On the properties of the solutions of the problem of cross- diffusion with the dual nonlinearity and the convective transfer

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Abstract. In this work, special methods for studying nonlinear parabolic equations were developed that allow a fairly detailed study of nonlinear problems based on self-similar and approximately self-similar solutions and the construction of an analogue Zeldovych-Kompanees type solution for a cross system, since the study of self-similar equations is relatively simpler in comparison with equations in private derivatives. Studied the properties of solutions to the problem of a biological population of the Fisher-Kolmogorov type in the case of cross-diffusion with the dual nonlinearity and the convective transfer. An estimate of the solutions is obtained, and on the basis of it the problem of choosing the initial approximation for the numerical solution of the Cauchy problem is solved.

1. Introduction

Terms of cross-diffusion are widely used in the equations of reaction diffusion which are found in models from mathematical biology and in various engineering applications now.

In recent years considerably the theoretical analysis of mathematical models of reactionary and diffusive type in the presence of cross diffusion extended [1–3]. They began to pay special attention to unlimited solutions that are the cause of the presence of energy release, a chemical reaction, etc. They are solutions that arise in many physical processes (for example, combustion). In this regard, in recent years, theory of blow up solutions strongly developed in many works by A.A. Samarsky, S.P.Kurdyumov, A.P. Mikhailov, V.A. Galaktionova, S.Nimova and many others have been devoted to this issue. Blow up solutions called aggravated solutions. Special methods were developed for the study of nonlinear parabolic equations, which allow for a sufficiently detailed study of the blow up solutions of the heat equation with a source [1, 2, 3]. In order to study nonlinear problems, it has become intensive to engage in self-similar and approximately self-similar solutions, since the study of self-similar equations is relatively simpler compared to partial differential equations. Therefore, it is possible to study qualitative properties of the solutions of the original partial differential equations by constructing various self-similar equations. Using them on the example of nonlinear heat conduction, filtration and diffusion, new nonlinear effects were established.

In [2] for the case of fast diffusion established a two-way estimation of solution and [3] investigated quasilinear degenerate parabolic equation with inhomogeneous density. Cross diffusion is a process in which the gradient of concentration or density of one chemical or a species induces a stream, linear or nonlinear, other look. In molecular biology cross-diffusive processes arise in the multicomponent systems containing at least two dissolved components [4,5]. Modern biotechnology is a very high-tech industry that cannot be imagined without achieving not only biochemistry, microbiology, molecular biology, genetics, but also a number of other sciences, such as physics, chemistry, computer science. Mathematics is the indisputable basis and tool for research that has found application in biotechnology. The close interweaving of many branches of scientific knowledge allows a person to create amazing things and study them. In modern biotechnology, mathematical models are necessary, since they can be used to describe the most complex technological processes in biology and suggest their further development. Also, no production using living organisms can do without such calculations.

Multicomponent systems containing nanoparticles, surfactants, polymers and other macromolecules in solution play an important role in industrial applications and biological functions [4]. In developmental biology, the latest experimental results show that cross-diffusion can be very significant in creating a spatial structure [6]. The effect of cross-diffusion on pattern formation models has been studied in many theoretical papers, for example, see [7]. In addition to patterning in biology, there are other areas of application of reaction-cross-diffusion systems, which include cancer motility [8], finance [9], and biofilm [10]. It has been shown that the introduction of cross-diffusion in standard reaction-diffusion models prevents an explosion.

In this paper, we consider a model of a biological population consisting of parabolic systems of two quasilinear reaction-diffusion equations. Biological populations are a very attractive object for evolutionary research, since they have a high reproduction rate, biomass growth, and microevolutionary processes. One method for obtaining a self-similar system for solving the cross-diffusion problem with double nonlinearity and convective transfer is described. Numerical calculations are performed that preserve the properties of the finite velocity and spatial localization of the flare.

2. Problem definition.

Let's consider in Q={(t,x): 0< t < ∞, x∈ R^N } parabolic system of reaction diffusion

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = \nabla\left(D_1 u_2^{m_1-1}\left|\nabla u_1^k\right|^{p-2}\nabla u_1\right) + div(c(t)u) + k_1 u_1\left(1-u_1^{\beta_1}\right), \\ \dfrac{\partial u_2}{\partial t} = \nabla\left(D_2 u_1^{m_2-1}\left|\nabla u_1^k\right|^{p-2}\nabla u_2\right) + div(c(t)u) + k_2 u_2\left(1-u_2^{\beta_2}\right), \end{cases} \tag{1}$$

$$u_1\big|_{t=0} = u_{10}(x)\,,\ u_2\big|_{t=0} = u_{20}(x)\,, \tag{2}$$

which coefficients of mutual diffusion are respectively equal $D_1 u_2^{m_1-1}\left|\nabla u_1^k\right|^{p-2}\nabla u_1$, $D_2 u_1^{m_2-1}\left|\nabla u_2^k\right|^{p-2}\nabla u_2$ and convective transfer with a speed $c(t)$. $m_1, m_2, n, p, \beta_1, \beta_2$, D_1 , D_2 – positive real numbers, $\nabla(.) - grad_x(.)$, $\beta_1, \beta_2 \ge 1$, $x \in R^N$ $l > 0$; $u_1 = u_1(t,x) \ge 0$, $u_2 = u_2(t,x) \ge 0$ – required solutions.

