

# Deformation of flexible gear of harmonic drive

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**Abstract.** Harmonic drives have high load capacity, kinematic accuracy, and small weight and dimensions' parameters, compared to other transmissions. Due to the variable stiffness in the areas of the teeth and tooth sockets, the toothed rim of the harmonic drive during bending takes the shape of a polyhedron. This effect most strongly appears when the gear ratio is less than 60. For this reason the geometric synthesis of harmonic drives with gear ratios of less than 60 is an important task. In this paper, the law of deformation of the flexible gear is given by harmonic oscillations of the inner angles of the polygon. The dependences which allow the teeth of a rigid gear to be profiled, taking into account the effect of the "polyhedron" of the toothed rim of the flexible gear are obtained. The proposed dependences are the basis of the geometric synthesis of harmonic drive elements.

**Keywords:** harmonic drive, rigid gear's teeth profiling, flexible gear, form of deformation of a gear rim

## 1. Introduction

Harmonic mechanical drives have appeared rather recently. For the first time harmonic mechanical drives has been offered by A.I. Moskvitin in 1947 [1]. The harmonic gear drive with the mechanical generator of waves has been invented by V. Masserom (USA) in 1959 [2]. Since then constructive varieties of the harmonic drives one of which is harmonic chain drive [3] have been developed. The essential contribution to the working out of the theory of synthesis of harmonic drives was made by M.N. Ivanov and E.G. Ginzburg [1, 4]. Now the geometry and kinematics of harmonic drives are well enough studied [5, 6]. By this time there are two methods of definition of mechanical trajectories of points belonging to a flexible gear of harmonic drive. Trajectories of points of a flexible gear are the necessary condition of profiling of tooth gearing of rigid and flexible gear. The first method considers flexible gear's in the form of a shell. Therefore for the exposition of the shape of deformation of flexible gear the theory of shells is used, and for simplification of calculations the shell is substituted by ring and described by elementary functions [1]. The second method is grounded on the exposition of velocity constant curve of a flexible gear in the system of axes linked with rigid and flexible gear's [4]. When studying of laws of motion of a flexible gear in the form of chain, the new method of form defining the of flexible gear and laws of its motion [6] has been gained. This method can be applied to synthesis of harmonic drives with the traditional flexible gear designed with a shape of thin-walled glass or ring that is important for harmonic drives with low gear ratio. Harmonic drives were widely adopted in various areas of engineering [2, 7-9]. Harmonic drives possess a number of advantages



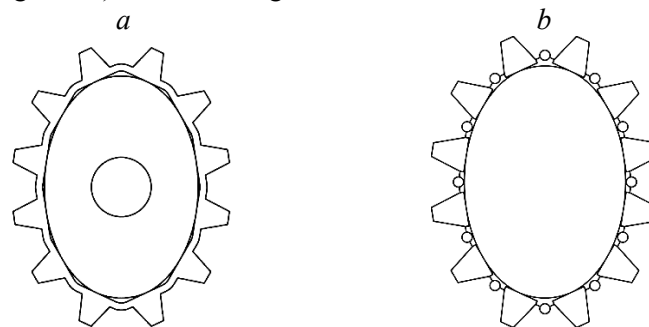
such as: multiple-tooth contact, high traffic capacity, and kinematic accuracy, small weight and overall size in comparison with other mechanical gears. Despite conclusive advantages, harmonic drives possess also disadvantages, such as: high value of the minimum gear ratio is 60 [7], the restricted rapidity of the input shaft 2000 – 6000 rpm, complexity of manufacturing of a flexible gear. Now there is a trend of expansion of a scope of harmonic drives to a zone of low gear ratio to 60. So *Harmonic Drive AG* company (Germany) mastered exhaustion of harmonic drives with the gear ratio equal to 30.

## 2. Problem statement.

Now the geometry and kinematics of harmonic drives are well enough studied [2, 5]. In theory of harmonic drive, flexible gears with the shape of shell are considered. Therefore to describe a form of deformation of flexible gear the theory of shells is used, and for simplification of calculations the shell is replaced with a ring and described by elementary functions. The existing theory demands specification in following cases: first, different rigidity of a gear rim on sites of teeth and hollows leads to that at gear rim bending its cylindrical surface takes the shape of a polyhedron [6, 7] that is typical for gearing with small number of teeth of a flexible gear, i.e. with the gear ratio less than 60; secondly, the theory of shells and rings is not applicable for wave chain gear in which the flexible gear is replaced by chain [7]. In this connection, synthesis of harmonic drives with the gear ratio less than 60 is an important problem.

