partial U_c}{\partial t} + \frac{\partial F_{c\epsilon}}{\partial s} = -\int vF_p p^2 dp d\mu, \tag{34}$$

where

$$U_c \equiv \int \epsilon f p^2\, dp d\mu, \tag{35}$$

$$F_{c\epsilon} \equiv \int v\mu\epsilon f p^2\, dp d\mu \tag{36}$$

are the cosmic-ray energy density and energy flux vector, respectively. The right-hand side of Equation (34) represents energy transfer between cosmic rays and waves due to scattering. It is shown in the Appendix that this can be written in terms of the wave energy densities and growth rates as

$$H_d \equiv \int vF_p p^2 dp d\mu = 2\int \Gamma_c(k) W_k dk, \tag{37}$$

which was first given for gyroresonant instability by Kennel & Engelmann (1966) and the notation $H_d$ is meant to suggest a plasma heating rate, because energy added to the waves must be transferred to the thermal plasma if the waves are in a steady state. Here, we evaluate the energy transfer term directly using Equation (33)

$$H_d = -\int v^2 \frac{pv_A}{v}\frac{(1-\mu^2)}{2}\frac{\partial f_c}{\partial s} p^2 dp d\mu = -v_A\frac{\partial P_c}{\partial s}, \tag{38}$$

where for streaming toward increasing $s$, $\partial P_c/\partial s < 0$.

Equation (38) agrees with the standard expression for the collisionless heating rate, and is notable for its simplicity and lack of any explicit dependence on wave properties. It does, however, have an implicit dependence in that the cosmic-ray pressure gradient is determined by transport, and the transport is determined, in part, by the degree of scattering. But as Equations (33) and (13) show, the scattering rate is linked to the wave growth rate. In a steady state, wave growth is balanced by wave damping by the thermal background. So ultimately, the pressure gradient and heating rate are determined by the properties of the cosmic-ray source and the background gas through which the cosmic rays stream. In particular, if the waves are heavily damped, the cosmic-ray anisotropy must be large to overcome wave damping. This corresponds to a long scattering mean free path or a large diffusion coefficient, which flattens the cosmic-ray pressure gradient and reduces the rate at which cosmic rays can heat and do work on the thermal gas.

### 4.2. Heating Due to Pressure Anisotropy

Now, we attempt to derive an expression for the heating rate due to pressure anisotropy instability that is as compact as Equation (38). Under the terms of our problem, Equation (24) is replaced by Equation (9) plus momentum space diffusion terms

$$\frac{\partial f}{\partial t} + \frac{\dot{B}}{3B}(P_0(\mu) - P_2(\mu))p\frac{\partial f_0}{\partial p} = \Delta\dot{f}(p) + \frac{\partial F_\mu}{\partial \mu} + \frac{1}{p^2}\frac{\partial}{\partial p}(p^2 F_p). \tag{39}$$

As discussed below Equation (22), waves of opposite circular polarization propagate in opposite directions, and particles resonate with waves traveling in the same direction they are (we assume the waves propagate in both directions with equal intensity). Accordingly, Equations (27) and (29) respectively can be used for $D_{\mu\mu}$ and $D_{pp}$, but

$$D_{\mu p} = D_{p\mu} = \frac{pv_A}{v}D_{\mu\mu};\ \mu > 0, \tag{40}$$

$$D_{\mu p} = D_{p\mu} = -\frac{pv_A}{v}D_{\mu\mu};\ \mu < 0. \tag{41}$$

The discontinuity in the off-diagonal terms $D_{\mu p}$ and $D_{p\mu}$ is only apparent, as $\nu \equiv 0$ for $\mu = 0$ for any distribution of waves with a short wavelength cutoff. Scattering mechanisms that supplement pitch angle scattering at small $\mu$, such as mirroring, have been proposed (Felice & Kulsrud 2001) but we ignore them here; we have also dropped terms of order $v_A/c$ in the diffusion tensor, which remove the singularity at $\mu = 0$ (Schlickeiser 1989). Importantly, because for pressure anisotropy $\partial f_c/\partial\mu$ is odd in $\mu$, $F_\mu$ is odd in $\mu$ while $F_p$ is even.

