Modeling of a profile of blades of the water-wheel with energy return optimization

A V Panichkin

Sobolev Institute of Mathematics, RAS, Omsk, st. Pevtsov, 13, Russia

Abstract. In work approach for numerical realization of definition of difficult borders with a current of a thin layer of ideal liquid on the surface of the working blade of the water-wheel is considered. The technique of creation of firm borders in two-dimensional and three-dimensional cases for streams of such layer of liquid with application of power optimization is presented. Test calculation of an optimum surface of blades of the water-wheel at the maximum return of kinetic and potential energy by liquid on the basis of use of the principle of a maximum was considered.

Keywords: current of a layer of ideal liquid, water-wheel blade, numerical modeling, algorithm of optimum control, principle of a maximum

1. Introduction

For determination of structure of blades of various turbines the set of techniques both with application of the simplified formulas [1], and with application of full calculation of the equations of hydrodynamics and gas dynamics is used. Definition of optimum surfaces of working blades of the water-wheel with return of the maximum kinetic and potential energy can be received from liquid within model of ideal liquid with attraction of a problem of optimum control on the basis of the interfaced differential equations of the movement.

In work [2] results of application of an algorithm of optimum control in the field of inclination with minimization of various functionalities were presented. This algorithm was applied to settling of an optimum surface of blades of the water-wheel with the movement of a thin layer of ideal liquid with free border where for these purposes the mathematical apparatus with the principle of a maximum of Pontryagin L. S. is used [3].

2. Problem definition

Let's consider on the rotating turbine one of blades with an uncertain surface. Let's define geometry of this surface in sense of optimum power return of a stream of the liquid sliding a thin layer on the surface moving to the field of inclination and causing rotation of the turbine with the set angular speed. For definition of such optimum surface of the separate blade of the water-wheel we will consider a number of trajectories of the movement on a neyzhidkost with an entrance at distance r = R from an axis of rotation of the turbine and with the set components of speeds V_{x0}, V_{r0}, V_{s0} along a rotation axis, in the radial and tangensalny directions, respectively. Let's consider that the turbine rotates with a constant angular speed ω_0 (see Fig. 1), and the movement of liquid will be considered in the accompanying mobile system of coordinates *(x, r, s)*, tied to the blade in a point of a start of motion of liquid on a surface with coordinates of $x_0=0$, $r_0=R$, $s_0=0$, with the further movement to edge of the blade down with *xk* coordinate. Inclination we will consider the field on *Ox* axis with continuous acceleration *of* g_0.

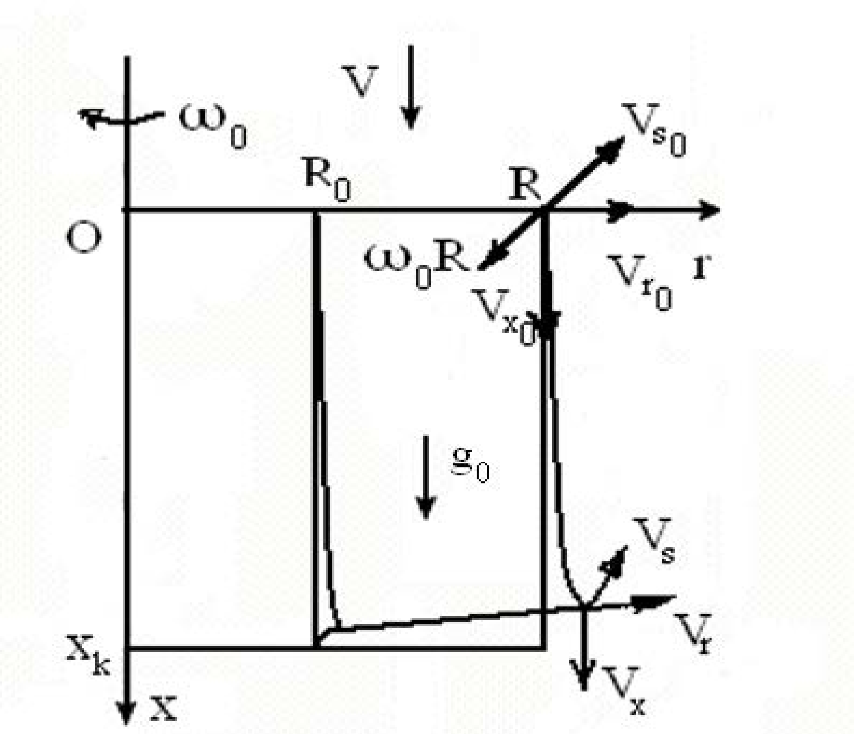

Figure 1. The image of the blade of the turbine and a separate trajectory of the movement of a drop of liquid (initial and final points are noted by speed vector components

