

# Dependence of Error Probability on the telegraph Distortion Value

A V Kosykh, D A Kryzhanovsky and V L Khazan

Omsk State Technical University, 11 Mira ave., Omsk, 644050, Russia

**Abstract.** The transmission of messages via non-stationary communication channels is made in conditions of adapting the receiving and transmitting equipment to the communication conditions. Mode of adaptation to communication conditions requires rapid assessment of the communication channel quality. The most obvious criterion for assessing the quality of a non-stationary communication channel is the constantly changing value of the elements error probability in the transmitted message. However, assessing the error probability of message elements requires significant time intervals, which may exceed the time of stationary state of the communication channel. It is known that the error probability of message elements is uniquely related to the value of telegraph distortions. The estimation of the telegraph distortion value requires much less time compared to the time required to estimate the probability of errors. In this case, the indirect estimation of the error probability by the value of the telegraph distortion can be made relatively quickly at an interval that is many times less than the stationary state interval of the communication channel. In this regard, the possibility of indirect estimation of error probability in terms of telegraph distortion is relevant for receiving and transmitting equipment with adaptation to communication conditions.

## 1. Introduction

Transmission of discrete messages via non-stationary radio channels is performed with the adaptation of the receiving and transmitting equipment to the existing communication conditions [1,2]. The methods of adaptation to communication conditions require rapid assessment of the quality of communication channels. The most obvious criterion of communication channel quality is the error probability of a message elements [3,4]. However, the error probability estimation requires relatively large time intervals, which depend on the error probability value and can be many times longer than the stationary state interval of the communication channel. Indeed, for a consistent estimation of the error probability it is required to implement up to 20 number of errors [5]. For example, if the probability of errors is 0.001, the number of accepted elements in its evaluation should be at least 20,000. If the manipulation speed is 100 Baud, then the time during which 20 errors appear is 200 seconds, i.e. more than 3 minutes. This interval can be many times greater than the stationary state interval of the communication channel. In [3, 4] the relationship between the error probability and the value of telegraph distortions is shown. Mind that the magnitude of telegraph distortion is estimated at the interval, which is many times less than the error probabilities estimation interval of the message elements and does not exceed the stationary interval of the channel, making this the criterion for assessing the quality of a communication channel indispensable in the design of receiving and transmitting equipment with adaptation to the communication conditions. It is especially important that the estimation interval of the average value of telegraph distortions does not depend on the magnitude of these distortions and is always made at the same time interval, where, for example, the



number of fronts of elementary telegraph parcels equal to 20 is realized. Since in a discrete message, when changing the values of elementary telegraph parcels, one front accounts for an average of two elements of the message, the number of elements that allow to obtain a consistent estimate of the average value of telegraphic distortions (AVTD) will be equal to about 40. Thus, in the case of speed manipulation equal to 100 Baud, the interval of AVTD and indirect estimation of the error probability will be not 200 s, like the estimation of errors probability, but only 0.4 s. This interval is usually many times less than the stationary state interval of the communication channel, which is a necessary condition for ensuring rapid adaptation of receiving and transmitting equipment to changing conditions.

## 2. Problem statement

The purpose of this article is to compare empirical analytical dependence between the values of AVTD and probability values of errors of discrete messages with the simulation results for a communication channel with telegraph distortion and rapid estimation of the probability of error in this channel.

