

Nonlinear net models of multi-parametrical systems and processes

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Abstract. This paper is devoted to the mathematical and computer simulation of multi-parameter systems. We show that the method and algorithm can easily be used in nonlinear net simulation of the systems. Simulation is based on experimental data and achieved by the variation of one-dimensional spline approximations. A set of variable one-dimensional splines is the result of simulation. Each of the splines is the image of a section of input parameters area. Software realization is based on the single algorithm that is used repeatedly. The method have been used in investigation some technological conditions of laminating fabrics systems. Specifically, we investigated the stability of gluing joints, hardness and bending of various parts of clothes. Also, we got the simulator of constituent elements of the mixture which may be used in label products as a temperature indicator. The examples eventually demonstrate the efficiency of the presented method.

1. Introduction

Multidimensional spaces have been studied for decades as one of the basic topics in multi-component systems simulation. Therefore a lot of important results are known today. Multidimensional spaces are often rather difficult to handle. Modeling them is an essential tool to study their geometry and hence it is widely used in Computer Aided Simulation in present.

There are many authors that have considered various methods for describing technological processes in terms of multidimensional geometric objects [1 – 3]. But standard methods and software have been usually developed for one class of problems are not intended for integrated tasks and so these methods often do not satisfy most of researchers. Moreover, there are computing difficulties. In this paper we present a new method in order to diminish the difficulties of multi-dimensional simulation. To achieve our purposes we consider multidimensional geometric models of technological processes as a set of variable one-dimensional spline approximations.

All our spline approximations are built by means of the same algorithms. It means that a knot model of multidimensional nonlinear area given by experiments is transformed into one-dimensional net model of the same area. In this paper we tried to motivate our proposals by simulation some technological problems.

First problem is connected with the technological process of laminating fabrics systems which are used in clothing industry. There is a problem of correct setting the parameters of technological conditions. Decision of the problem allows us to create a new system of materials having required form stability. Rational laminating conditions lead to effective gluing and high quality of clothing [4 – 7].

The other problem is referred to various new functional indicators using in printing industry [8 – 10]. Printing the various aimed indicators at the labels is a modern trend in the label industry development. In present there used indicators reacting to the gases and electromagnetic radiation like light and thermal energies.

In order to create an indicator it is necessary to blend some active substance and printing oil together. As a result one can get the substance like a printing ink which is put onto the label or other products



by means of printing methods. When the substance will become hard under the heating its surface will be rough because of gas escaping. This effect is used in our work.

2. General considerations

There are several ways to generate the model of some technological system that may be denoted by S . An entire system S is encoded as $S(\mathbf{X}, \mathbf{Y})$, where $\mathbf{X} = (x_1, \dots, x_n)$ is a vector of input parameters; $\mathbf{Y} = (y_1, \dots, y_m)$ is a vector of output parameters. We consider S as a determinate and static system. Initial data for simulation the system S are described by discrete experimental functions $\mathbf{Y} = \mathbf{Y}(\mathbf{X})$. The functions are considered over the rectangular box area of regular point set S^0 where $x_{i,1} < \dots < x_{i,k(i)}$ and $i = 1, \dots, n$.

There are several ways to create the geometric model of the system S . One approach is to consider some algebraic model S^n as follows $y = y(x_1, \dots, x_n, a_1, \dots)$, where a_1, \dots are the constant polynomial coefficients of the model. To define this concept more precisely, note that it is necessary to know general structure of the model, to provide a best estimate for coefficients and to optimize the general model. But it is not easy to do this, because we must examine a lot of variants of the model.

Another possibility would be to create the model of the form $y = y(x_1, \dots, x_k, a_1(x_{k+1}, \dots, x_n), a_2(x_{k+1}, \dots, x_n), \dots)$, where $k = 1, \dots, n-1$. We can realize this process only if we have the information about the functions $a_i = a_i(x_{k+1}, \dots, x_n)$. In the case $k = n-1$ such problems and several ramifications have been largely considered, whereas until recently very little was known for $k < n-1$. Theoretically, we can try to create two-parameter ordered sets of $(n-2)$ -dimensional special models or three-parameter $(n-3)$ -dimensional ones and so on. But that kind of models are very sensitive to little perturbing of the coefficients. We can define this property as stability of the model.

