

Decomposition method for real-time optimization problems in multilevel chemical process systems based on identification of compromise sets

S D Faruntsev

Omsk State Technical University, 11, Mira ave., Omsk, 644050, Russia

Abstract: Solving real-time optimization problem for multilevel chemical process systems (MCPS) is associated with the challenge of involving extensive computational resources required for prompt solution of high-dimension nonlinear optimization problems. The article introduces a method for decomposition of online optimization problem in multilevel chemical process systems with collector structure based on identification of compromise sets. The method allows bringing optimization search procedures to higher control levels leaving the tasks of process stabilization in calculated optimum points to the lower level subsystems. The article describes the scope of application for the method illustrated with a sample solution of the problem of online optimization of process unit parameters including the system of carbon black reactors operating in parallel for the common collector.

1. Introduction

The problems discussed in this article are considered as part of the class of process real-time optimization problems (RTO) that rank above the tasks of stabilization and advanced process control in the continuous process control hierarchy shown in figure 1 [1]:

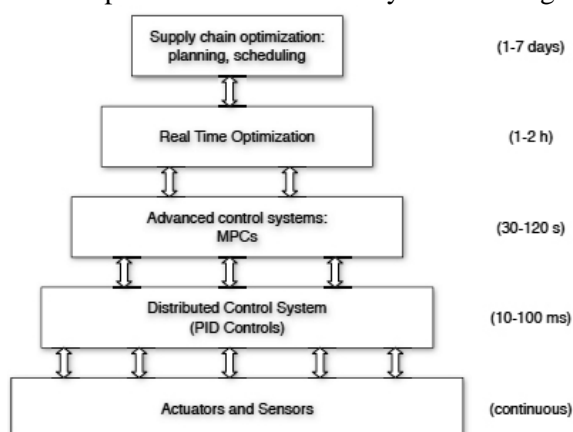


Figure 1. Hierarchy of control and optimization problems for continuous processes.

Advance achieved in the sphere of software and hardware for industrial automation allow modern controller to solve real-time optimization problems of significant complexity including the online mode optimization using technical and economic parameters based on process mathematical models that take into account the effects produced by disturbances. However, it is not possible to assert that



the problem of involving extensive computational resources required to solve online optimization problems for complex multiparameter objects is fundamentally solved. Besides, RTO developers need to exclude or minimize many problems encountered in “classical” or “academical” optimization of nonlinear systems, such as the solution dependence on initial approximation, algorithm looping, attribution of the solution to local optimum or degeneration of the solution, etc.

The method introduced in this publication is mostly in line with the decomposition ideas of the general optimization problem originally suggested by Dantzig and Wolf [2], i.e. the idea of breaking the problem down into stages in time (iteration) in the same way as, for instance, in method [3] that includes the model identification (correction) based on current conditions and subsequent process optimization. However, in the case considered it is suggested that, in addition to the stages performed at different time intervals, the problem should be also broken down by levels and subsystems in the variable space.

In several cases, the hierarchical multilevel topology of a chemical plant allows making this decomposition. This paper describes the model of a chemical plant with a collector structure where the lower subsystems serve as the sources and the higher subsystems are used as the receivers (collectors). The scheme of a multilevel CPS with a collector structure is shown in figure 2 in the form of a three-level complex (v is the level number) where rectangles are process subsystems; lines and arrows are links S between the subsystems of different levels; X^v, Y^v, Z^v are input and output variables of the subsystems.

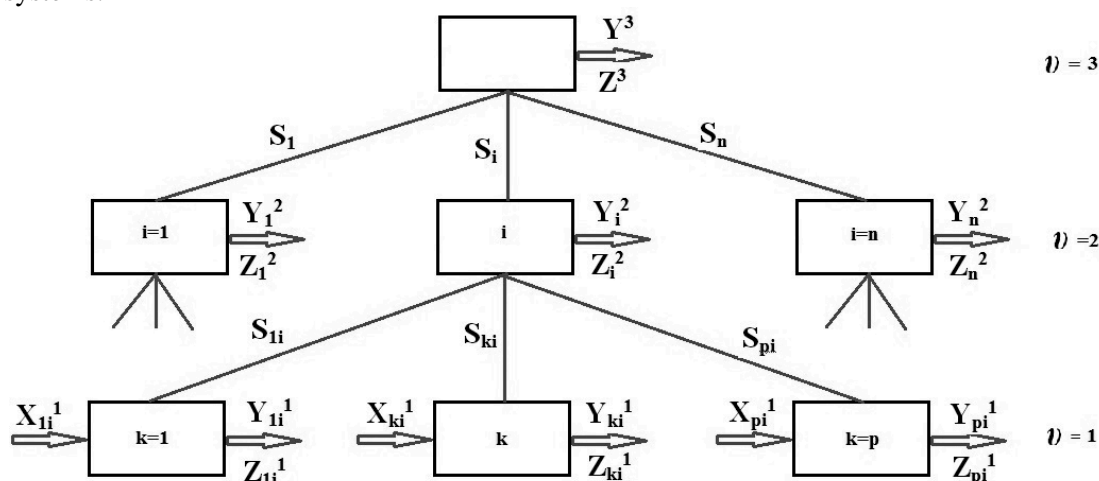


Figure 2. Scheme of a multilevel CPS with a collector structure.