Let's notice that replacement in

$$u_1(t,x) = e^{k_1 t} v_1(\tau(t),\xi)\,,\ u_2(t,x) = e^{k_2 t} v_2(\tau(t),\xi)\,,\ \xi = x - \int_0^t c(\eta)d\eta$$

will lead it to a look:

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau} = \nabla\left(D_1 v_2^{m_1-1}\left|\nabla v_1^k\right|^{p-2}\nabla v_1\right) + k_1 e^{[\beta_1 k_1-(p-2)kk_1-(m_1-1)k_2]t} v_1^{\beta_1+1}, \\ \dfrac{\partial v_2}{\partial \tau} = \nabla\left(D_2 v_1^{m_2-1}\left|\nabla v_2^k\right|^{p-2}\nabla v_2\right) + k_2 e^{[\beta_2 k_2-(p-2)kk_2-(m_2-1)k_1]t} v_2^{\beta_2+1}, \end{cases} \tag{3}$$

$$v_1\big|_{t=0} = v_{10}(x),\ v_2\big|_{t=0} = v_{20}(x). \tag{4}$$

If $k_1[(p-2)k-(m_1+1)] = k_2[(p-2)k-(m_2+1)]$, choosing that

$$\tau(t) = \frac{e^{[(m_1-1)k_2+(p-2)kk_1]t}}{(m_1-1)k_2+(p-2)kk_1} = \frac{e^{[(m_2-1)k_1+(p-2)kk_2]t}}{(m_2-1)k_1+(p-2)kk_2},$$

receive the following system of the equations:

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau} = \nabla\left(D_1 v_2^{m_1-1}\left|\nabla v_1^k\right|^{p-2}\nabla v_1\right) + a_1(t) v_1^{\beta_1+1}, \\ \dfrac{\partial v_2}{\partial \tau} = \nabla\left(D_2 v_1^{m_2-1}\left|\nabla v_2^k\right|^{p-2}\nabla v_2\right) + a_2(t) v_2^{\beta_2+1}, \end{cases} \tag{5}$$

where $a_1 = k_1\left((p-2)kk_1+(m_1-1)k_2\right)^{b_1}$, $b_1 = \dfrac{(\beta_1-(p-2)k)k_1-(m_1-1)k_2}{(p-2)kk_1+(m_1-1)k_2}$,

$$a_2 = k_2\left((m_2-1)k_1+(p-2)kk_2\right)^{b_2},\ b_2 = \frac{(\beta_2-(p-2)k)k_2-(m_2-1)k_1}{(m_2-1)k_1+(p-2)kk_2}.$$

When $b_i = 0$, and $a_i(t) = const$, $i = 1,2$, then the system has an appearance:

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau} = \nabla\left(D_1 v_2^{m_1-1}\left|\nabla v_1^k\right|^{p-2}\nabla v_1\right) - a_1 v_1^{\beta_1+1}, \\ \dfrac{\partial v_2}{\partial \tau} = \nabla\left(D_2 v_1^{m_2-1}\left|\nabla v_2^k\right|^{p-2}\nabla v_2\right) - a_2 v_2^{\beta_2+1}, \end{cases} \tag{6}$$

Below we describe one way to obtain a self-similar system for (5). First we find a solution to the following systems of equations

$$\begin{cases} \dfrac{d\overline{v}_1}{d\tau} = -a_1 \overline{\mathbf{v}}_{\mathbf{1}}^{\beta_1+1}, \\ \dfrac{d\overline{v}_2}{d\tau} = -a_2 \overline{\mathbf{v}}_{\mathbf{2}}^{\beta_2+1}, \end{cases}$$

in the form

$$\overline{v}_1(\tau) = (\tau(t))^{-\gamma_1},\ \gamma_1 = \frac{1}{\beta_1},\ \overline{v}_2(\tau) = (\tau(t))^{-\gamma_2},\ \gamma_2 = \frac{1}{\beta_2},$$

for a case $b_i = 0$, and $a_i(t) = const$, $i = 1,2$. And in the case $b_i \neq 0$, and $a_i(t) = const$, $i = 1,2$ find a solution to the system of ordinary differential equations

$$\begin{cases} \dfrac{d\overline{v}_1}{d\tau} = -a_1 \tau^{b_1} \overline{\mathbf{v}}_{\mathbf{1}}^{\beta_1+1}, \\ \dfrac{d\overline{v}_2}{d\tau} = -a_2 \tau^{b_2} \overline{\mathbf{v}}_{\mathbf{2}}^{\beta_2+1}, \end{cases}$$

look

$$\overline{v}_2(\tau)=(\tau(t))^{-\gamma_1},\ \gamma_1=\frac{b_1+1}{\beta_1},\ \overline{v}_2(\tau)=(\tau(t))^{-\gamma_2},\ \gamma_2=\frac{b_2+1}{\beta_2}\ .$$

then the solution of a system (5) is looked in the form

$$\begin{aligned} v_1(t,x)&=\overline{v}_1(\tau)w_1(\tau(t),\xi),\xi=\int_0^t c(y)dy-x\\ v_2(t,x)&=\overline{v}_2(\tau)w_2(\tau(t),\xi),\end{aligned} \tag{7}$$

and $\tau=\tau(t)$ is chosen so

$$\tau(\tau)=\int_0^\tau \overline{v}_1^{(p-2)k}(t)\overline{v}_2^{(m_1-1)}(t)dt=\int_0^\tau \overline{v}_2^{(p-2)k}(t)\overline{v}_1^{(m_1-1)}(t)dt$$

