

Application of Hermite splines for load movement on a flexible crane suspension through a curvilinear trajectory

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Abstract. Analytical expressions of the coordinates of the load, the suspension point and the rope deviation angle, as well as their first three time derivatives were obtained by setting the desired load movement trajectory on a pendulum type flexible crane suspension using two-point Hermite splines with the quartic highest derivatives. A well-known mathematical model of the oscillatory system described by a linearized differential equation was used. With the described movements, the uncontrollable pendulum oscillations of the load are absent. The load moves exactly along a reference trajectory. The superposition of load movements in two mutually perpendicular planes solves the problem of synthesising the suspension point trajectory, which ensures the movement of load along an arbitrary smooth curvilinear trajectory in the horizontal plane. The second horizontal coordinate of the load was represented as an interpolation polynomial from the first horizontal coordinate. The division of the movement trajectory along the axis of the second horizontal coordinate into several sections of the same length provide the determination of the load movement and its derivatives at reference points, as well as the calculated suspension point trajectory. The developed technique is promising for application in intelligent mechatronic control systems for travelling and bridge-travelling cranes.

Keywords: Hermite spline, load swinging, reference trajectory, pendulum suspension, travelling crane, oscillations, forced motion.

1. Introduction

In recent decades, a large number of studies have been devoted to a search for effective methods of automatic control of the moving links of travelling cranes (TC) [1] in order to provide partial or complete limitation (damping) of uncontrolled oscillations of the load on a flexible suspension [2]. Uncontrolled oscillations with non-optimal (including manual) control of the TC are characteristic both of the process of load moving and after stopping the moving links. They reduce the efficiency and safety of the TC application [3].

The group of methods for controlling the movement of TC open-loop moving links require no additional feedback sensors tracking the actual deviation angle of the elevator rope from the vertical or the coordinates of the load on a real object. Within this group of methods, two main approaches can be distinguished: 1) using input formers [4] particularly used in controllers of automatic control systems for links of real cranes [5] and in mathematical modelling of their movements [6]; 2) trajectory



planning [7], which can be applied both for lightly loaded systems [8] and systems with uncertainties [9].

In a number of works, the mathematical apparatus of fuzzy logic [10], including the adaptive one [11], and the apparatus of neural networks [12] are used to limit the oscillations of the TC load. In [13], the problem of obstacle avoidance by a load is considered, taking into account the prohibited zones around obstacles. The optimisation of the trajectory cost function by the energy criterion is performed. As a result, the oscillation movements of the load carrier are limited. In [14], an approach to planning a trajectory with an analytical expression of acceleration is proposed. Here, an analytical expression is obtained that relates the oscillations of the load and the acceleration of the TC load carrier. The use of smooth functions in a three-segment acceleration trajectory is proposed. The task of acceleration to a given speed of movement of the suspension point in the absence of residual angular oscillations of the rope is solved analytically in [15].

Most of the known techniques and algorithms have disadvantages. As a rule, they do not solve the problem of the accurate movement of the load along the reference trajectory. With a different degree of efficiency, another task of limiting the oscillations of the load is being solved. Most often, it is interpreted as a decrease in the amplitude of oscillations of a rope or load. The movement time in the specified task, as a rule, increases. Often, oscillations are not completely damped. The vast majority of methods do not take into account the attenuation coefficients of the pendulum oscillations of the load due to energy dissipation. In the known methods, characterised by high quality oscillation damping, a relatively complex and resource-intensive mathematical apparatus is used.

2. Formulation of the problem

To eliminate the drawbacks of the known methods, the article presents a simple analytical method for determining the movement trajectory of the load suspension point, ensuring the movement of the load along the reference curvilinear trajectory.

There is a curvilinear trajectory of load movement in the $Y = f(X)$ horizontal plane specified in the form of n reference points. The curve of the trajectory, described by an interpolation polynomial, passes through the X and Y specified horizontal coordinates of the reference points (figure 1).

