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Rational solutions and their interaction solutions for the (2+1)-dimensional dispersive long wave equation

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Abstract

The Hirota bilinear form of the (2+1)-dimensional dispersive long wave equation by the truncated painlevé series in this paper is obtained. Meanwhile, a pair of quartic-linear forms are also constructed by an appropriate selection of seed solution to explore the lump solutions of the (2+1)-dimensional dispersive long wave equation. Then some novel interaction solutions by combining quadratic functions and exponential functions are yielded. Finally, in order to better illustrate the features of the results, we draw the three-dimensional and two-dimensional figures.

Keywords: (2+1)-dimensional dispersive long wave equation, truncated painlevé series, rational solutions, interaction solutions

(Some figures may appear in colour only in the online journal)

1. Introduction

Seeking for exact solutions of NLPDEs plays an increasing significant part in the field of nonlinear science, such as dynamics, control processes, marine engineering, and so on [1–15]. And that some effective methods had been proposed to search the exact solutions of NLPDEs. For example, the Lie group method [16], the inverse scattering method [17], the Hirota bilinear method [18], etc [19–24]. In recent years, the lump solutions, the lump-type solutions and their interaction solutions have been used to explain various nonlinear phenomena, such as the rogue wave phenomena. In soliton theory, rogue waves not only have powerful and strange energy, but also generate tremendous destructive power [25]. Moreover, rogue wave is a special kind of lump solutions, it is rationally localized in all directions of the space [26]. Therefore the lump solutions catch the attention of many scholars, and that the lump solutions have been investigated in fluid, plasma, and optic media [27–32]. Especially, the Darboux transformation method and the Hirota bilinear method are two directly powerful approaches to construction of lump solutions [33–38]. In addition, the study of interaction solutions contributes to some problems for the exact solutions of NLPDEs.

In this article, we will analyze the (2+1)-dimensional dispersive long wave equation [39]

$$\begin{cases} u_{yt} + \eta_{xx} + \frac{1}{2}(u^2)_{xy} = 0, \\ \eta_t + (u\eta + u + u_{xy})_x = 0, \end{cases} \tag{1.1}$$

where u and η are the wave amplitude functions, x, y, t are independent variables. Equation (1.1) had been better studied in the last years, such as Boiti *et al* [40] constructed a compatibility by using a weak Lax pair, Tang and Lou [39] obtained plentiful related structures. Paquin and Winternitz [41] showed that the symmetry algebra of this considered equation is infinite-dimensional and possesses a Kac–Moody-Virasoro structure. Meanwhile, by using symmetry algebra and the classical theoretical analysis method, the special similarity solutions were found in [41]. Moreover, the more general symmetry algebra and ω_∞ symmetry algebra, were constructed in paper [42].

The main purpose of the paper is to obtain the abundant localized excitations by using the Painlevé-Bäcklund transformations and multi-linear variable separation methods. Besides, multiple soliton solutions and fusion interaction phenomena under the help of Bäcklund transformations method and the Hirota bilinear method are derived.

2. Lump solutions

Applying the painlevé analysis method, the Painlevé-Bäcklund transformation [43, 44] of the (2+1)-dimensional dispersive long wave equation is written as follows

$$u = \frac{u_0}{\phi} + u_1,\ \ \eta = \frac{\eta_0}{\phi^2} + \frac{\eta_1}{\phi} + \eta_2, \tag{2.1}$$

where ϕ is an arbitrary function of variable x, y and t, the functions u_1 and η_2 are solutions of equation (1.1). Substituting equation (2.1) into (1.1), then balancing the coefficient ϕ^{-4}, we derive

$$u_0 = 2\phi_x,\ \ \eta_0 = -2\phi_x\phi_y. \tag{2.2}$$

Next balancing the coefficient ϕ^{-3}, we have

$$u_1 = -\frac{\phi_t + \phi_{xx}}{\phi_x},\ \ \eta_1 = 2\phi_{xy}. \tag{2.3}$$

Besides, substituting equations (2.2) and (2.3) into the transformation (2.1) and letting the seed solution $u_1 = \eta_2 = 0$, the Hirota bilinear form of equation (1.1) can be found

$$\begin{aligned}&-2\phi_{xt}\phi_y - 2\phi_{xy}\phi_t - 2\phi_x\phi_{yt} - 2\phi_{xx}\phi_{xy}\\ &\quad -2\phi_x\phi_{xxy} - 2\phi_x^2 - 2\phi_{xxx}\phi_y = 0.\end{aligned} \tag{2.4}$$

In order to obtain the lump solutions of equation (1.1), we assume a quadratic function for ϕ

$$\begin{aligned}&\phi = g^2 + h^2 + a_9,\\ &g = a_1x + a_2y + a_3t + a_4,\\ &h = a_5x + a_6y + a_7t + a_8.\end{aligned} \tag{2.5}$$

Substituting equation (2.5) into (2.4) and collecting the different powers of x, y and t, we derive the solutions of the parameters

$$a_1 = 0,\ \ a_5 = 0,\ \ or\ a_2 = 0,\ \ a_6 = 0. \tag{2.6}$$

Therefore, through the above standard bilinear form, we fail to obtain any non-trivial lump solution.

