Computer modeling of filtration processes with piston extrusion

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Abstract. In this work, computer simulation of filtration processes based on the laws of hydrodynamics is considered. The adequacy of the developed models was verified by conducting a series of computational experiments. Created mathematical and software serves the purpose of research, forecasting and decision-making in the development and design of oil and gas fields.

1. Introduction

Achieving energy independence is one of the main goals of any state. Currently, the main sources of energy are oil and gas. However, the limited scope of these products, as well as their search and retrieval, are quite complex issues that require increasingly effective solutions.

In order to satisfy the needs of the real sector of the economy, this resource requires the acceleration of the processes of design, development and commissioning of new oil and gas fields, as well as the most complete extraction of products from old deposits.

Achieving this goal is impossible without developing appropriate mathematical models, effective conservative finite-difference methods and software for a comprehensive study of the processes occurring under a variety of natural and artificial conditions of influence on productive strata.

In [1], a mathematical model of displacement of the front of a multicomponent system in a porous medium was considered. The system is a filtered immiscible (including abnormal) fluids. In real applications, a qualitative description is given of the displacement process in a multi-component system with an intermediate "piston" agent. A mathematical model of a class of problems of frontal displacement for multicomponent systems is formulated as a variational inequality and provides a simple numerical implementation.

In [2], a more complex model of "piston" displacement of oil by water is considered, which takes into account differences in viscosity and density of both liquids. It is assumed that the reservoir is developed using a group of a finite number of production and injection wells (doubly periodic cluster) that periodically repeats in two directions. The filtration of liquids is described by the Darcy law. Fluids are considered weakly compressible, and the pressure in the reservoir - satisfying the quasistationary piezoconductivity equation. The model of piston extrusion leads to a discontinuity of the tangent component of the velocity vector at the boundary of the oil-water contact. Using the apparatus of the theory of elliptic functions together with generalized Cauchy type integrals allows reducing the problem of finding the current boundary of an oil-water contact to a system of singular integral equations for the tangent and normal components

of the velocity vector and the Cauchy problem for integrating the differential equation of motion of the boundary of an oil-water contact. An algorithm for the numerical solution of this problem has been developed. The monitoring of the movement of the contact boundary was carried out for various schemes of flooding (linear in-line, four-point, five-point, seven-point, nine-point and others).

In [3], the results of micro-rheological (on a pore scale) and filtration experiments are presented. The study of the extrusion coefficient in highly permeable rock was carried out by the express centrifugal extrusion method. As a result, it was found that the bulk of the oil is extracted in an anhydrous mode, providing a high level of oil displacement. With this level of oil recovery, it is obvious that the use of secondary methods is impractical. The experiment considered in the article with the following indicators: a small thickness of the boundary layers (~ 0.1 μm at $67°$C), a low viscosity ratio of fluids and a high reservoir temperature showed that these conditions provide a rather favorable displacement process in more permeable rock, anhydrous displacement coefficient close to the end, and the mode of extrusion to the piston. Including for this reason, the article concludes that the use of any additional effects in this case does not give a tangible effect.

The article by N.R. Batrakova [4] is devoted to the development of a mathematical model of three-phase three-component filtration of the "supercritical fluid-water-oil" system in the range of formation permeabilities less than 0.01 Darcy, formation water-cut more than 90%, oil viscosity more than 60 MPa. The model proposed by the authors describes the process of displacing oil from a watered reservoir with supercritical carbon dioxide, taking into account the presence of threshold limitations inherent in traditional methods of enhanced oil recovery.

The study of capillary pressures corresponding to the triangular capillary diffusion tensor in a three-phase fluid is the work of V. V. Shelukhina [5]. Filtering with such a tensor is described by a parabolic system of equations that degenerates on solutions. This system is integro-differential, since the total flow rate and the distribution of phase saturations under the conditions of a given pressure drop in one of the phases at the boundaries of the flow region are required. It is shown that in the problem of capillary extrusion a degenerating system can be investigated on the basis of a special maximum principle.

M.G. Lozhkin [6] created a mathematical model that allows to obtain phase permeabilities for gas, water and condensate when gas is displaced by water and condensate, as well as for oil, gas and water when oil is displaced by gas and water.