Both direct and indirect material merging of streams can occur in the collectors, for example, by summing up the company income from individual business units. The last observation imputes a relatively universal character to the CPS model considered in addition to the popular operating process when several units use a common collector, which takes place, for example, in petroleum products compounding (mixing) processes or in case chemical reactors are parallelly connected to the common collector.

Solving the task of online optimization in such systems where local process criteria have to be agreed with the corporate goals of the higher level and must take into account the evolving external conditions is associated with significant computational challenges arising out of the high dimensionality of the problem and, in general, nonlinearity of the mathematical description. Even if it is possible to create a system of linear equations that correctly describe the process statics for individual units, nonlinearities usually emerge at higher levels due to the nature of mixing and interaction of products in the collectors [4-6].

In many cases it turns out that, despite the significant computational resources involved (time, computer memory), traditional methods of nonlinear programming used to solve optimization

problems with a large number of variables and limitations do not reach the global optimum due to the creation of gaps in the search space or for other reasons [7].

The decomposition approach can be considered as one of the main methods used to solve optimization problems in complex multilevel CPSs; this method is being constantly supplemented with new ideas and achievements [8, 9]. In recent years, the methodology of multicriteria (vector) optimization has been proposed with increasing frequency as the practical tool for solving control tasks in multilevel systems [10, 11].

The theory of multicriteria (vector) optimization originates in the works of prominent economists of the late 19th and early 20th centuries F. Edgeworth and V. Pareto and was developed in numerous subsequent studies [12–14]. One of the key tenets of the theory formulated by Pareto comprises the definition of the compromise set of mutually unimprovable criteria. Later on, this set was often referred to by the author's name as "Pareto set" or "Pareto area", or "Pareto boundary".

At the present time, the vector optimization methodology is usually used in preproject studies. This paper suggests using several important concepts and methods of the vector optimization in the practice of solving operative tasks; an appropriate method has been formulated and its efficiency is demonstrated by a particular model example of solving a mode optimization problem in a system of technical carbon reactors operating in parallel for one collector.

2. General formulation of MCPS optimization problem

The mathematical model of a multilevel CPS with a collector structure includes equations between the input and output variables of blocks of different levels:

$$Y^3 = Y^3(Y_1^2, \dots, Y_n^2) \quad (1)$$

$$Z^3(Y_1^2, \dots, Y_n^2) = 0 \quad (2)$$

$$Y_i^2 = Y_i^2(Y_{li}^1, \dots, Y_{pi}^1) \quad (3)$$

$$Z_i^2(Y_{li}^1, \dots, Y_{pi}^1) = 0, \quad i = \overline{1, n} \quad (4)$$

$$Y_{ki}^1 = Y_{ki}^1(X_{ki}^1) \quad (5)$$

$$Z_{ki}^1(X_{ki}^1) = 0, \quad k = \overline{1, p} \quad (6)$$

We assume that there are no connections between the input and output variables of the same level (which is a peculiarity of the MCPS model considered) and these connections are realized through the common collector:

$$X_{li}^1 \neq Y_{ei}^1 \quad l, e \in \{k : k = \overline{1, p}\} \quad l \neq e \quad (7)$$

The problem of process mode optimization for the facility in question can be formulated as the requirement to optimize a certain function $\Psi(Z^3, Y^3)$ that depends on parameters of the higher level:

$$\underset{D}{opt} \Psi(Z^3, Y^3), \quad (8)$$

where opt is the optimization (maximization or minimization) operator Ψ on the admissible set D described in the form of a system of equations (1) - (7) and additional limitations imposed by the requirements of process regulations and equipment design on control and output variables.

Considering the aforementioned computational problems, even if all mathematical expressions and values of factors are known, it is advisable to supplement conditions (1) – (8) written in a general form by indicating a specific method (algorithm) that allows solving the problem most efficiently.

3. Method for decomposition of the optimization problem in MCPS with a collector structure based on multilevel approximation of compromise sets

The compromise set (more precisely, "strict compromise set") of the problem of vector optimization of criterion $F = [f_1(x), \dots, f_m(x)]$ in region $D_x \subset E_x$ is subset $X^c \subset D_x$, where in any point ($\varepsilon_x \subset D_x$) of ε -neighborhood there is no improvement of at least one j -th criterion while other criteria preserve constant values.

