

Hydroelastic Response of an End Wall Interacting with a Vibrating Stamp via a Viscous Liquid Layer

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Abstract. The mathematical modelling of a vibrating stamp interaction with a flexible restrained end wall (bellows) of a narrow channel via a viscous incompressible fluid is carried out. The narrow channel formed by two parallel walls and filled with a viscous liquid is investigated. The liquid motion in the channel is considered as a laminar one. The bottom channel wall is absolutely rigid, and the top one is the absolutely rigid vibrating stamp. At the right edge of the channel, the end wall with flexible restraint (bellows) is installed, and at the left one, there is a free liquid flow. The problem of longitudinal hydroelastic vibrations of the channel end wall is formulated and solved analytically. The distribution laws of velocities and pressure in the fluid layer along the channel are found. The motion law of the channel end wall is determined. The amplitude frequency and phase frequency responses of the end wall for steady-state harmonic oscillations are constructed. The mathematical modelling has shown the possibility of damping the channel end wall (bellows) vibrations by changing the distance between the channel walls or by changing the liquid viscosity in the channel.

Keywords Hydroelasticity, Vibrations, Stamp, End wall, Viscous liquid, Narrow channel.

1. Introduction

Nowadays, investigation of the dynamic interaction of the liquid with flexible structural components is an actual problem for modern mechanical engineering, technology and production processes [1]. One of the first works in this field is [2], in which the free oscillations of a circular plate interacting with an ideal liquid were considered. The hydroelasticity model of a beam filled with an ideal fluid for the study of a pipeline transverse vibration was considered in [3]. In reference [4], a hydroelastic oscillations model of a cylinder liner of an internal combustion engine with water cooling was proposed. The nonlinear behaviour of an elastic pipe conveying fluid and supported at both ends was investigated in [5]. In reference [6], the analogous problem was studied for the case the pipe possesses flexibly supported ends. The dynamic behaviour of an annular channel conveying ideal fluid and formed by two cylindrical shells for the case it subjects to the external gas flow was considered in [7]. In reference [8], the walls hydroelastic vibrations of an annular channel filled with pulsating viscous incompressible fluid were considered. The study on the outer wall oscillations of an annular channel filled with a viscous liquid under the inner channel wall vibration for the case this channel is surrounded by elastic medium was carried out in [9]. Oscillations of a circular plate surrounded by an ideal fluid located in a rigid cylinder were studied in [10]. The dynamics and stability of a plate, which is part of the wall separating two viscous liquids, were considered in [11]. The vibration damping of a beam lying on a viscous liquid layer was studied in [12]. The bending vibrations problem of a



cantilever beam surrounded by an unlimited volume of a viscous fluid was solved in [13]. A similar problem for a piezoelectric beam in a viscous incompressible flow was considered in [14]. In reference [15], a mathematical model was developed for the study of plate streamwise vibrations in parallel-walled channel conveying a viscous fluid due to forced transverse vibrations of this plate. Longitudinal and transverse vibrations of a flexibly restrained wall of a narrow tapered channel with a viscous fluid were studied in [16, 17]. The influence simulation of the end seal presence and the end discharge features of the viscous fluid on the disturbing moments in the float gyroscope was carried out in [18]. However, the parallel-walled channel possessing flexibly restrained end wall (bellows) was not considered in the above-mention papers.

2. Statement of the Problem

Let us consider a narrow channel formed by two rigid parallel walls in accordance with the scheme shown in Fig. 1. We associate the Cartesian coordinate system center with the bottom motionless wall of the channel. The upper channel wall is a stamp oscillating along the z -axis. The sizes in the plan view of the bottom and upper channel walls are $2\ell \times b$, let us assume that $b \gg 2\ell$ and consider the plane problem. The distance between the channel walls is δ_0 and $2\ell \gg \delta_0$. The channel is filled with a viscous incompressible fluid. The oscillation amplitude of the stamp is $z_m \ll \delta_0$. At the channel left end, the viscous fluid discharges into the same fluid possessing constant pressure p_0 . At the right one, there is an end wall with flexible restraint (bellows), i.e. this end wall can move along the x -axis with the amplitude of x_m . Further, we will take into account that the pressure in cross section at the channel left end coincides with the constant one p_0 . At the channel right end, the volume flow rate coincides with one due to the end wall vibration (volume flow rate in the bellows).

