Method for defining residual stresses in arched profiles

E A Seitov[1], S A Makeev[2]

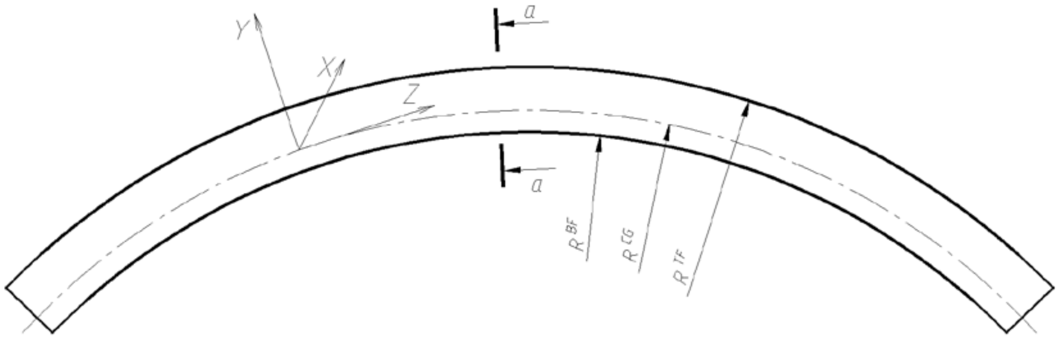
[1]The-Siberian State Automobile and Highway University, 5, Mira Ave., Omsk, 644080, Russia
[2]Omsk State Technical University, 11, Mira Ave., Omsk, 644050, Russia

Abstract. In the process of manufacturing arched steel thin-walled cold-formed profiles with trapezoidal corrugations, by rolling a flat profile into the arch through a system of rollers, in the normal section profile longitudinal technological residual normal stress are formed, unevenly distributed over the height of the section. Currently, these residual stresses are not taken into account in the design, which leads to errors in the calculations for strength and local stability of arched structures made of arched profiles. There is a number of ways for defining the residual stresses in steel structures, including arched thin-walled profiles. Still, they have a number of significant disadvantages associated with the low accuracy of the values obtained and the complexity of stress calculations. The purpose of this work is to develop a method for defining the residual longitudinally oriented normal stresses in the flanges of arched cold–formed profiles with trapezoidal corrugations, combining the simplicity of the measurement technique, calculation and high accuracy of the obtained stress values. As a result, we obtained a method of defining the residual longitudinally oriented normal stresses in cross-sections of the arched profile as in the compressed and stretched flanges for easy calculation and high enough for engineering calculations, the accuracy of measurements of parameters of the profile to calculate stresses. The proposed method can be used to define the longitudinal residual stresses in normal sections of arched profiles with subsequent consideration of these stresses in the design of structures.

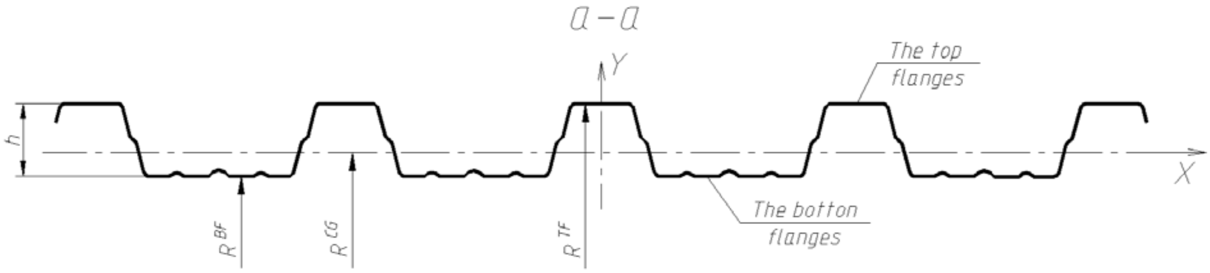
1. Introduction

In steel thin-walled cold-deformed elements residual stresses occur as a result of production processes: coiling and uncoiling of sheet steel [1], [2], stamping and profiling along the radius in the cross-section [3], [4], [5].

