

Presentation of preferences in multi-criterional tasks of decision-making

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Annotation. Fuzzy relationship preferences are widely used in multi-criteria decision-making problems. Although the fuzzy relationship of preference has been intensively researched, and they are very useful in making decisions, there are still problems that need to be investigated. The paper proposes conditions under which fuzzy relationship preferences improve the quality of decision-making.

1. Introduction

A large class of complex systems and processes and a modern information communication system, characterized by integrated, multi-level, the distribution and diversity performance indicators. In fact, the design of such systems, assessment of their structural-functional characteristics and management of ongoing processes are carried out under conditions of informational, procedural, functional, parametric and criterial uncertainties of various types [1]. In particular, these uncertainties apply fuzzy (vague) uncertainty characterized by incompleteness, inaccuracy and linguistic vagueness (fuzziness) present in the source information, the criteria and evaluations of customers and developers, as well as in used models and procedures for description and evaluation of alternatives analyzed variants of objects and their States. The necessity of taking into account in the selection process of the best options of several criteria, including the preferences of decision makers (DM), also characterizes one of uncertainty. This is due to the feasibility of developing and using models and methods of description and assessment of the options (alternatives) of the analyzed objects as well as decision-making (PR) on the choice of the best variant in the conditions of fuzzy uncertainty, which represent a special class of PR task, called unstructured or semi-structured [2]. In these tasks, alternative decisions are evaluated on the basis of the analysis of the soft estimates of indicators of effectiveness of implementation (outcomes) and the values of the risks, corresponding to various outcomes of decisions. The theoretical and methodological apparatus solving such problems is a means of intellectual information technology "Soft Computing" – "Soft computing" [3–7].

In the present paper fuzzy set approaches to the construction of models of the description and evaluation of alternatives and objectives semi-structured decision (SERPS) under fuzzy uncertainty.

We introduce definitions of key terms used in the task.

The alternative is one option of many possible decisions. The outcome – a possible result of the implementation alternatives, i.e., the consequence (object state) occurring as a result of the implementation of the decision. The criterion and performance indicator – the type and characteristics measures, which assesses the effectiveness of the outcomes and their corresponding alternatives. The preferences of the decision maker's subjective criteria based on experience and personal assessment of decision-makers, both internal and external current situation of the environment in which the analyzed



function objects (systems and processes of different nature). Problem situation – a set of alternatives, their outcomes, i.e. the state of the analyzed objects and their corresponding types and estimates performance indicators. Environment – the totality of types of uncertainties, in terms which are rating analyze problem situations and decision making. In this paper we consider a fuzzy environment.

The task of decision-making is formulated as follows. There are many possible solutions (alternatives), the implementation of which leads to the occurrence of some outcomes: one under certainty and several possible – in the face of uncertainty. The outcome may be characterized, for example, a status value, which will move the object in the result of implementation of this alternative. There are further indicators and criteria of efficiency, and, importantly, the subjective preferences of the decision maker. Evaluation of outcomes for selected performance criteria determines the degree of preference corresponding to the outcomes of the alternatives. You want to build the strategy of choice alternatives, the best in accordance with the performance criteria of outcomes and the decision maker's preference. Fuzzy relationship of preferences is one of the most widely used structures for presenting preferences in multicriteria decision-making problems [1]. Various types of preferences have been proposed in the literature, such as fuzzy [2], multiplicative [3], and linguistic preferences [4]. Sometimes decision-makers are not confident in the information provided about preferences due to limited experience in the area of concern. In these situations, experts can provide their information about preferences in the form of incomplete preference relationships, that is, some of its elements are missing [5].

To cope with incomplete relations, stochastic Monte Carlo simulation was considered in [6]. In [7], an iterative procedure was proposed for assessing missing values in relation to an incomplete fuzzy preference of a decision maker. In [8], a method was proposed to minimize the measure of global inconsistency in such a way as to obtain the optimal ratio of preferences. In [9], an interactive algorithm for calculating interval weights for incomplete paired comparison matrices in large problems is presented. In [10], a displacement matrix method was proposed for estimating the missing data. In [11], a procedure is described for solving a mixed problem with missing values. In [12], the linear programming problem was considered to evaluate the missing values in incomplete preference relations. In [13], a method for introducing quality functions was considered, introducing a new approach to decision making in a group that takes into account incomplete information of decision makers using the theory of fuzzy sets.