Calculation of this integral gives

$$\tau(\tau)=\begin{cases}\dfrac{(T+\tau)^{1-[\gamma_1(p-2)k+\gamma_2(m_1-1)]}}{1-[\gamma_1(p-2)k+\gamma_2(m_1-1)]}, & if\ \ 1-[\gamma_1(p-2)k+\gamma_2(m_1-1)]\neq 0,\\ \ln(T+\tau), & if\ \ 1-[\gamma_1(p-2)k+\gamma_2(m_1-1)]=0,\\ (T+\tau), & if\ \ p=2\ \ and\ \ m_1=1,\end{cases}$$

if $\gamma_1(p-2)+\gamma_2(m_1-1)=\gamma_2(p-2)+\gamma_1(m_2-1)$.

Then for $w_i(\tau,x),\ i=1,2$ receive system of equations:

$$\begin{cases}\dfrac{\partial w_1}{\partial\tau}=\nabla_\xi\left(D_1w_2^{m_1-1}\left|\nabla_\xi w_1^k\right|^{p-2}\nabla_\xi w_1\right)+\psi_1(w_1-w_1^{\beta_1+1}),\\ \dfrac{\partial w_2}{\partial\tau}=\nabla_\xi\left(D_2w_1^{m_2-1}\left|\nabla_\xi w_2^k\right|^{p-2}\nabla_\xi w_2\right)+\psi_2(w_2-w_2^{\beta_2+1}),\end{cases} \tag{8}$$

where

$$\psi_1=\begin{cases}\dfrac{1}{(1-[\gamma_1(p-2)k+\gamma_2(m_1-1)])\tau}, & if\ \ 1-[\gamma_1(p-2)k+\gamma_2(m_1-1)>0,\\ \gamma_1c_1^{-(1-[\gamma_1(p-2)k+\gamma_2(m_1-1)])}, & if\ 1-[\gamma_1(p-2)k+\gamma_2(m_1-1)=0,\end{cases}$$

$$\psi_2=\begin{cases}\dfrac{1}{(1-[\gamma_2(p-2)+\gamma_1(m_2-1)])\tau}, & if\ \ 1-[\gamma_2(p-2)+\gamma_1(m_2-1)]>0,\\ \gamma_2c_1^{-(1-[\gamma_2(p-2)+\gamma_1(m_2-1)])}, & if\ 1-[\gamma_2(p-2)+\gamma_1(m_2-1)]=0.\end{cases}$$

If $1-[\gamma_1(p-2)k+\gamma_2(m_1-1)=0$, self-similar solution of a system (8) has an appearance

$$w_i(\tau(t),x)=f_i(\xi),\ i=1,2, \tag{9}$$

$$\xi=(\int_0^t c(y)dy-x)/\tau^{\frac{1}{p}}.$$

Then substituting (9) in (8) relatively to $f_i(\xi)$ receive the system of the self-similar equations

$$\begin{cases} \xi^{1-N}\dfrac{d}{d\xi}(\xi^{N-1}f_2^{m_1-1}\left|\dfrac{df_1^k}{d\xi}\right|^{p-2}\dfrac{df_1}{d\xi})+\dfrac{\xi}{p}\dfrac{df_1}{d\xi}+\mu_1 f_1(1-f_1^{\beta_1})=0, \\ \xi^{1-N}\dfrac{d}{d\xi}(\xi^{N-1}f_1^{m_2-1}\left|\dfrac{df_2^k}{d\xi}\right|^{p-2}\dfrac{df_2}{d\xi})+\dfrac{\xi}{p}\dfrac{df_2}{d\xi}+\mu_2 f_2(1-f_2^{\beta_2})=0. \end{cases} \tag{10}$$

where $\mu_1=\dfrac{1}{(1-[\gamma_1 k(p-2)+\gamma_2(m_1-1)])}$ and $\mu_2=\dfrac{1}{(1-[\gamma_2 k(p-2)+\gamma_1(m_2-1)])}$.

System (10) has approximate solution of a look

$$\overline{f}_1=A(a-\xi^{\gamma})_+^{n_1},\ \gamma=p/(p-1),\ \overline{f}_2=B(a-\xi^{\gamma})_+^{n_2},$$

where A and B are constant and

$$n_1=\frac{(p-1)[k(p-2)-(m_1-1)]}{\left[k(p-2)\right]^2-(m_1-1)(m_2-1)},\quad n_2=\frac{(p-1)[k(p-2)-(m_2-1)]}{\left[k(p-2)\right]^2-(m_1-1)(m_2-1)}.$$

In this paper, we used the asymptotic representation of the solution that we found as an initial approximation.

