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Nonlinear Reconnection in Magnetized Turbulence

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Abstract

Recent analytical works on strong magnetized plasma turbulence have hypothesized the existence of a range of scales where the tearing instability may govern the energy cascade. In this paper, we estimate the conditions under which such tearing may give rise to full nonlinear magnetic reconnection in the turbulent eddies. When those conditions are met, a new turbulence regime is accessed where reconnection-driven energy dissipation becomes common, rather than the rare feature that it must be when they are not. We conclude that while such conditions are very stringent for fluid-scale eddies, they are easily met for kinetic-scale eddies; in particular, we suggest that our arguments may help explain recent *Magnetospheric Multiscale* (*MMS*) observations of (so-called) electron-only reconnection and of energy dissipation via electron Landau damping in the Earth's magnetosheath.

Unified Astronomy Thesaurus concepts: Plasma astrophysics (1261); Space plasmas (1544); Plasma physics (2089); Interplanetary turbulence (830); Magnetic fields (994)

1. Introduction

Magnetized plasmas are abundant in the universe. Examples include the Earth's magnetosphere, the solar wind and the solar corona, as well as many others, more distant and often more exotic, such as accretion disks around massive central objects, astrophysical jets, pulsar wind nebulae, etc. Many such environments, including all those just listed, are turbulent—a natural consequence of large-scale (roughly system-size) energy injection, and relatively infrequent collisions between the particles constituting those plasmas.

Beyond its interest as a fundamental physics problem, an understanding of turbulence in those and other environments is believed to be crucial to address longstanding, fundamental processes such as dynamo action, enhanced loss of angular momentum in accretion disks, electron-to-ion energy partition, and particle energization.

The modern understanding of (strong) plasma turbulence in the fluid approximation, though still incomplete, rests on a few qualitative ideas for which there is compelling observational and numerical evidence (e.g., Biskamp 2008; Chen 2016; Davidson 2016; A. A. Schekochihin 2020, in preparation). Among these are: (i) a Kolmogorov-like cascade of energy from large to small scales; (ii) the concept of critical balance (Goldreich & Sridhar 1995)—essentially a causality argument relating turbulent dynamics parallel and perpendicular to the local mean magnetic field; and (iii) the notion of dynamic alignment of the turbulent fluctuations (Boldyrev 2006; Chandran et al. 2015; Mallet et al. 2015), which determines constraints imposed on the turbulence by the active alignment between velocity and magnetic field fluctuations.

One direct consequence of the combination of these three concepts is the prediction that turbulent eddies should be anisotropic in all directions with respect to the local mean magnetic field; in particular, they should resemble current sheets—localized regions of intense electric current—in the field-perpendicular plane, whose aspect ratio increases with perpendicular wavenumber. Current sheets are, indeed, almost ubiquitously observed in direct numerical simulations of forced, three-dimensional magnetohydrodynamic (MHD) turbulence (e.g., Maron & Goldreich 2001; Biskamp 2008; Zhdankin et al. 2013).

The extension of these ideas to the kinetic range of plasma turbulence—relevant in weakly collisional plasmas of which the solar wind is the prototypical example—is, predictably, not straightforward. However, again, there is abundant numerical evidence for the formation of current sheets in this range (e.g., TenBarge & Howes 2013; Wan et al. 2015; Grošelj et al. 2018). Why this should be so has not been established on general theoretical grounds, but Boldyrev & Loureiro (2019) have recently advanced a possible explanation applicable to plasmas where $\beta_i \sim 1 \gg \beta_e$, such as found, for example, in the Earth's magnetosheath (i.e., for so-called inertial kinetic-Alfvén wave turbulence; Chen & Boldyrev 2017; Passot et al. 2017, 2018; Roytershteyn et al. 2019).

Current sheets being traditionally associated with magnetic reconnection (Biskamp 2005; Priest & Forbes 2000; Zweibel & Yamada 2009; Yamada et al. 2010), it is unsurprising that this process has acquired significant prominence as a potential key mechanism in magnetized turbulence (e.g., Matthaeus & Lamkin 1986; Retinò et al. 2007; Sundkvist et al. 2007; Servidio et al. 2009; Osman et al. 2014; Wan et al. 2015; Cerri et al. 2017; Shay et al. 2018). Fundamentally, reconnection leads to the conversion and dissipation of magnetic energy; thus, one expects that if it is indeed active in turbulence it may qualitatively impact the dynamics and observational signatures.