## 3. Theory

The moments of time of changing elements signs of in real communication channels under the influence of various kinds of interference are ahead or late relative to their true position by the value  $\Delta T$ . This value  $\Delta T$  is distributed relatively to the average position of the fronts of the transmissions according to the normal law with the standard deviation (SD)  $\sigma_{\Delta T}$  [6]:

$$P(\Delta T) = \frac{1}{\sigma_{\Delta T} \sqrt{2\pi}} e^{-\frac{\Delta T^2}{2\sigma_{\Delta T}^2}}.$$

AVTD, measured in per cent, is the ratio of the mean value of the modulus of deviation of the fronts of message items  $\Delta T_{aver}$  on their average position on a given time interval to the duration of the message element T, multiplied by 100:

$$AVTD = \frac{\Delta T_{aver}}{T} 100\% = \frac{\sum_{n=k}^{k+K} |\Delta T(n)|}{KT} \cdot 100\%.$$

Here K is the number of fronts of elementary sendings at a given time interval of about  $2TK$ . In [4] dependence of error probability on AVTD, which is described by the following equation is empirically obtained

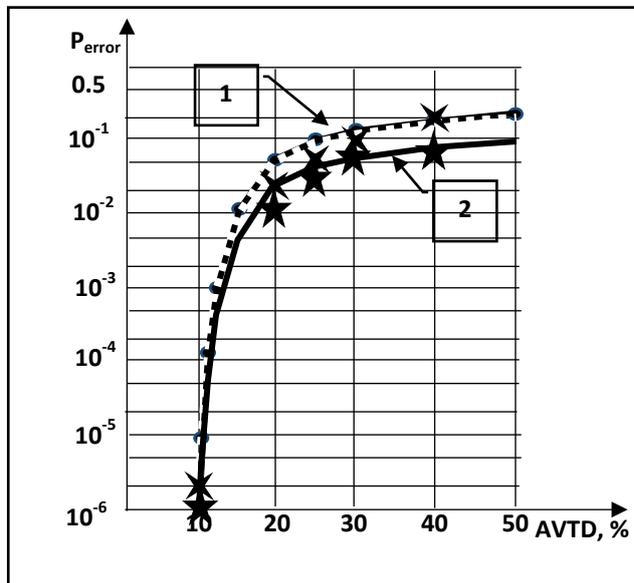
$$P_{error} \approx \frac{1}{2} e^{-10 \frac{1.44}{10} \frac{AVTD}{10} - 4}. \quad (1)$$

The corresponding graph of this dependence is shown in Fig. 1 as curve 1. However, as shown by the simulation, this error probability value, estimated by the magnitude of telegraphic distortions, is valid for the special case when a discrete message is a binary sequence of the form "meander", in which the number of fronts is equal to the number of elements in the received sequence. This kind of binary sequence is used as a part of the preamble, transmitted at the beginning of messages for clock synchronization. In general, when the number of fronts, as mentioned above, is on average two times less than the number of elements in the received information message, the dependence of the error probability on the average value of telegraphic distortions should be described by the following expression:

$$P_{error} \approx \frac{1}{4} e^{-10 \frac{1.44}{10} \frac{AVTD}{10} - 4}, \quad (2)$$

which corresponds to the curve 2 in Figure 1.

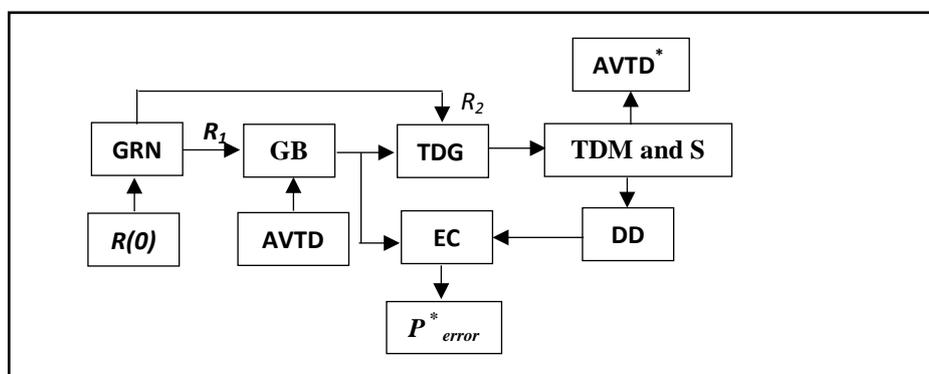
The obtained analytical dependence of the error probability on the AVTD was tested using a simulation model.



