

Methods of the analytical study of vibratory drum forced vibrations during the soil compaction

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Abstract. The fundamental principles of the vibration theory as a classical science are widely used for the description of movements of technical objects and mechanical systems. Vibrations of real objects are complicated by non-linear characteristics of stiffness of the medium to be compacted and viscous friction forces. Usable vertical movements of a vibratory drum have been determined whereby the technological compaction operation of materials and soils is performed. The methods of the vibratory drum vertical movement determination have been developed by means of considering the vibratory drum downward movement during the semi-period of one rotation of the eccentric weight. The operating process of the road roller vibratory drum is composed of sequential periodic transient operating processes of the material compaction and idle cycles in each turn of the eccentric weight rotation. Within the process of the study the operating process is considered of the vibratory drum downward movement during the material compaction, consisting of free damped and undamped forced downward movements of the vibratory drum. For the first time the damping time of transient processes during the analysis of forced oscillatory processes is regulated by the semi-period time of the eccentric weight rotation $t = 0.5T$.

Keywords: vibratory drum, vibration exciter, vertical movement amplitude, resonance.

1. Introduction

The theory of structures to be created and characteristics of vibration rollers lags behind the technical development and cannot explain the physical processes of the vibratory drum interaction with the material being compacted by an analytical method [1-6].

In the fifties of the last century the design bases relative to parameters and operating modes of vibration rollers were formulated, which have remained relevant at present [7-15]. The relevance of this type of machinery is confirmed by a high cost and demand in construction operations. Many problems of dynamic studies control over processes of the material and soil compaction by vibratory drums remain insufficiently studied. This hinders the further development of vibration machinery and its improvement.

2. Problem statement

The problems of studying the vibratory drum vertical movements by means of a detailed examination of the vibratory drum downward movement during the semi-period of the eccentric weight rotation have been methodically clarified. The relevance of such approach is explained by the fact that while moving downwards the vibratory drum performs the useful job – material compaction, and while moving upwards an idle accompanying action of the vibration exciter is performed.



The dynamic non-symmetry of a working downward movement and idle upward movement is taken into account. Vibratory drum downward and upward movements are performed with different values of movements during the time period of one turn of the vibration exciter eccentric weight rotation. The frequency of vertical vibrations of modern vibration rollers is represented by $f = 20 \div 80$ Hz. The duration of the material compaction cycle is equal to a half of the eccentric weight rotation period $t=0.5T$ and for modern vibration rollers it is equal to $0.5T = 0.0167 \div 0.00625$ s [9].

The periodic dynamic process with a small repetition period of compaction cycles is considered. The compaction process within the specified operating mode is implemented with the constant frequency f . During each semi-period of the eccentric weight rotation, free vibrations appear and energy dissipation processes of oscillating bodies occur. While moving the vibratory drum downwards the dissipation process is considered that occurs during the semi-period of movement $0.5T$, methods of the vibratory drum vertical movement calculation within a steady oscillatory operating mode of the vibration exciter during the material compaction have been developed.

3. Theory

In the modern classic theory of vibrations, free, damped and forced vibrations are considered for simple single-mass mechanical systems [7, 8, 9]. By default, modern papers consider symmetric oscillatory processes aimed at damping oscillations or stabilizing. The analysis of forced vibrations is restricted to obtaining a complete analytical solution of the differential equation of the oscillating body movement containing several components, to which the conditions of their existence are imposed. For example, in papers [7, 8] it is indicated that components of free vibrations are damping during more or less long period of time, after which they can be neglected. It means that the duration of the transient damping process in these cases is not made more specific and is not considered.

In this paper for the first time the transient dynamic process damping time has been made more specific by the duration of one half-turn of the eccentric weight rotation $t=0.5T$, which represents a very small value for modern vibration rollers, and therefore it's not required for the study of fast dynamic processes. Such approach to the study of the vibratory roller oscillatory process is caused by many reasons.

