COMPUTING FUZZY INTEGRAL OF THE BASIS OF FUZZY MESURE

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Abstract. The fuzzy integral of a function with a fuzzy measure is a general estimate in the form of a nonlinear convolution of particular estimates of information elements, and the possibility of interconnection of information elements is not excluded. Here is an approach that allows the expert to best formalize his fuzzy ideas, transforming the language of words into a language of quantitative estimates.

Introduction

A large class of complex systems and processes, including modern information and communication systems, is characterized by integration, multilevel, distributed and a variety of performance indicators. In reality, the design of such systems, the evaluation of the quality of their structural and functional characteristics, and the management of the processes, [1].

In particular, such uncertainties include fuzzy uncertainty, characterized by incompleteness, inaccuracy and linguistic vagueness (indistinctness) present in the initial information, criteria and assessments of customers and developers, as well as in the models and procedures used to describe and evaluate alternatives to the analyzed variants of objects and their states. The necessity to take into account several criteria, including the preferences of decision-makers (decision makers), also characterizes one of the conditions of uncertainty in the process of choosing the optimal options. This makes it expedient to develop and use models and methods for describing and evaluating options (alternatives) for the analyzed objects, as well as making decisions (MD) for choosing the best option under fuzzy uncertainty, which is a special class of PR problems, called unstructured or weakly structured [2]. In such tasks, alternatives to decisions are evaluated on the basis of an analysis of soft estimates of the performance indicators of the results of implementation of decisions (outcomes) and the values of the risks of losses corresponding to certain outcomes of decisions. The theoretical and methodological device for solving such problems is the means of intellectual information technology "Soft Computing" – "Soft Computing" [3–7].

In this paper, it consider fuzzy-multiple approaches to constructing models for describing and evaluating alternatives, as well as the tasks of adopting weakly structured solutions (AWSS) under conditions of fuzzy uncertainty.

The main goal of the present work is to develop an algorithm for selecting breeding varieties of cotton with the best biological and technological indicators in conditions of indistinctly given initial information.

And under such initial conditions, it is required to choose the most acceptable alternative: variety for given seeding conditions, cultivation (agrotechnological regimes, fertilizer dose components, irrigation, boundary conditions for these varieties and soil types).

The choice of the alternative can be done with a fuzzy integral. To this end, we give some definitions.

2. A fuzzy measure

We introduce the definitions of the main elements of the problem under consideration, a fuzzy measure.

Let be $\Gamma = \{x_1, x_2, ..., x_n\}$ – a set of elements $x_1, x_2, ..., x_n$. The construction of the set Γ depends on the specific task. For example, in the problem under consideration, the elements of this set can display the type of cotton variety or their yield under different conditions of sowing. In set-setve Γ construct all possible subsets-tion $\beta(\Gamma)$. For example, for a sets $\Gamma = \{x_1, x_2, x_3\}$ elements $\beta(\Gamma)$ are: $\boldsymbol{A}_1 = \{x_1\}$; $\boldsymbol{A}_2 = \{x_2\}$; $\boldsymbol{A}_3 = \{x_3\}$; $\boldsymbol{A}_4 = \{x_1, x_2\}$; $\boldsymbol{A}_5 = \{x_1, x_3\}$; $\boldsymbol{A}_6 = \{x_2, x_4\}$; $\boldsymbol{A}_7 = \Gamma = \{x_1, x_2, x_4\}$; $A_8 = \{\varnothing\}$.

The main property of a set $\beta(\Gamma)$ is that it is closed with respect to the operations of union, addition and intersection: $A_i \bigcup A_j \in \beta(\Gamma)$, $\bar{A}_i \in \beta(\Gamma)$, $A_i \bigcap A_j \in \beta(\Gamma)$. For example, $A_1 \bigcup A_2 = A_4 = \{25;30\} \in \beta(\Gamma)$.

In the sets $\beta(\Gamma)$ a fuzzy measure is introduced, defined as follows.

Definition 1. A fuzzy measure is a set function g defined on a set and $\beta(\Gamma)$ satisfying the following conditions [3, 13]:

1. Limited. $g(\varnothing) = 0$, $g(\Gamma) = 1$; $\Gamma = \{x_1, x_2, ..., x_n\}$;
2. Monotony. If then $g(A) \leq g(B)$ do $g(A) \leq g(B)$;
3. Continuity. Let the sequence, $\{A_i\} \in \beta(\Gamma)$, $1 \leq i \leq \infty$, then if then $A_i \supseteq A_z \supseteq .....A_n \supseteq ...$, do $g(\bigcap_{i=1}^{\infty} A_i) = \lim_{i\to\infty} g(A_i))$.

The triple (T, $\beta(\Gamma)$, g) is called a fuzzy measure space.

