

# Mathematical modeling of a hydraulic flume for carrying out numerical experiments on coastal waves and erosion of cohesive soil

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**Abstract.** Hydrowave flumes are applied in laboratory experiments in order to study coastal wave impact on shore facilities. The realization of such laboratory experiments is costly and time-consuming concerning its preparation. Thus, it is pressing to apply a mathematical model. The paper presents a material hydrowave flume mathematical model that significantly broadens the capability of the research on coastal wave propagation and its impact on sea floor cohesive soil. The paper considers a solitary wave climb to the shore in the areas of different configuration including sea floor irregularities and a layer of cohesive soil. The wave propagation on the surface and soil shift in the water are modelled by three-component viscous incompressible fluid where air, water and soil are considered as components of non-homogeneous medium. The solution is obtained through finite difference numerical algorithm based on splitting scheme by physical factors and prediction correction method. Results of numerical calculations are presented in the paper.

## 1. Introduction

Currently, it is a highly topical problem to define qualitative and quantitative behavior of a solitary wave propagation to the shore or a restriction. This is related to the possible destructive effect that can be implied by a long wave on the coastal structures. The sources of such waves are earthquakes, eruptions of underwater volcanos, landing slips, landslides, etc. [1]. Frequent origination and advance of a long wave of tsunami is studied by means of laboratory experiments [2–5]. However, undertaking of such experiments is costly and time-consuming considering its preparation. This is the reason why mathematical modeling is applied in order to receive results for different problems in a relatively shorter period as well as to broaden the capabilities of laboratory experiments.

There are several basic approaches for mathematical modeling of the problems of origination and advance of a wave on the free water surface [6, 7]. As for the waves of tsunami type, they are presented, for example, in the papers [8–10].

A hydrowave flume (see its detailed description in [11]) is applied for studying of a solitary wave behavior in «23 GMPI» of the branch of OAO «31 GPISS» (Saint Petersburg). A solitary wave in it is originated by a vacuum wave maker which is a sealed closed reservoir connected to a flume. The reservoir is equipped with a pump; the upper lid has a valve. The air is evacuated by a valve in order to reach the required height of the liquid. In the beginning of the experiment, the valve is opened, the air



enters the reservoir, due to the gravity the water discharges into the flume that creates a solitary wave and accompanying wave train of a lesser amplitude. In close proximity to the wave maker, there is an area of a flat bottom where the formation of a created wave is completed. Further, the bottom becomes a smooth slope.

The paper [11] introduces a mathematical model of a hydrowave flume «23 GMPI» and its validation based on the experimental data; the following study of the model was continued in the paper [12]. In order to define the motion of open surface water the papers apply the model of multicomponent viscous incompressible liquid which was also used for problems of erosion of cohesive soil [13, 14], surface wave propagation [15], and wave origination as a result of underwater land slide [16].

Under laboratory conditions, the geometry change of a hydrowave flume may require significant time and financial costs, and replenishment of the cohesive soil to the bottom is technically impossible due to design features.

The goal of the paper in continuation [17] is to broaden the capabilities of the mathematical model of a hydrowave flume «23 GMPI» in order to solve the problems that are difficult or impossible to study under the laboratory settings. For instance, the paper presents the numerical solution considering the problem of erosion of sea floor cohesive soil by an ingoing solitary wave.

## 2. Mathematical model and problem-solving methods

We consider the motion of multicomponent viscous incompressible medium, which viscosity and density depend upon the concentration of components corresponding to air, water and soaked cohesive soil. Each component is presented by viscous incompressible liquid with its own values of viscosity and density, and thus, with the possibility of mass diffusion between these components. The motion of such multicomponent medium is described by nonstationary system of Navier-Stokes equation taking into account the above-mentioned effects; and the transfer of medium components – by diffusion-convection equation and formulas for definition of density and viscosity [16]:

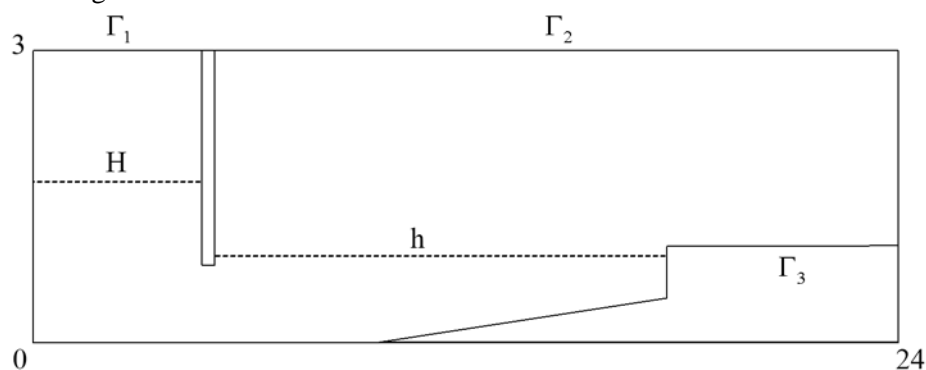
$$\left\{ \begin{array}{l} \frac{d(\rho \bar{V})}{dt} = -\nabla p + \operatorname{div}(\mu \mathbf{D}) + \rho \bar{f}, \\ \operatorname{div} \bar{V} = 0, \\ \frac{dC_1}{dt} = D_{12} \Delta C_1, \\ \frac{dC_3}{dt} = D_{23} \Delta C_3, \\ C_2 = 1 - C_1 - C_3, \\ \mu = \frac{\mu_1 \mu_2 \mu_3}{\mu_2 \mu_3 C_1 + \mu_1 \mu_3 C_2 + \mu_1 \mu_2 C_3}, \\ \rho = \rho_1 C_1 + \rho_2 C_2 + \rho_3 C_3, \end{array} \right. \quad (1)$$

where  $\bar{V}(\bar{x}, t) = (v_1, v_2, v_3)$  – velocity vector of the medium at the point  $\bar{x} = (x_1, x_2, x_3)$  and the time moment  $t$ ,  $\mu(\bar{x}, t)$  – dynamic viscosity,  $\rho(\bar{x}, t)$  – density,  $C_1(\bar{x}, t)$ ,  $C_2(\bar{x}, t)$ ,  $C_3(\bar{x}, t)$  – volume concentrations of components with constant densities  $\rho_1$ ,  $\rho_2$ ,  $\rho_3$  and viscosities  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$ ,  $\bar{f} = (f_1, f_2, f_3)$  – vector of mass force,  $p$  – pressure,  $\mathbf{D}$  – strain rate tensor which components are equal to  $\tau_{ij} = \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$ ,  $D_{12} = \text{const}$  – diffusion coefficient between the first and second component,  $D_{23} = \text{const}$  – diffusion coefficient between the second and third component.

For numerical implementation of the model, we apply the finite-difference method at the rectangular staggered grid [18]. Concurrently, Navier-Stokes equation is approximated by the splitting scheme by physical factors [19] taking into consideration density variable; and the predictor-corrector methods are applied to solve the diffusion-convection equation. The detailed description of this numerical algorithm is presented in the article [16].

### 3. Statement of a problem

We consider the problem of a solitary wave propagation in the hydrowave flume. In order to model the motion of water surface, we apply the three-component model of viscous incompressible liquid (1), where the first component is air, the second one – water, and the third one – soaked cohesive soil. We consider two dimensional statement of the problem as the width of the flume is much less than its length, and side faces do not produce significant impact on the flow behavior. The boundaries of the set are presented at figure 1.



**Figure 1.** Scheme of the flume set.

At this point:

$\Gamma_1$  – boundary that models the behavior of the wave maker valve

$$\left. \frac{\partial v_1}{\partial y} \right|_{\Gamma_1} = 0, \quad \left. \frac{\partial v_2}{\partial y} \right|_{\Gamma_1} = 0, \quad p|_{\Gamma_1} = P_{wm}(t), \quad \left. \frac{\partial C_1}{\partial y} \right|_{\Gamma_1} = 0, \quad \left. \frac{\partial C_3}{\partial y} \right|_{\Gamma_1} = 0.$$

$\Gamma_2$  – boundary of free discharge with a set of air-pressure

$$\left. \frac{\partial v_1}{\partial y} \right|_{\Gamma_2} = 0, \quad \left. \frac{\partial v_2}{\partial y} \right|_{\Gamma_2} = 0, \quad p|_{\Gamma_2} = 101325 \text{ Pa}, \quad \left. \frac{\partial C_1}{\partial y} \right|_{\Gamma_2} = 0, \quad \left. \frac{\partial C_3}{\partial y} \right|_{\Gamma_2} = 0.$$

$\Gamma_3$  – fixed boundary with a no-slip condition

$$v_1|_{\Gamma_3} = 0, \quad v_2|_{\Gamma_3} = 0, \quad \left. \frac{\partial p}{\partial n} \right|_{\Gamma_3} = 0, \quad \left. \frac{\partial C_1}{\partial n} \right|_{\Gamma_3} = 0, \quad \left. \frac{\partial C_3}{\partial n} \right|_{\Gamma_3} = 0.$$

Pressure value  $P_{wm}(t)$  at the boundary  $\Gamma_1$  is determined by the solution of the additional problem about air inflow into the wave maker through the open gate [20]. It is necessary to generate a wave according to the same principle which is applied under the laboratory conditions that allows to receive wave amplitudes congruent to the experiment [11].