The differential equations of the vibratory drum forced movements appear to be true only for the process of the vibratory drum moving downwards, and vibratory drum upward movements are described by another differential equation.

For the study of the vibratory drum oscillatory process the assumption relative to the constant angular velocity of the vibration exciter eccentric weight $A_{\text{rotation}} = p = \text{const}$ has been made. With a uniform rotation of the eccentric weight the dynamic damping processes occur during each semi-period $0.5T$ of the vibratory drum downward movement.

Figure 1 shows the vibratory drum consisting of three main masses: m_1 – unbalanced mass of the vibration exciter eccentric weight; m_2 – mass of the vibratory drum; m_3 – additional mass of the vertical cantledge on the vibratory drum axis from the roller frame, which is connected with the vibratory drum by rigid rubber-metal shock absorbers.

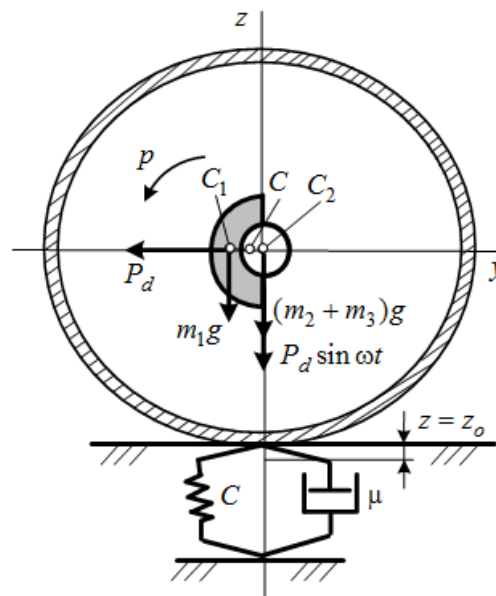


Figure 1. Vibratory drum on the ground surface.

In this paper the assumption has been made relative to the hard link of the masses m_2 and m_3 (see Figure 1). The mass m_3 is considered as a small value compared with the mass m_2 , i.e. $m_3 = (0,1 \div 0,3)m_2$. The compacted soil medium has elasticity with the stiffness factor C and viscous friction factor μ .

The eccentric weight rotates with the angular velocity p , radial driving inertia force P_d of the vibration exciter is determined by the formula of paper [7]

$$P_d = m_1 r_1 p^2, \quad (1)$$

where r_1 – is the eccentricity of the mass m_1 of the eccentric weight.

In Figure 1 the eccentric weight is located in the initial horizontal position, when the formation of the vertical downward directed driving force P_z starts, which implements the operating process of the material compaction

$$P_{dz} = P_d \sin pt, \quad 0 \leq t \leq 0.5T. \quad (2)$$

Beside the driving force of the vibration exciter P_d and gravity forces m_1g , m_2g , m_3g , the mechanical system is affected by the restoring force which is proportional to the movement z and viscous friction force which is proportional to the speed \dot{z} .

The novelty of the design diagram as opposed to known systems consists in the consideration of the eccentric weight as an element of a multi-mass mechanical system having the center of mass at the point C (see Figure 1).

Imagine the vibratory drum movement equation in the traditional form of the theory of forced vibrations [7-10]

$$\ddot{z} + 2n\dot{z} + \omega^2 z = h \sin pt, \quad 0 \leq t \leq 0.5T, \quad (3)$$

where T – is the period of one turn of the eccentric weight.

The novelty of writing equation (3) consists in the condition of limiting the time of transient processes of the vibratory drum downward movement damping by the semi-period time of vibrations $t = 0.5T$.

The general solution of equation (3) is considered as the sum of the general solution of equation (3) without the right-hand side and particular solution of equation (3) with the right-hand side.

