

Survivability assessment of damaged rod-type vibration systems

Yu F Filippova and S V Doronin

Institute of Computational Technologies SB RAS, PO box 25515, Krasnoyarsk, 660049, Russia

E-mail:filippovajf@ict.nsc.ru

Abstract. From the point of view of dynamics, structural damage manifests as a decrease of stiffness of the material of structural elements or a decrease of their masses (removal from the system due to destruction). The effect of damage on the behavior of vibration systems consists in redistribution of stiffnesses and masses and manifests as a change in natural frequencies and vibration modes. The survivability of damaged vibration systems is viewed as the sensitivity of their natural frequencies and vibration modes to various types of damage. The survivability analysis technology consists in a multivariate modal analysis of the structure with variation in its damage. The article considers the results of numerical simulation of frequencies and modes of natural vibrations of a high-pressure xenon tank mounted on the hull's primary structure after various types of damage to the latter. The tank is a part of the electric propulsion engine of a spacecraft. The results allow assessment of the sensitivity of dynamic characteristics of the tank to damage to the primary structure.

1. Introduction

Consideration of technical objects as vibration systems allows us to abstract from minor structural features and to study dynamic properties (natural vibration modes and frequencies) in connection with the distribution of mass and stiffness characteristics. The obvious relationship between dynamic properties, mass and stiffness characteristics is expressed by the equation in matrix form

$$(K - \lambda M)x = 0, \quad (1)$$

where K is the global stiffness matrix; M is the global mass matrix; $\lambda = \omega^2$; ω is the frequency; x is the mode eigenvector. This relationship is the basis for the formulation and solution of a large number of applied problems in the theory of vibrations of structures with defects and damages of various nature [1-21]. Defects and damages always lead to a local change (usually a decrease) of stiffness (a change in the components of the matrix K in equation (1)), and sometimes (in case of removal of damaged elements from the structure) – to a change in mass (a change in the components of matrix M in equation (1)). Direct tasks are to study the influence of defects and damage on the dynamic properties of structures, the inverse ones are to determine the location and size of defects and damage from the observed natural modes and frequencies.

Most of the research carried out in this subject field are devoted to the study of dynamic characteristics of damaged beams. This is explained both by the low dimensionality of the problems being solved and the high visibility of the description of the vibration modes in the form of a graph of



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displacements along the beam axis. In some cases, studies extend to two-dimensional [5] and three-dimensional objects (reinforced concrete structure of power lines [2], wooden frames of buildings [14], and spatial metal structures of masts [22]). Solutions can be obtained by analytical, numerical, and experimental methods, as well as their combination.

In most cases, the cracks are taken as the defects (single edge cracks in rectangular [3, 7, 9-12, 17] or circular [6] beams, and double cracks symmetrical along the beam axis [1]), although some corrosion can also be considered [18, 19] as well as a defect of any nature simulated by exclusion of the structural element or its part from the design model [14, 22]. The main result of solving such problems are the dependencies of the natural frequencies, modes, and damping coefficients of vibrations on the level of damage (localization and size of the defect).

The development of an idea of the relationship of defects, damage, and dynamic properties of structures led to the inclusion of this relationship in the formulation of problems of structures optimization and prediction of their residual life [15, 16]. In this paper, we consider a possible statement and solution of the problem of structural survivability analysis based on a numerical study of the effect of damage on the dynamic properties of the vibration system containing rod-type primary structures.

2. Formulating the Problem

As one of the possible (generally accepted) interpretations of the concept of “survivability” in this paper, we assume the property of the structure to keep limited operability in the presence of defects and damage of a certain type, as well as in the event of failure of some components. As applied to vibration systems, the sensitivity of parameter λ in Eq. (1) considered in [5] to the change in the stiffness matrix due to damage is closest to this interpretation:

$$\delta\lambda = \frac{x^T \delta K x}{x^T M x}.$$

Then the problem is to

- specify the concept of “survivability” for vibration systems incorporating rod-type structures;
- formulate a possible survivability criterion;
- develop and test methods for the quantitative analysis of survivability.

3. Criteria, indices and methods of survivability analysis

In many technical applications, as a condition for operability, the requirement is formulated that the lowest natural vibration frequency f_1 exceeds a predetermined critical value f^* . A variation of this requirement is to ensure that the natural frequencies f_i are inconsistent with the set of predetermined critical values f_i^* . These values, as a rule, are determined by the frequencies of possible disturbances. The requirements are aimed at eliminating the possibility of resonance phenomena. Then, by survivability of a vibration system with defects and damage, we mean its ability to keep

- natural frequencies within the allowable range and not to allow their matching the critical values (close approach to critical values);
- the rate of change of natural frequencies – sensitivity of dynamic characteristics to the presence of damage.