M. Sugeno proposed to construct a fuzzy join measure as follows $g(A \cup B)$ [3-4]:

$g(A \cup B) = g(A) + g(B) + \lambda g(A) g(B)$ here the parameter takes values in the interval, $-1 < \lambda < \infty$, $A \bigcap B = \varnothing$. The above expression is called a rule, and a fuzzy measure, respectively, is a measure of λ.

We consider in more detail the special case when the finite set $\Gamma = \{x_1, x_2, ..., x_n\}$ is used as Γ. The fuzzy measure g_λ in this case will be built on the basis of fuzzy density - measures. We denote it by $g(x_i) = g_\lambda(\{x_i\})$, $i = 1,2\ldots,n$. Further, we use the notation $g_i = g(x_i)$.

Assuming that fuzzy densities are given, $0 \leq g_i \leq 1$ the measure g_λ is constructed according to the λ-rule: $g_\lambda(\{x_1, x_j\}) = g_i + g_j + \lambda g_i g_i$. Summarizing, we can write that: $g_\lambda(\{x_1...x_k\}) = \sum_j^k g_i + \lambda \sum_{i_1=1}^{k-1} \sum_{i_2=i_1+1}^{k} g_{i_1} g_{i_2} + ... + \lambda^{k-1} g_1 g_2 ....g_k$. Or in the equivalent form:

$$g_\lambda(\{x_1, ..., x_k\}) = \begin{cases} \frac{1}{\lambda}(\prod_{i=1}^{k}(1+\lambda g_i) - 1), \lambda \neq 0), \\ \sum_{i+1}^{k} g_i, \quad \lambda = 0. \end{cases}$$

If $-1 < \lambda < 0$, то $\sum_{i+1}^{n} g_i > 1$, and in the case of $-1 < \lambda < \infty$, ever $\sum_{i=1}^{n} g_i < 1$.

In addition, if the value of any one fuzzy density is equal to one, then the values of the other fuzzy densities are always zero, i.e. if there exists an x_j such that $g = 1$, then for each, necessarily, $g_i = 0$.

In view of the foregoing, we describe the algorithm for constructing a fuzzy measure:

Step1. From the normalization $\frac{1}{\lambda}(\prod_{i11}^{n}(1+\lambda g_i)-1)=1$ condition, we calculate the parameter λ;

Step 2. The measure of any set is $A \in \beta(\Gamma)$ determined from the relation $g_\lambda(A)=\frac{1}{\lambda}(\prod_{x_i \in A}(1+\lambda g_i)-1)$ that satisfies the λ-Sugeno rule.

Definition2. Using the concepts and algorithm for constructing a fuzzy measure, we give the definition of a fuzzy integral Sugeno.
The fuzzy Sugeno integral of the function h on the set U is determined by fuzzy measure g by the expression [3].

$$\int h \circ g = \sup_\alpha(\alpha \wedge g(F_\alpha)),\ F_\alpha = \{x \in U : h(x) \geq \alpha\},\ \alpha \in (0,1].$$

Here is the symbol $\int$ means fuzzy integral, small circle $\circ$ - composition mark.

Note that if the integration is performed on the set $A \subseteq U$, then the fuzzy integral is determined by the expression

$$\int_A h(x) \circ g = \sup_\alpha(\alpha \wedge g(A \cap F_\alpha)),$$
$$\alpha \in (0,1].$$

We introduce the definition of a fuzzy integral for the case $U = \{x_1, x_2, \dots x_n\}$. If $h(x_1) \leq \dots \leq h(x_n)$ then the fuzzy integral is determined by the expression.

$$\int h(x) \circ g = \bigcup_{i=1}^{n}(h(x_i) \wedge g(E_i)),$$
$$E_i = \{x_i, \dots, x_n\},$$
$$\bigcup_{i=1}^{n} a_i = \max_i\{a_1\}$$

If $h(x_1) \geq \dots \geq h(x_n)$, then the fuzzy integral is determined by the expression $\int h(x) \circ g = \bigcup_{i=1}^{n} h(x_i) \wedge g(E_i),\quad E_i = \{x_1, \dots, x_i\}\cdot$

Thus, a fuzzy integral of the function h with respect to a fuzzy measure g is a general estimate in the form of a nonlinear convolution of particular estimates of information elements, and the possibility of interconnecting information elements is not excluded [13].

3. Computing a fuzzy integral

The calculation of a fuzzy integral consists of two parts: the definition of a parameter λ and the calculation of the integral itself from a given fuzzy density.

Definition 3. The fuzzy Sugeno integral of a function h on a set U with respect to a fuzzy measure g is defined by the expression [3]
$F_\alpha = \{x \in U : h(x) \geq \alpha\},\ \alpha \in (0,1].$ Here the symbol $\int$ means a fuzzy integral, a small circle is the $\circ$-sign of the composition.

