Investigation of the propagation of waves of sudden change in mass flow rate of fluid and gas in a "short" pipeline approach

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Abstract. The problem of the propagation of waves of sudden change in mass flow rates originated at the ends of a pipeline of a finite length is formulated and solved analytically in the paper. In the process of modeling, it was assumed that the route change in pressure is due to the local component of the inertia force, and the velocity of propagation of waves of small pressure disturbances depends on the elastic properties of the pipeline material and fluid. The Dirichlet problem with respect to mass flow rate is solved by the Fourier method and a periodic solution is obtained in the form of a functional series. Instead of solving the Neumann problem with respect to hydrostatic pressure, a pressure solution is obtained by integrating the mass conservation equation over time, where the known solution of the problem with respect to mass flow is used. The options for using the solution for cases of low and super compressible fluids, as well as for studying the propagation of longitudinal waves in an elastic rod, are discussed. According to analytical solutions, a series of calculations was carried out for the problems of instantly closing the inlet and/or outlet sections of the functioning area, as well as for the problem of gas pumping into the elementary section of the pipeline. The features of the propagation of compression and rarefaction waves in the context of the above tasks are investigated. Some graphical results are presented and analyzed.

Keywords: pipeline transport, compression wave, hyperbolic equation, the Dirichlet and Neumann problems, functional series, computational experiment

1. Introduction

The study of the cause-effect relation of a particular process allows us to make appropriate decisions on the management of the object. If the process is multi-factorial and there is a hierarchical relationship between the factors, then the study of the process based on field research becomes burdensome and the way out is sought for in mathematical and numerical modeling of the process.

The hydraulic system of pipeline transport of compressible and low-compressible media is such a complex system, the disadvantages of which are manifested mainly in the dynamic mode of operation. Failure or launch of new point or linear elements, for example, a separate parallel or serially connected supercharger, a point of concentrated withdrawal or pumping of the transported medium, parallel thread in a particular area leads to a redistribution of the transported medium indices over the entire network.

If we are talking about the contribution into the studies on pipeline networks, then it is necessary to mention the monograph (now being a classic) written by V.V. Grachev, S.G. Scherbakov S.G. and E.I. Yakovlev [1]. It is devoted to the development of the methods for calculating the network of gas pipelines in the dynamic mode of operation. The main point of the method developed in this paper is to link the linear and point network elements through images of the sought for functions, compiled using the Laplace transform over time.

The state of the linear section of the main gas pipeline (MGP) is described by quasi-one-dimensional equations of conservation of momentum and mass according to N.E. Zhukovsky. The nonlinear term, which takes into account friction resistance, is linearized according to I.A. Charny. Gas inertia is taken into account partially (local change in the mean velocity over time).

The paper presents the methods for representing the matrix entry for point elements and how to implement them for complex compounds. The difficult part of the implementation of this method is the direct and reverse transitions from the images to the originals of the pressure functions and mass flow functions   and vice versa, since depending on the network structure topology and the imposed conditions, functions of different complexity are expected.

Numerous examples of solving non-stationary problems are given, topological methods for describing MGP systems (directed graphs), generalized functions and functional series used in calculations, non-isothermal approaches and a lot of useful information are described.

The monograph by V.Ye. Seleznev, V.V. Aleshin and S.N.Pryalov [2] contains a detailed description of the new version of the concept and the new methods of high-precision numerical modeling of main pipeline systems. The proposed concept and methods are the basic elements of theoretical foundation of modern computer analytical tools for effective solution of a wide range of technical and technological problems of designing, constructing and operating pipeline transportation facilities throughout their life cycles. They confirmed the reliability and efficiency of their practical application in solving production problems characteristic of the fuel-energy complex.

In [3], the adequacy of the turbulence model was tested based on the Reynolds Averaged Navier-Stokes (RANS) equations. Results of calculation of a homogeneous fluid flow in pipes of different diameters were presented and analyzed using the Computational Fluid Dynamics (CFD) method. The main conclusion of the work is the proof of the permissibility of the transition to dimensionless variables when calculating the pipes of small, medium and large diameters.

In a number of papers by A. Bermúdez and co-authors the mathematical models of gas conveying networks were discussed with complicating factors: non-planar topography and a non-adiabatic model of a compressible gas flow and the methods for solving problems were suggested. In particular, in [4], a finite volume scheme for the numerical solution was proposed to solve the full Euler equations, which took into account the friction force, the variability of the route relief and heat exchange with the environment.

At equations discretization, the Q-scheme of van Leer was used - an analogue of the A.A.Samarsky anti-flow method in relation to the finite element method. A fairly accurate implementation of the law of conservation of mass in the computational domain was reached. It is planned to use the method for calculating a complex network of pipelines.