Today there are problems in the field of solving multicriteria decision-making problems.

2. Problem Statement

Matrix $A = (a_{ij})_{n \times n}$ called a multidimensional preference relation if $a_{ij} \cdot a_{ji} = 1$ and $a_{ij} > 0$ for $\forall i, j$, where a_{ij} indicates a preference intensity ratio alternative from x_i till x_j .

In a multi-dimensional relationship of preferences, the Saati scale is used from numerical values:

$$\left\{ \frac{1}{i}, 1, i \right\}, \quad i = 2, 3, \dots, 9. \quad (1)$$

Let $w = (w_1, w_2, \dots, w_n)^T$ is a priority vector of multidimensional preferences $A = (a_{ij})_{n \times n}$, where $w_i > 0$, $i = 1, 2, \dots, n$ and $\sum_{i=1}^n w_i = 1$. The priority vector characterizes a consistent multi-dimensional relation of preferences [14], that is, elements of the matrix $A = (a_{ij})_{n \times n}$ is $a_{ij} = a_{ik} \cdot a_{kj}$, $\forall i, j, k$,

where

$$a_{ij} = \frac{w_i}{w_j}, i, j = 1, 2, \dots, n. \quad (2)$$

Multidimensional preference relationships are generally incompatible. In these cases, the following expression

$$e_{ij} = a_{ij} \times \frac{w_j}{w_i}, i, j = 1, 2, \dots, n, \quad (3)$$

measures the error between the preference value and the corresponding agreed priority value constructed with the priority vector [14, 15]. Expression (3) can be equivalently written as

$$\log e_{ij} = \log a_{ij} - \log w_i + \log w_j, i, j = 1, 2, \dots, n. \quad (4)$$

So the priority vector $w = (w_1, w_2, \dots, w_n)^T$ can be obtained by solving finding the minimum of the quadratic model [15]:

$$\begin{aligned} \sum_{i=1}^n \sum_{j=1}^n (\log e_{ij})^2 &\rightarrow \min, \\ \begin{cases} \sum_{i=1}^n w_i = 1, \\ w_i > 0, \quad i = 1, 2, \dots, n. \end{cases} \end{aligned} \quad (5)$$

The optimal solution to this model is the geometric mean $A = (a_{ij})_{n \times n}$ [14]:

$$w_i = \frac{\left(\prod_{j=1}^n a_{i,j} \right)^{1/n}}{\sum_{k=1}^n \left(\prod_{j=1}^n a_{k,j} \right)^{1/n}}, \quad i = 1, 2, \dots, n. \quad (6)$$

A complete preference relationship requires the decision maker to provide $n(n-1)/2$ preference values. In real decision-making problems, decision-makers can provide their preference information in the form of incomplete preference relationships, that is, with the absence of some of its elements. Below we give a definition of incomplete preference multiplicative relations.

Matrix $A = (a_{ij})_{n \times n}$ the relation is called an incomplete multiplicative preference if some of its elements are absent and the elements provided satisfy $a_{ij} \cdot a_{ji} = 1$ and $a_{ij} > 0$ for $\forall i, j$.

Let $w = (w_1, w_2, \dots, w_n)^T$ is a priority vector of many alternatives $X = \{x_1, x_2, \dots, x_n\}^T$.

Let $w = (w_{ij})_{n \times n} = \left(\frac{w_i}{w_j} \right)$ is the characteristic matrix associated with the true priority weight

vector. The decision maker can only provide a preference relation based on a discrete scale of Saati values. However, in real life, it is difficult for decision-makers to choose the closest weight to an approximate preference relationship w_{ij} in all cases. The ranking of elements analyzed using the matrix of pairwise comparisons is based on the main eigenvectors obtained as a result of processing the matrices. Let a square matrix be given $A_{n \times n}$. Number λ is called an eigenvalue, and a nonzero vector W square matrix eigenvector A , if they are related by the ratio $AW = \lambda W$.

Eigenvalues of a square matrix $A_{n \times n}$ can be calculated as the roots of the equation $\det(A - \lambda E) = 0$, and eigenvectors as a solution to the corresponding homogeneous systems $(A - \lambda E)W = 0$.

