Simulation of vehicle collisions at an unregulated intersection based on the Monte Carlo method

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Abstract. Currently advanced driver-assistance systems develop actively, which allow to signal to the driver about danger of collision of vehicles, necessary braking or changes of motion parameters of the car to prevent dangerous situations and accidents. The effectiveness of advanced driver-assistance systems from the parameters of their work. Therefore, the actual task is to set the optimal parameters of advanced driver-assistance systems. The values of such parameters in general can be both statistical and independent of the current road situation, and dynamic and depend on the current road situation. This problem cannot be solved without modeling dangerous situations, creating models of motion and collision risks, the results of which can be given recommendations for setting the parameters of the advanced driver-assistance systems. The aim of the work is statistical modeling collision vehicles on the unregulated crossroads and the evaluation dependency of probability collision from the parameters of the road situation. An analytical modeling of the movement of vehicles at the crossroads was used and a mathematical model of their collisions was built. With the help of statistical modeling a probability of their collision with different parameters road situation was rated. Received result can be used to configuration parameters management vehicle advanced driver-assistance systems, including dynamic.

1. Introduction

More than 3 thousand people around the world die every day as a result of road accidents and about 100 thousand are seriously injured. Annually from 20 million to 50 million people receive various injuries in road accidents and more than 1,25 million people (186 thousand from them children) become victims, this indicator remains practically invariable since 2007 (according to the World Health Organization). Among all the cases of road accidents, the intersection has an important role, for example, in Russia in the first half of 2018 at the crossroads there were 11640 accidents (16.7%) , while due to the mismatch of speed – 6 182 (8.9%), due to the exit to the oncoming lane – 5 874 accidents (8.4%). Thus, every sixth accident in Russia occurs at the intersection.

Nowadays advanced driver-assistance systems (ADAS) develop actively, which allow to signal to a driver about danger of collision of vehicles and the necessity braking or changes of motion parameters of the car to prevent dangerous situations and accidents. The effectiveness of ADAS depends on the parameters of their work. For example, the most important parameter is the alarm time [1-3]. On the one hand, a short alarm time can lead to the fact that a driver will not have time to change the parameters of the movement. On the other hand, a large alarm time leads to excessive driver tension. Therefore, the actual task is to set the optimal parameters of ADAS. The values of such parameters in general can be both statistical and independent of the current road situation, and dynamic and depend on the current road situation. This problem cannot be solved without modeling dangerous situations, creating models of motion and collision risks, the results of which can help to give recommendations for setting the parameters of the ADAS. A similar situation will be observed in the case of autonomous changes in the ADAS parameters of the vehicle.

According to [4] the existing traffic description and collision risk assessment models can be divided into 3 groups.

1. Models in which vehicle's movements are described only by the laws of physics;

2. Models describing the movement of vehicles considering the maneuvers that the drivers intend to perform;

3. Traffic models which take into account the interaction between vehicles during the maneuver.

It is concluded that the motion models based on physical laws effectively calculate the risk, but they are suitable for short-term collision predictions. Maneuver traffic models provide a more reliable assessment of long-term traffic and risk but are not always reliable because they do not take into account how vehicle movements change in different road situations. Traffic models that take into consideration the interactions between vehicles often have high computational complexity, so they are not always suitable for real-time risk assessment [4].

We highlight some of the work that use certain methods of describing the movement and collision risk assessment of vehicles. We also highlight some areas in the development of this area. Most of the work is devoted to collision avoidance of intelligent vehicles that exchange information with each other about their traffic parameters and location.

In [5], Brännström M. et al. presented a method for assessing the avoidance of collision with another moving vehicle at an intersection using steering, braking or acceleration. The analytical solution of the multidimensional problem of threat assessment at discrete moments of time was used. For each moment of time, the coordinates of the center of the car were compared with the coordinates of the upper left corner of the other car and the coordinates which are not appropriate for the center of the car were determined.

