

## Early detection of the beginning of a gradual change in properties insulating materials

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**Abstract.** Procedure of early detection of a gradual change in insulating materials properties (disorder) is developed on the basis of thermal conductivity degradation under the hostility exposure. In the procedure of Bayesian statement development for early detection of a gradual disorder beside the initial data in which the disorder is searching it is proposed to use the additional information from indicators indirectly proving a possible disorder. The simulated results showed that the value of delay in the detection of gradual change beginning of thermal conductivity in various insulating materials is reduced by an average of 20% in comparison with the existing procedures. The worked out procedure is meant for implementation in information-measuring systems of materials properties and process characteristics control.

### 1. Introduction

Thermal insulation materials (TM) are widely used in various industries and transport. The problem of the effective use of TM is highly appointed when changing temperatures in a wide range, for example, in furnaces during the manufacture of products by founding, or in the Arctic, during the thermal protection of constructions, product pipelines, objects of mobile transport, etc. In particular, for thermal insulation of product pipelines the following materials are most widely used: foams, roofing felts, foamed concretes, foamed polyurethanes, rock wools, glass fiber mats, and etc. with a thermal conductivity of 0.02 ... 0.2 W / mK. In TM, which are capillary-porous bodies, heat transfer is carried out both due to thermal conductivity and due to convection of inter-pore gas, that inevitably leads to properties degradation because of the changes in the insulation material moisture. So, moistening the insulation on 2 ... 5% reduces its heat-insulating properties by 20 ... 30% or more, and during it the change in heat-insulating properties occurs gradually.

The practical problem of the gradual change in properties is typical for all types of TM due to the exposure of such factors as temperature and humidity drop, water phase transitions inside the insulation, compression / swelling of the insulation, polymer ageing, gases diffusion, oxidation, etc.

TM often perform the functions of outer protective sheaths of products and are also subjected to negative mechanical effects. A change in thermal insulation properties of a TM can occur both unevenly or gradually depending on the degree of negative mechanical effect. A stepwise change in the properties of TM is usually rapidly detected while a gradual change is difficult to detect and finally



leads to large economic losses. In this regard, the problem of rapid detection of a gradual negative change beginning in thermal insulation materials properties is topical.

The tasks of sequential detection of changes in random process properties, or the problems of disorder detecting, occur in the quality control of serial industrial products, the flowcharting for technological processes of electric power engineering, chemistry, metallurgy, technical and medical diagnostics, and other fields. Trouble-shooting tests, such as Schuhart graphs and the theoretical works of Page and Girshik–Rubin, are widely known and have gained practical application in this subject field. The term “disorder”, as well as the concept corresponding to it, was introduced by A. N. Kolmogorov together with A. N. Shiryayev and described in detail in [1]. A strict problem definition of rapid detection of Wiener process drift was given in this paper. The solution to this problem was worked out by A.N.Shiryayev [1]. The rigid problem statement of the early (promptly) detection of the Wiener process was described in the study.

The main characteristics of sequential methods for detecting the disorder of random processes are the mean time between a neighboring false alarms and the average delay time of detecting a disorder. Many authors have solved a number of intentional problems with definitions of disorder early detection that determine certain practical problems of systems functioning and the processes occurring in them, for example in [2]. The bibliography on the problems of rapid detection of disorders and their subsequent classification is very voluminous. Recently, an interest on method preparation for the rapid detection of changes in random process properties is increasing [3, 4].

Certain common limitations are common to many sequential methods of disorder detection. So, due to a gradual change in probabilistic characteristics of observations for example of meter defects, many methods are not able to detect a gradual change of process properties in time. When implementing Bayesian methods, it is necessary to specify a priori information on the distribution of disorder-inception time, while the accuracy of a priori information significantly affects the value of delay detection, which is often unacceptably large for practice. The presence of such imperfections, the need to reduce their influence upon the characteristics of sequential methods for the gradual disorder detection of random processes causes the topicality of enhancing the existing and finding new methods for solving various practical problems.

