Paper

Phys. Educ. **55** (2020) 035008 (5pp) iopscience.org/ped

Measuring coefficients of restitution with a piezo disk

Rod Cross

School of Physics, University of Sydney, Sydney, Australia

E-mail: rodney.cross@sydney.edu.au

CrossMark

Abstract

A method is described to measure the coefficient of restitution (COR) by dropping a ball on a piezoelectric disk. Multiple bounces can be observed at small drop heights, so the average COR over say ten bounces can be obtained from just one ball drop, without having to measure the bounce height or the bounce speed. The results show directly that the maximum impact force on the ball during each bounce decreases linearly with time until the ball stops bouncing.

Measurements of the coefficient of restitution (COR) for a vertically bouncing ball have been reported many times in teaching journals. The simplest technique is to measure the drop height, h_1, and the bounce height, h_2, in which case the COR, e, is given by $e = \sqrt{h_2/h_1}$. Alternatively a video camera can be used to measure the incident speed, v_1, and the rebound speed, v_2, in which case $e = v_2/v_1$. Another technique is let the ball bounce several times and to measure the impact sound with a microphone [1–5]. In that case, the flight time in the air can be calculated, giving an estimate of the vertical bounce speed between successive bounces. The flight time and bounce height could also be measured with a high speed video camera.

A variation of the microphone technique is to allow the ball to bounce on a piezoelectric disk mounted on a heavy metal block. The latter technique provides a much cleaner bounce signal since the voltage output of the disk is directly proportional to the impact force which is much larger than the force on the disk due to surface vibrations. By contrast, a microphone signal needs to be heavily filtered to estimate the impact and rebound times. Other problems with microphone recording are discussed in [5].

The main drawback with a piezo measurement is that a piezo disk is relatively small so the ball may bounce only once or twice on the disk if the ball is dropped from a large height. However, the latter problem can be avoided by dropping the ball from a small height, in which case e can be measured only at small impact speeds. The advantage is that it is easy to measure 5 or 10 successive bounces and it is easy to measure both the impact duration and the time interval between each bounce. It is also possible to measure the impact force, F, if the piezo disk is calibrated in terms of the incident and rebound velocity. If the ball mass is m then $F = m\mathrm{d}v/\mathrm{d}t$ so $\int F\,\mathrm{d}t = m(v_1 + v_2)$. Another approach is described in [6] where a piezo sensor is mounted underneath a metal plate, but a disadvantage is that the plate vibrates and introduces a spurious response in the piezo sensor.

A potential advantage of recording multiple bounces is that the bounce height after say the 5th bounce is more sensitive to the COR than the bounce height after the first bounce. For example, if the drop height is 1 arbitrary unit then the bounce height after one bounce is 0.81 if $e = 0.9$ or 0.64 if $e = 0.8$. After five bounces, the bounce heights are $0.81^5 = 0.349$ and $0.64^5 = 0.107$, respectively.

After one bounce, the two bounce heights differ by a factor of 1.26 but after five bounces they differ by a factor of 3.26. After ten bounces they differ by a factor of 10.5. Consequently, small differences in the COR can be detected more readily by recording the change in bounce height or speed from the first to the 5th or 10th bounce.

A common problem with COR measurements is that no two bounces are the same, so several if not many bounces may need to be recorded to obtain an average. Some authors appear to go overboard in that respect. Heckel *et al* [5] recorded 280 000 bounces of a small steel sphere to measure the COR as a function of incident speed. For most purposes, five bounces would be sufficient at any given speed, particularly in a student laboratory. Multiple bounces recorded with a piezo disk provide that information in just one ball drop. In addition, the variability in COR from one bounce to the next is easily extracted, without having to measure directly either the incident and rebound heights or speeds.

