

# About the election strength criteria in calculations for long-term strength for non-isothermal processes of loading

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**Abstract.** In the paper the research methods of axially symmetric elastic - plastic stress strain state of structures in the form of thin single and multiple layer shells of rotation with an account of material damage at creep is represented. The long-term strength criterion definition, on the basis of which the time to failure of the structure is calculated, is one of the important task. There are multiple long-term strength criteria, but very little detail recommendations on their usage and concretizing, which is what makes this problem so difficult to solve. The authors proposed a simple and easy-to-use method for concretizing the Pisarenko–Lebedev generalized long-term strength criterion. To verify the reliability of the proposed method of concretizing the long-term strength criterion, the calculations of time to failure a cylindrical shell under the action of internal pressure and a thin lamina with a round hole stretched by external radial forces are carried out. The calculation results obtained by the proposed method differ from the experimental ones by 1.8% and 3.1%. These results are the most reliable. Therefore, proposed method for calculating the long-term strength criterion enables more accurate damage and time-to-rupture predictions, compared with other calculation methods.

## 1. Introduction

The paper continues the research conducted by the authors and previously published in such works as [1 - 4]. In this works the research methods of axially symmetric elastic – plastic stress strain state of structures in the form of thin single and multiple layer shells of rotation with an account of material damage at creep is represented. The algorithm of task solution is realized as the software package in FORTRAN.

The structure is consist from isotropic materials and before loading is in undeformed state, and then it is affected by power and thermal loadings. The form of the shell is set by the parameters determining of its geometry configuration, the number of layers and the thickness of layers. Mechanical characteristics of materials are set as instant diagrams of strain, creep and long durability, linear thermal expansion coefficients and Poisson coefficients. The curvilinear orthogonal system of coordinates is used.

The task of structure calculation is solved in quasistatic and geometrically linear statement on the basis of elastic - plastic strain theory of solid state elements on the slight curvature trajectories.

Under these conditions in materials of the structure creep deformations can arise. The development of creep deformation will be lead by the elaboration of micro damages in the materials that can lead to its destruction.

At solve the task in view the along the meridian and across the thickness is divided into separate elements. The heating and loading process of the structure is devided into discrete loading stages so that the moments of time, separating the stages, in the best way coincided with the moments of the



direction change of the strain process. The duration of loading states is sets so to consider the time changes of external loadings, of the temperature field, and to investigate the history of stress – strain state to the point of structure destruction because of the creep.

To describe the processes of damage elaboration in the material, the kinetic equation of damage proposed by Yu.N. Rabotnov is used:

$$\frac{d\omega_c}{dt} = C \left( \frac{\sigma_{eq}}{1 - \omega_c} \right)^Q \quad (1)$$

where  $\omega_c$  is the scalar damage parameter;

$C$  and  $Q$  – parameters dependent on temperature and determined from long-term strength curves obtained from uniaxial tension tests of standard samples at fixed temperatures;

$\sigma_{eq}$  – equivalent stress, which is one of the criteria for long-term strength.

The equivalent stress definition, on the basis of which the time to failure of the structure is calculated, is one of the important and difficult problems. The equivalent stress is calculated by the long-term strength criterion. The most reasonable and universal criterion is Pisarenko–Lebedev generalized long-term strength criterion [5].

In [4], the authors proposed a simple and easy-to-use method to specifying the Pisarenko–Lebedev generalized strength criterion. In that method, the parameter which characterizes the degree of responsibility shear deformation for micro failure, creates favourable conditions for material failure and cracking [5], is taken to equal the relative residual contraction at break  $\psi$ . For lot of structural materials, reference books contain  $\psi$  value and other data which are required at long-term strength calculations [6]. In [3] the analysis of experimental data was carried out, and its showed that this approach enables more accurate determination of the equivalent stress. In addition, the stress – strain state of a flat lamina with a hole was calculated [4].

However, requires the comparison of calculation results obtained by the considered long-term strength criterion, with the results of in-site experiments, to assess the reliability of the proposed method for concretizing the Pisarenko–Lebedev generalized long-term strength criterion. This method will be sufficiently justified if the results of calculations and experiments are well coordinated only.