In [5], a new scheme for the equations discretization of natural gas transport in pipeline networks is presented using finite differences of the second order of accuracy. The system of equations takes into account the variability of the density, pressure and mass flow rate of gas and includes the equation of the state of gas - the ratio between the density, pressure and gas temperature. The method of approximation of equations allows accurate reproduction of conservation of mass equation. The advantage of the latter method is proved by a series of test problems using the method of explicit cleavage and other methods and by comparing calculation results with the results of the proposed explicit staggered-grid method.

The authors in [6] have introduced a model for gas flow in pipeline networks based on the Euler isothermal equations. At modeling the intersection of multiple pipes, an additional assumption on the

interface pressure was done. A method to obtain solutions to the gas network problem is described and numerical results for sample networks are presented.

The study in [7] presents two new methods for calculating the properties of natural gases. The first is an efficient empirical model to calculate compressibility and density of natural gases containing high amount of heptane plus and non-hydrocarbon components. The model is derived from 2400 measurements of compressibility and density of various gases presented in this study. Accuracy of the model is compared to various equations of state (EOS) and to empirical methods. The study shows that the new model is simpler and more efficient than EOS. It eliminates the numerous computations involved in EOS calculations. The new method also eliminates the characterization of the heptane plus fraction and estimation of binary interaction parameters needed for EOS calculations. Experimentally measured density of several gases has been used to study the validity of the proposed method. These measurements indicate that the new method successfully capture the physical trend of changing gas density as a function of pressure, temperature, and composition.

The second method is a modification of Lee–Gonzalez–Eakin gas viscosity correlation. The new method accounts for the presence of heptane plus, hydrogen sulfide, and carbon dioxide in natural gases. The proposed method is compared to other EOS-based viscosity model, corresponding methods of state, and correlations. The comparison indicates the preference of the new method over the other methods used to calculate viscosity of natural gases.

In [8], to specify the guiding parameters of the hydraulic lifting pipeline system for cut-suction extraction of natural gas hydrates (abbreviated as "hydrates") on the seabed, the decomposition characteristics of hydrates in hydraulic lifting pipelines were studied as well as the effect of flow parameters on the decomposition characteristics. The temperature – pressure model of the hydrate hydraulic lifting pipeline, the model of mass transfer of hydrate decomposition, and the multiphase model of the flow in the pipeline were established by the method of mathematical modeling in thermodynamics and fluid mechanics. Considering the phase transformations fluid-rigid body-gas, a number of the process features have been revealed, which are consistent with the results of the experiments.

An analysis of world experience in organizing the export of oil and gas from the Arctic offshore fields is given in [9]. Based on a detailed study of foreign experience in organizing the transportation of oil and gas from the Arctic offshore fields in the USA and Norway, the author critically assessed the activities of Russian companies in this field and gave practical recommendations for the further development of this sector. At the transportation of oil and gas, an account of heat exchange with the environment makes the problem the more acute one.

The studies in [10] were devoted to obtaining an analytical solution of a non-isothermal model of gas transportation along a linear section of a main gas pipeline. With the Laplace transform, an analytical solution of the linearized system of gas flow equations through the main gas pipeline was obtained, difficult to use when calculating real objects because of their complexity and specificity of various parameters entering these formulas.

In [11], various analytical and approximate methods for solving nonlinear equations of pipeline transport of compressible media were proposed. Mathematical models of gas pipelines that run through the seabed and through low-temperature zones were developed in [12, 13]. Various problems of pipeline transport and methods for their solution are given in [14-16].

According to the statement and the method of solution, the problem in question is close to the ones considered in [17, 18]. They deal with the problem of the propagation of waves of pulse disturbances in the conditions of connecting the pressure damper to the end of the area. The forces of friction, gravity and the local component of the inertia force are taken into account. The equations are linearized by introducing the mass flow and the A.I. Charny linearization [19] and are presented in the form of a telegraph equation for mass flow and pressure. The latter is solved by the Fourier method.

The solution was obtained in the form of a sum, where, unlike the study by Charny A.I., the amplitudes of the entire frequency spectrum, except for exponential multipliers, contain multiplicative components in the form of hyperbolic sine and cosine, linear function and trigonometric sine and

cosine of time. As for A.I. Charny's studies, as well as in the present paper, only trigonometric sine and cosine are taken into account in the expression of the perturbation amplitude.

When modeling linear elements of the pipeline network, to simplify the equations, the long and short pipelines approaches are used [1, 2, 19]. With long pipelines, and at small perturbations of the boundary conditions, the long pipeline approach can be used; ignoring the inertial terms in the momentum conservation equation. If the changes in the area are of dynamic nature or the pipeline is short, then in the object under study the role of the friction force will be insignificant and may be neglected.

A significant difference between the approaches is expressed in mathematical models of the process: a long pipeline approach leads to parabolic equations for mass flow rate and hydrostatic pressure, and a short pipeline approach leads to hyperbolic equations [1,18]. Simultaneous consideration of the friction and inertia forces leads to cut-off telegraph type equations, the solution of which is fraught with some difficulties, especially in the cases of discontinuous solutions.

