# Formalizing the search of targets with given coordinates distribution

**B K Nartov, A N Poluyanov**

Sobolev Institute of Mathematics SB RAS, 4 Acad. Koptyug avenue, Novosibirsk 630090, Russia

E-mail: nartov@ofim.oscsbras.ru

**Abstract.** The paper presents a new approach to formalizing the tasks of an optimal search of the targets with given coordinates distributions. The main problems of formalizing this task group are highlighted. The method is described, that allows one to avoid non-analytical limitations on search units control. The formalization scheme for optimal search planning problems is shown. The features of real-time search control are considered for the cases, when the moments and coordinates of target detection are known. The search simulation results partially verifying the proposed method are given.

*Keywords*: search control, stationary targets, formalization, imitational modeling

## 1. Introduction

The first mathematical statements for individual problems of searching stationary targets with specified coordinates distribution were given in traditional works of B. Kupman and V. I. Arkin, published in 1950s-1960s. In 1980s O. Hellman (see, for example, [1]) formalized some rather practical search problems but proposed no general methods in this field. Starting from 1990s, efficient neural network algorithms, though not guaranteeing exact solutions, have been used in search problems and systems together with traditional approaches. Current state of the search problems and their relevance are well shown in works of A. A. Strotsev et al. [2, 3] providing reasonably implementable search algorithms. However, general cases of optimal search problems in these publications are still not formalized. As a result, the authors are forced, for example, to specify the additivity for the tasks of managing several search units.

Further a method is presented that solves the main problem of formalizing this task class – the problem of analytical records of intersections and self-intersections of the search bands and tubes covered by the search units. Moreover, the study results of the main search problems for additivity and uniform optimality of control and the search simulation results are presented.

## 2. Method of elastic trace. Search planning

$K$ stationary targets with known coordinates distributions $p_k(x), 1 \le k \le K$. lie in plane search domain $G \subset E^2$. At the initial time moment $t$=0, $N$ search units (SU) are located in the domain. When moving, each SU is the centre of a circle with radius $a$, covering the search band with a width of 2$a$ in the domain $G$.

Let us introduce the following notations:

$\xi_i(t) = (\xi_1^i(t), \xi_2^i(t))$ is the trajectory of *i*-th SU, $1 \le i \le N, 0 \le t \le T$,

*T* is search time,

$p(x) = \sum_{k=1}^{K} p_k(x).$

Moreover $\int_G p(x)ds = K,$ where *ds* is the area element.

Let us set the following task: given the kinematic and spatial limitations on the SU control on the search interval (0,*T*), one should formalize the calculation of the search strategy $u^* = \{\xi_i^*(t) \mid 0 \le t \le T, 1 \le i \le N\}$ with free finite coordinates of SU maximizing the mathematical expectation for the number of the targets detected during the search.

Formalization scheme is as follows:

1. Predefine auxiliary function $\lambda$, associated with SU coordinates on the domain $G$:

$$\lambda(x, \xi(t)) = \begin{cases} 1, & if \quad |\xi(t) - x| < a, \\ 0, & if \quad |\xi(t) - x| \ge a. \end{cases}$$

2. Set differentiable bell-shaped function $F(x, \xi(t)),$ approximating the values of $\lambda$ arbitrarily closely. In this case, the specific form of *F* is quite arbitrary.

3. Consider the function $\tilde{F}(x, u, t)$ being the solution of the differential equation

$$\dot{\tilde{F}} = \alpha \sum_{i=1}^{N} \frac{F_i - \tilde{F}}{1 - F_i}, \tag{1}$$

where $\alpha$ is an adjustable positive parameter, $\tilde{F}(x, u, 0) = 0.$

It can be shown that for any initial conditions of the original problem there always are such $F_i$ and $\alpha$, that by the search end moment *T* the function $\tilde{F}$ from (1) realizes a differentiable profile, imitating SU movement, on the search domain *G*. In this case, the height of the profile with the specified accuracy will be equal to one above the examined domains, including the domains of intersections and self-intersections of search bands, and with the specified accuracy it equals zero outside the examined domains. The function $\tilde{F}$ rises quickly up to $M = \max(F_1, ..., F_N),$ if $\tilde{F} < M,$ and it decreases slowly to *M*, if $\tilde{F} > M$.

