

# Dynamics and Control of a Two-Spacecraft Coulomb Formation: Challenges and Prospects

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**Abstract.** The paper is devoted to the issue of active large space debris removal. The contactless method of influencing space debris through the electrostatic interaction is considered. The main advantages of this method are possibility of space debris removal in geosynchronous orbit with low energy costs and absence of a dangerous and complex stage of space debris capturing or docking. This paper introduces the trends and features of a relative attitude dynamics and control of the two-spacecraft coulomb formation. The advantages of the multi-sphere method are discussed for modeling and analyzing the attitude motion of space debris. Two towing schemes (push or pull) are studied in detail. It is shown that the pushing scheme has distinct advantages. The features of the attitude motion of the towed object are identified including chaotic phenomenas and control laws of attitude motion are presented. In the final, the possibilities removal of non-functioning GEO satellites of complicated configuration using electrostatic tug are discussed.

## 1. Introduction

Space debris is a growing concern for both low-Earth-orbit (LEO) and geosynchronous orbit (GEO) regimes [1–7]. The defunct GEO satellites tend to be very large, often reaching beyond 5–10 m in size, as well as rotating and tumbling [8]. The act of docking onto such large and tumbling space objects is very challenging; as a result, novel touchless debris removal or despinning solutions are being explored. The ion shepherd method uses the ion engine exhaust to push and/or despin a satellite [9, 10], whereas the laser ablation method uses the debris' own mass as a thruster fuel source [11]. A promising touchless and lowpower solution is the electrostatic tractor [12]. Here, active charge emission is used to both charge the tug or servicer vehicle as well as the debris object. Although the original concept uses an electron gun to charge the tug positive and debris negative, creating an attractive electrostatic tractor force, with auxiliary charge emission on the tug, it is also possible to charge both the servicer and debris to the same potential to create a repulsive force [13,14]. Most of the control research using the electrostatic tractor considers a pulling configuration to move GEO objects [15]. Earlier work considered the relative motion control for a pusher configuration but did not consider any attitude motion. Furthermore, modulated electrostatic tractor implementations were studied to detumble a space object without physical touch [16]. This enabled orbital servicing and docking missions to first remove a large amount of the rotational kinetic energy before physically docking and engaging with a satellite.

An important value for the study of the behavior of bodies under the influence of the Coulomb forces has the Multi-Sphere Method (MSM) which represents the complete spacecraft electrostatic charging model as a collection of spherical conductors dispersed through the body [17] to provide induced



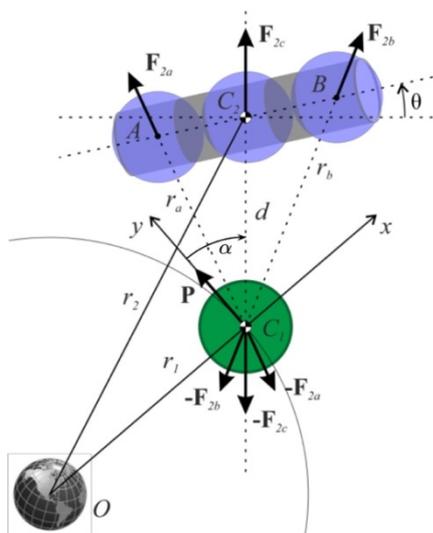
charging effects consistent with finite element methods. The Schaub & Stevenson method [17] for determining the Coulomb torques provides an opportunity to explore the dynamics of the defunct satellite in case the Coulomb interaction by analytical methods. Spacecraft using Coulomb Forces in Earth orbit is only viable in GEO due the electrical shielding properties of the space plasma [18]. The benefits of exploiting interspacecraft Coulomb forces are a virtually propellantless method to perform relative orbit corrections with the capability of operating at very high bandwidths. For small spacecraft separation distances of the order of tens of meters, the magnitude of the Coulomb force is as better than many high efficiency ion propulsion methods at a fraction of the power requirement.