An important point that needs to be introduced in our discussion at this stage is that of the relationship—and distinction—between magnetic reconnection and the tearing mode (Furth et al. 1963). The latter is an instability that manifests itself through the reconnection of magnetic field lines (and the consequent opening of magnetic islands (or flux ropes)). Strictly speaking, therefore, the tearing mode *can* be called magnetic reconnection; however, the term "reconnection" is most commonly used to refer to a strongly nonlinear plasma phenomenon associated with significant magnetic energy transfer and dissipation, as already mentioned. According to this classification, the deep nonlinear stage of evolution of the (strongly unstable, i.e., large instability parameter Δ')

tearing mode (Coppi et al. 1976; Waelbroeck 1989; Jemella et al. 2003; Loureiro et al. 2005) is appropriately referred to as reconnection; but not its linear and early nonlinear stages. Let us now see why this distinction matters.

Recent works (Boldyrev & Loureiro 2017, 2018, 2019; Loureiro & Boldyrev 2017a, 2017b, 2018; Mallet et al. 2017a, 2017b) have presented analytical arguments for the inevitability of the onset of the tearing mode (in either its resistive or collisionless forms, as appropriate) below a certain (so-called, critical) turbulence scale, $\lambda_{\rm cr} \ll L$, where L is the energy injection scale, in a wide variety of plasma regimes. It is argued by these authors that the effect of the tearing mode is to redefine the energy cascade rate (to become the tearing mode growth rate, see Section 2), resulting in a different energy spectrum and eddy anisotropy at scales $\lambda \ll \lambda_{\rm cr}$. Magnetohydrodynamic simulations performed subsequently appear to lend support to these ideas (Dong et al. 2018; Walker et al. 2018), as does a detailed analysis of solar wind data (Vech et al. 2018). Additional consistent numerical evidence has been reported by Arzamasskiy et al. (2019), Landi et al. (2019; specifically, the measurement of linear anisotropy of the turbulent fluctuations, $k_{\|} \sim k_{\perp}$, in the sub-ion range, as predicted for tearing-mediated inertial kinetic-Alfvén wave turbulence; Boldyrev & Loureiro 2019).

Tearing onset in turbulence thus appears to be in reasonably strong footing—prompting the important question of whether it can (and, if so, under what conditions) lead to a fully nonlinear reconnecting stage. Essentially, the reason this question is nontrivial is that the reconnection rate differs from the tearing mode growth rate (and, therefore, from the eddy turnover rate in the tearing-mediated turbulence range). The goal of this paper is to address this issue.[4]

2. Preliminaries

The key idea underlying the suggestion that the tearing mode is activated at turbulence scales $\lambda \ll \lambda_{\rm cr}$ derives from the observation that, at such scales, the tearing mode growth rate, $\gamma_t(\lambda)$, exceeds the eddy turn-over rate, $\tau_{nl}^{-1}(\lambda)$, that would otherwise pertain to those scales, i.e.,

$$\gamma_t \tau_{nl} \gg 1, \tag{1}$$

with $\lambda_{\rm cr}$ resulting from solving this condition in the case of approximate equality (Loureiro & Boldyrev 2017a; Mallet et al. 2017a). It is demonstrated in these references that the specific mode (wavenumber) that solves Equation (1), among all possible tearing-unstable modes, is the fastest-growing tearing mode (often dubbed the "Coppi" mode; Coppi et al. 1976).