3. Construction of the upper solution

Let's be engaged in creation of the upper solution for system (10) $\overline{f}_1(\xi),\ \overline{f}_2(\xi)$ possess properties

$$\overline{f}_2^{m_1-1}\left|\frac{d\overline{f}_1^k}{d\xi}\right|^{p-2}\frac{d\overline{f}_1}{d\xi}=-A^{k(p-2)+1}B^{m_1-1}\left(\gamma n_1 k\right)^{(p-2)}\gamma n_1\xi\,\overline{f}_1\in C(0,\infty)$$

$$\overline{f}_1^{m_2-1}\left|\frac{d\overline{f}_2^k}{d\xi}\right|^{p-2}\frac{d\overline{f}_2}{d\xi}=-A^{m_2-1}B^{k(p-2)+1}\left(\gamma n_2 k\right)^{(p-2)}\gamma n_2\xi\,\overline{f}_2\in C(0,\infty)$$

that $(\gamma-1)(p-1)=1,\ \gamma=\dfrac{p}{p-1},$

and

$$\begin{cases} \xi^{1-N}\dfrac{d}{d\xi}\left(\xi^{N-1}\overline{f}_2^{m_1-1}\left|\dfrac{d\overline{f}_1^k}{d\xi}\right|^{p-2}\dfrac{d\overline{f}_1}{d\xi}\right)=-A^{k(p-2)+1}B^{m_1-1}\left(\gamma n_1 k\right)^{(p-2)}\gamma n_1\left(N\overline{f}_1+\xi\dfrac{d\overline{f}_1}{d\xi}\right) \\ \xi^{1-N}\dfrac{d}{d\xi}\left(\xi^{N-1}\overline{f}_1^{m_2-1}\left|\dfrac{df_2^k}{d\xi}\right|^{p-2}\dfrac{df_2}{d\xi}\right)=-A^{m_2-1}B^{k(p-2)+1}\left(\gamma n_2 k\right)^{(p-2)}\gamma n_2\left(N\overline{f}_2+\xi\dfrac{d\overline{f}_2}{d\xi}\right) \end{cases}$$

Let's choose A and B from the system of the nonlinear algebraic equations

$$\begin{cases} A^{k(p-2)+1}B^{m_1-1}\left(\gamma n_1 k\right)^{(p-2)}\gamma n_1=1/p \\ A^{m_2-1}B^{k(p-2)+1}\left(\gamma n_2 k\right)^{(p-2)}\gamma n_2=1/p \end{cases}$$

i.e.

$$A=\left(\frac{n_1}{n_2}\right)^{\frac{(p-1)(k(p-2)+1)}{(k(p-2)+1)^2-(m_1-1)(m_2-1)}},\ B=\left[(p(\gamma n_1 k)^{p-2}\gamma n_1\left(\frac{n_1}{n_1}\right)^{\frac{(p-1)(k(p-2)+1)^2}{(k(p-2)+1)^2-(m_1-1)(m_2-1)}}\right]^{-\frac{1}{m_1-1}}.$$

Then the functions $\overline{f}_1$, $\overline{f}_2$ are solution profile like Zeldovich-Kompanees for a system (1) and in the area $|\xi| < (a)^{(p-1)/p}$ they satisfy the system of equations

$$\begin{cases} \xi^{1-N}\dfrac{d}{d\xi}\left(\xi^{N-1}\overline{f}_2^{\,m_1-1}\left|\dfrac{d\overline{f}_1^{\,k}}{d\xi}\right|^{p-2}\dfrac{d\overline{f}_1}{d\xi}\right)+\dfrac{\xi}{p}\dfrac{d\overline{f}_1}{d\xi}+\dfrac{N}{p}\overline{f}_1=0 \\ \xi^{1-N}\dfrac{d}{d\xi}\left(\xi^{N-1}\overline{f}_1^{\,m_2-1}\left|\dfrac{d\overline{f}_2^{\,k}}{d\xi}\right|^{p-2}\dfrac{d\overline{f}_2}{d\xi}\right)+\dfrac{\xi}{p}\dfrac{d\overline{f}_2}{d\xi}+\dfrac{N}{p}\overline{f}_2=0 \end{cases} \tag{11}$$

in classical sense.

Theorem 1. Let $u_i(0,x) \le u_{i\pm}(0,x),\ x \in R.$ Then for the solution of a task (1) in area Q takes place assessment

$$\begin{aligned} u_1(t,x) &\le u_{1+}(t,x) = e^{k_1 t}\overline{f}_1(\xi), \\ u_2(t,x) &\le u_{2+}(t,x) = e^{k_2 t}\overline{f}_2(\xi), \end{aligned} \qquad \xi = x/[\tau(t)]^{1/p}$$

where $\overline{f}_1(\xi)$, $\overline{f}_2(\xi)$ *u* $\tau(t)$ – functions defined above.

