

Dynamic model of the mechanical drive with automatic continuously variable transmission on the basis of Appell's equations

P D Balakin O S Dyundik and I P Zgonnik

Omsk State Technical University, 11, Mira ave., Omsk, 644050, Russia

Abstract. The design approach is based on to mechanical systems is implemented by adaptive properties to the actual parameters and to the operating conditions in the synthesis of the mechanical drive on design stage allows to implement many technical solutions of mechanical transmission with qualitatively new properties. Using for the adaptation additional motion of the links to the main one, it is possible and important to get auto-CVT with the transfer function dependent on the level of the transmitted power flow. The design of the auto-CVT which is potentially capable to provide full use of the available power in the mechanical drive, stationary and energetically perfect mode of operation of the engine at variable type of external loading is considered. Mathematical model of mechanical auto-CVT obtained by using Gibbs function – the acceleration energy. The results of the research are on the basis of which it is possible to perform design calculations of the links and connections of the auto-CVT, including its automatic control chain.

Key-words: dynamic model, design approach, control chain, auto-CVT

1. Introduction

The design approach to mechanical systems with adaptive properties to various types of errors and operating conditions is described in detail in [1].

Execution of the design approach gives the creation of many technical solutions with qualitatively new properties for mechanical drives. It is very important to create an automatic regulator of the power flow components between the engine and the operating member of the machine, that is the auto- CVT with the transfer function, depending on the level of the transmitted power flow. Using for adaptation additional motion of the links to the main one, capable of changing the kinematic dimensions of the links with the help of a hidden control chain.

Technical solutions of CVT with operator control are known: on the basis of the frontal scheme [2], V-belt [3], multi-disc [4], toroidal [5] and others [6].

2. Problem statement

Set and solved the problem of modeling the CVT which operates using only the laws of mechanics. The means of control of the transfer function is an additional motion of the links to the main one with the help of a built-in control chain, the operation of which is based on the use of a variable force component of the transformed power flow. Such a CVT will be potentially endowed with the ability to carry out a useful



evolution to achieve energy perfection of the drive and auto-control components of the transformed power.

Technical solutions such auto-CVT as, for example, [7] and [8], obtained on the basis of design approach of mechanical systems, based on the vesting of the systems designed by the property of adaptation to real parameters of the systems and to the modes of operation.

Modeling the work of the auto-CVT is important. This allows at an early stage of schematic design to obtain information about the power loading of links and connections, which is absolutely necessary for the design calculations of all elements of the auto-CVT's drive and especially the built-in control chain. For such the chain, it is necessary to choose a schematic solution and a characteristic of the sensing element, the deformation of which sets an additional motion of the links to the main one, fulfilling the useful evolution of the drive.

3. Theory

For example, the design of auto-CVT [9] and its modification has the potential to ensure full use of available power in mechanical drive and stationary and energy-perfect operation of the engine. With a variable type of the workflow, this is possible under the following conditions.

$$N_2 = N_1 \eta = M_1 \omega_1 \eta = M_2 \omega_2 = \text{const},$$

where N_1 is an engine power and N_2 is an operating member power; ω_1 is a velocity of a driving shaft auto-CVT and ω_2 is a velocity of a output shaft auto-CVT; η is a mechanical efficiency of a drive. The power transfer function of the drive under variable type load is determined by the dependent

$$U_{1,2} = \frac{M_2}{M_1 \eta},$$

where M_1 is a rotational moment of a driving shaft auto-CVT and M_2 is a rotational moment of a output shaft auto-CVT.

Stationary operation of the engine is provided when an additional condition is met $M_1 = \text{const}$ and $\omega_1 = \text{const}$. Receiving a signal for the automatic change of the transfer function from the power flow, the hidden control chain must realize a hyperbolic mutual dependence of the components M_2 and ω_2 of the output power at $M_1 = \text{const}$ and $\omega_1 = \text{const}$:

$$\omega_2 = \frac{M_1 \omega_1 \eta}{M_2}.$$

Researches have shown that the automatic controller of the power components due to the variation of the transfer function of the drive is technically easier to perform by means of a mechanical transmission with a rolling contact of active surfaces, the kinematic dimensions r and ρ of which, regardless of the performance, vary separately or in the aggregate. If, for example, $r = \text{const}$ and $\rho = \text{var}$ are taken then in addition to the angular coordinates φ_1 and φ_2 of the driving and output links, the transformation of the motion in the drive will be affected by the parameter ρ . Therefore, the behavior of the system can be characterized by three generalized coordinates φ_1 , φ_2 and ρ , but the differential equation of the connection the generalized coordinates can be composed of only one:

$$r \dot{\varphi}_1 = \rho \dot{\varphi}_2 \text{ or } U_{1,2} \dot{\varphi}_1 - \dot{\varphi}_2 = 0. \quad (1)$$

In the formula (1) for the variable value ρ , when ρ does not depend on time t and on the generalized coordinates φ_1 and φ_2 , equation (1) is not integrated, which indicates the presence in the structure of the CVT of a nonholonomic constraint between the input and output links. Such type joint is maintained and in a perfect CVT, which is completely absent of geometric and elastic slide in frictional contact.

However, if the condition of the problem ρ is taken to be dependent on the moment of loading, which can be determined by some function of time t , then the number of generalized coordinates in the system is reduced to two, for example, as coordinates φ_1 and φ_2 can be left. The actual mobility of the drive with the auto-CVT at $\rho = \rho(t)$ will be equal to one and will be determined by the general rule by subtracting

the number of nonholonomic constraints from the number of generalized coordinates. The presence of nonholonomic constraints introduced some features in the research of the dynamic behavior of machines with auto-CVT.