**Figure 1.** Dependence of error probability on AVTD  
 1-the curve corresponding to the equation (1);  
 ★the curve corresponding to the equation (2);  
 X - the result of simulation when receiving binary meander-type sequences;

#### 4. Results of computational experiments

The block diagram of the simulation model of the communication channel with telegraph distortions is shown in Figure 2.



**Figure 2.** Block diagram of the simulation model of the communication channel with telegraphic distortions

Figure 2 shows:

GRN is the generator of quasi-random numbers with a uniform distribution of their values in the range from 0 to 1;

GBS – a generator of random binary sequences;

TDG is the telegraph distortion generator;

TDM and S is the telegraph distortion meter and synchronizer;

DD-decision device;

EC is the error counter;

$R(0)$  is a number that puts the GRN in the initial state;

AVTD is a preset value of the telegraph distortion average;

AVTD\* is the measured value of the telegraph distortion average (AVTD estimation);

$P_{error}^*$  is the obtained estimate of the error probability.

The quasi-random number generator reproduces a sequence of random numbers  $R_1$  to simulate an information binary sequence and a sequence of random numbers  $R_2$  for a telegraph distortion generator.

The simulation algorithm of information binary sequence  $A(n)$  can be written as:

$$A(n) = \text{ent}[2R_1(n)].$$

A binary sequence is formed with the duration of elements, each of them containing 100 current counts out of the total number  $n$ .

If value  $X$  has a normal distribution, it is possible to model this value in two known ways: either using the central limit theorem of probability theory [6] or the Box-Muller method [7].

The first method has an algorithm that can be mathematically written as:

$$X = \sqrt{\left(\frac{12}{m}\right)} \left(\sum_{i=1}^m R_i - \frac{m}{2}\right),$$

where  $R_i$  are the numbers generated by GRN,

$m$  is the total number of summable quasi-random numbers.

At  $m$  equal to 12:

$$X = \sum_{l=1}^{12} R_l - 6. \quad (3)$$

SD of value  $X$  is equal to 1

In this case, to obtain one value of  $X$ , it is necessary to generate at least 12 quasi-random numbers evenly distributed over the interval from 0 to 1.

In the case of the Box-Muller method, only one quasi-random number is required to obtain each value of the normally distributed quantity  $X$ :

$$X_i = \sqrt{-2\ln R_j} \cdot \cos(2\pi R_i),$$

$$X_j = \sqrt{-2\ln R_j} \cdot \sin(2\pi R_i).$$

The first option, which corresponds to equation (2), is more simple, but has a disadvantage: the values of random variables with a low probability of occurrence can not be obtained if the number of summable numbers from the GRN is not large enough. Since very small error probabilities are irrelevant in the simulation of telegraph distortions when adapting to the coupling conditions, it is advisable to use the simplest algorithm corresponding to equation (2) to simulate a binary sequence with telegraph distortions.

In the case of discrete time, by specifying  $\sigma_{AT}$  as an integer percentage, we obtain random numbers  $Y = \sigma_{AT}X$ , which are rounded to the nearest integer  $S\%$ , which determines both the sign of the deviation of the front from its true position, and the number  $S$  of discrete time values  $T/100$ , by which the front deviates from this position.

The estimation of the telegraph distortions was carried out in accordance with the algorithm described in [4].

Figure 3 shows the sum of oscillations of 20 bipolar generators of binary periodic sequences at different values of telegraph distortions obtained by simulation using the algorithm described in [4].