Properties of $\tilde{F}$ allow the original task to be formalized as

$$J(u) = \int_G p(x)\tilde{F}(x, u, T)ds \to \max, \tag{2}$$

here *ds* - the area element, that is, it is written as the control performance functional including all possible intersections and self-intersections of the search bands.

## 3. Real-time search control

Let us consider the features of the real-time search control for the cases, when the moments and coordinates of the target detection are known.

It should be shown in advance that for the case $(K = 1, N \ge 1)$ and arbitrary discrete of plan recalculation, the optimal control coincides with the optimal plan calculated from (2) at the initial search moment.

Let arbitrary control be implemented on a certain interval $(0, t) \subset (0, T)$ and the target be undetected.

Let us introduce the following notations:

*p*(*x*) is target distribution density;

$G_1 \subset G$ is a domain examined at the control interval $(0, t)$;

$G_2 \subset G$ is a domain set at time moment *t* to be examined on the interval $(t, T)$;

$$I_1 = \int_{G_1} p(x)ds;$$

$$I_2 = \int_{G_2 \setminus (G_1 \cap G_2)} p(x)ds.$$

Let us record the problem of maximizing the conditional probability of the target detection on the interval $(t,T)$:

$$(1-I_1)(I_1+I_2) \to \max. \tag{3}$$

The first multiplier in expression (3) is the task constant which reduces (3) to the form of

$$(I_1+I_2) \to \max$$

or

$$\int_{G_1} p(x)ds + \int_{G_2 \setminus (G_1 \cap G_2)} p(x)ds \to \max$$

or, by combining integration domain,

$$\int_{G_1 \cup G_2} p(x)ds \to \max,$$

this preserves the trajectories optimality of task (2) search units on $(t,T)$.

It should be noted:

1. In the case of $(K=1, N \geq 1)$, optimal control of the real-time search (until the target is detected or the specified search time $T$ expires) coincides with the optimal blind search plan calculated from (2), under any parameters and initial conditions.

2. The case of $(K>1, N \geq 1)$ requires, generally speaking, a continuous disproportionate correction of the initial values $p_k, 1 \leq k \leq K,$ [4]. Here, control is a sequence of optimal plans recalculated after a designated time $\Delta t$, small compared to $T$, and at the targets' detection time. Uniform optimality of the control is not guaranteed.

3. The real-time search planning and search control tasks are additive only in the case of ($K$=1, $N$>1) under initial conditions and parameters that do not allow the intersections of search bands. In all other cases, the additivity of the problems is not guaranteed.

## 4. Verification of the results

Let us consider the results of numerical simulation of the "elastic" function $\tilde{F}$.

The following approximations and numerical values of the parameters were used to model the behavior of the function $\tilde{F}$.

$F = F_1 + F_2$, where

$F_1(x) = \varepsilon;$

$$F_2(x) = \begin{cases} 1-2\varepsilon, & |\xi(t)-x| < a \\ 0, & |\xi(t)-x| \geq a \end{cases};$$

$$\frac{d\tilde{F}}{dt} = \alpha \frac{F-\tilde{F}}{1-F};$$

$$\tilde{F}_{n+1} = \alpha \Delta t \frac{F_n - \tilde{F}_n}{1-F_n} + \tilde{F}_n;$$

$\tilde{F}(0) = 0;$

$\varepsilon = 0.001;$

at three values of $\alpha \Delta t$: 0.001, 0.0005 и 0.0001,

where $\Delta t$ is the time of the search unit shift by one point.

Search units with a radius of 50 points passed a rectangular search band divided into 1000x200 points at different rates. The values $\tilde{F}$ were recalculated for each band point at each shift.

Figure 1 shows the coloring of the intervals for the function $\tilde{F}$ values.

