Longitudinal stress waves in viscoelastic and plastic rods

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Abstract. Dynamic loading of rod construction from viscoelastic and plastic material is considered. For a task solution on distribution of a wave of tension of compression in a rod, the interconnected hypotheses, which allowed receiving an analytical solution, offered. Convenient analytical representation of tension distribution of a rod longwise for the wave of compressing tension of extending in viscoelastic and plastic material of a rod at blow to its end face tough weight is offered. Satisfactory approval of results of numerical calculations and data of the previous researchers that does use of this method practically feasible for a solution of applied tasks is shown.

1. Introduction

Rod constructions are widely applied in mechanical engineering, an aviation and shipbuilding, construction and other areas of the industry. Calculation of such constructions significantly becomes complicated if loading has dynamic character, and, in certain cases and shock. For example, in drives of locks of pipelines at a hydraulic shock the spindle [1], which represents the rod having a screw thread, is exposed to dynamic loading that in addition weakens this part [2, 3]. Shock loading can lead to loss of longitudinal stability a rod, to its bend, and, as a result, to execution failure by the mechanism of the functions [4-6]. At blow in material of a rod, longitudinal waves of normal stresses compression extend. Moreover, distribution of tension in a wave extremely unevenly longwise of a rod and quickly changes in time. For elastic waves, there is a known analytical solution. At the initial moment of blow, as a rule, tension reaches the great values significantly exceeding an elasticity limit. In this case, there is a viscos-plastic attenuation of tension amplitude and determination of real tension distribution longwise of a rod for applied engineering tasks becomes actual.

2. Definition of problem

The task solution on longitudinal waves distribution of normal compression stresses is under construction within a hypothesis of flat sections, and the stress-strain state of the rod material completely is described by the components of stress σ_{zz} (further - σ)and deformation ε_{zz} (further - ε) tensors. The system of equations describing a task includes the movement equation, a condition of continuity and the defining ratio [7]:

$$\begin{cases} \dfrac{\partial \sigma(z,t)}{\partial z} = \rho \dfrac{\partial V(z,t)}{\partial t} \\ \dfrac{\partial V(z,t)}{\partial z} = \dfrac{\partial \varepsilon(z,t)}{\partial t} \\ E\dfrac{\partial \varepsilon(z,t)}{\partial z} = \dfrac{\partial \sigma(z,t)}{\partial t} + E \cdot \Phi(\sigma,\varepsilon) \end{cases}, \quad (1)$$

where ρ - rod material density; $V(z,t)$ - speed of particles of a rod; $\Phi(\sigma,\varepsilon)$ - the function approximating properties of a rod material.
In an initial instant, a material rod not stressed, not deformed and stationary:

$$\sigma(Z,0) = \varepsilon(Z,0) = V(Z,0) = 0 .$$

Boundary conditions described by function $\sigma(0,t) = F(t)$, where $F(t)$ - beforehand given function, and $F(0) \neq 0$.
As indicial equation, type of dependency used:

$$E\frac{\partial \sigma(z,t)}{\partial t} = \frac{\partial \varepsilon(z,t)}{\partial t} + E\gamma(\sigma(z,t) - \sigma_S) \quad (2)$$

where γ - the coefficient of dynamic viscosity depends on properties of material and is defined by least squares approximation of the experimental curves of the dynamic loading of the material; σ_S - dynamic yield stress of material.
Figure 1 shows the dynamic loading curves of steel 30 (C1030) [8]. Solid lines are received experimentally, dashed - approximation of these data by the equation (2). Thus σ_S = 410 MPa, γ $=4{\cdot}10^{-6}{\cdot}(\mathrm{MPa{\cdot}s})^{-1}$. The agreement of the results with the experiment is satisfactory.