At such problem definition in the chosen system of coordinates of the equation of the movement will have the following appearance:

$$\frac{dV_x}{dt} = g_0 Cos(\psi)Cos(\psi) + f_1, \quad (1)$$

$$\frac{dV_s}{dt} = g_0 Cos(\psi)Sin(\psi) - \frac{V_s V_r}{r} - 2V_r\omega + f_2, \quad (2)$$

$$\frac{dV_r}{dt} = g_0 Cos(\varphi)Sin(\varphi) + \frac{(\omega r - V_s)^2}{r} + f_3, \quad (3)$$

$$\frac{dx}{dt} = V_x\ , \quad (4)$$

$$\frac{ds}{dt} = V_s\ , \quad (5)$$

$$\frac{dr}{dt} = V_r\ , \quad (6)$$

where ψ,φ – corners of a trajectory of the movement to *Ox* axis in the tangensalny and radial directions, or $\psi= arctg(V_s/V_x)$,$\varphi= arctg(V_r/V_x)$, and sizes f_1, f_2, f_3 – are accelerations from curvature of a trajectory of the movement ρ_s, ρ_r:

$f_1 = -(V_x^2+V_s^2)Sin(\psi)/\rho_s - (V_x^2+V_r^2)Sin(\varphi)/\rho_r$, $f_2 = (V_x^2+V_s^2)Cos(\psi)/\rho_s$, $f_3 = (V_x^2+V_r^2)Cos(\varphi)/\rho_r$, $\rho_s = (1+(\partial s/\partial x)^2)^{3/2}/\partial^2 s/\partial x^2$, $\rho_r =(1+(\partial r/\partial x)^2)^{3/2}/\partial^2 r/\partial x^2$.

For the system of the equations (1)-(6) all entry conditions of V_{x0}, V_{r0}, V_{s0} , x_0, r_0, s_0 at initial value of time of movement are set $t_0 = 0$. At the same time the unknown value of final time of the movement *of* t_κ *liquid* is determined by the known size x_κ after integration of the equation (4).

For creation of an optimum profile of the blade for which liquid will give as much as possible the kinetic energy composed from the set kinetic energy and the acquired potential energy in the direction

of travel in the field of inclination on *axis x* from x_0 to x_k, , to rotary motion of the turbine will decide from the joint solution of a system of the equations (1)-(6) and the systems of the interfaced equations at use of the principle of a maximum of Pontryagin for separate trajectories on initial values of coordinate r from R_0 to R (see Fig. 1). Sharing of systems is given in paragraphs 3 and 4.
Currents of incompressible liquid with mobile borders are peculiar to various technological processes. The analysis of earlier obtained experimental and theoretical data on the movement of a layer of viscous liquid between the rotating cylinders and its splitting at transfer from one cylinder on another showed that the problem is insufficiently studied. At the solution of such tasks it is labor-consuming that is caused, first, by nonlinearity and complexity of the mathematical equations, secondly, need to define a free surface at the solution of a system of the equations in private derivatives that is characteristic of currents in layers and films.
The solution of a problem of quantitative assessment of indicators of transfer of viscous incompressible liquid at change of width of a zone of the contacted cylinders in our opinion represents both practical, and scientific interest.

3. Optimization of trajectories of the movement

For carrying out optimization of separate trajectories we will enter functionalities from the principle of the maximum transition of kinetic energy of liquid to energy of rotation of the turbine. Taking into account minimization of kinetic energy we will consider function

$$f_H = (V_x^2 + (V_s - \omega_0 R)^2 + V_r^2)/2, \tag{7}$$

applied in Hamilton functionality by its optimization, here kinetic energy is considered from full speed in the motionless system of coordinates. Taking into account creation of the interfaced system of the differential equations the total them with the initial equations (1)-(6) will reach twelve. Let's consider some simplified option with creation of an expanded system of the equations for a task optimization at the assumption that radius at the movement on one trajectory does not change, and for this optimization perhaps practical application.
At such assumption the system of the equations (1)-(6) passes into following (here functions with points are derivatives on time):

$$\dot{V}_x = g_0 Cos(\psi) Cos(\psi) + f_1, \tag{8}$$

$$\dot{V}_s = g_0 Cos(\psi) Sin(\psi) + f_2, \tag{9}$$

$$\dot{x} = V_x, \tag{10}$$

$$\dot{s} = V_s, \tag{11}$$

for which regional conditions are set:

$$t_0 = 0: V_x = V_{x0}, V_s = V_{s0}, x = x_0, s = s_0; x = x_k \ (t = t_k). \tag{12}$$