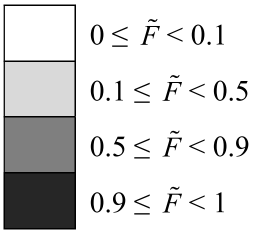


**Figure 1.** Intervals of the function $\tilde{F}$. values.

Figure 2 shows the behavior of the function $\tilde{F}$ when SU move at low speed.

The bands of medium and small values behind the leading search edge are barely expressed, but after passing SU, the function $\tilde{F}$ manages to decrease to medium and small values by the end of the search. The result is unsatisfactory.

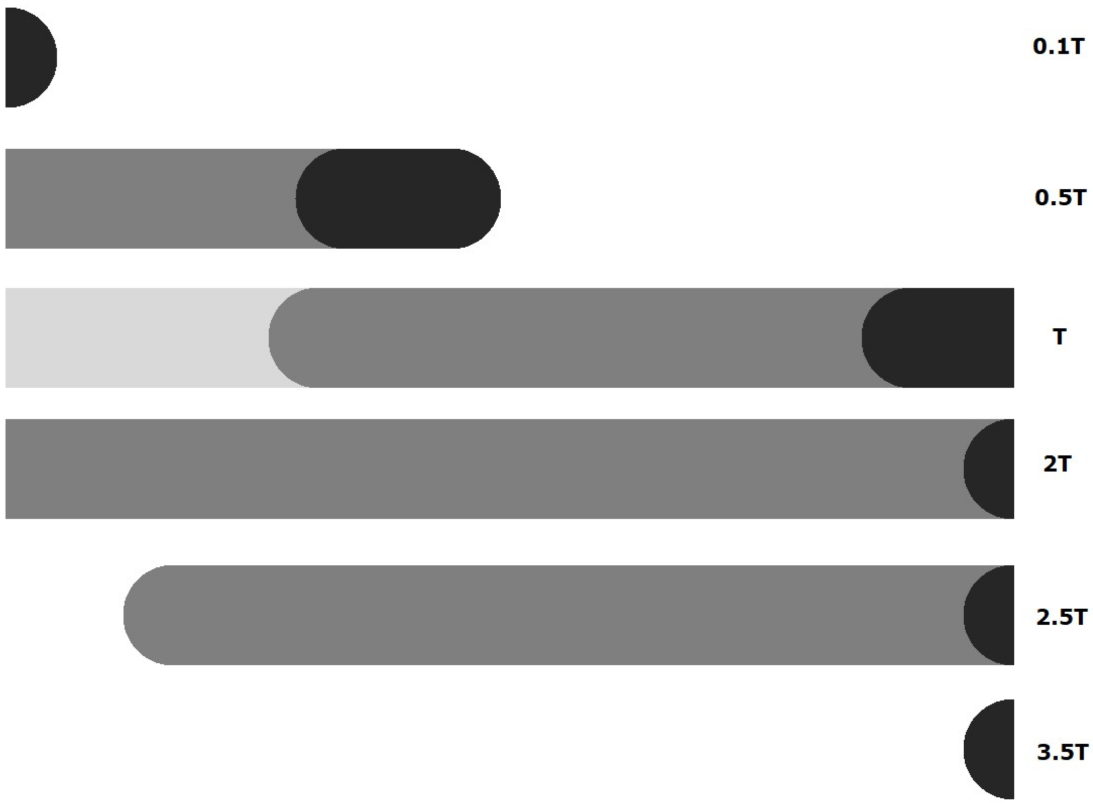


**Figure 2.** Function $\tilde{F}$ behavior at $\alpha\,\Delta t = 0.001$.

Figure 3 shows the behavior of the function $\tilde{F}$ when SU move at medium speed. The bands of medium and small values behind the leading search edge are more pronounced. After passing SU, the function $\tilde{F}$ manages to lower to the medium values by the end of the search. The result is unsatisfactory.