Arched steel cold-formed profiles with the trapezoidal corrugations (figure 1) [6] are obtained under longitudinal rolling of flat profile through a rolling-bending machine with a roller system [7], [8] together with the formation of the normal sections of the profile distributed along the section height of a self-balanced system of residual longitudinally oriented normal stresses.



(a)



(b)

Figure 1. A sketch of the arched profile with trapezoidal corrugations:
R^{CG} – known radius of the rolled profile through the center of gravity of the section; R^{BF} – known radius of the top surface of the bottom flange of the profile; R^{TF} – known radius of the top surface of the top flange of the profile; *X*, *Y*, *Z* – figure axis of the arched profile:
a) arched profile; b) section *a-a*.

As a result of implementing the technology of manufacturing arched profile self-balanced system of longitudinal normal residual stresses $\sigma_{z\ res}$ in the normal section in the general case for a symmetrical profile gives the effect of compressive stresses in the top convex flange profile and tensile at the bottom of the concave (figure 2).

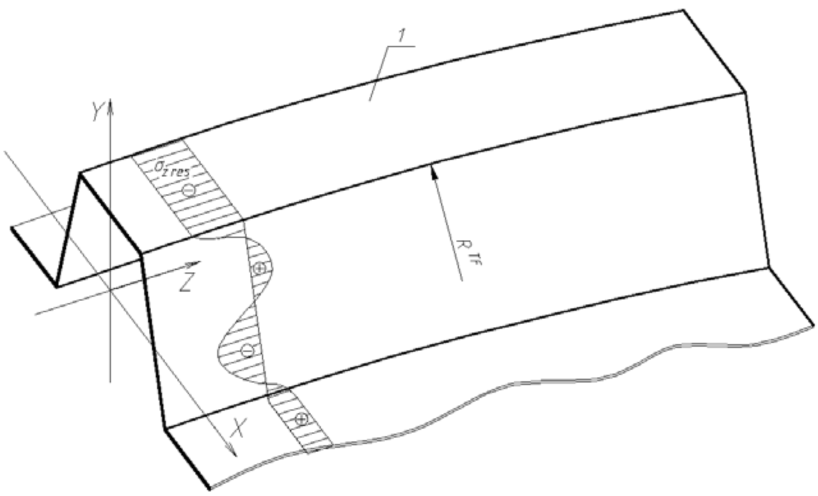


Figure 2. A fragment of an arched profile with a qualitative picture of distribution of longitudinal

residual normal stresses $\sigma_{z\ res}$ at the height of the normal cross-section of the profile: 1 – top convex section of the arched profile.

According to preliminary calculations [9] and experimental evaluation investigated residual normal stresses $\sigma_{z\ res}$ in the arched profiles depending on the magnitude of the residual radius and type of profile can in sections reach the values ±80 MPa or more, which is about one-third of the design strength of steel used [10]. Naturally, these stresses affect the strength and local stability of the arched profile elements when it is loaded in structures and they should be taken into account when calculating the bearing capacity of structures [11], [12]. At the moment, the residual longitudinal oriented normal stresses of the longitudinal rolling products are not taken into account in the calculations of loaded arched structures [13], [14], [15], [16].

There is also a method for defining residual stresses in the compressed top flange of an arched steel thin-walled cold-formed profile [17].

Its principle is as follows. In a compression convex top flange of the arched steel thin-walled cold-formed profile, with width *B* there perform two parallel through-cuts of length *L* 100 mm or more, spaced at distance *b*.