Estimation of SD and AVTD of telegraph distortions was carried out using the following algorithms:

$$SD \% = 100 \cdot (0.5T - \Delta t_{0.3}) / T,$$

$$AVTD \% = 0.798 \cdot SD \%$$

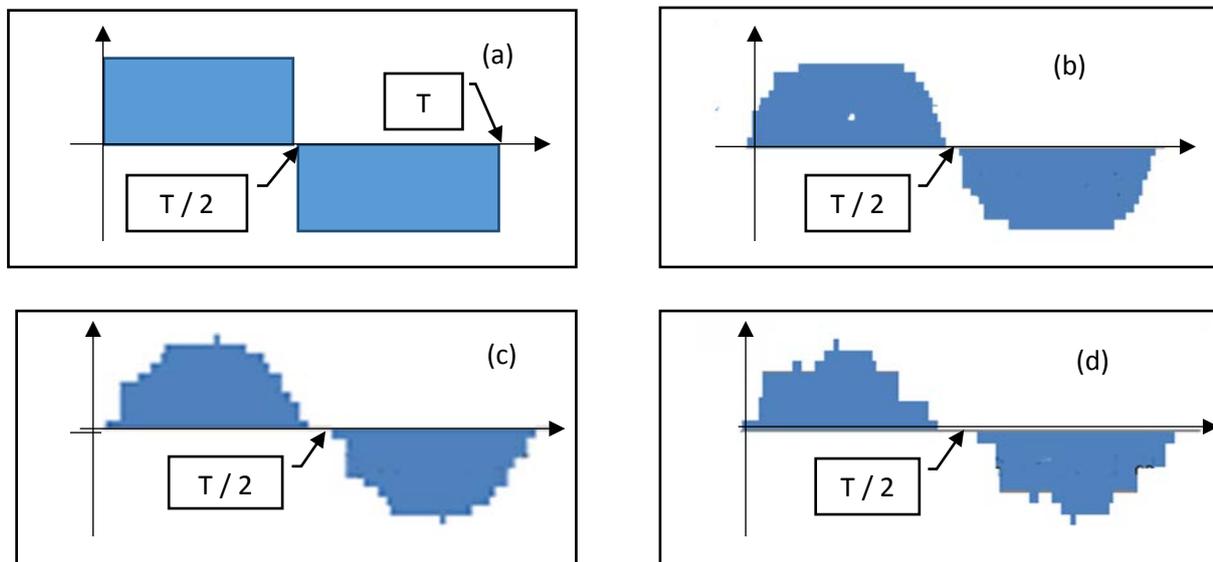
where  $\Delta t_{0.3}$  is the time interval when the sum of oscillations of bipolar generators is 0.3.

In determining the dependence of the error probability of the telegraph distortion value using simulation the value of AVTD binary sequence and time interval were set which were implemented on 20 fronts binary sequences, estimated the value of AVTD. The total number of errors in determining their probability for each value was 20.

The table 1 shows the specified values of AVTD, the values of the obtained estimates of AVTD\* and the corresponding values of the estimates of the elements error probabilities  $P_{\text{error}}^*$ .

**Table 1.** The accuracy of the AVTD\* and  $P_{\text{error}}^*$  estimates from the true value of AVTD.

AVTD	5	10	15	20
AVTD*	5	10	14	24
Relative error	0	0	0.067	0.2
$P_{\text{error}}^*$	$5.34 \cdot 10^{-12}$	$4.23 \cdot 10^{-6}$	$1.34 \cdot 10^{-3}$	$9.07 \cdot 10^{-3}$



**Figure 3.** The sum of the binary periodic oscillations from 20 bipolar generators at SD = 0 % (a), SD = 5 % (b), SD = 10 % (c) and SD = 15 % (d).

Figures 3(a), 3(b), 3(c) and 3 (d) show that the sum of binary oscillations clearly identifies the average position of the center of elementary premises. This feature of the telegraph distortion meter can be used for regeneration of message items of the received message.

## 5. Results and discussion

In figure 1 crosses mark the results of estimating the error probability by the average value of telegraph distortions when receiving a binary sequence of the "meander" type.