## 2. Problem setup

The possibility of detecting a gradual negative change in the thermal conductivity coefficient of TM as a result of exposure to adverse factors is considered. The information-measuring system (IMS) used for the quality control of the TM properties, including the thermal conductivity coefficient, may comprise the measuring transducers - measuring sensors of temperature, humidity, pressure, weight, etc., as well as measuring devices of thermal conductivity; vibration resistance; integrity (solidity) of materials, etc.

The general model of measurement process in the IMS of the TM properties has the form [5]:

$$Z_k = \tilde{N}_k X_k + N_k \zeta_k, \quad k = \overline{1, K}, \quad (1)$$

where  $Z_k$  – measurement vector, the components of which correspond to the controlled properties of TM;  $X_k$  – vector of controllable properties;  $C_k$  – matrix of observations;  $N_k$  – desired matrix corresponding to the accuracy characteristics of IMS meters;  $\zeta_k$  – measurement noise vector;  $k$  – current measurement;  $K$  – number of measurements in a series. The measurement process is multi-step, with the number of measurements  $K$  and the time intervals between measurements being determined including the following main factors – the intensity of degradation of TM properties (determined from field experience) and the cost of metering (by measuring devices).

The model of thermal conductivity measurements has the form

$$Z_{\partial k} = \varphi_{\partial j}(x_{\partial k}) + W_{\partial} \xi_{\partial k}, \quad j = 0, 1; \quad k = \overline{1, K}, \quad (2)$$

where  $x_{\partial k}$  – sequence of initial thermal conductivity data of the tested TM;  $\varphi_{\partial j}$  – known functions characterizing the thermal conductivity of TM;  $j=0$  corresponds to the original data in the absence of, and  $j=1$  – in the presence of the negative changes in thermal conductivity;  $\xi_{\partial k}$  – white discrete centered Gaussian noise of unit intensity;  $W_T$  – noise intensity;  $k$  – time stations separated by equal intervals  $\Delta t$ ;  $K$  – number of measurements in one series.

Threshold values of the output signals are established for meters from the IMS structure on the basis of operational practice, out of which a gradual change in the thermal conductivity of the tested TM is possible. When a signal of a meter (except the thermal conductivity meter) overruns a set threshold, such a meter in the detection procedure is considered as an indicator and its functioning model (1) is replaced by

$$Y_k = \Psi_{s,k}(Q) \quad (3)$$

where  $Y_k$  – the output signal of indicator meter ascribed to the concomitant of the beginning of thermal conductivity change  $Q$  character by the operator  $\Psi_{s,k}$ . The output signal of the meter indicator corresponds to the following values  $s$ :  $s=0$  corresponds to the absence of a concomitant character  $Q$  and  $s=1$  – the presence of the character  $Q$  under the source data  $x_{\partial k}$  control. The operator  $\Psi_{s,k}$  is defined by the conditional transition probability

$$\Psi_{s,k} = \Psi(s_k, k | s_{k-1}, Y_{k-1}, Q, k-1), \quad s = 0, 1; \quad k = \overline{1, K} \quad (4)$$

from state of  $s_{k-1}$  to  $s_k$  state. The description of the functioning of measuring indicators (3) and (4) is universal and does not depend on the physical nature of the measured value.