A feedback control law is proposed that stabilizes the relative motion of the space tug and space debris during active debris removal using the Coulomb force for the push scheme [19]. In [20] the pusher configuration is considered with taking into account an attitude motion. An influence of flexible appendages on satellite attitude motion in the Coulomb interaction is studied for the pusher configuration [21]. Moreover, for the push configuration equations of spatial attitude motion in the canonical form are deduced, and are obtained exact solutions using Jacobi elliptic functions, in the case where the distance between the space debris and the tug remains unchanged. If the distance (or) and charge voltage changes slowly over time, adiabatic invariants are found in terms of the complete elliptic integrals [22].

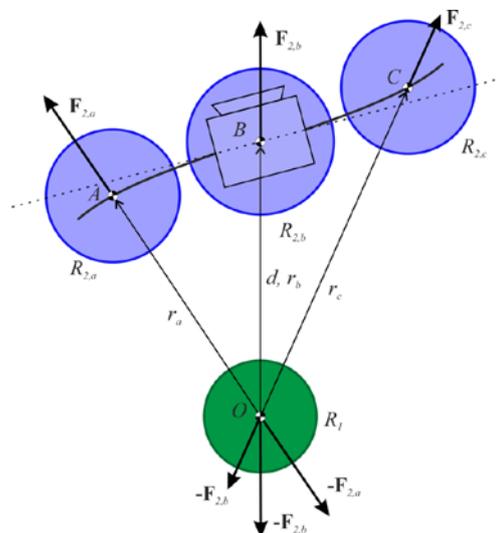
The goal of this paper is to investigate trends and features of a relative attitude dynamics and control of a two-spacecraft coulomb formation. This paper discusses various fundamental and applied aspects of achieving two-spacecraft Coulomb formations.

**2. Types of Space Objects Suitable for the Describe the Multi-Sphere Method**

The paper [17] devoted to the Multi-Sphere Method shows space objects of a very complex configuration with elastic elements, including solar batteries and antennas. However, in practice, in papers when considering an attitude dynamics by the Multi-Sphere Method, the active space object is represented as a conductor spherical in shape, and space debris as a body of very simple bodies in the form of a cylinder (Fig. 1) or a simple solid body with symmetrically fixed elastic panels (Fig. 2).



**Figure 1.** Space debris as a body of cylindrical shape [20].



**Figure 2.** Space debris as a body with symmetrically fixed elastic panels [21].

**3. Towing Principle: Push or Pull**

The paper [23] develops the deep space detumble attitude dynamics and stability arguments when a nominal attractive or repulsive potential is prescribed. The nominal potential serves as a tug or push to translate the entire system with the separation distance controlled by conventional servicer thrusting. Investigating the deep space scenario first provides two distinct benefits. First, operational electrostatic detumbling requires the space objects to be flying only multiple craft radii apart. Differential gravity

will have a small impact on the relative orbit and is absent in the deep space case. Further, the deep space scenario only requires knowledge of the relative orientation of the debris with respect to the servicer while maintaining a fixed relative position of the servicer allows for simplified servicer thrusting implementations.

There are two schemes of tugging (Figs. 3, 4). In the first scheme, the space tug and debris are charged electrically with different signs. An attractive Coulomb force exists between the tug and debris in this case. The debris ( $C_2$ ) is attracted to the space tug ( $C_1$ ) with the electrostatic force  $\mathbf{F}_{1,2}$ . The space tug is attracted to the debris with the force  $\mathbf{F}_{2,1} = -\mathbf{F}_{1,2}$ , which is equal in magnitude but opposite in direction. To transfer the whole system, the space tug should produce force (Fig. 3):

$$\mathbf{P} = \frac{1}{\cos \gamma} \left( 1 - \frac{m_1}{m_2} \right) |\mathbf{F}_{1,2}| \tag{1}$$

where  $m_1$  and  $m_2$  are the masses of the space tug and debris, respectively. The force vector  $\mathbf{P}$  should be inclined at  $\gamma$  to the direction  $C_1C_2$  to avoid unwanted pressure of the tug's engine flow on the debris.