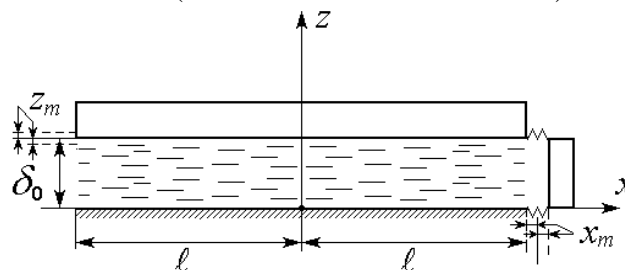


Figure 1. The narrow channel possessing the flexibly restrained wall at the right end.

According to [19], for the plane problem, the motion equations of a viscous fluid are

$$\frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial x} + V_z \frac{\partial V_x}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial z^2} \right), \quad (1)$$

$$\frac{\partial V_z}{\partial t} + V_x \frac{\partial V_z}{\partial x} + V_z \frac{\partial V_z}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 V_z}{\partial x^2} + \frac{\partial^2 V_z}{\partial z^2} \right),$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial z} = 0,$$

where V_x , V_z are the liquid velocity components along the coordinates axes, p is the pressure, ρ is the liquid density, ν is the kinematic viscosity coefficient of the liquid, t is the time.

The fluid motion equations (1) are complemented by boundary conditions, i.e. no-slip conditions at the bottom and upper channel walls

$$V_x = 0, V_z = 0 \text{ at } z = 0, \quad (2)$$

$$V_x = 0, V_z = \frac{dz_*}{dt} \text{ at } z = z_*.$$

In addition, we formulate the conditions at the ends of the channel. These conditions represent the pressure coincidence at the channel left end with the pressure of the surrounding liquid, as well as the volumetric flow rates equality at the channel right end and at the right cavity, i.e. equality of volumetric flow rates at the channel end and the bellows

$$p = p_0 \text{ at } x = -\ell, \quad (3)$$

$$\int_0^{z_*} u_x dz = \delta_0 \frac{dx_*}{dt} \text{ at } x = x_*,$$

Here z_* is the stamp motion law, x_* is the law of channel end wall motion (bellows), p_0 is the static pressure in the liquid. Note, in Eqs. (2), (3) we assume the stamp displacements and end wall one along the x and z axes are presented in the form of $x_* = \ell + x_m f_x(\omega t)$ and $z_* = \delta_0 + z_m f_z(\omega t)$.

3. Determining of the End Wall Response

Let us introduce dimensionless variables and small parameters of the problem

$$\psi = \frac{\delta_0}{\ell} \ll 1, \quad \lambda = \frac{z_m}{\delta_0} \ll 1, \quad \tau = \omega t, \quad \xi = \frac{x}{\ell}, \quad \zeta = \frac{z}{\delta_0}, \quad V_x = \frac{z_m \omega}{\psi} U_\xi, \quad V_z = z_m \omega U_\zeta, \quad (4)$$

$$p = p_0 + \frac{\rho V z_m \omega}{\delta_0 \psi^2} P,$$

where ψ, λ are small parameters characterizing the problem.