Proof. Upper solution of a task (1), (2) we will look for in the form of (7)

It is possible to be convinced by immediate check that functions

$$\overline{f}_1 = A(a-\xi^{\gamma})_+^{\,n_1},\ \gamma = p/(p-1),\ \overline{f}_2 = B(a-\xi^{\gamma})_+^{\,n_2},$$

are the generalized solution of the equation (11)

The proof of the theorem is based on a comparison theorem of solutions [14,15].

That

$$\begin{cases} \xi^{N-1}\overline{f}_2^{\,m_1-1}\left|\dfrac{d\overline{f}_1^{\,k}}{d\xi}\right|^{p-2}\dfrac{d\overline{f}_1}{d\xi} = -A^{k(p-2)+1}B^{m_1-1}\left(\gamma n_1 k\right)^{(p-2)}\gamma n_1 \xi^N \overline{f}_1 \\ \xi^{N-1}\overline{f}_1^{\,m_2-1}\left|\dfrac{d\overline{f}_2^{\,k}}{d\xi}\right|^{p-2}\dfrac{d\overline{f}_2}{d\xi} = -A^{m_2-1}B^{k(p-2)+1}\left(\gamma n_2 k\right)^{(p-2)}\gamma n_2 \xi^N \overline{f}_2 \end{cases}$$

function $\overline{f}_1(\xi)$, $\overline{f}_2(\xi)$ and streams possess the following tangential property

$$0 \le \overline{f}_1(\xi),\ \xi^{N-1}\overline{f}_2^{\,m_1-1}\left|\dfrac{d\overline{f}_1^{\,k}}{d\xi}\right|^{p-2}\dfrac{d\overline{f}_1}{d\xi} = -A^{k(p-2)+1}B^{m_1-1}\left(\gamma n_1 k\right)^{(p-2)}\gamma n_1 \xi^N \overline{f}_1 \in C\left(0,\infty\right),$$

$$0 \le \overline{f}_2(\xi),\ \xi^{N-1}\overline{f}_1^{\,m_2-1}\left|\dfrac{d\overline{f}_2^{\,k}}{d\xi}\right|^{p-2}\dfrac{d\overline{f}_2}{d\xi} = -A^{m_2-1}B^{k(p-2)+1}\left(\gamma n_2 k\right)^{(p-2)}\gamma n_2 \xi^N \overline{f}_2 \in C\left(0,\infty\right).$$

Let's choose A and B those that inequalities were carried out

$$\begin{aligned} A^{k(p-2)+1}B^{m_1-1}\left(\gamma n_1 k\right)^{(p-2)}\gamma n_1 &\ge 1/p, \\ A^{m_2-1}B^{k(p-2)+1}\left(\gamma n_2 k\right)^{(p-2)}\gamma n_2 &\ge 1/p. \end{aligned} \tag{12}$$

Then that (11)

$$f_1 > 0,\ \ f_2 > 0 \ \textit{в}\ C \in (0,\infty),$$

from (12) we have

$$\xi^{1-N}\frac{d}{d\xi}\left(\xi^{N-1-}\overline{f}_2^{\,m_1-1}\left|\frac{d\overline{f}_1^{\,k}}{d\xi}\right|^{p-2}\frac{d\overline{f}_1}{d\xi}\right)+\frac{\xi}{p}\frac{d\overline{f}_1}{d\xi}\le 0,$$
$$\xi^{1-N}\frac{d}{d\xi}\left(\xi^{N-1}\overline{f}_1^{\,m_1-1}\left|\frac{d\overline{f}_2^{\,k}}{d\xi}\right|^{p-2}\frac{d\overline{f}_2}{d\xi}\right)+\frac{\xi}{p}\frac{d\overline{f}_2}{d\xi}\le 0, \tag{13}$$
$$\xi\in(0,\infty).$$

It follows that $u_i(t,x)$ is bounded for all t> 0 and thus the global solvability of problem (1), (2) is established.

Theorem 1 is proved.

Research of qualitative properties of a system (1) allowed, to execute a numerical experiment depending on values, the logging-in numerical parameters. For this purpose as an initial approximation an asymptotics of solution was used.

Case $n_1>0, n_2>0,\ q>0$ (slow diffusion). Using the method [1] to solve equation (10), we obtain the following functions

$$\overline{\theta}_1(\xi)=A(a-\xi^{p/(p-1)})_+^{\ n_1},\ \overline{\theta}_2(\xi)=B(a-\xi^{p/(p-1)})_+^{\ n_2},$$

Finite solution of a system (10) at $\xi\to a_-$ has asymptotics $f_i(\xi)\sim\vartheta_i(\xi)$.

Case $n_1>0, n_2>0,\ q<0$ (fast diffusion). For (10) we have

$$\chi_1(\xi)=A_1(a+\xi^{p/(p-1)})^{n_1},\ \chi_2(\xi)=B_1(a+\xi^{p/(p-1)})^{n_2},$$

where $a>0$, $q=\left[k(p-2)\right]^2-(m_1-1)(m_2-1)$.