We solve for the anisotropy driven by the time varying background magnetic field by multiplying Equation (39) by $P_2(\mu)$ and integrating over $\mu$, making the same assumptions about the ordering of terms we made in deriving Equation (33) from Equation (24): we drop $\partial f/\partial t$, replace $f$ by $f_0$ in the $\dot{B}/B$ term, and drop $F_p$ but keep $F_\mu$ on the right-hand side. The result is

$$\frac{pv_A}{v}\int_0^1 \nu\mu(1-\mu^2)\left(\frac{\partial f}{\partial p} + \frac{v}{v_A p}\frac{\partial f}{\partial \mu}\right)d\mu = \frac{pv_A}{v}\int_0^1 \nu\mu(1-\mu^2)A[f, \omega, k]d\mu = \left(\frac{2}{45}\right)\frac{\dot{B}}{B}p\frac{\partial f_0}{\partial p}. \tag{42}$$

The left-hand side of Equation (42) is proportional to the anisotropy factor $A$ while the right-hand side of Equation (42) is positive, as expected for anisotropy driven instability. However, whereas Equation (33) gives $A$ directly as a function of $\nu$ and $p$, Equation (42) involves an integral of $A$ with $\nu$. The more important difference between Equations (33) and (42) is that the timescale on the right-hand side of Equation (33) is the light travel time $L_c/c$, while the timescale on the right-hand side of Equation (42) is the flux tube expansion timescale $|B/\dot{B}|$. We discuss the implications of Equation (42) for cosmic-ray self confinement in Section 5.

We derive an energy equation analogous to Equation (34) by multiplying Equation (39) by $\epsilon$ and integrating over momentum space. The result is

$$\frac{\partial U_c}{\partial t} - \frac{\dot{B}}{B}(U_c + P_{c\perp}) = \Delta \dot{U}_c - \int v F_p p^2 dp d\mu. \tag{43}$$

The first term on the right-hand side of Equation (43) is the cosmic-ray energy injection rate corresponding to the cosmic-ray pressure source. If the pressure were isotropic and we substituted $\dot{\rho}_g/\rho_g$ for $\dot{B}/B$, the left-hand side of Equation (43) would be in standard form for describing adiabatic expansion. Because in the absence of collisions, $P_\perp$ decreases slightly faster than $P_c$ itself, anisotropy slows the rate of energy loss. If we use the identity

$$P_{c\perp} = P_c + \frac{P_{c\perp} - P_{c\parallel}}{3}, \tag{44}$$

then Equation (43) can be written as

$$\frac{\partial U_c}{\partial t} - \frac{\dot{B}}{B}(U_c + P_c) = \frac{\dot{B}}{3B}(P_{c\perp} - P_{c\parallel}) + \Delta \dot{U}_c - \int v F_p p^2 dp d\mu. \tag{45}$$

Because pressure anisotropy is reversed in an increasing magnetic field, the anisotropy term on the right-hand side of Equation (45) is positive whatever the sign of $\dot{B}/B$. It shows that anisotropy reduces the rate of energy loss in a decreasing magnetic field and increases the rate of energy gain in an increasing field. In both cases, this is because the parallel momentum is fixed. Although the anisotropy term formally resembles gyroviscous heating (Kunz et al. 2011), it is not a true heating process because it is completely reversible.

Energy transfer to waves, however, is a true energy loss process. To evaluate it, we write out the diffusion term explicitly, using the even parity of $F_p$ noted below Equations (42) and (14). This gives

$$\int v F_p p^2 dp d\mu = \int p^2 dp v \left(\frac{p v_A}{v}\right)^2 \int_0^1 \nu(1-\mu^2) A(f, k v_A, k) d\mu. \tag{46}$$

We know from Equation (42) that

$$\left| \int_0^1 \nu(1-\mu^2)\left(\frac{\partial f_c}{\partial p} + \frac{v}{v_A p}\frac{\partial f_c}{\partial \mu}\right) d\mu \right| \geqslant \left| \int_0^1 \nu\mu(1-\mu^2)\left(\frac{\partial f_c}{\partial p} + \frac{v}{v_A p}\frac{\partial f_c}{\partial \mu}\right) d\mu \right| = \left| \frac{2}{45}\frac{v}{v_A p}\frac{\dot{B}}{B} p \frac{\partial f_c}{\partial p} \right| \tag{47}$$