The general solution of equation (3) of damped vibrations without the right-hand side is in the form as in paper [7, 8]

$$z_1 = e^{-nt} (C_1 \cos \omega_1 t + C_2 \sin \omega_1 t), \quad (4)$$

where ω_1 – the frequency of damped vibrations is determined by the formula of paper [7]

$$\omega_1 = \sqrt{\omega^2 - n^2}. \quad (5)$$

For the vibratory drum the damping factor n in equation (3) is in the following form

$$n = \frac{\mu}{2(m_1 + m_2 + m_3)}, \quad (6)$$

where μ – is the viscous friction factor of the soil.

The angular frequency of the natural vertical vibrations of the vibratory drum is determined by the formula of papers [7-10]

$$\omega = \sqrt{\frac{C}{m_1 + m_2 + m_3}}, \quad (7)$$

where C – is the soil stiffness factor.

The stiffness factor C in formula (7) while lowering down the vibratory drum is increased in the material compaction process. When the eccentric weight runs idle the compaction is stopped, parameter C as the factor of the vibratory drum interaction with the soil loses its physical meaning. For this reason the additional condition in equation (3) is substantiated.

In this paper for the heavy vibration roller SV 900T we take $\mu = 343985$ Ns/m; $C_{\min} = 25000$ kN/m, $C_{\max} = 97222$ kN/m.

In papers [7, 8] when determining the integration constants C_1, C_2 in equation (4) zero initial conditions are taken that are inappropriate for the vibration exciter (see Figure 1).

A new solution of equation (4) of damped vibrations has been obtained for the first time for the mechanical system with the vibration exciter. The design diagram of the mechanical system allows to determine the integration constants C_1 and C_2 for new initial conditions of the eccentric weight vibration exciter (see Figure 1).

For the determination of the integration constants C_1 and C_2 , we form the system of equations

$$\left. \begin{aligned} z_1 &= e^{-nt} (C_1 \cos \omega_1 t + C_2 \sin \omega_1 t); \\ \dot{z}_1 &= -ne^{-nt} (C_1 \cos \omega_1 t + C_2 \sin \omega_1 t) + e^{-nt} (-C_1 \omega_1 \sin \omega_1 t + C_2 \omega_1 \cos \omega_1 t). \end{aligned} \right\} \quad (8)$$

The initial conditions for the system of equations (8) are of the following form: at $t=0$ $z=z_0=0$; $\dot{z}=\dot{z}_0$ – the initial speed of the vibratory drum downward movement is determined by means of the theorem of the center of mass motion (see Figure 1)

$$\dot{z}_0 = r_c p, \quad (9)$$

where r_c – is the radius of the vibratory drum center of mass rotation, $r_c = CC_1$ (see Figure 1).

From the system of equations (8) after the insertion of the initial conditions, we find the integration constants

$$C_1 = 0; \quad C_2 = r_c \frac{p}{\omega} = \frac{m_1}{m_1 + m_2 + m_3} r_1 \frac{p}{\omega}.$$

The amplitude of free vibrations $A(z)$ is determined by the formula

$$A(z) = \frac{m_1 r_1}{m_1 + m_2 + m_3}. \quad (10)$$

The solution of equation (3) takes the following form

$$z_1 = e^{-nt} A(z) \sin \omega_1 t. \quad (11)$$

We'll represent the particular solution of equation (3) of forced vibrations in the traditional form [7, 8]

$$z_2 = A(z_f) \sin(pt - \varepsilon). \quad (12)$$

In equation (12) $A(z_f)$ is the amplitude of forced vibrations.

Inserting equation (12) into equation (3) we determine the amplitude of forced vibrations [7-10]

$$A(z_f) = \frac{h}{\sqrt{(\omega^2 - p^2)^2 + 4n^2 p^2}}. \quad (13)$$

In equation (13) the value h is determined by formula [7, 10]

$$h = \frac{P_d}{m_1 + m_2 + m_3} = \frac{m_1 r_1 p^2}{m_1 + m_2 + m_3}. \quad (14)$$

The angle ε of the vibration phase displacement relative the exciting force phase is determined by formula [7, 8]

$$\varepsilon = \arctg \frac{2np}{\omega^2 - p^2}. \quad (15)$$

In paper [7] additionally to two considered components of the equation solution (3) it's proposed to add an intermediate component, which takes into account additional information on additional damped free vibrations caused by forced vibrations. Such addition has been accepted in this paper.

