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Quantum coherence of multipartite W-state in a Schwarzschild spacetime

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Abstract – We study the quantum coherence and monogamy relationship of a tripartite W-state entangled system for Dirac fields in the background of a Schwarzschild black hole. We find that quantum coherence first decreases and then shows the phenomenon of freezing with the growth of the Hawking temperature. We also find that the l_1 norm of coherence is always equal to the sum of coherence of all bipartite systems for any Hawking temperature, while a similar monogamy relationship for the relative entropy of coherence is absent. Moreover, we extend the related investigations to the N-partite W-state systems. It is shown that a similar monogamy relationship for the l_1 norm of coherence is still satisfied, and the phenomenon of coherence freezing also exists.

Introduction. – It is well known that relativity theory and quantum mechanics are two fundamentals of modern physics. In order to solve the contradiction between them, quantum field theory (QFT) is born. In QFT, a very important prediction is the effect of Hawking radiation (Unruh effect) which tells us that a vacuum state observed by an observer who stays in flat Minkowski spacetime would be detected as a thermal state from another observer who hovers near the event horizon of the black hole with uniform acceleration [1,2]. Recently, quantum steering, entanglement and discord in curved spacetime have been studied extensively, and the results showed that they are reduced due to the loss of information caused by Hawking radiation [3–15]. It is obvious that these results not only help us understand the key of quantum information, but also play an important role in the study of the information paradox and entanglement entropy of black holes [16,17].

On the other hand, quantum coherence that derives from the quantum superposition principle is a fundamental aspect of quantum physics [18]. Quantum coherence is a common necessary condition for quantum correlations of multiple systems, but can exist in a single quantum system. Analogous to quantum steering, entanglement and discord, quantum coherence is also an important quantum resource in life sciences, quantum information, condensed matter physics and computation processing [19–25]. Nevertheless, the quantification of quantum coherence was missing until recently Baumgratz, Cramer and Plenio introduced two comprehensive measures, *i.e.*, the l_1 norm of coherence and the relative entropy of coherence (REC) [26]. Mathematically, for any quantum state ρ, the l_1 norm of coherence and the REC are defined as

$$C_{l_1}(\rho) = \sum_{i\neq j} |\rho_{ij}|, \tag{1}$$

and

$$C_{\mathrm{RE}}(\rho) = S(\rho_{\mathrm{diag}}) - S(\rho), \tag{2}$$

respectively. In eq. (2), $S(\rho)$ denotes the von Neumann entropy of quantum state ρ, and ρ_{diag} denotes the state obtained from ρ by deleting all off-diagonal elements.

Recently, tripartite quantum entanglement of scalar field in non-inertial frame [8] and tripartite quantum entanglement of Dirac field in curved spacetime [9–12] have been investigated extensively. However, few work involves the multipartite coherence in the relativistic

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framework. Also, most attention is focused on the multipartite Greenberger-Horne-Zeilinger (GHZ) state under the Hawking radiation or Unruh effect [9–11,27], and multipartite W-state is rarely studied because of the complexity of the calculations. Motivated by these facts, we here study the quantum coherence of multipartite W-state for Dirac fields in the background of a Schwarzschild black hole. We firstly focus our attention on the tripartite W-state systems of Dirac fields, and then extend the issues to N-partite systems. As the l_1 norm of coherence and the REC are the two typical measures for quantum coherence in the framework of resource theory, and the two measures are not exactly compatible [28], we thus combine them together for a comparative research.

The paper is organized as follows. In the next section, we study quantum coherence and the monogamy for a tripartite W-state when one observer hovers near the event horizon of the black hole. In the third section, we do the same research when two observers hover near the event horizon of the black hole. In the fourth section, we extend related issues to N-partite systems. The last section is devoted to a brief conclusion. For simplicity, we use the natural system of units $\hbar = G = c = k_B = 1$ throughout the paper.

