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Preferential attachment graphs as agent interaction structure

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Abstract. The preferential attachment graphs are considered as the basis for constructing agent interaction structures. The choice of such a structure has a great influence on the results of network processes simulation. Calibration (adjusting generation parameters) of the preferential attachment graphs improves the accuracy of the simulated processes results. An increase in the quality of calibration leads to the construction of adequate graph models. It can be used in processes simulation of telecommunication networks, social networks, cell molecular networks and other large networks of the modern networked world.

1. Introduction

Network modeling and network processes simulation are of great interest in such fields of science as statistical physics, bioinformatics, game theory and its applications for social and economic processes simulation. And the fact that random graph generation is also supported in simulation systems (such as most popular simulation system AnyLogic) is a consequence of the demand for models of this class. Random graphs are used as an agent interaction structure in simulation systems. However, the AnyLogic simulation system (like many other agent-based systems) supports a limited number of network models. For example, AnyLogic supports the generating of the Watts-Strogatz graph (‘Small World’), the Barabasi-Albert graph (‘Scale free’), the classical random graph (‘Random’), a random geometric graph, a lattice model in the form of a ring with a given degree of connected vertices (‘Lattice Ring’). These graph types are often not enough to build adequate models of large networks. So, in figure 1 are presented the results of simulating standard agent-based model BassDiffusionAgentBased. The number of agents who have adopted innovation as a result of the diffusion process is considered. The default model parameters were used. Constant representing advertising effectiveness *AdEffect* is equal 0.011, the proportion of contacts that are sufficiently persuasive to induce the potential adopter to purchase the product *AdoptionFraction* is equal 0.015, several potential adopters come into contact with real adopters *ContactRate* is equal 100. Various available in the AnyLogic system random graphs were used as the agent interaction structure. The parameters for random graphs generating are chosen so that the random graph and the network have the same number of vertices and edges, if possible. As a static model of the network was the Internet AS graph [1], containing 22963 vertices and 48436 edges. Clustering coefficient [2] of Watts-Strogatz graph is equal 0.12, which corresponds to the clustering coefficient of the Internet AS graph, also obtained if the Watts-Strogats graph parameter $p = 0.2$.

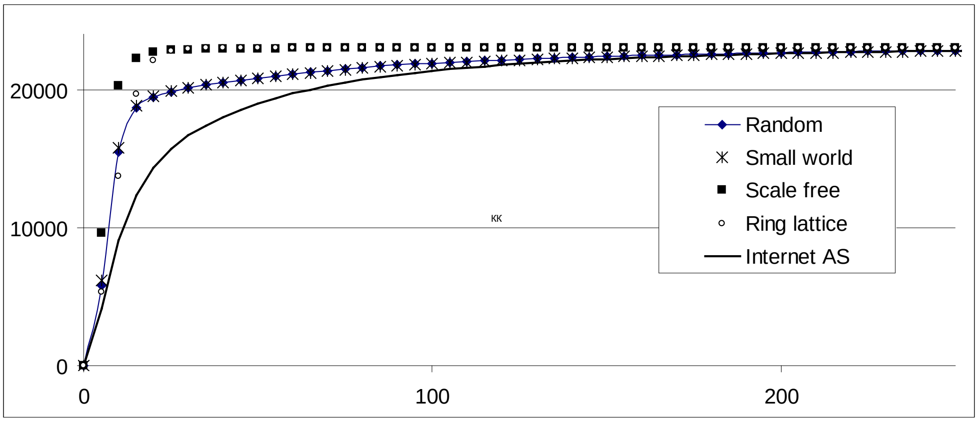


Figure 1. The number of adopted innovation agents (Adopters) in BassDiffusionAgentBased of the AnyLogic system. The various interaction structures between agents in the system are used