The onset of the tearing mode, *per se*, is not sufficient to interfere with the turbulent cascade. Its ability to be dynamically significant naturally hinges on whether it can attain a nonlinear amplitude. In this regard, the tearing mode is a somewhat peculiar instability in that it becomes nonlinear at very small amplitudes: i.e., as soon as the width of the magnetic island that it creates exceeds the thickness of the inner boundary layer (which is, forcefully, asymptotically smaller than the characteristic length scale of variation of the background magnetic profile; i.e., in this case, than the size of the eddy, λ).[5] As the tearing mode begins its nonlinear evolution, it continues to grow exponentially at the same rate as in the linear stage[6] (Wang & Bhattacharjee 1993; Porcelli et al. 2002; Loureiro et al. 2005). These notions imply that Equation (1) correctly represents the condition for the nonlinear tearing mode to affect the turbulent cascade (Boldyrev & Loureiro 2017; Loureiro & Boldyrev 2017a; Mallet et al. 2017a). Furthermore, the tearing mode onset implies that γ_t becomes the eddy turnover rate at those scales, with a consequent change in the turbulence spectrum and other properties.

However, and as we now explain, it is less clear—but, we will argue, critical—whether the (early) nonlinear stage of the tearing mode evolution has the chance to evolve toward the deep nonlinear (i.e., properly reconnecting) stage, whereupon a significant amount of the magnetic flux in the eddy is reconnected, and considerable magnetic energy dissipation and conversion occurs.

In the absence of background turbulence, the (strongly unstable, large Δ') tearing mode is known to transition to a fully nonlinear reconnecting state once its amplitude becomes sufficiently large (Waelbroeck 1989; Jemella et al. 2003; Loureiro et al. 2005, 2013). At this moment, the tearing rate will, in most cases, change to a different value, usually referred to as the (normalized) reconnection rate, $\mathcal{R}$. The current understanding of reconnection suggests the following. In resistive MHD, there are two possibilities for $\mathcal{R}$, depending on the value of the Lundquist number, $S = L_{\rm CS} v_{\rm A}/\eta$, where $L_{\rm CS}$ is the current sheet length, $v_{\rm A}$ the Alfvén speed based on the reconnecting component of the magnetic field, and η is the magnetic diffusivity. If $S \lesssim S_{\rm cr} \approx 10^4$ we have $\mathcal{R} = S^{-1/2}$ (i.e., the Sweet–Parker rate; Parker 1957; Sweet 1958). This is the only case where, in fact, the reconnection rate is the same as the tearing rate (of the most unstable tearing mode). However, this result is of limited applicability as, generally, $S \gg S_{\rm cr}$; in such cases, one instead has $\mathcal{R} = S_{\rm cr}^{-1/2} \approx 0.01$ (Loureiro et al. 2007, 2012; Bhattacharjee et al. 2009; Samtaney et al. 2009; Huang & Bhattacharjee 2010; Uzdensky et al. 2010; Loureiro & Uzdensky 2016). For collisionless reconnection, though absent a theoretical explanation (see, however, Liu et al. 2017), it is generally accepted that $\mathcal{R} \approx 0.1$ (Birn et al. 2001; Comisso & Bhattacharjee 2016; Cassak et al. 2017).

In addition to the difference in rates, collisional (resistive) and collisionless reconnection differ also in the ways in which the upstream magnetic energy may be channeled. In the MHD

[4] Hopefully, the reason for the somewhat tautological title of this paper is now clear. Reconnection is usually implicitly understood to be a nonlinear phenomenon. The specific phrasing of the title aims to stress the distinction between the linear and early nonlinear evolution of the tearing mode, on one hand, and its late, strongly nonlinear evolution on the other—the latter being what is meant here by the proper, or nonlinear, reconnection stage. This distinction is key, since the linear and early nonlinear stages of the tearing mode reconnect insignificant amounts of flux, and lead to negligible energy dissipation and conversion.

[5] To be specific: purely from geometric considerations and the definition of separatrix, one has that the full width of a magnetic island is given by $W = 4\sqrt{-\tilde{\psi}/\psi''_{\rm eq}}$, where $\tilde{\psi}$ is the perturbed (reconnected) flux, and $\psi''_{\rm eq} \approx B_{\rm eq}/a$ is the equilibrium current at the rational layer. The tearing mode becomes nonlinear when $W \approx \delta_{\rm in}$, where $\delta_{\rm in}$ is the width of the inner boundary layer that arises in the tearing mode calculation (Furth et al. 1963; Coppi et al. 1976); it scales with resistivity in the MHD regime, or with electron inertia in kinetic calculations and is, by definition, asymptotically smaller than a, the width of the background equilibrium. One thus finds that, in the early nonlinear regime, $\tilde{\psi}/\psi_{\rm eq} \approx 1/16\,(\delta_{\rm in}/a)^2 \ll 1$; note however that, numerical pre-factor aside, this condition implies that, for the Coppi mode, the perturbed and background currents are comparable at this stage.