3. Rational solutions of the (2+1)-dimensional dispersive long wave equation

In this section, we choose different seed solutions for u_1 and η_1 to obtain non-trivial quadratic function solutions. Firstly, we assume that

$$u = \frac{2\phi_x}{\phi} - \frac{\phi_t + \phi_{xx}}{\phi_x},\ \ \eta = \frac{-2\phi_x\phi_y}{\phi^2} + \frac{2\phi_{xy}}{\phi}, \tag{3.1}$$

with the seed solutions

$$u_1 = -\frac{\phi_t + \phi_{xx}}{\phi_x},\ \ \eta_2 = 0. \tag{3.2}$$

Substituting equation (3.1) into (1.1), we get the following two quartic-linear equations

$$\begin{aligned}&\phi_{xy}\phi_{tt} + 2\phi_{xy}\phi_{xxt} + 2\phi_{xt}\phi_{yt} + 4\phi_{xt}\phi_{xxy} + 2\phi_{xyt}\phi_t\\ &\quad + 4\phi_{xyt}\phi_{xx} + 2\phi_{xxx}\phi_{yt} + 4\phi_{xxx}\phi_{xxy} + 2\phi_t\phi_{xxxy}\\ &\quad + 4\phi_{xx}\phi_{xxxy} + \phi_{xx}\phi_t + \phi_{xy}\phi_{xxxx} + \phi_{xx}^2 = 0,\end{aligned} \tag{3.3}$$

$$\begin{aligned}&4\phi_{xt}\phi_{xy}\phi_t + 8\phi_{xt}\phi_{xy}\phi_{xx} + 4\phi_{xxx}\phi_{xy}\phi_t + 8\phi_{xxx}\phi_{xy}\phi_{xx}\\ &\quad + 8\phi_{xx}\phi_t\phi_{xxy} + 2\phi_{xx}\phi_t\phi_{yt} + 4\phi_{xx}\phi_{xx}\phi_{yt}\\ &\quad + 9\phi_{xx}\phi_{xx}\phi_{xxy} + \phi_t\phi_t\phi_{xxy} = 0.\end{aligned} \tag{3.4}$$

We construct a solution ϕ to satisfy equations (3.3) and (3.4) simultaneously. In order to derive the lump solutions of equation (1.1), we define a quadratic function solution for equations (3.3) and (3.4) as

$$\begin{aligned}&\phi = g^2 + h^2 + a_9,\\ &g = a_1x + a_2y + a_3t + a_4,\\ &h = a_5x + a_6y + a_7t + a_8,\end{aligned} \tag{3.5}$$

where $a_i(1 \leqslant i \leqslant 9)$ are parameters to be determined. Substituting equation (3.5) into equations (3.3) and (3.4), then collecting the coefficients of different powers of x, y, and t and setting them to zero. We can get the relationship among the parameters which satisfy equations (3.3) and (3.4) simultaneously. The parameters satisfy

$$a_i = a_i,\ \ a_2 = \frac{-a_6(a_1^2a_7 + 2a_1a_3a_5 + 3a_5^2a_7)}{3a_1^2a_3 + 2a_1a_5a_7 + a_3a_5^2}, \tag{3.6}$$

where a_i, $(i = 1, 2, 3, 4, 7, 8, 9)$ are arbitrary constants and $3a_1^2a_3 + 2a_1a_5a_7 + a_3a_5^2 \neq 0$, $a_9 > 0$ to guarantee the definition and positiveness of the resulting solutions. Substituting parameters (3.6) into equation (3.5), we can obtain the positive quadratic function solution as following

$$\begin{aligned}\phi = &\left(a_1x - \frac{a_6(a_1^2a_7 + 2a_1a_3a_5 + 3a_5^2a_7)}{3a_1^2a_3 + 2a_1a_5a_7 + a_3a_5^2}y + a_3t + a_4\right)^2\\ &+ (a_5x + a_6y + a_7t + a_8)^2 + a_9.\end{aligned} \tag{3.7}$$

Under the transformation (3.1), we obtain the rational solutions of equations (3.3) and (3.4) as follows

$$\begin{aligned}u &= \frac{4a_1g + 4a_5h}{g^2 + h^2 + a_9} - \frac{2a_3g + 2a_7h + 2a_1^2 + 2a_5^2}{2a_1g + 2a_5h},\\ \eta &= \frac{(2a_1g + 2a_5h)\left(\frac{-2a_6(a_1^2a_7 + 2a_1a_3a_5 + 3a_5^2a_7)g}{3a_1^2a_3 + 2a_1a_5a_7 + a_3a_5^2} + 2a_6h\right)}{(g^2 + h^2 + a_9)^2}\\ &\quad + \frac{-\frac{2a_1a_6(a_1^2a_7 + 2a_1a_3a_5 + 3a_5^2a_7)}{3a_1^2a_3 + 2a_1a_5a_7 + a_3a_5^2} + 2a_5a_6}{g^2 + h^2 + a_9},\end{aligned} \tag{3.8}$$