As can be seen in figure 1, the correspondence according to some structural characteristics of random graphs and the Internet AS graph is not enough for high-quality modeling of the network process. This is due to the mismatch of the graphs for other structural characteristics. Thus, although the random graphs provided by AnyLogic are popular and useful (using the considered network models, it was possible to explain the increased sensitivity of large networks to the spread of viruses, the resistance to random failures of network elements), their calibration capabilities (selection of generation parameters) are not enough to build an adequate model. The task is to develop such random graphs and their calibration tools that will allow you to build adequate network models and with which you can achieve better agreement on the structural characteristics of random graphs and graphs of the studied networks. One of such network models considered in this paper is a random graph with a non-linear rule of preferential attachment (NPA graph). An NPA graph allows you to implement graphs with a given vertex degree distribution (VDD vertices), a two-dimensional distribution of degrees of connectivity of end vertices of edges (EVDD edges) and a clustering coefficient.

2. Random graph with a non-linear rule of preferential attachment (NPA graph)

An NPA graph is generated step by step. At each generation step, an increment of the graph is added. A graph increment is a new vertex with a random number of *k* edges. The number of edges in the increment is determined by some discrete probability distribution $\{r_k\}$. The vertex *i* for joining the free ends of the edges of the increment is selected with probability pi proportional to some function *f* of the degree of connectivity k_i of the vertices:

$$p_i = f(k_i) / \sum_{j=1}^{N} f(k_j),$$

where *N* is the number of vertices in the graph at the next generation step. If the function $f(k) = k$, then the NPA graph is the Barabasi-Albert graph and the well-known rule “the rich becomes richer” is implemented. The calibration task for NPA graphs is to find the graph generation parameters $f(k)$ and $\{r_k\}$ for the implementation of the required structural characteristics. The calibration problem for VDD was solved in [3], and in contrast to available in AnyLogic random graphs, NPA graph with allows you to implement any required VDD. The graph obtained by applying the calibration method [3] will be called the NPA-1 graph. In [3], the following parameters were found for the NPA-1. The number of edges in the increment is determined probability distribution according $r_1= 0.3414$, $r_2= 0.4224$, $r_3= 0.0966$, $r_4= 0.0645$, $r_5= 0.0751$. The values of the preference function $f(1)$, …, $f(5)$ are 0, 0, 0, 0.6329, 3.8769 accordantly, and for k> 5 they are calculated by the formula $f(1)=0.8949\cdot k$. Figure 2 shows some VDD of the random graphs used as agent interaction structures, as well as the VDD of the Internet AS graph which is a model of the network of autonomous Internet systems.

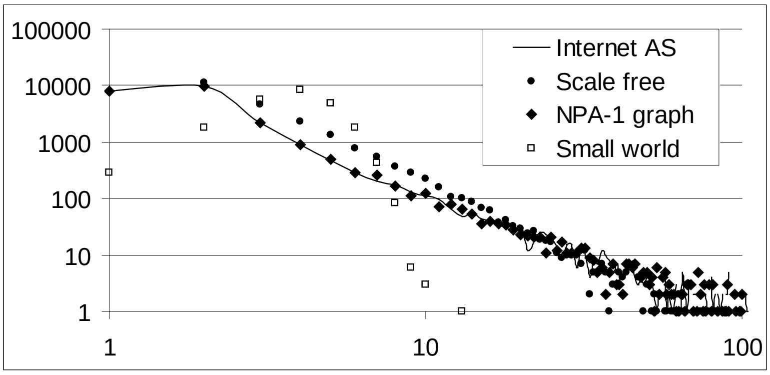


Figure 2. VDD of the graphs

The discrepancy between the VDD of the NPA-1 graph and the MBB of the Internet AS graph with large degrees is explained by the insufficient number of vertex implementations with a given degree of connectivity in the graphs.

The task of complex calibration of NPA graphs for realizing the required VDD and EVDD simultaneously (figure 3) was solved in [4].