The possibility of using ordinary differential equations as dynamic constraints and values of maximum speeds in the form of statistical constraints to describe the movement of vehicles is shown in [6]. Based on the solution of the optimization problem the model optimizing movement of automatic vehicles at intersections was created [6].

In [7], Zhu F. et al. used linear programming for automatic intersection control (LPAIC), considering the dynamics of the connected vehicle. A two-level optimization model was obtained to distribute of traffic flows and take into account the dynamic departure time, the dynamic route selection and the automatic intersection control in the context of the optimal network model of the system [7].

It is important to note the usage of machine learning methods for modeling dangerous situations. So in [8], Katrakazas C. et al. used the neural network and its integration with traffic models regarding Bayesian structure to predict the risk of collision of automatic vehicles. In [9], Katrakazas C. et al. used microsimulation traffic and traffic conflicts generating by a surrogate model of the assessment of safety. Further, the outcomes of the situations were estimated using classifiers based on the support vector machine, K-nearest neighbors, and random forest. The authors conclude that it is reasonable to use traffic micro-simulation along with a surrogate safety assessment model to predict collisions in real time [9].

Some works are devoted to the visualization of vehicle movement and simulation of their collision. So in [10], Scurlock B. estimated the accuracy and the possibility of reconstructing vehicle collisions based on the momentum model and virtual collision. The information about the orientation of the vehicles prior to impact, the impact area, the trajectory after impact, the rest area, the rest orientation and the damage location of each vehicle was used. In [11], Knight M. simulated and visualized a vehicle collision with a corner wall in real time. Two methods for calculating the reaction to a collision were used: the Restitution method and the Kinetic energy loss method.

In [12], Chouhan A.P. & Banda G. proposed a method permission conflicts of vehicles for safe and fast crossing the intersection. For this it was proposed to use the central planner vehicles, for example, one of the cars, which performs calculations time of arrival of the vehicle in a certain point and a conflict point on his way using the equations of motion Newton. The conflict-free movement management at the unregulated intersection of vehicles that exchange information, also described in [13]. Let's note one work [14], where technology connection vehicle-to-vehicle (V2V) for the implementation of the side computers computing effective decentralized algorithms joint prevent collisions two vehicles on the intersections is described.

In [15], Lee K. & Kum D. suggested the system of prevention or mitigation of a collision and maneuvering of a vehicle to a safer area. Probable local trajectory of the movement which then let select the safest trajectory calculated using the relative position, speed and acceleration.

A number of works is dedicated to assessment of the security of motion and changes of traffic lane for intellectual vehicles in dynamic conditions driving. So, in [16], Zheng H. et al. suggested a model of planning movement paying attention to the longitudinal and lateral movement in the process of maneuvering and change of a road lane. In [17], Nie J. et al. suggested the decentralized cooperative structure of decision-making about changing the road lane for connected automatic vehicles.

In [18], Ahn H. & Vecchio D.D. introduced the supervisory algorithm for ADAS' systems, which prevents the side collision between vehicles at the crossroads.

Thus, a large part of the work is dedicated to the development of models describing the movements and the prevention of collision for intellectual vehicles that exchange information with each other. In this connection different mathematical apparatus is used. However, if there is a task to use one of these methods in ADAS in real time for autonomous vehicles, not associated with each other, many of the existing methods won't be suitable. Let's take into account that the configuration of parameters of management of a vehicle ADAS is the task. In this regard, it may be interesting to present the usage of statistical modeling and method Monte Carlo [19-20].

The aim of the work is statistical modeling collision of vehicles at the unregulated crossroads and the evaluation dependency of probability collision from the parameters of the road situation. Let's use the analytical modeling of the movement of vehicles at the crossroads and build a mathematical model of their collisions. The one can evaluate the probability of their collision with different parameters of road situation with the help of statistical modeling. The received result can be used to configurate parameters of management a vehicle ADAS, including dynamic one.