where $g = a_1x - \frac{a_6(a_1^2a_7 + 2a_1a_3a_5 + 3a_5^2a_7)}{3a_1^2a_3 + 2a_1a_5a_7 + a_3a_5^2}y + a_3t + a_4$, and $h = a_5x + a_6y + a_7t + a_8$. In order to better describe this type of rational solutions, we take the parameters $a_1 = 3$, $a_3 = 2$, $a_4 = 1$, $a_5 = 3$, $a_6 = 1$, $a_7 = 1$, $a_8 = 2$, $a_9 = 2$. The following 3D-, 2D- plots in figures 1 and 2 are presented to illustrate the solutions (3.8). This type of solution u is

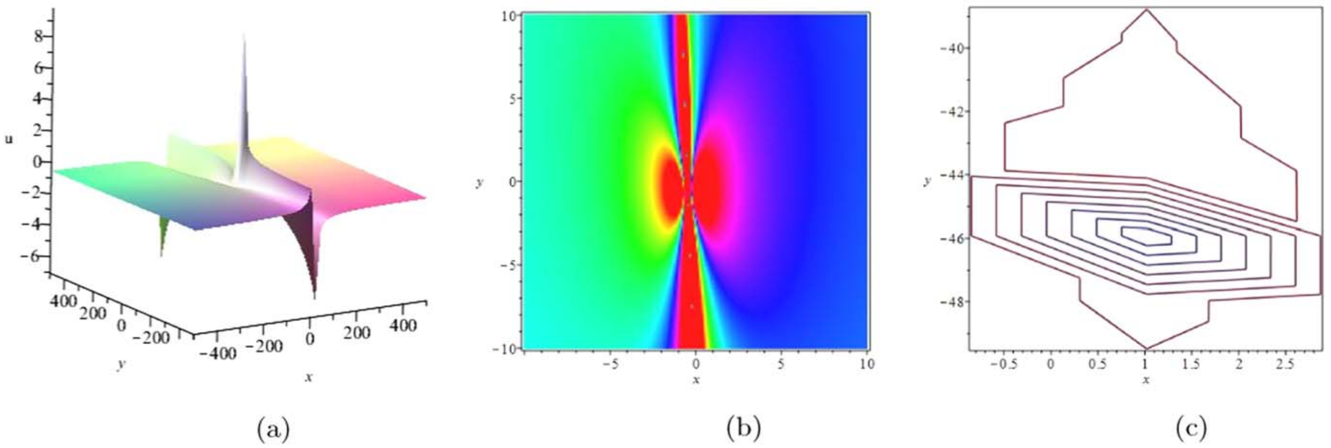

Figure 1. Three-dimensional diagram (a), two-dimensional density plot (b) and two-dimensional contour plot (c) of solution u with $a_1 = 3$, $a_3 = 2$, $a_4 = 1$, $a_5 = 3$, $a_6 = 1$, $a_7 = 1$, $a_8 = 2$, $a_9 = 2$ when $t = 0$ in the (x, y) plane.

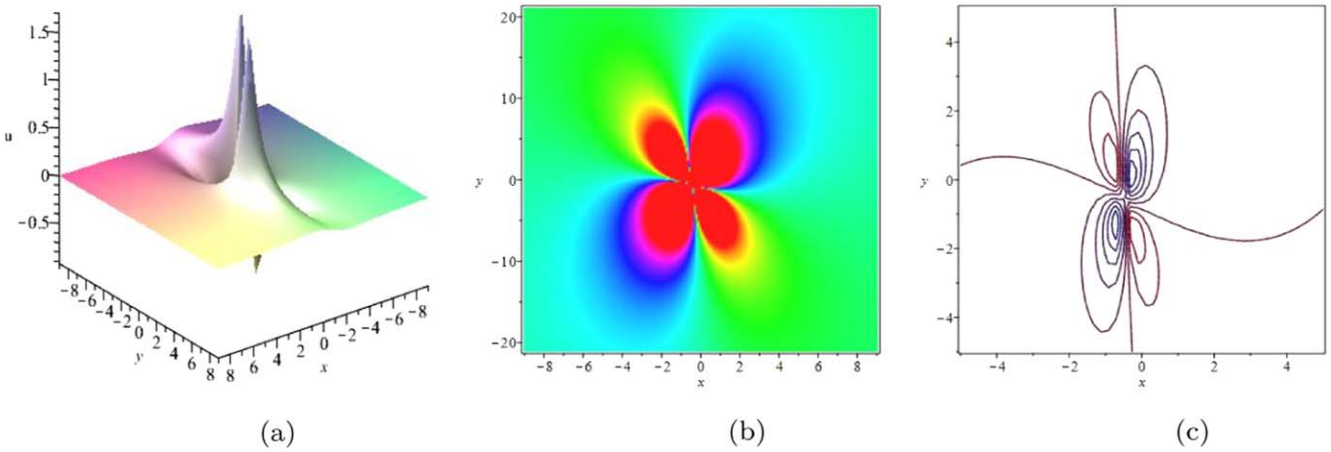

Figure 2. Three-dimensional diagram (a), two-dimensional density plots (b) and two-dimensional contour plot (c) of solution η with $a_1 = 3$, $a_3 = 2$, $a_4 = 1$, $a_5 = 3$, $a_6 = 1$, $a_7 = 1$, $a_8 = 2$, $a_9 = 2$ when $t = 0$ in the (x, y) plane.

different from general lump solutions, the traits of these rational solutions distinguish ones of lumps in the scheme of the second section [45–47].