At $\xi\to+\infty$ the tasks (10) disappearing on infinity solution has an asymptotics

$$f_i(\xi)\sim\chi_i(\xi),\quad i=1,2.$$

4. Computing experiment.

In the domain $Q=\{(t,x):t\in[0,T],\quad x\in[a,b]\}$ system of the quasilinear equations of parabolic type is considered

$$\begin{cases}\dfrac{\partial u_1}{\partial t}=\dfrac{\partial}{\partial x}\left(D_1u_2^{m_1-1}\left|\dfrac{\partial u_1^k}{\partial x}\right|^{p-2}\dfrac{\partial u_1}{\partial x}\right)+c(t)\dfrac{\partial u_1}{\partial x}+k_1(t)u_1\left(1-u_1^{\beta_1}\right),\\[2ex] \dfrac{\partial u_2}{\partial t}=\dfrac{\partial}{\partial x}\left(D_2u_1^{m_2-1}\left|\dfrac{\partial u_2^k}{\partial x}\right|^{p-2}\dfrac{\partial u_2}{\partial x}\right)+c(t)\dfrac{\partial u_2}{\partial x}+k_2(t)u_2\left(1-u_2^{\beta_2}\right),\end{cases} \tag{14}$$

with initial

$$\begin{aligned}u_1(0,x)=u_{10}(x)\ge 0,\quad x\in[a,b],\\ u_2(0,x)=u_{20}(x)\ge 0,\quad x\in[a,b],\end{aligned} \tag{15}$$

and boundary conditions

$$
\begin{aligned}
&u_1(t,a)=\varphi_1(t)\ge 0, \quad t\in[0,T],\\
&u_1(t,b)=\varphi_2(t)\ge 0, \quad t\in[0,T],\\
&u_2(t,a)=\psi_1(t)\ge 0, \quad t\in[0,T],\\
&u_2(t,b)=\psi_2(t)\ge 0, \quad t\in[0,T].
\end{aligned} \tag{16}
$$

Here m_1, m_2, β_1, β_2, p – positive constants, $u_{10}(x)$ and $u_{20}(x)$ – initial distribution respectively for the first and second component, $\phi_1(t)$ – value of the first component on the left-hand border, $\phi_2(t)$ – value of the first component on the right border, $\psi_1(t)$ and $\psi_2(t)$ – respectively for the second component.

Problem (14)-(16) describes many physical processes, for example, of diffusion, heat conductivity of the bipropellant environment: temperature and pressure, salt and moisture etc.

Let's construct the uniform grid on t and x

$\overline{\omega_{\tau h}}=\{t_j=j\tau,\, j=0,1,...,m,\tau m=T;x_i=a+ih,i=0,1,...,n,h=\dfrac{b-a}{n}\}$ approximate a task (14)-(16) method of balance (the integro-interpolation method)

$$
\begin{cases}
\dfrac{y_i^{j+1}-y_i^j}{\tau}=\dfrac{1}{h}\left(a_{i+1}\dfrac{y_{i+1}^{j+1}-y_i^{j+1}}{h}-a_i\dfrac{y_i^{j+1}-y_{i-1}^{j+1}}{h}\right)+\\
+c_i^{j+1}\dfrac{y_{i+1}^{j+1}-y_{i-1}^{j+1}}{2h}+k_{1i}^{j+1}y_i^{j+1}\left(1-\left(y_i^j\right)^{\beta_1}\right)\\
\dfrac{w_i^{j+1}-w_i^j}{\tau}=\dfrac{1}{h}\left(b_{i+1}\dfrac{w_{i+1}^{j+1}-w_i^{j+1}}{h}-b_i\dfrac{w_i^{j+1}-w_{i-1}^{j+1}}{h}\right)+\\
+c_i^{j+1}\dfrac{w_{i+1}^{j+1}-w_{i-1}^{j+1}}{2h}+k_{2i}^{j+1}w_i^{j+1}\left(1-\left(w_i^{j+1}\right)^{\beta_2}\right)
\end{cases}, \tag{17}
$$

where a_i and b_i calculated in ways

1) $a_i(y)=0.5D_1\left[\left(w_{i+1}^j\right)^{m_1-1}\left|\dfrac{\left(y^k\right)_{i+1}^{j+1}-\left(y^k\right)_i^{j+1}}{h}\right|^{p-2}+\left(w_i^j\right)^{m_1-1}\left|\dfrac{\left(y^k\right)_i^{j+1}-\left(y^k\right)_{i-1}^{j+1}}{h}\right|^{p-2}\right]$ and

$$
b_i(w)=0.5D_2\left[\left(y_{i+1}^{j+1}\right)^{m_2-1}\left|\frac{\left(w^k\right)_{i+1}^{j+1}-\left(w^k\right)_i^{j+1}}{h}\right|^{p-2}+\left(y_i^{j+1}\right)^{m_2-1}\left|\frac{\left(w^k\right)_i^{j+1}-\left(w^k\right)_{i-1}^{j+1}}{h}\right|^{p-2}\right],
$$

