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Lorentz-symmetry violating physics in a supersymmetric scenario in $(2+1)$-D

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Abstract – We study the dimensional reduction for a $(3+1)$-dimensional Lorentz violating Lagrangian in the matter and gauge sectors in a supersymmetric scenario. We thus obtain an effective model for photons induced by the effects of supersymmetry in our framework with Lorentz-symmetry violation. This model is discussed in connection with the attainment of a static potential between charged particles. Our calculation is done within the framework of the gauge-invariant, but path-dependent variable formalism. We thus find that the interaction energy displays a screening part, encoded by Bessel functions, and a linear confining potential.

Introduction. – The Standard Model of Particle Physics (SM) provides a very satisfactory description of fundamental processes within a scale of a few hundred GeVs. Appealing TeV accessible energies, the LHC has been scrutinizing the SM searching for new physics which may stem from higher energies; it is the core of supersymmetry (SUSY), quantum gravity effects and extradimensions.

The intensive activity on Lorentz-symmetry violation (LSV) encompasses a broad variety of phenomena from ground-based experiments such as atomic physics, accelerator physics and astrophysics. LSV takes place at very high energies, close to the Planckian scale, but it may be felt at accessible regions of our observations and may point to new patches to understand more fundamental physics beyond the SM [1–19].

It is reasonable to set up the discussion of LSV in a context where SUSY provides a good scenario. Our present understanding is that SUSY is broken at a lower scale, so that LSV takes place at a scale where SUSY should not be disregarded. In the works of refs. [20–23] we quote a number of papers where LSV is inspected in close connection with SUSY.

In the present paper we pursue an investigation that connects LSV and SUSY no more in $(1+3)$-D, as is currently done. We focus our attention on a $(1+2)$-dimensional space-time, so that planar phenomena may be contemplated. Though planar physics must directly interest condensed matter phenomena and those, in principle, do not immediately concern Lorentz symmetry, we know that non-relativistic effects derived from Lorentz-invariant models are relevant for Condensed Matter Physics.

On the other hand, more recently, SUSY has appeared as an emergent symmetry in Condensed Matter Systems [24–27]. In view of that we wish, in this letter, to report on a study we have undertaken which relates LSV, SUSY and $(1+2)$-D physics. We wish to understand how physical parameters associated to LSV and SUSY may interfere on planar models which might be of interest for describing low-dimensional Condensed Matter physics.

In previous publications [22,23], we have worked out effective photonic actions upon integrating over SUSY degrees of freedom, like the photino, for example. In the present letter, we consider LSV in the matter sector (the specific model shall be presented in the sequel) in four space-time dimensions in the presence of SUSY. The supersymmetric LSV model will undergo supersymmetric dimensional reduction to $(1+2)$ dimensions and the reduced 3D action will exhibit anisotropic parameters that are inherited from the 4D mother model.

An effective photonic action is considered to discuss the interaction energy for two probes charges. In other words, our purpose here is to further elaborate on the physical content of this new electrodynamics (photonics action) on a physical observable. To do this, we shall work out the static potential for this electrodynamics along the lines of [28,29]. The advantage of using this development lies in the fact that the interaction energy between two static charges is obtained once a judicious identification of the physical degrees of freedom is made. As will be seen, this new electrodynamics is analogous to that encountered in $D = 3$ models of antisymmetric tensor fields that emerges from the condensation of topological defects, as a consequence of the Julia-Thoulousse mechanism [30]. This same potential profile is obtained in the case of condensation of charged scalars in $D = 3$ dimensions [31]. In other terms, in this work we are concerned with the physical content associated with a "sort of duality", where duality refers to an equivalence between two or more quantum field theories whose corresponding classical theories are different.

We would like to point out a number of interesting articles that discuss relevant physical aspects of Lorentz-symmetry violating models in $(1+2)$ dimensions, both without [32–38] and with [21,39–41] SUSY taken into account.

Our work is organized to the following outline: in the following section, we perform the dimensional reduction for a $(3+1)$-dimensional LV-Lagrangian in the matter and gauge sectors along the lines of [42,43]. In the third section, we compute the interaction energy for a fermion-antifermion pair in this new electrodynamics. Interestingly enough, for this new electrodynamics, the static potential profile contains a linear term, leading to the confinement of static charges. Finally, some concluding remarks are presented in the fourth section. An appendix follows where we cast our conventions and notations.

In our conventions the signature of the metric is $(+1, -1, -1)$.

SUSY matter Lorentz-breaking Lagrangian. –

Matter Lagrangian. We start off from the superspace action

$$S = \int \mathrm{d}^4x\mathrm{d}^4\theta[(R+\bar{R})(\bar{\Phi}_1\Phi_1 - \bar{\Phi}_2\Phi_2)] + \int \mathrm{d}^4x\mathrm{d}^4\theta[(S+\bar{S})(\bar{\Phi}_1\Phi_1 + \bar{\Phi}_2\Phi_2)], \quad (1)$$

where θ is the spinorial coordinate of superspace. R, S, Φ_1 and Φ_2 are all chiral superfields. The LV background is accommodated in R and S; the matter degrees of freedom are contained in Φ_1 and Φ_2.

The respective field-component contents of the superfields above are:

$$R = \{r, \xi, H\}, \quad S = \{s, \chi, G\}$$

and

$$\Phi_1 = \{\phi_1, \psi_1, F_1\}, \quad \Phi_2 = \{\phi_2, \psi_2, F_2\}. \quad (2)$$

In all the superfields above the components in the brackets are organized such that the first field is a complex scalar, the second partner is a Majorana fermion and the third component is a complex scalar auxiliary field. We take r and s to be purely imaginary, $r + r^* = s + s^* = 0$, and we define $a_{\hat{\mu}} = \frac{i}{\sqrt{2}}\partial_{\hat{\mu}}(r - r^*)$ and $b_{\hat{\mu}} = \frac{i}{\sqrt{2}}\partial_{\hat{\mu}}(s - s^*)$.