The process of displacing oil with a water-gas mixture, taking into account the formation of microbubbles through the use of the foaming properties of oil, was considered by VN Kutrunov and Ye.A. Padin [7]. The authors compared the effectiveness of various technologies of pumping a water-gas mixture and obtained the optimal ratio of water and gas in the mixture, which allows to achieve maximum oil displacement in comparison with standard methods.

M. V. Vasilyeva and T. P. Everstova [8] proposed a mathematical model of two-phase filtration. To solve the problem, the authors constructed difference schemes for differential equations under the corresponding initial and boundary conditions. The results obtained in the implementation of the model to describe the displacement of oil by water are consistent with the results of the work of other authors.

The analysis of these sources showed that the authors' studies did not consider the process of two-way displacement of oil by gas and water from two sides, as a result of which zones of pure gas, a mixture of oil-gas-water, and pure oil are formed. In this paper, efforts have been made to fill this gap. As mentioned above, the mathematical model of the process occurring in reservoir conditions must be formulated on the basis of the provisions of the mechanics of multiphase media in the form of a problem like Stefan with unknown phase boundaries.

2. Methods

In this paper, we study the complex dynamic processes occurring in reservoir conditions when oil is forced out by gas or water in a one-dimensional formulation.

For ease of understanding, let us represent a one-dimensional reservoir in the form shown in Fig. 1.
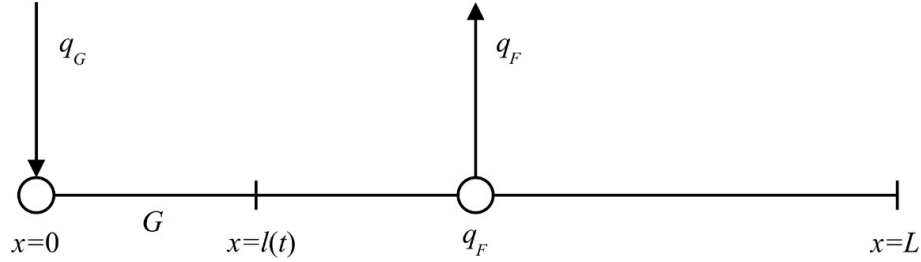

Figure 1. Schematic representation of a one-dimensional reservoir gas-liquid-water.

Gas (water) is introduced into the cross section $x = 0$ with intensity q_G. Oil is withdrawn from the cross section $x = \zeta$ with intensity q_F. Forming $x = L$ the boundary of the porous reservoir. The boundary between the injected gas (water) and oil is variable $x = l(t)$. Using the laws of gas-hydrodynamics, we can formulate a mathematical model of the process of influencing a reservoir with gas volume and fluid advancement in a reservoir, which can be solved to solve the following system of nonlinear differential equations:

$$\frac{\partial}{\partial x}\left(P_{gas}\frac{K}{\mu_G}\frac{\partial P_{gas}}{\partial x}\right) = m\frac{\partial P_{gas}}{\partial t} \ \ with \ \ 0 < x < l(t), \tag{1}$$

$$\frac{\partial}{\partial x}\left(\frac{K}{\mu_O}\frac{\partial P_{oil}}{\partial x}\right) = m\frac{\partial P_{oil}}{\partial t} + F \ \ with \ \ l(t) < x < L(t), \tag{2}$$

Equations (1) and (2) we also write in this form:

$$\frac{\partial}{\partial x}\left(KA\frac{\partial P}{\partial x}\right) = m\frac{\partial P}{\partial t} + F, \tag{3}$$

where

$$K, A, P = \begin{cases} \dfrac{K}{\mu_G}, P_{gas}, P_{gas}, \ 0 \le x \le l(t), \\[2mm] \dfrac{K}{\mu_O}, 1, P_{oil}, \ l(t) \le x \le L, \end{cases} \tag{4}$$

$$F = A_2 q_F \delta(x - \zeta_i). \tag{5}$$

These equations are integrated under the following boundary and internal conditions:

$$\frac{\partial P}{\partial x}\Big|_{x=0} = -A_1 q_G, \tag{6}$$

$$P(x,t) = f(x,t) \ \ with \ \ x = L, \ \ t > 0. \tag{7}$$

The following conditions are set at the moving interface:

$$S_0 \frac{dl}{dt} = -K\frac{\partial P}{\partial x}\Big|_{x=l(t)-0}, \tag{8}$$

$$\frac{K}{\mu_G}\frac{\partial P}{\partial x}\Big|_{x=l(t)-0} = \frac{K}{\mu_O}\frac{\partial P}{\partial x}\Big|_{x=l(t)+0}\,, \tag{9}$$

$$P_{gas}\Big|_{x=l(t)-0} = P_{oil}\Big|_{x=l(t)+0}\,. \tag{10}$$

At the beginning of development, the distribution of pressure and phase saturation are known, as well as the position of the interface:

$$P(x,0) = P_0^0,\ \ l(0) = l^0,\ \ 0 < x < L. \tag{11}$$

In formulas (1) - (11), the following notation is used: S_0 – saturation of the rock with oil; K – absolute permeability of the rock; μ_G, μ_O – the viscosity of the gas and oil, respectively; $P(x,0) = P_0$ – initial pressure distribution; ρ_G, ρ_O – the density of gas and oil, respectively; T – absolute temperature; P_{oil}, P_{gas}– the pressure of oil and gas, respectively; ζ_i– internal special point (injection or production well); $l(t)$– movable interface; L – reservoir length; q_G, q_F– well work intensity; A_1, A_2– some constant values.

To solve the problem, we first go to dimensionless variables, taking

$$x^* = \frac{x}{L},\ \ l^* = \frac{l(t)}{L},\ \ P^* = \frac{P}{P_0},\ \ t^* = \frac{\rho_O K_G RT}{m\mu_O L^2}t.$$

In the dimensionless form, the boundary-value problem (1) - (11), omitting the asterisks, is rewritten as follows:

$$\frac{\partial}{\partial x}\left(P_{gas}\frac{K}{\mu_G}\frac{\partial P_{gas}}{\partial x}\right) = \frac{\partial P_{gas}}{\partial t},\ \ 0 < x < l(t)\,, \tag{12}$$

$$\frac{\partial}{\partial x}\left(\frac{K}{\mu_O}\frac{\partial P_{oil}}{\partial x}\right) = \frac{\partial P_{oil}}{\partial t} + F\ \ with\ l(t) < x < L, \tag{13}$$

$$\frac{\partial P}{\partial x}\Big|_{x=0} = -A_1 q_G, \tag{14}$$

$$P(x,t) = f(x,t),\ \ x = 1,\ \ t > 0, \tag{15}$$

$$F = A_2 q_F \delta(x - \zeta_i), \tag{16}$$

$$S_0\frac{dl}{dt} = -\frac{\partial P}{\partial x}\Big|_{x=l(t)+0}\,, \tag{17}$$

$$P_{gas}\Big|_{x=l(t)-0} = P_{oil}\Big|_{x=l(t)+0}\,, \tag{18}$$

$$\frac{K}{\mu_G}\frac{\partial P}{\partial x}\Big|_{x=l(t)-0} = \frac{K}{\mu_O}\frac{\partial P}{\partial x}\Big|_{x=l(t)+0}\,, \tag{19}$$

$$P(x,0) = P_I,\ \ l(0) = l^0,\ \ 0 < x < 1. \tag{20}$$

Thus, a closed system of nonlinear differential equations was obtained, describing the operation of the "Plast-bore" system. The boundary-value problem describing the filtering process in question relates to tasks like Stefan.

For the numerical solution of the problem in question, we apply the method of rectifying phase fronts. The system of equations describing the formulated problem is non-linear with respect to the desired functions $l(t)$, $P(x,t)$. Therefore, it is impossible to obtain an exact analytical solution of the problem. To solve it, we use the finite difference method [9-11].

The discrete algorithm for solving problem (12) - (20) is based on the use of an integro-interpolation method, which allows us to construct a conservative difference scheme that satisfies the law of conservation at each node of the space-time grid.

We solve the problem using the integro-interpolation method and the average theorem.

The driving factors and the interface are determined using the appropriate formulas. The obtained nonlinear system of equations is solved by the sweep method using the simple iteration method in each time step.