Another approach for defining the multidimensional models would be to regard the model not as a global function, but as a piecewise approximation by polynomials in x_1, \dots, x_n . Unfortunately, only some of the methods are based on a well developed mathematical theory. In higher dimensions the problem is much harder.

In many cases one can use a parametric mapping. The realization of this process may be fulfilled by the following three steps: 1) The discrete model $\mathbf{Y} = \mathbf{Y}(\mathbf{X})$ is maintained; 2) The rectangular box $[x_j, x_{j+1}]_i$ is selected from the area S^0 ; 3) The n -variant interpolation is defined over the rectangular box $[x_j, x_{j+1}]_i$.

There is only one piece of n -variant hyper-surface given over the rectangular box by means of n -linear function. The function is a rational one and it has n order with respect to x_i . The coefficients of the function are calculated by the system of linear equations. Clearly, one can also consider the selection of rectangular box using three points $[x_{j-1}, x_j, x_{j+1}]_i$. This case leads to the n -variant interpolation by the pieces of hyper-surfaces. The degree of hyper-surface is equal 2^n in x_i . Such local n -variant approximation provides the coincidences of values y at the vertices and the edges of the rectangular box only.

Our approach is based on the following two principles: 1) We give up the one-piece polynomial S^n in x_1, \dots, x_n and the multi-piece polynomials over the parametrical rectangular box area; 2) We apply the one-dimensional spline approximation only, but this approximation is generated by reiterative process. The geometrical idea of suggested method is based on the variable sections of given rectangular box area S^0 and is realized by simple repeated calculation of spline knot values.

3. Theory

Generally speaking, we have two different n -dimensional spaces. First space is a space of input parameters and it is formed as a rectangular n -dimensional box. This space we may define as a space of originals. Second space is a space of output parameters and it is considered as n -dimensional approximant of a hyper-surface over the rectangular box. This space we may define as a space of images. One can see that the best situation is the one where two spaces are separated. But sometimes there are technological processes and systems where we have to consider both spaces as superposed ones. In the following description first consideration is more convenient for us.

Let's consider the process of variable one-dimensional spline approximation in detail. Let y be an output parameter.

Step 1: Let indices $i = 1, \dots, n-1$ and $j = 1, \dots, k(i)$ be chosen in succession.

Step 2: For all indices i, j the set of one-dimensional splines $k(1) \times \dots \times k(n-1)$ is built. Let us denote them by $S^1_{n-1}(x_n)$.

Step 3: Let indices $i = 1, \dots, n-2, n$ and $j = 1, \dots, k(i)$ be chosen in succession again.

Step 4: For all indices i, j next set of one-dimensional splines $k(1) \times \dots \times k(n-1)$ is built. Let us denote them by $S^1_{n-1}(x_{n-1})$. And so on.

There are several standard problems of simulations. For example, direct and inverse problems, problems of optimization, problems of controls and so on. If we want to solve the direct problem that is to find the value y_0 for given point $A(x_{i,0})$, taking into account that it is not the point of S_0 , we may use the following algorithm.

Step 1: Let index $i = 1$ be fixed. The $(n-1)$ -dimensional section of $S^1(\mathbf{X}, y)$

$$\text{Sec}_{n-1,1} S^1(\mathbf{X}, y) = S^1(\mathbf{X}, y) \cap (x_{1,0}) = \{A(x_{1,0}, x_2, \dots, x_n)\}$$

is built.

Step 2: After using the above algorithm for $i > 1$ one-dimensional sections $S^1_n \cup \dots \cup S^1_2$ are determined.

Step 3: Let index $i = 2$ be fixed. The $(n-2)$ -dimensional section of

$$\text{Sec}_{n-2} S^1(\mathbf{X}, y) = S^1(\mathbf{X}, y) \cap (x_{1,0}, x_{2,0}) = \{A(x_{1,0}, x_{2,0}, \dots, x_n)\}$$

is built.

Step 4: After using the above algorithm for $i > 2$ one-dimensional sections $S^1_n \cup \dots \cup S^1_3$ are determined. And so on.

Step 5: The one-dimensional section of $S^1(\mathbf{X}, y)$

$$\text{Sec}_{n-2,2} S^1(\mathbf{X}, y) = S^1(\mathbf{X}, y) \cap (x_{2,0}) = \{A(x_1, x_{2,0}, x_3, \dots, x_n)\}$$

is built. And so on.