It is also convenient to define (four-component) Majorana spinors associated to the LV background:

$$\Lambda_+ = \xi + \chi, \quad \Lambda_- = -(\xi - \chi); \quad (3)$$

they are going to appear in condensates in both the boson and sermonic matter Lagrangians to be presented in the sequel.

By projecting the superfield Lagrangian above in terms of component fields, eliminating the auxiliary fields, F_1 and F_2, with the help of their equations of motion and carrying out some Fierzings, we get to the fermionic Lagrangian, $\mathcal{L}_f$, given by

$$\mathcal{L}_f = \bar{\Psi}\Gamma^{\hat{\mu}}(i\partial_{\hat{\mu}} + \bar{a}_{\hat{\mu}} + \bar{b}_{\hat{\mu}}\Gamma_5 - m)\Psi, \quad (4)$$

where $\bar{b}^{\hat{\mu}} = \frac{1}{4}(W+V)^{\hat{\mu}} - b^{\hat{\mu}}$, $\bar{a}^{\hat{\mu}} = \frac{1}{4}(W-V)^{\hat{\mu}} - a^{\hat{\mu}}$, $W^{\hat{\mu}}(V) = \bar{\Lambda}_{+(-)}\Gamma^{\hat{\mu}}\Gamma_5\Lambda_{+(-)}$. As anticipated above, $a_{\hat{\mu}}$ and $b_{\hat{\mu}}$ carry the background fermion condensates built up in terms of Λ_+ and Λ_-. Besides, the bosonic part is given by

$$\begin{aligned}\mathcal{L}_b = &-\frac{1}{2}\phi_1^*\Big(\partial^{\hat{\mu}}\partial_{\hat{\mu}} + m^2 + i2\sqrt{2}(a+b)^{\hat{\mu}}\partial_{\hat{\mu}}\Big)\phi_1 \\ &-\frac{1}{2}\phi_2^*\Big(\partial^{\hat{\mu}}\partial_{\hat{\mu}} + m^2 + i2\sqrt{2}(a+b)^{\hat{\mu}}\partial_{\hat{\mu}}\Big)\phi_2. \quad (5)\end{aligned}$$

It should be further noted that beyond the quadratic terms, this method brings us a different kind of interaction between scalar and fermionic fields through the following interaction Lagrangian:

$$\mathcal{L}_{int} = \Phi^\dagger v + h.c., \quad (6)$$

where $\Phi^\dagger = (\phi_1^* \quad \phi_2^*)$ and $v = (i\bar{\Lambda}\Gamma\cdot\partial + \frac{m}{2}\bar{\Lambda}_R)\begin{pmatrix}\Psi \\ \Gamma_5\Psi\end{pmatrix}$.

We can now apply the dimensional reduction [42,43] in the total Lagrangian $\mathcal{L}_{tot} = \mathcal{L}_f + \mathcal{L}_b + \mathcal{L}_{int}$.

Following the conventions given in the appendix, the corresponding dimensional reduction of the Lagrangian is carried out with $\Psi = (\Psi_1 \quad \Psi_2)^T$. We thus obtain

$$\begin{aligned}\mathcal{L}_f = &(\bar{\Psi}_1 \quad \bar{\Psi}_2) \\ &\times\begin{pmatrix} i\gamma\cdot\partial + \gamma\cdot\bar{a} - \bar{b}_3 + m & -i\gamma\cdot\bar{b} + i\bar{a}_3 \\ -i\gamma\cdot\bar{b} + i\bar{a}_3 & -i\gamma\cdot\partial - \gamma\cdot\bar{a} + \bar{b}_3 + m\end{pmatrix} \\ &\times\begin{pmatrix}\Psi_1 \\ \Psi_2\end{pmatrix}, \quad (7)\end{aligned}$$

where $\bar{\Psi}_1 = \Psi_1^\dagger\gamma^0$. The bosonic and the mixed part can be evaluated together and they are given by

$$\mathcal{L}_{b+int} = \Phi^\dagger O(\partial)\Phi + \Phi^\dagger v + v^\dagger\Phi, \quad (8)$$

where

$$O(\partial) = \frac{1}{2}\Big(\partial^\mu\partial_\mu + m^2 + i2\sqrt{2}(a+b)^\mu\partial_\mu\Big) \quad (9)$$

and

$$v = \left(i\bar{\Lambda}_1\gamma\cdot\partial + \frac{m}{2}\bar{\Lambda}_{1R}\right)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} + \left(i\bar{\Lambda}_2\gamma\cdot\partial + \frac{m}{2}\bar{\Lambda}_{2R}\right)\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} = \begin{pmatrix}\bar{Q}_1 & \bar{Q}_2\\ -\bar{Q}_2 & \bar{Q}_1\end{pmatrix}\begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix}. \quad (10)$$

The $\bar{Q}_1$ and $\bar{Q}_2$ terms are given by $\bar{Q}_1 = (i\bar{\Lambda}_1\gamma\cdot\partial + \frac{m}{2}\bar{\Lambda}_{1R})$ and $\bar{Q}_2 = (i\bar{\Lambda}_2\gamma\cdot\partial + \frac{m}{2}\bar{\Lambda}_{2R})$.