Due to the presence of compressive residual longitudinal stresses, the resulting strip between the cuts will bend, become even more convex than the flange itself. Having measured the deflection in the middle of the span of the strip *V* relative to the surface of the flange, we calculate the residual stresses according to the formula:

$$\sigma_{z\ res} = -\frac{V \cdot E}{L \cdot k}, \tag{1}$$

where *V* is the maximum deflection of the cut steel strip, mm;

E is the normal modulus of steel elasticity, MPa;

L is the length of the cut steel strip, mm;

k is the coefficient, which depends on the geometric characteristics of the arched steel thin-walled cold-formed profile, which is a complex trigonometric formula.

The width of the strip *b* formed when performing cuts, depending on the width of the compressed flange profile *B*, is defined by the ratio $b \leq 0.1 \cdot B$.

The length of the cut steel strip *L*, formed when performing cuts depending on its width *b*, is defined by the ratio $L \geq 10 \cdot b$.

The width of the cuts does not exceed 1-2 mm.

The disadvantages of this method are:

– ability to measure residual normal stresses only in the compressed top flanges of arched steel thin-walled cold-formed profiles;

– the complexity and inaccuracy of measuring the deflection *V* of the cut strip due to its large bending malleability;

– labour intensity and complexity of calculating the coefficient *k* in formula (1), requiring a large amount of time.

Among the known technical solutions, the closest in terms of the set of essential features to the proposed method is the method for defining the residual stresses in flat steel structures, which presupposes measuring the deformations that occur when cutting the investigated element and calculating the residual stresses:

To define the residual stresses, measuring base is placed in steel plates. The length of the base *B* is selected in the range of 100 mm or more. The width of base *b* should be as small as possible. According to the results of measurements of the base before and after cutting, residual stresses are defined by the formula:

$$\sigma_{res} = -E \cdot \varepsilon = -E \cdot \frac{B_2 - B_1}{B_1}, \tag{2}$$

where E is the normal metal modulus of elasticity;
ε is the relative deformation of the metal resulting from cutting;
B_1 is the length of the base before the cutting;
B_2 is the base length after cutting.

The disadvantage of this method is the inability to use formula (2) for steel arched structures, due to the fact that the surfaces of the flanges of the arched profile are not flat and are cylindrical surfaces (figure 1).

Thus, this disadvantage of the known method leads to the impossibility of defining the residual longitudinally oriented normal stresses in the flanges of arched steel thin-walled cold-formed profiles with trapezoidal corrugations.

To assess the residual longitudinally oriented normal stresses in the arched profile with the possibility of taking them into account in the design of arched structures, it is necessary to develop a simple and affordable way to define them.

2. Problem statement

The objective of this work is to develop a method for defining the longitudinally oriented residual stresses in the flanges of arched steel thin-walled cold-formed profiles with trapezoidal corrugations, which allows to define the residual stresses in both compressed and stretched flanges of arched profiles and provides simplicity of measurement and calculation methods, reliability and high accuracy of the obtained stress values.

3. Implementation of the method

Implementation of the method: in the flange *1* with the known radius of the top surface of the top (bottom) R^{TF} (R^{BF}) of the arched steel thin-walled cold-formed profile with trapezoidal corrugations by two base holes *2* are perform with a diameter d = 3-5 mm, placed along the Z-axis with distance along the arc L_1 = 100-200 mm, then the chord length c_1 between the base holes *2* is measured (figure 3).

