

Visualization of the Ruled surfaces of General Type

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Abstract. A vivid example of the ruled surfaces of general type are the carriageaway surfaces and surfaces of roadsides. They are both projected and realised as the ruled surfaces, and since one of the guides - the axis of the road - is a spatial line of general type and the generatrix slope depends on the curvature of the axis plan, it is not possible to classify the surface. Visualization of the road surface is an urgent task, since it is connected with the definition of such basic characteristics of the road as its visual clarity and smoothness. The automation of obtaining dynamic changes of the visual smoothness of the road is of particular importance.

Keywords: surface; ruled surface; surface of general type; surface visualization.

1. Introduction

Ruled surfaces are used everywhere, including design and construction of highways. Later on, the discussion will be focused specifically on highways, since they, when they bend around the Earth's surface, are an example of the ruled surfaces of general type. The question arises: why are the roads formed of precisely ruled surfaces. It happens, because highways, according to the existing requirements set forth in the textbooks [1] and the highways' Building Regulations [2], are realised as compartments of the ruled surfaces. There is no specific name for the "ruled surface", but many facts allow to interpret the answer unambiguously. Here the transverse profiles of the road should be taken into account, compiled as segments of four straight lines (two segments for the roadway and two for roadsides) [1], regulated by the Building Regulations [2] slopes of these segments (there is no other interpretation, since the concept of "slope" is applicable specifically to straight lines), as well as methods of creating the surfaces themselves with the help of bulldozers and profiler machines that have cutting edges in the form of straight lines. It is also necessary to mention the rollers compacting the surface of embankments, the working surfaces of which represent the rotating cylinders. The straight line, moving in space, creates a ruled surface, which is taken into account.

The ruled surface of general type is characterized by the mutual independence of three variable parameters, each has its own functional connection with a certain parameter t , if the existing functional connections are given in a parametric form.

Designing, construction and reconstruction of an existing road are closely related to the visualization of ruled surfaces. In our country, roads and bridges are built everywhere, that is why various models - geometric and mathematical - to simplify the designing and construction stages are relevant.

2. Formulation of the problem

The perspective of representation of the road is quite significant for evaluating the project in road design due to the fact that visual smoothness, visual clarity, its external harmonicity are determined by it, and they also solve the problem of the design and placement of orientation tools for drivers [1; 6-11]. Therefore, consideration of the visualization of the road through the automatic construction of a perspective representation is quite reasonable in the design. An inspection or review of the projected



road before construction is a very important task for road builders: after all, the above-mentioned criteria of the evaluation of the road based on the differential geometric characteristics of the five main lines (road axis, edges of the carriageway, shoulders) make it possible to accurately determine the visual smoothness. Visual clarity and visual smoothness play a vital role in ensuring road safety.

Based on the above explanation, there is a practical need of visualization of the highway – both designed and reconstructed – to obtain the characteristics of visual smoothness and visual clarity, and as a result, to develop the safest road design.

3. Theory

The ruled surface when forming it in R^3 by means of a moving straight line has three guides [3; 4]. For the surface of the carriageway, this is: the axis of the road, the edge of the carriageway and ∞^1 planes normal to the horizontal projection of the road axis; for the roadside surface this is: the edge of the carriageway, the roadside shoulder and the same ∞^1 planes. Thus, ∞^4 straight lines have three fixed parameters [2] and there remain ∞^1 straight lines, in other words, a ruled surface. We will consider the compartments of the ruled surfaces enclosed between the axis of the road and the edges of the carriageway, as well as the edges of the carriageway and the roadside shoulders.

Let there already be systems of parametric equations that are expressions of the road axis q (1), the edges of the carriageway (2) and the roadside shoulders (3):

$$\begin{cases} X_q = X_1(t); \\ Y_q = Y_1(t); \\ Z_q = Z_1(t). \end{cases} \quad (1)$$

$$\begin{cases} X_k = X_2(t); \\ Y_k = Y_2(t); \\ Z_k = Z_2(t). \end{cases} \quad (2)$$

$$\begin{cases} X_b = X_3(t); \\ Y_b = Y_3(t); \\ Z_b = Z_3(t). \end{cases} \quad (3)$$

In [5], specific mathematical models of the systems of equations (1) - (3) are given. Having the equations of all five ruled forms of a road, it is possible to obtain the coordinates of points of transverse profiles with the necessary pitch that form the frame of surface forms. It should be taken into account that the plane to which the transverse profile belongs, passes normally to the horizontal projection of the road axis.

Figure 1 shows the longitudinal profile of the road section, which is a net of a cylindrical surface passing through the axis of the road.