Note that if integration is performed on a set $A \subseteq U$, then the fuzzy integral is determined by expression

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$$\int_A h(x) \circ g = \sup_\alpha (\alpha \wedge g(A \cap F_\alpha)),\ \alpha \in (0,1]$$

We introduce the definition of a fuzzy integral for the case $U = \{x_1, x_2, \ldots x_n\}$. If $h(x_1) \le \ldots \le h(x_n)$, then the fuzzy integral is determined by expression

$$\int h(x) \circ g = \bigcup_{i=1}^{n} (h(x_i) \wedge g(E_i)),\ \ E_i = \{x_i, \ldots, x_n\},\ \bigcup_{i=1}^{n} a_i = \max_i \{a_1\}\ .$$

If $h(x_1) \ge \ldots \ge h(x_n)$, then the fuzzy integral is determined by expression

$$\int h(x) \circ g = \bigcup_{i=1}^{n} h(x_i) \wedge g(E_i),\ \ E_i = \{x_1, \ldots, x_i\}\ .$$

Thus, the fuzzy integral of the function *h* by a fuzzy measure g is a general estimate in the form of a nonlinear convolution of particular estimates of information elements, and the possibility of interconnection of information elements is not excluded.

The calculation of the fuzzy integral consists of two parts: the determination of the parameter λ and the calculation of the integral itself with respect to a given fuzzy density.

The parameter λ by definition [3] belongs to the domain $(-1, \infty)$. Equality means $\lambda = 0$, that is fulfilled additivity condition $\sum_j g(d_j) = 1$, this is corresponds to a probability measure. In the case when the arithmetic sum of fuzzy densities $\sum_j g(d_j) \in (0,1), or > 1,\ \ then\, \lambda \ne 0.$

The numerical value of the parameter is λ from the solution of equation

$$\frac{1}{\lambda}\left[\prod_{i=1}^{m}(1+\lambda \cdot (\omega_i)) - 1\right] = 1,\quad m = 2^N\ .$$

4. Computing experiment

Suppose given:

- A sets of alternatives (selection varieties of the cotton depending on the type of soil and the application of fertilizers);

- feature sets (biological and technological characteristics, which are used to select an acceptable sort); acceptable variety

The experiment was carried out for the selection problem of four selection sorts: C-4727, Tashkent 1, 159-F, 108-F cotton ($X = \{x_1, x_2, \ldots, x_4\}$) better in the following characteristics ($P = \{p_1, p_2, \ldots, p_4\}$): yield, fiber length, fiber strength, seed oil [2].

The importance of each feature is given and expressed through fuzzy densities

$$g_1 = 0,66,\ \ \mathbf{g_2 = 0,89},\ \ \mathbf{g_3} = 0,96,\ \ g_4 = 0,93$$

$$h_1 = 0,19,\ \ h_2 = 0,21,\ h_3 = 0,22,\ \ h_4 = 0,24$$

$$g_\lambda(x_1, x_2, x_3, x_4) = 1.$$

$$g_1 g_2 g_3 g_4 \lambda^3 + (g_1 g_2 g_3 + g_1 g_2 g_4 + g_1 g_3 g_4 + g_2 g_3 g_4)\lambda^2 +$$
$$(g_1 g_2 + g_1 g_3 + g_1 g_4 + g_2 g_3 + g_2 g_4 + g_3 g_4)\lambda + g_1 + g_2 + g_3 + g_4 = 1.$$

$$0,524\lambda^3 + 2,49\lambda^2 + 4,409\lambda + 2,44 = 0.$$

$$\lambda^3 + 4,75\lambda^2 + 8,41\lambda + 4,66 = 0.$$

$$\lambda = -0{,}96 .$$

$$g_\lambda(x_1, x_2, x_3) = g_1 g_2 g_3 \lambda^2 + (g_1 g_2 + g_1 g_3 + g_2 g_3)\lambda + g_1 + g_2 + g_3 =$$
$$-0{,}96^2 \times 0{,}66 \times 0{,}89 \times 0{,}96 - (0{,}66 \times 0{,}89 + 0{,}66 \times 0{,}96 + 0{,}89 \times 0{,}96) \times 0{,}96$$
$$+0{,}66 + 0{,}89 + 0{,}96 = 1{,}03$$

$$g_\lambda(x_1, x_2, x_4) = g_1 g_2 g_4 \lambda^2 + (g_1 g_2 + g_1 g_4 + g_2 g_4)\lambda + g_1 + g_2 + g_4 =$$
$$-0{,}96^2 \times 0{,}66 \times 0{,}89 \times 0{,}93 - (0{,}66 \times 0{,}89 + 0{,}66 \times 0{,}93 + 0{,}89 \times 0{,}93) \times 0{,}96$$
$$+0{,}66 + 0{,}89 + 0{,}93 = 1{,}04$$