## 2. Research Goal

The goal hereof is to:

- perform verification calculations of the time to failure of simple axisymmetric structures for which there are experimental values at the time of failure (for example, annular laminas and pipes);
- make a conclusion about the reliability of the of the proposed method for concretizing the Pisarenko–Lebedev generalized long-term strength criterion, on the basis of comparison of the obtained results.

## 3. Materials and Methods

To determine the equivalent stress by the Pisarenko–Lebedev generalized long-term strength criterion the following formula is used [5]:

$$\sigma_{eq} = \chi \sigma_i + (1 - \chi) \cdot \sigma_1 \quad (2)$$

For concretizing that criterion is required to determine the  $\chi$  parameter. In [4] the methods of determining the  $\chi$  parameter for different types of loading are described in detail.

This paper proposes a simple and easy-to-use method to find the  $\chi$  parameter in practical calculations. In this case, the  $\chi$  parameter for this or that material is deemed to equal the relative

residual contraction at break  $\psi$  obtained by testing it for creep and long-term strength at constant stress and a fixed temperature T.

Innumerable practical tests of standard cylindrical samples for creep and long-term strength, made of metals and alloys and carried out in different years (e.g. [7]), are confirmed the possibility of applied of this approach.

Analysis of the results is showed: at large stress levels substantially exceeding the yield strength of the material, the inside-granular destruction (viscous or plastic destruction) dominates, which accompanied by localized intensively creep, whereby a contraction emerges. The relative residual contraction of the standard specimen, obtained at creep tests, is practically equal to the relative residual contraction value obtained at the tensile test of standard specimens, in the same conditions.

Meanwhile, at low stress slightly exceeds the creep strength of the material, inter-granular strain and fracturing (brittle fracture) dominates, with the material being embrittled without contraction; fracturing occurs at low cumulative creep deformation and over a long time. The deformation of the specimen is almost uniform along the entire length. The accumulated irreversible deformation value will be small, and the time to failure – large, about thousands hours, and in some cases – thousands tens hours. The relative residual contraction is nearly zero. In the conditions of real structures operations, this is a serious danger, because the material is destroyed suddenly and without visible deformation.

Based on these results and on the first-approximation assumption that the relative residual creep contraction of a fractured specimen is linearly dependent on the tensile stresses at which the tests occur, one can construct a simple dependence for fixed temperatures of the following type [4]:

$$\psi_c = \frac{\sigma}{\left(\sigma_s - \sigma_{0.2/10^5}\right)} \psi \quad (3)$$

where  $\psi_c$  is the relative residual contraction of a standard specimen of circular cross-section, obtained by testing for creep and long-term strength at a constant stress  $\sigma$  at a fixed temperature T;

$\psi$  – the relative residual contraction, obtained by stretching a standard specimen of circular cross-section at its instantaneous deformation at the same temperature;

$\sigma_s$  – the material strength limit;

$\sigma_{0.2/10^5}$  – the material creep limit at a given temperature.

Thus, the approach proposed herein implies using the following formula to find the equivalent stress for each element of structure:

$$\sigma_{eq} = \psi_c \sigma_i + (1 - \psi_c) \cdot \sigma_1 \quad (4)$$

Here,  $\psi_c$  is found as per (3). The statement of problem is complemented with the following mechanical properties of the material as observed at fixed temperature within its normal operating range for this structure: ultimate strength, creep strength, and relative residual contraction. These data are available in the reference literature for most structural materials, e.g., [8].

In [4] the analysis of simulation results obtained using the proposed approach was carried out. This analysis showed is that the chosen long-term strength criterion affects the estimated time to rupture. This effect is greater for lower loads and longer time of loading. The proposed variant to concretize the Pisarenko–Lebedev generalized long-term strength criterion allows to account for the altered plastic properties of the material (the embrittlement); such alteration occurs during the development of creep deformations.

The next stage of research is to verify the reliability of the proposed method of concretizing the Pisarenko–Lebedev generalized long-term strength criterion.

A large number of papers devoted to experimental and theoretical methods of studying the accumulation of damage in crystalline bodies and their destruction have been published by now.

For example, in [9], a continuum damage mechanics (CDM)-based creep model was proposed to study the creep behavior of T91 and T92 steels at high temperatures. It is shown that the developed model is able to predict creep data for the two ferritic steels accurately up to tens of thousands of hours.

