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Three-voltage linear method to determine ion recombination in proton and light-ion beams

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Abstract

A new practical method to determine the ion recombination correction factor (k_s) for plane-parallel and Farmer-type cylindrical chambers in particle beams is investigated.

Experimental data were acquired in passively scattered and scanned particle beams and compared with theoretical models developed by Boag and/or Jaffé. The new method, named the three-voltage linear method (3VL-method), is simple and consists of determining the saturation current using the current measured at three voltages in a linear region and dividing it by the current at the operating voltage (V) (even if it is not in the linear region) to obtain k_s.

For plane-parallel chambers, comparing k_s-values obtained by model fits to values obtained using the 3VL-method, an excellent agreement is found. For cylindrical chambers, recombination is due to volume recombination only. At low voltages, volume recombination is too large and Boag's models are not applicable. However, for Farmer-type chambers (NE2571), using a smaller voltage range, limited down to 100 V, we observe a linear variation of k_s with $1/V^2$ or $1/V$ for continuous or pulsed beams, respectively. This linearity trend allows applying the 3VL-method to determine k_s at any polarizing voltage.

For the particle beams used, the 3VL-method gives an accurate determination of k_s at any polarizing voltage. The choice of the three voltages must to be done with care to ensure to be in a linear region. For Roos-type or Markus-type chambers (i.e. chambers with an electrode spacing of 2 mm) and NE2571 chambers, the use of the 3VL-method with 300 V, 200 V and 150 V is adequate. A difference with the 2V-method and some 3V-methods in the literature is that in the 3VL-method the operational voltage does not have to be one of the three voltages. An advantage over a 2V-method is that the 3VL-method can inherently verify if the linearity condition is fulfilled.

1. Introduction

In this study, we focus on ion recombination correction factors (k_s) for proton and light-ion beams. Recombination of positive ions with free electrons or negative ions before they can be captured on the collecting electrode, results in charge loss and ion collection inefficiency influencing the response of air-vented ionization chambers. k_s is defined as the reciprocal of the ion collection efficiency, f, and must be applied to the response of

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the ionization chamber as recommended by international dosimetry protocols (Almond *et al* 1999, Andreo *et al* 2000). Recombination processes are complex because they depend on many parameters (e.g. type of radiation, time structure of the beam, experimental conditions, etc). Two main ion recombination pathways can be distinguished: one, in which ions created by different primary particles recombine and which therefore depends on the dose rate (or dose per pulse), named volume recombination, and the second in which ions created by the same ionizing particle recombine and which therefore depends on the ionization density within one particle track, named initial recombination. Another process, back diffusion against the electric field, can have an impact on the ion collection efficiency, but at low voltages, only.

The models currently used in most dosimetry recommendations to describe volume recombination have been published by Boag or Boag and Wilson (Boag 1950, 1982, Boag and Wilson 1952, Boag *et al* 1987). In these papers, to first order, a linear relation is predicted between k_s and $1/V$ or $1/V^2$ for pulsed or continuous beams, respectively. These conclusions have been confirmed numerous times experimentally in photon and electron beams (e.g. Boutillon (1998), Burns and McEwen (1998) and Palmans *et al* (2010)). Based on those assumptions of linearity, Boag and Currant (1980) derived a practical method to determine volume recombination, which is widely used in dosimetry protocols (Almond *et al* 1999, Andreo *et al* 2000): the two-voltage method (2V-method). It makes use of the ionization current determined at the operating voltage and the ionization current determined at a lower voltage to make an analytical extrapolation based on the assumed linearity. The 2V-method has been validated in different beams and is accurate within clinically acceptable uncertainties although Boag's theory alone is never sufficient to describe the total ion recombination mechanism. With the increasing number of particle therapy centres, experimental studies show that this inadequacy of Boag's theory becomes more pronounced in particle beams with ions heavier than protons because of the enhanced contribution of initial recombination (Kanai *et al* 1998, Kaiser *et al* 2012). Different theories have been published to describe the main initial recombination mechanism, columnar recombination, among which the theory of Jaffé (1913). In this model, dedicated to plane-parallel ionization chambers, Jaffé predicts a logarithmic dependence of the columnar initial recombination correction factor against $1/V$. Jaffé's theory has been confirmed experimentally for passively scattered carbon ion beams characterised by high linear energy transfer (LET) and low radiotherapeutic dose rates, so that ion recombination is dominated by initial recombination (Kanai *et al* 1998, Rossomme *et al* 2016).