$$g_\lambda(x_1, x_3, x_4) = g_1 g_3 g_4 \lambda^2 + (g_1 g_3 + g_1 g_4 + g_3 g_4)\lambda + g_1 + g_3 + g_4 =$$
$$-0{,}96^2 \times 0{,}66 \times 0{,}96 \times 0{,}93 - (0{,}66 \times 0{,}96 + 0{,}66 \times 0{,}93 + 0{,}96 \times 0{,}93) \times 0{,}96$$
$$+0{,}66 + 0{,}96 + 0{,}93 = 1{,}05$$

$$g_\lambda(x_2, x_3, x_4) = g_2 g_3 g_4 \lambda^2 + (g_2 g_3 + g_2 g_4 + g_3 g_4)\lambda + g_2 + g_3 + g_4 =$$
$$-0{,}96^2 \times 0{,}89 \times 0{,}96 \times 0{,}93 - (0{,}89 \times 0{,}96 + 0{,}89 \times 0{,}93 + 0{,}96 \times 0{,}93) \times 0{,}96$$
$$+0{,}89 + 0{,}96 + 0{,}93 = 1{,}042$$

$$g_\lambda(x_1, x_2) = g_1 g_2 \lambda + g_1 + g_2 = -0{,}96 \times 0{,}66 \times 0{,}89 + 0{,}66 + 0{,}89 = 0{,}99 ,$$

$$g_\lambda(x_1, x_3) = g_1 g_3 \lambda + g_1 + g_3 = -0{,}96 \times 0{,}66 \times 0{,}96 + 0{,}66 + 0{,}96 = 1{,}02$$

$$g_\lambda(x_1, x_4) = g_1 g_4 \lambda + g_1 + g_4 = -0{,}96 \times 0{,}66 \times 0{,}93 + 0{,}66 + 0{,}93 = 1{,}01$$

$$g_\lambda(x_2, x_3) = g_2 g_3 \lambda + g_2 + g_3 = -0{,}96 \times 0{,}89 \times 0{,}96 + 0{,}89 + 0{,}96 = 1{,}03$$

$$g_\lambda(x_2, x_4) = g_2 g_4 \lambda + g_2 + g_4 = -0{,}96 \times 0{,}89 \times 0{,}93 + 0{,}89 + 0{,}93 = 1{,}02$$

$$g_\lambda(x_3, x_4) = g_3 g_4 \lambda + g_3 + g_4 = -0{,}96 \times 0{,}96 \times 0{,}93 + 0{,}96 + 0{,}93 = 1{,}05$$

$$h_1 = 0{,}19, \quad h_2 = 0{,}21, \; h_3 = 0{,}22, \; h_4 = 0{,}24 .$$

$$i{=}1\text{: } h(x_1) \wedge g(x_1, x_2, x_3, x_4) = 0{,}19 \wedge 1.0 = 0{,}19 ,$$

$$i{=}2\text{: } h(x_2) \wedge g(x_2, x_3, x_4) = 0{,}21 \wedge 1{,}042 = 0{,}21 ,$$

$$i{=}3\text{: } h(x_3) \wedge g(x_3, x_4) = 0{,}22 \wedge 1{,}05 = 0{,}22 ,$$

$$i{=}4\text{: } h(x_4) \wedge g(x_4) = 0{,}24 \wedge 0{,}93 = 0{,}24 ,$$

$$\int h \circ g = \bigcup_{i=1}^{4} (h(x_i) \wedge g(E_i)) = \max(0{,}19;\ 0{,}21;\ 0{,}22;\ 0{,}24) = 0{,}24$$

$$x_4 = 0{,}24 .$$

Thus, the results of the ranking of all selection sorts showed that variety 108-F is the best among the offered selection sorts of cotton, because the resulting value of the degree of belonging of this variety to the fuzzy set is the largest (0.24).

Conclusion

The algorithm for solving the problem of selection using a fuzzy measure and a fuzzy integral is considered on the example of choosing the best cotton variety that provides optimal values of agrotechnological parameters in various conditions: sowing, growing, vegetation and harvesting. The proposed

algorithm is an addition to the fuzzy-set algorithms of the PSSR considered in [3, 13] in a fuzzy environment.
A promising direcion of research in this area is the development of methods for solving problems of the AWSS using a combination of "Soft Computing" technologies: fuzzy sets, neural networks, genetic algorithms, evolutionary modeling and programming. It is advisable to involve in practice the methods of analysis based on obtaining qualitative data estimates that would be based on modern world trends in economic science, one of which is rightly considered the direction of soft computing (Fuzzy technology).

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