Regular linear surfaces in architecture and construction

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Abstract. Linear surfaces are widely used in engineering, architecture and construction, which is explained by their simplicity and adaptability. The linear surface is formed by the movement of a straight line sliding along curvilinear guides. The shape of the guides is determined by the architect-designer and depends on the geometric boundary conditions. At the same time, the surface must meet certain aesthetic criteria. As a result, the shape of the curved guides may be overly complex, and the designed surface becomes irregular, that is, it does not have an exact analytical description. For visualization of the irregular surface, two-dimensional spline approximation methods (Koons surface, Bezier, NURBS surfaces) are used. The technology for making such a surface is, as a rule, extremely complicated. The article proposes a simplified graphical algorithm for constructing a regular linear surface, given by flat guides of arbitrary shape. The algorithm is based on approximation of the guide lines by arcs of curves of the second order. To build a linear surface that passes through given guides, the method of dividing the chords into proportional parts is used. In the general case, a proportional or projective correspondence is established between the points of the chords. The points of division are transferred to curvilinear guides by the method of central or parallel projection, after which the corresponding points are connected by segments of straight lines. In addition to general surfaces, the article considers oblique transition surfaces and wedge-shaped surfaces. The surface of an oblique transition rests on arcs of circles of the same radius. The wedge-shaped surface contains one or two rectilinear guides. The use of such surfaces in the practice of architectural and construction design allows to create a variety of forms while ensuring good technological conditions for the manufacture of the supporting framework of the designed surface.

1. Introduction

Linear surfaces are widely used in architectural and construction design, not only as a constructive tool, but also as a functional and artistic tool of shaping [1-4]. The world-famous architect Gaudi created on the basis of linear surfaces his own architectural language, which found its application in the 21st century [5, 6]. The peculiarity of the Gaudi style lies in the fact that architectural forms having similarities with natural plastics are practically realized due to the wide use of linear surfaces. Straight lines can generate a wide variety of curved surfaces. For example, the wave-shaped roof of theschool near the temple of the Holy Family (Figure 1) is formed by the movement of a straight line along a curved guide. Linear surfaces are studied in descriptive geometry [7-11]. Architects from different times have always realized the connection between nature and geometry. There is a widely known aphorism: "The book of nature is written with geometric symbols." Perhaps this aphorism goes back to the wisdom of ancient Egypt.

The linear surface is generally defined by three guide lines. In the practice of design using private methods of forming linear surfaces, more convenient from an engineering point of view. Particularly, it is necessary to single out the method of proportional breakdown of the guides, which allows replacing irregular surfaces with an ordered (regular) linear surface [12, 13].

Figure 1. The linear wave-shaped roof of the school near the SagradaFamilia (Barcelona).

2. Statementoftheproblem

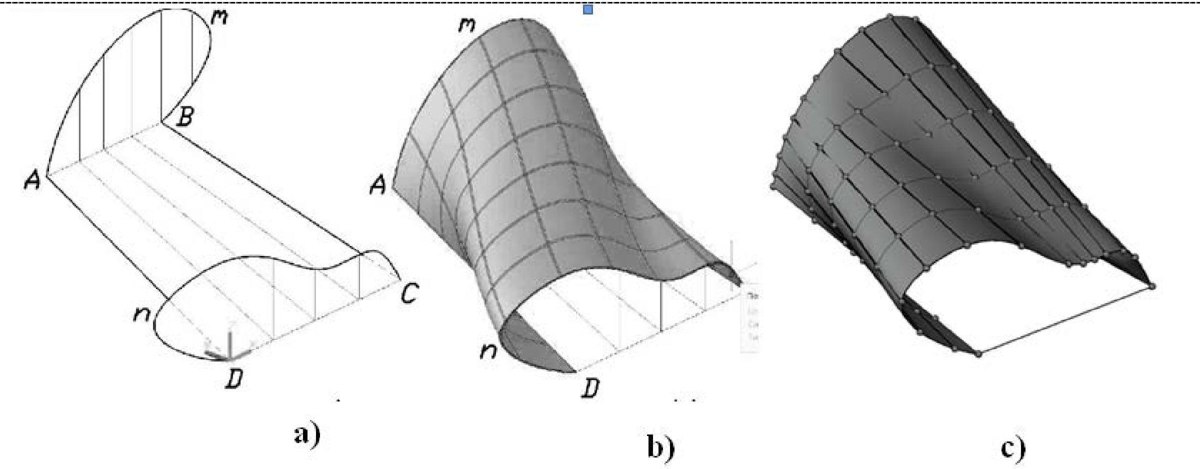

Figure 2. Irregular contour: a) support contour; b) spline approximation (“loft”); c) linear contour.