Subset $X^w = D_x \setminus X^c$ of space E_x where simultaneous improvement is possible for several (at least two) criteria is referred to as the “agreement region” $D_x = X^c \cup X^w$, $X^c \cap X^w = \emptyset$ [14].

Figure 3 shows an example of relative positions of X^c and X^w in admissible region D_x of two criteria formed by functions F_1 and F_2 that must be maximized and are linearly dependent on parameters X_1 and X_2 . The figure shows lines F_1 and F_2 of equal levels and the directions of their gradients in the form of vectors $\text{grad}F_1$ and $\text{grad}F_2$. In this case, the compromise set (Pareto region) is formed by curve segment ab . The agreement region X^w includes all points D_x with the exception of X^c (segment ab).

In case functions F_1 and F_2 are not linearly dependent on parameters X_1 and X_2 , the compromise set may be located both on the border of the admissible region and inside the region as shown by figure 4 illustrating the case of two maximization criteria set by convex functions F_1 and F_2 on admissible set D_1 or D_2 .

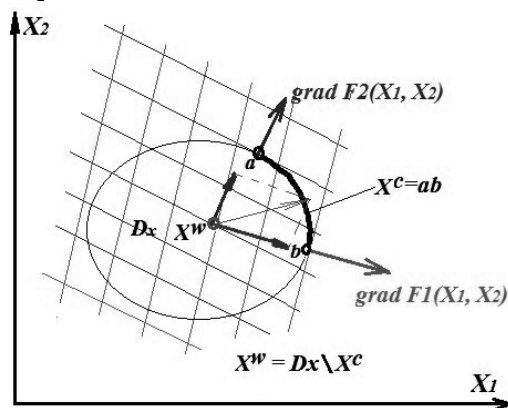


Figure 3. Example of relative position of compromise set X^c and agreement region X^w in admissible region D_x of criteria F_1 and F_2 that are linearly dependent on parameters X_1 and X_2

In region D_1 , compromise set X^c is on the border on the straight line a_1b_1 ; in region D_2 it is partially on the border (segments of the straight lines a_2b_2 and d_2e_2) and partially inside region D_2 (segment of curve $b_2c^*d_2$), while in total it belongs to segment $a_2b_2c^*d_2e_2$.

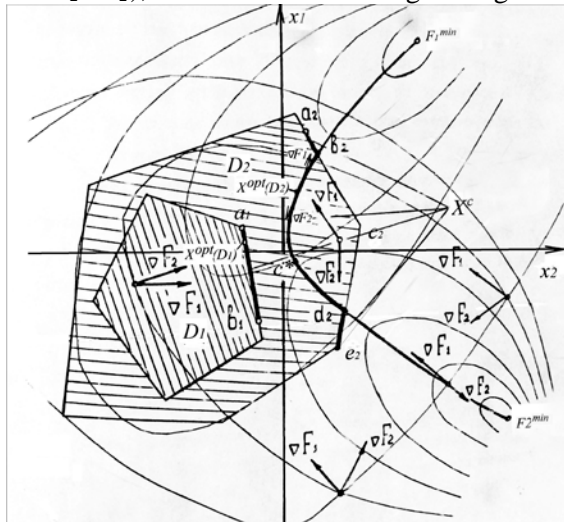


Figure 4. Example of relative position of compromise set X^c of criteria F_1 and F_2 that are not linearly dependent on parameters X_1 and X_2 for two cases of admissible region D_1 and D_2 .

The case when the compromise set is fully within the internal area of the admissible region is an example of unconditional vector optimization problem that can be identified by analyzing the relative position of the gradient vectors of local criteria, which are linearly dependent in the points of the compromise sets, and is solved by one of the unconditional vector optimization methods [15, 16].

The method of identification (and subsequent approximation) of the compromise set is selected depending on many factors including the type of admissible set (either convex or nonconvex), type of criteria dependence on the problem parameters (linear and nonlinear), and other factors. The example

given below describes case D_I of the compromise set belonging to the boundary of the admissible region.

According to the idea of the proposed method, in the course of solving the single-criterion problem of scalar criterion Ψ optimization (1-8), the problem is transferred to the sphere of vector optimization. In this case, a system of lower level local criteria is selected where the compromise set (Pareto region) X^c would totally contain the points of probable solutions by criterion Ψ .

The system of local criteria is selected considering the external conditions that change most frequently, such as the change of feed batch, commissioning/decommissioning of equipment units, or changes in limitations for feed stock, and are taken into account in formulation (1) - (8) as the problem constraints. In this case, the constraint equations of the initial scalar optimization problem are transferred to the rank of local criteria of vector optimization problem, for which points X^c of compromise set are searched.