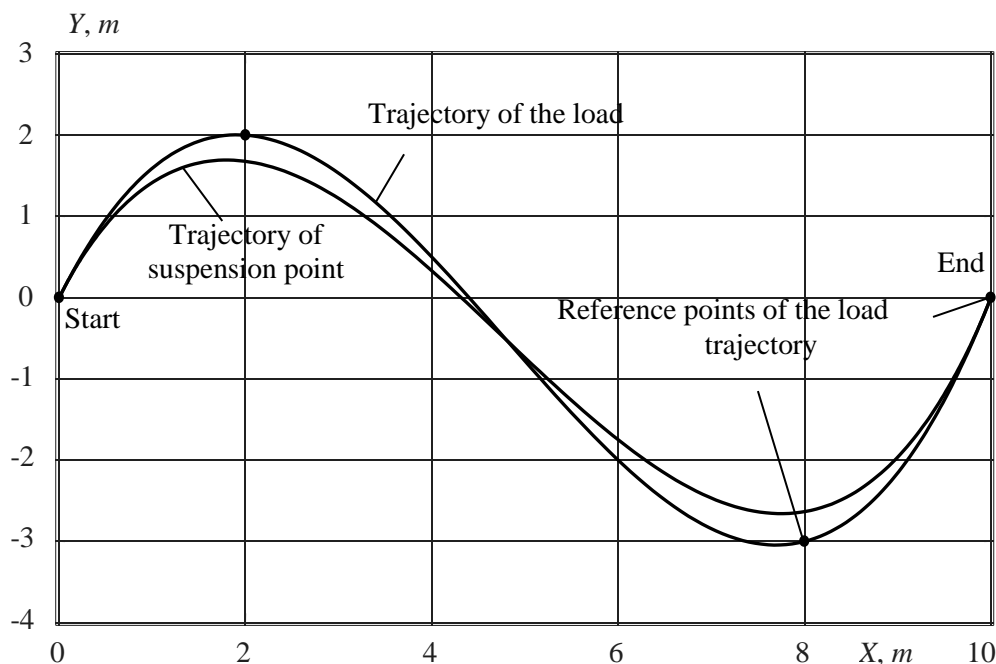


Figure 1. Schematic view of an cubic interpolation polynomial of the spatial load trajectory passing through 4 reference points and the corresponding trajectory of the suspension point.

Using the mathematical model of the TC with a rope suspension of the load (figure 2), it is necessary to determine such a trajectory of the suspension point of the load (bridge and load carrier of the TC moving along the X and Y horizontal axes, respectively), which will allow the reference movement trajectory of the load to be realised during the T time with the observance of additional conditions of equality to zero of the speeds and accelerations of the load and the suspension point both at the initial and at the final moments of time.

3. Theoretical part

The method is intended for low-loaded TCs (moving loads of the mass comparable to the mass of the TC carrier), which do not experience significant wind loads (indoor operation). The assumption is made about the negligible influence of the load mass on the controlled parameters of movement, speed and acceleration of the TC moving links, such as bridge and load carrier. The scope of application of the method is limited to small angles of deviation of the elevator rope from the vertical (not more than 5°) and a constant length of the rope in the process of load movement.

Taking into account the smallness of the deviation angles of the elevator rope, the motion of the TC spatial dynamic system (see figure 2(a)) was presented as a superposition of plane oscillations in two mutually perpendicular planes. These are the plane of movement of the crane bridge with a load (along the X axis, angle θ_x) and the plane of movement of the TC carrier with load (along the Y axis, angle θ_y). The scheme of moving a flexible suspension point of TC with a load in a separate plane (for example, the plane of motion for the TC crane bridge) is shown in figure 2(b).

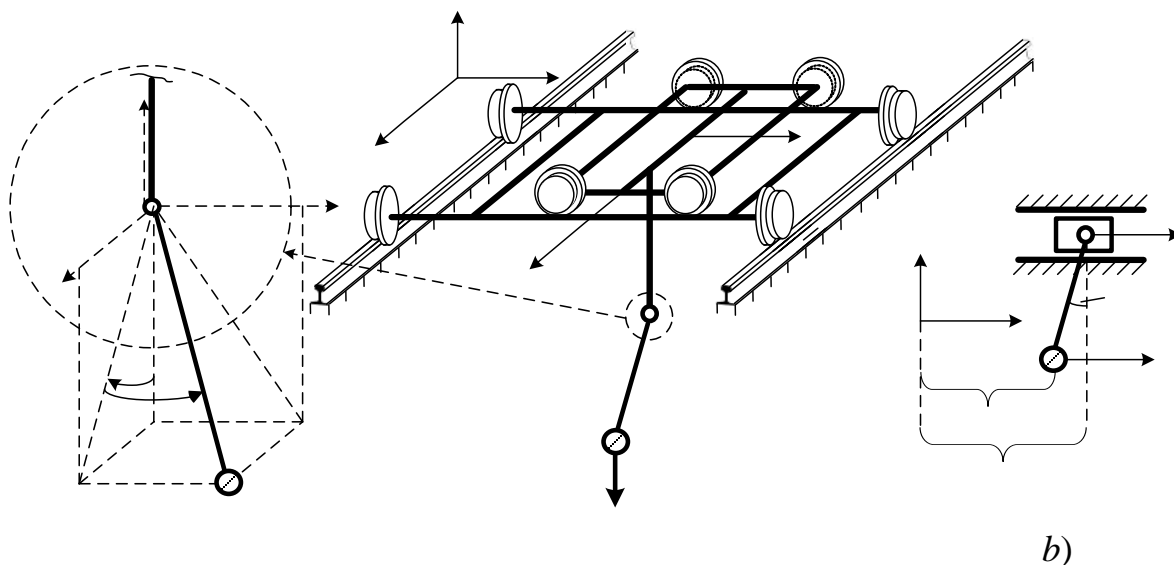


Figure 2. Bridge crane with rope suspension: (a) – spatial diagram; (b) – the scheme of movement in a separate plane.