It is worth noting here that by making use of the shift in the field $\Phi \to \Phi + O(\partial)^{-1}v$, the previous equation can be written alternatively in the form

$$\mathcal{L}_{b+int} = \Phi^\dagger O(\partial)\Phi - v^\dagger O(\partial)^{-1}v, \quad (11)$$

where

$$v^\dagger O(\partial)^{-1}v = \begin{pmatrix}\bar{\Psi}_1 & \bar{\Psi}_2\end{pmatrix} \times \begin{pmatrix}Q_1\bar{Q}_1 - Q_2\bar{Q}_2 & Q_1\bar{Q}_2 + Q_2\bar{Q}_1\\ -Q_1\bar{Q}_2 - Q_2\bar{Q}_1 & Q_2\bar{Q}_2 - Q_1\bar{Q}_1\end{pmatrix} \times O^{-1}(\partial)\begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix}, \quad (12)$$

and $O^{-1}(\partial) = \left(\partial^\mu\partial_\mu + m^2 + i2\sqrt{2}(a+b)^\mu\partial_\mu\right)^{-1}$.

We thus find that the total Lagrangian becomes

$$\mathcal{L}_{tot} = (\bar{\Psi}_1 \quad \bar{\Psi}_2) \times \begin{pmatrix} i\gamma\cdot\partial + \gamma\cdot\bar{a} - \bar{b}_3 + m & -i\gamma\cdot\bar{b} + i\bar{a}_3\\ -i\gamma\cdot\bar{b} + i\bar{a}_3 & -i\gamma\cdot\partial - \gamma\cdot\bar{a} + \bar{b}_3 + m\end{pmatrix} \times \begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} + \sum_{i=1}^{2}\phi_i O(\partial)\phi_i + (\bar{\Psi}_1 \quad \bar{\Psi}_2)\Omega(\partial,\Lambda)O^{-1}(\partial)\begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix}, \quad (13)$$

where $\Omega(\partial,\Lambda) = \begin{pmatrix}Q_1\bar{Q}_1 - Q_2\bar{Q}_2 & Q_1\bar{Q}_2 + Q_2\bar{Q}_1\\ -Q_1\bar{Q}_2 - Q_2\bar{Q}_1 & Q_2\bar{Q}_2 - Q_1\bar{Q}_1\end{pmatrix}$.

We now want to extend what we have done to the gauge sector.

CPT-odd gauge Lagrangian. By the introduction of a background scalar superfield

$$S = s + \sqrt{2}\theta\chi + i\bar{\theta}\sigma^{\hat{\mu}}\theta\partial_{\hat{\mu}}s + \theta^2 F + \frac{i}{\sqrt{2}}\theta^2\bar{\theta}\bar{\sigma}^{\hat{\mu}}\partial_{\hat{\mu}}\chi - \frac{1}{4}\bar{\theta}^2\theta^2\Delta\, s, \quad (14)$$

with the properties $(s + s^*) = 0$, $(s - s^*) = -\frac{i}{2}x_{\hat{\mu}}\xi^{\hat{\mu}}$ and $\partial_{\hat{\mu}}\chi = 0$, we are able to write a LV-action. These properties mean that the SUSY breaks down and generates a non-null vector background ξ and a non-null fermionic parameter χ. The action is given by

$$S_{CPT-odd} = \int d^4x d^4\theta \Big(W^\alpha(D_\alpha V)S + W^{\dot{\alpha}}(D_{\dot{\alpha}}V)S\Big), \quad (15)$$

where V is the vector superfield in the Wess-Zumino gauge and $W^\alpha = -\frac{1}{4}(\bar{D})^2 D^\alpha V$. Rewriting in terms of the component fields the total Lagrangian is written as $\mathcal{L}_{tot\text{-}gauge} = \mathcal{L}_A + \mathcal{L}_{ph} + \mathcal{L}_{int\text{-}gauge}$, where

$$\mathcal{L}_A = -\frac{1}{4}F_{\hat{\mu}\hat{\nu}}{}^2 + \frac{1}{2}\epsilon^{\hat{\mu}\hat{\nu}\hat{\alpha}\hat{\beta}}\xi_{\hat{\mu}}A_{\hat{\nu}}F_{\hat{\alpha}\hat{\beta}}. \quad (16)$$

For the photino (λ) we have (with fermionic Lorentz-breaking parameter χ)

$$\mathcal{L}_{ph} = -\frac{i}{2}\bar{\lambda}\Gamma^{\hat{\mu}}\partial_{\hat{\mu}}\lambda + \phi\bar{\lambda}\lambda - i\rho\bar{\lambda}\Gamma_5\lambda - \bar{V}_{\hat{\mu}}\bar{\lambda}\Gamma^{\hat{\mu}}\Gamma_5\lambda, \quad (17)$$

with $\phi = [Re(F) + \frac{1}{4}\bar{\chi}\chi]$, $\rho = [Im(F) + \frac{i}{4}\bar{\chi}\Gamma_5\chi]$ and $\bar{V}_{\hat{\mu}} = \frac{1}{4}[V_{\hat{\mu}} + \bar{\chi}\Gamma_{\hat{\mu}}\Gamma_5\chi]$. This procedure also gives a new interaction term between the photon and the photino field, and this term is given by

$$\mathcal{L}_{int\text{-}gauge} = \sqrt{2}\bar{\lambda}\Gamma^{\hat{\mu}\hat{\nu}}\Gamma_5\chi F_{\hat{\mu}\hat{\nu}}. \quad (18)$$

Now, we apply the dimensional reduction to the Lagrangian $\mathcal{L}_{tot\text{-}gauge} = \mathcal{L}_A + \mathcal{L}_{ph} + \mathcal{L}_{int\text{-}gauge}$.