The final solution of differential equation (3) of forced vibrations using the additional damped transition component is of the following form

$$z_o = e^{-nt} A(z) \sin \omega_1 t + A(z_f) \sin(pt - \varepsilon) + A(z_f) e^{-nt} \sin(\omega_1 t - \varepsilon), \quad 0 \leq t \leq 0.5T, \quad (16)$$

where z_o – is the vibratory drum downward movement.

A new concept of using equation (16) has been proposed. The well-known methods of using equation (16) lie in the fact that during more or less long period of time the damping of the first and second components of the equation occurs. In the result the third component remains, which is considered as the steady-state forced vibration of the object under consideration [7, 8]. Such approach does not allow to obtain useful information regarding the material and soil compaction process by the vibratory drum. It is proposed to consider the transition process of the vibratory drum forced movement according to equation (16) only for the downward movement during the time $t = 0.5T$ of the semi-period of the eccentric weight rotation.

The vibratory drum downward movement z_o according to equation (16) is a generalized movement, which is defined as the sum of the considered movements

$$z_o = z_1 + z_2 + z_3, \quad (17)$$

where z_3 – is an additional damped component of the free vibration movement, which is determined by the formula

$$z_3 = A(z_f) e^{-nt} \sin(\omega_1 t - \varepsilon), \quad 0 \leq t \leq 0.5T. \quad (18)$$

Studies have shown that the components of equation (17) to some extent are involved in the formation of the vibratory drum generalized movement z_o . In each working cycle of subsequent eccentric weigh revolutions the vibratory drum downward movement and material compaction are performed. During the time $t = 0.5T$ the dissipation energy transient processes are implemented according to equation (16). In this connection it has been found that upward and downward movements are not symmetrical, i.e. the vibratory drum upward and downward movements represent different physical processes.

New concepts have been introduced into the vibratory drum movement theory. Along with the amplitude $A(z)$ of natural damped vibrations and amplitude of forced vibrations $A(z_f)$, which is not damped, a new concept is introduced – the generalized vertical movement of the vibratory drum downwards z_o during the transient process time equal to the semi-period of the eccentric weight rotation $t = 0.5T$.

4. Results discussion

The considered theory has allowed us to receive new knowledge on the vibratory drum vertical movements during the material compaction.

The studies have been conducted for the heavy vibration roller SV 900 T (Japan), having the following technical parameters: overall mass is 19700 kg, vibration exciter frequency $f = 28$ Hz; eccentric weight driving force $P_d = 343$ kN; eccentric weight mass eccentricity $r_1 = 0.091$ m; weight of the eccentric weight $m_1 = 121.7$ kg; weight of the vibratory drum with the eccentric weight $m_1 + m_2 = 5000$ kg; vertical cantledge mass $m_3 = 1850$ kg.

Figure 2 shows the ε angle dependences of the phase error of the driving force P_d and vibratory drum vertical movement for the soil with the stiffness factor $C = 35111$ kN/m (Figure 2,a) and for the soil with the stiffness factor $C = 97222$ kN/m (Figure 2,b) according to data of paper [11]. In the below resonance mode at $p/\omega \leq 1$ the ε phase value is positive which is monotonically increased to 90° .

At resonance the ε function discontinuity occurs which changes the plus sign to (-90°) , then in the operating mode the ε value is monotonically increased maintaining a negative value. Such results have been obtained for the vibratory drum for the first time.