Assuming damage to the rod elements of the vibration system to occur, quantitative survivability indices S are proposed to be determined in physical or relative values as follows:

$$S^a = f_{1j} = h(d); \quad S^a = f_{ij} = h(d); \quad (2)$$

$$S^r = f_{1j} / f_1 = h(d); \quad S^r = f_{ij} / f_i = h(d); \quad (3)$$

$$S^v = (f_{1j} / f_1)' = h(d); \quad S^v = (f_{ij} / f_i)' = h(d); \quad (4)$$

where f_{1j} and f_{ij} are natural vibration frequencies when the j -th defect (damage) occurs; $i=1,m$; m is the number of regulated (unacceptable) natural frequencies; $j=1,n$; n is the number of possible damages taken into consideration in accordance with a certain scenario of their occurrence and development; $h(d)$ are the functions determined by calculation or experimental methods; d is the degree of damage, which is discrete or continuous, depending on the physical and technical nature of the damage:

$$d = \sum_{j=1}^n d_j; \quad d = \int_{j=1}^n d_j. \quad (5)$$

A discrete interpretation of the degree of damage is applicable with a complete loss of stiffness of the rod element because of its failure or loss of stability. In this case, it is removed from the design model with the appropriate changes to the global stiffness matrices K and mass M . A continuous interpretation of the degree of damage is possible in case of a gradual decrease in the stiffness of the rod element, e.g. due to the development of a crack in the section of the element or its corrosion. This is reflected in the adjustment of the global stiffness matrix K .

Survivability indices (2) - (4) reflect changes due to accumulation of damage:

- dependencies (2) – for absolute values of natural frequencies;
- dependencies (3) – for relative values of natural frequencies in comparison with the intact structure;
- dependencies (4) – for the intensity of changes in relative indices (3).

Quantitative criteria for survivability can be written as

$$S^a > f^*, \quad S^a > f_i^*, \quad (6)$$

$$S^r > (f_{1j} / f_1)^*, \quad S^r > (f_{ij} / f_i)^*, \quad (7)$$

$$S^v < (f_{1j} / f_1)^{r*}, \quad S^v < (f_{ij} / f_i)^{r*}. \quad (8)$$

Justification of the limiting values of f^* , f_i^* in (6) and their derivatives $(f_{1j} / f_1)^*$, $(f_{ij} / f_i)^*$ in (7) usually does not cause difficulties since they are determined by possible disturbance frequencies and are contained in the technical requirements for the designed object. Justification of the limiting values of the intensity of changes in relative indices in (8) is much more complicated and requires special research. However, as a preliminary basis for judgments about the object's survivability, one can consider the points (sections) of a tipping point (a sharp change) of the dependencies (2), (3) graphs – they correspond to a sharp change in the S^v values and indicate a significant increase in the sensitivity to damage of this degree and, accordingly, a sharp decrease in the ability to keep stable natural frequencies.

As additional qualitative criteria for survivability, we can consider the absence of unacceptable modes of vibrations in the event of defects and damage arising from the physical and technical features of the system.

The survivability analysis method is formulated in general terms as follows:

- determination (justification) of possible scenarios of the occurrence and accumulation of damage in the rod elements of the vibration system;
- calculated or experimental implementation of the scenarios of sequentially introducing n damages d_j into the system with fixing the natural vibration frequencies f_{1j} , f_{ij} ;
- determination of the conditions for fulfilling the survivability criteria (6)-(8);
- calculation of quantitative survivability indices in accordance with (2)-(4);
- development of recommendations to ensure or increase survivability.

This method is detailed in connection with the physical and technical nature and design features of the vibration system. Further, we consider individual aspects of applying the method to a specific vibration system.

4. Numerical example

As an object of survivability analysis, we consider a metal composite high-pressure tank of electric propulsion spacecraft engines designed to store a working medium (xenon). The weight of the tank itself is 30 kg, the weight of the working medium is 570 kg, and thus the total weight of the tank with the working medium is 600 kg. The tank is suspended using a system of pre-tensioned composite slings on the primary structure of the hull (PSH), which is a cylindrical shell made of carbon fiber rods (Figure 1). PSH is rigidly fixed in its lower part. The ends of each group of four slings are mounted in one of twenty-four attachment points on the primary structure of the hull (Figure 2).