In [10], Manson-Haferd model was employed to predict the creep life of T91 and T92 steels. The results of calculation and experiments demonstrate that the Manson-Haferd model is suitable for the two steels.

In [11], creep rupture behavior heat-resistant steel under uniaxial and multiaxial state of stress was investigated. Creep tests were conducted at the temperature of 923K under the stress 125MPa. Specimens for testing were U-notched cuts, which would create local stress concentration. The creep rupture mechanism was investigated based on the SEM fractography analysis. The ductility of steel gets worse due to the multiaxial state of stress. Fracture appearance of the creep rupture specimens was found to be dependent on the notch geometry. It implies that the ductile fracture and intergranular brittle fracture co-exist under this test condition during creep.

Most of these works are devoted to the study of damage to cylindrical samples, or other structures with a simple geometric shape. However, to verify the reliability of the proposed method of concretizing the Pisarenko–Lebedev generalized long-term strength criterion, the experimental results on the study of creep and fracture of thin shells of rotation are required.

Unfortunately, these experiments are practically not carried out at present. Therefore, the research team tested the experimental results published in previous years, e.g. [12, 13]. Then, we are consider these experiments in more detail.

#### 4. Experimental Results

The results of two experiments – a cylindrical shell under the action of internal pressure and a thin lamina with a round hole stretched by external radial forces will be considered in that paper. Information about the time of structural failure and deformation values at some points in time is available for both experiments.

The accuracy of determining the time of structural failure is estimated by comparing the results of calculation and experiment.

Steel 20 was taken as a material in both structures. The mechanical characteristics of that material are taken from the reference literature [8]. The research team used these characteristics earlier in the calculations of other designs.

In this paper we will consider the calculations results of the time to failure of structures and their compliance with the experimental results. Analysis of structural failure kinetics, deformation development and damage accumulation of these structures will be analyzed by the research team in other publications later.

The time to fracture of both structures was studied in terms of four different long-term strength criteria:

- maximum principal stress (the Johnson's criterion):  $\sigma_{eq} = \sigma_1$ ;
- stress intensity (the Katz's criterion):  $\sigma_{eq} = \sigma_i$ ;
- the Sdobyrev's criterion:  $\sigma_{eq} = 0,5 \cdot (\sigma_1 + \sigma_i)$ ;
- the Pisarenko–Lebedev's generalized long-term strength criterion:  $\sigma_{eq} = \psi_c \sigma_i + (1 - \psi_c) \cdot \sigma_1$ .

Calculations using the Johnson, Katz, and Sdobyrev's criterion were carried out by the authors earlier in [1] and [2].

It is established that usage the Johnson's criterion allows the minimum estimated time to rupture (brittle fracture), and usage the Katz's criterion returns the maximum estimated time to rupture (plastic destruction) [2, 14]. Thus, using any long-term strength criterion will produce a value within the range limited by the Johnson's and Katz's criteria values. The Sdobyrev's criterion is a half-sum of these criteria, and the contribution of viscous and brittle character of destruction in the overall picture needs to be equal [4].

4.1. Creep and destruction of a cylindrical shell under the action of internal pressure (S.N.Katz's experiment)

In this experiment the creep process and fracture of a thin cylindrical shell under a constant internal pressure was investigated. This experiment by S. N. Katz [12] in 1955 was performed. The experiment goal was to study the reliability of the long-term strength criterion ().The ends of the shell are muffled by the bottoms. Therefore, in its elements, there are meridional stress, in addition to the district. The Steel 20 shell had the following dimensions: average diameter 35,7 mm; wall thickness 1,7 mm. Calculations were run at a constant temperature  $T=500^{\circ}\text{C}$  and a constant internal pressure  $P=10.9$  MPa.