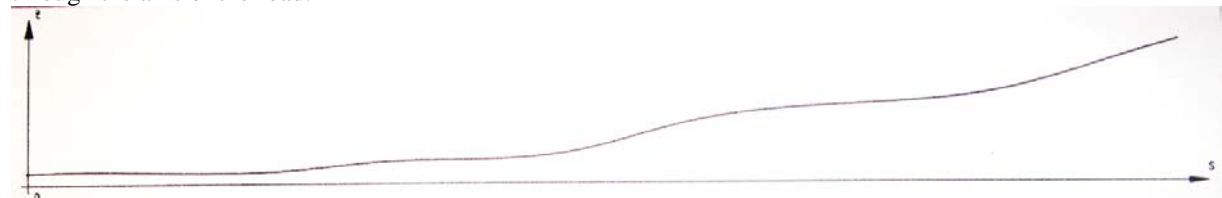


Figure 1. The longitudinal profile of the road.

Let's look at the scheme of obtaining the perspective of the road (Figure 2).

The choice of the point of view of T must meet the requirements for perspective images of the road [11; 12]. For the perspective of the road, it is necessary for the point of view T to be located at a

distance of 2m from the right edge of the carriageway with two lanes, at the height of 1.2m above the road surface. In other words, at the level of the passenger car driver's eye.

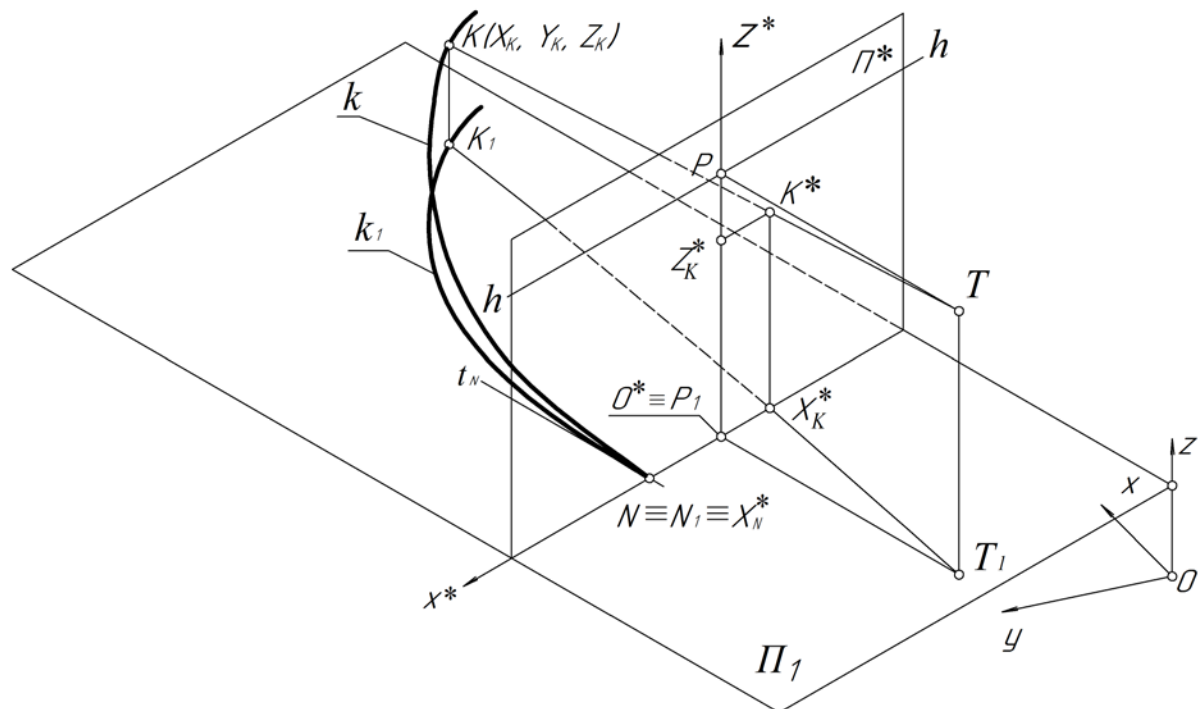


Figure 2. The scheme of obtaining the perspective of the road.

Let the point K belonging to the curve k in the system $Oxyz$ have coordinates X_K, Y_K, Z_K . The Fig. 2 represents the scheme of constructing a perspective of a road surface frame represented by systems (1), (2) and (3). Specifically Fig. 2 shows the perspective view of the point K of the line k , which is one of the five basic lines of the road.

Let's examine the algorithm for detection of the point's perspective. A point K is selected on the axis of the road k , from which the construction of the perspective should begin. The image plane Π^* is defined as the normal plane to the horizontal projection kI , at the point NI . In this case, the plane Π^* will pass through the transverse profile of the road containing the point N . The ray TP will pass parallel to the tangent t_N . We should choose it so that the distance $X_K^*O^*$ should be:

$$X_K^*O^* = b - 2,$$

where b is the width of the carriageway, m.