The water depth in the water maker  $H = 2 \text{ m}$ , the water level in the flume  $h = 1 \text{ m}$ . Dynamic viscosity and density of the components have the following values: for air  $\mu_1 = 10^{-5} \text{ Pa} \cdot \text{s}$ ,  $\rho_1 = 1 \text{ kg/m}^3$ ; for water  $\mu_2 = 10^{-3} \text{ Pa} \cdot \text{s}$ ,  $\rho_2 = 1000 \text{ kg/m}^3$ ; for soil  $\mu_3 = 10 \text{ Pa} \cdot \text{s}$ ,  $\rho_3 = 1500 \text{ kg/m}^3$ . The boundary line of the components takes place when the concentration values is  $C = 0.5$ .

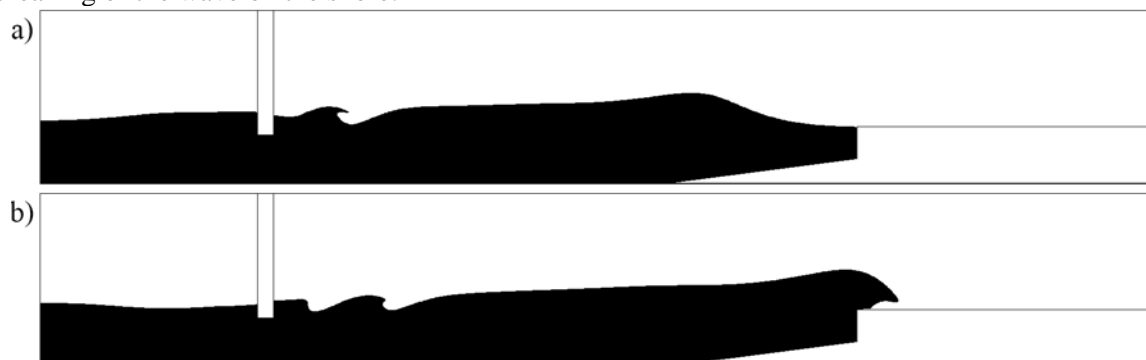
#### 4. Solitary wave propagation in the flume with different bottom shape

In the hydrowave flume «23 GMPI» a wave is originated as a result of water discharge from the wave maker. Figure 1 represents the area of the wave propagation. When reaching the shore the wave breaks making force action. Figure 2 represents the motion of the wave in the area which is usually used in laboratory experiments. Letters indicate time moments corresponding to a) initial position of the liquid, b) wave origination c) wave motion on the slope d) breaking of the wave on the shore.



**Figure 2.** Motion at the time moments (in seconds): a) 0, b) 1.2, c) 2.9, d) 3.7.

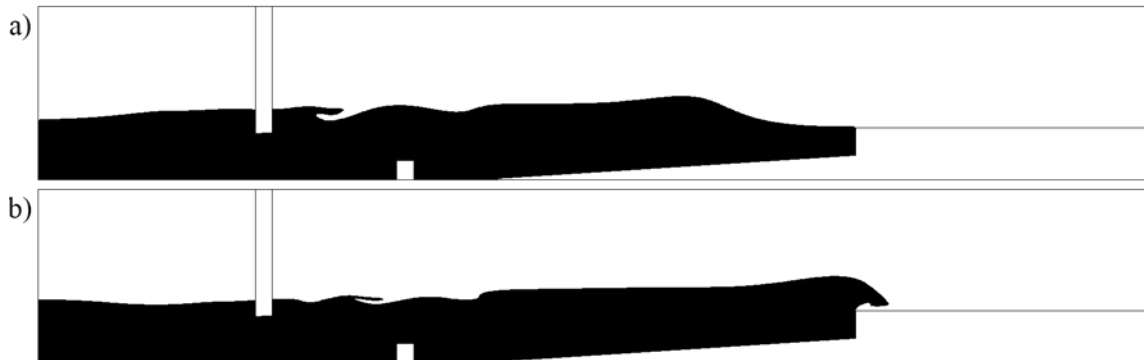
The shape and level of the sea floor in the coastal area may have a decisive impact on the character of propagation and breaking of a long wave. In order to define the degree of such an impact, we made calculations having increased the step steepness by two times. Figure 3 has numbers indicating surface of the liquid at the time moments corresponding to a) wave originating while moving on the slope, b) breaking of the wave on the shore.