[6] This is not generally true for all wavenumbers unstable to tearing, but it is true for the most unstable (Coppi) mode.

case, only ohmic and viscous dissipation are possible, in addition to the kinetic energy in the reconnection outflows, and the reconnected magnetic-field energy. In the kinetic case, however, super-thermal particle acceleration, as well as electron Landau damping, replace ohmic and viscous heating. Pertinently to our discussion here, reconnection-driven energy dissipation occurs predominantly in strongly localized current sheets in the MHD case; but it takes place at scales comparable to the background reconnecting field (the eddy, in the context of turbulence) in the kinetic case: particle energization can happen via the trapping of particles inside of plasmoids (e.g., Drake et al. 2006), whose widths are comparable to the scale of the background reconnecting field; and electron Landau damping induced by reconnection also seems to cover a wide range of spacial scales (Loureiro et al. 2013; Numata & Loureiro 2015). These notions inform how the qualitative properties of turbulence might change if full reconnection is permitted. For example, on the basis of this discussion, one may expect a steepening of the spectrum in the kinetic case, and enhanced intermittency in the MHD case (as discussed below).

For use in what follows, let us define the reconnection time as

$$\tau_{\rm rec} = \mathcal{R}^{-1}\tau_{{\rm A},\lambda}, \tag{2}$$

where $\tau_{{\rm A},\lambda} = \lambda/v_{{\rm A},\lambda}$. The physical meaning of $\tau_{\rm rec}$ is that it is the time that it takes to reconnect the magnetic flux contained in an eddy of size λ and reconnecting field $B_\xi(\lambda)$. The question which we wish to address is whether this reconnection rate is larger than the tearing rate, that is, whether an eddy distorted by the tearing instability may end up reconnecting a significant amount of the magnetic flux in the eddy where it occurs. We propose that this will only happen if

$$\gamma_t \tau_{\rm rec} \ll 1. \tag{3}$$

In other words, a typical eddy at scales $\lambda \ll \lambda_{\rm cr}$ exists for a time of order γ_t^{-1}. The tearing mode occurring within such an eddy, therefore, has a finite probability of reaching the deep nonlinear stage, whereupon it may transition to the reconnection regime. If condition (3) is met, then the reconnection time is much shorter than the eddy turnover time, and it is thus expected that full reconnection will occur. Concurrent energy dissipation or conversion may then lead to a steepening of the spectrum beyond the predictions of the tearing-mediated cascade, and/or an increase in the intermittency of the turbulence. Otherwise, reconnection is slower, and the eddy will cease to exist without significant reconnection having taken place; in this case, the tearing-mediated cascade is unaltered.

We now proceed to compute this condition, and discuss its implications, in three different cases: the pure MHD case, Section 3; and the cases when tearing, and reconnection, are enabled by kinetic physics (electron inertia) instead of resistivity, and the eddies in which they happen are above (Section 4.1) or below (Section 4.2) the ion kinetic scales.

3. The Magnetohydrodynamic Case

The onset of tearing in MHD turbulence has been addressed by Loureiro & Boldyrev (2017a), Mallet et al. (2017a), Boldyrev & Loureiro (2017). These authors find

$$\lambda_{\rm cr}/L \sim S_{\rm L}^{-4/7}, \tag{4}$$

where $S_{\rm L} = LV_{{\rm A},0}/\eta$ is the outer scale Lundquist number, and $V_{{\rm A},0}$ is the Alfvén velocity based on the background (mean) field B_0. Below this scale, the eddy turnover time becomes the growth rate of the fastest growing tearing mode:

$$\gamma_t \sim \tau_{{\rm A},\lambda}^{-1}(\lambda v_{{\rm A},\lambda}/\eta)^{-1/2} \tag{5}$$

where (Boldyrev & Loureiro 2017)

$$v_{{\rm A},\lambda} \sim \varepsilon^{2/5}\eta^{-1/5}\lambda^{3/5}, \tag{6}$$

with $\varepsilon = V_{{\rm A},0}^3/L$ the injected power.