The eigenvector corresponding to the maximum eigenvalue is called the main eigenvector.

The resulting main eigenvector ranks the alternatives and assigns weights to them. Note that the sum of the coordinates of the resulting vector is equal to unity. Thus, we can talk about the relative importance of one or another compared criterion or alternative.

A square matrix has at most n different eigenvalues. Calculate the main eigenvector of a positive square matrix A up to a certain constant factor, it is possible by the formula:

$$\lim_{k \rightarrow \infty} \frac{A^k e}{e^T A^k e} = CW,$$

where $e = (1, 1, \dots, 1)^T$ – vector composed of n units.

The maximum eigenvalue is calculated by the formula:

$$\lambda_{\max} = e^T A W.$$

As can be seen from the above example, the calculation of the eigenvectors and eigenvalues "head-on" is not a trivial task. When calculating the maximum eigenvalue of matrices of the order of more than two, it is almost always necessary to resort to approximate methods. This approach significantly complicates the task, since in the case of one hierarchy the number of matrices of pairwise comparisons can be very large. In the case when a person does not own numerical methods, the hierarchical hierarchy method can be rejected by him at all.

To calculate the eigenvectors and eigenvalues of the matrices, it is advisable to use computational tools and modern software products. However, in the absence of computing power, the approximate value of the main eigenvector can be obtained by summing the elements of each row and then dividing each sum by the sum of the elements of the entire matrix.

The approximate value of the maximum eigenvalue can be found by the formula $\lambda_{\max} = e^T A W$, considered above:

With such a calculation of the main eigenvector and the maximum eigenvalue, it may turn out that the matched in reality matrix is inconsistent in calculations and vice versa.

With a greater error in the method of calculating the main eigenvector, the consistency ratio of the matrix of pairwise comparisons could be greater.

It is advisable to use the procedures for finding exact eigenvalues and matrix vectors. Such a wish turns into a demand for particularly responsible tasks.

Let given:

- many alternatives – $X = \{x_1, x_2, \dots, x_n\}$;
- feature sets – $P = \{p_1, p_2, \dots, p_k\}$;
- degree of fuzzy relationship preference signs $p_j \in P$ on many alternatives X , described by membership functions $\mu_{R_j}(x_{i_1}, x_{i_2})$;
- degree of importance $p_j \in P$, described by membership functions $\mu_R(p_j, p_l)$, $p_j, p_l \in P, j \neq l$ in alternative $x_i \in X$;
- fuzzy attitude of preference alternatives (x_i, x_l) according to $p_j - R_j$;
- fuzzy relationship of feature preferences in alternatives – R .

The degrees of fuzzy relationship of non-dominated preferences, non-dominated alternatives according to the relevant criteria in the alternatives are set by experts.

It is required in the formulated conditions of fuzzy non-domination of alternatives and signs to choose the most acceptable alternative based on the totality of all the signs.

This is achieved using the following algorithm [16].

3. Algorithm for solving the problem

1. Preference matrices are built R_k – considering only a sign p_k .

2. Assuming that the characteristics in question have a different degree of importance: some of them are the most significant, others play a secondary role, the importance of the characteristics is characterized by a fuzzy relationship of preference for the characteristics R.

3. The intersection of fuzzy relationships R_1, \dots, R_k , which is denoted by Q_1 :

$$Q_1 = R_1 \cap R_2 \cap \dots \cap R_k.$$

3.1. For Q_1 found an undeniable set of alternatives Q_1^{UD} .

3.1.1. Inverse matrix is determined Q_1^{-1} .

3.1.2. From each element of the matrix Q_1^{-1} the corresponding matrix element is subtracted Q_1 . Moreover, if the result is a negative number, then it is replaced by zero. The result is a matrix Q_1^0 .

3.1.3. In each row of the matrix Q_1^0 maximum value is found $r(x_i), i = 1, 2, \dots, n$.

3.2. The resulting values are subtracted from unity. The result is calculated $\mu_{Q_1^{UD}}(x_i)$ – desired degrees of ownership of the resulting non-dominated alternatives Q_1^{UD} .