Further, points X^c are sequentially mapped in the upper level regions (since functional relationships exist that connect the variables of adjacent levels) including the highest level. i.e. identification of Y^c is performed. After that, it becomes possible to find optimum Ψ (vector Y^{opt}) not in the initial region D_x but among the points of the revealed compromise set Y^c :

$$\underset{Y^c}{opt} \Psi(Z^3, Y^3)$$

After that, the backward mapping of Y^{opt} solution is performed sequentially into the lower level regions including the first region (calculation of X^{opt}). This means that solving the optimization problem for a multilevel system comprises computational procedures performed sequentially in counter directions.

It is suggested that a well-known method should be used to perform the identification and, if needed, approximation of the compromise set [16, 17].

Introducing additional constraints identifying the compromise set X^c allows reducing the number of degrees of freedom of the initial problem (1) – (8), i.e. the optimization problem dimensionality. Since it is originally assumed that the lower level objects are locally independent, the admissible sets and compromise sets of individual objects turn out to be disjoint, which ensures the decomposition effect.

The method is mainly applied for multilevel systems where the issue of repeated solution of the optimization problem formulated as (1) - (8) is relevant due to the frequently changing external conditions. In such systems, each manifestation of one of the above-mentioned external factors entails the need to use significant computational resources in order to solve the initial complex multidimensional optimization problem.

According to the method under consideration, it is suggested that the resource-intensive problems of identification and approximation of compromise sets X^c and Y^c and the problem of optimization (search for Y^{opt} among Y^c points) be solved considering the nature of the changing external factors in routine offline mode at the upper control level which is usually equipped with powerful computational resources.

It is assumed that it will be possible to solve the operational tasks of optimization in the region of compromise sets X^c и Y^c identified in offline mode and perform the mode stabilization on the lower control levels. It is understood that the dimensionality of the operational optimization tasks and, consequently, their computational resource-intensity are significantly lower than in case of the original formulation (1) - (8).

The issue of theoretical justification of the applicability limits of the method suggested is generally beyond the scope of this paper, which is focused on demonstrating the idea and proving the efficiency of the method for the case of MCPS with a collector structure (as formulated in (1) – (8)) as well as illustrating them with particular example of a compromise set localization on the boundary of the admissible set.

4. Sample solution of the problem of online process mode optimization of reactor section in carbon black production plant

Let us illustrate the essence of the given method using a model example that describes the problem of online optimization for one flow of the reactor section in a carbon black production plant. This object can be represented in the form of a two-level system consisting of a group of units operating in parallel for one collector (refer to figure 5).

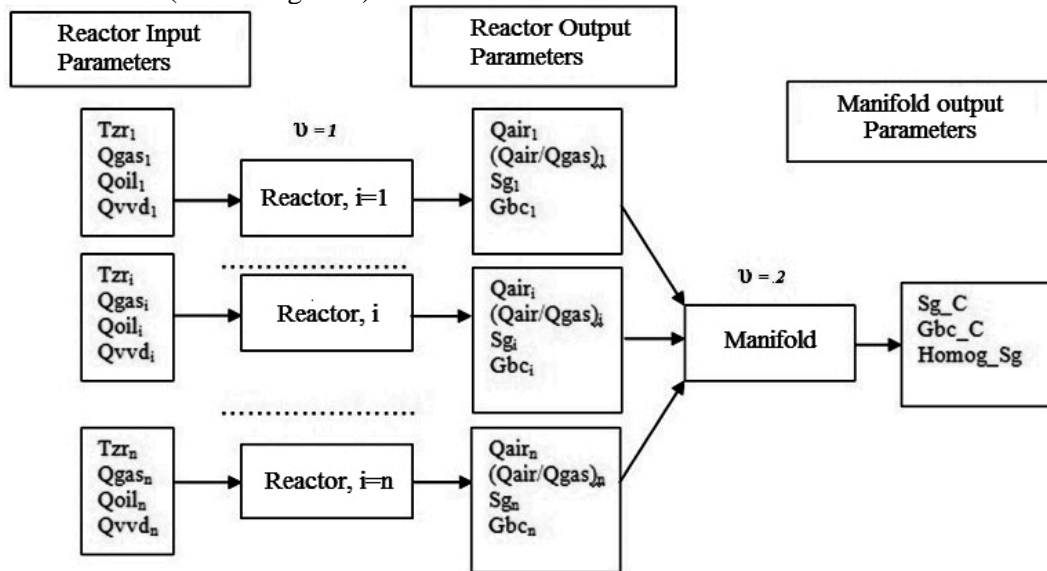


Figure 5. Diagram of flow in reactors operating for one collector.