Coherence for one observer near black hole. – The line element of the Schwarzschild black hole is given by

$$\begin{aligned} ds^2 = & -\left(1-\frac{2M}{r}\right)dt^2+\left(1-\frac{2M}{r}\right)^{-1}dr^2 \\ & +r^2(d\theta^2+\sin^2\theta d\varphi^2), \end{aligned} \tag{3}$$

where M denotes the mass of the black hole. Solving the Dirac equation near the event horizon, we can obtain a set of positive frequency outgoing solutions for the inside and outside regions of the event horizon [11,29]

$$\psi^+_{k,\mathrm{in}} \sim \mathcal{H}e^{i\omega u}, \tag{4}$$

$$\psi^+_{k,\mathrm{out}} \sim \mathcal{H}e^{-i\omega u}, \tag{5}$$

where $\mathcal{H}$ is a four-component Dirac spinor, ω is a monochromatic frequency, $u = t - r_*$ with $r_* = r + 2M \ln\frac{r-2M}{2M}$ being the tortoise coordinate. Making an analytic continuation for eqs. (4) and (5) according to Damour and Ruffini's suggestion, we get a complete basis of positive energy modes, *i.e.*, the Kruskal modes [30]. Expanding the Dirac field in terms of Schwarzschild and Kruskal modes, respectively, the Bogoliubov transformations for the creation and annihilation operators between the two modes [31] are established. After properly normalizing the state vector, in the single-mode approximation, the vacuum state and the excited state of the Kruskal particle can be expressed as

$$\begin{aligned} |0\rangle_K &= (e^{-\frac{\omega}{T}}+1)^{-\frac{1}{2}}|0\rangle_I|0\rangle_{II}+(e^{\frac{\omega}{T}}+1)^{-\frac{1}{2}}|1\rangle_I|1\rangle_{II}, \\ |1\rangle_K &= |1\rangle_I|0\rangle_{II}, \end{aligned} \tag{6}$$

where $T = \frac{1}{8M}$ is the Hawking temperature [32,33], $\{|n\rangle_I\}$ and $\{|n\rangle_{II}\}$ are the orthogonal bases for the outside region and inside region of the event horizon, respectively.

Quantum coherence. We assume that Alice, Bob and Charlie share initially a tripartite W-state entangled system defined in flat Minkowski spacetime,

$$|W\rangle_{ABC} = \frac{1}{\sqrt{3}}[|0_A0_B1_C\rangle + |0_A1_B0_C\rangle + |1_A0_B0_C\rangle]. \tag{7}$$

Letting Alice and Bob stay in flat Minkowski spacetime, while Charlie hovers near the event horizon of the black hole with uniform acceleration, then eq. (7) can be rewritten as

$$\begin{aligned} |\Phi\rangle_{ABC_IC_{II}} = & \frac{1}{\sqrt{3}}[\mathcal{S}|0_A1_B1_{C_I}1_{C_{II}}\rangle + \mathcal{S}|1_A0_B1_{C_I}1_{C_{II}}\rangle \\ & +\mathcal{C}|0_A1_B0_{C_I}0_{C_{II}}\rangle + \mathcal{C}|1_A0_B0_{C_I}0_{C_{II}}\rangle \\ & +|0_A0_B1_{C_I}0_{C_{II}}\rangle], \end{aligned} \tag{8}$$

with $\mathcal{C} = (e^{-\frac{\omega}{T}}+1)^{-\frac{1}{2}}$ and $\mathcal{S} = (e^{\frac{\omega}{T}}+1)^{-\frac{1}{2}}$. Note that $\mathcal{C}^2 + \mathcal{S}^2 = 1$, which is used later in the simplification of some expressions. Because the exterior region is causally disconnected from the interior region of the black hole, we obtain the reduced density matrix

$$\rho_{ABC_I} = \frac{1}{3}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \mathcal{C} & 0 & \mathcal{C} & 0 & 0 & 0 \\ 0 & \mathcal{C} & \mathcal{C}^2 & 0 & \mathcal{C}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{S}^2 & 0 & \mathcal{S}^2 & 0 & 0 \\ 0 & \mathcal{C} & \mathcal{C}^2 & 0 & \mathcal{C}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{S}^2 & 0 & \mathcal{S}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{9}$$