2. Simulation of the situation in the unregulated intersection

Consider the following model road of the situation in the unregulated intersection (see figure 1) The vehicle VA length l^A and width b^A is moving with a speed V^A at the traffic lane width w^A in the direction of the intersection. Its movement is carried out along the X-axis in the negative direction. The vehicle VB length l^B and width b^B is moving with the speed V^B at the road the lane width w^B to the junction perpendicular to the vehicle VA. It moves along the Y-axis in the negative direction. The intersection points of the X, Y axes will be the origin of the coordinate system (0; 0). We assume that the X and Y axes are in the center of the roadway. Suppose that at the initial time $t = 0$ vehicle VA is at a distance S^A from the intersection, and vehicle VB is at a distance S^B from the intersection. At this point in time, drivers can see a potentially dangerous situation and decide to change the parameters of the movement of their vehicles.

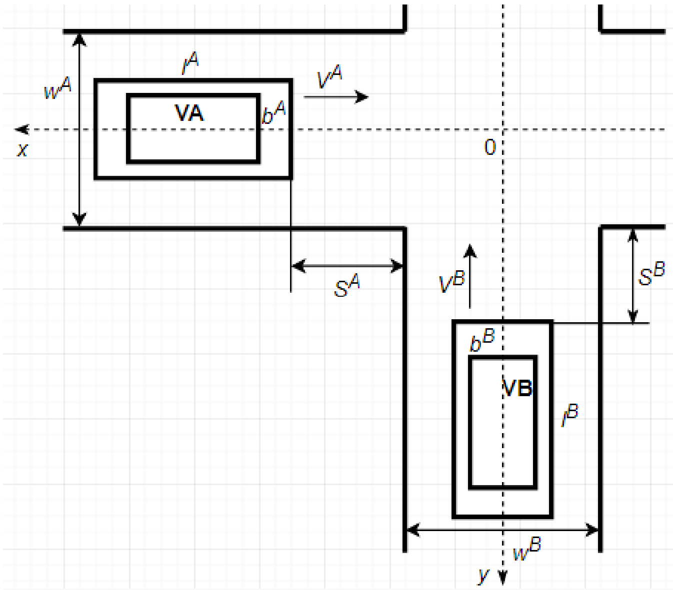


Figure 1. The road situation.

Let us denote the coordinates of the center point of the front bumper of the vehicle VA as (x^A; y^A), and the coordinates of the center point of the front bumper of the vehicle VB as (x^B; y^B). Then at the initial time $t = 0$ ($x^A(0)$; $y^A(0)$) = (S^A; 0), and ($x^B(0)$; $y^B(0)$) = (0; S^B). Given the size of the vehicles, we can conclude that if at some point in time $t = t_{col}$ condition

$$\begin{cases} -\left(b^B/_2 + l^A\right) < x^A(t_{col}) < b^B/_2, \\ -\left(b^A/_2 + l^B\right) < y^B(t_{col}) < b^A/_2, \end{cases} \tag{1}$$

is met, then there will be a collision of vehicles at this point in time. Accordingly, to assess the presence of a collision of vehicles, it is necessary to determine the coordinates x^A, y^B and check the condition (1) at each time. To determine the coordinates x^A, y^B, it is necessary to consider the parameters of the movement of vehicles.

Movement of vehicles will depend on the perception of danger this situation the drivers. They can decide either about the change of the parameters of the movement of his vehicle, particularly the braking, or about leaving them without changing. Based on the above there are possible 4 case of the development of events.

1. Only the driver of vehicle VA will start braking. In this case, the driver will press the brake after a time interval t_r^A of reaction and decision. The brake system will be actuated after the t_t^A delay time interval of the brake actuator operation. Since the braking value will not be set to the maximum value instantly, we introduce the t_d^A time interval of the deceleration rise. We assume that the braking value increases linearly. As a result, braking of vehicle VA will start at a time interval $t_r^A + t_t^A + {t_d^A}/{2}$. At the end of this time interval, the deceleration value of vehicle VA will be set to d^A. This value can be determined by the formula:

$$d^A = g\frac{K_{ch}^A}{K_e^A},$$

where g is the acceleration of gravity (9,8 m/s^2); K_{ch}^A is the coefficient of adhesion between tyre of vehicle the VA and a road; K_e^A – coefficient of efficiency of braking of the vehicle VA serves to reconcile the theoretical calculations and operational data and taking into account the unequal action of the wheel relative to the applied loads on them, and also technical condition of the braking system.