With introduction of the interfaced functions $\lambda 1$, $\lambda 2$, $\lambda 3$, $\lambda 4$ and functions Hamilton functionality (7) will take a form:

$$H = \lambda_1 (g_0 Cos(\psi) Cos(\psi) + f_1) + \lambda_2 (g_0 Cos(\psi) Sin(\psi) + f_2) + \lambda_3 V_x + \lambda_4 V_s - f_H . \tag{13}$$

The interfaced system of the equations to (8)-(11) will have an appearance:

$$\begin{aligned} \dot{\lambda}_1 = {} & \lambda_1 (-2 g_0 Cos(\psi) Sin(\psi)(V_s / V^2) + 2V_x Sin(\psi)/\rho_s - V_s Cos(\psi)/\rho_s) + \\ & + \lambda_2 (g_0 Cos^2(\psi) - Sin^2(\psi))(V_s / V^2) - 2V_x Cos(\psi)/\rho_s - V_s Sin(\psi)/\rho_s) - \\ & - \lambda_3 + V_x, \end{aligned} \tag{14}$$

$$\dot{\lambda}_2 = \lambda_1(2g_0 Cos(\psi)Sin(\psi)(V_x/V^2) + 2V_s Sin(\psi)/\rho_s + V_x Cos(\psi)/\rho_s) +$$
$$+\lambda_2(-g_0 Cos^2(\psi) - Sin^2(\psi))(V_x/V^2) - 2V_s Cos(\psi)/\rho_s + V_x Sin(\psi)/\rho_s) - \quad (15)$$
$$-\lambda_4 + (V_s - \omega_0 R),$$

$$\dot{\lambda}_3 = 0, \quad (16)$$

$$\dot{\lambda}_4 = 0, \quad (17)$$

where $V = (V_x^2 + V_s^2)^{1/2}$.

For the equations (14)-(17) initial data λ_{10}, λ_{20}, λ_{30}, λ_{40} are unknown, at the same time final values V_{xk}, V_{sk} , s_k , t_k, are not set , and for short circuit of boundary conditions it is possible to use transversality conditions at unknown $t = t_k$, when x_k is set. As variations from functionality $J(H) = \int_{t0}^{tk} H dt$ on t, V_x, V_s, s are not zero, for the requirement $\delta J(H)$=0 conditions have to be satisfied:

$$H = 0, \quad \frac{\partial H}{\partial \dot{V}_x} = 0, \quad \frac{\partial H}{\partial \dot{V}_s} = 0, \quad \frac{\partial H}{\partial \dot{s}} = \frac{\partial H}{\partial V_s} = 0.$$

These conditions take a form:

$$H = \lambda_1 \frac{dV_x}{dt} + \lambda_2 \frac{dV_s}{dt} + \left(\lambda_3 - \frac{V_x}{2}\right) V_x + \left(\lambda_4 - \frac{V_s}{2} + \omega R\right) V_s - (\omega R)^2/2 = 0,$$
$$\lambda_1 = 0, \lambda_2 = 0, \ \lambda_4 - V_s + \omega R = 0,$$

also four missing conditions for functions λ_i at t are defined by $t = t_k$:

$$\lambda_1 = 0, \lambda_2 = 0, \left(\lambda_3 - \frac{V_x}{2}\right) V_x + \frac{(V_s)^2}{2} - \frac{(\omega R)^2}{2} = 0, \ \lambda_4 - V_s + \omega R = 0. \quad (18)$$

In addition to conditions (12) it is possible to set value V_s at $t = t_k$:

$$V_s = \omega_0 R, \quad (19)$$

what is equivalent at which the transversality condition $\lambda_2 = 0$ on function V_s in (18) will be superfluous. This case corresponds to loss by a stream circular speed components that has to correspond to the maximum return of kinetic energy on tangensalny a speed component on the water-wheel blade. At the same time the value V_x will remain uncertain with a necessary condition $V_x > 0$ for a liquid descent from the blade down a vertical for removal of liquid.

4. The problem solution and results

For numerical calculations of a system of the equations (8)-(11),(14)-(17) the final and differential algorithm with the Runge-Kutta method of the fourth order applied to a similar task of definition of a trajectory with optimum control [2] is used. Conditions (18) which performance is reached by separate iterative process are added to boundary conditions (12).

On the basis of the carried-out calculations optimum forms of blades at the set initial radius from R_0 to R_k turn out and the assigned altitude of a working part of the turbine x_k. The surfaces of blades with optimum characteristics of power return change depending on the assigned speed of rotation of the turbine ω_0, the axial and radial size of the blade (R_0, R_k, x_k), the directions and sizes of speed of the arriving liquid (V_{x0}, V_{s0}) and also existence of the field of inclination of g_0. Comparative calculations were carried out.