Experimental procedure

The COR of several different balls was measured by dropping each ball on a 50 mm diameter, 8 mm thick piezoelectric ceramic disk. The disk itself was mounted on an 8 kg cylinder of copper, 100 mm in diameter, as shown in figure 1. A fine wire lead was soldered to the upper surface on the disk and the copper cylinder was used as the second electrode in order to measure the voltage induced in the disk. On its own, the disk generated a voltage of about 20 V when a steel ball was dropped on the disk from a small height. A 10 nF capacitor was connected across the two wire leads, in parallel with the piezo disk, to increase its capacitance. When a 10 MΩ voltage probe is connected to the capacitor, the charge q induced in the disk decays exponentially with a time constant RC = 0.1 s. The output voltage is then directly proportional to the force on the disk, at least for times up to about 10 ms. Since the largest impact time of interest was only 1.5 ms, the piezo responded faithfully to the impact force, the voltage dropping to zero (without reversing sign) when the ball bounced off the disk. The maximum output voltage was then typically about 0.3 V, since the voltage across the capacitor is q/C

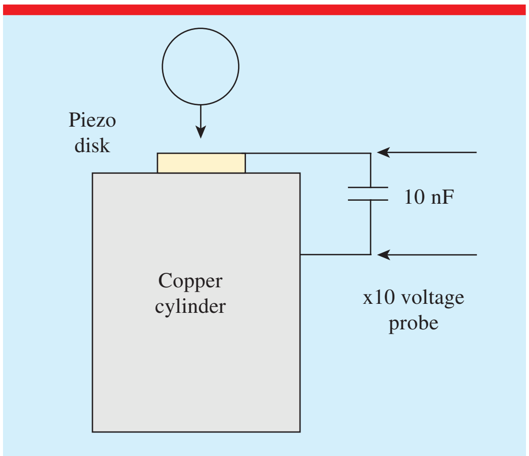

Figure 1. Experimental arrangement.

and since the voltage probe reduced the signal by a factor of 10.

The output voltage from the piezo was recorded with an inexpensive data acquisition system (PicoScope 2205A), sampling 8000 data points over the recording time. Each 1 ms duration impact was sampled with at least 5 data points so that the start and end time of the impact could be determined accurately. The bounce speed, v, immediately after each bounce was calculated from the relation $v = gT/2$ where T is the measured time interval from one bounce to the next.

Results with a golf ball

A raw piezo voltage signal obtained by dropping a 46.0 g golf ball on the piezo disk is shown in figure 2. The impact duration was 1.5 ± 0.2 ms, a very small fraction of the flight time of the ball in the air. The time between the first and second bounce is 113.8 ms, so the ball bounced vertically at 0.558 m s^{-1} after the first bounce. The time between the 2nd and 3rd bounce is 96.8 ms, so the ball bounced at 0.474 m s^{-1} after the second bounce, giving $e = 0.474/0.558 = 0.85$. Successive values of e for the first 12 bounces are shown in figure 3, giving an average and standard deviation value $e = 0.866 \pm 0.015$.

Results with a billiard ball

The raw piezo signal observed with a 50.8 mm diameter, 114.7 g billiard ball is shown in figure 4. The impact duration was 0.8 ± 0.1 ms