### 2.1. Theory

To describe the form of deformation of a flexible gear (figure 1a) with allowance for the effect of effect of the “polyhedron”, we will construct its geometrical model presented in the form of a convex polygon (figure 1a). Let's notice that such geometrical model completely describes a case of application of a chain (figure 1b) as a flexible gear of harmonic drive.



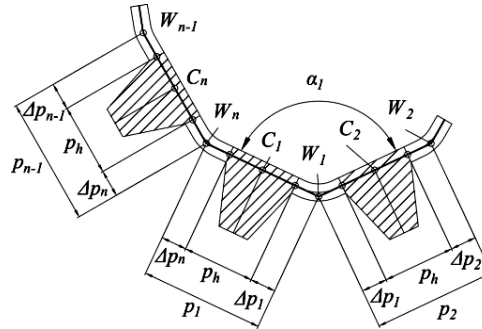
**Figure 1.** The shape of the deformed flexible gear and model of a flexible gear in the form of chain.

In the theory of harmonic drives the form of deformation of flexible gear is defined by the form of the generator of waves. But in case of representation of a flexible gear in the form of a polygon it starts to be impermissible in view of the inconstancy of the form of flexible gear deformation. Therefore, it is necessary to set the law of motion of a flexible gear not by cam form of a generator, but by the function of movement of each party of a polygon, and then to carry out a cam form of the generator and teeth of flexible and rigid gear.

For the most general description of cyclic movement of a flexible link we will accept the geometrical model shown on figure 1b. At representation of model of a flexible gear (figure 1a) in the form of geometrical model (figure 1b) we will accept following assumptions: 1) we will consider teeth as absolutely rigid (the shaded areas, figure 2); 2) sites of hollows between teeth we will consider as pliable. Then the step of a flexible gear, unlike a chain step will be variable and will be defined as:

$$p_i = p_h + \Delta p_i + \Delta p_{i-1}, \quad (1)$$

where  $p_h$  is the thickness of tooth,  $\Delta p_j$  and  $\Delta p_{j-1}$  are the intervals substituting the bending radius of the gap of flexible gear.



**Figure 2.** Flexible gear model.

The intervals  $\Delta p_j$  and  $\Delta p_{j-1}$  are variable and depend on angles  $\alpha_j$ ,  $\alpha_{j-1}$  and forms of deformation of a hollow between teeth of a flexible gear  $\Delta p_j = \alpha_j$ .

If one considers the deformation of gaps between teeth on a circle arch (figure 2) it is possible to write down:

$$\Delta p_j = \frac{p_f}{\pi - \alpha_j \tan \frac{\alpha_j}{2}}, \quad (2)$$

where  $p_f$  – the size of a hollow between teeth of a flexible gear which is defined as an arc length of median line gash between teeth.

It is known that the sum of internal angles of any convex polygon is equal  $\pi \cdot (n - 2)$ , where  $n$  – quantity of its sides. Then we will write down:

$$\alpha_1 + \alpha_2 + \dots + \alpha_j + \alpha_n = \pi(n - 2). \quad (3)$$

All internal angles  $\alpha$  of a polygon, for a case of not deformed flexible gear are equal and make:

$$\theta = \alpha_1 = \alpha_2 = \dots = \alpha_j = \alpha_n = \frac{\pi(n - 2)}{n}, \quad (4)$$

where  $\theta$  is the average value of concluded angle  $\alpha$ .

When using two-wave generator its opposite concluded angles  $\alpha$  of a polygon are equal. Let the corners  $\alpha$  change under the following law:

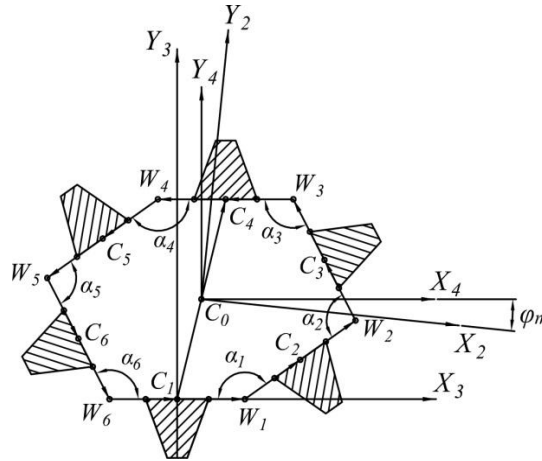
$$\left. \begin{aligned} \alpha_1 &= \theta + \chi \sin(\omega t + \varphi_0) \\ \alpha_2 &= \theta + \chi \sin\left(\omega t + \frac{4\pi}{n} + \varphi_0\right) \\ \alpha_3 &= \theta + \chi \sin\left(\omega t + 2\frac{4\pi}{n} + \varphi_0\right) \\ &\vdots \\ \alpha_j &= \theta + \chi \sin\left(\omega t + (j-1)\frac{4\pi}{n} + \varphi_0\right) \end{aligned} \right\}, \quad (5)$$

where  $\omega$  is the frequency of oscillation,  $s^{-1}$ ,  $t$  is the parameter (time),  $s$ ,  $\varphi_0$  is the initial phase of oscillations,  $\chi$  is the amplitude of oscillations.

It is necessary to notice that the law of  $\alpha$  angles change in expression (5) can be set not only by sinus, but also by cosine.

For definition of the form of deformation of a flexible gear at a given time  $t$  it is necessary to define relative positions of its teeth. It is necessary for the subsequent definition of profiles of teeth of rigid and flexible gears, and also a profile of a cam of the generator.

Let us connect coordinates system  $X_3O_3Y_3$  with any of teeth of a flexible gear, as it is shown in figure 3.



**Figure 3.** Calculation model.

Let us write down the equation of the vector contour constructed on the sides of a polygon:

$$\overline{C_1W_1} + \overline{W_1C_2} + \overline{C_2W_2} + \dots + \overline{C_jW_j} + \overline{W_jC_{j+1}} + \dots + \overline{C_nW_n} + \overline{W_nC_1} = 0 \quad (6)$$

Coordinates of vectors will be defined by formulas:

$$\left. \begin{aligned} x_{C_jW_j} &= \left( \frac{p_h}{2} + \Delta p_j \right) \cos \gamma_j, \\ y_{W_jC_{j+1}} &= \left( \frac{p_h}{2} + \Delta p_j \right) \sin \gamma_j, \\ x_{W_jC_{j+1}} &= \left( \frac{p_h}{2} + \Delta p_j \right) \cos \gamma_{j+1}, \\ y_{C_jW_j} &= \left( \frac{p_h}{2} + \Delta p_j \right) \sin \gamma_{j+1}, \end{aligned} \right\}, \quad (7)$$

where  $\gamma_j$  are the angles between corresponding vector and axis  $O_3X_3$ , which can be defined as follows:

$$\gamma_j = (j-1)\pi - \alpha_1 + \alpha_2 + \dots + \alpha_{j-1}. \quad (8)$$

Trajectories of movement of points  $C$  in system of coordinates  $X_3O_3Y_3$  will be defined as a trajectory of the corresponding ends radius-vectors:

$$\overline{OC_j} = \overline{C_1W_1} + \overline{W_1C_2} + \dots + \overline{C_jW_j}. \quad (9)$$

Then position of points  $C$  will be calculate by formulas:

$$\left. \begin{aligned} x_{3C_j} &= x_{C_1W_1} + x_{C_1W_2} + \dots + x_{C_jW_j}, \\ y_{3C_j} &= y_{C_1W_1} + y_{C_1W_2} + \dots + y_{C_jW_j}. \end{aligned} \right\} \quad (10)$$

Let us write down position of  $C$  points in coordinate system having the beginning on an axis of wave gear and parallel system  $X_3O_3Y_3$ . For this purpose we will use parallel carrying over of coordinate system. It is necessary to find the center of a flexible gear in coordinate system  $X_3O_3Y_3$ . As such point it is possible to consider the middle of any diagonal vector connecting points  $C$ . To simplify the calculation we take vector  $\overline{C_1C_{\frac{n}{2}+1}}$ .