(where the first inequality simply follows from $\mu \leqslant 1$). Therefore, we have a lower bound on the magnitude of cosmic-ray heating

$$\left| \int v F_p p^2 dp d\mu \right| \equiv H_{pa} \geqslant \left| \frac{2}{45} v_A \frac{\dot{B}}{B} \int p^4 \frac{df_c}{dp} \right| = \left| \frac{4}{15}\frac{v_A}{c}\frac{\dot{B}}{B} P_c \right| = \frac{4}{15}\frac{v_A}{c}\frac{P_c \Delta \dot{P}_c}{\gamma_c P_c + \gamma_m P_m + \gamma_g P_g}, \tag{48}$$

where in the last step we have used Equations (3) and (1). Equation (48) is only a lower bound because our analysis does not give the functional form of $\nu$. Experimentation with various trial functions for $\nu$ suggests that Equation (48) is unlikely to underestimate the heating by more than a factor of 2.

We can generalize Equation (48) by comparing the rates at which the cosmic rays are heating their environment to the rate at which they are doing work on it, which from Equation (4) is $-P_c \dot{B}/B$. Therefore, the rate of heating is the rate of work multiplied by a factor of order $v_A/c$.

### *4.3. Comparison of Drift and Pressure Anisotropy Heating*

The rate of cosmic-ray heating due to streaming anisotropy (Equation (38)) is proportional to $\nabla_\parallel P_c$, while the heating rate due to pressure anisotropy (48) is proportional to $\dot{P}_c$, so in order to compare them they must be given in the same dimensions.

Suppose cosmic rays are injected at $x = 0$ when the background magnetic field $\boldsymbol{B} = \hat{x}B$. We take $P_c(0, t)$ to be a given, increasing function of time and assume the cosmic rays stream away from the boundary at speed $v_A$. To keep the problem simple, we assume $P_c(0, t) = P_{c0} + \Delta P_c(t)$ with $\Delta P_c/P_{c0} \ll 1$ and ignore the compression and acceleration of the thermal gas by the cosmic rays. Then, $P_c$ at a point $x > 0$ can be found from

$$\frac{\partial P_c}{\partial t} + v_A \frac{\partial P_c}{\partial x} = 0, \tag{49}$$

the solution of which is

$$P_c(x, t) = P_c(0, t - x/v_A). \tag{50}$$

The heating rate $H_d$ is found directly from Equation (49)

$$H_d(x, t) = \frac{1}{v_A}\frac{\partial P_c(x, t)}{\partial t} = \Delta \dot{P}_c(0, t - x/v_A). \tag{51}$$

Comparing Equations (48) and (51), we see that heating due to pressure anisotropy is lower by a factor of order $v_A/c$, as suggested by the analysis in Section 4.2. The essential difference is that in the case of drift anisotropy, scattering balances the tendency of cosmic rays to stream down the magnetic field aligned component of their gradient at the speed of light, but in the case of pressure anisotropy, scattering balances the driving of pressure anisotropy by transverse expansion of the magnetic field at the magnetoacoustic speed. It should be borne in mind, however, that while we assumed uniform cosmic-ray injection along the flux tube for exploratory purposes, this condition is unlikely to occur in nature. In Section 7, we speculate on the joint effects of parallel and perpendicular cosmic-ray pressure gradients.

## 5. Implications for Cosmic Ray Self Confinement

Equations (33) and (42) constrain the product of the pitch angle scattering frequency $\nu$ and the anisotropy factor $A$, which appears in the growth rate $\Gamma_c$. Requiring that $\Gamma_c$ equals $\Gamma_d$, the rate of damping by the thermal background, yields an independent constraint on $A$. From this, we can estimate the scattering frequency $\nu$, from which we can derive the cosmic-ray spatial diffusivity $\kappa \sim v^2/\nu$ and check for self consistency of the frequent scattering/short mean free path assumption that underlies cosmic-ray hydrodynamics.