by taking the trace over the state of the interior region. According to eqs. (1) and (2), we can obtain the l_1 norm of coherence and the REC as

$$C_{l_1}(\rho_{ABC_I}) = \frac{1}{3}(4\mathcal{C}+2), \tag{10}$$

and

$$\begin{aligned} C_{RE}(\rho_{ABC_I}) = & \frac{2}{3}\mathcal{S}^2 + \frac{1}{3}(1+2\mathcal{C}^2)\log_2\frac{1}{3}(1+2\mathcal{C}^2) \\ & -\frac{2}{3}\mathcal{C}^2\log_2\frac{1}{3}\mathcal{C}^2 - \frac{1}{3}\log_2\frac{1}{3}, \end{aligned} \tag{11}$$

respectively. These measures of coherence obviously depend on the Hawking temperature T.

In fig. 1, we plot the l_1 norm of coherence $C_{l_1}(\rho_{ABC_I})$ and REC $C_{RE}(\rho_{ABC_I})$ as functions of the Hawking temperature T. It is shown that quantum coherence decreases with the growth of the Hawking temperature, meaning that the thermal noise introduced by Hawking radiation reduces the quantum coherence. Interestingly, when $T \to \infty$, the coherence has nonzero asymptotical values $C_{l_1}(\rho_{ABC_I}) = \frac{1}{3}(2\sqrt{2}+2)$ and $C_{RE}(\rho_{ABC_I}) = \frac{4}{3}$. We call this phenomenon freezing of quantum coherence.

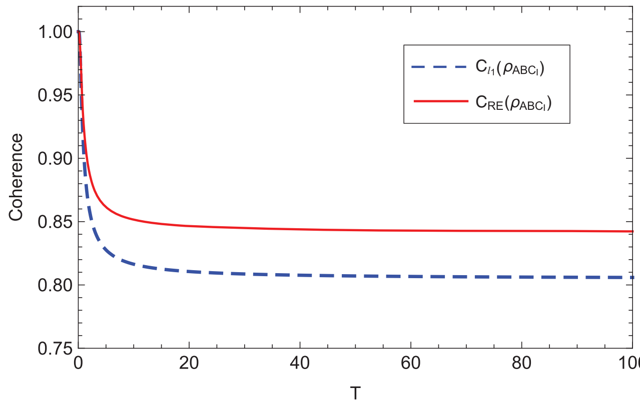

Fig. 1: The l_1 norm of coherence (blue dashed line) and REC (red solid line) for ρ_{ABC_I} as a function of the Hawking temperature T for fixed value $\omega = 1$. We normalize the maximum quantum coherence to **1**.

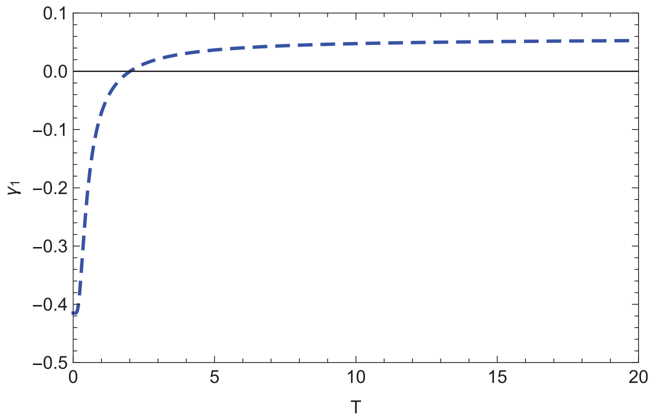

Fig. 2: The monogamy γ_1 as a function of the Hawking temperature T for fixed value $\omega = 1$.