By substituting variables (4) into fluid motion equations (1) and neglecting the small terms [20], we obtain the system

$$\frac{\omega \delta_0^2}{\nu} \frac{\partial U_\xi}{\partial \tau} + \frac{\partial P}{\partial \xi} - \frac{\partial^2 U_\xi}{\partial \zeta^2} = 0, \quad \frac{\partial P}{\partial \zeta} = 0, \quad \frac{\partial U_\xi}{\partial \xi} + \frac{\partial U_\zeta}{\partial \zeta} = 0. \quad (5)$$

Carrying out a similar substitution in the boundary conditions (2), (3) we obtain

$$U_\xi = U_\zeta = 0 \text{ at } \zeta = 0, \quad (6)$$

$$U_\xi = 0, \quad U_\zeta = \frac{df_z}{d\tau} \text{ at } \zeta = 1,$$

$$P = 0 \text{ at } \xi = -1,$$

$$\int_0^1 U_\xi d\zeta = \frac{x_m \psi}{z_m} \frac{df_x}{d\tau} \text{ at } \xi = 1.$$

The motion equation of the end wall with flexible restraint has the form

$$m \frac{d^2 x}{dt^2} + nx = \delta_0 b \left(p_0 + \rho V z_m \omega (\delta_0 \psi^2)^{-1} P|_{\xi=1} \right), \quad (7)$$

where m is the end wall mass, n is the stiffness coefficient of the end wall flexible restraint.

Solving (5), (6) using the iteration method we find that

$$U_\xi = \frac{\zeta^2 - \zeta}{2} \frac{\partial P}{\partial \xi} + \frac{\omega \delta_0^2}{\nu} \left(\frac{d^2 f_z}{d\tau^2} \xi - \left(\frac{d^2 f_z}{d\tau^2} + \frac{x_m \psi}{z_m} \frac{d^2 f_z}{d\tau^2} \right) \right) \frac{\zeta^4 - 2\zeta^3 + \zeta}{2}, \quad (8)$$

$$U_\zeta = \frac{3\zeta^2 - 2\zeta^3}{12} \frac{\partial^2 P}{\partial \xi^2} + \frac{\omega \delta_0^2}{\nu} \frac{d^2 f_z}{d\tau^2} \frac{5\zeta^4 - 5\zeta^2 - 2\zeta^5}{20},$$

$$P = \frac{\xi^2 - 2\xi - 3}{2} \left(\frac{6}{5} \frac{\omega \delta_0^2}{\nu} \frac{d^2 f_z}{d\tau^2} + 12 \frac{df_z}{d\tau} \right) - \frac{x_m \psi}{z_m} \left(\frac{1}{10} \frac{\omega \delta_0^2}{\nu} \frac{d^2 f_x}{d\tau^2} + 12 \frac{df_x}{d\tau} \right) (\xi + 1).$$

According to (8) the pressure in the channel cross section at the right end is

$$P|_{\xi=1} = - \left(\frac{12}{5} \frac{\omega \delta_0^2}{\nu} \frac{d^2 f_z}{d\tau^2} + 24 \frac{df_z}{d\tau} \right) - \frac{x_m \psi}{z_m} \left(\frac{2}{10} \frac{\omega \delta_0^2}{\nu} \frac{d^2 f_x}{d\tau^2} + 24 \frac{df_x}{d\tau} \right). \quad (9)$$

Taking into account (9) Eq. (7) we write as

$$(m + M_x) \frac{d^2 x}{dt^2} + K_x \frac{dx}{dt} + nx = \delta_0 b p_0 - K_z \frac{dz}{dt} - M_z \frac{d^2 z}{dt^2}. \quad (10)$$

Here $M_z = \frac{12}{5} \frac{b \rho \delta_0^2}{\psi^2}$, $M_x = \frac{1}{5} \frac{b \rho \delta_0^2}{\psi}$ are the added mass, $K_z = 24 \frac{b \rho \nu}{\psi^2}$, $K_x = 24 \frac{b \rho \nu}{\psi}$ are the damping coefficients due to the influence of a viscous fluid.