The diagram shows a conventional representation of n number of reactors (in the problem solved the number $n=6$). The following variables are independent (control) parameters of individual reactors: Tzr_i is temperature in the reaction zone, °C; $Qgas_i$ is fuel gas consumption in the reactor, m³/h; $Qoil_i$ – hydrocarbon feed consumption in the reactor, kg/h; $Qvvd_i$ – air consumed to spray feed in the reactor, m³/h.

The output reactor variables are Gbc_i – reactor output, kg/h; Sg_i – specific geometric surface of carbon black particles, m²/g; $Qair_i$ – air consumed for combustion, m³/h.

According to [6], the relationship between the reactor input and output parameters is as follows:

$$\begin{bmatrix} Gbc_i \\ Sg_i \\ Qair_i \end{bmatrix} = \begin{bmatrix} 0 & 3.05 & 0.9 & -0.128 & -0.128 & -130 \\ 0.13 & 0.08 & -0.07 & 0.09 & 0 & Sg_0i \\ 4.2 & 14.5 & 2.5 & -1.39 & 0 & Qair_0i \end{bmatrix} \times \begin{bmatrix} Tzri \\ Qgasi \\ Qoili \\ Qvvd_i \\ Qair_i \\ 1 \end{bmatrix}. \quad (9)$$

The individual peculiarities of the reactors are taken into account by supplementing the model with the following values of the adjustable absolute terms of equations Sg_0i and $Qair_0i$:

$$\begin{aligned} Sg_0i &= [-99; -100; -101; -102; -103; -104]; \\ Qair_0i &= [-6100; -6050; -6000; -5950; -5900; -5850]. \end{aligned} \quad (10)$$

Constraints for i -th reactor:

- The ratio of combustion air consumption to fuel gas consumption shall not be less than $Kmin=38$ determined based on the thermal strength of the reactor combustion chamber:

$$Kmin_i * Qgas_i - Qair_i \leq 0; \quad (11)$$

- Constraints on mode parameters imposed by the process regulations:

$$\begin{aligned} 1500 &\leq T_{zr\ i} \leq 1600\ ^\circ\text{C}; \\ 40 &\leq Q_{gas\ i} \leq 80\ \text{m}^3/\text{h}; \\ 500 &\leq Q_{oil\ i} \leq 700\ \text{kg}/\text{h}; \\ 280 &\leq G_{vvd\ i} \leq 320\ \text{m}^3/\text{h}. \end{aligned} \quad (12)$$

Collector parameters:

- Total reactors output Gbc_C :

$$Gbc_C = \sum_{i=1}^n Gbc_i \quad (13)$$

- Specific geometric surface of carbon black particles in the collector Sg_C :

$$Sg_C = \frac{\sum_{i=1}^n Sg_i * Gbc_i}{Gbc_C}; \quad (14)$$

- Dispersion (dissipation) of individual value indicators of specific geometric surface in reactors relative to the average D_Sg is calculated using the formula:

$$D_Sg = \frac{\sum_{i=1}^n (Sg_C - Sg_i)^2}{n-1}; \quad (15)$$

- Homogeneity of carbon black based on the specific geometric surface indicator $Homog_Sg$, i.e. the dispersion of the size of carbon black particles is calculate as the mean square deviation of the values of specific geometric surface by reactors:

$$Homog_Sg = \sqrt{D_Sg} \quad (16)$$

Constraints on general flow parameters

- Homogeneity shall not exceed the maximum allowable value of $2\ \text{m}^2/\text{g}$:

$$Homog_Sg \leq 2; \quad (17)$$

- Specific geometric surface of carbon black in the collector Sg_C shall not be less than the minimum allowable value of $95\ \text{m}^2/\text{g}$:

$$95 \leq Sg_C. \quad (18)$$

The task of the total output maximization in the reactors (13) on the admissible set specified by equations (9) - (12) and (14) - (18) is a nonlinear problem, where the dimensionality determined by independent variables (number of degrees of freedom) is $4n=24$. Solving this problem by traditional methods depends to a large extent on the initial approximation and, in case of a bad choice of initial data, may lead to one of the local minima. The authors of paper [6] had to significantly simplify the problem formulation in order to reduce the computational problems while solving a similar problem by traditional methods of nonlinear programming.