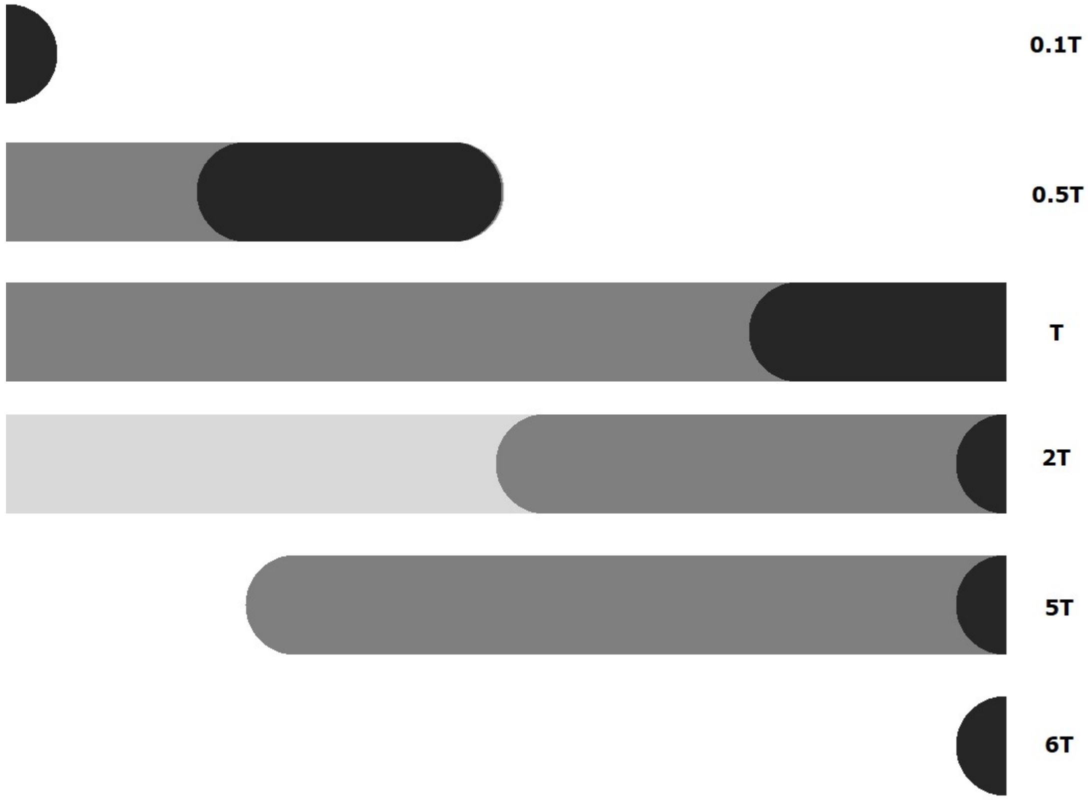


**Figure 3.** Function $\tilde{F}$ behavior at $\alpha\,\Delta t = 0.0005$.

Figure 4 shows the behavior of the function $\tilde{F}$ when SU move at high speed. The bands of medium and small values behind the leading search edge are pronounced. By the end of the search, the function fails to get out of the specified required values range (0.9,1), and the function is "ready for substitution" in the optimal search problem (2).