Due to the introduction of new types of accelerators in light-ion beam therapy (*such as* synchrocyclotrons) and the progress of therapeutic delivery techniques, instantaneous dose rates have increased strongly. Consequently, volume recombination can be of similar magnitude as initial recombination for high LET beams, as shown by Rossomme *et al* (2017). In that paper, good agreement has been demonstrated between experimental results obtained in different light-ion beams for plane-parallel ionization chambers and a model based on a combination of Boag's theory and Jaffé's theory. This led to the conclusion that initial recombination and volume recombination must be considered together in scanned particle beams. Therefore, due to the logarithmic variation of the initial recombination correction factor as a function of $1/V$, the assumption of linearity of k_s against $1/V^2$ is not valid, and the 2V-method for continuous beams recommended by dosimetry protocols cannot be applied for plane-parallel ionization chambers. Determining the saturation curves at different dose rates (Palmans *et al* 2006), and comparing experimental results with a theoretical model, the contributions of initial and volume recombination can be separated (Rossomme *et al* 2017). From a practical point of view, this experimental procedure to distinguish both recombination contributions is too cumbersome and too time consuming to be used on a routine basis in clinical situations. In addition, this procedure has a non-negligible financial cost due to the cost of beam time. The main reasons are: (1) the method requires measurements at many voltages, (2) the method requires measurements at many depths because recombination can be substantially different in the plateau and in the Bragg peak region due to the variation of dose rates (or dose per pulse) and (3) the method requires the use of beams at multiple dose rates (or dose per pulse), which may not always be clinically commissioned. Moreover, the separation of both recombination components is not needed for dosimetry if the combined effect can be determined accurately.

In light-ion beams, due to the higher uncertainty of the beam quality correction factors, k_Q (with reference to a Cobalt-60 calibration beam), cylindrical ionization chambers are recommended by the TRS-398 protocol for reference dosimetry in modulated beams (with SOBP width $\geqslant 2$ g cm^{-2}) (Andreo *et al* 2000). At shallow depth in single-layer scanned fields (Palmans and Vatnitsky 2016), plane-parallel ionization chambers are recommended because of the uncertainty of the effective point of measurement of cylindrical ionization chambers. Since, depending on the measurement conditions, both cylindrical and plane-parallel ionization chambers are recommended to be used in particle therapy, a method to determine recombination correction factors is needed for both types of ionization chambers.

In the current study, saturation curves are experimentally determined in two types of particles beams with different temporal structures using plane-parallel and cylindrical ionization chambers. Comparing experimental results to values obtained by model fits, our objective is to present that, in all analysed cases, a practical method based on the assumption of linearity between the reciprocal of the ionization current and $1/V$ (or $1/V^2$) in a voltage region provides an accurate way of determining k_s. Named the three-voltage linear method (3VL-method), this method allows determining k_s also for operating voltages that are not in the linear region as long as it is not included in the data points used for the linear fit. Three-voltage methods have been proposed before (Schechtman *et al* 1989, Pardo-Montero and Gómez 2009) but differ from the 3VL-method proposed in this work which is based on the variation of the reciprocal of the ionization current versus $1/V$ or $1/V^2$ at three high voltages.

Measurements presented in this study have been performed in different particle therapy or research centres. k_s-values have been obtained in proton and carbon ion beams generated by a cyclotron, a synchrotron or a synchrocyclotron. As reported in Palmans *et al* (2006), even if cyclotron and synchrotron beams are pulsed beams, they can be considered as continuous beams with respect to recombination. Results have either been published previously (Rossomme *et al* 2016, 2017) or that are newly measured for this work.

2. Method

2.1. Theoretical models

In this section, theoretical models for volume and initial recombination are presented.

