

Noise proof coding based on orthogonal functions

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Abstract. In article the encoding scheme for messages with use a family of orthogonal functions is suggested. For coding the message, the waiting sum of orthogonal functions is used. Weight coefficients are formed from bits of the transferred message. The received function is transferred on a communication channel as analog signal. For decoding the message, the orthogonality property is used. Integration is carried out numerically. Existence errors of decoding depends on accuracy of a numerical integration. This scheme allows to transfer messages on strongly noisy channels. At increase in noise level it is necessary to increase accuracy of an integration. The computer experiment is made. The dependence a step of a numerical integration on noise level is received.

1. Introduction

Recently schemes with using dynamic chaos for concealment the transfer the substantial message gained development. The useful signal is somehow mixed to a chaotic signal. For development a chaotic signal chaotic generators are used. Chaotic signal intensity considerably exceeds useful signal intensity. The main problem consists in extraction by host the useful component of the accepted signal. For this purpose, on a communication channel additional information on the chaotic generator is transferred. On the basis of additional information, the host self-contained generates a chaotic component and subtracts it from the received signal. This approach can be characterized as formation two bound identical chaotic generators. Several various approaches gained development: chaotic masking [1], switching the chaotic modes [2], nonlinear mix the transferred message to a chaotic signal [3], modulation the operating parameters of the chaotic generator [4].

The most prime in realization is the method of chaotic masking the message [1]. The transferred message $m(t)$ added with a chaotic signal $x(t)$. The received mixed signal of $m'(t)=m(t)+x(t)$ is transmitted to host. The primal problem consists in synchronization the chaotic generator $u(t)$ with the transferring site $u(t)=x(t)$. The transferred message can be received by a simple subtraction $m(t)=m'(t)-u(t)$. In chaotic masking schemes developed now noise level in comparison with the useful signal is 35–65 db [5]. The quality of the received signal strongly decreases if there is a mistiming the operating parameters of noise generators [6–8].



One of possible approaches to a subtraction a noise from transmitted signal is the orthogonalization the chaotic signals. Such schemes gained development within the general approach to modulation signals, watered the name DCSK (Differential Chaos Shift Keying) [9]. One of problems these systems is creation the orthogonal chaotic signals. For the solution this problem ordinary noise generators are used with application to them Gilbert transformation [10-12], Gram-Schmidt transformations [13-15] or Walsh's codes [16–18]. The low channel capacity is characteristic of schemes based on orthogonal chaotic signals. Each orthogonal chaotic frame has one bit of information.

In this article the signal encoding scheme steady against the strong noise is suggested. This scheme doesn't demand synchronization the noise generators.

2. Coding of the message

Let's present the message as the bit's sequence

$$M=b_0b_1...b_N.$$

Let's break the message into frames with length n . Quantity of frames is $k=[N/n]+1$. The message can be submitted as the frames sequence

$$M=M_0M_1...M_k.$$

Transfer of the message is carried out on one frame. It is enough to consider transfer the one frame. Let's consider further that the message consists one frame.

$$M=b_0b_1...b_n.$$

Let's choose family of orthogonal functions $f_i(t)$ ($i=1, ..., n$) in interval $[a, b]$. t – time axis. The condition of the orthogonality is

$$\int_0^b w(t) f_i(t) f_j(t) dt = \delta_{ij},$$

$$\delta_{ij} = \begin{cases} 0, & i \neq j, \\ 1, & i = j. \end{cases}$$

$w(t)$ - weight function.

Let's write down the weighting sum of orthogonal functions

$$F(t) = \sum_{i=0}^n c_i f_i(t).$$

Weight coefficients are calculated based on message bits

$$c_i = 2b_i - 1 \quad (i=1, \dots, n).$$

For $b_i=0$ we have $c_i=-1$. For $b_i=1$ we have $c_i=1$. Let's carry out sampling the function $F(x)$ with step h on an orthogonality interval $[a, b]$:

$$F_j = F(a+jh), \quad (j=0, \dots, s, \quad s = [(b-a)/h]).$$

The sequence F_j is transferred on information channel. If channel bandwidth is equal ν , then frame period is equal $T=s/\nu$.

For decoding the message, the property of an orthogonality is used. Let's calculate values

$$d_i = \int_a^b w(t) F(t) f_i(t) dt.$$

It will be ideally received

$$d_i = \sum_{j=0}^n c_j \int_a^b w(t) f_j(t) f_i(t) dt = \sum_{j=0}^n c_j \delta_{ij} = c_i.$$

The value of the function $F(x)$ is known only in some finite number of points in interval of an orthogonality $[a, b]$. For calculation the values d_i only the numerical integration can be used. Results will be received with some error. Therefore, for calculation coefficients c_i it is necessary to use the threshold scheme:

$$c_i = \begin{cases} 1, & d_i \geq 0, \\ 0, & d_i < 0. \end{cases}$$

Accuracy of calculation for integrals depends on the used numerical integration algorithm and sampling step h . In our scheme the sampling step is set at a stage of formation the transferred sequence F_j .

The submitted scheme is steady against transmission the messages on strongly noisy channels. Let at information channel there is the uniform noise $H(t)$. The host will receive a signal

$$G(t)=F(t)+H(t).$$

Coefficients will be calculated

$$e_i=d_i+h_i,$$

$$e_i = \int_a^b w(t)G(t)f_i(t)dt.$$

$$h_i = \int_a^b w(t)H(t)f_i(t)dt.$$

For truly accidental noise at an analytical integration equalities have to be carried out:

$$h_i=0 \ (i=0,...,n).$$

Some coefficients will be nonzero because of numerical integration inaccuracy ($h_i \neq 0$). If intensity of noise is higher, than it is more than nonzero values h_i . Accuracy of calculations can be increased having reduced a sampling step. It will lead to increase the frame period. The message will be longer transferred.