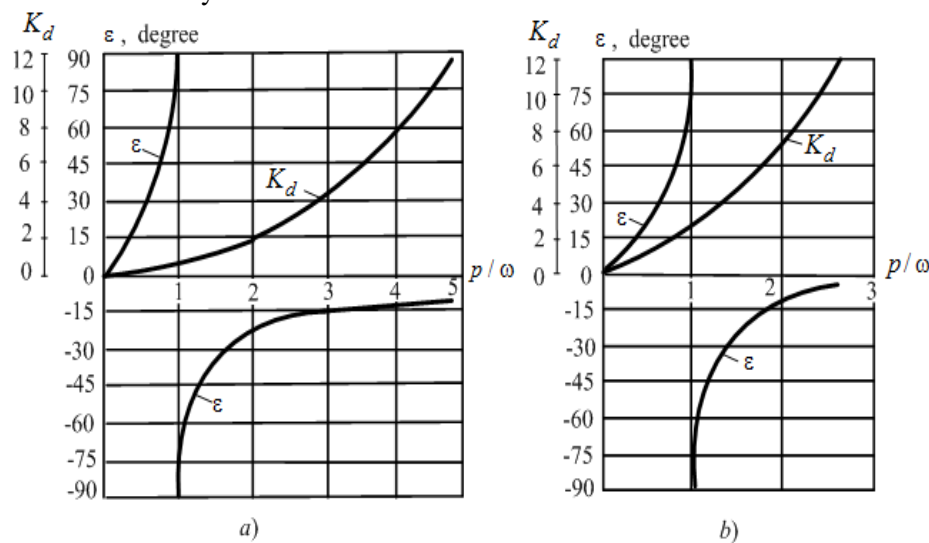


Figure 2. Dependences of the ε angle of the driving force phase error on the relative frequency p/ω .

Figure 2 additionally shows the dependence of the vibratory drum dynamic factor K_d on p/ω . In this paper the dynamic factor is equal to the ratio of the eccentric weight driving force to the vibratory drum weight [10]

$$K_d = \frac{P_d}{(m_1 + m_2 + m_3)g}.$$

Relative to the vibration roller the manufacturer declared the nominal frequency $f = 28$ Hz. In Figure 2,a $K_d = 4.45$ corresponds to this frequency.

When the vibratory drum works on the heavy soil at $C = 97222$ kN/m (Figure 2,b) the dynamic factor is also equal to $K_d = 4.45$ at the frequency $f = 28$ Hz.

Figure 3 – Figure 6 show the vibratory drum frequency characteristics for soils with different stiffness factors: $C = 25000$ kN/m (Figure 3), $C = 35111$ kN/m (Figure 4); $C = 74444$ kN/m (Figure 5); $C = 97222$ kN/m (Figure 6). The range of stiffness factor C is adopted according to data of paper [11]. In the presented figures (Figure 3 – Figure 6) it is apparent that in the resonance zone $p/\omega \leq 1$ the amplitude of forced vibrations $A(z_f)$ has a maximum within the range $A(z_f) = 2 \div 3.5$ mm

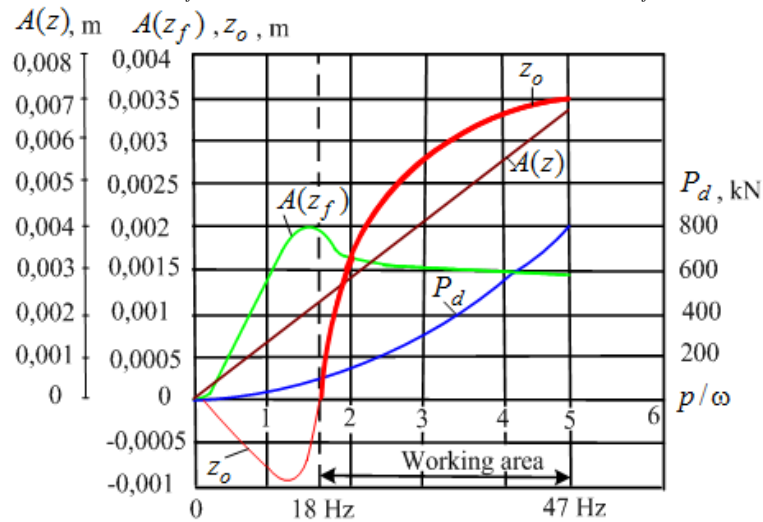


Figure 3. Frequency characteristics of the vibratory drum on the soil ($C = 25000$ kN/m).