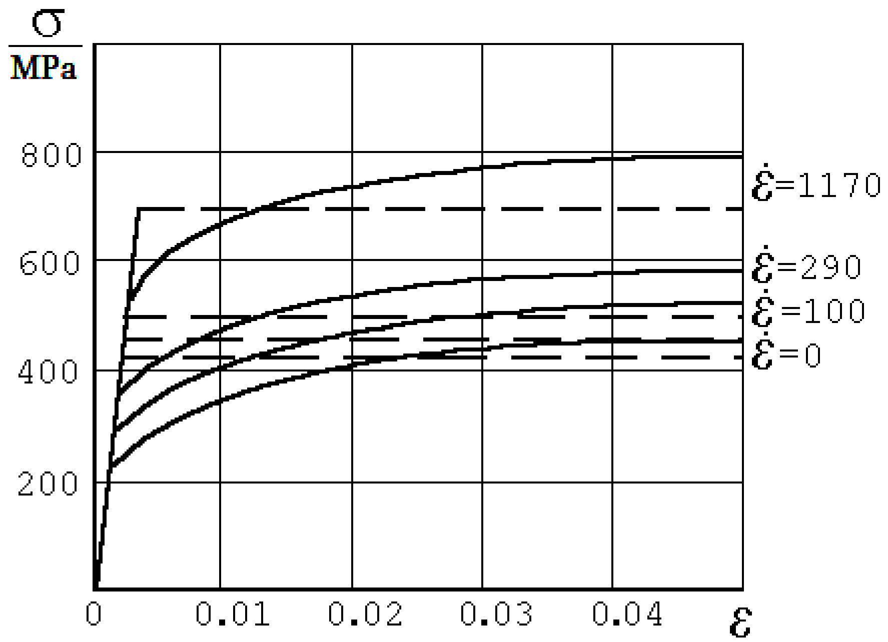

Figure 1. Curves of dynamic loading of steel 30 (C1030)

The system (1) represents system of quasilinear differential equations in partial derivatives of first order of hyperbolic type and therefore, it is reasonable to solve it by method of characteristics [9].
Let's add it with identical ratios for total differentials of required functions:

$$\left.\begin{aligned} d\sigma(z,t) &= \frac{\partial\sigma(z,t)}{\partial z}dz + \frac{\partial\sigma(z,t)}{\partial t}dt \\ d\varepsilon(z,t) &= \frac{\partial\varepsilon(z,t)}{\partial z}dz + \frac{\partial\varepsilon(z,t)}{\partial t}dt \\ dV(z,t) &= \frac{\partial V(z,t)}{\partial z}dz + \frac{\partial V(z,t)}{\partial t}dt \end{aligned}\right\} \tag{3}$$

As a result, received system of six linear algebraic equations (1) and (3) with unknown functions:

$$\frac{\partial\sigma}{\partial z},\ \frac{\partial\sigma}{\partial t},\ \frac{\partial\varepsilon}{\partial z},\ \frac{\partial\varepsilon}{\partial t},\ \frac{\partial V}{\partial z},\ \frac{\partial V}{\partial t}.$$

Let's demand that the received system had infinite set of solutions. For this purpose, it is necessary that its main determinant equaled to zero. From this condition received three families of the equations of characteristics: $dz = 0$, $dz = \pm a\,dt$. For the solution along the characteristics to be finite, it is necessary to consider the equality to zero of the other determinants of the system. For this purpose, we will replace the last column of the main determinant with the corresponding right parts of the equations (1), (3) and we will consider its equality to zero. Consistently substituting in the received equality of the equation of characteristics, we will construct differential ratios between required functions. Finally, ratios between differentials of required functions along the characteristic directions in the phase plane $z0t$ taking into account indicial equation (2), will register so [7]:
- along the characteristic direction $dz = 0$:

$$d\sigma(z,t) - Ed\varepsilon(z,t) + E\gamma(\sigma(z,t) - \sigma_s)dt = 0; \tag{4}$$

- along the characteristic direction $dz = adt$:

$$d\sigma(z,t) + a\rho dV(z,t) + E\gamma(\sigma(z,t) - \sigma_s)dt = 0, \tag{5}$$

- along the characteristic direction $dz = -adt$:

$$d\sigma(z,t) - a\rho dV(z,t) + E\gamma(\sigma(z,t) - \sigma_s)dt = 0. \tag{6}$$

Thus, integration of quasilinear system of differential equations of hyperbolic type (1) at the set initial and boundary conditions is reduced to integration of ratios (4)-(6) along the respective characteristic directions. In a general view the solution can be received only in number that significantly complicates its engineering application. Creation of analytical representation of distribution of tension longwise of with great dispatch loaded rod is an actual task and is a subject of researches of this article.