From the dimensional reduction scheme of the appendix, we write $A^{\hat{\mu}} = (A^\mu, \varphi)$, $\lambda = (\lambda_1 \quad \lambda_2)^T$ and $\chi = (\chi_1 \quad \chi_2)^T$, we have

$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\varphi}{4}\varepsilon^{\mu\nu\alpha}\xi_\mu\partial_\nu A_\alpha - \frac{1}{2}\xi_3\varepsilon^{\mu\nu\alpha}A_\mu\partial_\nu A_\alpha. \quad (19)$$

The photino sector is given by

$$\mathcal{L}_{ph} = (\bar{\lambda}_1 \quad \bar{\lambda}_2)\begin{pmatrix} -\frac{i}{2}\gamma\cdot\partial + \phi - \bar{V}_3 & \rho - i\gamma\cdot\bar{V}\\ -\rho - i\gamma\cdot\bar{V} & \frac{i}{2}\gamma\cdot\partial + \phi + \bar{V}_3\end{pmatrix} \times \begin{pmatrix}\lambda_1\\ \lambda_2\end{pmatrix}, \quad (20)$$

where $\bar{V}_\mu = \frac{1}{4}[V_\mu + \bar{\chi}\Gamma_\mu\Gamma_5\chi]$ and $\bar{V}_{\hat{\mu}} = \frac{1}{4}[V_{\hat{\mu}} + \bar{\chi}\Gamma_{\hat{\mu}}\Gamma_5\chi]$. The mixing terms can be rewritten as follows:

$$\mathcal{L}_{int\text{-}gauge} = \sqrt{2}i(\bar{\lambda}_1\gamma^{\mu\nu}\chi_2 + \bar{\lambda}_2\gamma^{\mu\nu}\chi_1)F_{\mu\nu} + \sqrt{2}(\bar{\lambda}_1\gamma^\mu\chi_2 + \bar{\lambda}_2\gamma^\mu\chi_1)\partial_\mu\varphi. \quad (21)$$

In a short way, we can rewrite the above equation as $\mathcal{L}_{int} = \bar{\Theta}\Upsilon$, where $\bar{\Theta} = (\bar{\lambda}_1 \quad \bar{\lambda}_2)$ and

$$\Upsilon = \sqrt{2}\begin{pmatrix}\gamma\cdot F & \gamma\cdot\partial\varphi\\ \gamma\cdot\partial\varphi & \gamma\cdot F\end{pmatrix}\begin{pmatrix}\chi_2\\ \chi_1\end{pmatrix}. \quad (22)$$

Here $\gamma\cdot F = \gamma^{\mu\nu}F_{\mu\nu}$ and $\gamma\cdot\partial\varphi = \gamma^\mu\partial_\mu\varphi$. Manipulating the equation above and applying the same kind of shift used in the matter sector, we can rewrite the photino and mixing terms as

$$\mathcal{L}_{ph+int} = \bar{\Theta}O(\partial)\Theta + \bar{\Theta}\Upsilon = \bar{\Theta}O(\partial)\Big(\Theta + \frac{1}{2}O^{-1}(\partial)\Upsilon\Big) + \frac{1}{2}\bar{\Theta}\Upsilon, \tag{23}$$

where $\tilde{O}(\partial) = \begin{pmatrix} -\frac{i}{2}\gamma\cdot\partial + \phi - \bar{V}_3 & \rho - i\gamma\cdot\bar{V} \\ -\rho - i\gamma\cdot\bar{V} & \frac{i}{2}\gamma\cdot\partial + \phi + \bar{V}_3 \end{pmatrix}$.

With the shift $\Theta \to \Theta + \frac{1}{2}\tilde{O}^{-1}(\partial)\Upsilon$ we have, finally,

$$\mathcal{L}_{ph+int} = \bar{\Theta}\tilde{O}(\partial)\Theta - \frac{1}{4}\bar{\Upsilon}\tilde{O}^{-1}\Upsilon. \tag{24}$$

Thus, the final action of the CPT-odd gauge sector will be given by

$$\begin{aligned}\mathcal{L}_{tot\text{-}gauge} = &-\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\varphi}{4}\varepsilon^{\mu\nu\alpha}\xi_\mu\partial_\nu A_\alpha \\ &-\frac{1}{2}\xi_3\varepsilon^{\mu\nu\alpha}A_\mu\partial_\nu A_\alpha \\ &-\frac{1}{4}\bar{\Upsilon}\tilde{O}^{-1}\Upsilon + \begin{pmatrix}\bar{\lambda}_1 & \bar{\lambda}_2\end{pmatrix}\tilde{O}(\partial)\begin{pmatrix}\lambda_1 \\ \lambda_2\end{pmatrix}.\end{aligned} \tag{25}$$

In summary then, we have obtained a complete $(2+1)$-dimensional Lagrangian, which defines a new electrodynamics. In the following section we compute the interaction energy between static point-like sources for this new electrodynamics. Following the same steps as the ones presented in our previous works [22,23], we get an effective photonics-scalar Lagrangian which shall be the matter in the coming section.

Interaction energy. – As we have already expressed before, we now proceed to calculate the interaction energy between static point-like sources for the model under consideration by using the gauge-invariant but path-dependent variables formalism to examine the interaction energy, along the lines of refs. [28,29]. In this case the corresponding theory is governed by the Lagrangian density

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}^2 + \frac{m}{4}\varepsilon^{\mu\nu\kappa}A_\mu F_{\nu\kappa} + m\varepsilon^{\mu\nu\kappa}v_\mu F_{\nu\kappa}\varphi \\ &+ \frac{1}{2}(\partial_\mu\varphi)^2 + t_{\mu\nu}F^{\mu\lambda}F^\nu_\lambda + \alpha t_{\mu\nu}F^{\mu\lambda}\frac{\Delta}{\bar{\Delta}}F^\nu_\lambda \\ &+ \beta t_{\rho\lambda}F^{\mu\lambda}\frac{\partial_\mu\partial_\nu}{\bar{\Delta}}F^{\nu\rho} + s^\mu F_{\mu\nu}\partial^\nu\varphi + s_\lambda F^{\mu\lambda}\frac{\partial_\mu\Delta}{\bar{\Delta}}\varphi,\end{aligned} \tag{26}$$