Exactly the same problem is solved in this paper; in its modeling the quasi-one-dimensional equations of conservation of momentum and mass are used, taking into account the deformable thin walls of the pipelines. The problem of transition from one stationary mode of operation of an elementary section of a horizontal pipeline to another stationary mode is considered; it is caused by a discontinuous change in mass flow rate at the ends of the section.

2. Problem Statement

In the isothermal mode, the state of an ideal gas medium conveyed through a horizontal short pipeline is described by a system of nonlinear equations [1,2]

$$-\frac{\partial p}{\partial x} = \frac{\partial (\rho w)}{\partial t}, \quad -\frac{1}{c^2}\frac{\partial p}{\partial t} = \frac{\partial (\rho w)}{\partial x}. \tag{1}$$

Here and hereinafter p, ρ, w are the pressure, density and average flow velocity of the transported medium in cross section x at time t, respectively; D, $f = \pi D^2 / 4$ are the diameter and cross-sectional area of the pipeline; c is the rate of small pressure disturbances in the medium-pipe system taken as $c = \left(\frac{\rho_0}{k_{ж c}} + \frac{2R_0\rho_0}{E\delta}\right)^{-1/2}$ [20], where ρ_0 is the density of undisturbed fluid; E is the modulus of elasticity of pipe material; R_0 is the inner radius of the pipe in an undisturbed state; δ is the wall thickness of a round pipe.

This system is linearized by introducing mass flow rate $M = \rho w f$ and is presented as a hyperbolic equation with respect to mass flow rate:

$$\frac{\partial^2 M}{\partial t^2} = c^2 \frac{\partial^2 M}{\partial x^2}. \tag{2}$$

Suppose that an elementary section of a length l up to time $t = 0$ worked with performance M_0. Beginning from the moment $t = 0$, at the inlet to the section, the mass flow rate M_H was established, and at its outlet - M_K. At this statement, at $M_H = M_K$, the problem is considered in the range $0 < t < +\infty$, otherwise - in a limited period of time.

Due to the change in mass flow rate at the end (or at the ends) of the area, the pressure and mass flow rate along the length of the area are rearranged. According to the statement of the problem, the boundary conditions of equation (2) are

$$M(x,0) = M_0 = const, \quad \frac{\partial M(x,0)}{\partial t} = 0 \text{ at } 0 \le x \le l, \quad t < 0; \tag{3}$$

$$M(0,t) = M_H = const, \quad M(l,t) = M_K = const \text{ at } t \geq 0.$$

A similar equation can be derived regarding the pressure:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2}.$$

Initial distribution of pressure and its time derivative in the area can be taken as

$$p(x,0) = p_0, \quad \frac{\partial p(x,0)}{\partial t} = 0.$$

The boundary conditions for pressure, according to the first equation of system (1), are

$$\frac{\partial p(0,t)}{\partial x} = \frac{\partial p(l,t)}{\partial x} = 0.$$

For the mass flow rate, the Dirichlet problem is obtained, and for pressure - the Neumann problem [20]. The use of the method of separation of variables in solving the problem of pressure led to the loss of some terms of the solution. The reason for this, in our opinion, is the zero value of the integral of the eigenfunction on x in the area of calculation. In this connection, the problem was solved initially with respect to mass flow rate, and the solution obtained was used to find the solution to the pressure problem.

3. Analytical Solution

The solution of the problem of mass flow rate is sought for in the form of a sum of stationary and non-stationary parts [20]

$$M(x,t) = U(x) + u(x,t).$$

The stationary part of the solution is taken as:

$$U(x) = M_H + \frac{x}{l}(M_K - M_H).$$

In this case, the equation relative to $u(x,t)$ has the form

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2},$$

the boundary conditions become uniform:

$$u(0,t) = u(l,t) = 0,$$

The initial conditions are

$$u(x,0) = M_0 - M_H - \frac{x}{l}(M_K - M_H) = A_0 + A_1 x, \quad \frac{\partial u(x,0)}{\partial t} = 0.$$

Here $A_0 = M_0 - M_H$, $A_1 = -\dfrac{M_K - M_H}{l}$.

The application of the Fourier method [20] leads to the solution of the form

$$u(x,t) = \sum_{n=1}^{\infty} \left(a_n \sin\frac{\pi n c t}{l} + b_n \cos\frac{\pi n c t}{l} \right) \sin\frac{\pi n x}{l},$$

Where eigenfunctions $X_n(x) = \sin\dfrac{\pi n x}{l}$ on x make up a countable set of orthogonal functions with the square of the norm $\left\| X_n^2(x) \right\| = l/2$.