Therefore, evaluation of Equation (3) yields the requirement

$$\lambda/L \gg \mathcal{R}^{-5/4}S_{\rm L}^{-3/4}. \tag{7}$$

It is necessary for the validity of this result that $\lambda_{\rm cr} \gg \lambda \gg \lambda_{\rm diss}$, where $\lambda_{\rm diss} \sim S_{\rm L}^{-3/4}L$ (Boldyrev & Loureiro 2017) is the dissipation scale. Since $\mathcal{R} < 1$, the second inequality is automatically satisfied. As to the first, we find that it implies

$$S_{\rm L} \gg \mathcal{R}^{-7}. \tag{8}$$

As mentioned before, as long as $S_\xi = \xi v_{{\rm A},\lambda}/\eta \gtrsim S_{\rm cr} \sim 10^4$, the reconnection rate is $\mathcal{R} \sim S_{\rm cr}^{-1/2} \sim 0.01$. Let us check that this is indeed true. Using Equation (6) and the scaling $\xi \sim L(\lambda_{\rm cr}/L)^{1/4}(\lambda/\lambda_{\rm cr})^{9/5}$ (Boldyrev & Loureiro 2017), both of which expressions are valid in the tearing-mediated turbulence range, we find that $S_\xi \gg S_{\rm cr} \Rightarrow \lambda/L \gg S_{\rm cr}^{5/12}S_{\rm L}^{-73/84}$. This is smaller than $\lambda_{\rm cr}/L$ if $S_{\rm L} \gg S_{\rm cr}^{7/5} \sim 10^{28/5}$, a condition which is superseded by Equation (8). We thus arrive at the conclusion that significant reconnection is only possible if $S_{\rm L} \gg 0.01^{-7} = 10^{14}$, a considerable demand even by the standards of astrophysical and space plasmas—and certainly one that direct numerical simulations cannot be imagined to meet anytime in the foreseeable future.

Still, one can speculate as to what might happen once that threshold is reached. In the usual Kolmogorov-type picture of turbulence, the dissipation of turbulent energy is assumed to be uniform in space and time, occurring in a space-filling manner at the scale of the smallest turbulent eddies. The triggering of full reconnection in inertial-scale eddies is expected to change this picture. In resistive MHD at high Lundquist numbers, reconnection leads to ohmic and viscous heating in sharply localized structures—small-scale current sheets that mediate the plasmoid chain. This fact, in combination with the idea that there is only an order unity chance (rather than a near certainty) that eddies will live long enough to reconnect, leads us to expect that at the very high values of S that Equation (8) points to, turbulence should become significantly more intermittent than below that threshold.

To estimate the effect of intermittency, we assume a simple intermittency model, the so-called β-model (e.g., Frisch 1995). The model assumes that the energy cascade is not space filling but rather occurs in the vicinity of structures with spatial dimension D. The requirement of a constant energy flux over scales then reads $p_\lambda v_\lambda^2\gamma_t = {\rm const}$, where $p_\lambda \sim \lambda^{3-D}$, and γ_t is given by Equation (5). Indeed, eddies of size λ occupying the regions around D-dimensional structures cover a fraction of the volume proportional to p_λ. From this we derive $v_\lambda \sim \lambda^{3/5} p_\lambda^{-2/5}$. The energy density then reads $E_\lambda \sim v_\lambda^2 p_\lambda \sim \lambda^{6/5} \lambda^{(3-D)/5}$. For two-dimensional structures, this gives $E_\lambda \sim \lambda^{7/5}$, which implies the Fourier energy spectrum $E(k_\perp) \sim k_\perp^{-12/5} = k_\perp^{-2.4}$, only somewhat steeper than the $k_\perp^{-11/5}$ spectrum predicted in the tearing-range excluding intermittency corrections (Boldyrev & Loureiro 2017; Mallet et al. 2017a).