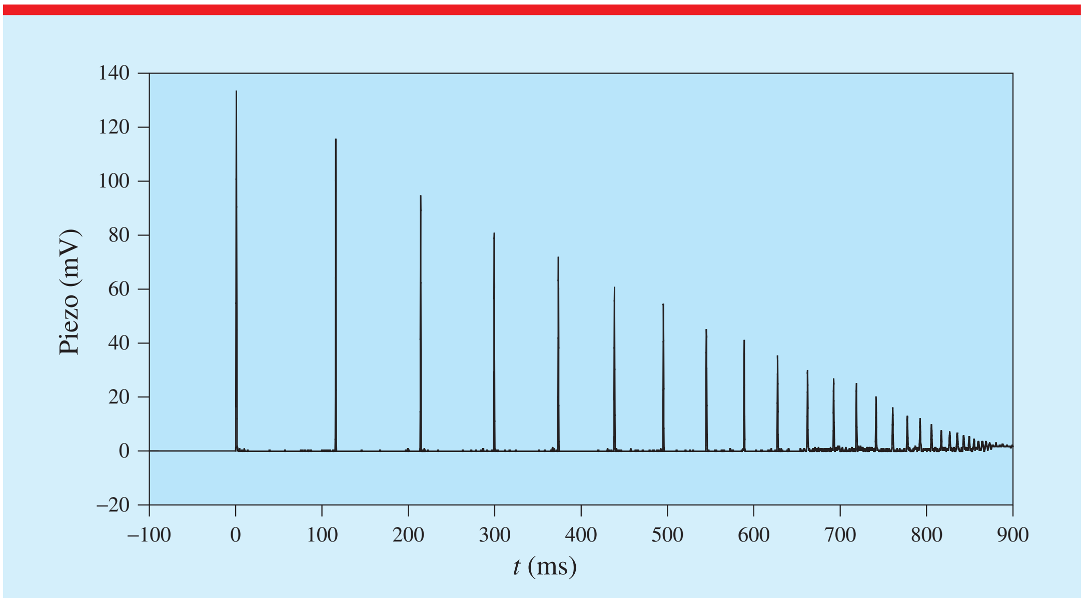

Figure 2. Multiple bounces of a golf ball on the piezoelectric disk.

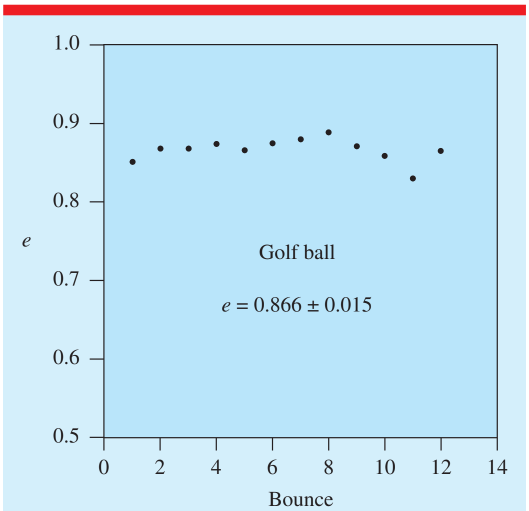

Figure 3. Value of e for 12 consecutive bounces of the golf ball.

for all bounces. The flight time in the air was 78.1 ms after the first bounce so the ball bounced at 0.383 m s^{-1} after the first bounce. The value of e for 16 consecutive bounces is shown in figure 5, giving $e = 0.932 \pm 0.015$.

Discussion

An interesting experimental result observed with both balls is that the peak force on the ball decreases after each bounce, and it decreases almost linearly with time. That result is consistent with a simple calculation, shown in figure 6. In that calculation, a ball was dropped from a height of 20 mm to land on a horizontal surface at a vertical speed 0.626 m s^{-1}. The resulting bounce speed was calculated for ten consecutive bounces, with $e = 0.85$ or $e = 0.9$, and the flight time after each bounce was also calculated. Figure 6 shows that the bounce speed decreases linearly with elapsed time. This result follows from the fact that the ball speed decreases from v to ev after each bounce, and the ball remains airborne for a time $t = 2v/g$. The slope of each graph in figure 6 is therefore $(e - 1)g/2$. For a given ball mass and a given impact time, the impact force is proportional to the incident ball speed. Consequently, the impact force decreases linearly with time, as observed.

The measured values of e do not necessarily represent the COR of either of the two balls examined. Rather, the COR represents the combined elastic properties of the ball and the piezo disk on which the ball bounces. That is, some of the energy that is lost during each bounce is lost in the ball, and some is lost in the piezo disk. The disk itself was a hard ceramic, so the measured COR values are probably close to those of the ball itself. The only way to be reasonably certain would be to bounce the balls on a hardened