4. Interaction between lumps and solitons

4.1. Between lumps and one line-soliton

In this sub-section, via the combine of the quadratic function and other type functions, we construct interaction solutions between lumps and several types of solitons. To obtain interaction solutions between lumps and one line-soliton, we presume an interaction solution as a sum of a quadratic function and an exponential function

$$\begin{aligned} \phi &= g^2 + h^2 + a_9 + k_1 \\ &\quad \times \exp(k_2 x + k_3 y + k_4 t + k_5), \\ g &= a_1 x + a_2 y + a_3 t + a_4, \\ h &= a_5 x + a_6 y + a_7 t + a_8, \end{aligned} \tag{4.1}$$

where k_i $(i = 1, 2, \cdots, 5)$ are five undetermined real parameters. Substituting equation (4.1) into equations (3.3) and (3.4) and collecting the same power terms of x, y and t, then we obtain the following three classes parameters values

Case I.

$$a_i = a_i,\ a_6 = -\frac{a_1 a_2}{a_5},\ a_7 = \frac{a_3 a_5}{a_1},\ k_2 = 0,\ k_4 = 0, \tag{4.2}$$

where a_i, $(i = 1, 2, 3, 4, 5, 8, 9)$ are arbitrary constants, and $a_1 a_5 \neq 0$, $a_9 > 0$, $k_1 > 0$ to guarantee that the corresponding solution ϕ is positive, analytical and localization in all directions in the (x, y)-plane. Then substituting equation (4.1) into equations (3.1) and (4.2), we get a type of interaction solutions of equation (1.1)

$$u = \frac{4a_1 g + 4a_5 h}{g^2 + h^2 + a_9 + k_1 e^f} - \frac{2a_3 g + \frac{2a_3 a_5 g}{a_1} + 2a_1^2 + 2a_5^2}{2a_1 g + 2a_5 h}, \tag{4.3}$$

$$\eta = \frac{-(4a_1 g + 4a_5 h)(2a_2 g - \frac{2a_1 a_2 h}{a_5} + k_1 k_3 e^f)}{(g^2 + h^2 + a_9 + k_1 e^f)^2}, \tag{4.4}$$

where

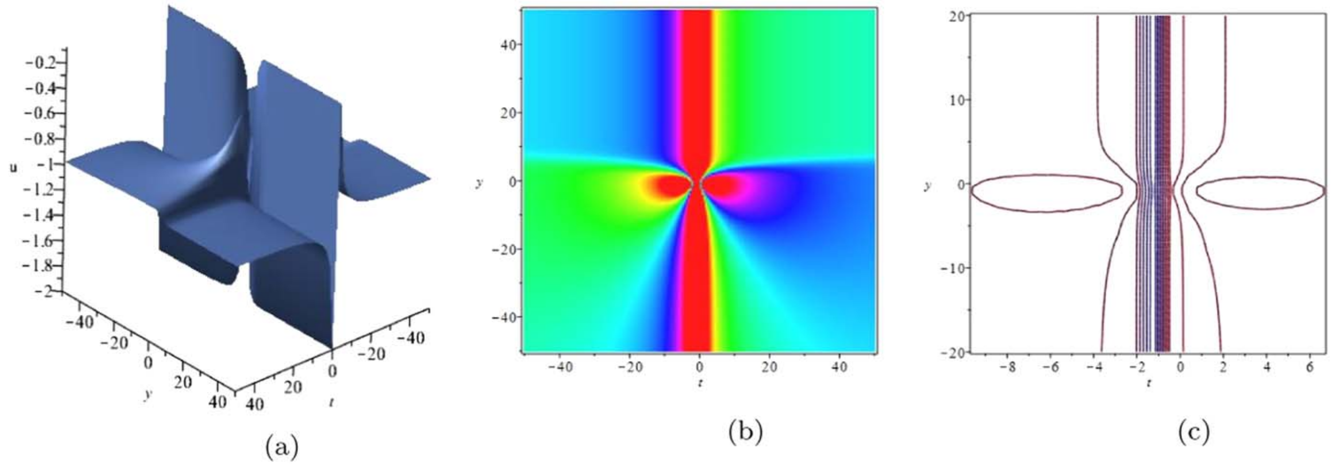

Figure 3. Three-dimensional diagram (a), two-dimensional density plot (b) and two-dimensional contour plot (c) of solution u with $a_1 = 1$, $a_2 = 1$, $a_3 = 1$, $a_4 = 1$, $a_5 = 1$, $a_8 = 1$, $a_9 = 3$, $k_1 = 1$, $k_3 = 1$, $k_5 = 2$ when $t = 0$ in the (x, y)-plane.