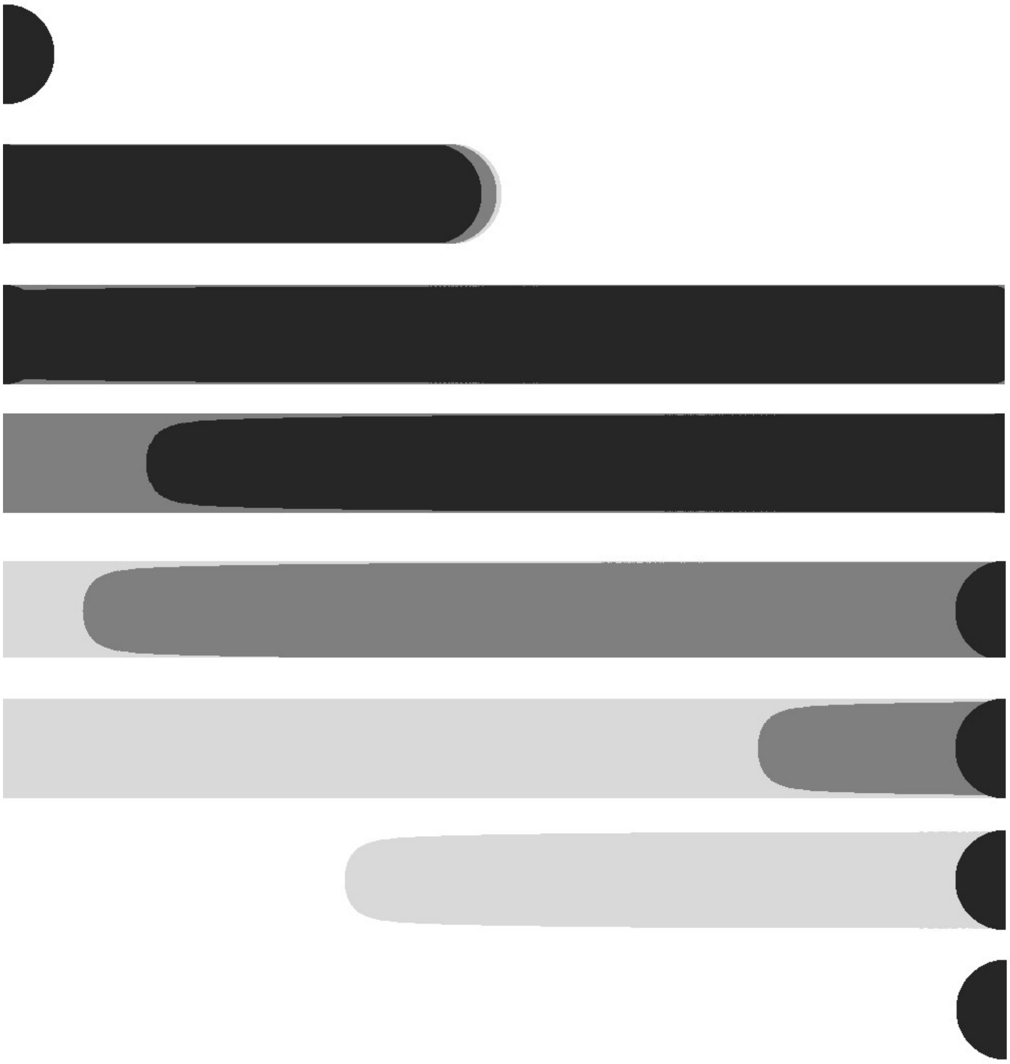


**Figure 4.** Function $\tilde{F}$ behavior at $\alpha \Delta t = 0.0001$.

## 5. Conclusions

The presented formalization method is easily extended to the search problems in three-dimensional space.

Search problems with the risk of search units destruction require a separate study on additivity and uniform optimality.

Application programs developed to verify the presented method can be used for fast selection of search algorithm parameters.

## Acknowledgment

Sections 1-2 of the work were supported by the program of fundamental scientific researches of the SB RAS № I.5.1., project № 0314-2019-0020.

Sections 3-5 of the work were supported by RFBR, projects № 18-08-01284, № 18-07-00526.

## References

[1] Hellman O 1980 *Introduction to optimal search theory* (Moscow: Nauka) 254

[2] Strotsev A A, Yunusmetov R A, Shvetsov D D 2009 Mathematical model of search efforts of group search system with spherical surveillance means *Bulletin of Rostov State Transport University* **35**(3) 62–70

Journal of Physics: Conference Series **1441** (2020) 012154 doi:10.1088/1742-6596/1441/1/012154

[3] Yanusmetov R A, Strotsev A A 2010 Mathematical model of search system with groups of search units *Proceedings of higher educational establishments. North Caucasian Region. Series: Technical sciences* **2** 17-23
[4] Murzin F A, Nartov B K 2018 *Problems of trajectory control. Formalization, surveillance, parallel calculations* (Novosibirsk: SB RAS) 90