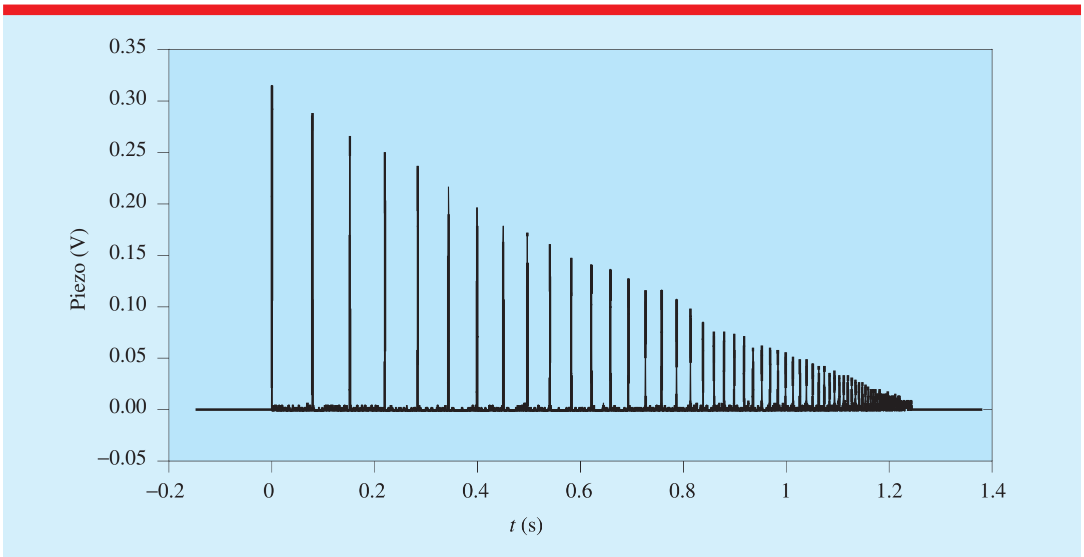

Figure 4. Multiple bounces of a billiard ball on the piezoelectric disk.

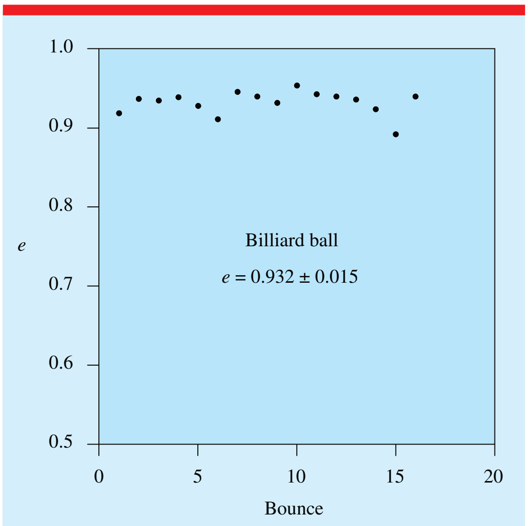

Figure 5. Value of e for 16 consecutive bounces of the billiard ball.

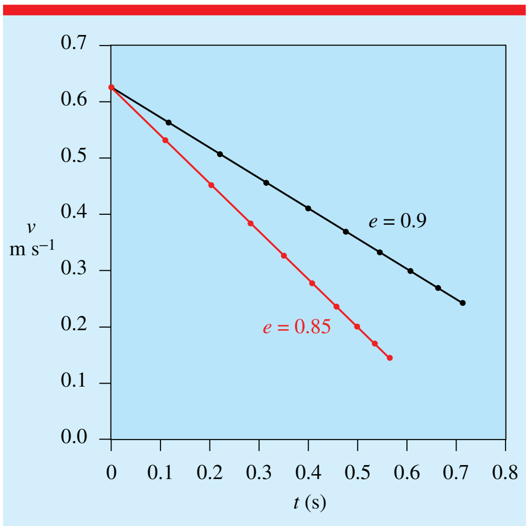

Figure 6. Theoretical impact speed for ten consecutive bounces, versus elapsed time, t, when a ball is dropped vertically from a height of 20 mm, with $e = 0.85$ or $e = 0.9$.

steel plate. However, it can be concluded from the present experiment that the COR for a low speed impact of a golf or billiard ball on a hard ceramic disk can be measured to within ± 0.015.