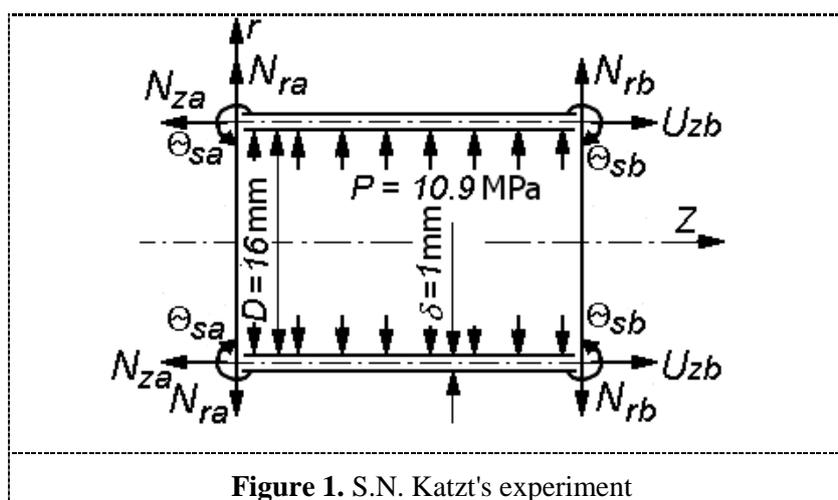


Figure 1. S.N. Katz's experiment

Table 1 shows the experimental and calculated values of the structure failure time; the values were obtained using four different long-term strength criteria.

Table 1 shows that the Johnson criterion allows to determine the time of shell failure with a sufficiently small error of 5.5%, compared to the experiment. At the same time, the Sdobyrev and Katz criterions gives significantly inflated the time of shell failure (respectively 47% and 137%), which is completely unacceptable.

Table 1. Results of S.N. Katz's experiment.

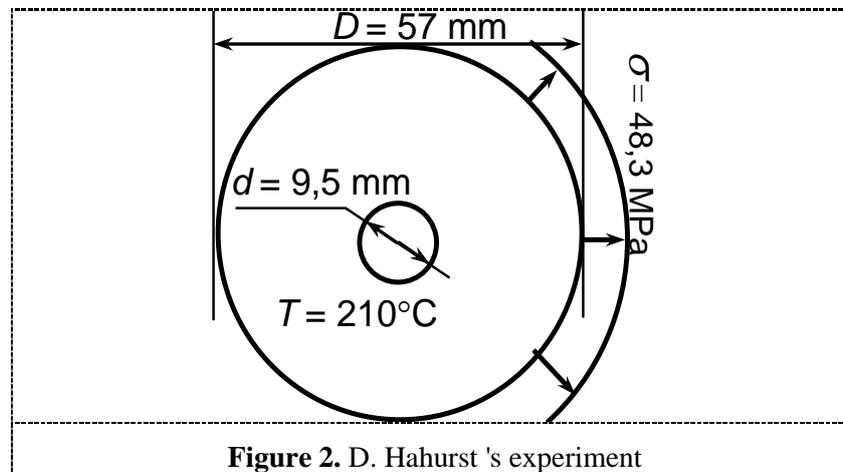
	Experiment	Results of calculations using the following criteria			
		$\sigma_{eq} = \sigma_1$	$\sigma_{eq} = \sigma_i$	$\sigma_{eq} = 0,5 \cdot (\sigma_1 + \sigma_i)$	$\sigma_{eq} = \psi_c \sigma_i + (1 - \psi_c) \cdot \sigma_1$
The time of the destruction (hour.)	1176	1112	2785	1732	1154
Error, %		5,5	137	47	1,8

The Pisarenko–Lebedev generalized long-term strength criterion allows to determine the result closest to the experimental one (about 1,8%).

4.2. Creep and destruction of a thin lamina with a round hole (D. Hahurst 's experiment)

In this experiment the change kinetics of stress-strain state and creep process of thin circular lamina with a central circular hole was investigated. The lamina had the following dimensions: external diameter 57 mm; internal diameter 9,5 mm. Calculations were run at a constant temperature  $T=500^{\circ}\text{C}$

and stretching load intensity of 48.3 MPa. The experiment on a specially designed installation and special samples was carried out. The results are given in [13].



Calculations showed that stresses affecting the most loaded points of the lamina at such load are about 150 MPa, its below the yield strength of the material. This fact shows that it is dominated by brittle fracture over viscous fracture.