We can already guess from the results of Section 4.3 that $\kappa_{\rm pa}$, the diffusivity due to self confinement by pressure anisotropy instability, is larger than $\kappa_d$, its counterpart in the drift case, by a factor of order $c/v_{\rm A}$. From Equation (30) we see that $\nu$ is directly proportional to the fluctuation spectrum $W$, while $\Gamma_c$ is now to be equated to $\Gamma_d$. If their product is smaller in the pressure anisotropy case by a factor of $v_{\rm A}/c$, the scattering rates themselves must be smaller by approximately the same factor. Here, we provide some background on the argument and show explicitly how $\nu$ can be estimated.

The most important damping mechanisms are thought to be ion-neutral friction (Kulsrud & Pearce 1969), nonlinear Landau damping by thermal ions (Lee & Völk 1973; Kulsrud 1978), damping by an ambient turbulent cascade (Yan & Lazarian 2002; Farmer & Goldreich 2004), and enhancement of turbulent damping by high plasma $\beta$ effects (Wiener et al. 2017). With the exception of ion-neutral friction, which appears to be strong enough to prevent cosmic-ray self confinement in dense, neutral gas entirely (Everett & Zweibel 2011), the other mechanisms suppress self confinement only above energies of about 100–300 GeV for Milky Way conditions. Although we will not have to make explicit calculations involving any damping mechanisms to compare $\kappa_{\rm pa}$ with $\kappa_d$, we provide a sample calculation for nonlinear Landau damping in Section 6.

From Equations (33) and (18) we estimate the diffusivity due to drift anisotropy instability as

$$\kappa_d \sim \frac{v^2}{\nu} \sim (v_{\rm A} L)\left(\frac{v_D}{v_{\rm A}} - 1\right), \quad (52)$$

which corresponds to a ratio of mean free path to length scale

$$\frac{\lambda_d}{L} \sim \frac{v_{\rm A}}{v}\left(\frac{v_D}{v_{\rm A}} - 1\right). \quad (53)$$

If we take $\kappa_d$ to be the widely accepted value $3 \times 10^{28}$ cm$^2$ s$^{-1}$, set $v_A = 100$ km s$^{-1}$ (corresponding to an ion density of 0.01 cm$^{-3}$ and $B = 5\,\mu$G) and take $L$ to be 1 kpc, then $v_D/v_{\rm A} - 1 \sim 1$.

From Equations (42) and (22), the diffusivity due to pressure anisotropy instability is

$$\kappa_{\rm pa} \sim v v_{\rm A} \tau_B\left(\frac{c\Delta P_c}{v_{\rm A} P_c} - 1\right), \quad (54)$$

where we have removed the spectrum-dependent factors from Equation (23) because they are of order unity. The corresponding ratio of mean free path to fiducial length $v_{\rm A}\tau_B$ is

$$\frac{\lambda_{\rm pa}}{v_{\rm A}\tau_B} \sim \left(\frac{c\Delta P_c}{v_{\rm A} P_c} - 1\right). \quad (55)$$

Due to the similarity between the drift and pressure anisotropy instability growth rates (Equations (19) and (23)), the drift and pressure anisotropy factors required to balance wave damping are probably about the same: namely, order unity. Therefore, the scattering mean free path due to pressure anisotropy is probably similar to the Alfvén travel length $v_{\rm A}\tau_B$.

## 6. Example: Application to Bottlenecks in Galactic Halos

One of the motivations for this study was the realization that models of cosmic-ray self confinement by drift anisotropy should be prone to the formation of bottlenecks. Here we apply the theory of self confinement by pressure anisotropy to the models of bottleneck formation in low density gas between a cosmic-ray source and a denser cloud such as shown in the top panel of Figure 1 (Wiener et al. 2017, 2019). That study focused on the effect of the cosmic-ray source on the cloud. Here we address the effects of pressure anisotropy on the intercloud medium for the range of parameters chosen for that study. In Wiener et al. (2019), the cosmic rays were injected in a pulse of duration comparable to the Alfvén and sound travel times from the source to the cloud, so spatial gradients were important and a steady state was never achieved. In order to minimize these effects,we imagine that the cloud is much closer to the source than the 1 kpc chosen in the original bottleneck studies.