**Figure 3.** Motion at the time moments (in seconds): a) 2.9, b) 3.7.

As tsunamis represent one of the most unpredictable and destructive calamities the solution for protection of onshore facilities is very pressing. One of the common ways of such protection is the installation of coastal barriers that ward off long waves of the specified height [21, 22]. The following calculation includes a rectangular barrier set in front of the step. Figure 4 has letters indicating the

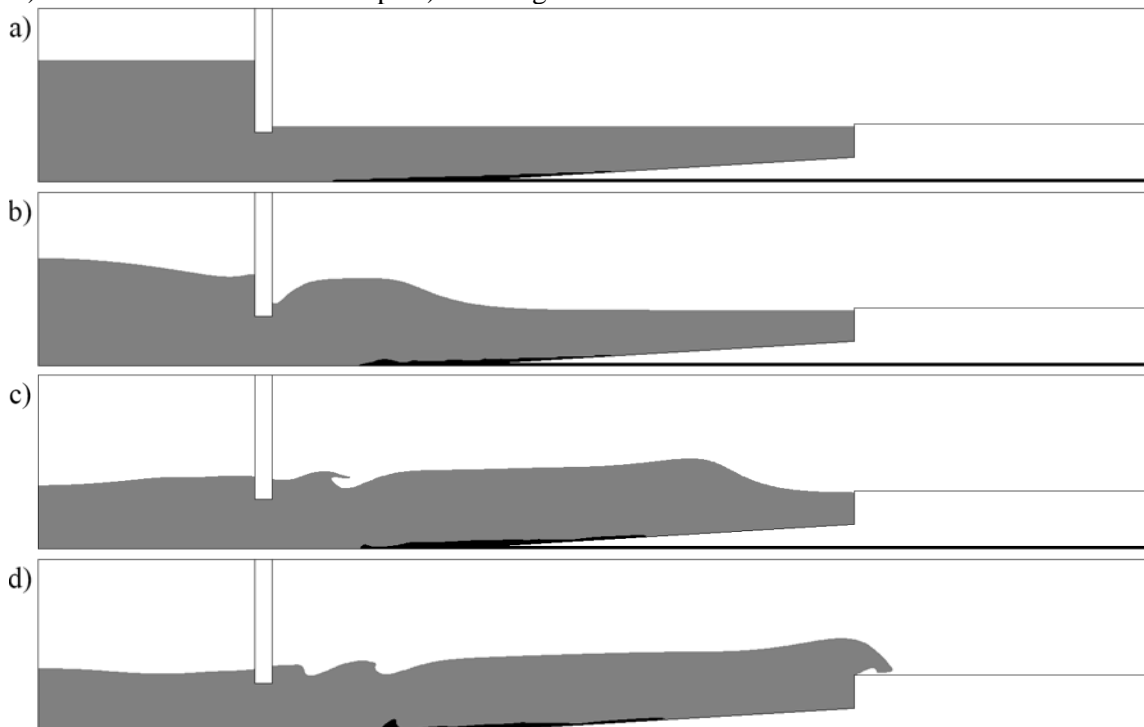
wave shape at the time moments corresponding to a) wave motion on the slope b) breaking of the wave on the shore.



**Figure 4.** Motion at the time moments (in seconds): a) 2.9, b) 3.7.

### 5. Erosion of sea floor cohesive soil

Within the frames of the set of a hydrowave flume «23 GMPI» it is technically impossible to undertake experiments on the interaction of the surface wave and cohesive soil on the sea floor as they have recycling water supply of a flute in the laboratory experiments. The represented in the paper mathematical model makes it possible to include the interaction with the seal floor cohesive soil into the problem of long surface wave propagation. The numerical experiments were undertaken in the area shown on the Figure 1, where the layer of the cohesive soil on the bottom is marked in black color (see Figure 5). Figure 5 has letters indicating the wave shape and position of the cohesive soil at the time moments corresponding to a) initial position of liquid and soil, b) wave origination taking the soil with it c) climb of the wave on the slope d) breaking of the wave on the shore.



**Figure 5.** Motion in the area with sea floor soil at the time moments (in seconds): a) 0, b) 1.2, c) 2.9, d) 3.7.

## 6. Conclusion

The conducted calculations show that: firstly, the value of wave setup on the rectangular shore depends upon the angle of slope and the shape of the adjoining sea floor; secondly, besides the fact that the cohesive soil follows the motion of the surface water, soil shifting also influences the water shape, thus we have their interaction here.

The introduced mathematical model of a hydrowave flume «23 GMPI» significantly broadens the capabilities of a physical experiment. The model can be applied in combination with laboratory studies for solving the problem of protection of coastal facilities and prognosis of silting of the offshore strip.

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