When measuring  $Z_{Tk}$  in accordance with (2), two hypotheses can take place:  $\Theta = 0$  and  $\Theta = 1$ . The hypothesis  $\Theta = 0$  corresponds to the absence of a change in the TM thermal conductivity, and  $\Theta = 1$  means the presence of a negative change in thermal conductivity, the beginning of which occurs due to the action of one or a combination of negative factors at a random time station  $n$ ,  $1 < n < N$  with a probability close to unity. The change in thermal conductivity occurs gradually and can be characterized by a change in the probability characteristics of the initial data sequence, for example, the law of variation in mathematical expectation. The a priori probability  $P_{k-1}(\Theta=1) = \overline{P}_{1,1}$  is determined from the results of TM exploitation analysis. The probability density function of measurements  $p_0(Z_{Tk}) = p(Z_{Tk} | \Theta=0)$ ,  $p_1(Z_{Tk}/n) = p(Z_{Tk} | \Theta=1)$  in the case of absence and in presence of changes in tested TM thermal conductivity at station  $n$  are also determined by the results of the TM exploitation analysis. According to the results of processing the output signals (2) and (3), it is required in consecutive order to determine the presence or absence of the beginning of a change in sequence  $x_{\partial k}$  properties.

### 3. Theory

Considering the possibility of collecting and processing a sufficiently large amount of data on the change in thermal conductivity during the TM exploitation under various conditions, it is appropriate to develop a Bayesian procedure for detecting the beginning of a gradual negative change in the thermal conductivity of TM. In this case, along with acceptable by accuracy a priori information on thermal conductivity, it is possible to use indicator meters from the IMS of quality control of TM properties. Indicator meters can be used, inter alia, for the adjustment of comparison threshold for the a posterior probability of a thermal conductivity change, calculated on the basis of the output signal from measuring instrument of a thermal conductivity at each  $k$ -th step of current set of measurements. This approach is conditioned by the possibility of accumulating a large amount of a priori data during the TM exploitation, both on the exploitation conditions of the TM and on all kinds of losses from incorrect decisions, including losses associated with the cost of measurements.

It will be considered that the losses of decision making  $u_n = j$  about the absence ( $j=0$ ) or presence ( $j=1$ ) of the beginning of TM thermal conductivity change at the  $k$ -th step, having the form

$$g(\Theta, u_k, n, k) = \begin{cases} g_{0j}(k) & \text{if } \Theta = 0, k < K, \\ g_{1j}(k) + C(k-n) & \text{if } \Theta = 0, k < K, \\ \tilde{g}_{0j}(k) & \text{if } \Theta = 0, k = K, \\ \tilde{g}_{1j}(k) & \text{if } \Theta = 1, k = K, \end{cases} \quad (5)$$

where  $g_{00}(k)$  – losses associated with the correct non-detection of changes. These losses are determined by the cost of measurements;  $g_{01}(k)$  – loss caused by false alarm. These losses are usually associated with the need for additional checks of equipment;  $g_{11}(k)$  – losses associated with the correct detection of the beginning of a change in thermal conductivity. Despite the detection of TM defects, these losses are minimal;  $C$  – losses caused by a one-step delay in detection;  $g_{10}(k)$  – losses arising from the non-detection of changes in TM thermal conductivity. These losses lead to significant costs not only for restoring the TM properties, but also due to possible defects in the stored objects and, as a rule, are the largest;  $g_{0j}(K)$  and  $g_{1j}(K)$  – losses due to a decision made at the end of current set of measurements, similar in content to  $g_{0j}(k)$  and  $g_{1j}(k)$ . With a gradual change in thermal conductivity for almost all types of TM, the following relations are valid

$$\begin{aligned} g_{00}(k) &<< g_{01}(k) < g_{10}(k); \\ g_{11}(k) + C(k-n) &<< g_{01}(k) < g_{10}(k); \\ \tilde{g}_{11}(K) &<< \tilde{g}_{10}(K) < \tilde{g}_{01}(K); \\ \tilde{g}_{0j}(K) &< g_{0j}(k); \quad g_{1j}(k) < \tilde{g}_{1j}(K). \end{aligned} \quad (6)$$