We can record changes in the coordinates $x^A(t)$, $y^B(t)$:

$$\begin{cases} x^A(t) = S^A + \dfrac{w^B}{2} - V^A t, \quad if \quad 0 \le t \le t_r^A + t_t^A + \dfrac{t_d^A}{2}, \\ x^A(t) = S^A + \dfrac{w^B}{2} - V^A\left(t_r^A + t_t^A + \dfrac{t_d^A}{2}\right) - \left(V^A\left(t - \left(t_r^A + t_t^A + \dfrac{t_d^A}{2}\right)\right) - \dfrac{d^A}{2}\left(t - \left(t_r^A + t_t^A + \dfrac{t_d^A}{2}\right)\right)^2\right), \\ \qquad\qquad if \quad t_r^A + t_t^A + \dfrac{t_d^A}{2} < t \le t_r^A + t_t^A + \dfrac{t_d^A}{2} + \dfrac{V^A}{d^A}, \\ x^A(t) = S^A + \dfrac{w^B}{2} - V^A\left(t_r^A + t_t^A + \dfrac{t_d^A}{2}\right) - \dfrac{\left(V^A\right)^2}{2d^A}, \quad if \quad t > t_r^A + t_t^A + \dfrac{t_d^A}{2} + \dfrac{V^A}{d^A}, \\ y^B(t) = S^B + \dfrac{w^A}{2} - V^B t. \end{cases}$$

2. Only the driver of the car VB will start braking. The movement of car VB will be similar to the movement of car VA in the considered case 1, and the movement of car VA will be similar to the movement of car VB in case 1. To describe the movement of car VB, we will use indicators similar to those used in case 1 for car VA: the time interval t_r^B of reaction and decision, the time interval t_t^A of

the delay of the brake actuator operation, the time interval t_d^A of the deceleration rise, the value of the deceleration d^B, the coefficient K_{ch}^B of tire traction; the coefficient K_e^B of braking efficiency.

We can record changes in the coordinates $x^A(t)$, $y^B(t)$:

$$\begin{cases} x^A(t) = S^A + \dfrac{w^B}{2} - V^A t, \\ y^B(t) = S^B + \dfrac{w^A}{2} - V^B t, \quad if \quad 0 \le t \le t_r^B + t_t^B + \dfrac{t_d^B}{2}, \\ y^B(t) = S^B + \dfrac{w^A}{2} - V^B\left(t_r^B + t_t^B + \dfrac{t_d^B}{2}\right) - \left(V^B\left(t - \left(t_r^B + t_t^B + \dfrac{t_d^B}{2}\right)\right) - \dfrac{d^B}{2}\left(t - \left(t_r^B + t_t^B + \dfrac{t_d^B}{2}\right)\right)^2\right), \\ \qquad\qquad if \quad t_r^B + t_t^B + \dfrac{t_d^B}{2} < t \le t_r^B + t_t^B + \dfrac{t_d^B}{2} + \dfrac{V^B}{d^B}, \\ y^B(t) = S^B + \dfrac{w^A}{2} - V^B\left(t_r^B + t_t^B + \dfrac{t_d^B}{2}\right) - \dfrac{\left(V^B\right)^2}{2d^B} \quad if \quad t > t_r^B + t_t^B + \dfrac{t_d^B}{2} + \dfrac{V^B}{d^B}. \end{cases}$$