System of schemes (17) is non-linear concerning function y^{j+1} and w^{j+1}. For finding of its solution the method of iterations is used. We build a repetitive process as follows:

$$\begin{cases} \dfrac{\overset{s+1}{y}{}_i^{j+1} - y_i^j}{\tau} = \dfrac{1}{h}\left(\overset{s}{a}_{i+1} \dfrac{\overset{s+1}{y}{}_{i+1}^{j+1} - \overset{s+1}{y}{}_i^{j+1}}{h} - \overset{s}{a}_i \dfrac{\overset{s+1}{y}{}_i^{j+1} - \overset{s+1}{y}{}_{i-1}^{j+1}}{h} \right) + c_i^{j+1} \dfrac{\overset{s+1}{y}{}_{i+1}^{j+1} - \overset{s+1}{y}{}_{i-1}^{j+1}}{2h} + \\ +k_{1i}^{j+1} \overset{s+1}{y}{}_i^{j+1} \left(1 - \left(y_i^j\right)^{\beta_1}\right) \\ \dfrac{\overset{s+1}{w}{}_i^{j+1} - w_i^j}{\tau} = \dfrac{1}{h}\left(\overset{s}{b}_{i+1} \dfrac{\overset{s+1}{w}{}_{i+1}^{j+1} - \overset{s+1}{w}{}_i^{j+1}}{h} - \overset{s}{b}_i \dfrac{\overset{s+1}{w}{}_i^{j+1} - \overset{s+1}{w}{}_{i-1}^{j+1}}{h} \right) + c_i^{j+1} \dfrac{\overset{s+1}{w}{}_{i+1}^{j+1} - \overset{s+1}{w}{}_{i-1}^{j+1}}{2h} + \\ +k_{2i}^{j+1} \overset{s+1}{w}{}_i^{j+1} \left(1 - \left(w_i^{j+1}\right)^{\beta_2}\right) \end{cases} \tag{18}$$

Concerning function $\overset{(s+1)}{y}{}^{j+1}$ and $\overset{(s+1)}{w}{}^{j+1}$ difference schemes (18) it appears the linear. As initial iteration functions y and w of the previous step on time undertake: $\overset{(0)}{y}{}^{j+1} = y^j$ and $\overset{(0)}{w}{}^{j+1} = w^j$. For convergence of iteration demand realization of a condition

$$\max_i \left| \overset{(s+1)}{y_i} - \overset{(s)}{y_i} \right| \le \varepsilon \text{ and } \max_i \left| \overset{(s+1)}{w_i} - \overset{(s)}{w_i} \right| \le \varepsilon .$$

For the solution of the linear scheme (18), with conditions (15)-(16) on a grid, the sweep method is used.

Now we will give results of numerical experiments. Repetitive process is under construction by Picard's method, Newton and express. Results of computing experiments show that all iterative methods are suitable for the constructed scheme. For achievement of identical accuracy Newton's method (with square convergence) demands less iterations, than Picard's (Table 1-4) method.

Table 1

Fast diffusion. Values of parameters has to be

$$n_1 > 0, n_2 > 0,\ q < 0,\ 1 - [\gamma_1(p-2)k + \gamma_2(m_1 - 1)] \ne 0 : \tau(t) = \frac{(T+\tau)^{1-[\gamma_1(p-2)k+\gamma_2(m_1-1)]}}{1-[\gamma_1(p-2)k + \gamma_2(m_1-1)]}.$$

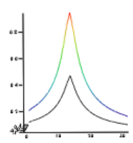
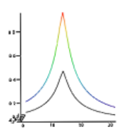
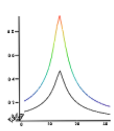
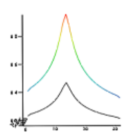
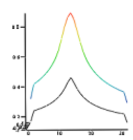
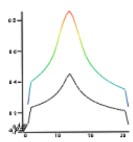
Parameter values	$t_{\max} = 0.5,\ x_{1\max} = 1.373,$ $x_{2\max} = 1.373$	$t_{\max} = 10,\ x_{1\max} = 2.045,$ $x_{2\max} = 2.045$	$t_{\max} = 40\ x_{1\max} = 2.924$ $x_{2\max} = 2.924$
$m_1 = 4.1,\ m_2 = 4.0,\ p = 4.4\ \ eps = 10^{-3}$ $n_1 = 0.822 > 0\ \ n_2 = 0.779 > 0$ $q = -7.86 < 0\ \beta_1 = 1,\ \beta_2 = 1\ \ k = 0.5$ n=3			
$m_1 = 5.7,\ m_2 = 5.4,\ p = 3\ \ eps = 10^{-3}$ $n_1 = 0.291 > 0$ $n_2 = 0.24 > 0$ $q = -11.68 < 0\ \beta_1 = 2,\ \beta_2 = 2$ $k = 3$, n=3			

Table 2

Fast diffusion. Parameter values should be

$n_1 > 0, n_2 > 0,\ q < 0\ 1-[\gamma_1(p-2)k+\gamma_2(m_1-1)] = 0:\ \tau(t) = \ln(t+T)$, T>0.