Table 2 shows that Johnson's criterion allowed to determine the fracture time of the lamina with a sufficiently small error of 15%, compared to the experiment (similar to the first experiment). And the Sdobyrev and Katz criterions showed the result of much more the actual durability of the lamina (respectively, 2.7 times and 1.5 times). The results also confirm the higher reliability of the results obtained used the Pisarenko–Lebedev generalized long-term strength criterion (error is 3.1%).

**Table 2.** Results of D. Hahurst's experiment.

Experiment	Results of calculations using the following criteria				
	$\sigma_{eq} = \sigma_1$	$\sigma_{eq} = \sigma_i$	$\sigma_{eq} = 0,5 \cdot (\sigma_1 + \sigma_i)$	$\sigma_{eq} = \psi_c \sigma_i + (1 - \psi_c) \cdot \sigma_1$	
The time of the destruction (hour.)	410	348	1118	602	397
Error, %		15,1	172,7	46,8	3,1

Thus, the research team concluded that the use in the calculations of long-term strength criteria that do not correspond to the prevailing mechanisms of destruction, can lead to significant errors.

## 5. Conclusions

In the course of research the adequacy of the proposed method for concretizing the Pisarenko–Lebedev generalized long-term strength criterion was confirmed. The obtained results coincide with similar experimental results. Therefore, proposed method for concretizing the Pisarenko–Lebedev generalized long-term strength criterion enables more accurate damage and time-to-rupture predictions, compared with other long-term strength criteria. However, this paper considers only two variants of structures, and one material. Therefore, it is impossible to assert the universality of the considered long-term strength criterion on the basis of the obtained results. The research team behind this paper plans to continue out such research further on.

Authors will be grateful to all specialists performing similar structural strength calculations for any constructive comments and recommendations.

## 6. References

- [1] Belov A V 1989 Axially symmetric elasto-plastic stressed – strained state of shells of rotation with an account of material damage at a creep, abstract of dissertation, Kiev.
- [2] Polivanov A A 2004 Axially symmetric elasto-plastic deformation of multilayered shells of rotation with an account of material damage at a creep, abstract of dissertation, Volgograd.
- [3] Belov A V, Neumoina N G 2014 On the use of Pisarenko–Lebedev generalized strength criterion in strength calculations for non-isothermal loading processes *International Journal of Applied and Basic Research* **9** p 8–10
- [4] Belov A V, Neumoina N G, Polivanov A A 2019 About the choice of strength criterion in calculations for long-term strength in non-isothermal loading processes. *Modern high technologies* **1** p 20–25
- [5] Lebedev A A 2010 The development of theories of strength in the mechanics of materials. *Strength problems* **5** p 127–146
- [6] Shevchenko Y. N, Terekhov R G, Braikovskaya N S, Zakharov S M 1994 Investigation of the processes of destruction of body elements as a result of material damage during creep. *Applied mechanics* V **30** 4 p 21–30
- [7] Volegov P S, Gribov D S, Trusov P V 2015 Damage and destruction: classical continual theories *Physical mesomechanics* V **18** 4 p 68–87
- [8] Pisarenko G S, Yakovlev A P, Matveev V V 2008 Reference mechanical characteristics of materials (Kyiv) 816 p
- [9] J P Pan, S H Tu, XW Zhu, L J Tan, B Hu and Q Wang 2017 Creep Tests and Modeling Based on Continuum Damage Mechanics for T91 and T92 Steels *IOP Conf. Series: Materials Science and Engineering* **281**
- [10] J P Pan, S H Tu, G L Sun, X W Zhu, L J Tan and B Hu 2018 High-temperature creep properties and life predictions for T91 and T92 steels *IOP Conf. Series: Materials Science and Engineering* **292**
- [11] Facai Ren, Xiaoying Tang 2017 Creep behavior of Grade 91 steel under uniaxial and multiaxial state of stress *IOP Conf. Series: Materials Science and Engineering* **242**
- [12] Katz S N 1955 The study of long-term strength carbon steel tubes *Thermal engineering* **2** p 37–40
- [13] Hahurst D 1972 Creep rupture under multi – axial states of stress *J. of mech. and phys. of solids* **20** p 381–390
- [14] Aliev M M, Shafieva S V, Gilyazova S R 2015 Criterion of long-term strength for materials of different resistance Materials of the scientific session of scientists of Almet'yevsk State Oil Institute **1** 1 p 254–257