In the mathematical model and on the plane oscillation scheme (see figure 2(b)), the following designations of the parameters of the dynamic system are set (using the example of motion along the X axis): L is the length of the pendulum suspension of the load (i.e., elevator rope) from the suspension point of the TC load carriage to the mass centre of the load, m; q is the angle of deviation of the TC elevator rope from the vertical, rad; m is the load mass, kg; g is the gravitational acceleration, m/s^2 ; t is the current time of the moving process, s; T is the end time of the moving process, s; $b=B/(mL^2)$ is the oscillation damping factor, s^{-1} ; B is the drag torque factor of the angular rotation of the rope relative to the gravitational vertical, $\text{kg}\cdot\text{m}^2/\text{s}$; x_t is the linear horizontal coordinate of the suspension point, m; $x_{t0}=0$ is the linear horizontal coordinate of the load at the $t=0$ initial time, m; x_{tT} is the linear horizontal coordinate of the load at the T end time, m. Top points above all variable parameters denote their time

derivatives. When considering the movement along the Y axis, all parameters containing the x -axis symbol will correspond to the parameters containing the y -axis symbol.

The basis of the mathematical description of the process of TC load moving was formed on the well-known differential equation of the pendulum oscillations with a moving suspension point [16]. In the case of small angles and taking into account the damping of oscillations, this linearized differential equation has the form

$$\ddot{q} + \ddot{x}_l / L + b\dot{q} + q \cdot g / L = 0 \quad (1)$$

Differential equation (1) is obtained by linearizing the nonlinear differential equation of large oscillations in the plane.

The linearized equations of the relations of movements, velocities, accelerations and snatches of the suspension point and the load point at small values of the q angle have the form

$$x_l(t) = x_l(t) - L \cdot q(t); \dot{x}_l(t) = \dot{x}_l(t) - L \cdot \dot{q}(t); \ddot{x}_l(t) = \ddot{x}_l(t) - L \cdot \ddot{q}(t); \ddot{x}_l(t) = \ddot{x}_l(t) - L \cdot \ddot{q}(t). \quad (2)$$

When substituting the expression for the $\ddot{x}_l(t)$ acceleration of the suspension point from (2) in the second-order differential equation (1), the latter is reduced to the first order. The differential equation of the first derivative of the \dot{q} angle in the Cauchy form is formed as:

$$\dot{q} = -(\ddot{x}_l + q \cdot g) / (L \cdot b) \quad (3)$$

When substituting expressions of the $\ddot{x}_l(t)$ acceleration of the load in the form of smooth functions to the differential equation (3), in some cases an analytical solution of this differential equation can be obtained.

In order to set the reference trajectory of the load, this article proposes to apply Hermite splines. Using the well-known general derivation formulas [17, 18], an expression was obtained for the two-point Hermite spline with the highest order of derivatives at the $m=4$ nodal points, given below. For the m order being less than 4, snatches in the acceleration of the suspension point at the initial and final moments of the process are observed. The m order greater than 4, in turn, significantly increases the complexity of the resulting analytical expressions and makes them rather cumbersome. Therefore, it was decided to settle upon the $m=4$ value. Two-point Hermite spline that defines the trajectory of the load at $m=4$ has the form [17, 18]:

$$x_l(t) = s_9 t^9 + s_8 t^8 + s_7 t^7 + s_6 t^6 + s_5 t^5 + s_4 t^4 + s_3 t^3 + s_2 t^2 + s_1 t \quad (4)$$

where $s_1 \dots s_9$ are constant coefficients determined by the specified values of the time of movement and derivatives of the linear movement of the load at the starting and ending nodal points:

$$\begin{aligned} s_1 &= \dot{x}_{l0}; \quad s_2 = \frac{3\ddot{x}_{l0}T^3 + 30\dot{x}_{l0}T^2}{6T^3} - \frac{5\dot{x}_{l0}}{T}; \quad s_3 = \frac{10\ddot{x}_{l0}}{T^2} - \frac{15\ddot{x}_{l0}T^3 + 150\dot{x}_{l0}T^2}{6T^4} + \frac{\ddot{x}_{l0}T^3 + 15\ddot{x}_{l0}T^2 + 90\dot{x}_{l0}T}{6T^3}; \\ s_4 &= \frac{15\ddot{x}_{l0}T^3 + 150\dot{x}_{l0}T^2}{3T^5} - \frac{10\dot{x}_{l0}}{T^3} - \frac{5\ddot{x}_{l0}T^3 + 75\ddot{x}_{l0}T^2 + 450\dot{x}_{l0}T}{6T^4} + \frac{5\ddot{x}_{l0}T^2 + 45\ddot{x}_{l0}T + 210\dot{x}_{l0}}{6T^3}; \\ s_5 &= \frac{-\ddot{x}_{lT}T^3 + (21\ddot{x}_{lT}T^2)/2 - 56\dot{x}_{lT}T + 126x_{lT}}{T^5} - \frac{15\ddot{x}_{l0}T^3 + 150\dot{x}_{l0}T^2}{3T^6} + \\ &+ \frac{5\dot{x}_{l0}}{T^4} + \frac{5\ddot{x}_{l0}T^3 + 75\ddot{x}_{l0}T^2 + 450\dot{x}_{l0}T}{3T^5} - \frac{25\ddot{x}_{l0}T^2 + 225\ddot{x}_{l0}T + 1050\dot{x}_{l0}}{6T^4}; \\ s_6 &= \frac{196\dot{x}_{lT} - (77T\ddot{x}_{lT})/2 + (23T^2\ddot{x}_{lT})/6 - (420x_{lT})/T}{T^5} + \frac{15\ddot{x}_{l0}T^3 + 150\dot{x}_{l0}T^2}{6T^7} - \\ &- \frac{\dot{x}_{l0}}{T^5} - \frac{5\ddot{x}_{l0}T^3 + 75\ddot{x}_{l0}T^2 + 450\dot{x}_{l0}T}{3T^6} + \frac{25\ddot{x}_{l0}T^2 + 225\ddot{x}_{l0}T + 1050\dot{x}_{l0}}{3T^5}; \end{aligned}$$