3. Theory
In certain cases, for example, when modeling destructions of screws of locks drives of pipelines at a hydraulic shock [10] or losses of longitudinal stability with great dispatch loaded rod [11] it is enough to receive approximate solution of a task about longitudinal waves of tension in rods in a convenient analytical form. Let's accept the following interconnected hypotheses:

1. Strain rate of material of with great dispatch loaded rod in neighborhoods of a contact area (the loaded end face) is so high that material there (especially in an initial stage of blow) within the constitutional ratio of Malvern-Sokolowski [9] behaves as linearly elastic.
2. Nature of tension change along the positive characteristic direction of a grid of characteristics is similar to corresponding change on a leading wave front of compression tension for which initial condition is the condition at the loaded rod end face at the time of the blow beginning.
As boundary condition we will accept blow tough weight on a rod end face. Thus, initial conditions remain zero, and boundary register in the form of a Cauchy problem for ordinary differential equation of first order:

$$M\frac{dV(0,t)}{dt} = -\sigma(0,t)\cdot S\,. \tag{7}$$

Then, in an instant $t=0$ from origin of coordinates $z=0$ along a rod the direct wave, starts extending at the front which undergo a rupture of tension of the first sort, deformations and the speed of particles. Thus, values of ruptures cannot be set randomly as they are subject to some conditions following from hypotheses of mechanics of a deformable solid and laws of classical mechanics. Using a hypothesis of continuity of material in the course of deformation and material momentum conservation law, received conditions of a dynamic and kinematic continuity in a type:

$$\sigma = -a\cdot\rho\cdot V \text{ and } V = -a\cdot\varepsilon\,. \tag{8}$$

At the front of the wave the ratio (5) is true. Substituting the first of conditions (8) in this equation, we receive the ordinary differential equation describing tension variation on a leading wave front:

$$2d\cdot\sigma(z,t) = -E\cdot\gamma(\sigma(z,t)-\sigma_s)dt\,. \tag{9}$$

The right member of equation (9) is a consequence of the first hypothesis.
The solution of the equation (7) for tension variation on a contact area in time registers so:

$$\sigma(0,t) = \sigma_0\cdot e^{-\frac{\sqrt{E\cdot p}}{M}\cdot S\cdot t}, \tag{10}$$

where $\sigma_0 = a\cdot\rho\cdot V_0$ - tension at the loaded end face at the time of blow.
Using the second hypothesis, a ratio (8) and the equation (10), and also the equation describing tension variation on a leading wave front (9), we receive dependence for determination of tension of compression in any point of the phase plane:

$$\sigma(z,t) = \left(\sigma_0\cdot e^{-\frac{a\cdot\rho\cdot S}{M}\left(t-\frac{z}{a}\right)} - \sigma_S\right)\cdot e^{-\frac{1}{2}a\cdot\rho\cdot\gamma\cdot z} + \sigma_S\,. \tag{11}$$

4. Discussion of results

Results of numerical calculation and its comparison to a solution of a wave task [7] (solid line), are presented on Figure 2. The shaped line illustrates a solution of a wave task in a form (10) within the accepted system of hypotheses at the following input data: M = 0.015 kg, V_0 = 1000 m/s, S =2·10^{-5}m^2, t =2.89·10^{-6} s.