where $\Delta \equiv \partial_\mu\partial^\mu$, $v_\mu \equiv \xi_\mu$, $t_{\mu\nu}$ and s_μ are given in appendix B of the work in ref. [23]. It is also important to observe that when we carry out the integration over the φ-field, we find the following effective theory:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}\Big(1 - \frac{4m^2v^2}{\Delta}\Big)F^{\mu\nu} + \frac{m}{4}\varepsilon^{\mu\nu\kappa}A_\mu F_{\nu\kappa} \\ &+ t_{\mu\nu}F^{\mu\lambda}F^\nu_\lambda + \alpha t_{\mu\nu}F^{\mu\lambda}\frac{\Delta}{\bar{\Delta}}F^\nu_\lambda + \beta t_{\rho\lambda}F^{\mu\lambda}\frac{\partial_\mu\partial_\nu}{\bar{\Delta}}F^{\nu\rho} \\ &+ 2v_\mu v_\nu F^{\mu\lambda}\frac{m^2}{\Delta}F^\nu_\lambda - m\varepsilon_{\rho\xi\sigma}v^\xi s_\lambda F^{\mu\lambda}\Big(\frac{1}{\bar{\Delta}} - \frac{1}{\Delta}\Big)\partial_\mu F^{\rho\sigma} \\ &+ \frac{1}{2}s_\lambda s_\nu F^{\mu\lambda}\Big(\frac{\Delta}{(\bar{\Delta})^2} - \frac{2}{\bar{\Delta}} + \frac{1}{\Delta}\Big)\partial_\mu\partial_\rho F^{\nu\rho}.\end{aligned} \tag{27}$$

However, as was mentioned before, this paper is aimed at studying the static potential of the above theory, a consequence of this is that one may replace Δ by $-\nabla^2$ in eq. (27). Furthermore, we recall that the only non-vanishing $t_{\mu\nu}$-terms are the diagonal ones, since, as already anticipated, $t_{\mu\nu}$ can be brought into a diagonal form. Without loss of generality, we may always choose $t_{00} \neq 0$. Restricting our considerations to the $v^i \neq 0$ and $v^{ij} = 0$ ($v_0 = 0$) case (referred to as the space-like background in what follows), the effective Lagrangian becomes

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}\Big(1 - \frac{4m^2\mathbf{v}^2}{\nabla^2}\Big)F^{\mu\nu} + \frac{m}{4}\varepsilon^{\mu\nu\kappa}A_\mu F_{\nu\kappa} \\ &+ t_{00}\frac{(A_2+\alpha)}{A_2}F^{i0}\mathcal{O}F^{i0} + -\beta\frac{t_{00}}{A_2}F^{i0}\frac{\partial_i\partial_j}{(\nabla^2 - X^2)}F^{j0} \\ &+ \frac{2m}{A_2}(\mathbf{v}\cdot\mathbf{s})F^{j0}\mathcal{O}'\partial_j B + B\mathcal{O}''B - A_0 J^0,\end{aligned} \tag{28}$$

where B is the magnetic field ($B = \varepsilon_{ij}\partial^i A^j$), $A_1 = \mu^2$ and $A_2 \equiv (c^{ii} - 1) = (k-1)$. Notice that these A_1 and A_2 are not to be confused with the components of the photon field. Nevertheless $\mathcal{O} \equiv [\frac{\nabla^4 - p\nabla^2 - q}{\nabla^2(\nabla^2 - X^2)}]$, $\mathcal{O}' \equiv [\frac{(1-A_2)\nabla^2 + A_1}{\nabla^2(\nabla^2 - X^2)}]$ and $\mathcal{O}'' \equiv [\frac{-\bar{v}\nabla^2 + w}{\nabla^2(\nabla^2 - X^2)}]$. Here $p = \frac{(t_{00}A_1 - 2m^2\mathbf{v}^2A_2)}{t_{00}(A_2+\alpha)}$, $q = \frac{(2m^2\mathbf{v}^2A_1)}{t_{00}(A_2+\alpha)}$, $X^2 = \frac{A_1}{A_2}$, $\bar{v} = 4m^2\mathbf{v}^2$ and $w = 4m^2\mathbf{v}^2X^2$.

Having characterized the model under study, we shall now examine the interaction energy. To this end, we shall first consider the Hamiltonian framework for this model. We thus find that the canonical momenta are found to be $\Pi^\mu = (1 - \frac{4m^2\mathbf{v}^2}{\nabla^2})F^{\mu 0} + \frac{m}{2}\varepsilon^{\mu 0\lambda}A_\lambda + 2t_{00}\frac{(A_2+\alpha)}{A_2}F^{\mu 0} + \frac{2\beta t_{00}}{A_2}\frac{\partial^\mu\partial_i}{(\nabla^2 - X^2)}F^{i0} - \frac{4m}{A_2}(\mathbf{v}\cdot\mathbf{s})O'\partial^\mu B$. From this expression it follows that $\Pi^0 = 0$, which is the usual primary constraint equation. It should be further noted that the remaining non-zero momenta are $\Pi^i = \{(1 - \frac{4m^2\mathbf{v}^2}{\nabla^2} + 2t_{00}\frac{(A_2+\alpha)}{A_2}\mathcal{O})\delta^{ij} - \frac{2\beta t_{00}}{A_2}\frac{\partial^i\partial^j}{(\nabla^2 - X^2)}\}F^{j0} + \frac{m}{2}\varepsilon^{ij}A_j - \frac{4m}{A_2}(\mathbf{v}\cdot\mathbf{s})\mathcal{O}'\partial^i B$.