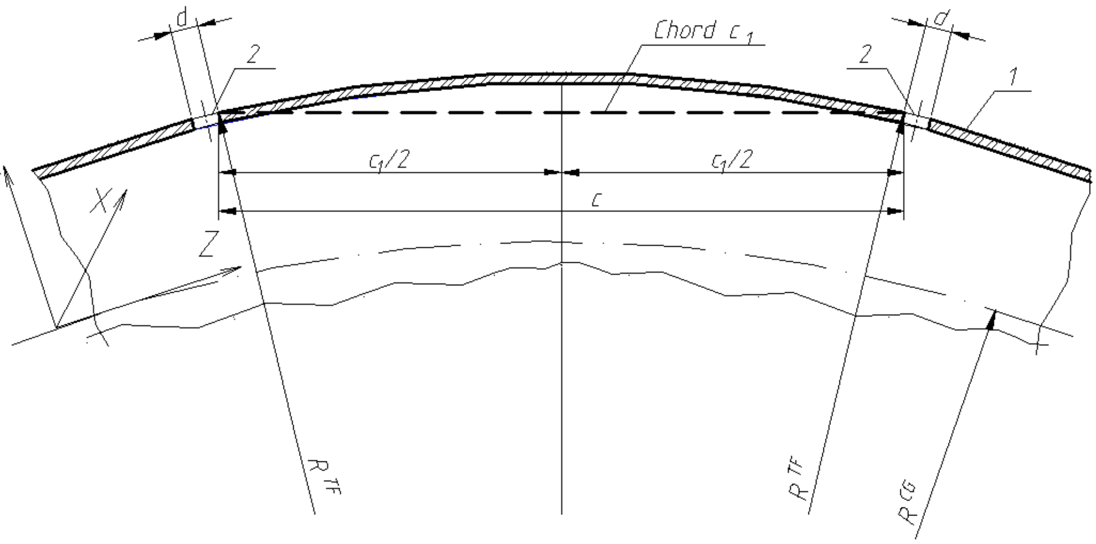


Figure 3. Measuring the length of the chord before performing the cut: *1* – top convex flange of the arched profile; *2* – base holes with diameter d; R^{TF} – known radius of the top surface of the top flange of the arched profile; L_1, c_1, φ_1 – arc length, chord length and central angle between the base holes before performing a U-shaped cut in the flange.

Further, with a symmetrical coverage of the base holes *2*, a through cut of the U-shape width b = 8-12 mm in the investigated flange is performed (figures 4, 5) followed by repeated measurement of the length of the chord c_2 between the base holes *2* when fixing the cut strip in the surface of the flange (figure 6).

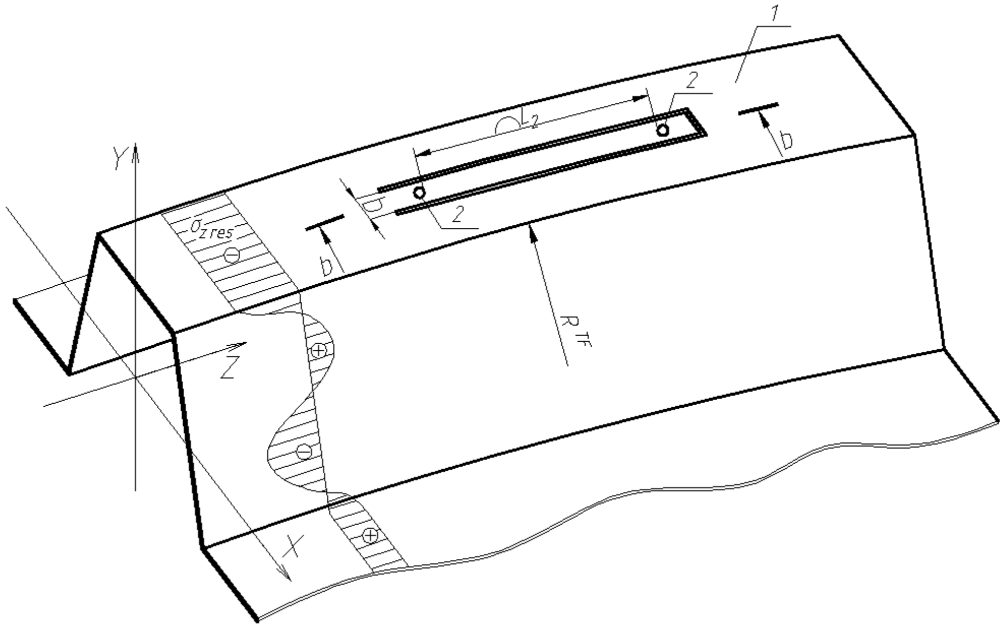


Figure 4. A profile piece with the base holes, and a U-shaped cut in the flange on the example of the top flange of a convex arched profile: *1* – top convex flange of the arched profile; *2* – base holes with diameter d; R^{TF} – known radius of the top surface of the top convex flange of the arched profile; b – width of the U-shaped cut in the flange; L_2 – the length of the arc between the base holes after performing the cut in the flange.