The main feature is the impossibility of creating a single-mass dynamic model of the drive, since the reduction of forces and masses through a nonholonomic constraint is incorrect. Therefore, the dynamic model can only be dual-mass with a separate reduction of forces and masses to the two reduction links from the branches of the kinematic chain and located on different sides of the nonholonomic constraint. In this case, the motions of the reduction links are not free and are related by the differential equation (1).

A dynamic model of a mechanical system with nonholonomic constraint can be constructed on the basis of Appell's equation, which is a further generalization of Lagrange's power models relating energy transformations in a closed mechanical system with its motion [10]

In Appell's equations, the motion parameters are connected by Gibbs function – the acceleration energy, the partial derivative of which by acceleration is the inertial component of the equation and the Gibbs function has the form:

$$S = \sum_{i=1}^n \frac{J_i \ddot{\varphi}_i^2}{2}$$

where S is the Gibbs function (the acceleration energy); φ_i are the generalized coordinates of system. Appell's equation in general for the angular generalized coordinates will be as follows:

$$\frac{\partial S}{\partial \ddot{\varphi}_i} = M_i$$

Compose Gibbs function of connecting the two reduction links align with a driving 1 and output 2 links (shafts):

$$S = \frac{J_1 \ddot{\varphi}_1^2}{2} + \frac{J_2 \ddot{\varphi}_2^2}{2} \quad (2)$$

For mechanical systems of general form with links with a complex motion, in the preparation of the Gibbs function will have some difficulties.

Let's take an independent coordinate φ_1 and differentiate (2) by $\ddot{\varphi}_1$. As a result

$$\frac{\partial S}{\partial \ddot{\varphi}_i} = M_1^{re} + M_2^{re} U_{2,1}$$

or in expanded form

$$\frac{1}{2} \frac{\partial}{\partial \ddot{\varphi}_i} (J_1 \ddot{\varphi}_1^2 + J_2 \ddot{\varphi}_2^2) = M_1^{re} + M_2^{re} U_{2,1} \quad (3)$$

Replace in (3) $\ddot{\varphi}_2$ from the nonholonomic relation equation (1):

$$\ddot{\varphi}_2 = \dot{U}_{2,1} \dot{\varphi}_1 + U_{2,1} \ddot{\varphi}_1$$

then

$$\frac{1}{2} \frac{\partial}{\partial \ddot{\varphi}_i} (J_1 \ddot{\varphi}_1^2 + J_2 (\dot{U}_{2,1} \dot{\varphi}_1 + U_{2,1} \ddot{\varphi}_1)^2) = M_1^{re} + M_2^{re} U_{2,1}$$

After differentiation, the equation of motion of the driving link 1 will take the form

$$(J_1 + J_2 U_{2,1}^2) \ddot{\varphi}_1 + J_2 U_{2,1} \dot{U}_{2,1} \dot{\varphi}_1 = M_1^{re} + M_2^{re} U_{2,1} \quad (4)$$

The equation of motion of the output link 2 is similar, only for an independent coordinate should take φ_2 and differentiate Gibbs function by $\ddot{\varphi}_2$

$$(J_2 + J_1 U_{1,2}^2) \ddot{\varphi}_2 + J_1 U_{1,2} \dot{U}_{1,2} \dot{\varphi}_2 = M_1^{re} U_{1,2} + M_2^{re} \quad (5)$$

Equations (4) and (5) are correct dynamic model of the mechanical drive with auto-CVT. They take into account the features of nonholonomic constraint and are second-order differential equations with variable coefficients. The equations are solvable only numerically. The method of solution (4) and (5) is described in detail in [11].

Thus, under the terms of the terms of reference for the design of the auto-CVT for transport machines at preservation $M_1 = \text{const}$ and $\dot{\varphi}_1 = \omega_1 = \text{const}$, the coordinate φ_1 have $\ddot{\varphi}_1 = 0$ and the expression (4) is simplified to the form:

$$J_2 U_{2,1} \dot{U}_{2,1} \dot{\varphi}_1 = M_1 + M_2 U_{2,1}$$

or

$$\dot{U}_{2,1} = \frac{M_1 / U_{2,1} + M_2}{J_2 \dot{\varphi}_1} \quad (6)$$

Since the transfer function $U_{2,1}$ is linear with respect to M_2 , it can be represented as $U_{2,1} = kM_2$ and in (6) the only variable is M_2 . If the law of change M_2 is known in time, the expression (6) is integrated. With this in mind, the problem of the motion of the CVT of the transmission of transport machines becomes solvable by quadratures. Knowledge of $U_{2,1} = kM_2$ allows you to synthesize a built-in drive control chain auto-CVT and perform structural calculations of the chain's elements.

4. Results discussion

Regularity changes $U_{2,1} = U_{2,1}(M_2)$ is the source for the design of control chain auto-CVT. In the numerator (6) the sign "+" means the algebraic addition of the multidirectional moments M_1 and M_2 , and when the numerator turns to zero, the value of the transfer function becomes $U_{2,1} = \text{const}$, which corresponds to the end of the transition process.

The research of the motion model for (4) and (5) shows that the behavior of the main links is influenced not only by the transfer function, but also by the speed of its change. For the transmission of transport machines, this effect has a damping nature and the sharper the transfer function changes, the less obediently the speed of the branches of the system will change.

Since in (6) the numerator is the difference of the force characteristic before and after the nonholonomic constraint, then, in fact, this difference is the excess torque, which at $M_1 = \text{const}$, is completely determined by $M_2(t)$.

5. Conclusions

The obtained regularities serve as the basis for designing links and joints of a mechanical auto-CVT. It specially applies to the synthesis of the built-in control chain of gear ratio of the adaptive controller of power components in the drive of transport machines.

6. References

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