Let us demonstrate how the global optimum can be reached using the suggested method. A preliminary analysis showed that indicators Gbc_C and Sg_C are competing criteria in the formulation under consideration, i.e. the maximum total output of the reactors and the output of each reactor taken separately is achieved at the minimum size of specific surface and vice versa. Thus, first of all, it is necessary to identify the compromise set to maximize the specified criteria in the parameter space of each reactor. To this end, let us perform a linear convolution of criteria Gbc_i and Sg_i , i.e. form the following specific problem of maximization of the weighted sum of these criteria:

$$\max_{D_i} [\alpha_i * Gbc_i + (1 - \alpha_i) * Sg_i], \quad \alpha_i \geq 0, \ i = \overline{1, n} \quad (19)$$

in the admissible set D_i determined by equations and inequations (9) - (12) and solve it sequentially by linear programming method for each of the reactors at different values of factor α_i that varies from 0 to 1 with the interval of 0.1. If $\alpha_i = 1$, the problem (19) is identical to the problem of maximization of

Gbc_i ; if $\alpha_i = 0$, it is identical to the problem of maximization of Sg_i ; if α_i is between 1 and 0, the optimum will be in intermediate points of the compromise set.

5. Discussion

The results of problem (19) solution performed using Optimization Toolbox MATLAB software are summarized in table 1.

Table 1. Values of the compromise set points in parameter space X_i^c and criteria space Y_i^c .

#	α_i		X_i^c						Y_i^c	
			Tzr_i	$Qgas_i$	$Qoil_i$	$Qvvd_i$	$Qair_i$	$Qair_i/Qgas_i$	Sg_i	Gbc_i
i=1	0.0	a1	1600.0	62.7	500.0	320.0	2382.2	38.0	110.6	165.3
	0.1	b1	1600.0	80.0	662.7	320.0	3040.0	38.0	101.0	280.4
	0.2	c1	1578.0	80.0	700.0	320.0	3040.0	38.0	95.6	313.9
	0.4	d1	1500.0	66.0	700.0	320.0	2506.7	38.0	84.3	339.4
	1.0	e1	1500.0	68.3	700.0	280.0	2596.6	38.0	80.9	340.2
i=2	0.0	a2	1600.0	64.8	500.0	320.0	2463.0	38.0	109.8	161.5
	0.1	b2	1600.0	80.0	642.7	320.0	3040.0	38.0	101.4	262.4
	0.2	c2	1566.1	80.0	700.0	320.0	3040.0	38.0	93.1	313.9
	0.4	d2	1500.0	68.1	700.0	320.0	2587.6	38.0	83.5	335.5
	1.0	e2	1500.0	70.5	700.0	280.0	2677.5	38.0	80.1	336.3
i=3	0.0	a3	1600.0	66.9	500.0	320.0	2543.9	38.0	108.9	157.6
	0.1	b3	1600.0	80.0	622.7	320.0	3040.0	38.0	101.7	244.4
	0.2	c3	1555.4	80.0	700.0	320.0	3040.0	38.0	90.5	313.9
	0.4	d3	1500.0	70.2	700.0	320.0	2668.4	38.0	82.7	331.7
	1.0	e3	1500.0	72.6	700.0	280.0	2758.3	38.0	79.3	332.5
i=4	0.0	a4	1600.0	69.1	500.0	320.0	2624.7	38.0	108.1	153.7
	0.1	b4	1600.0	80.0	602.7	320.0	3040.0	38.0	102.1	226.4
	0.2	c4	1554.3	80.0	700.0	320.0	3040.0	38.0	88.0	313.9
	0.4	d4	1500.0	72.3	700.0	320.0	2749.3	38.0	81.8	327.8
	1.0	e4	1500.0	74.7	700.0	280.0	2839.2	38.0	78.4	328.6
i=5	0.0	a5	1600.0	71.2	500.0	320.0	2705.6	38.0	107.3	149.9
	0.1	b5	1600.0	80.0	582.7	320.0	3040.0	38.0	102.4	208.4
	0.2	c5	1553.1	80.0	700.0	320.0	3040.0	38.0	85.4	313.9
	0.4	d5	1500.0	74.5	700.0	320.0	2830.1	38.0	81.0	323.9
	1.0	e5	1500.0	76.8	700.0	280.0	2920.0	38.0	77.6	324.8
i=6	0.0	a6	1600.0	73.3	500.0	320.0	2786.5	38.0	106.4	146.0
	0.1	b6	1600.0	80.0	562.7	320.0	3040.0	38.0	102.8	190.4
	0.2	c6	1551.9	80.0	700.0	320.0	3040.0	38.0	82.9	313.9
	0.4	d6	1500.0	76.6	700.0	320.0	2911.0	38.0	80.2	320.1
	1.0	e6	1500.0	79.0	700.0	280.0	3000.9	38.0	76.8	320.9

Points $a_1...e_1, ...a_6...e_6$ are the nodes of the corresponding compromise sets for each of the reactors. Figure 6 includes a graphic illustration of the solution obtained in the three-dimensional space of independent parameters of reactors Tzr_i , $Qgas_i$, $Qoil_i$. The cube faces in the figure are created by constraints (12).

Let us map points $a_1...e_1, ...a_6...e_6$ in the criteria space of indicators Gbc_i, Sg_i , i.e. plot the graphs of the last two columns of table 1 that create sets Y^c_i . The result is shown in figure 6.