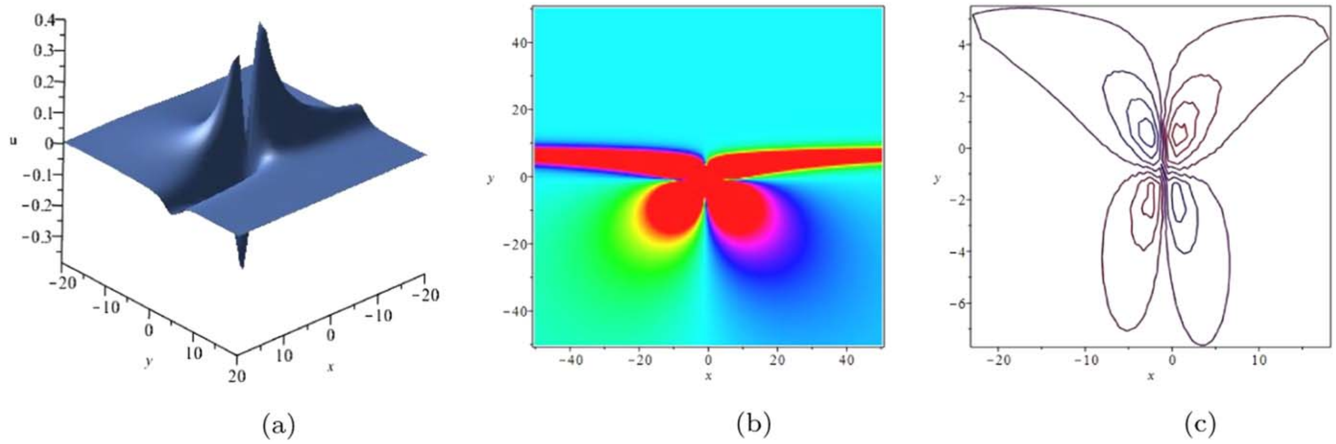

Figure 4. Three-dimensional diagram (a), two-dimensional density plot (b) and two-dimensional contour plot (c) of solution η with $a_1 = 1$, $a_2 = 1$, $a_3 = 1$, $a_4 = 1$, $a_5 = 1$, $a_8 = 1$, $a_9 = 3$, $k_1 = 1$, $k_3 = 1$, $k_5 = 2$ when $t = 0$ in the (x, y)-plane.

$$\begin{aligned} \phi &= g^2 + h^2 + a_9 + k_1 \exp(f),\\ g &= a_1 x + a_2 y + a_3 t + a_4,\\ h &= a_5 x - \frac{a_1 a_2}{a_5} y + \frac{a_3 a_5}{a_1} t + a_8,\\ f &= k_3 y + k_5. \end{aligned} \tag{4.5}$$

The dynamical characters of solutions (4.3) and (4.4) are illustrated by figures 3, 4.

Case II.

$$\begin{aligned} &a_i = a_i,\ \ a_2 = -\frac{a_6(a_1^2 a_7 + 2a_1 a_3 a_5 + 3a_5^2 a_7)}{3a_1^2 a_3 + 2a_1 a_5 a_7 + a_3 a_5^2},\\ &k_2 = 0,\ \ k_4 = 0, \end{aligned} \tag{4.6}$$

where a_i, $(i = 1, 3, 4, 5, 6, 7, 8, 9)$ are arbitrary constants, according to analytical method of the Case I, we can get $3a_1^2 a_3 + 2a_1 a_5 a_7 + a_3 a_5^2 \neq 0$, $a_9 > 0$, $k_1 > 0$. Substituting equation (4.1) into equations (4.6) and (3.1), we obtain another type of interaction solutions between lumps and one line-soliton of equation (1.1)

$$u = \frac{4a_1 g + 4a_5 h}{g^2 + h^2 + a_9 + k_1 e^f} - \frac{2a_3 g + 2a_7 h + 2a_1^2 + 2a_5^2}{2a_1 g + 2a_5 h}, \tag{4.7}$$

$$\begin{aligned} \eta = &\frac{-\left((4a_1 g + 4a_5 h)\left(\frac{-2a_6(a_1^2 a_7 + 2a_1 a_3 a_5 + 3a_5^2 a_7) g}{3a_1^2 a_3 + 2a_1 a_5 a_7 + a_3 a_5^2} + 2a_6 h + k_1 k_3 e^f\right)\right)}{(g^2 + h^2 + a_9 + k_1 e^f)^2}\\ &+ \frac{2\left(\frac{-2a_1 a_6(a_1^2 a_7 + 2a_1 a_3 a_5 + 3a_5^2 a_7)}{3a_1^2 a_3 + 2a_1 a_5 a_7 + a_3 a_5^2} + 2a_5 a_6\right)}{g^2 + h^2 + a_9 + k_1 e^f}, \end{aligned} \tag{4.8}$$