The first initial condition from (3) leads to the following equation

$$u(x,0) \equiv A_0 + A_1 x = \sum_{n=1}^{\infty} b_n \sin\frac{\pi n x}{l}.$$

The application of the orthonormality of eigenfunctions $X_n(x)$ to this equality allows us to obtain the formula

$$b_n = \frac{2}{\pi n}\left\{ \left[(-1)^n - 1 \right]\left(M_H - M_0 \right) + (-1)^n \left(M_K - M_H \right) \right\},$$

in the derivation of which, the orthonormality of the eigenfunctions $X_n(x)$ and the following values of the integrals were used

$$\int_0^l \sin\frac{\pi n x}{l}\,dx = -\left[(-1)^n - 1 \right]\frac{l}{\pi n},$$

$$\int_0^l x \sin\frac{\pi n x}{l}\,dx = -(-1)^n \frac{l^2}{\pi n}.$$

The implementation of the second initial condition leads to dependency:

$$\frac{\partial u(x,0)}{\partial t} \equiv 0 = \sum_{n=1}^{\infty} \frac{\pi n c}{l} a_n \sin\frac{\pi n x}{l}.$$

Hence, $a_n = 0$. So, non-stationary part of the solution is

$$u(x,t) = \sum_{n=1}^{\infty} b_n \cos\frac{\pi n c t}{l}\sin\frac{\pi n x}{l}.$$

Reverse transition to mass flow rate gives a periodic solution over time.

$$M(x,t) = M_H + \frac{x}{l}\left(M_K - M_H \right) + \sum_{n=1}^{\infty} b_n \cos\frac{\pi n c t}{l}\sin\frac{\pi n x}{l}. \tag{4}$$

Finding the value of $M(x,t)$, proceed to the determination of pressure. The second equation of system (1) is integrated over time, and the result is written in the form

$$p(x,t) = p(x,0) - \frac{c^2}{f}\int_0^t \frac{\partial M(x,\xi)}{\partial x}\,d\xi.$$

Calculations of the derivative and the integral lead to the solution

$$p(x,t) = p_0 + \frac{M_H - M_K}{lf}c^2 t - \frac{c}{f}\sum_{n=1}^{\infty} b_n \sin\frac{\pi n c t}{l}\cos\frac{\pi n x}{l}. \tag{5}$$

Here, the first term represents the initial distribution of pressure, the next term represents the increase or decrease of hydrostatic pressure over time depending on the difference in inlet and outlet mass flow rates $M_H - M_K$, i.e. on the positivity or negativity of changes in the mass of the medium in the area. The sum represents the periodic part of the solution.

At statement and solution of the problem, the mass flow rate of the medium $M = \rho w f$ was used. In the incompressible medium approach, when compressibility is observed only under the influence of a pulse wave, $\rho \approx const$ is accepted. Then the average flow rate over the cross section of the pipeline is defined as [1]

$$w(x,t) = \frac{1}{\rho f} M(x,t).$$

If the medium consists of gas, then its state is described by the equation [2, 19]

$$p = Z\rho RT,$$

where Z is the gas supercompressibility factor; R, T are the reduced gas constant and gas temperature. The expression of mass flow rate becomes nonlinear, since the gas density entering this expression is a variable. In this regard, to determine the gas velocity, the following formula is used

$$w(x,t) = \frac{ZRT}{f} \frac{M(x,t)}{p(x,t)}, \qquad (6)$$

where average or exact values of Z, R, T and f are used.

Hyperbolic equations are used in electrodynamics, in the theory of a deformable rigid body and in other branches of science. In particular, expressions (4) and (5) can be interpreted as a solution to the problem of longitudinal wave propagation in elastic rods, when the propagation velocity of small perturbations is represented as $c = \sqrt{E/\rho}$, where E is the Young modulus of rod material [20]. The result $M(x,t)$ corresponds to the displacements of the rod points in the case when $M_0 = M_H = 0$, and the instantaneous displacement of the second end of the rod at $t = 0$ is $M_K - M_H$. At the same time, the result $p(x,t)$ expresses the displacement of the rod points at $p_K = 0$, when the velocity of the point $x = 0$ is zero, and the point $x = l$ at $t = 0$ instantly gains the velocity $\frac{2a}{f}(M_H - M_K)$.

4. Results and Discussion
A calculation program was drawn up on the basis of solutions (4) and (5), formula (6). Below the results of a computational experiment on super-compressible gas are presented.

Two series of calculations have been conducted. In the first series, a functioning area of the main gas pipeline was considered and the gas-dynamic state of the section was examined at the closing of the inlet and/or outlet areas. The second series of calculations related to the first stage of testing the suitability of a gas pipeline section for operation — gas pumping into the elementary area.

Calculations were conducted for the constants $R = 528 \, J/(K \cdot kg)$, $c = 400 \, m/s$, $D = 0,992 \, m$, $T = 297,15 \, K$, $Z = 0,920$ and different values of mass flow rate and initial pressure in the area. The number of terms in the sums was 200. The length of the section l was 0,5, 1, 2, 5 and 10 km. The calculation step along the length was $l/50$. The time scale was the conditional period $2l/c$ of the process during which the wave returns to the starting point. The calculations were performed with a time step $\tau = l/(10 \, s)$, i.e. 10τ was needed for a wave to pass from the beginning of the area to its end or back.