Monogamy relation. Next, we study the distribution of quantum coherence of the tripartite W-state in curved spacetime. The relation between the total quantum coherence of a compound system and the coherence of all its subsystems is usually called monogamy. After tracing over the modes A, B or C_I from ρ_{ABC_I} respectively, we can obtain the bipartite reduced density matrices as

$$\rho_{AB} = \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{12}$$

$$\rho_{AC_I} = \rho_{BC_I} = \frac{1}{3}\begin{pmatrix} \mathcal{C}^2 & 0 & 0 & 0 \\ 0 & 1+\mathcal{S}^2 & \mathcal{C} & 0 \\ 0 & \mathcal{C} & \mathcal{C}^2 & 0 \\ 0 & 0 & 0 & \mathcal{S}^2 \end{pmatrix}. \tag{13}$$

The corresponding quantum coherences are given by

$$C_{l_1}(\rho_{AB}) = C_{RE}(\rho_{AB}) = \frac{2}{3}, \tag{14}$$

$$C_{l_1}(\rho_{AC_I}) = C_{l_1}(\rho_{BC_I}) = \frac{2}{3}\mathcal{C}, \tag{15}$$

$$\begin{aligned} C_{RE}(\rho_{AC_I}) &= C_{RE}(\rho_{BC_I}) \\ &= \sum_{i=1}^{4} \lambda_i(\rho_{AC_I}) \log_2 \lambda_i(\rho_{AC_I}) \\ &\quad - \sum_j \beta_j(\rho_{AC_I}) \log_2 \beta_j(\rho_{AC_I}), \end{aligned} \tag{16}$$

where $\lambda_i(\rho_{AC_I})$ are the eigenvalues of density matrix ρ_{AC_I}

$$\begin{aligned} \lambda_1(\rho_{AC_I}) &= \frac{1}{3}\mathcal{C}^2, \\ \lambda_2(\rho_{AC_I}) &= \frac{1}{3}\mathcal{S}^2, \\ \lambda_3(\rho_{AC_I}) &= \frac{1}{3}(1 - \sqrt{1-\mathcal{C}^2\mathcal{S}^2}), \\ \lambda_4(\rho_{AC_I}) &= \frac{1}{3}(1 + \sqrt{1-\mathcal{C}^2\mathcal{S}^2}), \end{aligned} \tag{17}$$

and $\beta_j(\rho_{AC_I})$ is the diagonal elements of ρ_{AC_I}.

For the l_1 norm of coherence, an interesting monogamy relation,

$$C_{l_1}(\rho_{ABC_I}) = C_{l_1}(\rho_{AB}) + C_{l_1}(\rho_{AC_I}) + C_{l_1}(\rho_{BC_I}), \tag{18}$$

is found, which says that the total l_1 norm of coherence is always equal to the sum of l_1 norm of coherence of all the bipartite systems for any Hawking temperature T. This monogamy relation describes in some sense the distribution of quantum coherence for the W-state in the curved spacetime.

For the relative entropy coherence, however, it is not so simple. For convenience of study, we define REC monogamy

$$\gamma_1 = C_{RE}(\rho_{ABC_I}) - C_{RE}(\rho_{AB}) - C_{RE}(\rho_{AC_I}) - C_{RE}(\rho_{BC_I}). \tag{19}$$

Figure 2 shows how the Hawking temperature T influences the monogamy γ_1. We find that γ_1 changes from negative to positive with the growth of the Hawking temperature T. For T smaller than a certain value, the negative γ_1 means that the total REC is smaller than the sum of REC of all bipartite systems; For T bigger than the certain value, the total REC is bigger than the sum of REC of all bipartite systems. This result suggests that the Hawking radiation can change the distribution of REC —from negative monogamy in the flat Minkowski spacetime to the positive monogamy in the curved spacetime. Note that the different monogamy relations for l_1 norm of coherence and REC reveal the difference between the two measures of coherence, and the different evolutions of monogamy *vs.* temperature reveal the different affections that the Hawking radiation imposes on the two measures.

Coherence for two observers near black hole. – Now, we continue to study the effect of Hawking radiation on quantum coherence and monogamy relation of the tripartite W-state system in another case where two observers hover near the event horizon of the black hole. The method is similar, but the calculation is more complicated.