Then coordinates of the center  $C_0$  belonging to an axis of wave transfer:

$$x_{C_0} = \frac{x_{3C_{\frac{n}{2}+1}}}{2}, \text{ and } y_{C_0} = \frac{y_{3C_{\frac{n}{2}+1}}}{2}. \quad (11)$$

Coordinates of points in coordinate system  $X_4O_4Y_4$  which is parallel to the system  $X_3O_3Y_3$  and having the beginning in point  $C_0$  will be:

$$\left. \begin{aligned} x_{4C_j} &= x_{3C_j} - x_{C_0}, \\ y_{4C_j} &= y_{3C_j} - y_{C_0}. \end{aligned} \right\} \quad (12)$$

It is necessary to write down the equations of  $C$  points movement in the coordinate system  $X_2O_2Y_2$  having the beginning in point  $C_0$  and turning through  $\varphi_m$  angle during movement of  $C$  points with respect to coordinate system  $X_4O_4Y_4$ . Angle  $\varphi_m$  we will find by the integration of average arithmetic sum of instant angular speeds  $\omega_j$  vectors  $\overline{W_jW_{j-1}}$ :

$$\varphi_m = \int \frac{\omega_1 + \varphi_1 + \dots + \omega_j + \dots + \omega_n}{n} dt = \frac{\gamma_1 + \gamma_1 + \dots + \gamma_j + \dots + \gamma_n}{n}. \quad (13)$$

Then coordinates of  $C$  points in system  $X_2O_2Y_2$  are:

$$\left. \begin{aligned} x_{2C_j} &= x_{4C_j} \cos \varphi_m + y_{4C_j} \sin \varphi_m, \\ y_{2C_j} &= -x_{4C_j} \sin \varphi_m + y_{4C_j} \cos \varphi_m. \end{aligned} \right\} \quad (14)$$

To further synthesize harmonic drive links, due to the similarity of the trajectories, it is sufficient to know the law of motion of only one point  $C$  in the  $X_2O_2Y_2$  system. For example the  $C_1$  point will be satisfactory for this purpose.

To define the trajectory of  $C_1$  point movement in coordinate system which is connected with a rigid gear we will use coordinate transformation formulas:

$$\left. \begin{aligned} x_1 &= x_2 \cos \frac{\omega t}{-2i_{hg}^k} + y_2 \sin \frac{\omega t}{-2i_{hg}^k}, \\ y_1 &= -x_2 \sin \frac{\omega t}{-2i_{hg}^k} + y_2 \cos \frac{\omega t}{-2i_{hg}^k}. \end{aligned} \right\} \quad (15)$$

where  $i_{hg}^k$  is the gear ratio from the generator to a rigid gear when the flexible gear is motionless.

Let us define trajectories of  $C$  points in the system of coordinates  $XOY$  connected rigidly with the generator using the coordinate transformation equation:

$$\left. \begin{aligned} x &= x_2 \cos \frac{\omega t}{-U} + y_2 \sin \frac{\omega t}{-U}, \\ y &= -x_2 \sin \frac{\omega t}{-U} + y_2 \cos \frac{\omega t}{-U}. \end{aligned} \right\} \quad (16)$$

where  $U$  is the quantity of waves of deformation.

The received trajectory represents the closed oval curve.

The tooth profile equation of a flexible gear we will set in local coordinate system  $X_5O_5Y_5$  where the axis  $O_5X_5$  is precisely vector  $\overline{C_jW_j}$ . Considering the repeatability of trajectories of the points belonging to each vector  $\overline{C_jW_j}$  we will consider movement only one vector. Expressions (15) define only trajectories of  $C$  points (the beginning of system of coordinates  $X_5O_5Y_5$ ) in coordinate system  $X_1O_1Y_1$ . To describe the movement of coordinate system  $X_5O_5Y_5$  in  $X_1O_1Y_1$  system it is necessary to know the relative rotation  $\varphi_5$  angle of  $X_5O_5Y_5$  system in  $X_1O_1Y_1$  system. Let us define at first angle changing between  $X_5O_5Y_5$  and  $X_2O_2Y_2$  coordinate systems:

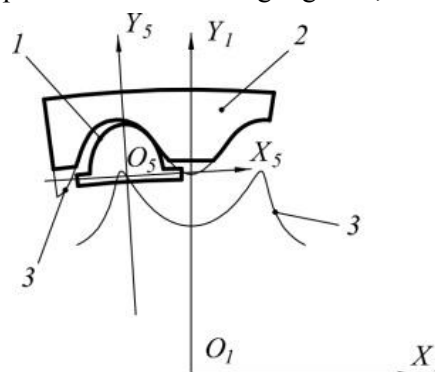
$$\varphi_6 = -\varphi_m + \pi \left( 1 - \frac{1}{n} \right), \text{ then } \varphi_5 = -\varphi_6 + \frac{\omega t}{2i_{hg}^k}.$$

Let the profile of tooth of a flexible gear is set in  $X_5O_5Y_5$  coordinate system by an equation of the form:

$$\left. \begin{aligned} x_5 &= \Omega_x(\tau), \\ y_5 &= \Omega_y(\tau), \end{aligned} \right\} \quad (17)$$

where  $\tau$  is the parameter of the parametrical equation of a profile of tooth of a flexible gear.