We thus find that the canonical Hamiltonian takes the form

$$\begin{aligned}H_C = &\int d^2x\left\{-A_0\left(\partial_i\Pi^i + \frac{m}{2}\varepsilon^{ij}\partial_i A_j - J^0\right)\right\} \\ &+ \int d^2x\left\{\frac{1}{2}E_i\Lambda D_{ij}E_j + \frac{2m}{A_2}(\mathbf{v}\cdot\mathbf{s})E^iO'\partial_i B\right\} \\ &+ \int d^2x\left\{\frac{1}{2}B\left(1 - \frac{4m^2v^2}{\nabla^2} - 2O''\right)B\right\},\end{aligned} \tag{29}$$

where $\Lambda = \frac{[\nabla^4 - a\nabla^2 + b]}{\nabla^2(\nabla^2 - X^2)}$, whereas $a = X^2 + 4m^2\mathbf{v}^2$ and $b = 4m^2\mathbf{v}^2 A_1 + 2t_{00}\frac{(A_2+\alpha)}{A_2}$.

Preservation in time of the primary constraint, Π_0, leads to the usual secondary constraint (Gauss's law) $\Gamma_1 \equiv \partial_i \Pi^i + \frac{m}{2}\varepsilon^{ij}\partial_i A_j - J^0 = 0$ and together displays the first-class structure of the theory. It should be further noted that the extended (first-class) Hamiltonian that generates the time evolution of the dynamical variables has the form $H = H_C + \int \mathrm{d}^2x (c_0(x)\Pi_0(x) + c_1(x)\Gamma_1(x))$, where $c_0(x)$ and $c_1(x)$ are arbitrary functions of space and time. It is also important to observe that $\Pi^0 = 0$ for all time and $\dot{A}_0(x) = [A_0(x), H] = c_0(x)$, which is completely arbitrary. Hence we discard A^0 and Π^0. In other words, it is redundant to retain the term containing A_0 because it can be absorbed by redefining the function $c_1(x)$. We can, therefore, write

$$\begin{aligned} H = & \int \mathrm{d}^2x \left\{ c(x) \left(\partial_i \Pi^i + \frac{m}{2}\varepsilon^{ij}\partial_i A_j - J^0 \right) \right\} \\ & + \int \mathrm{d}^2x \left\{ \frac{1}{2} E_i \Lambda D_{ij} E_j + \frac{2m}{A_2}(\mathbf{v}\cdot\mathbf{s}) E^i O' \partial_i B \right\} \\ & + \int \mathrm{d}^2x \left\{ \frac{1}{2} B \left(1 - \frac{4m^2 v^2}{\nabla^2} - 2O'' \right) B \right\}, \end{aligned} \tag{30}$$

where $c(x) = c_1(x) - A_0(x)$.

Since there is one first class constraint $\Gamma_1(x)$ (Gauss's law), according to the usual procedure, we impose a gauge condition such that the full set of constraints becomes of second class. A convenient choice is [44]

$$\Gamma_2(x) \equiv \int_{C_{\zeta x}} \mathrm{d}z^\nu A_\nu(z) \equiv \int_0^1 \mathrm{d}\lambda x^i A_i(\lambda x) = 0, \tag{31}$$

where λ $(0 \leq \lambda \leq 1)$ is the parameter describing the space-like straight path $x^i = \zeta^i + \lambda(x-\zeta)^i$, and ζ is a fixed point (reference point). There is no essential loss of generality if we restrict our considerations to $\zeta^i = 0$. We thus obtain the only non-vanishing equal-time Dirac bracket for the canonical variables

$$\begin{aligned} \{A_i(\mathbf{x}), \Pi^j(\mathbf{y})\}^* = & \ \delta_i^{\ j}\delta^{(2)}(\mathbf{x}-\mathbf{y}) \\ & - \partial_i^x \int_0^1 \mathrm{d}\lambda x^j \delta^{(2)}(\lambda\mathbf{x}-\mathbf{y}). \end{aligned} \tag{32}$$

Making use of this last equation, we can rewrite the Dirac brackets in terms of the magnetic field

$$B = \varepsilon_{ij}\partial^i A^j, \tag{33}$$

and electric field

$$\begin{aligned} E_i = & \ \Lambda^{-1}\left(\delta_{ij} + \frac{\partial_i\partial_j}{(\gamma^2\Lambda - \nabla^2)} \right) \\ & \times \left(\Pi_j + \frac{4m}{A_2}(\mathbf{v}\cdot\mathbf{s})\varepsilon_{kl}\partial^k\partial_j A^l - \frac{m}{2}\varepsilon_{jk}A^k \right). \end{aligned} \tag{34}$$

We thus find

$$\begin{aligned} \{E_i(\mathbf{x}), E_r(\mathbf{y})\}^* = & \ \frac{2m}{A_2}\Lambda^{-2}(\mathbf{v}\cdot\mathbf{s})\mathcal{O}'(\varepsilon_{kr}\partial^k\partial_i - \varepsilon_{pi}\partial^p\partial_r) \\ & \times \left(1 + \frac{\nabla^2}{\Omega} \right)\delta^{(2)}(\mathbf{x}-\mathbf{y}) \\ & + m\Lambda^{-2}D_{ij}^{-1}D_{rn}^{-1}\varepsilon_{nj}\delta^{(2)}(\mathbf{x}-\mathbf{y}), \end{aligned} \tag{35}$$

where $D_{ij}^{-1} = \delta_{ij} + \frac{\partial_i\partial_j}{(\Lambda\gamma^2 - \nabla^2)}$, $1 + \frac{\nabla^2}{\Omega} = 1 + 2\beta t_{00}\frac{\nabla^2}{A_2\gamma^2(\nabla^2 - X^2) - 2\beta t_{00}\nabla^2}$ and $\gamma^2 = \frac{A_2}{2\beta t_{00}}(\nabla^2 - X^2)$.