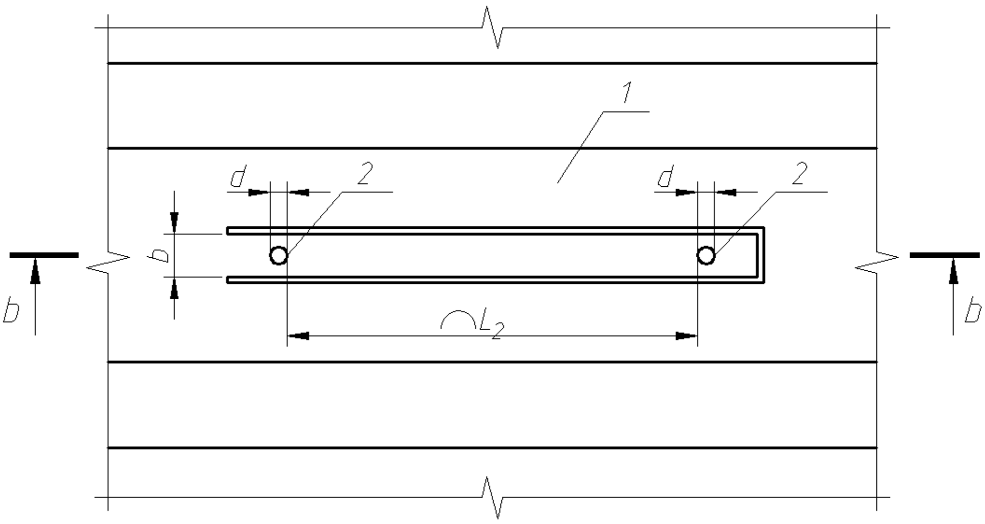


Figure 5. Top view of the arched steel thin-walled cold-formed profile at the site of the basic holes and U-shaped cut in the flange in figure 4.

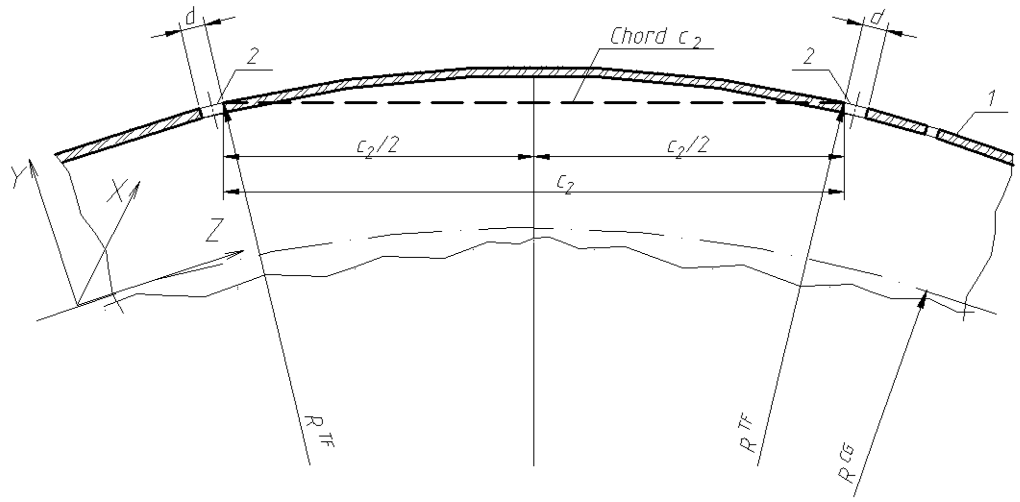


Figure 6. Measuring the length of the chord after the cut:
1 – top convex flange of the arched profile; *2* – base holes with diameter *d*; R^{TF} – known radius of the top surface of the top flange of the arched profile; L_2, c_2, φ_2 – arc length, chord length and central angle between the base holes after performing a U-shaped cut in the flange.