It is required to construct a smooth contour stretched over a curved bearing contour ABCD (Figure 2, a). Flat curved guides m, n are located in parallel planes. Modern computer graphics tools allow to automatically perform surface visualization using the “loft” command (Figure 2, b). Computer visualization based on two-dimensional spline approximation (Koons surface, Bezier, NURBS surface) can be effectively used for computer animation, computer games and the formation of virtual 3D objects [14-16]. But for the practical construction of a real physical contour, completely different geometric modeling tools are required, and above all the simplest lines: straight lines and curves of the second order [17–19]. Constructive modeling methods based on the use of linear surfaces make it possible to “pull” a simple linearcontour to a given ABCD contour (Figure 2, c).At the same time, the regular linear surface is slightly different from the irregular contour obtained by computer graphics methods. To solve the problem, various geometrical methods are used: proportional division of the guides, use of additional rectilinear guides and other methods [20].

3. Method of proportional division of guides

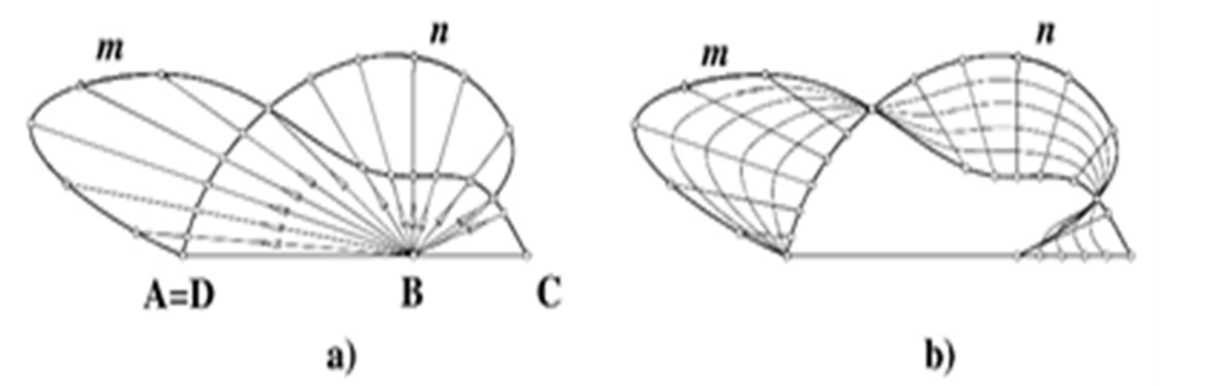

Figure 3. Linearization of irregular surface: a) one-to-one correspondence m≈n; b) forming a linear surface.

Combining the plane of the guide m, n, from the corner point B we draw projecting lines that intersect with the guide curves at the pairwise corresponding points (Figure 3, a). The corresponding points are joined by straight line segments (Figure 3, b). We obtain a linear surface based on a given ABCD contour. The guide curves m, n in the general case cannot be described using the equation, therefore the resulting linear surface is not regular. To obtain a regular surface, you should use simple guide lines consisting of arcs of circles and curves of the second order.

4. Approximation of the guide lines by the arcs of curves of the second order

It is required to construct a surface passing through the curved contour of ABCD and through the predetermined straight line generators AD, BC, a, b, c (Figure 4, a). Replace guides m, n with arcs of the ellipsesm^2, n^2. An arbitrary cutting plane intersects the five given generators at points 1, 2, 3, 4, 5, through which a single second-order curve passes. We draw a set of section planes parallel to the planes of the guides m, n. In each cutting plane, we obtain a second-order curve defined by five points [21, 22]. A set of second-order curves forms the skeleton of a linear surface (Figure 4, b). We obtain a regular contour formed by a continuous motion of a second-order curve sliding along five straight guides (Figure 4, c). The shape of this curve changes from the arc of the DEC ellipse to the arc of the AB ellipse (Figure 5).