Based on the developed mathematical model (12) - (20) and a numerical algorithm for solving the problem of nonlinear oil-gas-water filtration in a porous medium, a software tool was developed to determine the main parameters and their ranges of changes for the purpose of designing and developing oil and gas fields.

The functional properties of the program include the following: analyzing the dynamic processes of the reservoir filtration system, determining the change in pressure fields and saturations in time and space depending on the hydrodynamic parameters of the object.

The program was developed in the Embarcadero Delphi XE3 environment and consists of the following modules: a module for data entry (Fig. 3); calculation module; a module for analyzing and interpreting the results of the performed numerical calculations (Fig. 4).

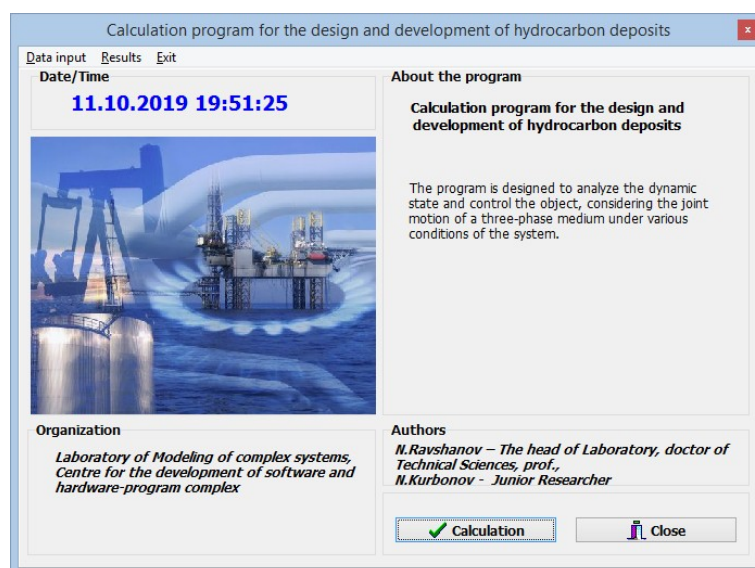After launching the software, the main program window appears on the screen (Fig. 2).

Figure 2. Main window of program.

The main window of the program contains information about the name, functionality, organization and authors - developers. Also in this window are the following interface elements: the menu "Data Input", "Results" and "Exit"; "Calculation" and "Close" buttons.

When you click the "Data Input" menu, a dialog box opens in which the user can maintain and change the necessary data online (Fig. 3).

The "Input Data" dialog box contains fields for entering the following parameters: initial pressure; reservoir porosity; filtration coefficient; thickness, length and width of the reservoir; oil and gas viscosity coefficients; the amount of produced fluid, oil and gas; oil density; atmospheric pressure and calculation accuracy. To reset the values and new data entry, you must click the "Clear data" button. The "Exit" button closes the dialog box.

After entering the parameters, the user must click the "Calculation" button in the main window of the program. The program performs calculations based on the entered object parameters and records the results of the performed numerical calculations in the form of files containing pressure changes in the considered area by day. The program also creates a separate file containing pressure changes for the entire period of time under consideration.

Figure 3. Data entry window.

The program provides an opportunity to get acquainted with the results of calculations in text and graphic forms. To do this, the user needs to select the "Results" menu. The program provides two ways to present the results in text form: in the Notepad system utility and directly in the program interface.

The program also provides the ability to display and analyze the results in graphical form. To do this, the user must select the menu "In graphical view". A corresponding dialog box opens where the user needs to select files for visualization and analysis. You can choose to compare five different files with the results by day. When you click the "Build" button, the results will be visualized as a graph (Fig. 4).
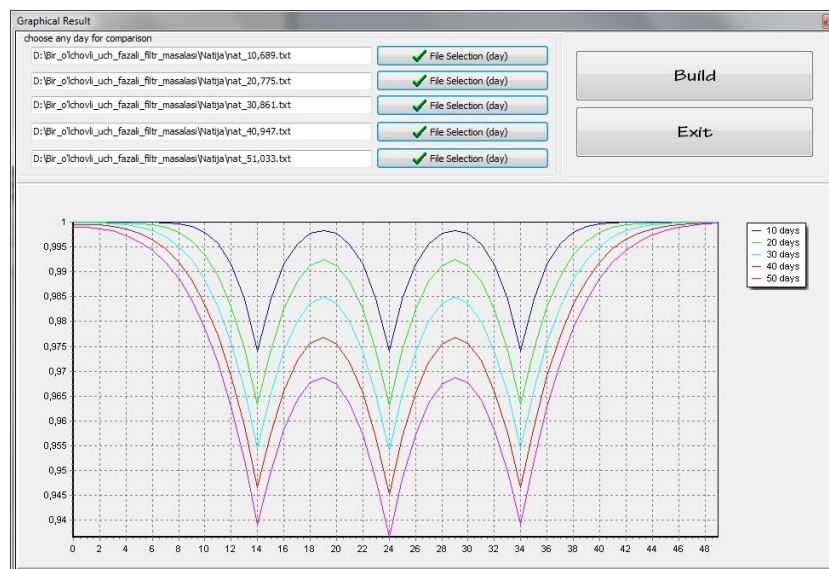