The options for instantly closing the outlet of the area, the inlet area and both ends of the area were considered.

The first calculations were related to the data: $l = 500\,m$, $p_0 = 5,0\,MPa$, $M_0 = M_H = 250\ kg\,/\,s$, $M_K = 0\ kg\,/\,s$, when the outlet section of the working area was instantly closed.

Fig. 1 shows the mass flow rate curves for the first 11 steps in time. The upper limit of the figure reflects the mass flow rate until the closing the end of the section. The right curve represents the mass flow rate at $t = 0$. The second curve to the right corresponds to time $t = \tau$ (in this case $\tau = 1,25\ s$), the third curve - to time $t = 2\tau$, etc. The left curve is obtained at $t = 10\tau$. Besides the curves obtained at $t = 0$ and $t = 10\tau$, the curves turned out to be three-link: on the left is the link of the boundary mass flow rate M_H, in the middle there is a sudden change in mass flow rate, on the right there is the decelerated part of the gas.
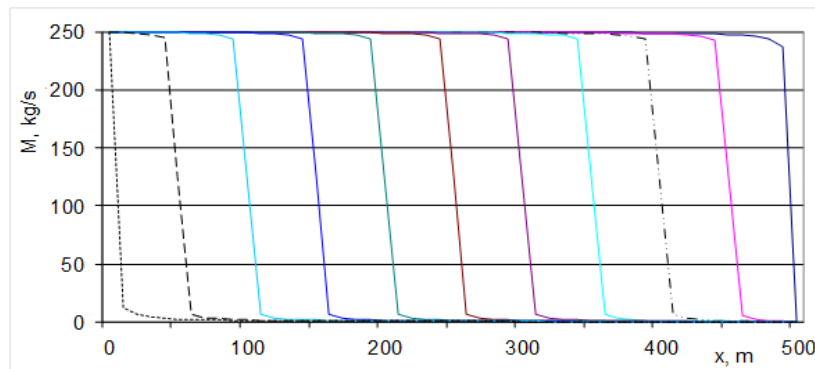

Figure 1. Mass flow rate curves for the first eleven time steps $\tau = 1,25\ s$ at instant closing of the inlet section. $l = 500\,m$, $M_0 = M_H = 250\ kg\,/\,s$, $M_K = 0\ kg\,/\,s$.

The wave reaching the beginning of the area at $t = 10\tau$, gradually returns to the end of the section. The point of discontinuity of the mass flow rate makes a "shuttle" motion between the end points of the area at a constant velocity c.

With instantaneous closing of the inlet section of the area $M_0 = 250\ kg\,/\,s$, $M_H = M_K = 0\ kg\,/\,s$, a pattern similar to mass flow is observed, but the segments of mass flow rate curves are arranged in reverse order.

Fig. 2 shows the mass flow rate curves of the first conditional period at instant closing of the two ends of the area.

The process begins with sudden change in mass flow rate at the two ends of the area: mass flow rate drops to zero. A kind of mass flow rate trapezium is formed, the upper base of which lies on the straight line $M_0 = 250\ kg\,/\,s$ and the lower base lies on the coordinate axis. The sides of the trapezoid replace the disturbance waves: a compression wave occurs to the right, and a rarefaction wave - to the left. The waves gradually propagate to the middle of the area. At $t = 5\tau$ they meet in the middle of the area. In this case, the rarefaction and compression waves are mutually compensated, and the mass flow rate is represented by a straight line $M = 0$.

At $t > 5\tau$ the trapezium-graphics are formed, large bases of which are located on the x-axis, and small bases - on the line $M = -250\ kg\,/\,s$. At $t = 10\tau$ the section returns to its original state in mass flow rate. Further the process reiterates in the cyclic mode.

Besides the times dividable by 5τ, the curves of the mass flow rate are five-link ones. The conditional period of the process in this case is not $2l\,/\,c$, but $l\,/\,c$.