Also, it may be stated that

$$\{B(\mathbf{x}), B(\mathbf{y})\}^* = 0, \tag{36}$$

and

$$\{E_i(\mathbf{x}), B(\mathbf{y})\}^* = -\Lambda^{-1}\varepsilon_{ij}\partial_j\delta^{(2)}(\mathbf{x}-\mathbf{y}). \tag{37}$$

Making use of the foregoing results, we obtain the following equations of motion for the magnetic and electric fields:

$$\dot{B}(\mathbf{x}) = -\varepsilon_{ij}\partial_i E_j(\mathbf{x}) \tag{38}$$

and

$$\begin{aligned} \dot{E}_i(\mathbf{x}) = & \ \frac{2m}{A_2}\Lambda^{-1}(v\cdot s)\mathcal{O}'(\varepsilon_{kr}\partial_i - \varepsilon_{ki}\partial_r)\left(1 + \frac{\nabla^2}{\Omega} \right) \\ & \times D_{rb}\partial^k E_b(\mathbf{x}) \\ & + \Lambda^{-1}\varepsilon_{ij}\left(1 - \frac{4m^2\mathbf{v}^2}{\nabla^2} - 2\mathcal{O}'' \right)\partial_j B(x) \\ & + \frac{2m}{A_2}\Lambda^{-1}\varepsilon_{ij}\partial_j\partial_k\mathcal{O}'E_k(\mathbf{x}). \end{aligned} \tag{39}$$

It is also straightforward to observe that Gauss's law for the present theory reads

$$D_{ij}\Lambda\partial_i E_j - mB - \frac{4m}{A_2}(\mathbf{v}\cdot\mathbf{s})\mathcal{O}'\nabla^2 B = J^0, \tag{40}$$

where $D_{ij} = (\delta_{ij} - \frac{\partial_i\partial_j}{\Lambda\gamma^2})$.

Next, it is to be specially noted that by taking into account the assumed conditions of static fields, eqs. (38) and (39) must vanish. We accordingly express the magnetic field in the form

$$B = -\frac{m[(\mathbf{v}\cdot\mathbf{s}) - 1]}{A_2}\frac{[(1-A_2)\nabla^2 + A_1]}{(\nabla^2 - X^2)(\nabla^2 + 4m^2\mathbf{v}^2)}\partial_i E_i. \tag{41}$$

Inserting eq. (41) into eq. (40), we find that the static electric field can be brought to the form

$$\begin{aligned} E_i(\mathbf{x}) = & \ \frac{1}{g_1}\partial_i\left\{ \frac{[\nabla^4 + (4m^2\mathbf{v}^2X^2)\nabla^2 - 4m^2\mathbf{v}^2X^2]}{\nabla^2[\nabla^4 + \frac{g_2}{g_1}\nabla^2 + \frac{g_3}{g_1}]} \right\} \\ & \times(-J^0). \end{aligned} \tag{42}$$

Here we have simplified our notation by setting $g_1 = 1 + 2t_{00} + \frac{2t_{00}(\alpha-\beta)}{A_2}$, $g_2 = (1+2t_{00})X^2 + \{[1-(\mathbf{v}\cdot\mathbf{s})]\frac{(1-A_2)}{A_2} - 4\mathbf{v}^2(1 + 2t_{00} + 2t_{00}\frac{(\alpha-\beta)}{A_2})\}$ and

$g_3 = m^2 X^2[(\mathbf{v}\cdot\mathbf{s}) - 4\mathbf{v}^2(1+2t_{00}) - 1]$. By a further manipulation of the terms, we can write eq. (42) also as

$$\begin{aligned} E_i(\mathbf{x}) =& -\frac{1}{g_1}\frac{1}{(M_1^2-M_2^2)}\partial_i\left[\frac{\nabla^2}{(\nabla^2-M_1^2)} - \frac{\nabla^2}{(\nabla^2-M_2^2)}\right] \\ &\times(-J^0) \\ &+ \frac{1}{g_1}\frac{(X^2-4m^2\mathbf{v}^2)}{(M_1^2-M_2^2)}\partial_i\left[\frac{1}{(\nabla^2-M_1^2)} - \frac{1}{(\nabla^2-M_2^2)}\right] \\ &\times(-J^0) \\ &+ \frac{(4m^2v^2X^2)}{g_1(M_1^2-M_2^2)}\frac{\partial_i}{\nabla^2}\left[\frac{1}{(\nabla^2-M_1^2)} - \frac{1}{(\nabla^2-M_2^2)}\right] \\ &\times(-J^0), \end{aligned} \tag{43}$$

where $M_1^2 = -\frac{1}{2}\frac{g_2}{g_1} + \frac{1}{2}\sqrt{\frac{g_2^2}{g_1^2} - 4\frac{g_3}{g_1}}$ and $M_2^2 = -\frac{1}{2}\frac{g_2}{g_1} - \frac{1}{2}\sqrt{\frac{g_2^2}{g_1^2} - 4\frac{g_3}{g_1}}$. For $J^0(\mathbf{x}) = q\delta^{(2)}(\mathbf{x})$, expression (43) becomes

$$\begin{aligned} E_i(\mathbf{x}) =& -\frac{q}{g_1}\frac{1}{(M_1^2-M_2^2)}\partial_i\{\nabla^2 G_1(\mathbf{x}) - \nabla^2 G_2(\mathbf{x})\} \\ &+ \frac{q}{g_1}\frac{(X^2-4m^2\mathbf{v}^2)}{(M_1^2-M_2^2)}\partial_i\{G_1(\mathbf{x}) - G_2(\mathbf{x})\} \\ &+ \frac{q}{g_1}\frac{(4m^2\mathbf{v}^2X^2)}{(M_1^2-M_2^2)}\partial_i\left\{\frac{G_1(\mathbf{x})}{\nabla^2} - \frac{G_1(\mathbf{x})}{\nabla^2}\right\}, \end{aligned} \tag{44}$$

where $G_1(\mathbf{x}) = -\frac{\delta^{(2)}(\mathbf{x})}{\nabla^2 - M_1^2} = \frac{1}{2\pi}K_0(M_1|\mathbf{x}|)$ and $G_2(\mathbf{x}) = -\frac{\delta^{(2)}(\mathbf{x})}{\nabla^2 - M_2^2} = \frac{1}{2\pi}K_0(M_2|\mathbf{x}|)$.