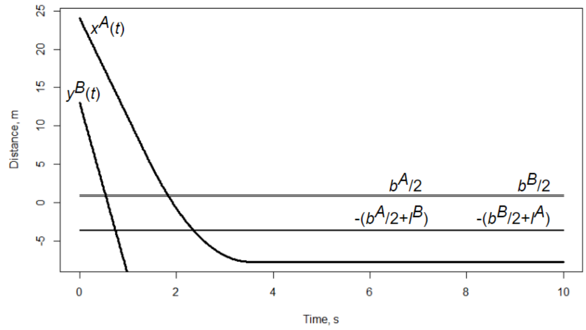
3. Drivers of cars VA and VB will start braking. Similar to cases 1 and 2, we can record changes in the coordinates $x^A(t)$, $y^B(t)$:

$$\begin{cases} x^A(t) = S^A + \dfrac{w^B}{2} - V^A t, \quad if \quad 0 \le t \le t_r^A + t_t^A + \dfrac{t_d^A}{2}, \\ x^A(t) = S^A + \dfrac{w^B}{2} - V^A\left(t_r^A + t_t^A + \dfrac{t_d^A}{2}\right) - \left(V^A\left(t - \left(t_r^A + t_t^A + \dfrac{t_d^A}{2}\right)\right) - \dfrac{d^A}{2}\left(t - \left(t_r^A + t_t^A + \dfrac{t_d^A}{2}\right)\right)^2\right), \\ \qquad\qquad if \quad t_r^A + t_t^A + \dfrac{t_d^A}{2} < t \le t_r^A + t_t^A + \dfrac{t_d^A}{2} + \dfrac{V^A}{d^A}, \\ x^A(t) = S^A + \dfrac{w^B}{2} - V^A\left(t_r^A + t_t^A + \dfrac{t_d^A}{2}\right) - \dfrac{\left(V^A\right)^2}{2d^A}, \quad if \quad t > t_r^A + t_t^A + \dfrac{t_d^A}{2} + \dfrac{V^A}{d^A}, \\ y^B(t) = S^B + \dfrac{w^A}{2} - V^B t, \quad if \quad 0 \le t \le t_r^B + t_t^B + \dfrac{t_d^B}{2}, \\ y^B(t) = S^B + \dfrac{w^A}{2} - V^B\left(t_r^B + t_t^B + \dfrac{t_d^B}{2}\right) - \left(V^B\left(t - \left(t_r^B + t_t^B + \dfrac{t_d^B}{2}\right)\right) - \dfrac{d^B}{2}\left(t - \left(t_r^B + t_t^B + \dfrac{t_d^B}{2}\right)\right)^2\right), \\ \qquad\qquad if \quad t_r^B + t_t^B + \dfrac{t_d^B}{2} < t \le t_r^B + t_t^B + \dfrac{t_d^B}{2} + \dfrac{V^B}{d^B}, \\ y^B(t) = S^B + \dfrac{w^A}{2} - V^B\left(t_r^B + t_t^B + \dfrac{t_d^B}{2}\right) - \dfrac{\left(V^B\right)^2}{2d^B} \quad if \quad t > t_r^B + t_t^B + \dfrac{t_d^B}{2} + \dfrac{V^B}{d^B}. \end{cases}$$

4. None of the drivers will slow down. In this case, the changes of $x^A(t)$, $y^B(t)$ will be written as follows:

$$\begin{cases} x^A(t) = S^A + {w^B}/{2} - V^A t, \\ y^B(t) = S^B + {w^A}/{2} - V^B t. \end{cases}$$