Prior to cosmic-ray injection, the intercloud medium has thermal pressure $P_g = 3.2 \times 10^{-13}$ dyne cm$^{-2}$, magnetic pressure $P_m = 3.97 \times 10^{-14}$ dyne cm$^{-2}$, and negligible cosmic-ray pressure. A heating rate $\Gamma_0$ of $1.0 \times 10^{-25}$ erg s$^{-1}$ per hydrogen atom is included to balance radiative losses at the initial temperature $T = 1.1 \times 10^6$ K, giving a volumetric heating rate of about $9.0 \times 10^{-29}$ erg cm$^{-3}$ s$^{-1}$.

The fiducial cosmic-ray energy flux into the domain rate $\Delta\dot{P}_c$ is $1.67 \times 10^{-25}$ erg cm$^{-3}$ s$^{-1}$. From Equation (3), the characteristic flux tube transverse expansion time $\tau_B$ is $3.7 \times 10^{12}$ s. Although the steady state cosmic-ray pressure of $8.2 \times 10^{-14}$ dyne cm$^{-2}$ derived from the simulation parameters is less than 25% of the original pressure, because $v_{\rm A}/c \sim 2 \times 10^{-4}$, the resulting distention of the magnetic field is more than enough to excite the cosmic-ray pressure anisotropy instability.

From Equation (48), the lower bound on the heating rate $H_{\rm pa}$ is $1.2 \times 10^{-30}$ erg cm$^{-3}$ s$^{-1}$, slightly more than 1% of the heating rate that offsets radiative cooling. Even in models with 10 times the fiducial source strength, heating would be a relatively weak effect compared to the heating required to offset radiative cooling.

The mean free path for scattering, however, is more interesting. From Equation (42), we estimate $\nu$ by approximating Equation (42) as

$$\nu\frac{v_{\rm A}}{c}p\frac{df_0}{dp}\left(\frac{\Delta P_c}{P_c}\frac{c}{v_{\rm A}} - 1\right) \sim \frac{2}{45}\frac{\dot{B}}{B}p\frac{df_0}{dp}, \quad (56)$$

from which it follows that

$$\nu\left(\frac{c\Delta P_c}{v_{\rm A} P_c} - 1\right) \sim \frac{2c}{45 v_{\rm A}\tau_B} \sim 6 \times 10^{-9}\ {\rm s}^{-1}, \quad (57)$$

corresponding to a mean free path of about 1.6 pc if the anisotropy factor is unity. Bearing in mind that this value of $\nu$ is weighted by $\mu(1 - \mu^2) < 1$, and that if wave damping is weak the anisotropy factor could be significantly stronger, we conclude that pressure anisotropy instability could well be adequate to couple the cosmic rays to the intercloud medium.

It is also instructive to repeat the estimate of $\nu$ if nonlinear Landau damping is the primary dissipation mechanism. For cosmic rays near the mean energy, the damping rate $\Gamma_{\rm nlld}$ is

$$\Gamma_{\rm nlld} \sim \nu \frac{v_i}{c}, \tag{58}$$

where $v_i$ is the thermal ion velocity. Equating $\Gamma_{\rm nlld}$ to $\Gamma_{\rm c}$, which we estimate from Equation (42), gives

$$\nu \frac{v_i}{c} \sim \Gamma_{\rm c} \sim \frac{\Omega_0}{\nu \tau_B} \frac{n_{\rm c}}{n_i} \frac{c}{v_{\rm A}}. \tag{59}$$

Solving Equation (59) for $\nu$, we find

$$\nu \sim \left( \frac{\Omega_0}{\tau_B} \frac{n_{\rm c}}{n_i} \frac{c^2}{v_i v_{\rm A}} \right)^{1/2}. \tag{60}$$

For the parameters assumed in Wiener et al. (2019), Equation (60) gives $\nu \sim 3.7 \times 10^{-8}\ {\rm s}^{-1}$. The corresponding mean free path is $\lambda \sim 0.26$ pc. Although this is a rough estimate, it suggests good confinement. Our estimate for the heating rate is unchanged.

According to these results, waves driven by the pressure anisotropy instability would provide a short scattering mean free path and lock the cosmic rays to the thermal fluid. However, the cosmic-ray pressure profile would be quite flat and the cosmic rays would transfer little heat or momentum to the gas.