Figure 4. Results visualization example.

3. Results and discussion

The developed program and the mathematical apparatus implemented in it can be used by specialists of organizations engaged in the extraction of hydrocarbons in order to increase the

efficiency of the fields.

With the help of the program a number of computational experiments were carried out. The experiments were carried out with the following input values: L=10 km – the length of the reservoir; B=6 m – power; H=700 m – width; μ_O=4 cP – oil viscosity; the amount of produced fluid - q_m=1000 t/day; m=0.2; K=0,1 Darsy; μ_G=0.2 cP; P_0=200 at; ρ_O=0.85 g/s^3; R=8.31 $J/(mol \cdot K)$; T=273 K; Z=40; N=50; ε=0.0001.

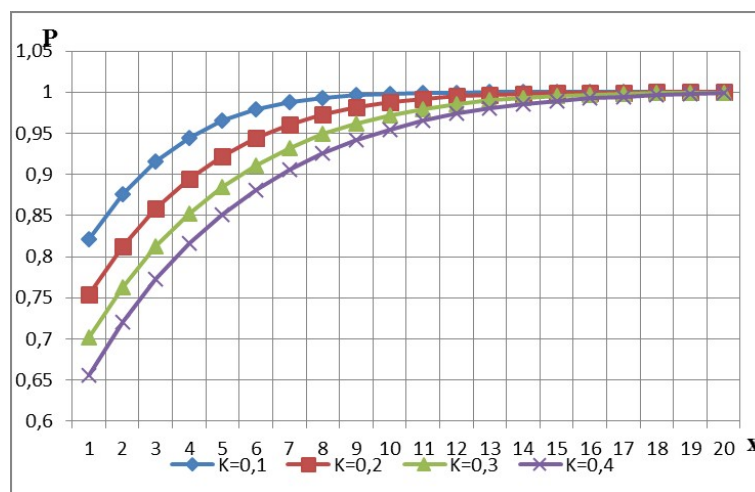The results of the performed computational experiments are given in the works and are shown in Fig. 5-8.

Figure 5. The change in pressure along the length of the reservoir depending from filtration ratio.
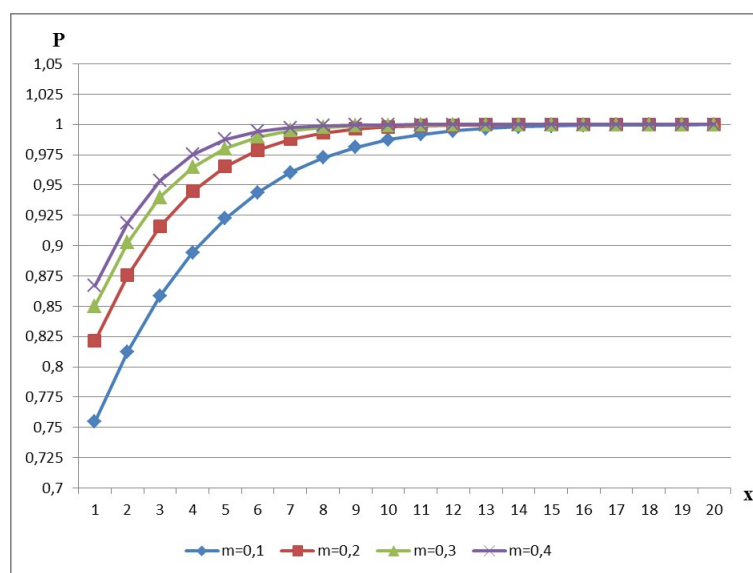