A graph obtained by applying the complex calibration method [4] will be called an NPA-2 graph. The following generation parameters were obtained for such a graph in [4]. The distribution of the number of edges in the increment is $\{r_1, ..., r_{20}\}$ = {0.37009; 0.45798; 0.106; 0.0331; 0.00214; 0.00819; 0.00384; 0; 0.00386; 0; 0; 0.0038; 0.00173; 0; 0; 0; 0; 0.00093; 0.00047; 0.00788}. The preference function $f(k)$ is given by the series $f(1), ..., f(20)$ = 0.08389; 0.15189; 0.76084; 1.75237; 2.52761; 3.5848; 4.25984; 4.5506; 5.43441; 5.71515; 5,92737; 7.25874; 8.181; 8.51582; 8.806; 9.0512; 9.25108; 10.0695; 10.6298; 17.8403, and $f(k) = 0.89202\,k$, if $k \geq 21$.

As can be seen in figure 3, the NPA-2 graph allows one to implement with sufficient accuracy the EVDD (figure 3d) corresponding to the EVDD of the Internet AS graph (figure 3a). It should also be noted that all EVDD of the graphs available in AnyLogic are very different from the EVDD of the Internet AS graph.

By some modification of NPA-graphs and refinement of the complex calibration method a new calibration method was proposed in [5]. The method allows one to implement required VDD, EVDD, and the clustering coefficient. A graph obtained by using the new calibration method [5] is called an NPA-3. The preferential attachment function $f(1), \ldots, f(31)$ are defined as 0.0737, 0.1416, 0.7014, 1.455, 2.546, 3.393, 4.198, 4.490, 5.122, 5.783, 6.047, 7.216, 8.516, 8.861, 9.306, 9.704, 10.07, 10.57, 11.06, 11.49, 11.92, 12.18, 12.44, 12.66, 12.86, 13.43, 13.98, 14.11, 14.27, 20.21, 26.2 accordantly, and $f(k) = 0.8294 \cdot k$, if $k \geq 32$.