$$\begin{aligned}
s_7 &= \frac{53\ddot{x}_{IT} - \frac{11T\ddot{x}_{IT}}{2} - \frac{260\dot{x}_{IT}}{T} + \frac{540x_{IT}}{T^2}}{T^5} - \frac{3\ddot{x}_{I0}T^3 + 30\dot{x}_{I0}T^2}{6T^8} + \\
&+ \frac{5\ddot{x}_{I0}T^3 + 75\dot{x}_{I0}T^2 + 450\dot{x}_{I0}T}{6T^7} - \frac{25\ddot{x}_{I0}T^2 + 225\dot{x}_{I0}T + 1050\dot{x}_{I0}}{3T^6}, \\
s_8 &= \frac{\frac{7\ddot{x}_{IT}}{2} - \frac{65\ddot{x}_{IT}}{2T} + \frac{155\dot{x}_{IT}}{T^2} - \frac{315x_{IT}}{T^3}}{T^5} - \frac{\ddot{x}_{I0}T^3 + 15\dot{x}_{I0}T^2 + 90\dot{x}_{I0}T}{6T^8} + \\
&+ \frac{25\ddot{x}_{I0}T^2 + 225\dot{x}_{I0}T + 1050\dot{x}_{I0}}{6T^7}, \\
s_9 &= \frac{\frac{15\ddot{x}_{IT}}{2T^2} - \frac{5\ddot{x}_{IT}}{6T} - \frac{35\dot{x}_{IT}}{T^3} + \frac{70x_{IT}}{T^4}}{T^5} - \frac{5\ddot{x}_{I0}T^2 + 45\dot{x}_{I0}T + 210\dot{x}_{I0}}{6T^8}. \quad (5)
\end{aligned}$$

Thus, a separate movement of the load from the start to the end point is characterised by a set of boundary conditions at the initial and final time points in the form of coordinate values, its first three derivatives, as well as the initial value of the q_0 angle of inclination of the rope.

The expressions of the first three time derivatives of the Hermite spline (4), determining the specified speed, acceleration and snatch of the load, are of the form:

$$\dot{x}_l = 9s_9t^8 + 8s_8t^7 + 7s_7t^6 + 6s_6t^5 + 5s_5t^4 + 4s_4t^3 + 3s_3t^2 + 2s_2t + s_1; \quad (6)$$

$$\ddot{x}_l = 72s_9t^7 + 56s_8t^6 + 42s_7t^5 + 30s_6t^4 + 20s_5t^3 + 12s_4t^2 + 6s_3t + 2s_2; \quad (7)$$

$$\ddot{x}_l = 504s_9t^6 + 336s_8t^5 + 210s_7t^4 + 120s_6t^3 + 60s_5t^2 + 24s_4t + 6s_3. \quad (8)$$

The derived coordinates of the load (6) - (8), obtained by time differentiating expressions (4), provide the expressions of speeds, accelerations and snatches of the load suspension point from equations (2). This is possible in the presence of analytical expressions of the first three derivatives of the rope angle, as will be discussed below.

For differential equation (3), after substituting into it the expression of the second derivative of the Hermite spline (7), a solution can be obtained in the analytical form. The solution consists in a time dependence of the deflection angle of the load rope:

$$\begin{aligned}
q(t) &= C_1 e^{-\frac{gt}{Lb}} - \frac{72s_9t^7 + 56s_8t^6 + 42s_7t^5 + 30s_6t^4 + 20s_5t^3 + 12s_4t^2 + 6s_3t + 2s_2}{g} + \\
&+ \frac{362880L^7b^7s_9}{g^8} + \frac{6Lb(84s_9t^6 + 56s_8t^5 + 35s_7t^4 + 20s_6t^3 + 10s_5t^2 + 4s_4t + s_3)}{g^2} - \\
&\frac{24L^2b^2(126s_9t^5 + 70s_8t^4 + 35s_7t^3 + 15s_6t^2 + 5s_5t + s_4)}{g^3} + \\
&+ \frac{5040L^5b^5(36s_9t^2 + 8s_8t + s_7)}{g^6} + \frac{120L^3b^3(126s_9t^4 + 56s_8t^3 + 21s_7t^2 + 6s_6t + s_5)}{g^4} - \\
&- \frac{40320L^6b^6(s_8 + 9s_9t)}{g^7} - \frac{720L^4b^4(84s_9t^3 + 28s_8t^2 + 7s_7t + s_6)}{g^5}. \quad (9)
\end{aligned}$$