## 7. Summary and Conclusions

Cosmic ray propagation and cosmic-ray hydrodynamics—the fluid description of how cosmic rays exchange energy and momentum with magnetized thermal plasma—depend on kinetic scale plasma processes. One of the most potent and best studied of these is the drift, or streaming instability, of Alfvén waves with a wavelength of order the cosmic-ray gyroradius. In a steady state, which is assumed to be reached on a timescale short compared to the macroscopic dynamical time, energy and momentum transferred from the cosmic rays to the waves is absorbed by the thermal gas such that cosmic rays exert a parallel force of magnitude $\nabla_\| P_{\rm c}$ on the thermal gas, heat the gas at a rate of magnitude $|v_{\rm A} \nabla_\| P_{\rm c}|$, and drift down their pressure gradient at velocity $\boldsymbol{v}_{\rm A}$ relative to the thermal background.

In order for the instability to operate, the parallel cosmic-ray pressure and thermal gas density gradients must point in the same direction. If this condition is not met, a cosmic-ray "bottleneck" forms, in which the cosmic-ray pressure is constant and the cosmic rays do not exchange momentum or energy with the ambient medium. This can fundamentally alter the impact of cosmic rays on gas dynamics and thermodynamics, as speculated by Skilling (1971) and first demonstrated by Wiener et al. (2017).

In this paper, we have investigated a complementary mechanism for cosmic-ray self confinement and coupling to the thermal gas based on an instability driven by cosmic-ray pressure anisotropy. The pressure anisotropy instability, like the drift instability, is triggered by anisotropy of order $v_{\rm A}/c$, destabilizes hydromagnetic waves with wavelength of order the cosmic-ray gyroradius, and has a growth rate proportional to the ratio of cosmic ray to thermal gas density, scaled by the nonrelativistic ion cyclotron frequency (Equations (19) and (23)).

In previous studies of this instability (LB06; Yan & Lazarian 2011), pressure anisotropy was assumed to be driven by compressive interstellar turbulence. In this situation, cosmic rays mediate the dissipation of turbulent energy as heat, and absorb some of that energy themselves through second order Fermi acceleration. As such, the cosmic rays create an energy sink for the turbulent cascade at larger spatial scales than would be expected due to dissipative processes in the thermal gas alone.

We considered anisotropy driven by a slowly changing magnetic field, but assumed the energy source for changing the magnetic field to be the cosmic rays themselves, which we modeled as simple injection (Section 2; see Figure 1). The anisotropy destabilizes circularly polarized waves, which propagate in both directions (Section 3). In Section 4, we calculated the relationship between the force exerted by the cosmic rays on the medium and the rate at which they heat it. Whereas in the case of parallel streaming the magnitudes of the force and the heating rate are $\nabla_\| P_{\rm c}$ and $v_{\rm A} \nabla_\| P_{\rm c}$, we found that for a transverse force $\nabla_\perp P_{\rm c}$ the heating rate is only of order $(v_{\rm A}/c) C_{\rm ma} \nabla_\perp P_{\rm c}$, where $C_{\rm ma}$ is the magnetoacoustic speed defined in Equation (1). The underlying reason for the difference in heating rates for drift and pressure anisotropy is that in the drift anisotropy case, scattering balances the streaming of cosmic rays down the magnetically aligned component of their pressure gradient at the speed of light, but in the pressure anisotropy case, in which expansion is perpendicular to $\boldsymbol{B}$, the cosmic rays must overcome the inertia of the thermal gas and scattering need only balance the driving of pressure anisotropy by expansion at the much lower magnetoacoustic speed. We showed in Section 5 that the weaker scattering rate corresponds to weaker self confinement.

In Section 6, we applied the pressure anisotropy model to one of the situations that motivated this paper: the formation of a bottleneck between an interstellar cloud and a cosmic-ray source (Wiener et al. 2017, 2019). We showed that although the heating rate is only a small perturbation to the thermodynamics, the scattering rate could be enough to prevent the cosmic ray–thermal gas decoupling that would occur if a bottleneck formed. In this section we also showed how $\nu$ can be estimated when nonlinear damping is the main source of thermal dissipation; this led to an estimate for $\lambda$ somewhat shorter than the estimate based on linear damping.