In the development procedure, the Bayesian sequential rule for detecting the beginning of a change in the sequence  $Z_{Tk}$ ,  $k = \overline{1, K}$ , which minimizes the average risk  $r_k(n) = M[g(\Theta, u_k, n, k)]$ , is found on the basis of minimizing a posteriori risk  $r_k(Z_{\dot{O}k}, u_{k-1}) = r_k(Z_{\dot{O}k})$  by selection  $j \in \overline{0, 1}$  [6]. The detection rule takes into account the fact that the posterior risk does not depend on  $u_1, \dots, u_{k-1}$ , as these decisions are attributed with the continuation of measurements because of the non-detection of changes in them in this series. Thus, the equation of the a posteriori risk is given by

$$r_k(Z_{\dot{O}k}) = \min \left\{ \inf_{u_k \in U_{\zeta}} M[g(\Theta, u_k, n, k) | Z_{\dot{O}k}, u_k], \inf_{u_k \in U_{\bar{\zeta}}} M[r_k(Z_{\dot{O}k+1}, u_k | Z_{\dot{O}k}, u_k)] \right\} \quad (7)$$

where  $U_{\zeta}$  and  $U_{\bar{\zeta}}$  – decision region that determine the completion and continuation of measurements, respectively.

Using the formula of full mathematical expectation, as well as the function (5), which takes into account losses from decisions made both during and after the current sets of measurements, and relation (6), we can determine that

$$M[g(\Theta, u_k, n, k) | Z_{\dot{O}k}, u_k] = \begin{cases} g_{10}(k)P_{1,k} + g_{00}(k) & \text{if } u_k = 0, \\ g_{01}(k)(1 - P_{1,k}) + g_{00}(k) & \text{if } u_k = 1, \end{cases} \quad (8)$$

where  $P_{1,k} = P(\Theta = 1 | Z_{\dot{O}k})$  – a posteriori probability of the beginning of a gradual change in the initial data sequence of tested TM thermal conductivity. The resulting solution  $\delta_k(Z_{\dot{O}k}) = u_k \in U_{\zeta}$ , in which losses (8) take the smallest value

$$\inf_{u_k \in U_{\zeta}} M[g(\Theta, u_k, n, k) | Z_{\dot{O}k}, u_k] = q(P_{1,k}) + g_{00}(k), \quad (9)$$

$$q(P_{1,k}) = \min \{g_{10}(k)P_{1,k}; g_{01}(k)(1 - P_{1,k})\} \quad (10)$$

has the form

$$\delta_k(Z_k) = \begin{cases} j=0 & \text{if } g_{10}(k)P_{1,k} < g_{01}(k)(1-P_{1,k}), \\ j=0 & \text{if } g_{10}(k)P_{1,k} > g_{01}(k)(1-P_{1,k}). \end{cases} \quad (11)$$

For the second minimized component in (7) related to the continuation of measurements, by analogy with (7) - (10), the following dependences will be valid

$$\begin{aligned} \rho_K(Z_{\dot{O}k}) &= M[r_k(Z_{k+1}, u_k | Z_{\dot{O}k}, u_k)] = \\ &= \min\{q(P_{1,k}) + \tilde{g}_{00}(K), M[r_K(Z_{\dot{O}k+1}, u_k | Z_{\dot{O}k}, u_k)]\}, \quad k = K-1, \dots, 1, \\ q(P_{1,k}) &= \min\{\tilde{g}_{10}(K)P_{1,K}; \tilde{g}_{01}(K)(1-P_{1,K})\}, \\ M[r_K(Z_{\dot{O}K+1}, u_k | Z_{\dot{O}k}, u_k)] &= \begin{cases} \tilde{g}_{10}(K)P_{1,k} + \tilde{g}_{00}(K) & \text{if } u_K = 0, \\ \tilde{g}_{01}(K)(1-P_{1,k}) + \tilde{g}_{00}(K) & \text{if } u_K = 1. \end{cases} \end{aligned}$$

By the sense  $\rho_K(Z_{Tk})$  is the smallest future posterior risk. The moment of measurements stop is defined as

$$\tau_\zeta = \inf\{k\Delta t : q(P_{1,k}) = \rho_K(Z_{\dot{O}K})\} \quad (12)$$