For completeness, we note that the calculation leading to condition (7) can be straightforwardly adapted to the case when the profile of the reconnecting magnetic field in the eddy is better represented by $\sin(x/\lambda)$, instead of the Harris profile ($\tanh(x/\lambda)$) that we consider by default in this paper. In that case, $\gamma_t \sim \tau_{A,\lambda}^{-1}(\lambda v_{A,\lambda}/\eta)^{-3/7}$, with $v_{A,\lambda} \sim \varepsilon^{7/18}\eta^{-1/6}\lambda^{5/9}$ (Boldyrev & Loureiro 2017). Equation (7) is then replaced by $\lambda/L \gg \mathcal{R}^{-3/2} S_L^{-3/4}$. The estimation of the dissipation scale is unchanged from that pertaining to the Harris sheet; but, in this case, $\lambda_{cr}/L \sim S_L^{-6/11}$. Therefore, Equation (8) is instead $S_L \gg \mathcal{R}^{-22/3}$. The intermittency-corrected spectrum in this case is $E(k_\perp) \sim k_\perp^{-42/15} = k_\perp^{-2.8}$.

4. The Collisionless Case

Let us now examine the same question in a plasma where collisions are sufficiently rare that the breaking of frozen flux condition required to enable the tearing mode and the subsequent nonlinear reconnection stage is due to electron inertia (active at the electron skin-depth scale, $d_e = c/\omega_{pe}$), rather than resistivity as considered in the previous section; i.e., in this case, $\mathcal{R} \approx 0.1$.

As documented in Loureiro & Boldyrev (2017b), there are two cases that need considering: the first, somewhat simpler to address, is when the critical scale at which the tearing mode onsets is in the MHD range (i.e., λ_{cr} is larger than the ion kinetic scales)—even though, to repeat, the tearing and reconnection themselves require kinetic effects. This is treated in Section 4.1. The second case is when the onset of tearing only occurs for scales smaller than the ion kinetic scales. This is discussed in Section 4.2.

4.1. Reconnection at Fluid Scales

Several cases are possible, depending on plasma parameters (Loureiro & Boldyrev 2017b). There is no need here to be exhaustive: for any particular case, the calculation proceeds in a qualitatively similar way. Therefore, let us consider, as an example, a low beta plasma (see Sections 2 and 3 of Loureiro & Boldyrev 2017b)—we choose to analyze this particular case because of its potential relevance to solar wind observations (-Vech et al. 2018), and perhaps also to the solar corona. In this case, in the tearing-mediated range, the eddy turnover rate becomes the growth rate of the fastest growing tearing mode as given by $\gamma_t \sim v_{A,\lambda} d_e \rho_s / \lambda^3$, where ρ_s is the ion sound Larmor radius. Evaluation of Equation (3) then yields:

$$\lambda \gg \mathcal{R}^{-1/2}(d_e \rho_s)^{1/2}. \tag{9}$$

This expression only applies in the range of scales $\lambda_{cr} \gg \lambda \gg \rho_s$ where, for this case, $\lambda_{cr}/L \sim (d_e/L)^{4/9}(\rho_s/L)^{4/9}$ (Loureiro & Boldyrev 2017b; Mallet et al. 2017b); this translates into

$$\mathcal{R} \ll \frac{d_e}{\rho_s} \ll \mathcal{R}^9 \left(\frac{L}{\rho_s}\right)^2. \tag{10}$$

The left inequality is probably not satisfied in the (pristine) solar wind at $\sim$1 au (it requires $\beta_e \ll 2(m_e/m_i)\mathcal{R}^{-2} \approx 0.1$, which may be too low). In that case, one concludes that reconnection in current sheets should not be a main energy dissipation mechanism in that turbulent environment at these scales. The opposite situation, however, should pertain to the solar corona: using standard parameters there is no difficulty in concluding that both inequalities in Equation (10) should hold comfortably.[7] From the point of view of numerical simulations, this result, like Equation (8), unfortunately places close to impossible demands.