Note that equation (10) is true both for an arbitrary law of stamp vibration and for a harmonic one. Next, we consider the harmonic law of stamp vibration, i.e. $z = \delta_0 + z_m f_z(\omega t)$, $f_z(\omega t) = \sin(\omega t)$. In this case, the solution to equation (10) is written as

$$x = \frac{\delta_0 b}{n} p_0 - z_m \left[\frac{(n - (m + M_x) \omega^2) K_z \omega + M_z \omega^2 K_x \omega}{(n - (m + M_x) \omega^2)^2 + (K_x \omega)^2} \cos \omega t - \frac{(n - (m + M_x) \omega^2) M_z \omega^2 - K_z K_x \omega^2}{(n - (m + M_x) \omega^2)^2 + (K_x \omega)^2} \sin \omega t \right]. \quad (11)$$

Using this solution, we construct the amplitude and phase responses of the end wall, i.e. we write (11) in the form

$$x = \frac{\delta_0 b}{n} p_0 - z_m A(\omega) \sin(\omega t + \varphi(\omega)), \quad (12)$$

$$A(\omega) = \sqrt{\frac{(M_z \omega^2)^2 + (K_z \omega)^2}{(n - (m + M_x) \omega^2)^2 + (K_x \omega)^2}},$$

$$\varphi(\omega) = -\arctg \left(\frac{(n - (m + M_x) \omega^2) K_z + M_z \omega^2 K_x}{(n - (m + M_x) \omega^2) M_z \omega - K_z K_x \omega} \right).$$

4. Calculation results

We carried out calculations of the channel end wall amplitude responses using the developed mathematical model. The following data were used in the simulation $\ell = 0,1$ m, $\delta_0/\ell = 1/10$, $b/\ell = 5$, $m = 0,5$ kg, $n = 300$ kg/sec². Two liquids with different physical properties are considered. The first one is hydraulic oil (AMG-10) with the following parameters $\rho = 840$ kg/m³, $\nu = 2 \cdot 10^{-5}$ m²/sec, and the second one is water for which $\rho = 1000$ kg/m³, $\nu = 10^{-6}$ m²/sec. In the course of modelling, the dimensionless frequency was introduced as the ratio of the current frequency ω to the eigenfrequency one for the system without damping $\sqrt{n/m}$, i.e. $\eta = \sqrt{\omega^2 m/n}$. Thus, the dimensionless amplitude responses were considered

$$A(\eta) = \sqrt{\frac{\eta^2 K_z^2/(mn) + (\eta^2 M_z/m)^2}{(1 - \eta^2(1 + M_x/m))^2 + \eta^2 K_x^2/(mn)}}.$$

The calculation results of $A(\eta)$ for hydraulic oil (AMG-10) with decreasing dimensionless channel width δ_0/ℓ are shown in Fig. 2. Similar calculations of $A(\eta)$ for water are shown in Fig. 3.

The calculation results showed the occurrence of significant vibration amplitudes of the channel end wall near the eigenfrequencies of the system without damping. In addition, the possibility of damping these oscillations due to decreasing the liquid layer thickness or increasing the kinematic viscosity is shown.

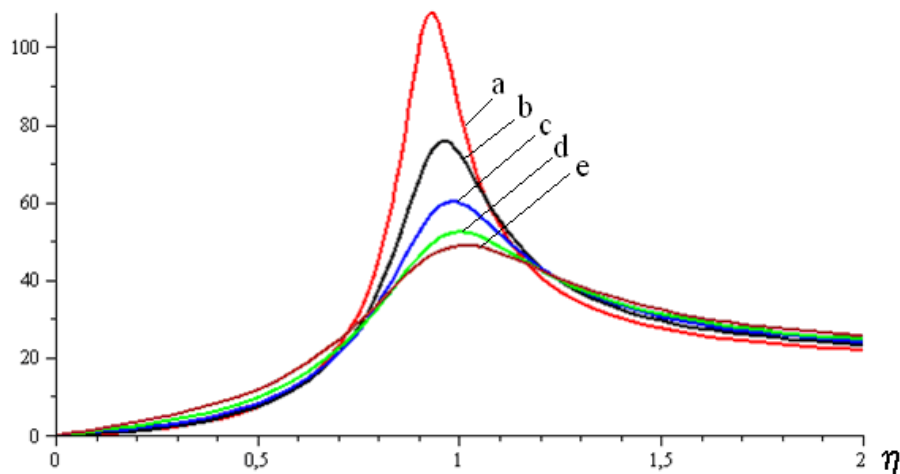


Figure 2. Charts of $A(\eta)$ for hydraulic oil (AMG-10): (a) $\delta_0/\ell = 10^{-1}$, (b) $\delta_0/\ell = 15^{-1}$, (c) $\delta_0/\ell = 20^{-1}$, (d) $\delta_0/\ell = 25^{-1}$, (e) $\delta_0/\ell = 30^{-1}$.