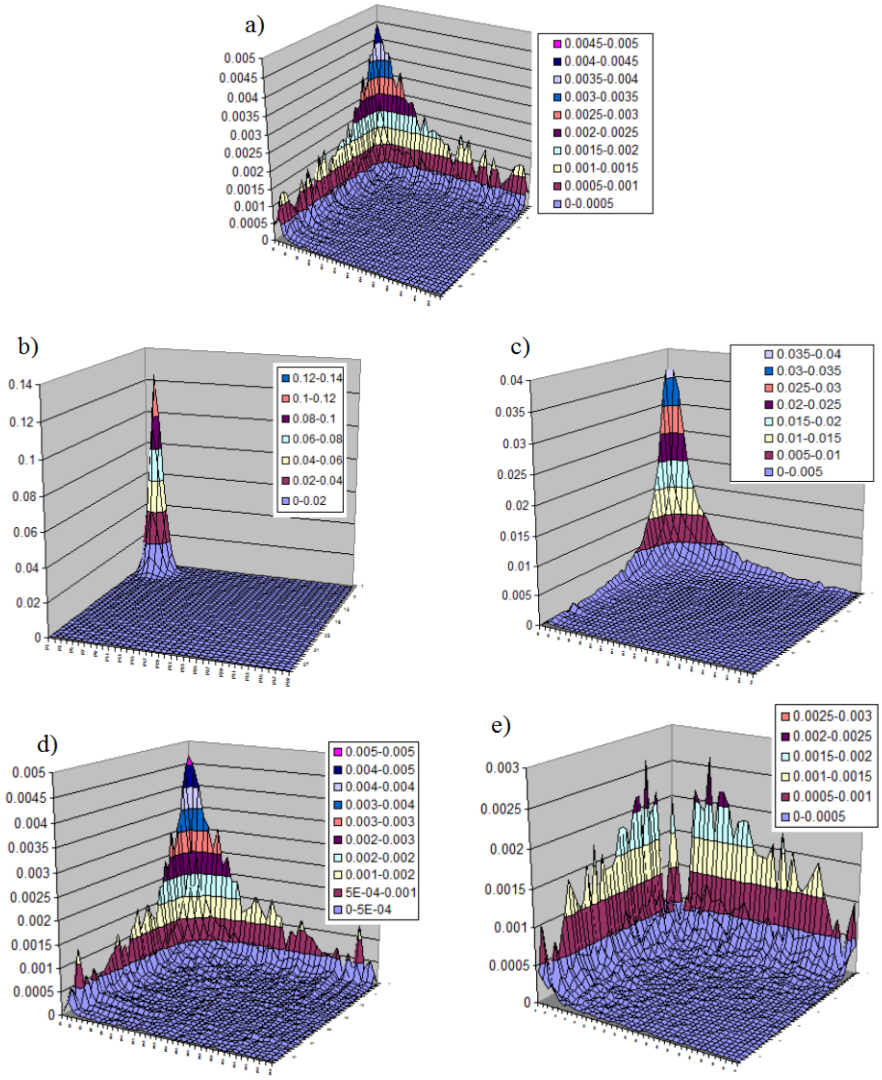
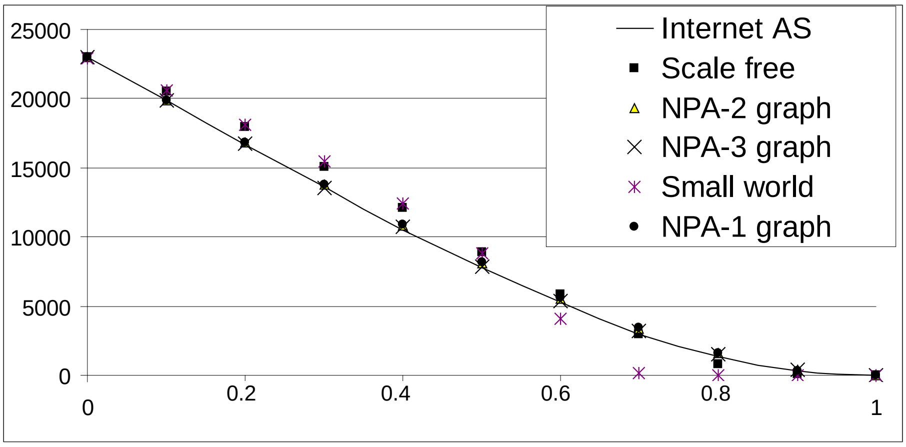


Figure 3. Graph EVDD a) Internet AS graph, b) Watts-Strogatz graph («Small world»), c) Barabasi-Albert graph (‘Scale free’), d) NPA-2 graph, e) NPA-1 graph

3. Simulation of random failures

We take as a measure of the reliability of the graph the proportion of vertices in the maximum connected component after random deleting of vertices / edges to the total number of vertices in the graph. It is known from percolation theory that when lattices (that is, graphs with a regular infinite structure) have threshold values for fraction of randomly removed vertices/edges. When the value is exceeded, the lattice breaks up into disconnected components [6]. At the same time, it was theoretically established [7] that such a threshold does not exist if Barabasi-Albert graph is analyzed. That means the graphs are unprecedentedly resistant to random removed of its elements. But, the threshold exists [8] and can be obtained using simulation if we analyze finite graph, for example Barabasi-Albert graphs containing a given number of vertices. In figure 4 presents the results of the simulation of random failures using 100 statistical experiments. As network models, Internet AS graphs were used, as well as its models based on random graphs.