So set Q^{UD} is a collection of elements x_1, x_2, \dots, x_n , each of which has its own degree of affiliation $\mu_{Q^{UD}}(x_i)$ fuzzy set Q^{UD} .

4. For R, an undominated set R^{UD} . Degrees of Membership $\mu_{R^{UD}}(p_1), \mu_{R^{UD}}(p_2), \dots, \mu_{R^{UD}}(p_k)$ are denoted by l_1, l_2, \dots, l_k , c using which weights are calculated $\lambda_m, m = \overline{1, m}$ or each of the signs.

5. Matrix compiled Q_2 , elements of which are calculated by the formula:

$$\mu_{Q_2}(x, y) = \sum_{m=1}^k \lambda_m \mu_{R_m}(x, y) \quad 6. Q_2^{UD} \text{ is find according to the algorithm described above.}$$

7. $Q = Q_1^{UD} \cap Q_2^{UD}$ crossing is construct.

The choice of an alternative having the maximum value of the degree of belonging in Q is considered rational.

4. Computational Experiment

Experiment was carried out for the task of choosing from four breeding varieties: S-4727, Tashkent 1, 108-F, 159-F cotton ($X = \{x_1, x_2, \dots, x_4\}$) best on the following characteristics ($P = \{p_1, p_2, \dots, p_5\}$): productivity, fiber length, fiber strength, absolute weight of seeds, oil content of seeds [17-19].

Quality of cotton is determined by several biological and technological characteristics. For the textile industry the most important technological properties of cotton fiber are: length, metric number, strength, breaking length, maturity and other. These properties largely depend on the characteristics of variety, its ability to absorb and use the nutrients on the soil type, etc. the Need for breeding varieties for nutrients is determined by the interaction between varieties and fertilizers in the environment "variety – soil – fertilizer". The result of this interaction is reflected not only on the quantity of the cotton crop, but also on the technological properties of the fiber.

Many researchers studied the effect of fertilizers on yield of some varieties of cotton. However, these studies were conducted for only one soil type and one variety. The results of these studies are not sufficient for judgments about the responsiveness of different cotton varieties at different doses and

ratios of fertilizers on their productivity in different soil types. The fact that each variety responds differently to those conditions of supply that are created in a particular soil of a difference after application of fertilizers as a result of their specific interaction. Naturally, every soil has a system "soil-fertilizer". Therefore, the response of cotton varieties to fertilizer should only be judged for each particular soil.

Study of changes in the technological properties of cotton fiber depending on the variety, soil conditions and fertilizers allows us to determine the optimal dose and fertilizer application ratio on a particular soil, at which the corresponding grade will give a high fiber yield with the best technological properties. In real conditions, the parameters of the regimes of sowing and agricultural techniques of cotton cultivation are set approximately (vaguely) according to experts. This necessitates the construction of a fuzzy mathematical model that describes the indicated relationship between the studied input and output parameters. This model is the basis for solving the decision-making problem (PR) for choosing an acceptable variety.

Main goal of this work is to develop an algorithm for selecting breeding varieties of cotton with the best biological and technological indicators in the conditions of fuzzy given initial information. In the initial conditions, the alternatives are not clearly dominant in any of the quality indicators individually, nor in their totality.

And under such initial conditions, it is necessary to choose the most acceptable alternative: a variety for the given conditions of sowing, cultivation (agro-technological regimes, components of the dose of fertilizing, irrigation, boundary conditions for these varieties and types of soil).

1. According to the proposed scheme for the formulation and solution of the problem under consideration, fuzzy information about the source data is presented in the form of the following preference matrices R_1, R_2, \dots, R_5 :

$$R_1 = \begin{bmatrix} 1 & 0,78 & 0,66 & 0,61 \\ 1 & 1 & 0,87 & 0,80 \\ 1 & 1 & 1 & 0,92 \\ 1 & 1 & 1 & 1 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 1 & 1 & 0,99 & 0,98 \\ 0,98 & 1 & 0,98 & 0,97 \\ 1 & 1 & 1 & 0,99 \\ 1 & 1 & 1 & 1 \end{bmatrix}, R_3 = \begin{bmatrix} 1 & 1 & 0,99 & 0,99 \\ 1 & 1 & 0,99 & 0,99 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix},$$

$$R_4 = \begin{bmatrix} 1 & 1 & 0,96 & 0,95 \\ 0,98 & 1 & 0,94 & 0,93 \\ 1 & 1 & 1 & 0,99 \\ 1 & 1 & 1 & 1 \end{bmatrix}, R_5 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0,98 & 1 & 0,97 & 1 \\ 0,99 & 0,98 & 1 & 1 \\ 0,95 & 0,97 & 0,96 & 1 \end{bmatrix}.$$