If to ignore the second term in the solution (5) (or at $M_H = M_K$), then the pressure is also periodic. The second term in the solution expresses an increase (at $M_H > M_K$), a decrease (at $M_H < M_K$) and a constancy (at $M_H = M_K$) of mass over time in the considered area. At the instantaneous closure of the outlet section, the sudden change in pressure increase at the ends of the area to which the wave has reached is $c(M_H - M_K)/f$.
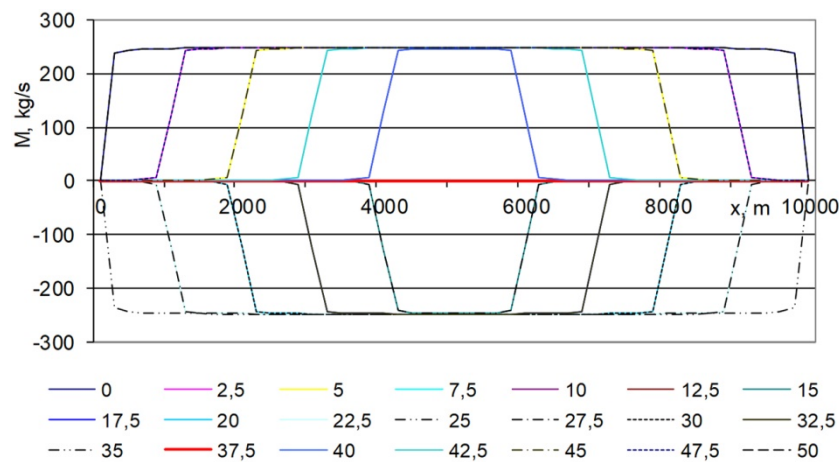

Figure2. Change in gas mass flow rate at the first period with instant closing of both ends of the area. $l = 10,0\ km$, $p_0 = 0,1\ MPa$, $M_0 = 250\ kg/s$, $M_H = M_K = 0\ kg/s$.

Fig. 3 shows the pressure curves for the 9th period. The graphs remained the same as were received in the first, second ... conditional periods, but they shifted upward. If in the first period the lowest pressure value was 5,0 MPa, then for the 9th period it increased to 7,3289 MPa. And for the 12th period, this figure was 8,1053 MPa.
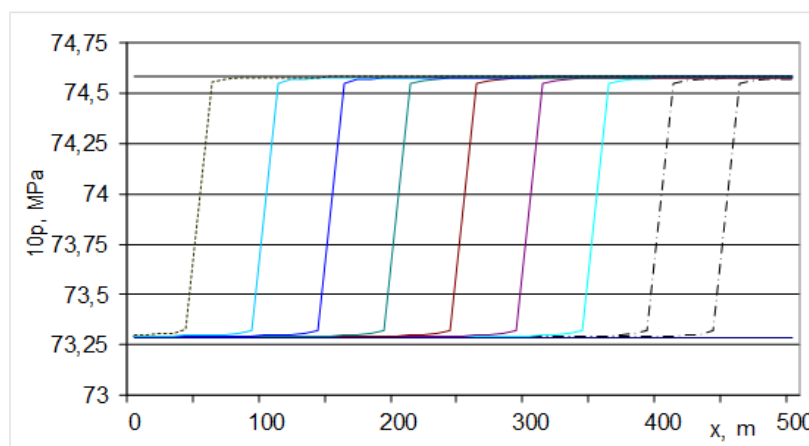

Figure 3. Pressure curves for the 9th conditional period after the closure of the outlet section. $p_0 = 5,0\ MPa$, $l = 500\,m$, $M_0 = M_H = 250\ kg/s$, $M_K = 0\ kg/s$.

Similar curves were obtained at closing the inlet section, when the pressure curves shifted lower. In both cases, the solution describes the process for a small period of time, as mentioned above, i.e. pressure constraints are not included into the model. In this sense, time is not limited to the case when both ends of the section are instantly blocked.

Fig. 4 shows the pressure curves for the first period l/c. On the right, on the upper part, there are the graphs of the gas compression wave, formed by gas deceleration, and on the lower left are the graphs of the rarefaction wave, formed by blocking the gas supply to the area. At $t = 5\tau$ the waves of compression and rarefaction are mutually "annulled" and the pressure graph in the area coincides with the initial pressure. But the energy accumulated there provokes the formation of reverse waves, which in the same way return to their ends.
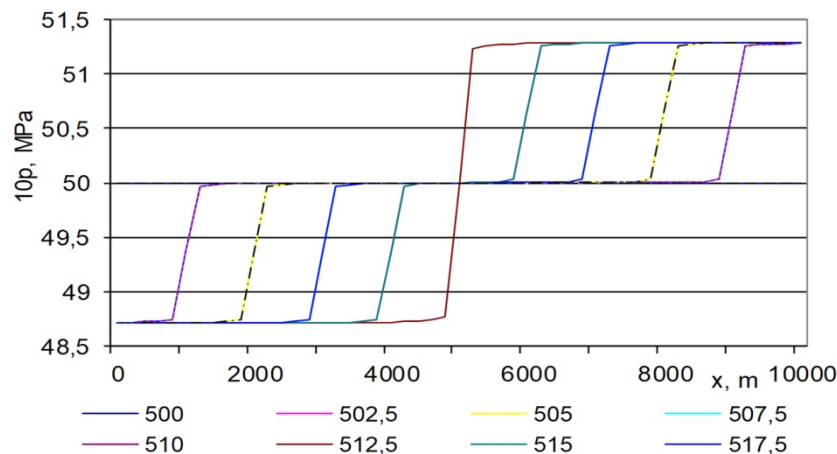

Figure 4. Pressure change in the first half of the period at instant closing of the ends of the area. $l = 10,0\ km$, $p_0 = 0,1\ MPa$, $M_0 = 250\ kg/s$, $M_H = M_K = 0\ kg/s$.