The expression of the C_1 constant coefficient, determined by substituting the initial value of the q_0 angle to (9), has the form:

$$C_1 = q_0 + \frac{2s_2}{g} + \frac{24L^2b^2s_4}{g^3} - \frac{120L^3b^3s_5}{g^4} + \frac{720L^4b^4s_6}{g^5} - \frac{5040L^5b^5s_7}{g^6} + \frac{40320L^6b^6s_8}{g^7} - \frac{362880L^7b^7s_9}{g^8} - \frac{6Lbs_3}{g^2}. \quad (10)$$

By time differentiating the formula (9), analytical expressions of the first three derivatives of the rope angle are obtained:

$$\begin{aligned} \dot{q}(t) = & \frac{5040L^5b^5(8s_8 + 72s_9t)}{g^6} - \frac{362880L^6b^6s_9}{g^7} - \\ & - \frac{504s_9t^6 + 336s_8t^5 + 210s_7t^4 + 120s_6t^3 + 60s_5t^2 + 24s_4t + 6s_3}{g} + \\ & + \frac{120L^3b^3(504s_9t^3 + 168s_8t^2 + 42s_7t + 6s_6)}{g^4} - \frac{720L^4b^4(252s_9t^2 + 56s_8t + 7s_7)}{g^5} + \\ & + \frac{6Lb(504s_9t^5 + 280s_8t^4 + 140s_7t^3 + 60s_6t^2 + 20s_5t + 4s_4)}{g^2} - \\ & - \frac{24L^2b^2(630s_9t^4 + 280s_8t^3 + 105s_7t^2 + 30s_6t + 5s_5)}{g^3} - \frac{C_1ge^{-\frac{gt}{Lb}}}{Lb}; \end{aligned} \quad (11)$$

$$\begin{aligned} \ddot{q}(t) = & - \frac{3024s_9t^5 + 1680s_8t^4 + 840s_7t^3 + 360s_6t^2 + 120s_5t + 24s_4}{g} + \frac{362880L^5b^5s_9}{g^6} - \\ & - \frac{720L^4b^4(56s_8 + 504s_9t)}{g^5} + \frac{6Lb(2520s_9t^4 + 1120s_8t^3 + 420s_7t^2 + 120s_6t + 20s_5)}{g^2} - \\ & - \frac{24L^2b^2(2520s_9t^3 + 840s_8t^2 + 210s_7t + 30s_6)}{g^3} + \\ & + \frac{120L^3b^3(1512s_9t^2 + 336s_8t + 42s_7)}{g^4} + \frac{C_1g^2e^{-\frac{gt}{Lb}}}{L^2b^2}; \end{aligned} \quad (12)$$

$$\begin{aligned} \ddot{\ddot{q}}(t) = & \frac{120L^3b^3(336s_8 + 3024s_9t)}{g^4} - \frac{362880L^4b^4s_9}{g^5} - \\ & - \frac{15120s_9t^4 + 6720s_8t^3 + 2520s_7t^2 + 720s_6t + 120s_5}{g} + \\ & + \frac{6Lb(10080s_9t^3 + 3360s_8t^2 + 840s_7t + 120s_6)}{g^2} - \\ & - \frac{24L^2b^2(7560s_9t^2 + 1680s_8t + 210s_7)}{g^3} - \frac{C_1g^3e^{-\frac{gt}{Lb}}}{L^3b^3}. \end{aligned} \quad (13)$$

The movement of the suspension point and its derivatives can be determined by substituting the values of the rope angle and its first three derivatives obtained in (9), (11), (12) and (13) directly into the constraint equations (2).

All the above analytical expressions describe the movement of the pendulum system of the load with a moving suspension point in a separate plane. Thus, the obtained expressions allow analytical expressions of the trajectory of the load suspension point, given by equations (4), (6) - (8) for the load, to be obtained, i.e. to obtain the expressions of the working parts of TC: bridge or load carrier. Moving the latter along the trajectory (2) promote the realisation of the given trajectory of the load in the plane in the absence of uncontrolled vibrations, subject to the rest conditions at the initial and final moments of time.

4. Experimental results

Next, let us consider the method of using the obtained expressions to study the spatial movements of the load along a curve located in the horizontal plane (with the restriction in the form of a L constant rope length). It is necessary to set the curved trajectory of the load in the horizontal plane, and then present it as a superposition of two plane controlled vibrations. The simplest and most universal way of defining a curved trajectory of the load movement in the XY plane as a $Y = f(X)$ function is to specify the X and Y coordinates for n reference points through which the curve should pass, and the subsequent generation from them of coefficients of the interpolation polynomial of $n-1$ degree by known numerical methods [19]. The only necessary condition is the following. It is required that the values of the X coordinate in the sequence of reference points of the trajectory monotonously increase. The values of the Y coordinate can be changed arbitrarily.