In the second scheme, the space tug and debris are charged the same (equal sign). The space tug pushes the debris, and the tug's thrust should be

$$\mathbf{P} = \left( 1 - \frac{m_1}{m_2} \right) |\mathbf{F}_{1,2}|$$

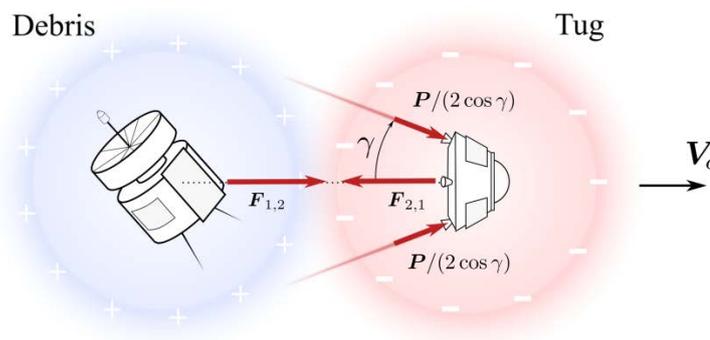


Figure 3. Pull scheme [19].

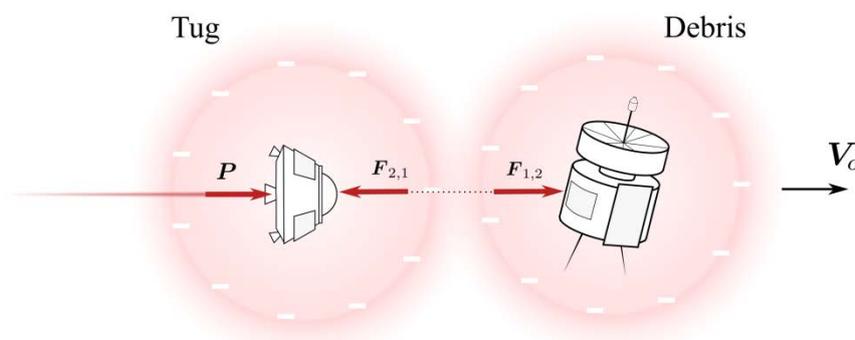


Figure 4. Push scheme [19].

We suppose that the push scheme has the following advantages over the pull scheme. The push scheme is naturally stable in terms of distance between the space tug and debris. In case of a change of

the tug’s thrust or the charge of the tug or debris, the new stable relative distance is established in a natural way. In the pull scheme, the change in the charges of the bodies or variation of the tug’s thrust can lead to increasing distance between the space tug and debris, decreasing the Coulomb force, and so the control force should be applied to maintain the condition of stationary motion, which is expressed by Eq. (1). Hence, the pull scheme imposes heavy demands on the tug’s control system in comparison with the push scheme. In the push scheme, the tug’s thrust can be applied along the line between the tug and debris, but in the pull scheme, the tug’s thrust should be inclined to that line (angle  $\gamma$  in Fig. 3) to avoid unwanted pressure of the tug’s engine flow on the debris. Therefore, the effectiveness of the pull scheme is less than the push scheme. The potential difference between bodies in the push scheme is less than in the pull scheme, and so the first one is safer in case of contact between the bodies (Fig. 5). For these reasons, we suppose that the space tug and the debris have the same sign of charge, and the space tug pushes the debris.

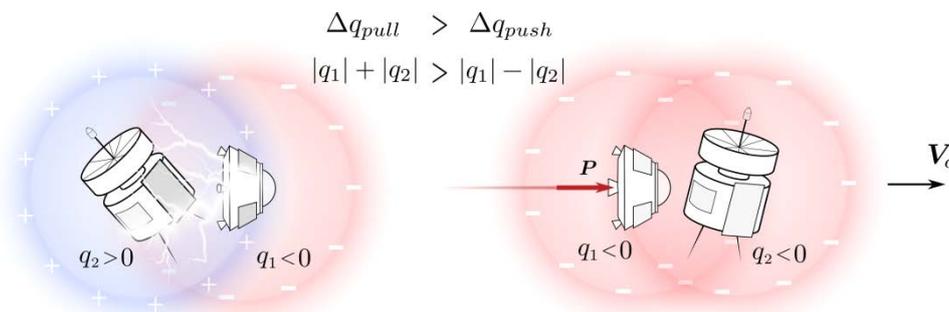


Figure 5. Difference between the charge for push and pull scheme [19].