For an incompressible fluid, the flow velocity is proportional to the mass flow rate: dividing the mass flow rate by the fluid density and the cross-sectional area of the pipe, the flow rate is obtained. So, the flow rate graphs reflect velocity graphs, but with a different scale along the vertical axis. For super-compressible gas, according to the above formula, the flow rate from period to period either increases (at $M_H < M_K$) or decreases (at $M_H > M_K$) or remains at the same level (at $M_H = M_K$). In the latter case, the flow rate is periodic in nature and has similar graphs, shown in Fig. 2.

Calculation results for a non-active area refer to the first step of the integrity checking of the area. In the first step of testing, one end of the area is tightly closed, and air is pumped into the area from the other end of the section. Pumping continues until an average pressure in the area is 20% higher than the expected working pressure. Then, the pumping stops and the area stays at rest for 30 minutes (the second step). If during this period the pressure in the area does not decrease, then the area is considered serviceable and the excess air is released into the atmosphere (the third step).

It was assumed that the performance of the supercharger is constant and is $M_H\ (>0)$ by mass. The initial state of gas is determined by conditions $M_0 = 0$ and $p(x,0) = p_0 = const$.

For mass flow rate, the features described in the previous section, when comparing with the shuttle motion, remain valid.

According to the solution of problem (5), the sudden change in pressure increase when the wave reaches one of the ends of the area is cM_H/f. Between the jumps, the pressure value is almost constant. Consider the pressure change over time at inlet to the area (Fig. 5).

A similar graph, but with l/c displacement to the left is obtained for the outlet section of the area. This character of pressure change is effective for the operation of pneumatic and hydraulic actuators: a rhythmic "shock" is formed over the area of the working unit.

Under the conditions of a periodic change in mass flow rate and a cyclic increase in pressure, a peculiar dynamics of the flow velocity in the area and its ends is formed.

Fig. 6 shows a graph of the flow rate at the inlet to the area. At t=0, the inlet velocity is 466,9 m/s, and after a period it is 130,139 m/s. Then 75,606, 53,280 m/s and so on ... At 600th second of pumping,

the inlet flow velocity is 14,576 *m/s*. If to shift this graph to the left for a time l/c, a velocity graph at the outlet area is obtained.

Fig. 7 shows the velocity graphs at the first conditional period with a time step $\tau = l/(10c)$. Thin lines refer to the first half-period, and thick lines - to the second half-period.
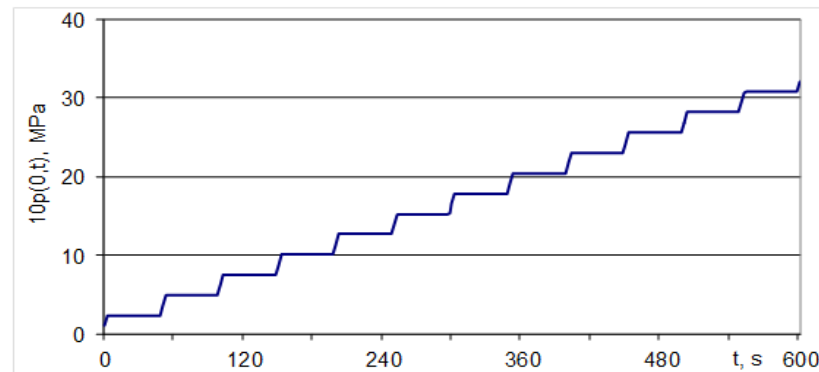

Figure 5. Change in gas pressure at the inlet to the area over time during gas pumping.
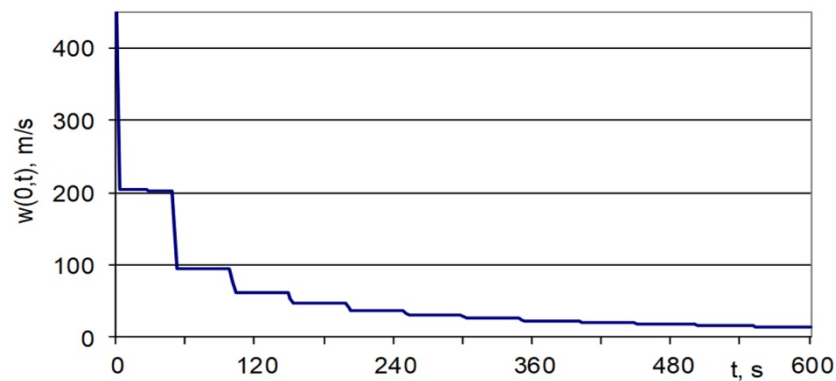$l = 10,0 \ km$, $p_0 = 0,1 \ MPa$, $M_0 = M_K = 0 \ kg/s$, $M_H = 250 \ kg/s$.