Let's define the correspondence of points of the perspective coordinates system $O^*x^*z^*$ with points of the orthogonal coordinates system $Oxyz$. In this case, we obtain the transformation formulas of orthogonal coordinates in perspective for any point located on any of the five basic lines, and in general, on any point that belongs to the road.

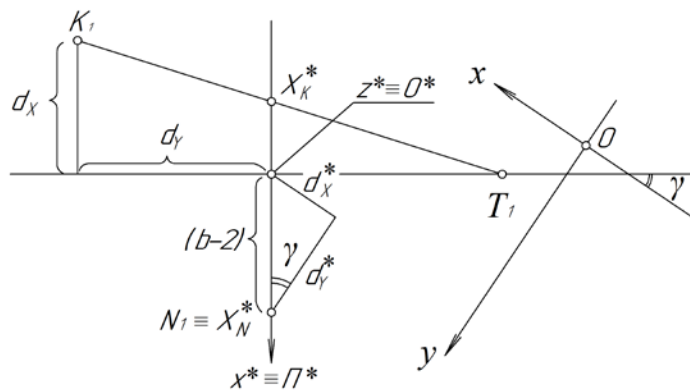


Figure 3. The projection of the perspective apparatus on the vertical plane.

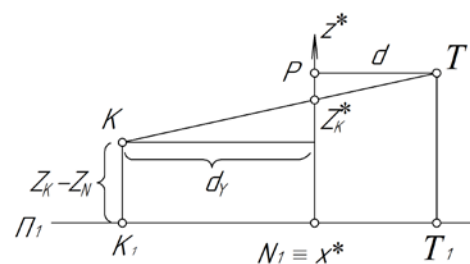


Figure 4. The projection of the perspective on Π_1 .

Figures 3 and 4 show the projections on different planes. Figure 3 shows a projection on the horizontal (objective) plane Π_1 . On Figure 4 – projection on the vertical plane that goes through the main ray TP . From Figure 3 it can be seen that the perspective coordinate X_K^* of point K on the picture plane Π^* can be obtained from the condition:

$$X_K^* = \frac{T_1 O^*}{d_Y + T_1 O^*} d_X.$$

Since $T_1 O^* = TP + b$ (see Figure 4), then:

$$X_K^* = \frac{b}{d_Y + b} d_X \quad (4)$$

From Figure 4 the coordinate Z_K^* of point K can be determined:

$$Z_K^* = T_1 T - \frac{T_1 T + Z_N - Z_K}{TP(Y_K + TP)^{-1}}$$

Considering that $T_1 T = 1.2$ m, $TP = d$ (distance),

$$Z_K^* = 1,2 - \frac{d}{Y_K + d} (1,2 + Z_N - Z_K). \quad (5)$$

Thus, we obtain the system of equations (6), which will be the system of transformation of Cartesian coordinates into perspective ones. Then we should connect this system with the initial Cartesian coordinate system $Oxyz$.

$$\begin{cases} X_K^* = \frac{b}{d_Y + b} d_X; \\ Z_K^* = 1,2 - \frac{d}{Y_K + d} (1,2 + Z_N - Z_K). \end{cases} \quad (6)$$

Let's transform the system (6). To do this, we find the location of the main ray TP and the picture plane Π^* in the coordinate system $Oxyz$.

From Figure 2 and 3 we can find the equation of the line t :

$$\begin{cases} Z = Z_N; \\ Y = Y_N - (Y_N)'(X - X_N) = 0, \end{cases} \quad (7)$$

where X_N , Y_N , Z_N are the coordinates of point N . in the $Oxyz$ system.

The straight line TP is parallel to the straight line (7), which means that its equation will be (Figure 2 and 3):

$$\begin{cases} Z = Z_N + 1, 2; \\ Y = Y_N - d_Y^* - (Y_N)'(X - X_N + d_X^*) = 0. \end{cases} \quad (8)$$

From Fig. 3 follows $tg \gamma = (Y_N)'$, therefore, $d_Y^* / d_X^* = (Y_N)'$ or

$$d_X^* = (Y_N)' d_Y^*.$$

It should be emphasized that the segments d_X^* and d_Y^* are parallel to x and y axes of the coordinate systems $Oxyz$.