With the foregoing information, we can now proceed to obtain the energy interaction. As already mentioned, in order to accomplish this purpose we shall use the gauge-invariant, but path-dependent, variables formalism [44]

$$V \equiv q(\mathcal{A}_0(\mathbf{0}) - \mathcal{A}_0(\mathbf{y})), \tag{45}$$

where the physical scalar potential is given by

$$\mathcal{A}_0(\mathbf{x}) = \int_0^1 \mathrm{d}\lambda x^i E_i(\lambda\mathbf{x}), \tag{46}$$

and $i = 1,2$. As was shown in [44], this follows from the vector gauge-invariant field expression

$$\mathcal{A}_\mu(x) \equiv A_\mu(x) + \partial_\mu\left(-\int_\xi^x \mathrm{d}z^\mu A_\mu(z)\right), \tag{47}$$

where the line integral is along a space-like path from ξ to x, on a fixed time slice. It may be noted that these variables (47) commute with the sole first-class constraint (Gauss's law). From this it follows that these variables are physical variables.

With the aid of eq. (44), eq. (46) becomes

$$\begin{aligned} \mathcal{A}_0(\mathbf{x}) =& -\frac{q}{g_1}\frac{1}{(M_1^2-M_2^2)}(\nabla^2 G_1(\mathbf{x}) - \nabla^2 G_2(\mathbf{x})) \\ &+\frac{q}{g_1}\frac{(X^2-4m^2\mathbf{v}^2)}{(M_1^2-M_2^2)}(G_1(\mathbf{x}) - G_2(\mathbf{x})) \\ &+\frac{q}{g_1}\frac{(4m^2\mathbf{v}^2X^2)}{(M_1^2-M_2^2)}\left(\frac{G_1(\mathbf{x})}{\nabla^2} - \frac{G_2(\mathbf{x})}{\nabla^2}\right), \end{aligned} \tag{48}$$

after subtracting the self-energy terms.

We accordingly express the potential for two opposite charges located at $\mathbf{0}$ and $\mathbf{y}$ in the form

$$\begin{aligned} V =& -\frac{q^2}{2\pi g_1}\frac{(X^2-4m^2\mathbf{v}^2)}{(M_1^2-M_2^2)}(K_0(M_1L) - K_0(M_2L)) \\ &+ \frac{q^2}{g_1}\frac{m^2\mathbf{v}^2X^2}{M_1M_2(M_1+M_2)}L \\ &+ \frac{q^2}{2\pi g_1}\frac{1}{(M_1^2-M_2^2)}(\nabla^2K_0(M_1L) - \nabla^2K_0(M_2L)), \end{aligned} \tag{49}$$

where $L \equiv |\mathbf{y}|$. In this last line, we have assumed that $\frac{G_1(\mathbf{x})}{\nabla^2} = \frac{|\mathbf{x}|}{4M_1}$ and $\frac{G_2(\mathbf{x})}{\nabla^2} = \frac{|\mathbf{x}|}{4M_2}$.

Concluding remarks. – The goals of this contribution were twofold:

a) to write down the LSV fermonic matter Lagrangian of fermonic equation (5) in a supersymmetric scenario along the lines followed in the series of papers cast in refs. [22,23,45], and

b) to inspect how the SUSY LSV terms worked out in $(1+3)$ dimensions may go down, by means of dimensional reduction, and affect the interparticle potential of planar electrodynamics.

We thus find that the three terms on the right-hand side of expression (49) display a screening part, encoded in the Bessel functions and their derivatives, and the linear confining potential. We readily verify that the linear potential disappears when m, X^2, or $\mathbf{v} \to 0$. Mention should be made, at this point, to the fact that the two first terms on the right-hand side of expression (49) is exactly the result obtained for $D = 3$ models of antisymmetric tensor fields that emerges from the condensation of topological defects, as a consequence of the Julia-Thoulousse mechanism [30]. This same potential profile is obtained in the case of condensation of charged scalars in $D = 3$ dimensions [31].

A point that we intend next to inspect regards the spin-orbit interaction in two-dimensional materials in the presence of SUSY and an external anisotropy as it appears in the Lagrangian of eq. (25) above. We shall soon report on this issue elsewhere.

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Appendix: conventions and notation for the dimensional reduction. – Let us start our analysis by introducing the way in which the dimensional reduction process will be implemented. Hat indexes are used in $(3+1)$-D, $\hat{\mu} = 0,1,2,3$, whereas normal indexes are used for the (2 + 1)-D case, $\mu = 0, 1, 2$. Also, we mention that the z-component of all vector fields V will be represented by a scalar field, *i.e.*, $V^{\hat{\mu}} = (V^\mu, V^3 = \zeta)$. Besides, the Dirac matrices will be rewritten in the form:

$$\Gamma^{\hat{\mu}} = \begin{pmatrix} \gamma^\mu & 0 \\ 0 & -\gamma^\mu \end{pmatrix}; \quad \hat{\mu} = 0, 1, 2 \tag{A.1}$$

$$\Gamma^{\hat{\mu}} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}; \quad \hat{\mu} = 3 \tag{A.2}$$

$$\Gamma_5 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \tag{A.3}$$

$$\Gamma_{R/L} = \frac{1}{2} \begin{pmatrix} 1 & \pm i \\ \pm i & 1 \end{pmatrix}. \tag{A.4}$$

It should be noted that is implicit the 2×2 identity matrix inside the matrices, and $\gamma^0 = \sigma_y$, $\gamma^1 = \sigma_x$, $\gamma^2 = i\sigma_z$. Let us also mention here that the Dirac spinor Ψ will be split into two components, $\Psi = (\Psi_1 \quad \Psi_2)^T$.

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