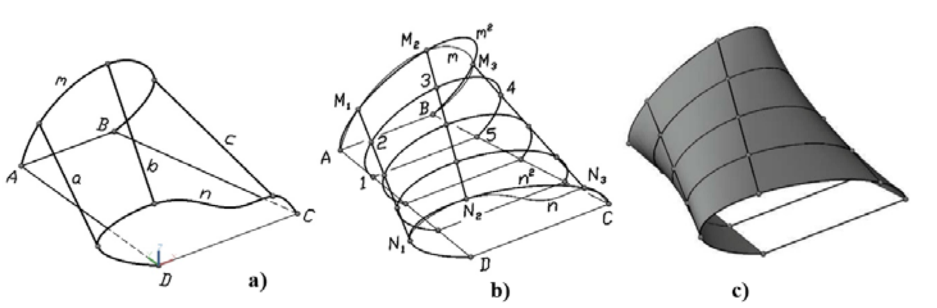

Figure 4. Natural contour: a) linear guides; b) contour casing; c) spatial image of the linear surface

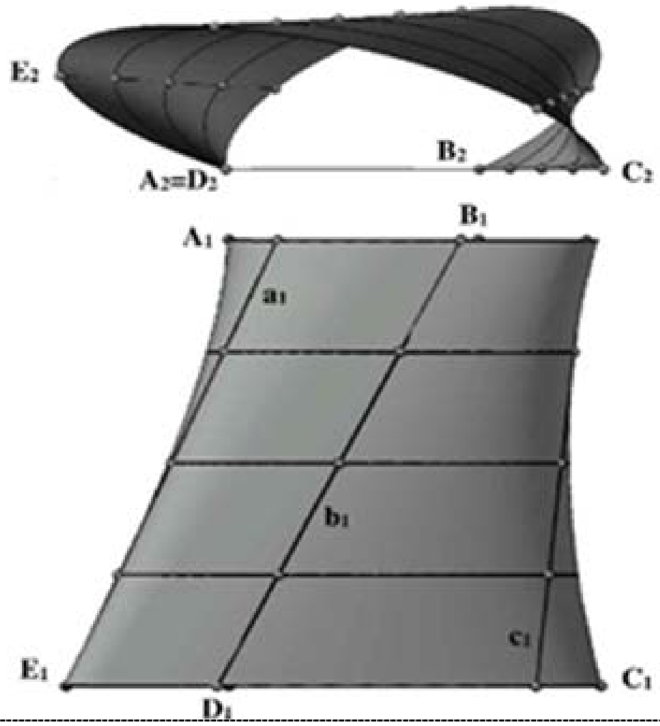

Figure 5. Natural contour.

5. Wedge-shapedsurfaces

The linear surface is fully defined by three guiding lines. One of the guides can be a straight line. In this case, we obtain a wedge-shaped surface (Figure 6).

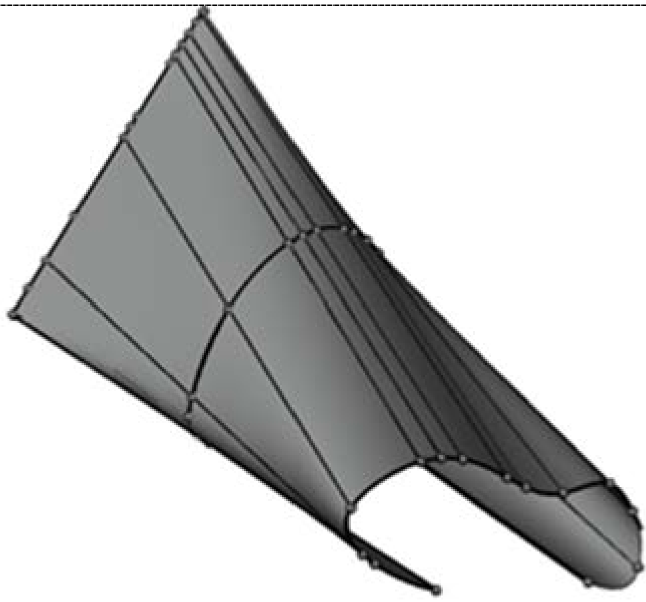

Figure 6. Wedge-shaped surface.