3. Computer experiment

The computer experiment was made for definition the resistance this scheme to noise of various intensity. Testing was held for family of orthogonal functions

$$f_j(t) = \sqrt{2} \cos(\pi j t).$$

The weight function for this family of orthogonal functions is equal to unit $w(t)=1$. Orthogonality interval is $[0,1]$. Length of the message was equal $n=8$. As messages all eight-bit values from 0 to 255 sequentially got over. Noise was generated by means of the linear congruent generator for the pseudorandom sequences. Each message was coded. After that noise was imposed and decoding was made. For each message and each intensity of noise the experiment repeated 100 times. Between the coded and decoded message Hamming's distance was calculated. Noise level changed in the range from 0 up to 50 db. For an integration the method of trapezes was used. At zero intensity of noise the message is decoded without mistakes at sampling step $h < 0.1$. Dependence the average Hamming distance on an integration step at various noise level are presented in figure 1.

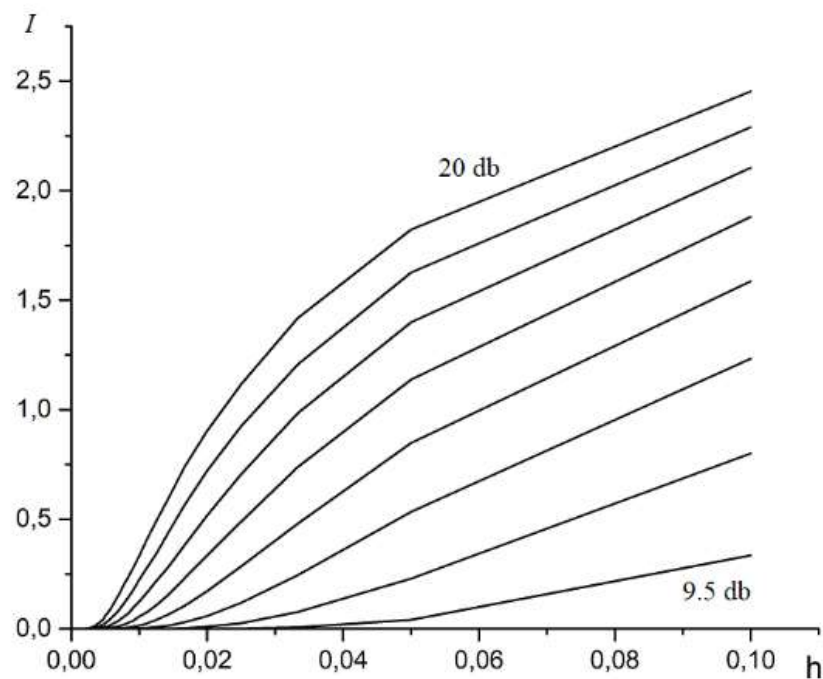


Figure 1. Dependence the average Hamming distance between the coded and decoded message from a sampling step at various intensity of noise.

Apparently from the figure 1 step of sampling at which the message is decoded without mistakes decreases with increase in noise level. Dependence the minimum quantity of function values in a frame from noise level it is presented in figure 2.

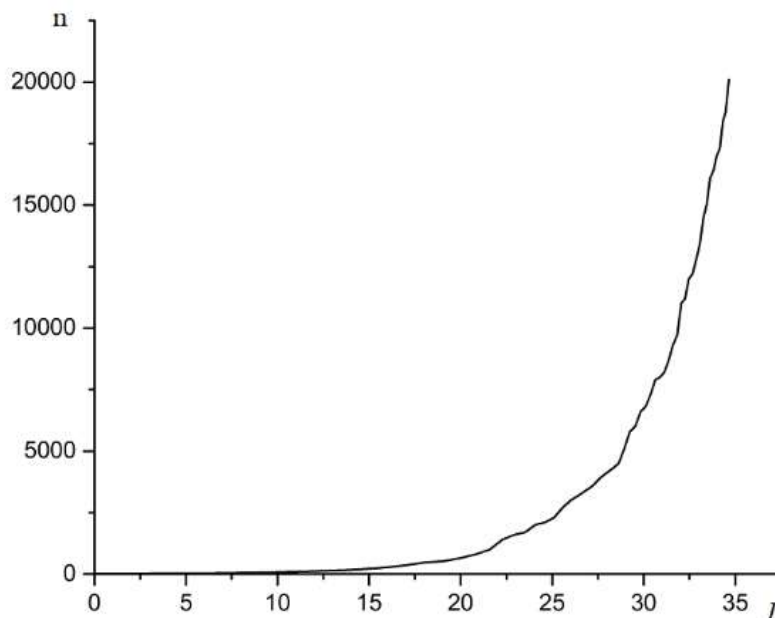


Figure 2. Dependence the minimum quantity of function values in a frame s from noise level I for decoding without mistake.

Apparently from figure 2 that at increase in intensity of noise it is necessary to increase frame's transfer period for steady decoding.

4. Conclusion

The method for coding the messages suggested in this article allows to realize the chaotic masking scheme of a signal without coordination the chaos generators. As a result, resistance to the accidental noise in communication channels significantly increases. The main problem for creation this scheme is ensuring high precision of a numerical integration. Accuracy of an integration affects to correctness of extraction the transferred message. Mistake size at integrals calculation can vary by means of a function sampling step. Decrease the sampling step leads to decrease in number of inaccurately taken bits.

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