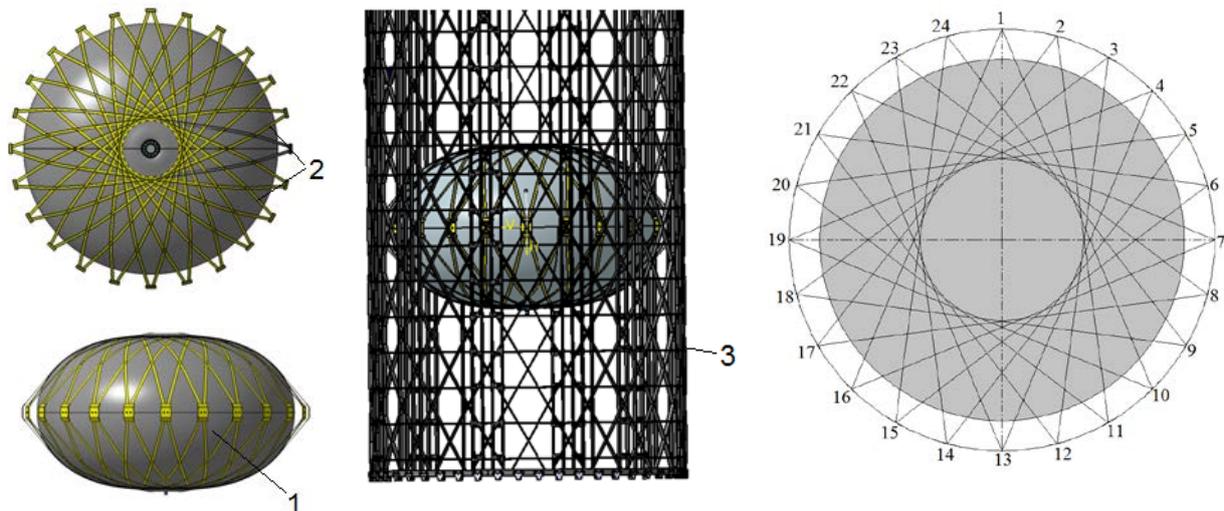


Figure 1. Overall view of the object: 1 – tank; 2 – slings; 3 – a fragment of the primary structure of the hull.

Figure 2. Diagram of attaching slings to the bearing structure.

The object in question experiences significant unsteady loads during the launch of the spacecraft into orbit. In this regard, its behavior as a vibration system with its dynamic properties determining its operability is of interest. A hypothetically possible damage to individual structural elements of the PSH obviously leads to a local decrease in stiffness and, as a consequence, a change in the dynamic properties of the vibration system. The goal of the survivability analysis of the object under consideration is to study the effect of damage to the PSH, modeled by removing its structural elements from the design model, on the lowest natural frequency of vibrations.

Justification of possible scenarios of the occurrence and accumulation of damage in the vibration system is as follows. The bulk of the weight of the vibration system is concentrated in tank 1, while its stiffness is provided mainly by the PSH 3 structure and the stiffness of its elements. During dynamic loading, inertial force effects from the tank are transmitted via slings to the zones of their attachment to the PSH. Therefore, the PSH structural elements in these zones are the most loaded and can be damaged. This is confirmed by mechanical tests of the bearing capacity with increased horizontal loads on the tank: damage occurred to the PSH structural elements on which the attachment points of the slings were mounted. Thus, a single damage in this work means the destruction of four rod structural elements of the PSH, which make up one of the twenty-four attachment points of the tank suspension sling to the PSH.

Consider two fundamentally different scenarios of damage accumulation. In the first of them (I), the structural elements are destroyed so that the remaining connections in the horizontal plane are located symmetrically with respect to the vertical axis passing through the tank's center of gravity. The second scenario (II), on the contrary, assumes successive occurrence of the damage in the circumferential direction. The quantity and numbers of destroyed nodes in accordance with the diagram in Figure 2 for each of the scenarios are presented in the table.

In accordance with the table, for the finite element model of the vibration system, five computational experiments for scenario I and twenty-one computational experiments for scenario II were implemented, during each of which the corresponding rod elements were removed from the model, and repeated modal analysis was performed with recording of the lowest frequency.

The lowest natural vibration frequency of the undamaged structure f_1 is 19.1 Hz. As individual structural elements of the PSH are removed, the frequency gradually decreases. As a quantitative characteristic of survivability, we consider the dependence in the form (3) for two scenarios of damage development (Figure 3). In accordance with (5), in this case the accumulated number of destroyed attachment points of the slings on the PSH was considered as the degree of damage.

The approximation of the obtained dependences by polynomials allowed us to obtain the following expressions with the concretization of the form of function $h(d)$:

for scenario I $S^r = -8 \cdot 10^{-7} d^6 + 5 \cdot 10^{-5} d^5 - 0.0011 d^4 + 0.0118 d^3 - 0.0595 d^2 + 0.1032 d + 1$;

for scenario II $S^r = 10^{-5} \cdot d^4 - 0.0005 \cdot d^3 + 0.002 \cdot d^2 + 0.0092 \cdot d - 0.9957$.