After performing a U-shaped cut, longitudinally oriented residual normal stresses in the flange are released. In the general case of a profile symmetric cross-section these are compressive stresses when performing the cut in the convex top flange, while the cut strip is extended or tensile stresses at the section in the extended bottom concave flange, the cut strip is shortened (figure 4).

According to the results of measurements of chord lengths c_1 and c_2 between the base holes before and after performing a U-shaped cut in the flange, the values of longitudinally oriented normal residual stresses $\sigma_{z\,res}$ in the flange of the arched steel thin-walled cold-formed profile with trapezoidal corrugations are defined by the formula:

$$\sigma_{z\,res} = -E \cdot \left(\frac{\arcsin\left(\frac{c_2}{2 \cdot R^{TF}}\right)}{\arcsin\left(\frac{c_1}{2 \cdot R^{TF}}\right)} - 1 \right),$$

where E is the normal modulus of elasticity of the steel used, MPa;

c_1 is the chord length between the base holes before performing a U-shaped cut in the flange, mm;

c is the chord length between the base holes after performing a U-shaped cut in the flange, mm;

R^{TF} (R^{BF}) is the known radius of the top surface of the top convex (concave bottom) flanges of the arched profile, mm.

Here it is assumed that the residual radius arched profile R^{TF} (R^{BF}) do not change when performing a local U-shaped cut in one flange profile.

4. Mathematical basis for calculations

At known arc lengths between the base holes L_1 and L_2, respectively, before and after the U-shaped cut in the flange (figures 3-6), the required residual stresses $\sigma_{z\,res}$ in the flange of the arched profile can be defined by the formula known from Hooke's law:

$$\sigma_{z\,res} = -E \cdot \varepsilon_z = -E \cdot \frac{\Delta L}{L_1} = -E \cdot \frac{L_2 - L_1}{L_1}, \qquad (3)$$

where E is the normal modulus of elasticity of the steel used, MPa;

Journal of Physics: Conference Series **1441** (2020) 012113 doi:10.1088/1742-6596/1441/1/012113

ε_z is the longitudinal deformation of the strip formed as a result of U-shaped cut in the flange;

L_1 is the length of the arc between the base holes before performing a U-shaped cut in the flange, mm;

L_2 is the length of the arc between the base holes after performing a U-shaped cut in the flange, mm.

In view of the fact that it is technically impossible to measure the lengths of arcs L_1 and L_2 or is rather difficult, the authors proposed to measure the chords c_1 and c_2, then to move to the corresponding lengths of arcs under the assumption of constancy of the residual radius of the arched profile.

Based on figure 3, the length of the arc L_1 and the corresponding chord c_1 between the base holes before performing a U-shaped cut in the flange can be expressed from the usual geometric relations:

$$L_1 = \varphi_1 \cdot R^{TF}, \tag{4}$$

$$c_1 = 2 \cdot R^{TF} \cdot \sin\frac{\varphi_1}{2}, \tag{5}$$

where L_1 is the length of the arc between the base holes before performing a U-shaped cut in the flange, mm;

R^{TF} (R^{BF}) is the known radius of the top surface of the top convex (concave bottom) flange of the arched profile, mm.

c_1 is the chord length between the base holes before performing a U-shaped cut in the flange, mm;

φ_1 is the central angle before performing a U-shaped cut in the flange, deg.