Quantum coherence. We assume that both Bob and Charlie hover near the event horizon of the black hole with uniform acceleration and Alice stays in flat Minkowski spacetime. Using eq. (6), we obtain

$$\begin{aligned}
&|\Phi\rangle_{AB_IB_{II}C_IC_{II}} = \frac{1}{\sqrt{3}}[\mathcal{C}|0_A0_{B_I}0_{B_{II}}1_{C_I}0_{C_{II}}\rangle \\
&+\mathcal{S}|0_A1_{B_I}1_{B_{II}}1_{C_I}0_{C_{II}}\rangle + \mathcal{C}|0_A1_{B_I}0_{B_{II}}0_{C_I}0_{C_{II}}\rangle \\
&+\mathcal{S}|0_A1_{B_I}0_{B_{II}}1_{C_I}1_{C_{II}}\rangle + \mathcal{C}^2|1_A0_{B_I}0_{B_{II}}0_{C_I}0_{C_{II}}\rangle \\
&+\mathcal{CS}|1_A0_{B_I}0_{B_{II}}1_{C_I}1_{C_{II}}\rangle + \mathcal{CS}|1_A1_{B_I}1_{B_{II}}0_{C_I}0_{C_{II}}\rangle \\
&+\mathcal{S}^2|1_A1_{B_I}1_{B_{II}}1_{C_I}1_{C_{II}}\rangle]. \qquad (20)
\end{aligned}$$

After tracing over the inaccessible modes B_{II} and C_{II}, the reduced density matrix reads

$$\rho_{AB_IC_I} = \frac{1}{3}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{C}^2 & \mathcal{C}^2 & 0 & \mathcal{C}^3 & 0 & 0 & 0 \\ 0 & \mathcal{C}^2 & \mathcal{C}^2 & 0 & \mathcal{C}^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\mathcal{S}^2 & 0 & \mathcal{CS}^2 & \mathcal{CS}^2 & 0 \\ 0 & \mathcal{C}^3 & \mathcal{C}^3 & 0 & \mathcal{C}^4 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{CS}^2 & 0 & \mathcal{C}^2\mathcal{S}^2 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{CS}^2 & 0 & 0 & \mathcal{C}^2\mathcal{S}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{S}^4 \end{pmatrix}.$$

According to the measures of coherence, we obtain the l_1 norm of coherence and the REC of this state,

$$C_{l_1}(\rho_{AB_IC_I}) = \frac{2}{3}(2\mathcal{C}+\mathcal{C}^2), \qquad (21)$$

$$\begin{aligned}
C_{RE}(\rho_{AB_IC_I}) &= \sum_{i=1}^{4}\lambda_i(\rho_{AB_IC_I})\log_2\lambda_i(\rho_{AB_IC_I}) \\
&\quad -\sum_j \beta_j(\rho_{AB_IC_I})\log_2\beta_j(\rho_{AB_IC_I}), \qquad (22)
\end{aligned}$$

where $\lambda_i(\rho_{AB_IC_I})$ are the nonzero eigenvalues

$$\begin{aligned}
\lambda_1(\rho_{AB_IC_I}) &= \frac{1}{3}\mathcal{S}^4, \\
\lambda_2(\rho_{AB_IC_I}) &= \frac{1}{3}\mathcal{C}^2\mathcal{S}^2, \\
\lambda_3(\rho_{AB_IC_I}) &= \frac{1}{3}\mathcal{S}^2(2+\mathcal{C}^2), \\
\lambda_4(\rho_{AB_IC_I}) &= \frac{1}{3}\mathcal{C}^2(3-\mathcal{S}^2),
\end{aligned} \qquad (23)$$

and $\beta_j(\rho_{AB_IC_I})$ is the diagonal elements of $\rho_{AB_IC_I}$.

In fig. 3, we plot the l_1 norm of coherence $C_{l_1}(\rho_{AB_IC_I})$ and REC $C_{RE}(\rho_{AB_IC_I})$ as functions of Hawking temperature T. We find that fig. 3 and fig. 1 are very similar, but they have two visible difference: One is that the coherence in fig. 3 reduces faster than fig. 1, because now there are more observers (both Bob and Charlie) suffering from Hawking radiation. The other difference is that the decaying speeds of $C_{l_1}(\rho_{AB_IC_I})$ and REC $C_{RE}(\rho_{ABC_I})$ in fig. 3 are reversed. When $T\to\infty$, we obtain $C_{l_1}(\rho_{AB_IC_I}) = \frac{1}{3}(2\sqrt{2}+1)$ and $C_{RE}(\rho_{AB_IC_I}) = \frac{\log_2 3125}{6} - 1$, meaning that quantum coherence in this case also has the freezing phenomenon.