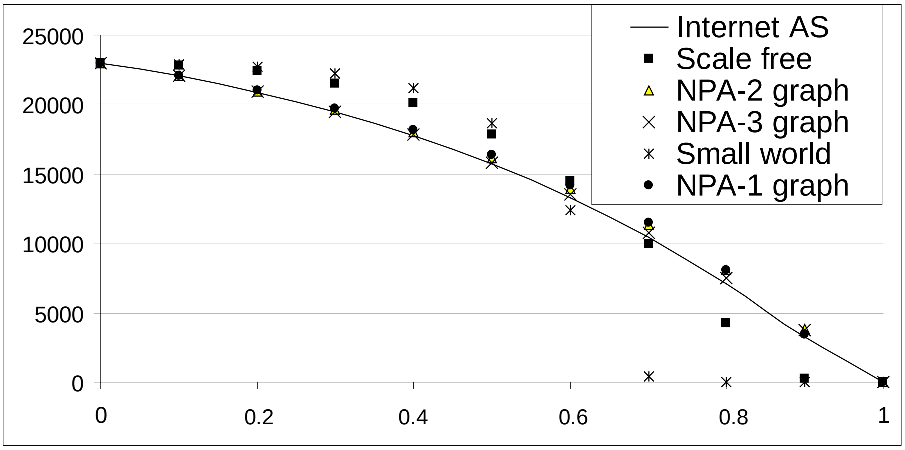


Figure 4. Reliability of graphs at random deleting of vertices (top) and edges (bottom). The share of vertices in the largest connected component is considered

As you can see, the simulation results on the Internet AS graph are most accurately reproduced in the model based on the graph with NPPS-3, with a sufficient degree of adequacy, you can use the graph with NPPS-1 and the graph with NPPS-2.

4. Modeling the spread of the virus

The proportion of 'infected' agents depending on the probability of 'infection' of the agent from another infected agent using the well-known agent epidemiological model of 'susceptible' – 'infected' – 'susceptible' (SIS) is shown in figure 5. As structures of agent interaction, the preferential attachment graphs were used. The implementation of the model in which the agents infected in the previous step at the next step is transferred with probability one to the 'susceptible' state is considered. It is the parameters of the SIS model that it was applied in [9] where it was theoretically established that at viruses cannot die in Barabasi-Albert graph if any probability infection speeding to neighboring nodes does. And all viruses, even those that are weakly contagious, will not disappear in the networked system.

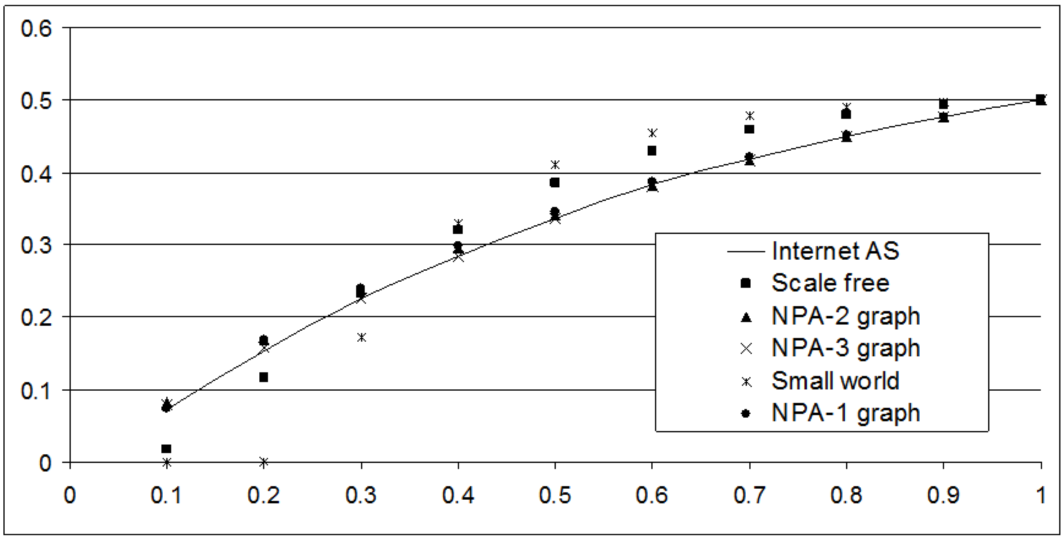


Figure 5. The proportion of agents in the 'infected' state at various values of the infection spread coefficient

Studies of the proportion of infected agents in the SIS model at various probabilities to 'infect' neighbor agents also indicate that taking into account various structural characteristics in random graph generation increases the accuracy of network process simulation results.