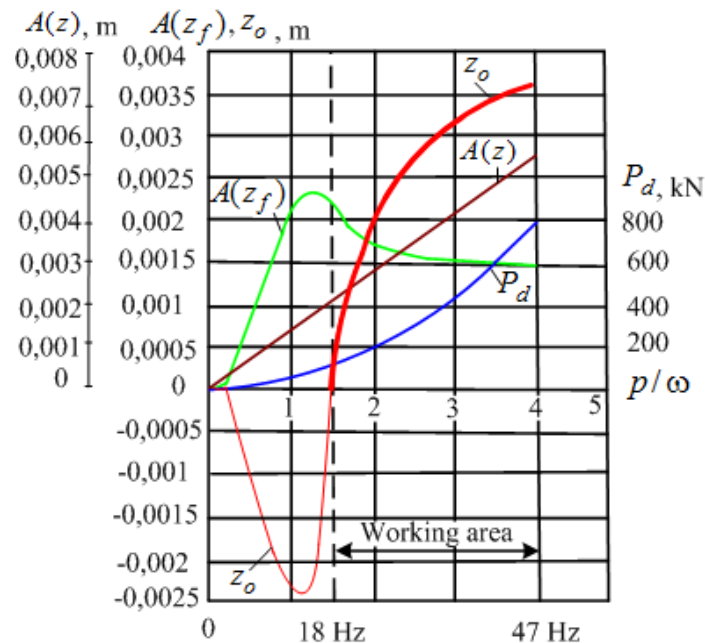


Figure 4. Frequency characteristics of the vibratory drum on the soil ($C = 35111$ kN/m).

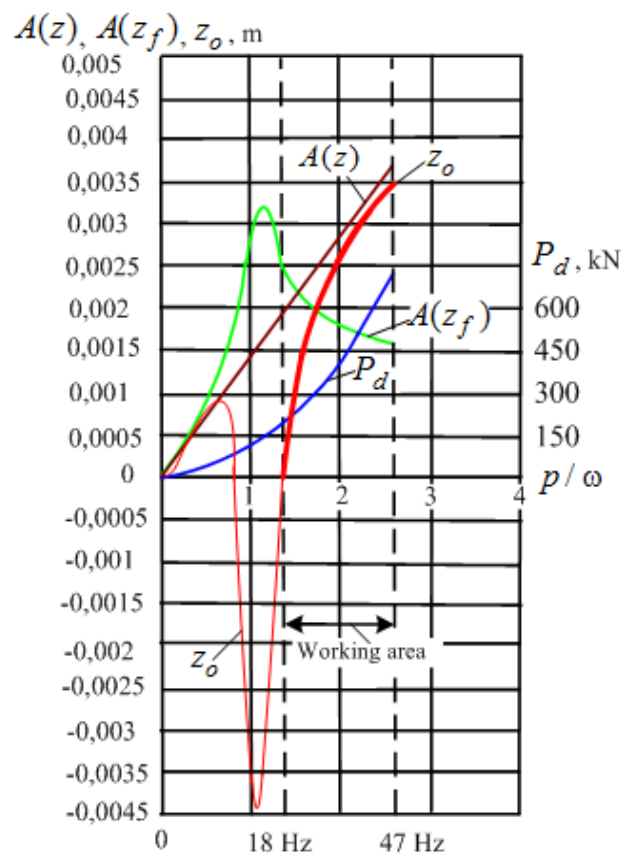


Figure 5. Frequency characteristics of the vibratory drum on the soil ($C = 74444 \text{ kN/m}$).