For test calculation the following introduction data were taken: ω_0 = 10 radian/s ÷ 20 radian/s, $V0 = 50$ m/s ÷100 m/s, R_0=1 m, R_k = 2 m, x_k = 2 m. The initial vector of speed (V_{x0}, V_{s0}) in the accompanying system of coordinates was coordinated from the tangensalny komponenty speed of the turbine at the radius *of* R equal $\omega_0 R$, i.e. $V_0 = (V_{x0}^2 + (V_{s0} - \omega_0 R)^2)^{1/2}$. So, at ω_0 = 20 radian/s, R = 1.5 m, V_0 = 50 m/s can put V_{x0} = 40 m/s and V_{s0} = 0 m/s, further for R = 2 m V_{s0} = 10 m/s and for R = 1 m - V_{s0} =-10 m/s turns out. For these data with x_k = 2 m and other sizes R (1 and 2 m) are carried out calculations with a step for time τ = 0.001 s. For them s(x) values for optimum trajectories of liquid for which at the exit the minimum absolute speeds of the movement turn out are given in table 1 at values of acceleration of gravity of g_0 $=0$ and 9.81 sq.m/s, as provides a maximum of transformation of kinetic energy of liquid to energy of rotation of the water-wheel at the set angular speed ω_0.

On the discrete values of size s(x,r) given in table 1 in the radial and axial directions it is possible to restore the optimum surface of the blade giving for the water-wheel the maximum return of kinetic and potential energy from currents of thin layers of liquid at rotation with the set angular speed which in this test example ω_0 = 20 radian/s.

Table 1. Results of calculation of an optimum surface of the blade of the turbine (size *s(x)*, *x* from 0 to 2 m) at values of radius of R = 1 m, 1.5 m, 2 m, angular speed ω_0 = 20 radian/s, entrance speed to the turbine *V0* = 50 m/s for two cases with acceleration of gravity (*g0* = 9.81 sq.m/s and 0 sq.m/s).

value *x*, м	ω_0 = 20 radian/s, V_0 = 50 m/s, g_0 = 9.81 sq.m/s				ω_0 = 20 radian/s, V_0 = 50 m/s, g_0 =0 sq.m/s			
	dependence of s(x,r), m on r, m			V, m/s at R=1.5 m	dependence of s(x,r), m on r, m			V, m/s at R=1.5 m
	1.0	1.5	2.0		1.0	1.5	2.0	
0.25	-0.11	0.05	0.09	50.12	-0.09	0.07	0.14	49.21
0.50	0.04	0.15	0.22	49.37	0.06	0.17	0.26	47.53
0.75	0.21	0.32	0.46	47.89	0.19	0.30	0.45	45.61
1.00	0.35	0.49	0.76	45.17	0.33	0.48	0.74	43.09
1.25	0.54	0,73	1.17	41.85	0.51	0,71	1.14	40.38
1.50	0.69	1.13	1.55	38.75	0.67	1.10	1.51	37.63
1.75	1.02	1.58	2.28	34.27	0.99	1.54	2.23	33.85
2.00	1.50	2.35	3.24	30.64	1.45	2.29	3.17	30.48
t_k, c :	.080	0.094	0.119	-	0.084	0.098	0.124	-

5. Conclusions

Approach is developed for numerical definition of a difficult surface of the working blade of the water-wheel for a current of a thin layer of ideal liquid on the basis of power optimization by the principle of a maximum of Pontryagin L. S.

Problem definition in two-dimensional and three-dimensional cases for currents of a thin layer of liquid on the water-wheel blade with carrying out test calculation of an optimum surface of blades of the water-wheel is considered. Thus, possibilities of numerical optimization at design of water-wheels with the improved characteristics when the maximum return of kinetic and potential energy liquid is possible at an economic expense in the form of thin layers of its current are shown.

Journal of Physics: Conference Series **1441** (2020) 012158 doi:10.1088/1742-6596/1441/1/012158

6. References

[1] Markov N M 1966 *The theory and calculation of the scapular device of axial turbomachines (Moscow:Mechanical engineering)* pp 239

[2] Trushlyakov V I and Panichkin A V 1982 The optimum movement of the aircraft with unregulated draft *J. RST* **3** 71-77

[3] Pontryagin L S, Boltyansky V G, Gamkrelidze R V, Mishchenko E F 1969 *Mathematical theory of optimum processes* (Moscow: *Science)* pp 384