2.1.1. Volume recombination

Volume recombination is described by different theoretical models, in particular the models of Mie and Boag. Neglecting space charge effects, for continuous beams, Boag and Wilson derived the following relation to calculate k_s in continuous beams (Boag and Wilson 1952):

$$k_s^{\mathrm{Cont}} = \frac{1}{f} = \frac{1+\sqrt{1+4\ {}^2}}{2} \text{ with } \xi^2 = \frac{B\ I_{sat}}{V^2} \text{ and } B = \frac{m^2 d^3}{6}, \tag{1}$$

where m is a gas/chamber specific constant, d is the electrode spacing, V is the polarizing voltage and I_{sat} is the saturation current. Note that the definition of the parameter ξ in equation (1) differs from that given in Boag and Wilson (1952) by a factor $\sqrt{6}$ for simplicity of the equations. Using the first-order term of a series expansion around $1/V = 0$ of relation (1) for large voltages it can be written as

$$k_s^{\mathrm{Cont}} = 1 + \xi^2 = 1 + \frac{BI_{sat}}{V^2}. \tag{2}$$

This relation confirms Mie's model, which also describes volume recombination (Mie 1904). Boag's and Mie's models have been developed for continuous beams, for which the radiation time is much longer than the charge collection time. For pulsed beam, i.e. if (1) the charge collection time is much shorter than the pulse repetition period and (2) the radiation pulse duration time is much shorter than the charge collection time, Boag derived another relation between the ion collection efficiency and the operating voltage (Boag 1950). Using the first-order term of a series expansion around $1/V = 0$, the ion recombination correction factor can be written as

$$k_s^{\mathrm{Pulsed}} = 1 + \frac{\lambda I_{sat}}{V} \text{ with } \lambda = \frac{m' d}{2}, \tag{3}$$

with m' a gas/chamber specific constant. All the models described in equations (1)–(3) have been developed for plane-parallel ionization chambers. For cylindrical ionization chambers, Boag established an equivalent electrode spacing allowing the use of his models: $d \equiv (a-b)\sqrt{\frac{a+b}{a-b}\frac{\ln(a/b)}{2}}$, with a being the radius of the cavity and b the radius of the electrode (Boag and Currant 1980). Based on the assumption of linear relationships between k_s and $1/V$ for pulsed beams and k_s and $1/V^2$ for continuous beams, Boag and Currant derived a simple method to determine k_s: the 2V-method (Boag and Currant 1980). This method is recommended by international dosimetry protocols to determine ion recombination correction factors (Almond *et al* 1999, Andreo *et al* 2000). The 2V-method requires the measurement of the response of the ionization chamber (M) at two different voltages (V_1, the operating voltage and V_2, a lower voltage). As described in TRS-398, two simple relations allow determining k_s depending on the beam delivery method (Andreo *et al* 2000):

$$-\text{for continuous beams}: k_s = \frac{\left(V_1/V_2\right)^2 - 1}{\left(V_1/V_2\right)^2 - (M_1/M_2)}. \tag{4}$$

$$-\text{for pulsed or pulsed scanned beams}: k_s = a_0 + a_1\frac{M_1}{M_2} + a_2\left(\frac{M_1}{M_2}\right)^2, \tag{5}$$

where the coefficients a_i depend on the voltage ratio V_1/V_2. These values are given in Andreo *et al* (2000). M represents the reading of the ionization chamber.

2.1.2. Initial recombination

Different theoretical models have been published to describe the initial recombination for plane-parallel ionization chambers, among which the theory of Jaffé (1913). Depending on the orientation of the ionization chamber with respect to the direction of the ion tracks, Jaffé obtained two relations between k_s and the polarizing voltage. When the ion tracks are parallel to the electric field, Jaffé found that the ion collection efficiency can be expressed as:

$$f = \frac{e^{-1/g}}{g\ q}\left(li\left(e^{\frac{1}{g}+\ln(1+q)}\right) - li\left(e^{1/g}\right)\right) \text{ with } q = \frac{2\ d\ D}{k\ b^2\ E} \text{ and } g = \frac{\alpha\ N}{8\ \pi\ D}, \tag{6}$$

where α is the recombination coefficient, D the ion diffusion coefficient, k the ion mobility, b the initial mean-square radius of the Gaussian distribution, E the electric field strength, d the electrode spacing of the ionization chamber and N the ionization density along the track. The function $li(e^p)$ is the logarithmic integral of p. Using the first-order term of a series expansion around $1/V = 0$, the ion recombination correction factor can be written as a linear function of $1/V$. No model exists for cylindrical ionization chambers.