On figure 4 a profile of tooth of a flexible gear 1, tooth of a rigid gear 2, a trajectory of a  $C$  point in system of coordinates  $X_1O_1Y_1$ , a profile of tooth of a rigid gear 4, are shown.



**Figure 4.** Gear mesh of flexible and rigid gears.

Let us write down the equations of family of the curves representing a profile of tooth of a flexible gear in system of coordinates  $X_1O_1Y_1$ :

$$\left. \begin{aligned} x_s = \Psi_x(t, \tau) &= x_2 + \Omega_x(\tau) \cos \varphi_5 - \Omega_y(\tau) \sin \varphi_5, \\ y_s = \Psi_y(t, \tau) &= y_2 + \Omega_y(\tau) \sin \varphi_5 - \Omega_x(\tau) \cos \varphi_5. \end{aligned} \right\} \quad (18)$$

The equation for definition bending around look like:

$$\left| \frac{\frac{\partial \Psi_x(t, \tau)}{\partial t} \frac{\partial \Psi_y(t, \tau)}{\partial t}}{\frac{\partial \Psi_x(t, \tau)}{\partial t} \frac{\partial \Psi_y(t, \tau)}{\partial t}} \right| = 0. \quad (19)$$

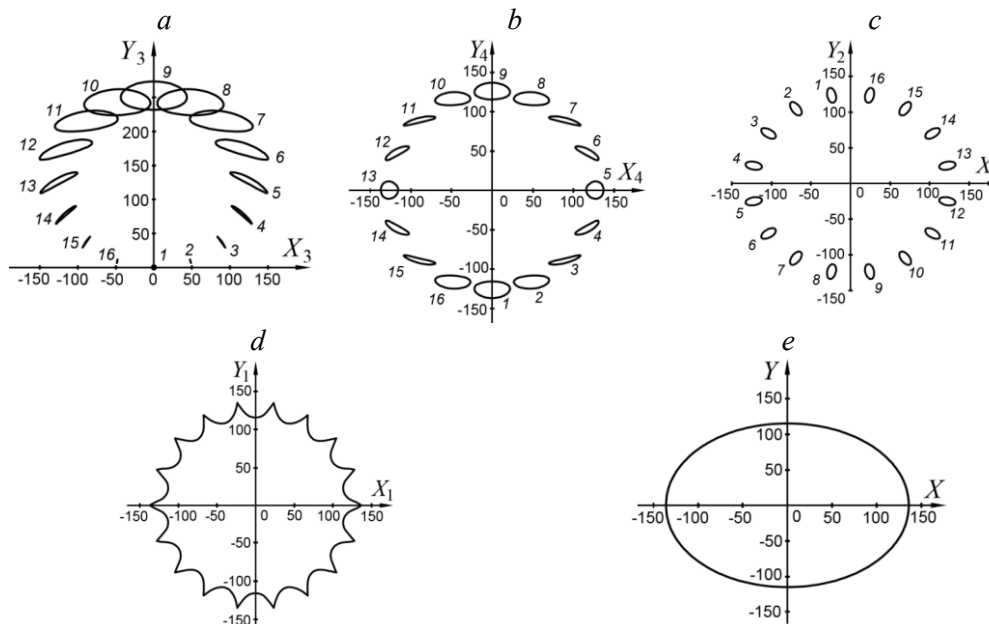
The equations of a profile of tooth of a rigid gear we will find, solving in common system of the equations (18) and (19):

$$\left. \begin{aligned} x_{pl} &= \Phi_x(t), \\ y_{pl} &= \Phi_y(t). \end{aligned} \right\} \quad (20)$$

The solution it is received by a numerical method as  $\tau$  from (19) in an explicit form is not expressed.