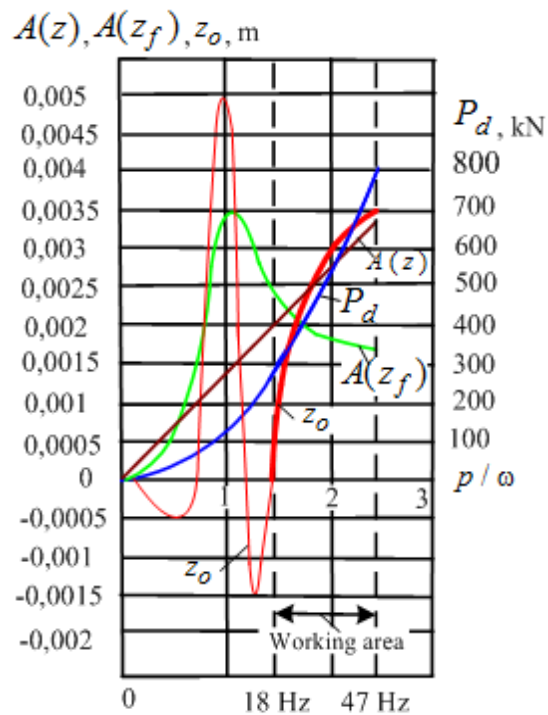


Figure 6. Frequency characteristics of the vibratory drum on the soil ($C = 97222 \text{ kN/m}$).

5. Consideration of the results

In this case in the above resonance zone the amplitude of forced vibrations $A(z_f)$ varies within the range $A(z_f) = 1.5 \div 1.7$ mm. For different soils the amplitude of forced vibrations $A(z_f)$ is determined by formulas (13), (14).

The driving force of the vibration exciter P_d depends on the rotation frequency of the eccentric weight p and is the quadratic function $P_d = f(p)$ according to equation (1). The amplitude of free vibrations $A(z)$ is a linear function of the frequency p according to equation (10).

The maximum value of the free vibration amplitude $A(z)$ for different soils is within the range $A(z) = 6.7 \div 3.3$ mm, i.e. it is decreased with increasing the soil strength.

The dependences of the generalized movements of the vibratory drum z_o during the material compaction on the relative frequency p/ω are the most important in the presented graphs. In all cases in the above resonance zone a stable zone of increasing the vibratory drum vertical movements z_o is observed with increasing the relative frequency p/ω .

In the above resonance zone at $p/\omega > (1.4 \div 1.5)$ there is a working stable zone of the vibratory drum positive downward movements, which represent the generalized movement z_o . The positive values of z_o according to equation (17) contain a negative value of the movement z_2 from the amplitude of forced vibrations $A(z_f)$. This means that in the above resonance zone the forced vibrations z_2 are in the antiphase with the forcing influence of P_{dz} of the vibration exciter. However, at the expense of other components equation (17) forms the vibratory drum positive downward movement z_o , which is in phase with the forced influence of P_{dz} and performs the useful effect - soil compaction.

These dependences confirm the possibility of realizing the significant driving force P_d for the implementation of an efficient compaction process with the amplitude $z_o = 3.5$ mm.

Within the frequency range indicated in Figure 3 – Figure 6 the stable control of the vertical movements z_o is possible by varying the frequency p of the driving force.

6. Conclusion

Thus, it has been established that the real vertical movements z_o are formed by three oscillatory functions in equation (16), in this case two functions have the damping time of transient processed equal to the semi-period of the eccentric weight rotation $t = 0.5T$, and the third component depends on ε and has a negative sign in the above resonance zone of the vibratory drum operation.

The developed methods allow to select rational operating modes and parameters of the vibration roller ensuring required efficiency of the material and soil compaction process. New knowledge has been obtained relative to the nature of changes of the vibratory drum movements while changing the vibration frequencies.

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