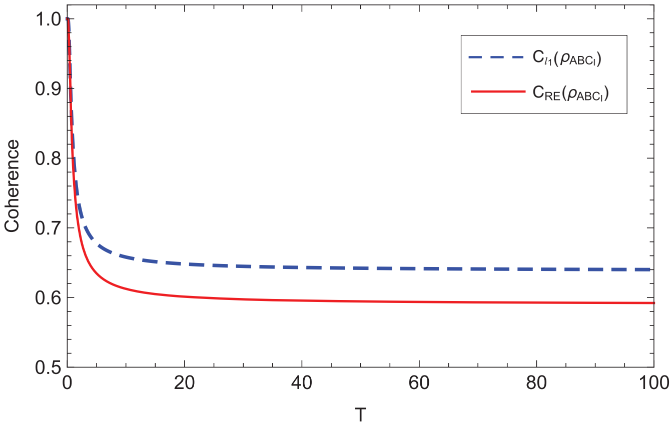

Fig. 3: The l_1 norm of coherence (blue dashed line) and REC (red solid line) for $\rho_{AB_IC_I}$ as functions of Hawking temperature T for fixed value $\omega = 1$. We normalize the maximum quantum coherence to **1**.

Monogamy relation. In order to study monogamy relation, we need to obtain the reduced states ρ_{AB_I}, ρ_{AC_I}, $\rho_{B_IC_I}$ and calculate their coherence. Note that now the reduced states ρ_{AB_I} and ρ_{AC_I} should have the same matrix form according to the symmetry of the system. After tracing over the mode A, we obtain $\rho_{B_IC_I}$ as

$$\rho_{B_IC_I} = \frac{1}{3}\begin{pmatrix} \mathcal{C}^4 & 0 & 0 & 0 \\ 0 & \mathcal{C}^2+\mathcal{C}^2\mathcal{S}^2 & \mathcal{C}^2 & 0 \\ 0 & \mathcal{C}^2 & \mathcal{C}^2+\mathcal{C}^2\mathcal{S}^2 & 0 \\ 0 & 0 & 0 & 2\mathcal{S}^2+\mathcal{S}^4 \end{pmatrix}, \qquad (24)$$

and the corresponding quantum coherence reads

$$\begin{aligned}
C_{l_1}(\rho_{B_IC_I}) &= \frac{2}{3}\mathcal{C}^2, \qquad (25) \\
C_{RE}(\rho_{B_IC_I}) &= \frac{1}{3}\mathcal{C}^2\mathcal{S}^2\log_2\left(\frac{1}{3}\mathcal{C}^2\mathcal{S}^2\right) \\
&\quad +\frac{1}{3}\mathcal{C}^2(2+\mathcal{S}^2)\log_2\left[\frac{1}{3}\mathcal{C}^2(2+\mathcal{S}^2)\right].
\end{aligned}$$

For the l_1 norm of coherence, we still find a compact monogamy relation

$$C_{l_1}(\rho_{AB_IC_I}) = C_{l_1}(\rho_{AB_I}) + C_{l_1}(\rho_{AC_I}) + C_{l_1}(\rho_{B_IC_I}), \qquad (26)$$

i.e., the total l_1 norm of coherence is equal to the sum of coherence of all bipartite system for any Hawking temperature T.