2.1.3. Combination of initial and volume recombination

In proton and light-ion beams, we must consider both initial and volume recombination to describe ion recombination correctly. The total ion collection efficiency of an ionization chamber can be expressed as $f^{\text{tot}} = f^{\text{ini}} f^{\text{vol}}$ and the ion recombination correction factor as $k_s^{\text{tot}} = k_s^{\text{ini}} k_s^{\text{vol}}$. In 1967, using a combination of the first-order term of a series expansion of Jaffé's model (i.e. k_s^{ini} is a linear function of $1/V$) and the first-order term of a series expansion of Boag's model for continuous beams (equation (2)) around $1/V = 0$, Niatel developed a general equation/model:

$$k_s^{\text{tot}} = 1 + \frac{A}{V} + \frac{B}{V^2} I_{sat}, \tag{7}$$

where the second term and the third term on the right-hand side represent initial recombination and volume recombination, respectively (Niatel 1967). B is defined in equation(1). Based on the first-order term of a series expansion around $1/V = 0$ of the Jaffé's relation, A depends on parameters g and q defined in equation (6). A and B values can be determined by applying the Niatel's method which consists of separating initial and volume recombination using different dose rates (Niatel 1967). This model has been validated in passively scattered proton beams by Palmans *et al* (2006) and pulsed high-energy photon beams by Palmans *et al* (2010). However, in light-ion beams, the first-order term of a series expansion around $1/V = 0$ of Jaffé's expression does not allow describing initial recombination contribution over the entire voltage range and the full Jaffé's relation (equation (6)) must be used. Such a model has been validated in helium, carbon ion and oxygen ion beams by Kanai *et al* (1998) and Rossomme *et al* (2017).

2.2. Experimental method

Investigations were carried out using different types of plane-parallel ionization chambers and Farmer-type cylindrical ionization chambers: IBA PPC40 (SN 512), PTW-34001 (Roos—SN 1684), NE2571 (SN 1940). For each ionization chamber tested, a second similar type of ionization chamber (to have the same sensitive volume), used at a fixed voltage, has been used as monitor to compensate for beam fluctuations. All ionization currents presented in the remainder of this paper represent the ionization current of the chamber under investigation divided by the monitor current.

The experimental procedure in this study consisted of the determination of the plot of the reciprocal of the ionization current, $1/I$, against the reciprocal of the polarizing voltage applied to the ionization chamber under test, $1/V$, or against $1/V^2$. Measurements have been performed at different dose rates (or dose per pulse), as described in the Niatel's method to separate initial and volume recombination. The beam characteristics used in each particle centre are given in table 1.

- Centre antoine lacassagne (CAL) (Nice, France), where passively scattered proton beams are produced by the cyclotron 'MEDICYC' and pulsed PBS (i.e. pencil beam scanning) proton beams are generated by a synchrocyclotron (IBA S2C2). Results have been published in Rossomme *et al* (2017).
- Proton Therapy Center of Azienda Provinciale per i Servizi Sanitari (APSS) (Trento, Italy), where PBS proton beams are generated by a cyclotron (IBA C230). Results have never been published.
- Heidelberg Ion-Beam Therapy Center (HIT) (Heidelberg, Germany), where scanned particle beams (i.e. proton, helium, carbon ion and oxygen ion beams) are produced by a synchrotron. Results have been published in Rossomme *et al* (2017).

Table 1. Characteristics of the proton and carbon ion beams used in this study.

	Centre	Modality	Energy	Depth	Field
Proton beams					
Passively scattered	CAL	Isochronous cyclotron (frequency: 25 MHz)	62.5 MeV	Plateau	3 cm diameter
PBS	APSS	Isochronous Cyclotron (frequency ~ 100 MHz)	100 MeV	Plateau	$10 \times 10\,\mathrm{cm}^2$ (41×41 spots)
Pulsed PBS	CAL	Synchrocyclotron (frequency: 1 kHz pulse length: 7–10 μs)	96.17 MeV	Close to the peak	$10.2 \times 10.2\,\mathrm{cm}^2$ (51×51 spots)
Carbon ion beams					
Passively scattered	LNS	Isochronous cyclotron (frequency: 15–48 MHz)	62 MeV/n	Plateau	2.5 cm diameter
Scanned	CNAO	Synchrotron (spill duration: 1–3 s pause ~4.5 s)	115 MeV/n	Plateau	$5 \times 5\,\mathrm{cm}^2$ (25×25 spots)
Scanned	HIT	Synchrotron (spill duration: ~5 s pause: 4–4.5 s)	115 MeV/n	Plateau	$4 \times 7\,\mathrm{cm}^2$ (21×36 spots)