Based on the calculations in this paper, we can say that pressure anisotropy instability and drift anisotropy instability are not equivalent and not interchangeable as far as cosmic-ray confinement and cosmic-ray coupling to thermal gas on global (kiloparsec) scales are concerned. On intermediate, or mesoscales, the anisotropy drive may be strong enough to confine the cosmic rays and provide momentum transport, but not a significant amount of heating.

In general, cosmic-ray sources are localized in space, in which case we would expect both drift and pressure anisotropy to be present. For definiteness, suppose the direction of streaming is such that the drift anisotropy $f_1 > 0$, while $f_2 > 0$ due to cosmic-ray expansion of the flux tube onto which cosmic rays are injected. The waves for which drift and pressure anisotropy are both destabilizing have the largest growth rate. These waves propagate in the same direction as the cosmic-ray streaming and have $\mu_r > 0$. If $f_1$ and $f_2$ adjust such that cosmic-ray excitation balances thermal damping, then waves propagating in the opposite direction, or with the opposite sense of polarization, are damped. Therefore, the cosmic rays will be convected at the Alfvén speed, and their pressure will vary as $\rho_g^{\gamma_c/2}$ along the magnetic flux tube. If $\rho_g$ increases along the flux tube, this is exactly the condition for bottleneck formation. But due to pressure anisotropy, the cosmic rays do not decouple completely. Rather, as the pressure gradient flattens, counterpropagating waves of opposite polarization will become unstable, and the system will resemble the one studied here, with a short cosmic-ray mean free path but little momentum or heat transfer. There would be very little difference between this bottleneck and the original one based on drift anisotropy alone. In future work, we hope to explore this complex picture through analysis and numerical simulations.

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## Appendix

Here we sketch proof that resonant scattering from an ensemble of randomly phased, small amplitude waves transfers energy between waves and cosmic rays in a way that is consistent with wave growth and damping. For general discussions of wave/particle energetics see Kennel & Engelmann (1966), Kulsrud & Pearce (1969), and Stix (1992); here, we consider only parallel propagating Alfvén waves, which is the relevant case for our problem.

Consider a wave of wavenumber $\boldsymbol{k} = k\boldsymbol{B}/B$ with electric field $\delta\boldsymbol{E}_k$

$$\delta\boldsymbol{E}(k) = \frac{1}{2}(\delta\boldsymbol{E}_k e^{i\psi} + \delta\boldsymbol{E}_k^* e^{-i\psi^*}), \tag{61}$$

where $\psi \equiv kz - \omega t$. We will assume $\omega = \omega_r + i\Gamma_c$, with $|\Gamma_c/\omega_r| \ll 1$. The spectral energy density $\delta W_k$ is the sum of the electric, magnetic, and background plasma kinetic energy densities; the magnetic and kinetic energies are equal and larger than the electric energy density by a factor of $(c/v_A)^2$. Then,

$$\delta W_k = 2\frac{c^2}{v_A^2}\frac{1}{4 \times 8\pi}\langle(\delta\boldsymbol{E}e^{i\psi} + \delta\boldsymbol{E}^* e^{-i\psi^*})^2\rangle = \frac{c^2}{v_A^2}\frac{\delta\boldsymbol{E}_k\delta\boldsymbol{E}_k^*}{8\pi}, \tag{62}$$

where $\langle\rangle$ denotes an average over the phase $\psi$.

Let the perturbed cosmic-ray distribution function produced by the wave be $\delta f_{ck}$. In the quasilinear approach used here, diffusion in momentum space is produced by the interaction of each wave electromagnetic field with the distribution function it produces, integrated over all $k$

$$\begin{aligned}\frac{Df_c}{Dt} &= -\frac{1}{4}\int\langle q(\delta\boldsymbol{F}_k e^{i\psi} + \delta\boldsymbol{F}_k^* e^{-i\psi^*})\\ &\quad\cdot\frac{\partial}{\partial\boldsymbol{p}}(\delta f_{ck}e^{i\psi} + \delta f_{ck}^* e^{-i\psi^*})\rangle dk\\ &= -\frac{1}{4}\int\delta\boldsymbol{F}_k\cdot\frac{\partial\delta f_{ck}^*}{\partial\boldsymbol{p}} + \delta\boldsymbol{F}_k^*\cdot\frac{\partial\delta f_{ck}}{\partial\boldsymbol{p}}dk,\end{aligned} \tag{63}$$