For the REC, we define the monogamy

$$\begin{aligned}
\gamma_2 &= C_{RE}(\rho_{AB_IC_I}) - C_{RE}(\rho_{AB_I}) - C_{RE}(\rho_{AC_I}) \\
&\quad - C_{RE}(\rho_{B_IC_I}). \qquad (27)
\end{aligned}$$

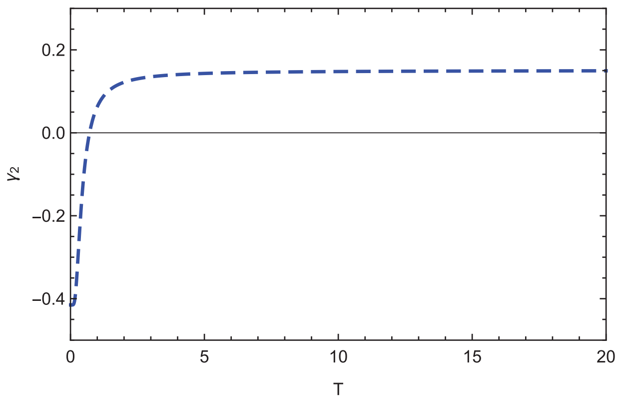

Fig. 4: The monogamy γ_2 as a function of the Hawking temperature T for fixed value $\omega = 1$.

Figure 4 shows how the Hawking radiation affects the monogamy γ_2. Comparing it with fig. 2, we find that $\gamma_2 = \gamma_1$ for $T = 0$, *i.e.*, the monogamy of the initial state equation (7) in the flat Minkowski spacetime is the same. However, with the increase of Hawking temperature, the curve goes up faster and the asymptotic value is bigger than fig. 2. The reason can be explained by the Hawking radiation: Hawking radiation leads to γ increasing as Hawking temperature increases. The case where two observers hover near the event horizon has stronger Hawking radiation than where one observer hovers near the event horizon.

Extended to N-partite systems. – In this section, we extend the results of the tripartite system to the N-partite systems. The N-partite W-state can be written as follows:

$$W_{123...N} = \frac{1}{\sqrt{N}}(|10...00\rangle + |01...00\rangle + ... + |00...01\rangle), \quad (28)$$

where the mode i ($i = 1, 2, ..., N$) is observed by observer O_i. We assume that one observer hovers near the event horizon of the black hole with uniform acceleration and the others stay in flat Minkowski spacetime. According to eqs. (1), we can obtain the l_1 norm of coherence

$$C^1_{l_1} = \frac{(N-1)(2\mathcal{C} + N - 2)}{N}. \quad (29)$$

Contrarily, if $N - 1$ observers hover near the event horizon of the black hole with uniform acceleration and one observer stays in flat Minkowski spacetime, then the l_1 norm of coherence can be obtained,

$$C^{N-1}_{l_1} = \frac{(N-1)[2\mathcal{C} + (N-2)\mathcal{C}^2]}{N}. \quad (30)$$

We are very interested in the effect of the particle number N on quantum coherence in the process of Hawking radiation. Figure 5 shows the l_1 norm of coherence $C^1_{l_1}$ and $C^{N-1}_{l_1}$ as functions of the Hawking temperature T for