For the $\sin(x/\lambda)$ profile, Equation (9) becomes instead $\lambda \gg \mathcal{R}^{-2/3} d_e^{4/9} \rho_s^{5/9}$; whereas Equation (10) is replaced by $\mathcal{R}^{3/2} \ll d_e/\rho_s \ll \mathcal{R}^{63/6}(L/\rho_s)^{9/4}$. The conclusions drawn above pertain equally for these estimates.

4.2. Reconnection at Kinetic Scales

Finally, we analyze the case when the tearing onset only occurs at scales below the ion kinetic scales. Let us consider here the analysis recently proposed by Boldyrev & Loureiro (2019) of sub-ion range turbulence in plasmas such that $\beta_i \sim 1 \gg \beta_e$. The relevant eddy turnover rate is

$$\gamma_t \sim \frac{v_{Ae,\lambda}}{\lambda}\left(\frac{d_e}{\lambda}\right)^2. \tag{11}$$

Therefore we find:

$$\gamma_t \tau_{rec} \ll 1 \Rightarrow \frac{\lambda}{d_e} \gg \mathcal{R}^{-1/2} \sim 3. \tag{12}$$

Repeating this derivation for the $\sin(x)$ magnetic field profile instead yields

$$\lambda/d_e \gg \mathcal{R}^{-2/3} \sim 5. \tag{13}$$

Unlike the two cases considered previously, an estimation of λ_{cr} is not available for this situation (a reflection of the fact that a detailed understanding of sub-ion range turbulence is still lacking). The only known constraint that applies to λ_{cr} is that it be smaller than $\min(d_i, \rho_i, \rho_s)$, which simply follows from the range of validity of the equations that are used by Boldyrev & Loureiro (2019) to compute Equation (11). At small scales, it is required that $\lambda \gg d_e$, which is (marginally) satisfied by Equation (12). Thus, unlike the regimes identified by Equations (8) and (9), the condition represented by Equation (12) may be unavoidable; that is, first-principles numerical simulations that address this plasma regime with adequate separation between the ion and the electron scales may inevitably satisfy the conditions for nonlinear reconnection in turbulent eddies at sub-ion scales. Indeed, the biggest such simulation that we are familiar with reports spectra steeper than $k_\perp^{-3}$ in the sub-ion range (Told et al. 2015), which could be a manifestation of reconnection-driven energy depletion in the eddies.

[7] Note that this is indeed the regime that we would expect to describe turbulence in the solar corona at these scales, rather than the MHD case of Section 3: for typical coronal conditions, i.e., $S_L \approx 10^{14}$, $L \approx 10^4$ km, Equation (15) of Loureiro & Boldyrev (2017a), describing the MHD case, yields an estimate of the width of the inner boundary layer of the tearing mode occurring on an eddy of width λ_{cr} of approximately 1 cm, smaller than the electron skin depth and showing, therefore, that the tearing mode at such scales is collisionless, as we consider in this section.

It is interesting to analyze this result in light of recent *MMS* observations of so-called electron-only reconnection in the Earth's magnetosheath (Phan et al. 2018; Stawarz et al. 2019). In Boldyrev & Loureiro (2019) we have estimated that the decoupling of the ions in these events requires $\lambda/d_e \ll \sqrt{d_i/d_e}$ or $\lambda/d_e \ll (d_i/d_e)^{2/3}$ depending on whether one assumes a $\tanh(x/\lambda)$ or $\sin(x/\lambda)$ magnetic field profile for the reconnecting field $B_\xi(x)$ in the eddy. These estimates range from ~6 to ~12; combined with Equation (12) or (13) they suggest a rather narrow range of scales where nonlinear "electron-only" reconnection in the eddies may be possible. Remarkably, Phan et al. (2018) report a current sheet thickness of ~4 d_e, strikingly consistent with these numbers and with Equation (12). This is certainly very encouraging, but one must also bear in mind that all our analytical results are only order of magnitude estimates which ignore order unity numerical prefactors.