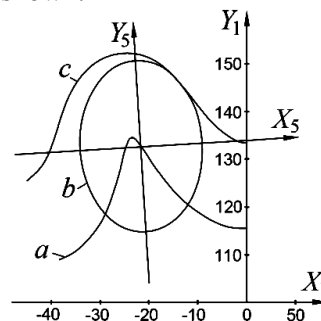
## 2.2. Results of calculating

According to equations (10) – (16) for a flexible gear with quantity of teeth 16 and parameters:  $\chi = 0.1$ ,  $\omega = 1 \text{ s}^{-1}$ ,  $\varphi_0 = 1.963$ ,  $p_h = 25 \cdot 10^{-3} \text{ m}$ ,  $p_f = 25 \cdot 10^{-3} \text{ m}$  trajectories of movement of points  $C$ , belonging to teeths of flexible gear are constructed. On figure 5 trajectories of points  $C$  in different systems of coordinates are shown:  $X_3O_3Y_3$ ,  $X_4O_4Y_4$ ,  $X_2O_2Y_2$ ,  $X_1O_1Y_1$  and  $XOY$ , accordingly. On fig. 5 *a* and *b* trajectories of all points  $C$  would coincide, and on fig. 5 *c* – *e* (poses. 1 – 16) trajectories of points  $C_1 - C_{16}$  are shown.



**Figure 5.** Trajectories of movement of points  $C$  of teeth of a flexible gear in different to systems of coordinates.

On figure 6 profiles of tooth of the interfaced gears: flexible (*b*) and rigid (*c*) where the profile of flexible gear has the ellipse form are shown.



**Figure 6.** Profiles of teeth of flexible and rigid gears and a trajectory of movement of points *C* in system of coordinates  $X_1O_1Y_1$ .

On figure 6 also it are shown (*a*) – a trajectory of movement of points *C* in system of coordinates  $X_1O_1Y_1$ .

### 3. Summary and conclusions

At synthesis of harmonic drive such cases require special attention: a) the flexible gears executed in the form of a flexible cylinder (ring) with teeth cut on it also have variable rigidity; b) the flexible gears executed with use of a chain and, possessing zero rigidity. As the flexible gear will not be turned in relation to the generator or will have superfluous degrees of freedom. In these cases the form of deformation of a flexible gear is presented by a polygon and it is difficult or impossible to set its form of the generator. Thereupon, it is offered to set the law of deformation of flexible gear changing interior angles of a convex polygon representing the model of a flexible gear (figure 1 and figure 2). It is necessary to consider that these angles change in accordance with the harmonious law. Such an approach makes it possible to solve only a small part of the problem, as can be seen in figure 5a trajectories of points *C*, belonging to teeth of a flexible gear are not identical to each other. From the theory of harmonic drives it is known that trajectories of these points should be identical to each other and in regular intervals settle down on a circle relative to the axis of harmonic drive, it is true of all types of harmonic drives. Therefore in the synthesis a flexible gear should be aligned with transfer axis. For this purpose the method of parallel carrying over of system of coordinates is applied, i.e. coordinates of *C* points were registered in system of coordinates beginning in the center of diagonal of polygon.

Thus, the binding of a flexible gear to an axis of harmonic drive is executed. Therefore we have received trajectories of points *C* located on a circle from an axis of harmonic drive, but these trajectories are not identical each other (figure 5b). It has been established that identical trajectories of points *C* can be received, writing down their coordinates in system with coordinates which turns in relation to this system by an angle depending on instant values of internal angles of a convex polygon. The result is shown on figure 5c by means of a method of transformation of coordinates trajectories of points *C* in systems of coordinates connected with a rigid gear (figure 5d) and the generator (figure 5e) are constructed. All points *C* in systems of coordinates connected with a rigid gear (figure 5d) and the generator (figure 5e) move same trajectory. The received trajectories are the basis for profiling of elements of harmonic drive. As a result of experiments the profile of tooth of a rigid gear based on elliptic profile of tooth of a flexible gear (figure 6) is constructed. Received results make it possible to conclude that the adopted method is efficient to synthesis of harmonic drive.

A new approach to determining the trajectory of movement of points of the flexible gear taking into account the effect of “polyhedron” was proposed.



The dependences making a basis of geometrical synthesis of elements of harmonic drive were proposed and allow to raise the accuracy of calculation. The possibility to create the harmonic drive with increased load capacity and resource at the gear ratio less than 60 was offered.

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