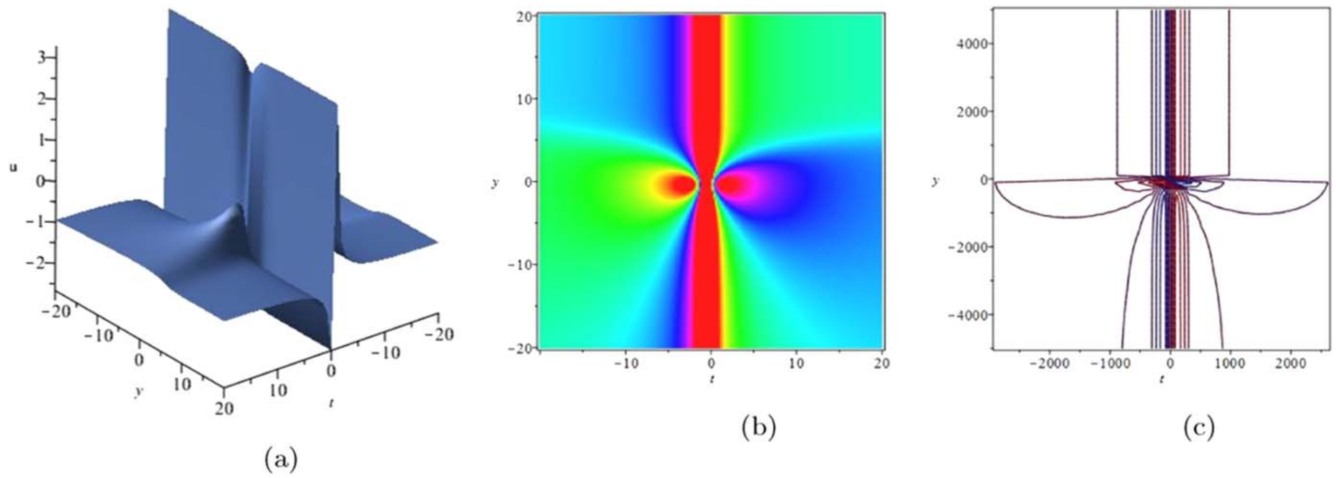

Figure 5. Three-dimensional diagram (a), two-dimensional density plot (b) and two-dimensional contour plot (c) of solution u with $a_1 = 1$, $a_3 = 1$, $a_4 = 1$, $a_5 = 2$, $a_6 = 1$, $a_7 = 2$, $a_8 = 1$, $a_9 = 3$, $k_1 = 1$, $k_3 = 1$, $k_5 = 2$, when $t = 0$ in the (x, y)-plane.

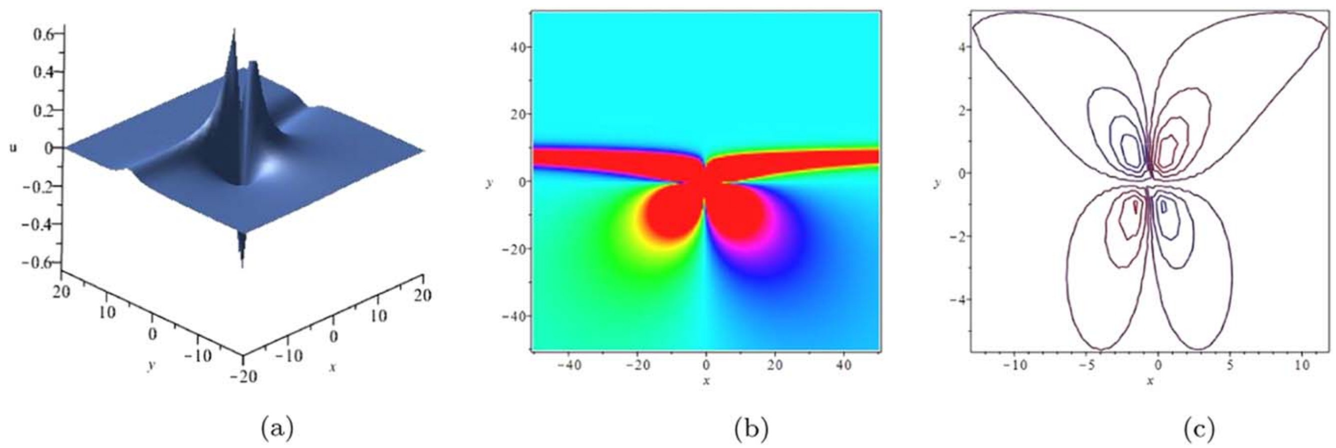

Figure 6. Three-dimensional diagram (a), two-dimensional density plot (b) and two-dimensional contour plot (c) of solution η with $a_1 = 1$, $a_3 = 1$, $a_4 = 1$, $a_5 = 2$, $a_6 = 1$, $a_7 = 2$, $a_8 = 1$, $a_9 = 3$, $k_1 = 1$, $k_3 = 1$, $k_5 = 2$, when $t = 0$ in the (x, y)-plane.

where

$$\begin{aligned}
\phi &= g^2 + h^2 + a_9 + k_1 \exp(f),\\
g &= a_1 x - \frac{a_6(a_1^2 a_7 + 2a_1 a_3 a_5 + 3a_5^2 a_7)y}{3a_1^2 a_3} + a_3 t + a_4,\\
h &= a_5 x + a_6 y + a_7 t + a_8,\\
f &= k_3 y + k_5.
\end{aligned} \tag{4.9}$$

We can derive dynamical characters of solutions (4.7) and (4.8) by figures 5, 6.