The final stage is to solve the maximization problem (13) under constraints (14) - (18) in the admissible set $Y^c_i, i=\overline{1,n}$ created by the segments of lines $a_1...e_1, ...a_6...e_6$, figure 7.

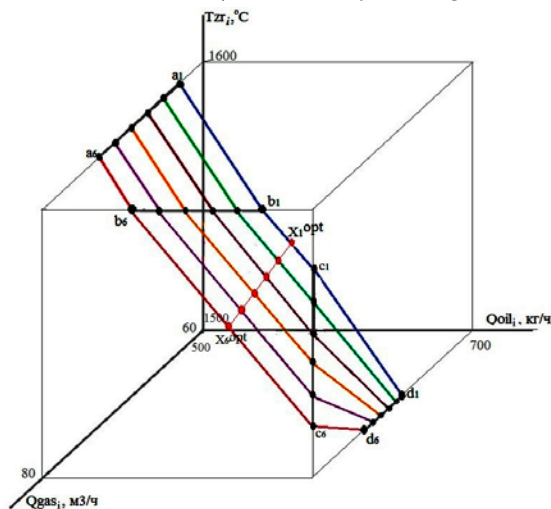


Figure 6. Location of the compromise set points in the three-dimensional space of independent reactor parameters.

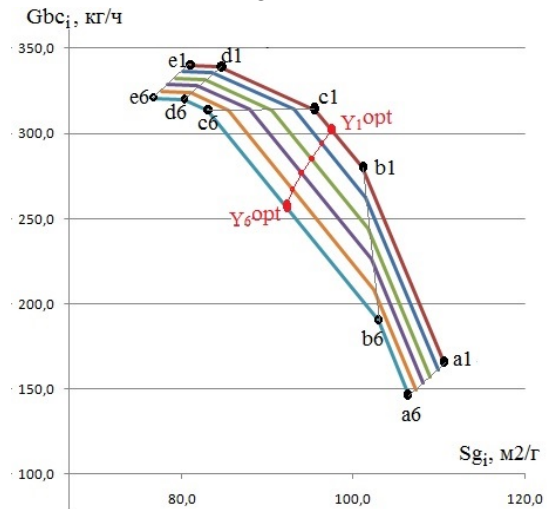


Figure 7. Mapping of the points of compromise set Y^c_i in the parameter space Gbc_i, Sg_i .

It was also possible to preliminary formulate and solve the problem of approximating the broken lines by curve segments that form nonlinear dependences $Gbc_i = f(Sg_i)$, for example, by regression analysis or neural networks in order to add the identified relationships to the constraint system (14) - (18). However, in the specific case under consideration, an algorithm was identified that allows avoiding the procedure of compromise sets approximation in the criteria space and addition of additional nonlinearities by searching for the optimum directly in set $Y^c_i, i=\overline{1,n}$ defined by broken lines segments $a_1...e_1, ...a_6...e_6$.

This problem was solved using MATLAB tools in the space of independent variables $n=6$, i.e. the search was performed by 6 independent variables (as opposed to $4n=24$ variables of the original problem) and, despite its nonlinear character arising from the type of dependencies (13) - (16), and led to global optimum Y^{opt} with the points shown in figure 6. Figure 6 also shows the mapping of Y^{opt} in the space in independent reactor variables in the form of points X^{opt} .

Table 2 summarizes the final results of solving the problem of indicator (13) optimization on the admissible set defined by equations and inequations (9) - (12) and (14) - (18).

Table 2. Results of the problem solving in parameter space X^{opt} and criteria space Y^{opt} .

#	X^{opt}						Y^{opt}	
	Tzr_i	$Qgas_i$	$Qoil_i$	$Qvvd_i$	$Qair_i$	$Ratio_i$	Sg_i	Gbc_i
i=1	1588.7	80.0	686.6	320.0	3040	38.0	97.6	301.7
i=2	1580.2	80.0	676.4	320.0	3040	38.0	96.5	292.7
i=3	1574.2	80.0	666.2	320.0	3040	38.0	95.4	283.4
i=4	1568.6	80.0	656.0	320.0	3040	38.0	94.4	274.4
i=5	1562.6	80.0	645.7	320.0	3040	38.0	93.3	265.0
i=6	1557.1	80.0	635.5	320.0	3040	38.0	92.2	256.0

The values of general flow optimum parameters:

- $Gbc_C = 1673.1 \text{ kg/h}$;
- $Sg_C = 95.0 \text{ m}^2/\text{g}$;
- $Homog_Sg = 2 \text{ m}^2/\text{g}$.

As evident from the numerical values of the last two inequations, the potentialities provided by constraints (17) and (18) were totally exhausted in the course of searching for the optimum solution. The solution is found in the extreme point of the admissible region determined by these constraints.