#### 4. Regular and Chaotic Attitude Motions

The attitude motion equations of the debris, even for describing a plane motion, are very difficult. This is due to the fact that, according to the multi method of calculating electrostatic forces, it implies inverting of matrices, for example, for a three-point model we have matrixs of 4x4 dimension [17]. So according to [17] the three-sphere Multi- Sphere Method is a means to approximate the electrostatic interactions between conducting objects with generic geometries. Fig. 1 depicts a cylindrical satellite, modeled by 3 optimally placed spheres, in the vicinity of the active satellite as a thrust source. Both objects are assumed for now to be conducting and reside at voltage levels  $\Phi_1$  and  $\Phi_2$ . The voltage  $\Phi_i$  on a given sphere is a function of the charge  $q_i$  on that sphere and the charges on its neighboring spheres. This relation is governed by Eq. (4) Ref. [17], where  $R_i$  represents the radius of the sphere in question and  $r_{i,j} = r_j - r_i$  is the center-to-center distance to each neighbor. The constant  $k_c = 8.99 \cdot 10^9 \text{ Nm}^2 / \text{C}^2$  is Coulomb’s constant

$$\Phi_i = k_c \frac{q_i}{R_i} + \sum_{j=1, j \neq i}^m k_c \frac{q_j}{r_{i,j}}$$

These relations can be combined for each sphere to obtain the matrix equation

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_2 \\ \Phi_2 \end{bmatrix} = k_c [C_M]^{-1} \begin{bmatrix} q_1 \\ q_a \\ q_c \\ q_b \end{bmatrix}$$

where

$$[C_M]^{-1} = \begin{bmatrix} 1/R_1 & 1/r_a & 1/r_c & 1/r_b \\ 1/r_a & 1/R_{2,a} & 1/l & 1/2l \\ 1/r_b & 1/l & 1/R_{2,c} & 1/l \\ 1/r_c & 1/2l & 1/l & 1/R_{2,b} \end{bmatrix}$$

where  $R_1$  is the radius of the tug. By inverting  $[C_M]^{-1}$ , the charge on each sphere is determined at any instance of time. The charge redistribution and interaction with the space environment are assumed to be orders of magnitude faster than the spacecraft motion. The total electrostatic force is then given by the summations [17]

$$\mathbf{F}_{E2} = -\mathbf{F}_{E1} = k_c |q_1| \sum_{i=a}^c \frac{q_i}{r_i^3} \mathbf{r}_i$$

The attitude motion equation of the debris can be simplified if to assume the cylinder length  $l$  is much less than the distance between the centers of mass of two bodies  $d$  (Fig.1)

$$\lambda = l / (2d) \ll 1 \tag{2}$$

In this case we get [20]

$$J(\ddot{\theta} - \ddot{\alpha}) = \lambda^3 \frac{3R_1}{2k_c d^2} \Phi_1 \left( \frac{R_1}{3d} \Phi_1 + \Phi_2 \right) \sin 2\theta$$

where  $J$  is moment of inertia of the space debris,  $\alpha$  is angle between local horizontal line of the tug and the line connecting the tug and debris,  $\theta$  is pitch attitude angle,  $R_1$  is the spherical tug radius.