The optimal rule for detecting gradual negative changes in the initial data can be represented as [6]

$$\delta^* = \begin{cases} u_k & \text{if } k\Delta t < \tau_\zeta; \\ j=0 & \text{if } k\Delta t = \tau_\zeta, P_{1,k} < c_\zeta; \\ j=1 & \text{if } k\Delta t = \tau_\zeta, P_{1,k} > c_\zeta, \end{cases} \quad (13)$$

where the value of the threshold  $c_\zeta$  at the moment of measurements stop is determined as

$$c_\zeta = g_{10}(k) / [g_{10}(k) + g_{01}(k)] \quad (14)$$

When a meter-indicator registers a sign of a possible negative change in the initial data, regardless of the decision made in (13), after a moment (12) a new series of measurements begins.

The posterior probability  $P_{1,k}$  of the beginning of a gradual change in the sequence of the initial data of the tested TM thermal conductivity  $Z_{Tk}$  at the  $k$ -th step is calculated by the Bayes formula

$$P_{1,k} = [P_{1,k-1}\Lambda_k] / [P_{1,k-1}\Lambda_k + (1-P_{1,k-1})] \quad k = \overline{1, K}, \quad P_1(k=1) = P_{1,1}, \quad (15)$$

where  $\Lambda_k = p_{1,k}(Z_{\dot{O}k} | Z_{\dot{O}k-1}) / p_{0,k}(Z_{\dot{O}k} | Z_{\dot{O}k-1})$ .

The process of fixing the obtaining of the threshold value by any of the indicator meters in (3) is incomparably shorter in time in comparison with the process of accumulation the defects in TM, leading to a gradual disorder of thermal conductivity. In this problem, indicator meters, with the exception of a device that measures the TM thermal conductivity, can be considered as non-inertial. Then, at the moment  $n$  of the corresponding signal change at the output of the indicator meter, we can assume that the changes in thermal conductivity will occur with probability  $P(\Theta=1)=1$ . This means that it is advisable to compare a posterior probability (15) with a threshold value that takes into account losses from decisions at the end point pre-determined by the duration of a set of measurements

$$c_\zeta = \tilde{g}_{10}(K) / [\tilde{g}_{10}(K) + \tilde{g}_{01}(K)] \quad (16)$$

while the rule for detecting the beginning of a negative change will be

$$\delta_n^* = \begin{cases} j=0 & \text{if } P_{1,k} < c_{\zeta n}; \\ j=1 & \text{if } P_{1,k} > c_{\zeta n}. \end{cases} \quad (17)$$

After the moment matched with the fixing by the meter-indicator the possible beginning of the thermal conductivity change character, the measurements in the current set should be continued until

the time  $\tau_3$ , determined in accordance with (12), if no changes have been detected before that according to rule (17). When comparing the threshold values (14) and (16) and taking into account the specified relations (6), it is clear that when using information from the character of conditionally non-inertial indicator meter, a significant delay reduction in determining the beginning of a negative change in TM thermal conductivity due to decrease of the threshold level is possible.

#### 4. Results and discussion

The study of the designed procedure for detecting the beginning of a gradual change in TM thermal conductivity was carried out by simulation.

As the test feature the model (2) was accepted, in which  $\varphi_{\partial 0}(x_{\partial k}) = l_0 k$ ,  $\varphi_{\partial 1}(x_{\partial k}) = l_1 k$  are the linear segments with angular coefficients  $l_0$  and  $l_1$  characterizing the absence and presence of negative changes in thermal conductivity, respectively. Change in thermal conductivity was modeled by mutual replacements of  $l_0$  and  $l_1$  at random time stations. The output signal of the indicator meter was modeled by a Markov circuit into two states  $s_k=0$  and  $s_k=1$ , corresponding to the absence and presence of the characteristic  $Q$  in the sequence  $x_{Tk}$ . It was considered that at the moment of the beginning of the change and up to the decision was made, the character  $Q$  is present in  $x_{Tk}$  with probability of 1. The Markov sequence  $x_{Tk}$  when modeling the functioning of indicator meters was set by conditional transfer probabilities