Another observationally based result which we interpret to be consistent with our analysis is the recent claim by Chen et al. (2019) that energy dissipation at kinetic scales in the magnetosheath is dominated by linear electron Landau damping (the energy dissipation rate via that channel being comparable to the energy cascade rate). Indeed, Equation (12) demonstrates that full reconnection in sub-ion scale eddies is permitted for typical magnetosheath parameters; and previous investigations of heating in (strong guide-field) collisionless reconnection (Loureiro et al. 2013; Numata & Loureiro 2015) show that when $\beta_e \ll 1$ linear electron Landau damping is by far the dominant energy dissipation channel. Furthermore, Chen et al. (2019) report a steepening of the spectrum at the scales where Landau damping is prominent, consistent with our expectations of the effects of active collisionless reconnection in the cascade.

4.3. Low-beta Pair Plasmas

For completeness, we address here the case of low-beta pair plasmas, for which governing equations and turbulence scalings are given in Loureiro & Boldyrev (2018).

In the MHD regime the same equations as for an electron-ion plasma apply; the discussion is thus unchanged from that of Section 3.

In the case of a collisionless pair plasma, the criterion stated in Equation (3) yields $\lambda/d_e \gg \mathcal{R}^{-1/2}$ (exactly as in Equation (12) since the scaling for γ_t is the same). This needs to be reconciled with the requirements for the existence of a tearing range in the first place: for a Harris-type reconnecting profile, these are (Loureiro & Boldyrev 2018):

$$(d_e/L)^{8/9} \gg \lambda/L \gg d_e/L. \tag{14}$$

Therefore, the realizability of reconnection requires

$$L/d_e \gg \mathcal{R}^{-9/2} \approx 3 \times 10^4 \tag{15}$$

(as well as simply that $\mathcal{R}^{-1/2} \gg 1$, a criterion which we have also encountered in Section 4.2 and which we assume to be marginally, though perhaps not asymptotically, satisfied). An entirely similar calculation for the case of a sinusoidal reconnecting field (instead of a Harris profile) yields instead the requirement that $L/d_e \gg \mathcal{R}^{-14/3} \approx 4.6 \times 10^4$. Both this and Equation (15) are easily satisfied in astrophysical systems, and are even reasonable enough that they might possibly be met by numerical simulations in the not-too-distant future.

5. Conclusion

This paper builds on previous recent work on the onset of the tearing instability in strong magnetic plasma turbulence, establishing the conditions under which this instability may develop into a deep nonlinear reconnecting state. The ability to do so may deeply change the relative efficiency of different energy dissipation channels, as well as the intermittency of the turbulence. We think this has profound implications for turbulent systems. For example, in weakly collisional plasmas, reconnection is a well known efficient particle acceleration mechanism (e.g., Guo et al. 2014; Sironi & Spitkovsky 2014; Dahlin et al. 2015; Werner et al. 2017), and heats different species at different rates (e.g., Numata & Loureiro 2015; Shay et al. 2018). Therefore, if reconnecting eddies are a common occurrence—the conditions for which are worked out in this paper—then one might expect turbulence to be more efficient at generating non-thermal populations and different electron-to-ion temperature ratios, which are indeed observed or expected in different space and astrophysical plasmas (see, e.g., Schekochihin et al. 2019, and references therein). Moreover, the very observability of reconnecting turbulence depends, obviously, on whether truly reconnecting eddies are the norm or an exception.

Yet another consequential implication of the analysis carried out in this paper stems from the fact that neither Equation (8) nor Equation (10) have ever been met in computer simulations conducted to date, nor is that likely to happen in the near future. It thus follows that *all* observations of reconnecting current sheets in (three-dimensional) numerical simulations of strong turbulence in the plasma regimes to which those equations pertain are bound to be relatively rare or transient events, with no significant impact on the nature of energy dissipation. In the kinetic case, one immediate consequence of this is that particle energization rates obtained in simulations of magnetic turbulence may be severely underestimated with respect to the environments that such simulations aim to study. One way to remedy this situation might be to hard-wire, in numerical simulations, energy dissipation prescriptions based on the energetics of reconnection (specific to the particular plasma parameters under study) at scales where the turbulent cascade is tearing-dominated.

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