The attitude motion equation of the debris with flexible appendages (Fig.2), then even with assumption (2) we already get the system of equations written as [21]

$$a_{\theta p} \ddot{p}_1 + a_{\theta p} \ddot{p}_2 + a_{\theta} \ddot{\theta} = Q_{\theta} \tag{3}$$

$$a_p \ddot{p}_1 + a_{\theta p} \ddot{\theta} = Q_{p_1} - c_p p_1 \tag{4}$$

$$a_p \ddot{p}_2 + a_{\theta p} \ddot{\theta} = Q_{p_2} - c_p p_2 \tag{5}$$

where  $(p_1, p_2)$  are the modal coordinates,  $Q_{\theta}, Q_{p_1}, Q_{p_2} = f(\theta, p_1, p_2)$  are the generalized forces [21].

The solution of Eq. (3) has the unstable position and the presence of small periodic perturbations caused by oscillations flexible appendages, chaos occurs [21].

### 5. Control Laws of the Translational and Attitude Motion

The control laws of the translational motion of the tug-debris system are written as [20]

$$u_x = c_{\alpha} \sin \alpha + c_{\dot{\alpha}} \dot{\alpha}, u_y = c_d (d - d_s) + c_{\dot{d}} \dot{d}$$

where  $c_{\alpha}, c_{\dot{\alpha}}, c_d, c_{\dot{d}}$  are feedback control coefficients.

Obviously, that it is possible to control of the attitude motion using only the magnitude and sign of the electrical charge of the tug. The control law of the electrical change is given as [20]

$$\Phi_1 = \bar{\Phi}_1 (1 + \kappa \theta \sin 2\theta) \quad (\bar{\Phi}_1 = const < 0)$$

where  $\kappa$  are feedback control coefficient.

### 6. Multi-Sphere Method for GEO Satellites of Complicated Configuration

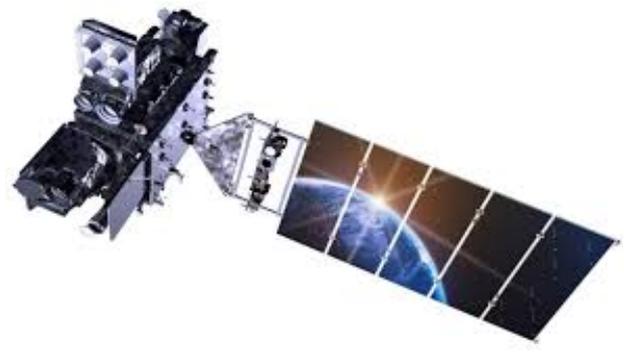
It is enough to look at the current GEO satellites (Figs. 6-9) and analyze their configuration from the point of view of using the Multi-Sphere Method, to understand that the difficult challenge faces us is how to models the attitude motion of these satellites.



**Figure 6.** Satellite AEHF4.

Launch date: October 17, 2018.

<https://www.n2yo.com/satellite/?s=43651>



**Figure 7.** Satellite GOES 17.

Launch date: March 1, 2018.

<https://www.n2yo.com/satellite/?s=43226>



**Figure 8.** Satellite INTELSAT-37E.

Launch date: September 29, 2017.

<https://www.n2yo.com/satellite/?s=42950>



**Figure 9.** Satellite TELSTAR 18V.

Launch date: September 7, 2018.

<https://www.n2yo.com/satellite/?s=43611>

New ideas are needed by the method of transferring electrostatic charge to such complicated satellites. Perhaps, it is necessary to transfer the charge not to all attached elements, but only to some of them. It may also be that we should use a few tugs and synchronize their work to the charge transfer at some fixed points of the satellite.

## 7. Conclusion

This paper introduces the trends and features of a relative attitude dynamics and control of the two-spacecraft coulomb formation. The advantages of the multi-sphere method were discussed for modeling and analyzing the attitude motion of space debris. It was shown that this method is suitable for describing the electrostatic interaction not only of rigid bodies of a cylindrical shape, but also of bodies with attached flexible elements. Two towing schemes (push or pull) have been studied in detail. It was shown that the pushing scheme has distinct advantages. With the use of the simplified mathematical models the features of the attitude motion of the towed object were identified including chaotic phenomena. The control laws of attitude motion were presented. In the final, we discussed the possibilities removal of non-functioning GEO satellites of complicated configuration using electrostatic tug.

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