2. Preference matrices are built

$$R = \begin{bmatrix} 1 & 1 & 1 & 0,9 & 1 \\ 1 & 1 & 1 & 0,9 & 1 \\ 1 & 1 & 1 & 0,9 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0,9 & 1 \end{bmatrix}.$$

3. $Q_1 = R_1 \cap \dots \cap R_5$ is find, preference matrices are built Q_1^{-1} and calculated Q_1^0 :

$$Q_1 = \begin{bmatrix} 1 & 1 & 0,66 & 0,61 \\ 0,98 & 1 & 0,87 & 0,80 \\ 0,99 & 0,98 & 1 & 0,92 \\ 0,95 & 0,97 & 0,96 & 1 \end{bmatrix}, Q_1^{-1} = \begin{bmatrix} 1 & 0,98 & 0,99 & 0,95 \\ 1 & 1 & 0,98 & 0,97 \\ 0,66 & 0,87 & 1 & 0,96 \\ 0,61 & 0,80 & 0,92 & 1 \end{bmatrix},$$

$$Q_1^0 = \begin{bmatrix} 0 & 0 & 0,33 & 0,34 \\ 0,02 & 0 & 0,11 & 0,17 \\ 0 & 0 & 0 & 0,04 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

3.2. Matrix is found Q_1^{UD} :

$$Q_1^{UD} = [0,66 \quad 0,83 \quad 0,96 \quad 1].$$

4. According to the same scheme is found R^{UD} and weights coefficient are calculated λ_i :

$$\lambda = [0,19 \quad 0,19 \quad 0,19 \quad 0,24 \quad 0,19].$$

5. Calculated Q_2 :

$$Q_2 = \begin{bmatrix} 1 & 0,95 & 0,9 & 0,89 \\ 0,97 & 1 & 0,93 & 0,92 \\ 0,99 & 0,99 & 1 & 0,96 \\ 0,98 & 0,99 & 0,98 & 1 \end{bmatrix}.$$

6. Q_2^{UD} is found:

$$Q_2^{UD} = [0,91 \quad 0,93 \quad 0,98 \quad 0,93].$$

7. Crossing is construct

$$Q = Q_1^{UD} \cap Q_2^{UD}:$$

$$Q = [0,66 \quad 0,83 \quad 0,96 \quad 0,93].$$

Thus, the ranking results of all breeding varieties showed that the variety 108-F is the best among the proposed breeding varieties of cotton, since the resulting value of the degree of belonging of this variety to the fuzzy set Q is the largest (0.96).

5. Conclusion

In this paper we propose an approach based on the principle of rationality, for comparison among the performance of preference relations. If you increase the size of preference relations using incomplete preference relations have significantly improved the quality of priority vectors. In most cases a relationship of trust with the confidence performs better than incomplete preferences. In addition, the small size of the preferred relations clearly increased the effectiveness of the confidence-dependent relationship.

Analysis and design of complex processes and systems and managing in real conditions, as a rule, occurs in the presence of non-stochastic uncertainty with fuzzy, vague. The typical uncertainties that are present in the process of evaluating their structural and functional characteristics are fuzzy, vague uncertainty, for example, the incompleteness and vagueness of many of the original data, qualitative and subjective nature of the assessment criteria, the heuristic nature of the model source models of the designed systems and processes that contribute to the application of soft treatments for the analysis, prediction, evaluation, choice and decision-making. In these cases, classical statistical methods of operations research do not provide finding and making correct decisions. For the development of these promising methods is the use of intelligent information technologies soft

A promising direction of research on these problems is to develop problem-solving techniques poorly structured decision making using a combination of funds, "Soft Computing" technologies: fuzzy sets, neural networks, genetic algorithms, evolutionary modeling and programming.

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