The surface may contain two rectilinear guides. In particular, a fourth-order linear algebraic surface is completely defined by a second-order guiding curve and two crossed rectilinear guides [23-26]. The construction of the framework of such a surface is performed on the basis of the following theorem [27].

Theorem. Let a fourth-order linear surfaceΘ^4given by the second order guide curve$e^2$and two straight guidesu, v, intersecting with plane Σ of the conic sectione^2at points G, Q. Then an arbitrary plane Δ passing through the points G, Q intersects with the surface Θ along a curve of the second order.

Substantiation. Thestraightline GQ crosses the guide line.e^2at two points (real or imaginary), and also intersects guides g, q. It follows that the direct GQ consists of two matched forming surfacesΘ^4. The arbitrary plane Δ passing through the straight line GQ intersects this surface along a fourth-order curve that breaks into a double-counted straight line GQ and into a second-order curveg^2 (Figure 7, a). The theorem is proved. The arbitrary plane passing through a straight line$q=\Pi_1\cap\Pi_2$, intersects a linear surface along the second order curve (Figure 7, b).

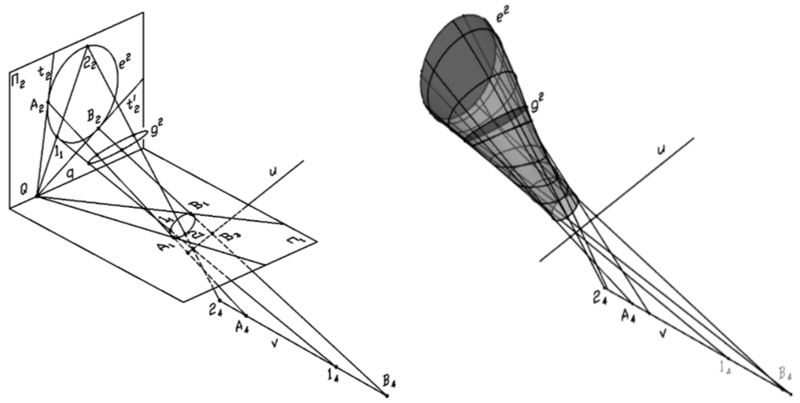

Figure 7. Algebraic surface of the fourth order: a) scheme; b) visualization.

6. Surface of the oblique transition

For the formation of the surface of an oblique transition set two guides of the circle of the same radius with the centersO_1, O_2. The circles are located in the parallel planes$\Gamma_1//\Gamma_2$. The third guide is a straight line passing through the middle of the segment$O_1O_2$and perpendicular to the planesΓ_1, Γ_2.An arbitrary cutting plane intersects the resulting surface along an 8-order algebraic curve. Oblique transition surfaces are used in architectural and construction design.

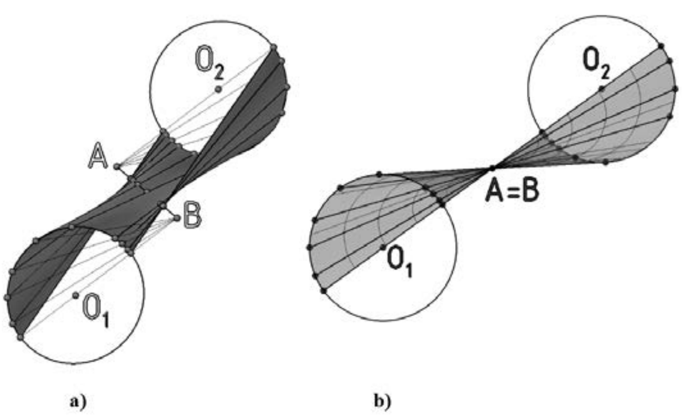

Figure 8. Surface of the oblique transition: a) axonometry; b) orthogonal projection in the direction AB.

7. Conclusion

The regular linear surfaces are widely used in the practice of architectural and construction design. Such surfaces are not only easy to manufacture, but also allow to create nature-like architectural forms. If second-order straight lines and curves are used as guide lines, the linear surface is described by an algebraic equation of the fourth or eighth order. The possibility of obtaining a simple analytical description of a surface is its additional advantage over non-one-dimensional surfaces.

8. References

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