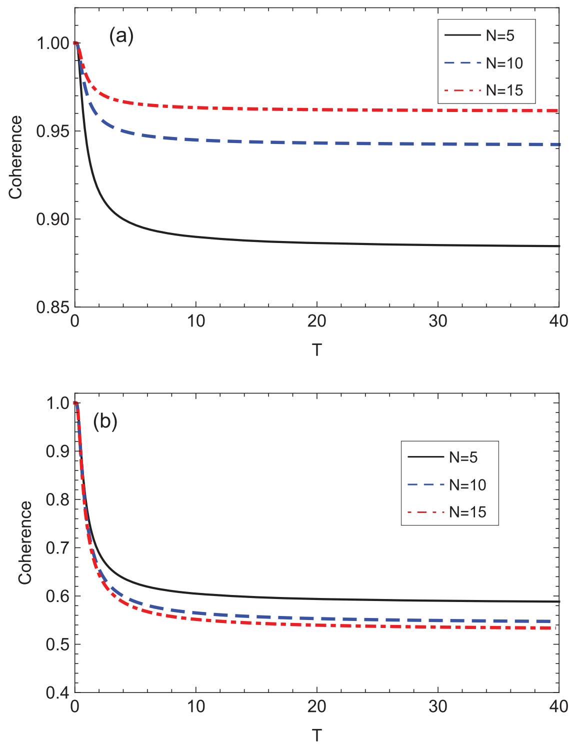

Fig. 5: The l_1 norm of coherence $C^1_{l_1}$ (a) and $C^{N-1}_{l_1}$ (b) as a function of the Hawking temperature T for fixed value $\omega = 1$. $N = 5$ (solid black line); $N = 10$ (dashed blue line); $N = 15$ (dotted red line). We normalize the maximum quantum coherence to 1.

different numbers of particles. We find that as the number of particles increases, the coherence $C^1_{l_1}$ increases and $C^{N-1}_{l_1}$ decreases. This result is still in agreement with the fact that the Hawking radiation destroys the quantum coherence, since with the increase of N, the number of particles that are influenced by Hawking radiation increases for $C^{N-1}_{l_1}$ and remains unchange for $C^1_{l_1}$. Note that this result is very different from the case of GHZ-state of Dirac fields where the coherence is independent of the particle number N [34]. In addition, we also find that both $C^1_{l_1}$ and $C^{N-1}_{l_1}$ are decreasing functions of Hawking temperature T, and have nonzero asymptotical values when $T \to \infty$ (freezing phenomenon).

Next, we study the monogamy relation of the N-partite W-state. For the case where one observer hovers near the event horizon of the black hole, we obtain

$$\sum_{n,m=1;n<m}^{N-1} C_{l_1}(\rho_{nm}) + \sum_{n=1}^{N-1} C_{l_1}(\rho_{nm_I}) = C^1_{l_1}, \quad (31)$$

and for the case where $N - 1$ observers hover near the event horizon of the black hole, we have

$$\sum_{m_I=1}^{N-1} C_{l_1}(\rho_{nm_I}) + \sum_{n_I,m_I=1;n_I<m_I}^{N-1} C_{l_1}(\rho_{n_I m_I}) = C^{N-1}_{l_1}. \quad (32)$$

Here ρ_{nm}, ρ_{nm_I} and $\rho_{n_I m_I}$ represent, respectively, the reduced bipartite states for the inertial observers n and m, the inertial observer n and non-inertial observer m_I, and the non-inertial observers n_I and m_I, and the total number of particles fits $N \geq 3$. Equation (31) is the generalization of eq. (18), which says that the total coherence is equal to the sum of all the bipartite coherence between two inertial observers, and between inertial and non-inertial observers. Equation (32) can be seen as the generalization of eq. (26), which says that the total coherence is equal to the sum of all the bipartite coherence between two non-inertial observers, and between non-inertial and inertial observers.

Conclusions. – In the background of a Schwarzschild black hole, we have studied the effect of Hawking radiation on quantum coherence and monogamy relationship of a tripartite W-state entangled system for Dirac fields. We have considered two cases: One observer and two observers hover near the event horizon of the black hole, and the remaining observers stay in flat Minkowski spacetime. The results shown that both the l_1 norm of coherence and REC are destroyed by the Hawking radiation, and asymptotically approach nonzero values for infinite Hawking temperature (freezing of coherence). In the case that one observer hovers near the horizon, the decaying rate of the l_1 norm of coherence is greater than that of REC; while for the case that two observers hover near the horizon, the result is reversed. Moreover, we have found that the l_1 norm of coherence of the tripartite W-state is always equal to the sum of coherence of all bipartite systems for any Hawking temperature, while a similar result is absent for REC.

We have generalized the relevant investigations to the N-partite W-state systems, and found that the monogamy relation for the l_1 of norm coherence still holds. In the case that only one observer hovers near the event horizon and all other observers stay in flat Minkowski spacetime, the l_1 norm of coherence increases when the number of total particles N increases; while in the case that $N-1$ observers hover near the horizon, the l_1 norm of coherence is a decreasing function of N. In both cases, the asymptotical coherence for $T \to \infty$ is nonzero (freezing of coherence).

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