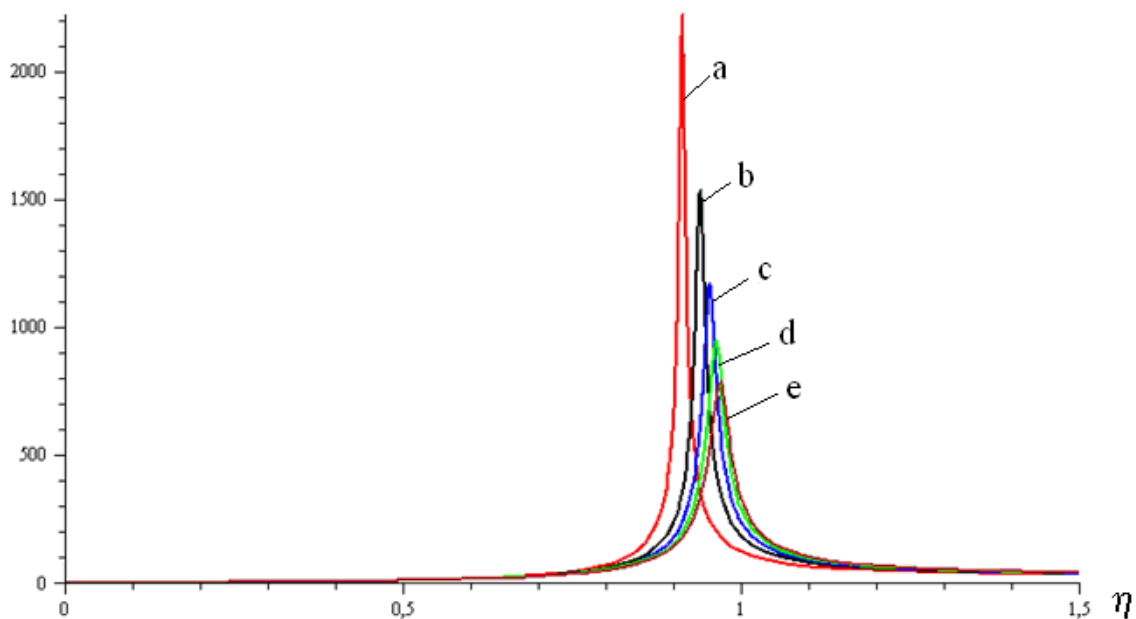


Figure 3. Charts of $A(\eta)$ for water: (a) $\delta_0/\ell = 10^{-1}$, (b) $\delta_0/\ell = 15^{-1}$, (c) $\delta_0/\ell = 20^{-1}$, (d) $\delta_0/\ell = 25^{-1}$, (e) $\delta_0/\ell = 30^{-1}$.

5. Summary and Conclusion

Thus, the mathematical model has been developed for the study of longitudinal vibrations of the end wall with flexible restraint (bellows) due to vibrations of the stamp (the channel upper wall). The elaborated mathematical model can be used both for the harmonic law of stamp vibration and for arbitrary one. In the latter case, if the stamp motion law is given non-linearly, the motion equation of the end wall with flexible restraint must be solved by appropriate methods, for example, numerically. The obtained results can be used in practice to predict the resonant frequencies of bellows vibrations and calculate the amplitudes of bellows vibrations at resonance, as well as to study the fluid pressure distribution along the channel in lubrication systems, hydraulic drive, cooling, etc.

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