As an example, with the number of reference points of the trajectory equal to 4 (see figure 1), the expressions of the interpolation polynomial and its first three derivatives take the form:

$$\begin{aligned} y_l &= p_1 x_l^3 + p_2 x_l^2 + p_3 x_l + p_4; \quad \dot{y}_l = 3p_1 \dot{x}_l x_l^2 + 2p_2 \dot{x}_l x_l + p_3 \dot{x}_l; \\ \ddot{y}_l &= 6p_1 \dot{x}_l^2 x_l + 2p_2 \dot{x}_l^2 + 3\ddot{x}_l p_1 x_l^2 + 2\ddot{x}_l p_2 x_l + \ddot{x}_l p_3; \\ \dddot{y}_l &= 6p_1 \dot{x}_l^3 + 18p_1 \ddot{x}_l \dot{x}_l x_l + 3p_1 \ddot{x}_l x_l^2 + 6p_2 \ddot{x}_l \dot{x}_l + 2p_2 \ddot{x}_l x_l + p_3 \ddot{x}_l, \end{aligned} \quad (14)$$

where p_1, p_2, p_3, p_4 are the interpolation polynomial coefficients sorted in decreasing order.

5. Results and discussion

For the example shown in figure 1, four reference points of a given load trajectory have $X = [0; 2; 8; 10]$ m, $Y = [0; 2; -3; 0]$ m coordinates. The coefficients of the interpolation polynomial take the values of $p_1 = 0.052083$; $p_2 = -0.75$; $p_3 = 2.291667$; $p_4 = 0$.

As an example of one of the methods for setting the spatial trajectory, let the movement of the load along the X axis be set using the only Hermite spline of the form (4), determined by the following values of the constant parameters:

$$\begin{aligned} m &= 1000 \text{ kg}; B = 5 \text{ kg} \cdot \text{m}^2/\text{s}; T = 30 \text{ s}; q_0 = 0^\circ; x_{l0} = 0 \text{ m}; \dot{x}_{l0} = 0 \text{ m/s}; \ddot{x}_{l0} = 0 \text{ m/s}^2; \dddot{x}_{l0} = 0 \text{ m/s}^3; \\ x_{lT} &= y_l(T) = 10 \text{ m}; \dot{x}_{lT} = 0 \text{ m/s}; \ddot{x}_{lT} = 0 \text{ m/s}^2; \dddot{x}_{lT} = 0 \text{ m/s}^3. \end{aligned} \quad (15)$$

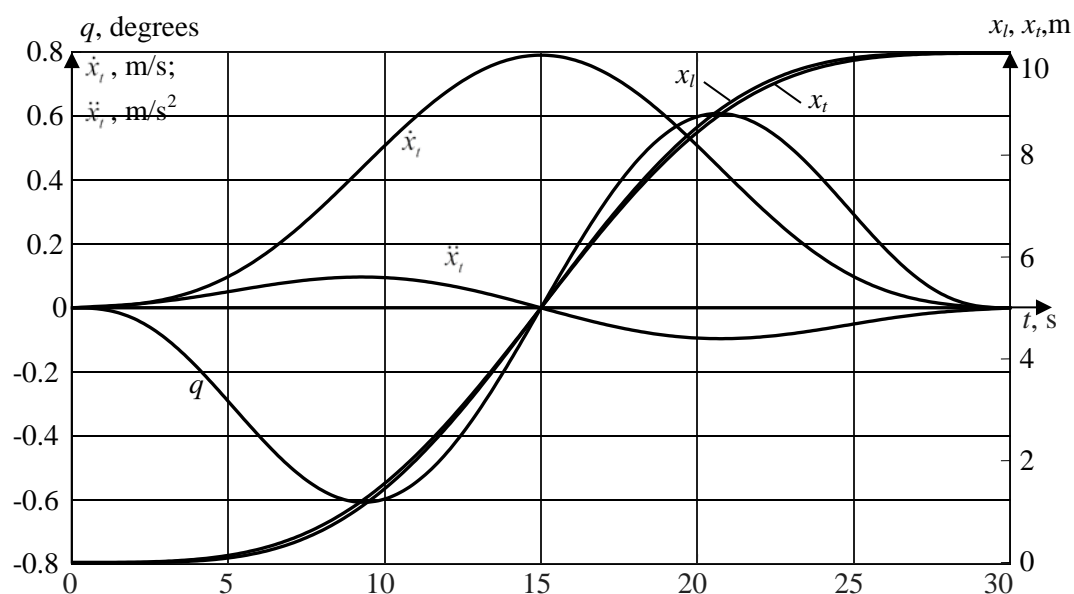


Figure 3. Time dependences of a number of variable parameters during the load movement along the X axis: x_l denote load coordinates, x_t are coordinates of the suspension point, q is the angle of the deviation of the rope from the vertical, \dot{x}_t is the speed of the suspension point, \ddot{x}_t is the acceleration of the suspension point (example).