As can be seen from the obtained dependencies, in the first variant of the damage accumulation scenario, the lowest natural frequency is characterized by low sensitivity to destruction of individual structural elements: when 12 attachment points are destroyed, the frequency decreases only by 6% ($f_{1d=12} / f_1 = 0.94$), 16 nodes – by 14% ($f_{1d=16} / f_1 = 0.86$). With further increasing the degree of damage, the frequency begins to decrease rapidly, at $d > 20$ it drops to almost 0. In the second scenario, the sensitivity of the lowest natural frequency to damage is higher: a decrease of 6% is already observed when six attachment points are destroyed ($f_{1d=6} / f_1 = 0.94$), when 12 nodes are destroyed, the frequency drops by 38% ($f_{1d=12} / f_1 = 0.62$).

Table 1. The quantity (numbers) of the destroyed attachment points at various stages of development of damage accumulation scenarios.

Stage	Scenario	
	I	II
1	4 (1, 7, 13, 19)	1 (1)
2	8 (1, 4, 7, 10, 13, 16, 19, 22)	2 (1-2)
3	12 (2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24)	3 (1-3)
4	16 (2, 3, 5, 6, 8, 9, 11, 12, 14, 15, 17, 18, 20, 21, 23, 24)	4 (1-4)
5	18 (2-4, 6-8, 10-12, 14-16, 18-20, 22-24)	5 (1-5)
6	20 (2-6, 8-12, 14-18, 20-24)	6 (1-6)
7	22 (2-12, 14-24)	7 (1-7)
8		8 (1-8)
9		9 (1-9)
10		10 (1-10)
11		11 (1-11)
12		12 (1-12)
13		13 (1-13)
14		14 (1-14)
15		15 (1-15)
16		16 (1-16)
17		17 (1-17)
18		18 (1-18)
19		19 (1-19)
20		20 (1-20)
21		21 (1-21)

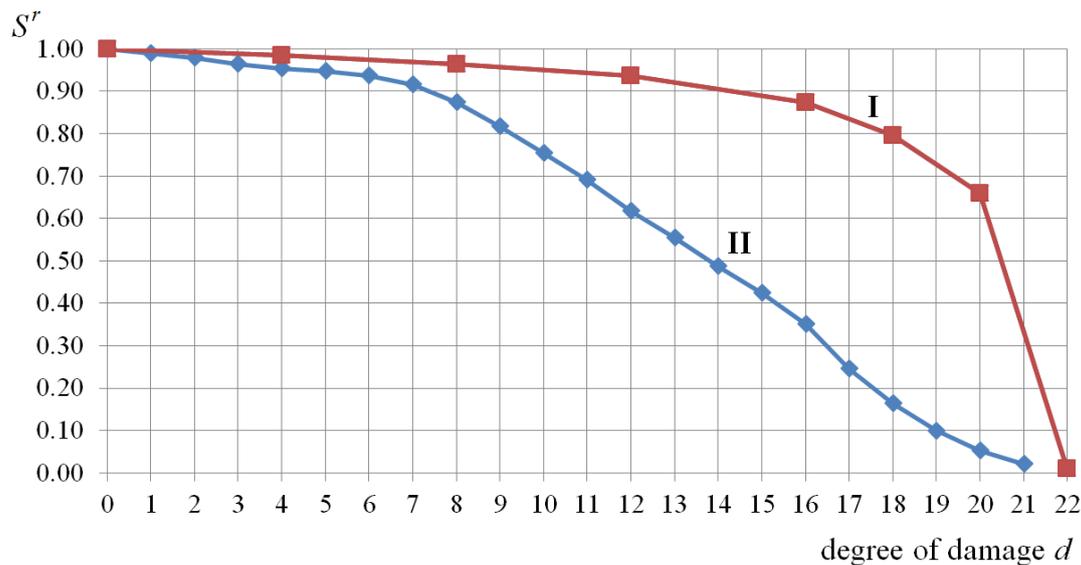


Figure 3. Dependencies $S^r = f_{1j} / f_1 = h(d)$ for two damage accumulation scenarios. Thus, in the first scenario of damage accumulation, the structure survivability turns out to be much higher since the dynamic characteristics of the vibration system remain sufficiently stable when a large enough number of PSH attachment points are destroyed.

5. Conclusion

The survivability of vibration systems can be estimated by establishing and analyzing the dependencies of the natural vibrations frequencies on the degree of structural damage. For vibration systems that include rod structures, it is reasonable to consider the quantity of destroyed structural elements (removed from the design model) as this degree. The most important part of survivability analysis procedure is damage accumulation scenarios: the same object can have completely different survivability characteristics with different scenarios.

The considered approach to assessing survivability was successfully tested in the analysis of the dynamic properties of a high-pressure xenon tank being a part of the electric propulsion engine of a spacecraft and mounted on the hull's primary structure, after various types of damage to the latter.

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