Based on equation (5), we express the central angle between the base holes before performing a U-shaped cut in the flange:

$$\varphi_1 = 2 \cdot \arcsin\frac{c_1}{2 \cdot R^{TF}}. \tag{6}$$

Given equation (6), equation (4) takes the form:

$$L_1 = 2 \cdot R^{TF} \cdot \arcsin\frac{c_1}{2 \cdot R^{TF}}. \tag{7}$$

Similarly, from figures 4-6 we express the length of the arc L_2 and then the chord of this arc c_2:

$$L_2 = \varphi_2 \cdot R^{TF}, \tag{8}$$

$$c_2 = 2 \cdot R^{TF} \cdot \sin\frac{\varphi_2}{2}, \tag{9}$$

where L_2 is the length of the arc between the base holes after performing a U-shaped cut in the flange, mm;

R^{TF} (R^{BF}) is the known radius of the top surface of the top convex (concave bottom) flange of the arched profile, mm.

c_2 is the chord length between the base holes after performing a U-shaped cut in the flange, mm;

φ_2 is the central angle after the U-shaped cut in the flange, deg.

From equation (9), we express the central angle of the arc φ_2:

$$\varphi_2 = 2 \cdot \arcsin\frac{c_2}{2 \cdot R^{TF}}. \tag{10}$$

Given equation (10), equation (8) has the following form:

$$L_2 = 2 \cdot R^{TF} \cdot \arcsin \frac{c_2}{2 \cdot R^{TF}}. \tag{11}$$

Given equations (7) and (11), equation (3) takes the form:

$$\sigma_{z\ res} = -E \cdot \left(\frac{\arcsin\left(\frac{c_2}{2 \cdot R^{TF}} \right)}{\arcsin\left(\frac{c_1}{2 \cdot R^{TF}} \right)} - 1 \right), \tag{12}$$

where E is the normal modulus of elasticity of the steel used, MPa;
c_1 is the chord length between the base holes before performing a U-shaped cut in the flange, mm;
c_2 is the chord length between the base holes after performing a U-shaped cut in the flange, mm;
R^{TF} (R^{BF}) is the known radius of the top surface of the top convex (concave bottom) flange of the arched profile, mm.

5. Discussion

The method for defining residual longitudinally oriented normal stresses $\sigma_{z\ res}$ in the flanges with known residual radius of the top surface of the top convex (concave bottom) flange R^{TF} (R^{BF}) of the arched steel thin-walled cold-formed profile with trapezoidal corrugations, including two base holes, a U-shaped cut in the studied flange with subsequent measurement of the physical parameters and calculation of residual stresses according to the formula, which differs from the prototype by the fact that the measured chord lengths c_1 and c_2 between the base holes oriented along the Z-axis, before and after execution of the U-shaped cut with subsequent calculation of residual longitudinally oriented normal stresses $\sigma_{z\ res}$ by the formula (12).

Assuming that the value of the residual longitudinally oriented normal stresses in the top compressed flange of the steel arched profile $\sigma_{z\ res}$ is -80 MPa at the arc between the base holes of length 150...200 mm, the measured lengthening of the strip ΔL (3) after the U-shaped cut will be:

$$\Delta L = \frac{L_1 \cdot |\sigma_{z\ res}|}{E} = \frac{[150...200] \cdot 80}{210000} = [0.0571...0.0762]\,mm = [57...76]\,\mu m. \tag{13}$$

The values of the difference between the measured chords c_1, c_2 will be the equal to the values in the same ranges.

Such values of the linear measurements are accurately recorded by Russian measuring tools such as micrometers, type MKC-200 0.001 KLB* (listed in the state register № 56668-14, measurement range from 0 mm to 200 mm), measurement accuracy is 1.0 µm [18]. For the example (13), the measurement error will be (±3.5...2.6) %.

6. Conclusion

Thus, a method for defining the longitudinally oriented normal residual stresses in the flanges of arched steel thin-walled cold-formed profiles with trapezoidal corrugations is proposed, which allows to define the residual stresses in both compressed and stretched flanges and provides simplicity of calculations and high accuracy of measurements sufficient for engineering calculations.