5. Conclusion

The development of large networks models is one of the urgent problems of the theory of agent-based simulation. This problem is quite successfully solved for Euclidean graphs (GIS, flat lattice models, random geometric graphs). In this paper, we consider the possibilities of using random graphs without space binding, namely, the preferential attachment graphs and their possibilities for network processes simulation are considered. Preferential attachment graphs are also interesting in that they predict network growth. As the results of random failures simulation and virus spreading process have shown, the finding of the parameters of preferential attachment graph to realize required structural characteristics of the graphs makes it possible to increase the modeling accuracy. Similar results are obtained if other network processes are simulated (figure 6).

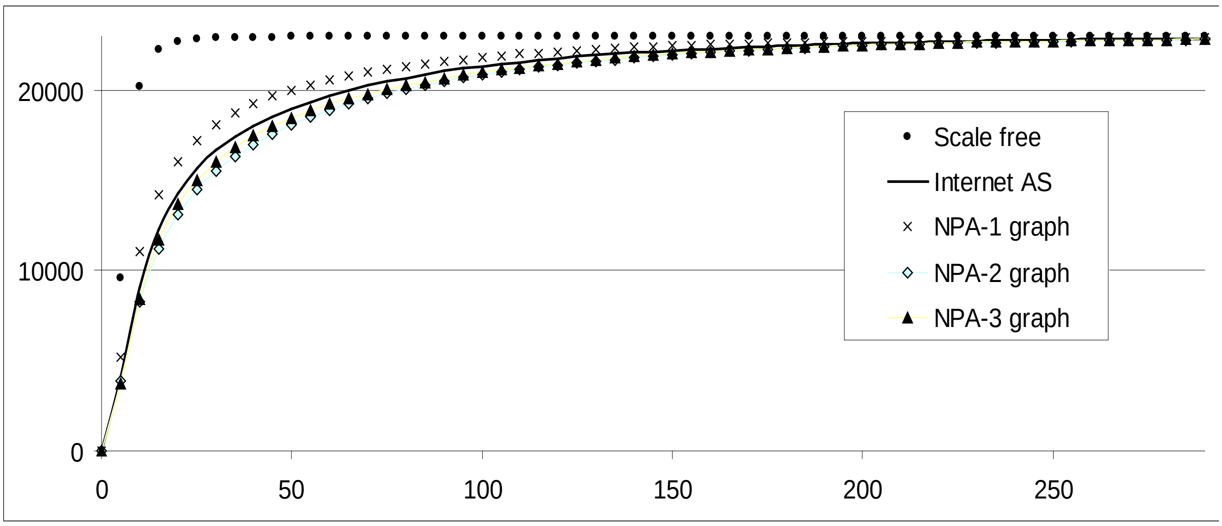


Figure 6. The number of adopted innovation agents (Adopters) in BassDiffusionAgentBased of the AnyLogic system. The various interaction structures between agents in the system are used

The preferential attachment graph calibration to realize required VDD is implemented in agent-based simulation system Simbigraph (https://sourceforge.net/projects/budnik/). The graph calibration by empirical VDD, EVDD and clustering coefficient can also be automated using formulas obtained in [3-5]. The implementation of calibration tools for preferential attachment graph in commercial simulation systems like Anylogic would allow us to solve the problems of studying “non-geometric” networks and network processes as Internet (including social networks) and molecular networks of cell (gene networks, networks of protein interactions) and other large networks of modern "Network society. The use of adequate models of network structures based on preferred attachment graphs can be useful in agent-based modeling in specialized agent-based simulation systems [10,11].

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