where $\boldsymbol{F}_k \equiv q(\delta\boldsymbol{E}_k + \boldsymbol{v}\times\delta\boldsymbol{B}_k/c)$ is the electromagnetic force due to the wave and $D/Dt$ is the convective derivative in phase space. We derive an energy equation, by multiplying Equation (63) by particle energy $\epsilon$ and integrating over momentum space. Here, we are mainly interested in the right-hand side, which we integrate by parts

$$\begin{aligned}\int\epsilon\frac{Df_c}{Dt}d^3p &= -\frac{1}{4}\int\epsilon\delta\boldsymbol{F}_k\cdot\frac{\partial\delta f_{ck}^*}{\partial\boldsymbol{p}} + \delta\boldsymbol{F}_k^*\\ &\quad\cdot\frac{\partial\delta f_{ck}}{\partial\boldsymbol{p}}dkd^3p\\ &= \frac{1}{4}\int q\boldsymbol{v}\cdot(\delta\boldsymbol{E}_k\delta f_{ck}^* + \delta\boldsymbol{E}_k^*\delta f_{ck})dkd^3p\\ &= \frac{1}{4}\int(\delta\boldsymbol{E}_k\cdot\delta\boldsymbol{J}_{ck}^* + \delta\boldsymbol{E}_k^*\cdot\delta\boldsymbol{J}_{ck})dk,\end{aligned} \tag{64}$$

where $\delta\boldsymbol{J}_{ck}$ and $\delta\boldsymbol{J}_{ck}^*$ are the perturbed cosmic-ray current Fourier component and its complex conjugate generated by the wave.

Each Fourier component of the cosmic-ray current $\delta\boldsymbol{J}_{ck}$ is related to the total wave current $\delta\boldsymbol{J}_k$ and the wave plasma current $\delta\boldsymbol{J}_{pk}$ by

$$\delta\boldsymbol{J}_{ck} = \delta\boldsymbol{J}_k - \delta\boldsymbol{J}_{pk}. \tag{65}$$

The relationship between $\delta\boldsymbol{E}_k$ and $\delta\boldsymbol{J}_k$ follows from combining Ampere's law and Faraday's law, and dropping the displacement current, which gives

$$\delta\boldsymbol{J}_k = \frac{ic^2k^2}{\omega}\frac{\delta\boldsymbol{E}_k}{4\pi}. \tag{66}$$

The plasma current $\delta\boldsymbol{J}_{kp}$ for an undamped, parallel propagating Alfvén wave can be shown to be

$$\delta\boldsymbol{J}_{pk} = \frac{i\omega c^2}{v_A^2}\frac{\delta\boldsymbol{E}_k}{4\pi}. \tag{67}$$

Substituting Equations (66) and (67) into Equation (65) gives the cosmic-ray current

$$\delta\boldsymbol{J}_{ck} = i\frac{i\delta\boldsymbol{E}_k}{4\pi}\frac{c^2}{v_A^2}\left(\omega - \frac{k^2v_A^2}{\omega}\right). \tag{68}$$

Taking the scalar product of Equation (68) with $\delta\boldsymbol{E}_k^*$, then taking the scalar product of the complex conjugate of Equation (68) with $\delta\boldsymbol{E}_k$ and its complex conjugate, subtracting one equation from the other, and using Equation (62) together with the assumptions $\omega_r^2 = k^2v_A^2$ and $|\Gamma_c/\omega_r| \ll 1$ gives

$$\frac{1}{4}(\delta\boldsymbol{E}_k\cdot\delta\boldsymbol{J}_{ck}^* + \delta\boldsymbol{E}_k^*\cdot\delta\boldsymbol{J}_{ck}) = -2\Gamma_c W_k. \tag{69}$$

Substituting Equation (69) into Equation (64) gives the result we sought

$$\int\epsilon\frac{Df_c}{Dt}d^3p = -\int 2\Gamma_c\delta W_k dk. \tag{70}$$

Equation (70) shows that the quasilinear force term represents the exchange of energy between the cosmic rays and the waves that scatter them. If $\Gamma_c > 0$ (unstable waves), the cosmic rays lose energy to the waves.

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