Figure 6. Change in gas velocity at the inlet to the area over time during gas pumping. $l = 10,0 \ km$, $p_0 = 0,1 \ MPa$, $M_0 = M_K = 0 \ kg/s$, $M_H = 250 \ kg/s$.
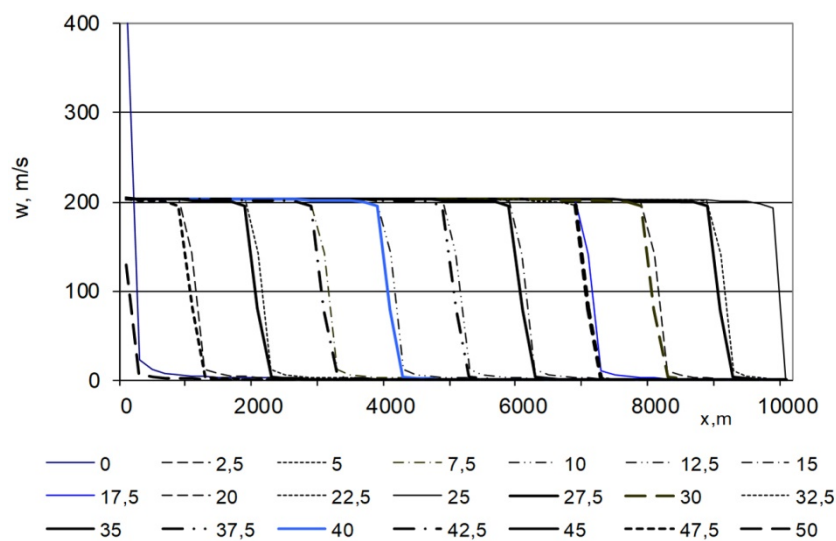
Figure 7. Change in the flow rate in the area at the first period of
gas pumping. $l = 10,0\ km$, $p_0 = 0,1\ MPa$, $M_0 = M_K = 0\ kg/s$, $M_H = 250\ kg/s$.

At $t = 0$ the velocity curve is reduced from 466.9 *m/s* to 0 *m/s* (the upper part of the graph is cut-off). At this point in time, the pressure increases abruptly.

Therefore, the upper limit of velocity for the rest interval of time period is set at about 200 *m/s*. Formed velocity protrusion expands to the right. Reaching the end cross section, the gas flow stops and the rarefaction wave flows back to the beginning of the area. But the last curve, obtained at $2l/c$, no longer reaches 200 *m/s* and stops at 130, 139 *m/s*, since at the given moment the pressure increases abruptly. Further, this process is repeated, but with less velocity interval.

Averaging the pressure expression (5) over the period, the formula for calculating the average pressure in the area is obtained depending on time

$$\overline{p}(t) = p_0 + \frac{M_H - M_K}{lf} c^2 t \ ,$$

where lf is the physical volume of the area under consideration.

If in calculating the value of velocity to use this formula instead of pressure, then we would come to the hyperbolic law of diminishing flow velocity in the area over time. On the other hand, if the left side of the equality is taken to be equal to the pressure value $\overline{p}$ at which the integrity of the area is checked, then an approximate formula for the pumping time is obtained at the supercharger capacity M_H:

$$t_* = \frac{\overline{p} - p_0}{c^2 M_H} lf \ .$$

5. Conclusion

The problems of sudden change in mass flow rate have been solved within the framework of the "short" pipeline approximation, when the pressure gradient is formed only by the local component of the inertia force. By introducing a mass flow rate, the problem is linearized and reduced to the Dirichlet and Neumann problems for the hyperbolic equation. These problems are not new, because there are their solutions obtained by the method of analytic continuation [20] using the generalized functions. However, the use of the Fourier method made it possible to clearly identify the non-periodic part of the solution with respect to hydrostatic pressure.

The options for applying the solutions obtained to study the flow of compressible and incompressible fluids, as well as the propagation of longitudinal waves in elastic rods are analyzed.

Numerical results related to the linear area of a functioning gas pipeline and to the first stage of integrity checking of a gas pipeline area are presented. Periodic changes in mass flow rates, as a whole, and in pressure and velocity in particular, are revealed.

The features of the dynamics of gas pressure and velocity under monotonic increase and decrease in mass in the pipeline section are discussed. An approximate formula is obtained to determine the duration of gas pumping until a certain average pressure is reached in the area.

The results obtained demonstrate the possibilities of mathematical modeling as applied to the problems with discontinuous solutions in the form of compression and rarefaction shocks. Mathematical model of the problem can be complicated with the addition of friction force and the route gradient, as was done in [17, 18].

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