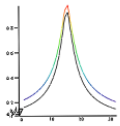
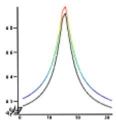
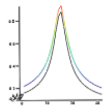
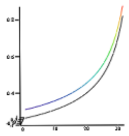
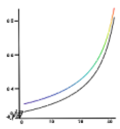
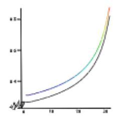
Parameter values	$t_{\max} = 0.5,\ x_{1\max} = 1.229,$ $x_{2\max} = 1.229$	$t_{\max} = 1,\ x_{1\max} = 1.444,$ $x_{2\max} = 1.444$	$t_{\max} = 15,\ x_{1\max} = 3.488,$ $x_{2\max} = 3.488$
$m_1 = 3,\ m_2 = 3.2,\ p = 3\ \ eps = 10^{-3}$ $n_1 = 0.588 > 0\ \ n_2 = 0.706 > 0$ $q = -3.4 < 0\ \beta_1 = 3,\ \beta_2 = 3\ \ k = 1$, n=7			
$m_1 = 5,\ m_2 = 5.2,\ p = 3\ \ eps = 10^{-3}$ $n_1 = 0.313 > 0$ $n_2 = 0.344 > 0$ $q = -12.8 < 0\ \beta_1 = 10,\ \beta_2 = 5$ $k = 2$, n=0.7			

Table 3

Slow diffusion. Parameter values should be

$$n_1 > 0, n_2 > 0, q > 0, 1-[\gamma_1(p-2)k+\gamma_2(m_1-1)] \neq 0:\ \tau(t) = \frac{(T+\tau)^{1-[\gamma_1(p-2)k+\gamma_2(m_1-1)]}}{1-[\gamma_1(p-2)k+\gamma_2(m_1-1)]}.$$

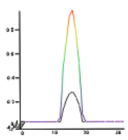
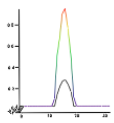
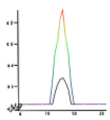
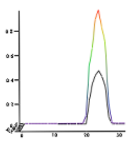
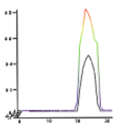
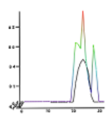
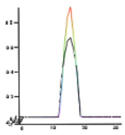
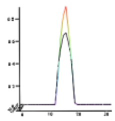
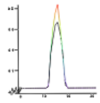
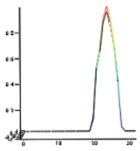
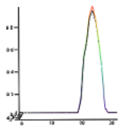
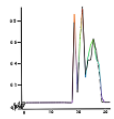
Parameter values	$t_{\max} = 0.5,\ x_{1\max} = 3.48,$ $x_{2\max} = 3.48$	$t_{\max} = 1,\ x_{1\max} = 1.444,$ $x_{2\max} = 1.444$	$t_{\max} = 15,\ x_{1\max} = 13.17,$ $x_{2\max} = 13.17$
$m_1 = 2.7,\ m_2 = 2.5,\ p = 5.4$ $eps = 10^{-3}$ $n_1 = 0.514 > 0$ $n_2 = 0.534 > 0$ $q = 43.69 > 0$ $\beta_1 = 3,\ \beta_2 = 2\ \ k = 2$, n=7			
$m_1 = 3.7,\ m_2 = 3.5,\ p = 7.4$ $eps = 10^{-3}$ $n_1 = 0.338 > 0$ $n_2 = 0.343 > 0$ $q = 255.69 > 0$ $\beta_1 = 2,\ \beta_2 = 3\ \ k = 3$, n=0,3			

Table 4

Slow diffusion. Parameter values should be

$$n_1 > 0, n_2 > 0, q > 0,\ 1-[\gamma_1(p-2)k+\gamma_2(m_1-1)]=0 : \tau(t)=\ln(t+T),\ \text{T>0}.$$

Parameter values	$t_{\max}=0.5,\ x_{1\max}=3.025,$ $x_{2\max}=3.025$	$t_{\max}=1,\ x_{1\max}=3.49,$ $x_{2\max}=3.49$	$t_{\max}=15,\ x_{1\max}=13.61$ $x_{2\max}=13.61$
$m_1=3,\ m_2=3.5,\ p=5$ $eps=10^{-3}$ $n_1=1>0$ $n_2=0.5>0$ $q=4>0$ $\beta_1=5,\ \beta_2=5\ k=1$, n=7			
$m_1=3,\ m_2=3.5,\ p=7$ $eps=10^{-3}$ $n_1=0.505>0$ $n_2=0.474>0$ $q=95>0$ $\beta_1=14,\ \beta_2=7\ k=2$, n=0,3			

Conclusion

Based on the obtained estimates of the solutions, it was established that the proposed nonlinear mathematical model of the biological population with double nonlinearity correctly reflects the physics of the flare of the process under study.

Using the presented model, it is possible to estimate the time required for a complete recovery of a population after exposure to an unfavorable factor. Using several similar models, competitiveness of competing populations is assessed. An analysis of the experimental data obtained using such models allows us to conclude that the combined action of populations. Based on this, the following conclusion is made: when several populations compete, the population that can better tolerate adverse factors will dominate.

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