Finally, we will have one-dimensional nets of the axis x_i and one-dimensional spline approximations $S^1_n(y), \dots, S^1_1(y)$. Obviously, $S^1_1(x_{1,0}, y) \neq S^1_2(x_{2,0}, y) \neq \dots \neq S^1_n(x_{n,0}, y)$. Denote $S^1_i(x_{i,0}, y)$ by $y_{i,0}$. Then we can find the value of y_0 as follows: $y_0 = (y_{1,0} + \dots + y_{n,0})/n$.

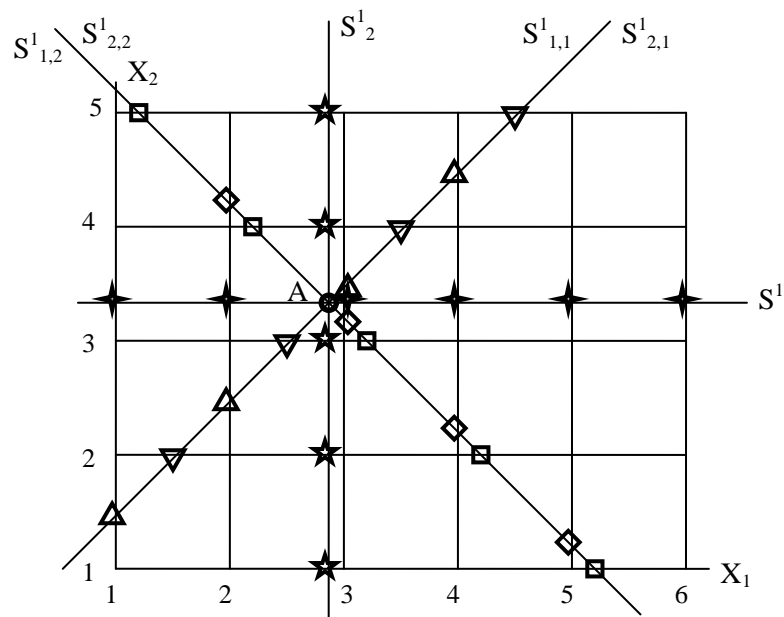


Figure 1. The sections of input parameter area in space of originals

As an example, let us now consider the case where $n = 2$, $k(1) = 6$, $k(2) = 5$. Evidently, we have the net of knots and a point $A(x_{1,A}, x_{2,A})$. Let us consider only four conditions. These conditions are given by $x_2 = c(x_1 - x_{1,A}) + x_{2,A}$ where $c = \{-1, 0, +1\}$ and $x_2 = x_{1,A}$. Hence, we get six sections $S^1_1, S^1_2, S^1_{1,1}, S^1_{1,2}, S^1_{2,1}, S^1_{2,2}$ as shown in Fig. 1.

Consequently, we get six one-dimensional spline approximations $S^1_1(y), S^1_2(y), S^1_{1,1}(y), S^1_{1,2}(y), S^1_{2,1}(y), S^1_{2,2}(y)$. The knots of the approximations are shown in Fig. 1 by means of various figures. The substitution $x_{1,A}, x_{2,A}$ gives six values $y_{1,A}, y_{2,A}, y_{1,1,A}, y_{1,2,A}, y_{2,1,A}, y_{2,2,A}$. The most probability value of y_A is the average value.

Note that we may extend the number of conditions. For example, additional set of conditions may be obtained when factor c is varied from $-c$ (the value of c is a minimum value) to $+c$ (the value of c is a maximum value) symmetrically.

4. Software realization

Software module consists of two sub-modules. The basic module is used for operations with data base and for building the sections of hyper-surfaces. The auxiliary module has three basic software groups which are used for analyses the plane sets of points and curves. The structure of software module is shown in Table 1.

In order to build a new curve in space of images we have to find the points of the section and to generate a new matrix that is an image of the section in new space. To build the intersections of two surfaces we have to work with two matrices having the same dimension. The result of calculation is a matrix of the same dimension and all elements of the matrix are coordinates of the points.

Table 1. Structure of the software modules

THE BASIC SUB-MODULES	THE AUXILLARY SUB-MODULES
Building the sections	Unitive algorithms
Building the intersections of hyper-planes and hyper-surfaces	Plane curves construction algorithms
Building the intersections of hyper-surfaces	Algorithms transforming broken line approximations into plane curves approximations.