Consider for example 2 traffic situations with different speeds of movement of vehicles and decision making in them. Let the first situation corresponds to case 1 and the second situation corresponds to option 3. We will carry out a simulation of the movement of vehicles at each time t and visualize the results. To do this, we postpone the time along the abscissa axis and we postpone the values of the coordinates $x^A(t)$, y^B(t) along the ordinate axis. Additionally, apply the values -($b^B/2+l^A$) and $b^B/2$ and -($b^A/2+l^B$) and $b^A/2$ (see figure 2). It can be seen from figure 2,a that in this situation the collision will not occur, because there is no point in time for which both the point of the curve $x^A(t)$ is between -($b^B/2+l^A$) and $b^B/2$, and the point of the curve y^B(t) is between -($b^A/2+l^B$) and $b^A/2$. In figure 2,b the point of curve $x_A(t)$ is between -($b^B/2+l^A$) and $b^B/2$, and the point of curve y^B(t) is between -($b^A/2+l^B$) and $b^A/2$ simultaneously at the time of t_{col} indicating the collision of vehicles at that time.



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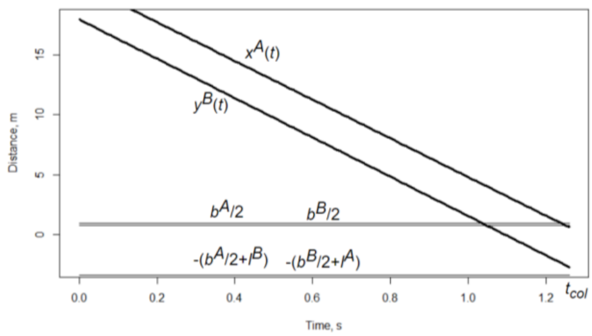

b

Figure 2. The simulation of the movement of vehicles.

3. Statistical modelling of the probability of collision of vehicles

To determine the probability of collision of vehicles, we will conduct statistical modeling using the Monte Carlo method. To do this, we will test the created model with a set of random signals with given probability densities. As a result of each experiment, we will fix two outcomes: the presence or absence of a collision. Then the probability P_{col} of collision will be defined as the ratio of the number N_{col} of collisions in N experiments to the number N:

$$P_{col} = \frac{N_{col}}{N}.$$

As random signals we will use the values of the model parameters generated by the uniform distribution law in the given ranges: V^A = [50; 100] km/h, V^B = [40; 80] km/h, d^A = [4.0; 6.5] m/s^2, d^B = [4.0; 6.5] m/s^2, b^A = [1.6; 2.1] m, b^B = [1.6; 2.1] m, l^A = [2.2; 2.8] m, l^B = [2.2; 2.8] m, t^A_r = [0.3; 1.5] s, t^A_t = [0.3; 1.5] s, t^A_d = [0.1; 0.6] s, t^B_r = [0.1; 0.6] s, t^B_t = [0.2; 0.8] s, t^B_d = [0.2; 0.8] s, w^A = [3.0; 3.75] m, w^B = [3.0; 3.75] m, S^A = [30; 60] m, S^B = [20; 40] m.

We define the probability of collision in each of the four options:

$$P_{col}^{1} = 0.189,\ P_{col}^{2} = 0.211,\ P_{col}^{3} = 0.194,\ P_{col}^{4} = 0.226.$$

Assuming that the appearance of variants is equally probable, we determine the probability of collisions in this case:

$$P_{col} = 0.205.$$

When determining the probabilities in each case, we will conduct $N = 10000$ experiments. Verification of the sufficiency of the number of experiments N for the given error ε and the confidence probability Q can be estimated using the inverse function F^{-1} Laplace formula:

$$N = \frac{P_{col}\left(1 - P_{col}\right)}{\varepsilon^2}\left(F^{-1}(Q)\right)^2 .$$

For the error value $\varepsilon = 0.01$ and the confidence probability $Q = 0.99$, at which $F^{-1}(Q = 0.99) = 2.58$, this number of experiments will be sufficient.

4. Determination of dependencies of collision probabilities on various parameters of road systems

One of the tasks of car collision modeling is to obtain dependences of the collision probability on various parameters of the road system (see figure 3 and figure 4). The usage of these dependencies allows you to consider the impact of various parameters on traffic safety and consider them when developing and configuring the parameters of ADAS.