Case III.

$$\begin{aligned}
&a_i = a_i,\ a_6 = 0,\ a_7 = -\frac{a_3(3a_1^2 + a_5^2)}{2a_1 a_5},\\
&k_2 = 0,\ k_4 = 0,
\end{aligned} \tag{4.10}$$

where a_i, $(i = 1, 2, 3, 4, 5, 8, 9)$ are arbitrary constants, according to similar analysis of the Case I, we have $a_1 a_5 \neq 0$, $a_9 > 0$, $k_1 > 0$. Substituting equation (4.1) into equations (4.10) and (3.1), we obtain the third type of interaction solutions between lumps and one line-soliton of equation (1.1)

$$\begin{aligned}
u &= \frac{4a_1 g + 4a_5 h}{g^2 + h^2 + a_9 + k_1 e^f}\\
&\quad - \frac{2a_3 g - \frac{a_3(3a_1^2 + a_5^2)h}{a_1 a_5} + 2a_1^2 + 2a_5^2}{2a_1 g + 2a_5 h},
\end{aligned} \tag{4.11}$$

$$\begin{aligned}
\eta &= \frac{-(4a_1 g + 4a_5 h)(2a_2 g + k_1 k_3 e^f)}{(g^2 + h^2 + a_9 + k_1 e^f)^2}\\
&\quad + \frac{4a_1 a_2}{g^2 + h^2 + a_9 + k_1 e^f},
\end{aligned} \tag{4.12}$$

where

$$\begin{aligned}
\phi &= g^2 + h^2 + a_9 + k_1 \exp(f),\\
g &= a_1 x + a_2 y + a_3 t + a_4,\\
h &= a_5 x - \frac{a_3(3a_1^2 + a_5^2)t}{a_1 a_5} + a_8,\\
f &= k_3 y + k_5.
\end{aligned} \tag{4.13}$$

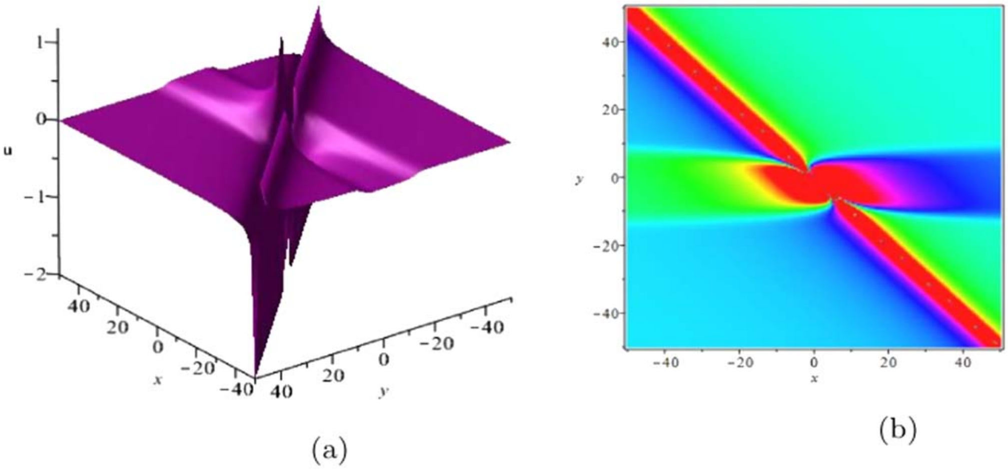

Figure 7. Three-dimensional diagram (a) and two-dimensional density plot (b) of solution u with $a_1 = 1$, $a_2 = 1$, $a_4 = 1$, $a_5 = 3$, $a_7 = 1$, $a_8 = 1$, $a_9 = 2$, $k_1 = 1$, $k_3 = 1$, $k_5 = 2$, $k_6 = 1$. when $t = 0$, in the (x, y)-plane.