Each set of points is presented by a coordinate matrix. Using the operations with data we can choose proper matrix and get the data in order to create a new plane curve in space of images. At the same time we have software resources which allow us to examine all other parameters. In most cases these resources are hidden ones and they may be used without our participation.

The algorithms uniting the points in ordered sets belong to the first software group. We attribute the algorithms of dividing the set of points into monotonous segments as the unitive algorithms. As the example, we use the unitive algorithms based on monotony changing of points along axes. The points are ordered in accordance with increasing of their coordinates. Also we consider the possibility of approximation by means of polynomials of odd order. In most cases the polynomials of three orders are used.

The algorithms of construction the plane curves at the given sets of points form the second group of auxiliary module. We use spline interpolations by the curves of three order and linear interpolation only. Also we can use a smooth approximation in spite of some additional errors. But the errors are compensated by experimental ones.

The third group consists of algorithms using transformations of broken line approximations into approximation by plane curves. The equations which have been received after previous processing are supplemented by their points of breaking. Classic approach to the problem is smoothing the broken lines by means of joining their partial derivatives at the units.

Using all these methods and algorithms we get a general algorithm as follows:

1. We set such values of output parameters as we need and build the sections of input parameters area by hyper-planes. Some sets of matrices are the results of the operations.
2. We find an area common to all intersections by means of examination the matrices in pairs.
3. If we want to fix the values of input parameters and diminish the area of optimization we may repeat the algorithm using the other hyper-planes.

So we get the software which allows us to construct hyper-surfaces as an approximants based on the experimental data. Moreover, the software is capable to show the results on the screen. Hence, we can receive graphical and numerical information about all variants, which have been considered before and we can fulfill their analyses and comparison. The software allows us to optimize the processes having rather great many parameters.

5. Applications

We consider two problems connected with light industry. First problem is of importance in researching optimal technological conditions of laminating fabrics systems. Input parameters are as follows: a temperature of working part (x_1), a time of pressing (x_2). Output parameters are as follows: stability of gluing joints (y_1), hardness and resilience in bending (y_2). To obtain the laminated fabrics systems corresponding to various parts of clothes we used the mixed costume fabrics and universal fabrics. Laminating was carried out under recommended conditions and various combinations of parameters. The temperature was varied from 140 °C to 160 °C at intervals 10 °C. The time of pressing and heating was varied from 10 to 60 seconds at intervals 5 seconds.

To reveal the interconnection of gluing conditions and gluing joint properties we used the multidimensional geometric model of laminating process. A piece of the model is shown in Figure 2. One can see the approximations of gluing joint stability with duration of pressing and heating.

To obtain the laminated fabrics system having desired stiffness we analyzed the conditions of laminating. The searching of the system having y_1 equal to 0.35 and y_2 equal to 18000 is illustrated in Figure 3. We set the desired output parameters and carried out the sections which determined the area of parameters (see lines 1 – 5). Parameters of laminating are determined by the follows lines: the line 1 corresponds to thermo-glutinous gasket materials based on knitted basis, the lines 2 and 3 correspond to thermo-glutinous gasket materials based on woven basis and the line 4 corresponds to thermo-glutinous gasket materials based on base-knitted basis.

The stiffness, we need, may be obtained only for thermo-glutinous gasket materials based on the knitted basic (see line 5). The lines 4 and 5 intersect each other at a point N. Coordinates of the point N determine the parameters, we need. Hence, we have the model of the laminated fabrics system with two output parameters.

Using the geometric model we can see that laminated fabrics system having y_1 more then 0.35 and y_2 equal to 18000 may be obtained for thermo-glutinous gasket materials based on base-knitted basis at the follows parameters: time is equal to 18 seconds, the temperature is equal to 158 °C and the pressure is equal to 0.36 MPa.

The second applied problem is connected with the printing of various function indicators onto the label products. To verify our theory we prepared a special printing ink having a temperature indicator. The ink was transferred onto the cardboard and tested by thermal process. Based on the experimental data the net simulator of constituent elements of mixture was created. Visual analysis of indicator shows the degree of ink coat destruction because of gas discharging. Hence, one can come to conclusion about influence of the temperature.

The input parameters are as follows: percentage of temperature indicator concentration (x_1), percentage of additional activator ZnO (x_2), percentage of white spirit as a solvent (x_3). The output parameter is a thickness of ink coat after heating.