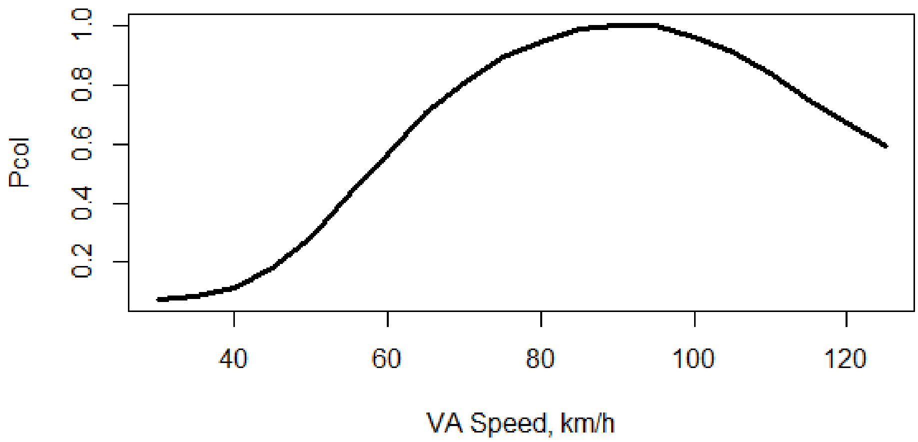

Figure 3. The dependence of the probability of the collision of VA and VB on the speed of vehicle VA.

It can be seen from figure 3 that the collision probability increases to the speed of car VA of 85 km/h, then does not change much, and then decreases. It can be seen from figure 4 that the collision probability increases to the distance of car VA from the intersection $t = 0$ of 55-60 m, and then decreases. Other dependencies can be built and analyzed similarly.

Using the dependencies obtained, you can customize how ADAS works. The combination of such dependencies will allow to find the area of safe functioning of the road system.

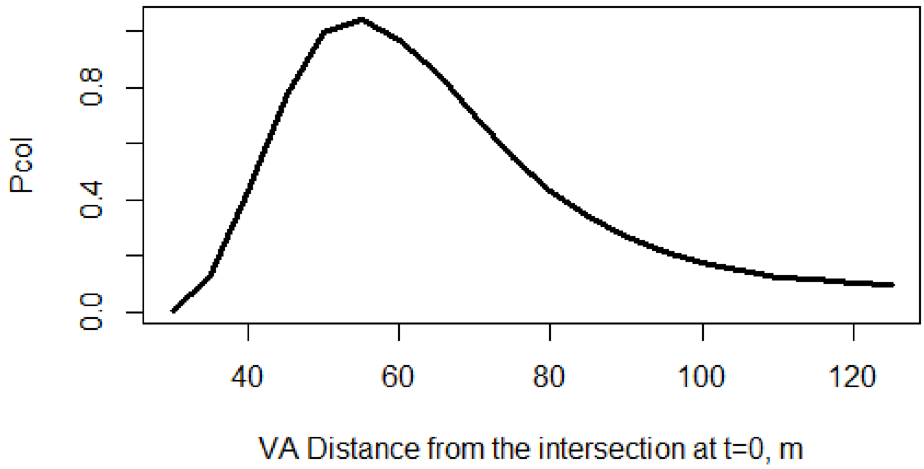

Figure 4. The dependence of the probability of the collision of VA and VB on the distance of vehicle VA from the intersection at $t = 0$.

5. Conclusion

In this paper, we have presented statistical modeling collision vehicles on the unregulated crossroads and the evaluation dependency of probability collision from the parameters of the road situation. The mathematical model of vehicles' movements and their collision at the crossroads was created. As a result, graphs of the coordinates of the vehicles during the collision and if there is no collision were obtained, and also a plot of the probability of collision to vehicle speed A at a fixed speed of vehicle B was obtained. So the distances from each car to the intersection within 10-20 meters were used for the simulation, that is why this method of determining the probability of collision can be used in ADAS systems to detect a dangerous situation and timely signaling about it to the driver of the vehicle.

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