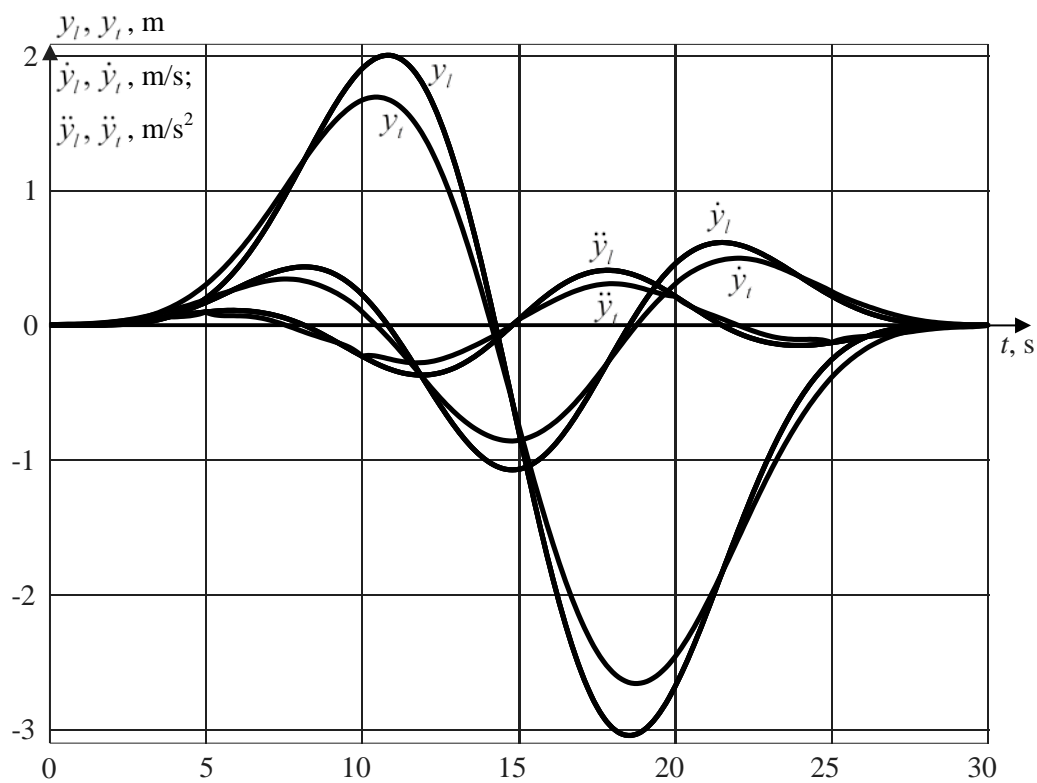


Figure 4. Time dependences of the load movements, speeds and accelerations ($y_l, \dot{y}_l, \ddot{y}_l$) of the load and load suspension point ($y_t, \dot{y}_t, \ddot{y}_t$) when moving along the Y axis (example).

Figure 3 present the time dependences of the variable parameters of the system, calculated for a single movement of the load along the X axis. Single acceleration and deceleration, similar to those shown in figure 3, is advisable to use for defining the movement along the X axis of the TC bridge, which has a large inertia in comparison with a load carrier.

Expressions (14) make it possible to determine the values of the load movement along the Y axis, as well as the first three derivatives of this displacement for an arbitrary number of reference points of the $Y(t)$ trajectory. The movement of the load along the Y axis has a variable direction. The $[0; T]$ total time interval of the movement can be divided into k gaps.

Figure 4 shows the time dependences of the movements, velocities and accelerations of the load and the load suspension point in the direction of the Y axis for the considered example with $k=6$. In this case, the $y_l(t)$ number of reference points of the trajectory is equal to $k+1$, i.e. 7. The final value of the angle of deviation of the load rope in each of the gaps acts as the initial q_0 value for the subsequent interval.