To create the model we carried out 27 experiments. Intervals of input parameters are as follows: x_1 was varied from 5 to 30 % at 12,5 % intervals, x_2 was varied from 0.25 to 0.375 % at 0.0625 % intervals, x_3 was varied from 1 to 5 % at 2 % intervals. The temperature was varied from 150 to 210 °C. The output parameter was measured from 0.08 to 0.098 mm.

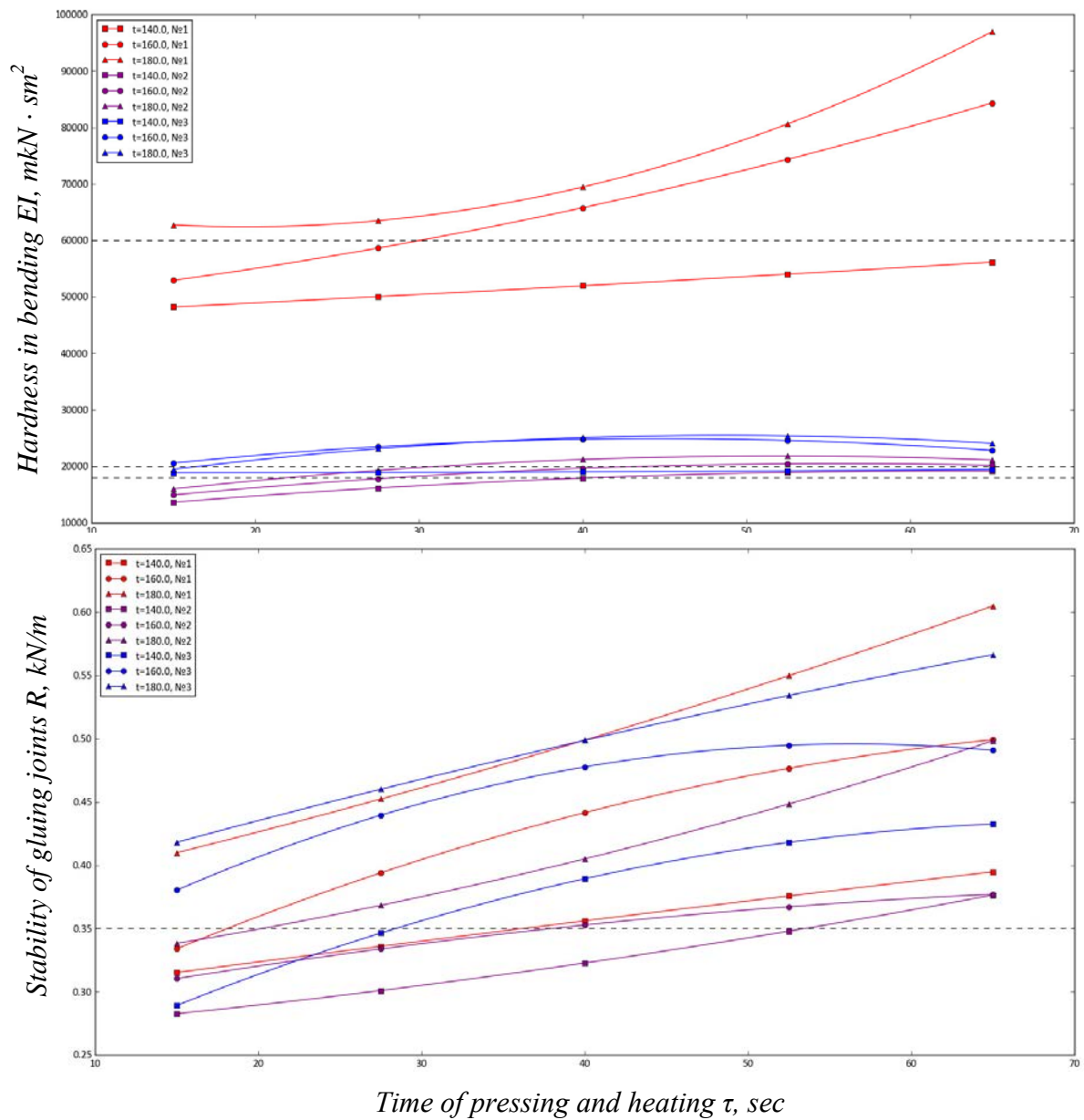


Figure 2. Geometric model of the system having two input and two output parameters

The net simulator consists of 54 one-dimensional spline approximations of second order. Calculation of the factors was fulfilled by the experimental data. Choice of a point inside the rectangular box area was made by means of interactive mode. Calculation of output parameter value in every point inside the area was fulfilled by 12 one-dimensional splines. Calculation of spline factors was carried out at points of 12 sections by computer program. The sections of the net model were given parallel to the faces and bisector planes of the area.