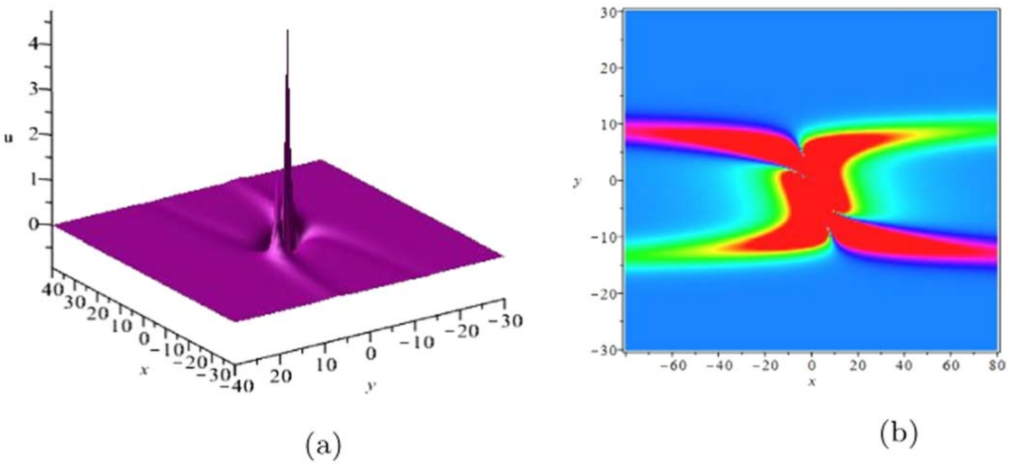

Figure 8. Three-dimensional diagram (a) and two-dimensional density plot (b) of solution η with $a_1 = 1$, $a_2 = 1$, $a_4 = 1$, $a_5 = 3$, $a_7 = 1$, $a_8 = 1$, $a_9 = 2$, $k_1 = 1$, $k_3 = 1$, $k_5 = 2$, $k_6 = 1$. when $t = 0$, in the (x, y)-plane.

Through the above analysis, we know that the dynamics of interaction solutions are partly different with different parameters.

4.2. Between lumps and a pair of line solitons

In this sub-section, we apply a quadratic function with two exponential functions to construct interaction solutions between lumps and a two-stripe solitary

$$\begin{aligned} &\phi = g^2 + h^2 + a_9 + k_1 \exp(f) + k_6 \exp(-f),\\ &g = a_1 x + a_2 y + a_3 t + a_4,\\ &h = a_5 x + a_6 y + a_7 t + a_8,\\ &f = k_2 x + k_3 y + k_4 t + k_5. \end{aligned} \tag{4.14}$$

Substituting equation (4.14) into equations (3.3) and (3.4) collecting the coefficients of x, y, t. Then letting each terms to zero, we get a type of solution of the parameters

$$\begin{aligned} &a_i = a_i,\ \ a_3 = -\frac{a_5 a_7}{a_1},\ \ a_6 = \frac{a_2 a_5}{a_1},\\ &k_2 = 0,\ \ k_4 = 0, \end{aligned} \tag{4.15}$$

where a_i, $(i = 1, 2, 4, 5, 7, 8, 9)$ are arbitrary constants, similar to the above, $a_1 \neq 0$, $a_9 > 0$. Substituting equations (4.15) and (4.14) into (3.1), we can obtain the interaction solutions

$$u = \frac{4a_1 g + 4a_5 h}{g^2 + h^2 + a_9 + k_1 e^f + k_6 e^{-f}} - \frac{\frac{-2a_5 a_7 g}{a_1} + 2a_7 h + 2a_1^2 + 2a_5^2}{2a_1 g + 2a_5 h}, \tag{4.16}$$

$$\eta = \frac{-(4a_1 g + 4a_5 h)(2a_2 g + \frac{2a_2 a_5 h}{a_1} + k_1 k_3 e^f - k_3 k_6 e^{-f})}{(g^2 + h^2 + a_9 + k_1 e^f + k_6 e^{-f})^2} + \frac{4a_1 a_2 + \frac{4a_2 a_5^2}{a_1}}{g^2 + h^2 + a_9 + k_1 e^f + k_6 e^{-f}}, \tag{4.17}$$

where

$$\begin{aligned}
&\phi = g^2 + h^2 + a_9 + k_1 \exp(f) + k_6 \exp(-f),\\
&g = a_1 x + a_2 y - \frac{a_5 a_7 t}{a_1} + a_4,\\
&h = a_5 x + \frac{a_2 a_5 y}{a_1} + a_7 t + a_8,\\
&f = k_3 y + k_5.
\end{aligned} \tag{4.18}$$

The characters of solutions (4.7) and (4.8) are presented by figures 7, 8.

5. Conclusions

The main work in this paper is to investigate the (2+1)-dimensional dispersive long wave equation with the help of the truncated painlevé series idea. As a result, rational solutions and their interaction solutions, which combine the lumps and solitons with quadratic function and the exponential functions, are yielded. They can be used to express more rich phenomena appear in fluid or plasma mechanics by 3D and 2D plots in detail. For the rational solutions (3.8), we can see that u, η possess different evolutionary form under the same parameters, the solution u is singular twins mainly, however, the solution η is double twins from figures 1, 2. As for the solutions (4.3) and (4.4), we can find out that the evolutions of u, η are basically similar, u is smoother than η merely from figures 3, 4. Moreover, we can see that the interaction solutions (4.16) and (4.17) show both localities of the lumps and non-locality of the a pair of line solitons from figures 7, 8. In addition, we can also investigate lumps and their interaction solutions by solving different multi-linear forms and that we will study more interactions of lumps in shorting time.

Acknowledgments

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