The asterisks in Fig. 1 mark the results of error probability estimation by the average value of telegraph distortions during the of binary sequence information reception.

The simulation results are in good agreement with the results (curves 1 and 2) obtained empirically in [4].

We estimate from the point of view of energy the difference that takes place between the of empirical dependence (2) and the simulation results. As it can be seen from Figure 1, in the worst case, these results of the error probability differ by a factor of 2. It is known [8, 9] that the dependence of the error probability on the signal / noise ratio is described by the expression:

$$P_{\text{ош}} = \frac{1}{2} e^{-\frac{h^2}{M}}, \quad (4)$$

where M depends on the type of the manipulation: M=4 for AM, M=2 for FM (frequency modulator) and M=1 for RFM (relative phase modulator), when the signal reception is carried out by comparison of the polarities.

From equation (4) we can obtain the following dependence of the signal-to-noise ratio  $h^2$  on  $R_{\text{error}}$ :

$$h^2 = -M \ln(2P_{\text{error}}).$$

Therefore:

$$10 \lg \frac{h_1^2}{h_2^2} = 10 \lg \frac{\ln(2P_{\text{error}1})}{\ln(2P_{\text{error}2})}.$$

In this case, the 2-times difference in error probability estimation at level of  $10^{-2}$ , obtained by different methods, inaccuracy in dB is calculated by the formula:

$$10 \lg \frac{h_1^2}{h_2^2} = 10 \lg \frac{\ln(2 \cdot 10^{-2})}{\ln(4 \cdot 10^{-2})} = 0.847 \text{ dB}.$$

Thus, from the energy point of view, the discrepancy in the estimates of error probabilities according to equations (1) and (2) and obtained by the simulation method does not exceed an error of 1 dB.

It is important to note that the described algorithm of the simulation model of the communication channel with telegraph distortions in terms of estimating the average value of telegraph distortions and clock synchronization can be used in real receiving equipment without any changes.

## 6. Conclusion

The resulting equation (2) for the rapid estimation of the error probability by the average value of telegraph distortion can be recommended for the development of algorithms for fast estimation of the received signal quality in the designed communication channels capable of adapting to rapidly changing communication conditions.

Besides, the described method of estimating the error probability by the average value of telegraph distortions solves the problem of regeneration of message items of the received message.

## References

- [1] Vasin V A and other 2004 Information technologies in radio engineering systems (Moskva: MGTU named after N.E. Bauman) p 768
- [2] Ruan D Gomes, Marcelo S Alencar, Diego V Queiroz, Iguatemi E Fonseca and Cesar Benavente-Peces 2017 Comparison Between Channel Hopping and Channel Adaptation for Industrial Wireless Sensor Networks. (URL: <https://www.researchgate.net/publication/313853257> Conference Paper February 2017) (assessed on 27.06.2019).
- [3] Kosykh A V and Khazan V L 2017 A method for assessing the quality of non-stationary radio channels in terms of Telegraph distortion *Dynamics of systems, mechanisms and machines* vol 5 No 4 pp 28–33
- [4] Kosykh A V and Khazan V L 2018 Estimation of Probability of Error on Degree Telegraph Distortion *Dynamics of systems, mechanisms and machines* vol 6 No 4 pp 32–39
- [5] Wenzel E C 2010 Probability theory (Moskva: KNORUS) p 664
- [6] Abdullaev D A and Aripov M N 1985 Transmission of discrete messages: problems and exercises. (Moskva: Radio and communications) p 128
- [7] Box G E P and Muller Mervin E. 1958 A note on the generation of random normal deviates *The Annals of Mathematical Statistics* vol 29 No 2 pp 610–611
- [8] Sklyar B 2007 Digital communication. Theoretical basis and practical application (Williams

publishing house) p 1104.

- [9] Vasiliev K K, Glushkov V A, Dormidontov A V and Nesterenko A G 2008 Theory of electrical communication (Ulyanovsk: UISTU) p 452