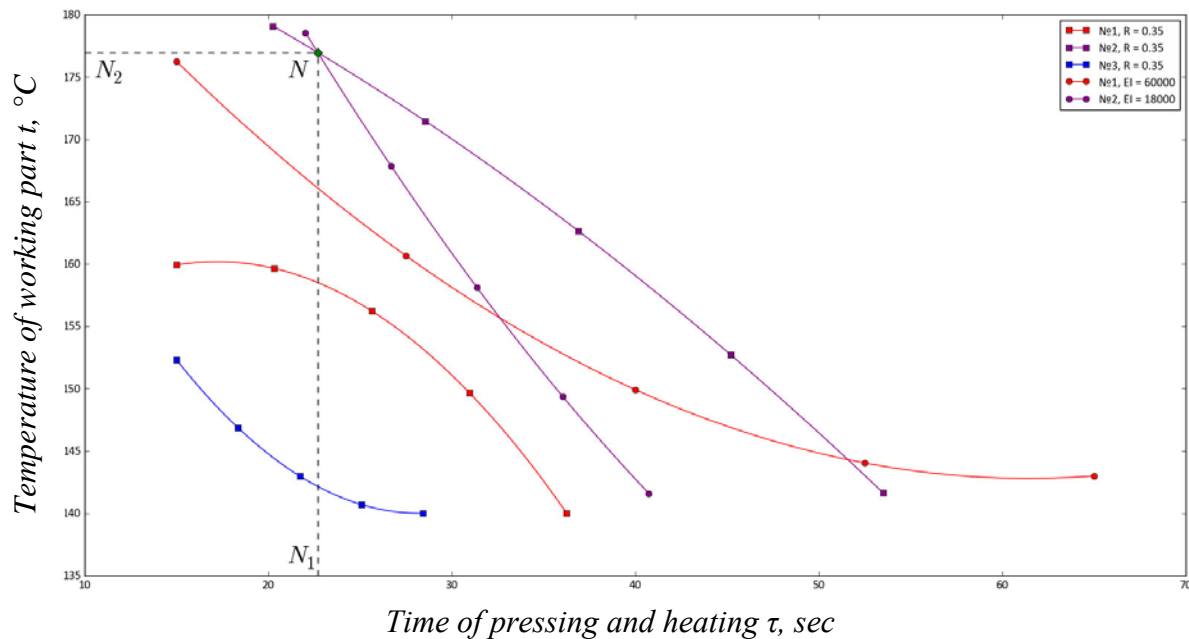


Figure 3. Searching of output parameters area for laminating conditions

6. Results and Discussion

Provided theoretical researches show the availability to simulate multi-parametrical systems or technological processes. Created software realization allows us to examine a point inside the multidimensional area of input parameters by means of interactive mode and to choose the values of output parameters.

Experimental researches allow us to find optimal conditions of technological processes in light and printing industries. The results are as follows. Firstly, the laminated systems having a value of stability more then 3.5 kN/m may be produced. The laminated systems based on the knitted fabrics have variable hardness which is varied from 12000 to 20000 mkN/sm². The laminated systems which hardness is varied from 20000 to 40000 mkN/sm² can not be produced. Secondly, the laminated systems having its stability of gluing joints more then 0.35 kN/m and having its hardness and resilience in bending equal to 18000 mkN/sm² can be produced when the time of pressing will be equal to 18 seconds and the temperature will be equal to 158 °C.

7. Conclusions

The mathematical simulator which is a set of one-dimensional spline approximations inside the area of parameters is described. The model may be applied for simulation multi-parametrical systems or technological processes of light industry. Simulation of the systems is based on a single algorithm that is used repeatedly. Effectiveness of the model is confirmed by applications to closing and printing industries. The model allows us to solve optimization problems and problems of processes control at interactive mode.

The basic idea of this paper can also be applied to other types of technological processes. So the method and algorithm which we have worked out in detail is not at all confined to light and printing industries. We do hope that our considerations will open a wide range of other applications.

8. References

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