As the need arises, the online optimization problem considered above can be formulated and solved at the level of the unit comprising all reactor flows and further at the level of the plant consisting of several units that produce different grades of carbon black.

In this case, the enterprise profit can be used as the global criteria while the cost indices for individual plants are added to the local criteria showing the costs incurred to maintain the specified quality of technical carbon on the reactor flows.

6. Conclusion and summary

The paper describes the controlled object represented with a multilevel chemical process system. The paper specifies the methods that can be used to solve static optimization problems for such objects and the issues associated with the problem solving.

A method was suggested that allows solving the problem of online optimization of an initially scalar criterion of MCPS with a collector structure by transferring the problem to the sphere of vector optimization through the selection of the local parameter system where the compromise set comprises the points of possible solutions required to optimize the specified criterion and, simultaneously, the parameters of measures disturbances.

After that, it is suggested that the optimum should be searched for not in the initial admissible region, but among the points of the identified compromise set whose collector property makes it possible to divide the compromise set into independent subsets. This produces the decomposition effect leading to the reduction of the optimization problem dimensionality and saves computational resources.

The problem of online process mode optimization in MCPS was formulated in accordance with described method.

A model example is given demonstrating the implementation of the method in solving the problem of online optimization of a process unit performance including the system of carbon black reactors operating in parallel for one collector. The example proves the effectiveness of the method suggested.

References

- [1] Marco Vaccari and Gabriele Pannocchia 2017 *Real-Time Optimization* MDPI AG vol **246** pp 71–91
- [2] Dantzig G B and Wolf Ph 1960 Decomposition Principle for Linear Programs *Operations Research* vol **8** No 1 pp 101–111
- [3] Chen C Y and Joseph B 1987 On-line optimization using a two-phase approach: An application study *Ind Eng Chem Res* **26** pp 1924–1930
- [4] Nikitin V A and Mussaev A A 2007 *Hydrocarbon Mixtures Compounding Optimization in a Flow with Nonlinear Component Interaction* Papers (Saint-Petersburg: Nauka) pp 327–336
- [5] Kirgina M V, Sakhnevich B V, Maylin MV, Ivanchina E. D and Chekantsev N V 2014 *Development of a Smart Computer System to Support the Motor Fuel Production Process* Chemistry and Chemical Technology vol **57** 11 pp 84–86
- [6] Faruntsev S D, Perov V L and Bobrov D A. 1981 Algorithm for Optimal Control of Parallel Operating Carbon Black Production Reactors *Automation and Instrumentation in Refining and Petrochemical Industries* (Moscow: CNIITNeftekhim) 1 pp 9–12

- [7] Zakharova E M and Minashina I K 2014 *Review of Multidimensional Optimization Methods* Information Processes vol **14** 3 pp 256–274
- [8] Lisitsyn N V 2003 *Optimization of oil Refining Production*. Saint-Petersburg: Khimizdat 180 p
- [9] Chekanina E A 2014 The Problem of Hierarchical Control of an Industrial Complex with Continuous Technology in the Class of Linear Models *Automation and Modern Technology* **7** pp 35–37
- [10] Faruntsev S D 1980 *The Multi-criteria Principle of Decomposition of Large-scale Optimization Problems in Multilevel Collector Systems* Optimization of ACS Structures in Mechanical Engineering Regional scientific and practical conference (Omsk, Russia, 1980) pp 88–89.
- [11] Chikunov P A and Krivodubsky O A 2014 Analysis of Modern Methods of Building Decision Support Systems for Multi-level Enterprises *Philosophical Problems of Information Technology and Cyberspace* vol **2** (8) pp 20–40
- [12] Wentzel E S 1988 *Operations Research: Objectives, Principles, Methodology* (Moscow: Nauka) 203 p
- [13] Podinovskiy V V and Nogin V D 2007 *Pareto Optimal Solutions for Multicriteria Problems* (Moscow: Fizmatlit) 256 p
- [14] Sorin Mihai Grad 2014 *Vector Optimization and Monotone Operators via Convex Duality: Recent Advances* (Springer) 269 p
- [15] Claus Hillermeier 2012 *Nonlinear Multiobjective Optimization: A Generalized Homotopy Approach* ISNM (Birkhäuser) 135 p
- [16] Lotov A V and Pospelova I I 2008 *Multi-Criteria Decision Making Problems* (Moscow State University) 197 p
- [17] Karpenko A P, Semenikhin A S and Mitina E V 2012 *Population Methods for Approximating the Pareto Set in a Multicriteria Optimization Problem*. Overview Science and Education: Electronic Scientific and Technical Publication vol **4** (<http://technomag.edu.ru/doc/363023.html>)