$$\Psi(s_k, k | s_{k-1}, Y_{k-1}, Q, k-1) = \begin{cases} \Psi(s_k | Y_k (1 - e^{-\Delta t/T})) & s \neq s_{k-1}, \\ \Psi(s_k | Y_k (1 - e^{-\Delta t/T}) + e^{-\Delta t/T}) & s = s_{k-1}, \end{cases}$$

where  $\Psi(s_k, k | Y_k)$  – characteristic of meter indicator:  $\Psi(0|0)$  – probability of correct non-detection;  $\Psi(0|1)$  – probability of false alarm;  $\Psi(1|0)$  – probability of skipping;  $\Psi(1|1)$  – the probability of correct detection of the attribute [7]. The statistics  $\Lambda_k$  in (15) was specified as

$$\Lambda_k = k\Delta t / W_{\partial} \left[ l_k \sum_1^k (Z_{\partial k} - l_{0k}) - 0,5 l_k^2 \right], \quad l_k = l_{1k} - l_{0k}, \quad k = \overline{1, K}.$$

The threshold value, in accordance with the procedure (8)-(15), was calculated by the formula

$$c_{\zeta} = 1 - 2W^2 g_{00}(k) / [l_k^2 - g_{10}(k)],$$

and when any of the meter-indicators at the  $k$ -th time station is responded – according to the formula

$$c_{\zeta k} = 1 - 2W^2 g_{00}(k) / [l_k^2 - \tilde{g}_{10}(K)]$$

Existing procedures for detecting gradual disorder in the source data flow are based on processing only the initial data itself. In the designed procedure for the early detection of gradual disorder beside the initial data in which the disorder is finding out it is proposed to use the additional information from meter-indicators, indirectly standing a possible disorder. This approach is new and should be thoroughly investigated in the future. The determination of meter indicators influence (their correlation score with the under study process and their probabilistic characteristics) on the delay value in decision-making on the disorder is most interesting and significant.

The simulated results performed in accordance with Section IV, and processed according to the Monte Carlo method, showed that the delay in detecting the beginning of a gradual change in thermal conductivity for various TM and various numerical ratios in (6) is on average reduced by 20% in comparison with the existing procedures. The economic effect on the implementation for the procedure worked out of early detection for gradual changes in the TM properties is to be evaluated using specific operational examples.

#### 5. Conclusion and summary

A procedure for detecting the beginning of a gradual negative change in the initial data has been developed in Bayesian formulation on an example of the analysis of TM thermal conductivity. Main feature of this procedure is the use of additional data from meter indicator, which allows to reduce the delay in detection. A variant of reducing the delay detection by lowering the threshold value for comparing the posterior probability of the beginning of a change at time fixing by the indicator-meter of the possible presence of a character in the initial data change is considered to reduce the delay in detection. The delay value in detection, as well as the number of false alarms, significantly depend on the assigned losses when making decisions. Also, the value of delay in detection depends on the characteristics of the indicator meter. It should be noted that a negative change in the initial data can also occur with an unregistered character, while fixing the possible presence of a character does not necessarily lead to a subsequent change in the initial data of the observed series. In cases when long-standing processes are analyzed, it is advisable to pre-assign the duration of measurements set. The reliability of detecting the beginning of change increases with sharing of several meter indicators, which do not necessarily simultaneously fix the corresponding change indicator. Using each specific indicator, due to its “correlations” with the process under study, the loss function (6) that affects the threshold (16) is individual. In an intelligence IMS another procedure for detection of the beginning of a gradual change in the TM thermal conductivity it is advisable to start whenever some meter-indicator is operated, and, according to the results of previous tests correct a priori information, in particular, the loss function (6).

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