Several authors [7, 8] claim that a bouncing ball bounces an infinite number of times in a finite time before it comes to a stop. Given the finite impact duration of the ball, it would actually take an infinite time to bounce an infinite number of times. An experimental result obtained with the billiard ball is shown in figure 7. In order to examine the last few bounces, the ball was dropped from a height of only 0.1 mm by resting it on a sheet of paper then removing the paper quickly. Initially, the ball impacted the piezo disk with a force much greater than the weight of the ball, the force decreasing to zero between bounces. The ball stopped bouncing when the impact force was

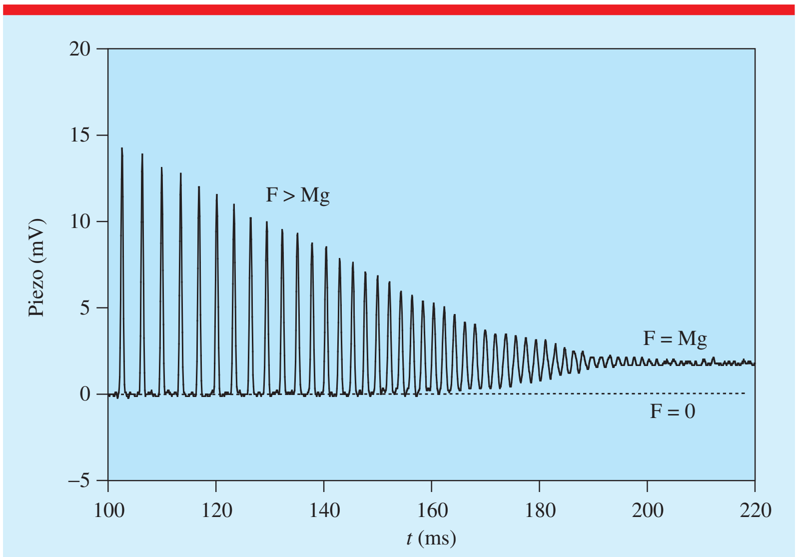

Figure 7. Multiple bounces of the billiard ball as it comes to a stop.

comparable with the weight of the ball. Instead, the ball vibrated with decreasing amplitude until the force on the piezo disk settled to a value equal to the weight of the ball.

ORCID iDs

Rod Cross https://orcid.org/0000-0001-9409-2791

Received 7 January 2020, in final form 23 January 2020
Accepted for publication 29 January 2020
https://doi.org/10.1088/1361-6552/ab7141

References

[1] Stensgaard I and Laegsgaard E 2001 Listening to the coefficient of restitution—revisited *Am. J. Phys.* **69** 301–5

[2] Agular C and Laudares F 2003 Listening to the coefficient of restitution and the gravitational acceleration of a bouncing ball *Am. J. Phys.* **71** 499–501

[3] Nunn J 2014 The bounce meter *Phys. Educ.* **49** 303–9

[4] Gonzalez M, Gonzalez M, Vegas J and Llamas C 2017 Measuring the coefficient of restitution and more: a simple experiment to promote students' critical thinking and autonomous work *Phys. Educ.* **52** 055002

[5] Heckel M, Glielmo A, Gunkelmann N and Poschel T 2016 Can we obtain the coefficient of restitution from the sound of a bouncing ball? *Phys. Rev.* E **93** 032901

[6] Falcona E, Laroche C, Fauve S and Coste C 1998 Behavior of one inelastic ball bouncing repeatedly off the ground *Eur. Phys. J.* B **3** 45–57

[7] Foong S, Kiang D and Lee P 2004 How long does it take a bouncing ball to bounce an infinite number of times? *Phys. Educ.* **39** 40–3

[8] Nordmark A and Essen H 2018 An impacting linear three body system *Eur. J. Phys.* **39** 015001

Rod Cross is an Honorary Assoc Prof in physics at Sydney University with interests in the physics of sport and physics experiments for students.