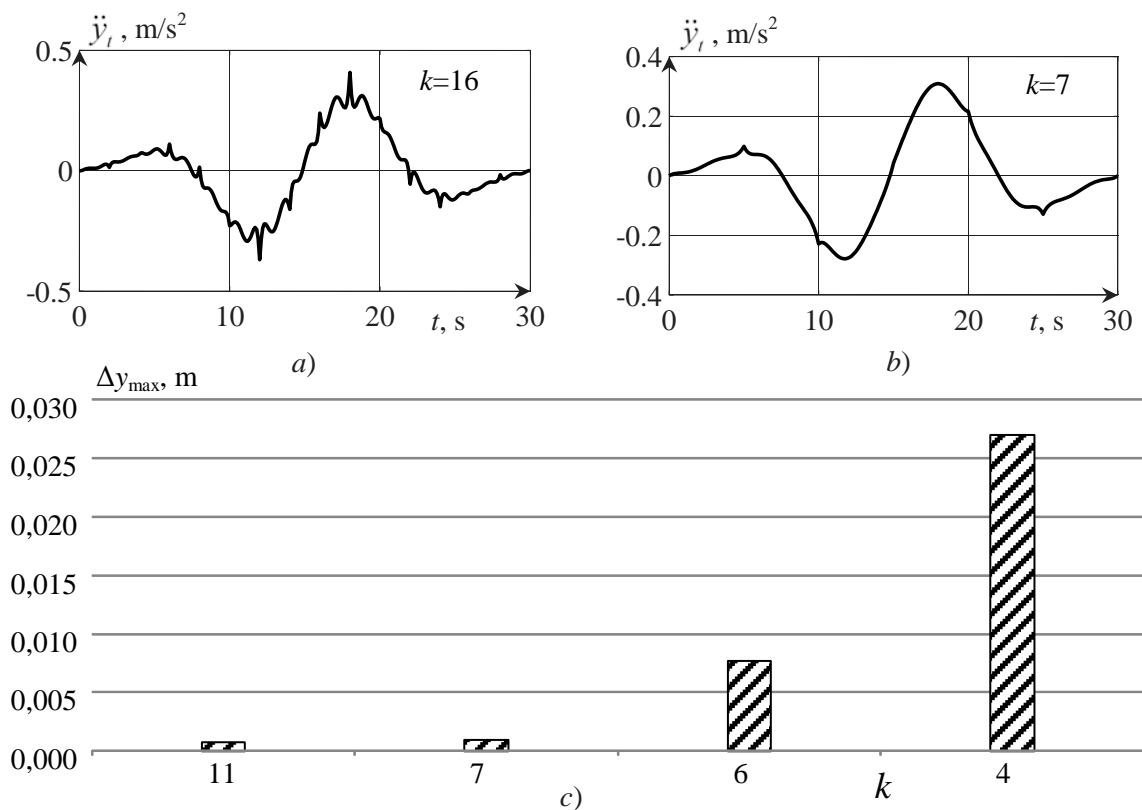


Figure 5. Synthesised time dependences of the \ddot{y}_l acceleration of the suspension point for the $k=16$ (a) and $k=7$ (b) number of reference points of the $y_l(t)$ trajectory; diagram of the values of the absolute error of the Δy_{\max} coordinate of the load for $k=11, 7, 6$ and 5 (c).

The parameters of the load trajectory specified by (14) and obtained by formulas (4), (6) and (7) visually coincide. For the parameters of the trajectory of the suspension point, the smoothness of the functions is observed only for y_l movement and \dot{y}_l speed. The \ddot{y}_l acceleration plot of the suspension point has break points with time values coinciding with the reference points of the $y_l(t)$ trajectory (figure 5(a), (b)).

An increase in the number of control points reduces the maximum absolute error in the implementation of a planned trajectory of the load along the Y axis (Figure 5(c)).

The absolute error was determined as the difference between the current values of the y_l coordinate of the load obtained by the formula (14) and the corresponding values obtained on the TC simulation model [20] of applying to it as input actions at the same time two movements of the suspension point obtained by formulas (2) along the X and Y axes. The simulation model allows large spatial movements of the load to be explored. Possible values of the deviation angles of the load rope in this model are not limited.

The Δy_{\max} maximum absolute error at $k \geq 7$ is less than 1 mm (see figure 5(c)), which can be considered a negligible value.

The maximum absolute error along the axis of movement of the bridge in this example took constant values: $\Delta x_{\max} = 0.55$ mm due to the movement of the load along the X axis of the bridge being specified by the same two-point Hermite spline in all the calculation cases.

6. Conclusions

1. The use of two-point Hermite splines with the highest order of derivatives, equal to 4, made it possible to obtain analytical expressions of load movements, its suspension point, the angle of deviation of the rope from the vertical, as well as their time derivatives. In this case, the movement of load in a spatial separate plane is carried out exactly along a projected trajectory, in the mode of absence of uncontrolled oscillations. When using the presented analytical expressions, there is no need for a numerical solution of the differential equations describing the considered plane oscillatory system of the load with a moving suspension point.

2. A superposition of load oscillations in two mutually perpendicular planes of the bridge movement and the movement of the load carrier is permissible for small deviation angles of the elevator rope from the vertical. It provides the salvation of the problem of moving load on a flexible pendulum suspension of constant length along a planned curvilinear trajectory, when the latter is set in the horizontal plane. For this, the coordinate of the load along the axis of movement of the load carrier can be represented as an analytical function (for example, an interpolation polynomial) from the coordinate of the load along the axis of movement of the crane bridge. As an example, the article gives the formulas of the interpolation polynomial of the third degree, as well as its first three time derivatives. This polynomial describes a trajectory possessing one inflection point and passing through four reference points with given geometric coordinates. Such a trajectory allows two obstacles to be avoided by the load. By dividing the time trajectory of the load along the axis of movement of the load carrier into several sections of the same duration, the necessary values of the movement of the load and its first three derivatives at the reference points can be calculated. Then, using the above analytical expressions for each section, the trajectory of the suspension point along the axis of movement of the load carrier can be determined.

3. For the considered interpolation polynomial of the third degree, which determines the dependence of the coordinate of the load along the movement axis of the carrier on the coordinate of the load along the movement axis of the bridge, it is advisable to use seven or more reference points of the time dependence for the load trajectory. This reduces the maximum absolute error of the method in the considered example to an insignificant value of less than 1 mm. The specified error was obtained by forcing the movements of the bridge and the load carrier in the spatial simulation model according to time trajectories obtained from the analytical expressions given in the article.

7. References

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