On the other hand: $(d_X^*)^2 + (d_Y^*)^2 = (X_N^* O^*) = (b-2)^2$, from where:

$$d_X^* = \sqrt{(b-2)^2 - (d_Y^*)^2}.$$

Comparing both results of the definition d_X^* , we have:

$$(Y_N)' d_Y^* = \sqrt{(b-2)^2 - (d_Y^*)^2} \quad \text{or:}$$

$$d_Y^* = \frac{b-2}{\sqrt{1+(Y_N')^2}} \quad (9)$$

and

$$d_X^* = \frac{(b-2)Y_N'}{\sqrt{1+(Y_N')^2}}. \quad (10)$$

Then we place the obtained values (9) and (10) into equations (8):

$$\begin{cases} Z = Z_N + 1, 2; \\ Y - Y_N - Y_N' (X - X_N) - (b-2)[1 + (Y_N')^2] = 0. \end{cases} \quad (11)$$

The equation of the plane Π^* is obtained from Figure. 2 and Fig. 3:

$$Y - Y_N + (Y_N')^{-1}(X - X_N) = 0$$

We define d_X as the distance from the horizontal projection K_I of the point K to the horizontal projection $T_1 O^*$ of the main ray TP , and d_Y as the distance from the horizontal projection K_I of the point K to the picture plane Π^* :

$$d_X = \frac{|Y_K - Y_N - Y_N'(X_K - X_N)|}{\sqrt{1+(Y_N')^2}} - (b-2); \quad (12)$$

$$d_Y = \frac{|Y_K - Y_N - Y_N'(X_K - X_N)|}{\sqrt{1+(Y_N')^2}} Y_N'. \quad (13)$$

Taking into account (12) and (13) in (6), we obtain the final version of the formulas for converting orthogonal coordinates of the highway's points of $Oxyz$ system into perspective $O^*x^*z^*$:

$$\begin{cases} X_K^* = \frac{d}{\alpha} \left[Y_K - Y_N' (X_K - X_N) - (b-2) \sqrt{1+(Y_N')^2} \right]; \\ Z_K^* = 1, 2 - \frac{d}{\alpha} (1, 2 + Z_K) \sqrt{1+(Y_N')^2}; \\ \alpha = Y_N (Y_K - Y_N) + X_K - X_N + d \sqrt{1+(Y_N')^2}. \end{cases} \quad (14)$$

We obtain systems of equations that provide mathematical support for calculating prospective

coordinates of points for highways, adding equations (14) to systems (1), (2) and (3).

4. Experimental result

In case of a perspective representation of a road, it is customary for designers to depict the nearest transverse profile far from a point T at a distance of 100 meters. Such removal of the observer makes the image inaccurate, misleading [12], since the perception of the nearest transverse profile is removed no further than 5 meters from the observer's point of view.

But since objects located closer than 10 meters are perceived in perspective with some distortions, and perspective correction can lead to disruption of the projection connection, it was decided to construct a perspective with a distance to the picture plane starting from 10 meters and a little further. Based on these considerations, a distance of $d = 20$ meters was taken to construct the perspective.

Figure 5 shows a perspective view of a road section, the longitudinal profile of which is shown on Figure 1.

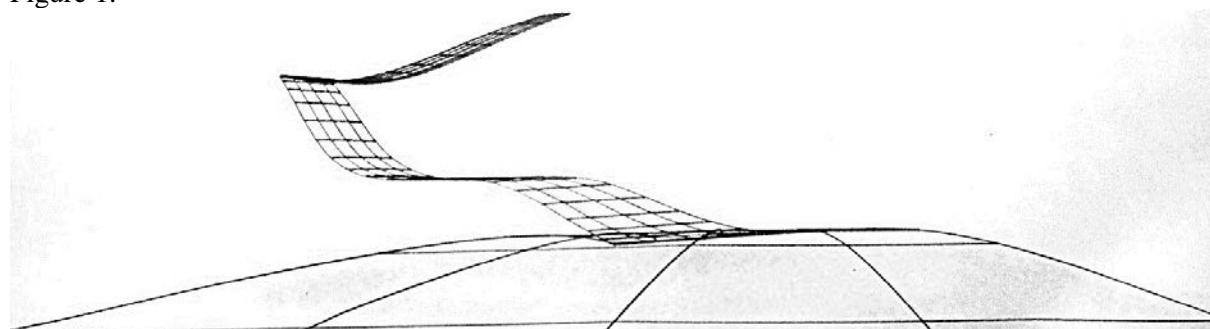


Figure 5. The perspective of the highway.

The adjustment of the longitudinal profile (Figure 8) leads to an improvement in the perspective representation of the road (Figure 9) and, accordingly, its visual smoothness and clarity.

5. Discussion of results

The system (11) also proved to be convenient for obtaining a cinema perspective, in other words, when a car moves along the highway, and this is the next step to obtain visual smoothness and clarity of the road in a dynamic mode, and thus to take the next step to improve the design of highways.

6. Conclusion

As a result of consideration of the available mathematical expressions of the five basic lines of the highway, a mathematical model for visualization of perspective representations of the highway was developed in order to obtain the closest approximation to the normal visual perception of a motor transport driver.

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