Figure 6. The dynamics of the redistribution of pressure in the reservoir at different values of the porosity coefficient.
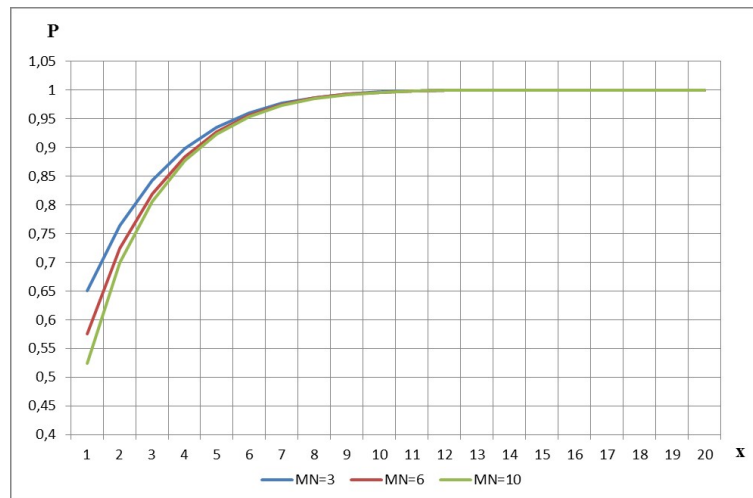
Figure 7. The dynamics of the redistribution of pressure in the reservoir at various values of viscosity coefficient.

The numerical calculations showed that the essential parameters affecting the technology for developing the production of hydrocarbons from reservoir systems are the filtration coefficients, viscosities and the structure of porous rocks (see Fig. 5 - 7).
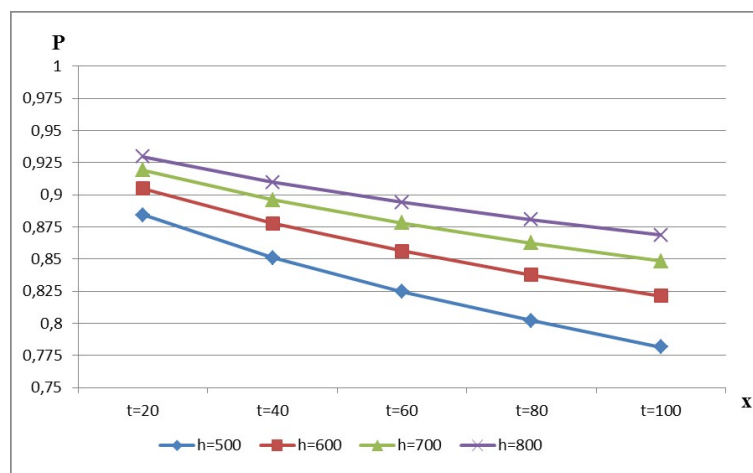

Figure 8. The dynamics of the redistribution of pressure in the reservoir at different values of reservoir width.

The higher the reservoir width, the slower the pressure drop of hydrocarbons in the reservoir during production (Fig. 8).

When fluid sampling occurs within the filtration area, a symmetric pressure drop is observed relative to the location of the wells.Computational experiments were performed at various well flow rates.

Analysis of the results of numerical experiments shows that with the same intensity of production, the pressure drop at the well is faster when filtering oil than when filtering with gas, and when considered in terms of the presence of gas in the oil composition, the fluidity of the mixture increases. The rate of pressure drop on the gallery at high viscosities of oil is getting

faster and faster in time, and at low viscosities of oil - at first faster, reaching some small value, then it starts to fall.

In general, it can be noted that the dynamics of the redistribution of the pressure of the reservoir significantly depends on the thickness of the reservoir. With increasing reservoir thickness, the pressure in the well and adjacent points decreases. And the time of exploitation of the productive formation substantially depends on its length, thickness, number of wells and their flow rates.

4. Conclusion

The developed program and the mathematical apparatus implemented in it can be used by specialists of organizations engaged in the extraction of hydrocarbons in order to increase the efficiency of the fields.

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