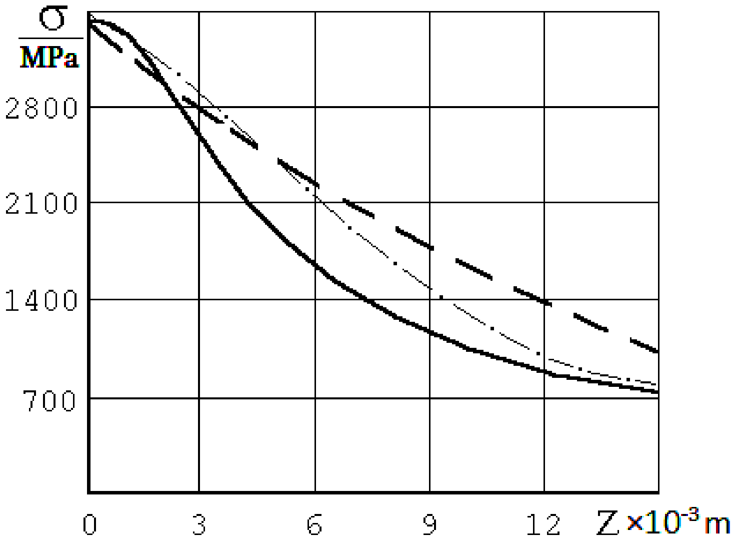

Figure 2. Different candidate solutions of a wave task

Discrepancy [7] (solid line) is connected with results of a numerical solution with a margin error approximation of properties of material (Figure 1) and can be reduced, on the basis of accounting of the heterogeneity of dynamic characteristics of material of a rod, introduced by inelastic deformation of material in a wave of tension. It is for this purpose offered in the defining equation (2) the dynamic yield point of material in the form of function considering reduction of a yield point longwise of a rod at strain rate reduction in indicial equation:

$$\sigma_s(z) = 800 - 3.79 \cdot 10^4 \cdot z, \text{ MPa},$$

and coefficient of dynamic viscosity in the form of dependence of distribution longwise a rod owing to reduction of viscos-plastic attenuation of amplitude of tension at strain rate reduction:

$$\gamma(z) = 3.78 + 110 \cdot z, \text{ (MPa·s)}^{-1}.$$

The dash-dot line in Figure 2 illustrates these changes in definition equation. Absence in the received diagrams so-called «a plateau of deformations» (section at the loaded end face with small attenuation of tension) which is observed at numerical calculations, is connected with impossibility the equations like Sokolowski-Malvern (with linear approximation of viscos-plastic properties of material) to describe this effect. For this purpose, it is necessary to use the defining ratios like P. Perzyna. Considering that this section makes only 1…2 mm, it is possible to draw a conclusion not only on quantitative, but also high-quality approval of results.

Tension variation longwise of a rod is presented on Figure 3 at the following input data: diameter of a rod d = 10 mm, the struck weight M = 0.061 kg, blow speed V_0 = 100 m/s at different coefficients of viscosity.

The analysis of drawing allows to draw a conclusion that attenuation of tension in a wave happens to reduction of coefficient of viscosity more slowly, but, under other equal conditions, tension reaches an elasticity limit in the same coordinate. It should be noted that the coefficient of viscosity has no impact on the maximum tapping interval.

Tension variation longwise of a rod is presented on Figure 4 at the same input data at different speeds of blow.

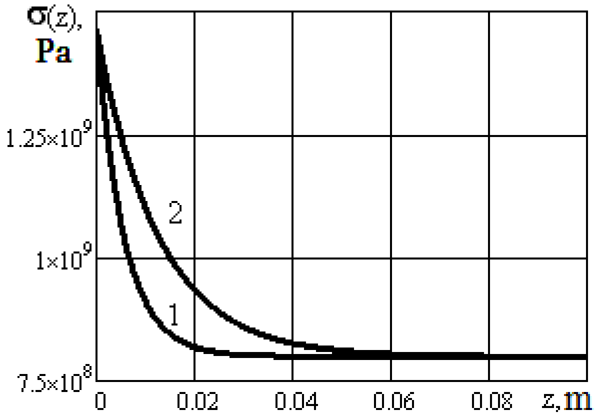

Figure 3. Tension variation longwise rod:
curve 1 - $\gamma = 10^{-5}$ (MPa·s)$^{-1}$,
curve 2 - $\gamma = 5\ 10^{-6}$ (MPa·s)$^{-1}$