References

[1] Quach W M, Teng J G, Chung K F 2004 Residual stresses in steel sheets due to coiling and uncoiling: a closed-form analytical solution *Engineering Structures* vol **26** Issue 9 pp 1249–59. DOI: 10.1016/j.engstruct.2004.04.005.

[2] Quach W M, Teng J G, Chung K F 2009 Residual stresses in press-braked stainless steel sections. I: Coiling and uncoiling of sheets *J. of Constructional Steel Res* vol **65** Issue 8-9 pp 1803-15. DOI: 10.1016/j.jcsr.2009.04.007.
[3] Quach W M, Teng J G, Chung K 2009 Residual stresses in press-braked stainless steel sections. II: Press-braking operations *J. of Constructional Steel Res* vol **65**, Issue 8–9 pp 1816–26 DOI: 10.1016/j.jcsr.2009.04.011.
[4] Moen C D, Igusa T, Schafer B W 2008 Prediction of residual stresses and strains in cold-formed steel members *Thin-Walled Structures* vol **46** Issue 11 pp 1274–89 DOI: 10.1016/j.tws.2008.02.002.
[5] Jacques R 1987 Residual stresses in cold-rolled profiles in *Construction and Building Materials* vol **1**, Issue 3. pp 150-64. DOI: 10.1016/0950-0618(87)90016-X.
[6] Technical instructions 1122-001-49529858-2005 2005 *Steel roll-formed sections with trapezoidal corrugations of the arch. Technical conditions. Intr. 2005-12-10* (Novosibirsk: Publishing house SibNIIstroy)
[7] Company Zeman Bauelemente. LEGATO – bending machine. URL: http://www.zebau.com/en/legato---bending-machine (accessed: 05.08.2019).
[8] Company Zeman Bauelemente. LEGATO – TWIN TASK machine. URL: http://www.zebau.com/en/legato---twin-task (accessed : 05.08.2019).
[9] Makeev S A and Kolmakov D M 2014 Modeling of residual stresses in arched thin-walled rolling of a trapezoidal section *Omsk scientific bulletin* **1** (127) pp 69–73
[10] GOST 9045-93. Cold-rolled sheet steel of low-carbon high-quality steel for cold stamping. Technical conditions. Intr. 1997-01-01
[11] Makeev S A, Grishaev N A 2010 Numerical analysis of strength and local stability of trapezoidal arched profiles taking into account residual stresses of longitudinal bending in *Construction mechanics and calculation of structures* vol **2** pp 37-40
[12] Kholkin E.G., Sokolovskiy Z.N. Design of trapezoid thin-walled profiles taking into account local loss of stability *Omsk scientific bulletin* 2010 vol **3** pp 50–53
[13] Patent 260.1325800.2016. Thin-walled steel structures made of cold-rolled galvanized profiles and corrugated sheets
[14] Krasotina LV, Krasnoshchekov Yu.V 2009 Mosenkis Yu.M. Use of arched corrugated board during the reconstruction of buildings *Vestnik SibADI* vol **4** (14) pp 41–45
[15] Makeev S A, Grishaev N A 2010 Numerical study of local stability of trapezoidal arched profiles *Omsk scientific bulletin* **1** (87) pp 93-97
[16] Krasotina L V 2016 Engineering method for determining critical stress local stability in the core zone modular cylindrical profiled bearing shells *Vestnik SibADI* **2** (48) pp 104–110.
[17] Makeev S A, Kuzmin D A, Grishaev N A 2012 Pat. 2455622 Russian Federation, IPC G 01 L 1/06. Method for defining residual stresses SibADI No 2011105715/28; Appl. 15.02.2011; publ. 10.07.2012, Bulletin No 19.
[18] Electronic digital micrometer MKC-200 0.001 KLB*. URL : https://www.tdkalibron.ru/i/mikrometr_mkts__200_0001_klb (accessed: 19.08.2019).