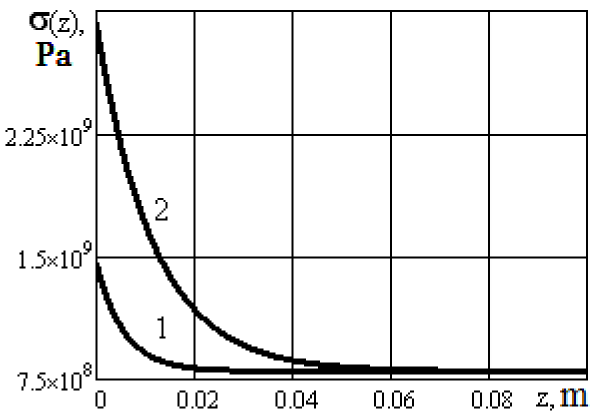

Figure 4. Tension variation longwise a rod at different speeds of blow:

curve 1 - V_0 = 100 m/s; curve 2 - V_0 = 200 m/s

The analysis of Figure 4 allows to draw a conclusion that with growth of speed of blow amplitude of tension increases, but thus speed does not influence intensity of attenuation. Similar results were received in works [12, 13].

5. Conclusions

Convenient analytical representation of distribution of tension longwise of a rod for the wave of compression tension, extending in elastic and viscos-plastic material of a rod at blow to its end face tough weight, is offered.

Satisfactory approval of results of numerical calculations and data of the previous researchers, that does use of this method practically feasible for a solution of applied tasks, is shown.

6. References

[1] Lopa I V, Efimova A I and Zhukaev A I 2015 Definition of differential pressure when closing a gate lock *News of Tula State University. Technical Science* **11-1** pp 186-191

[2] Lopa I V, Patrikova T S and Patrikova E N 2011 Definition of moment of inertia of lateral section of the screw *News of Tula State University. Technical Science* **2** pp 236-247

[3] Lopa I V, Patrikova T S and Efimova A I 2011 A cross bend of the screw taking into account change of moment of inertia on its length *News of Tula State University. Technical Science* **2** pp 241-245.

[4] Proskuriakov N E, Lopa I V 2016 Calculation of spindle of pipeline fittings on the longitudinal stability *Procedia Engineering*. **152C.** pp. 265-269.

[5] Hirofumi M H, Seifried R, Eberhard P and Kawamura S 2011 Analysis of repeated impacts on a steel rod with visco-plastic material behavior *European Journal of Mechanics - A/Solids* **30**, Issue 3, May-June. pp. 336-344.

[6] Wang H, Yin X, Deng Q, Yu B, Hao Q and Dong X 2017 Experimental and theoretical analyses of elastic-plastic repeated impacts by considering wave effects *European Journal of Mechanics - A/Solids*, **65**, September-October. pp. 212-222

[7] Baranov V L and Lopa I V 1993 Longitudinal elastic viscos-plastic waves in rods of the final length *Izvestiya VUZov. Mashinostroyeniye* Issue 1, p 54.

[8] Baranov V L, Lopa I V, Chivikov Z Ch and Simeonov P S 1997 Stability of with great dispatch loaded rods (Tula: Tula State University Press). 128 p.

[9] Baranov V L and Lopa I V 1989 Radial waves of torsion and longitudinal shift in an elastic and viscos-plastic thick plate in no isothermal setting *Izvestiya VUZov. Mashinostroyeniye*. Issue 7. pp. 27-35.

[10] Proskuriakov N E, Lopa I V and Trapeznikov E V 2017 Control of influence of a thread on a bending of screw *Journal of Physics: Conf. Series* **1088** 012028.

[11] Baranov V L and Lopa I V 1995 Instability of with great dispatch loaded rods *Izvestiya VUZov. Mashinostroyeniye* Issue 3 p. 45.

[12] Patent 2064647 Russian Federation, MPK F 41 H 1/02 5/04. Multilayer armor / Avraamov G V, Baranov V L, Gevorkyan A M and Lopa I V, publ. 27.07.96, Bulletin No. 15.

[13] Baranov V L, Zubachev V I, Lopa I V and Schitov V N 1996 Some questions of design of bullets of small arms (Tula: Tula State University Press). 116 p