- ¤ Istituto Nazionale di Fisica Nucleare—Laboratori Nazionali del Sud (INFN-LNS) (Catania, Sicily), where passively particle beams are generated by a K800 superconducting cyclotron, using the '0-degree' beam line. Results have been published in Rossomme *et al* (2016).
- ¤ Centro Nazional di Adroterapia Oncologica (CNAO) (Pavia, Italy), where scanned carbon ion beams are generated by a synchrotron. Results have been published in Rossomme *et al* (2017).

Based on the criterion given by Boag (see section 2.1.1) and Karsch (2016), from a recombination point of view, all beams can be considered as continuous beams, except the pulsed PBS proton beams produced at CAL.

2.3. Practical method: three-voltage linear method

For all results we obtained in particles beams, we observed that the first-order term of a series expansion around $1/V = 0$ of Jaffé's theory and/or Boag's theory is valid to describe at least a part of the saturation curve. Defining a specific range of voltages, we used this characteristic to establish a new procedure to derive k_s: the 3VL-method.

The 3VL-method uses the dependence of the reciprocal of the ionization current, $1/I$, on the reciprocal of the polarizing voltage, $1/V$, or the reciprocal of the square of the polarizing voltage, $1/V^2$. In a first step the saturation current, I_{sat}, is calculated using an extrapolation of a linear fit of $1/I$ versus $1/V$ or $1/V^2$ measured at three high polarizing voltages. k_s is then determined at any operating voltage (included or not in the linear region) as the ratio of I_{sat} and the ionization current measured at the operating voltage $I(V)$: $k_s(V) = \frac{I_{sat}}{I(V)}$. This method is only valid under two conditions:

(1) absence of charge multiplication at the operating voltage and any of the voltage points used for the linear fit and
(2) assumption of linearity between k_s and $1/V$ (or $1/V^2$) over a certain voltage range.

Due to type-A uncertainties, the more voltages that are used for the linear interpolation, the more accurate it will be. Three points have been used because it is a good compromise between accuracy and measurement time. In principle, two voltage points would be sufficient if one is sure of being in a linear region but the use of three points gives additional confirmation that the linearity condition is satisfied. If an inadequate choice is made and the three points are in a non-linear region because the voltages are too high (due to charge multiplication) or too low (due to an increase of higher-order terms in the volume recombination or initial recombination) the experimental data will reveal this prompting that I_{sat} must be determined using another set of voltages where the dependence is linear. This is very useful in the case a substantial number of measurement points has to be covered by the ionization chamber such as in the measurement of a depth dose distribution. The dose rates or dose per pulse can be substantially higher in the Bragg peak and thus recombination can also be substantially different, affecting the peak to plateau ratio which is an important characteristic to get right in the commissioning of the beam models in treatment planning systems. It is almost impossible in the clinic to measure a full Jaffé plot for a sufficient number of points along the Bragg curve. If you have already proven linearity over a certain voltage range for one point, then the 3VL-method is very useful to provide assurance that for other points the same range of voltages is also in the linear region. Note also that the operating voltage does not need to be one of the three voltages used for the linear fit which is an important distinction with the traditional 2V method. Figure 1 illustrates this new approach.

3. Results

The numerical values presented in this study depend on the facilities and experimental conditions, such as dose per pulse or dose rate, which varied between different beams. Consequently, no conclusions can be drawn from comparing the k_s-values obtained in different centres as numerical values are expected to be very different due to experimental conditions.

3.1. Saturation curves

3.1.1. Plane-parallel ionization chambers

Most of results used in this section are part of full studies on ion recombination in particle beams that have been published previously (Palmans *et al* 2006, Rossomme *et al* 2017). In this study, to consider the main temporal structure of proton and carbon ion beams, we acquired new data in PBS proton beams, which are presented in section 3.1.1.1. Section 3.1.1.2 shows experimental data used to validate the 3VL-method.

3.1.1.1.PBS proton beams

Figure 2 shows the result obtained in a 100 MeV APSS PBS proton beam with an IBA PPC40 Roos-type ionization chamber (SN 512), using three different beam intensities. Figure 2(A) shows a Jaffé plot while figure 2(B) presents

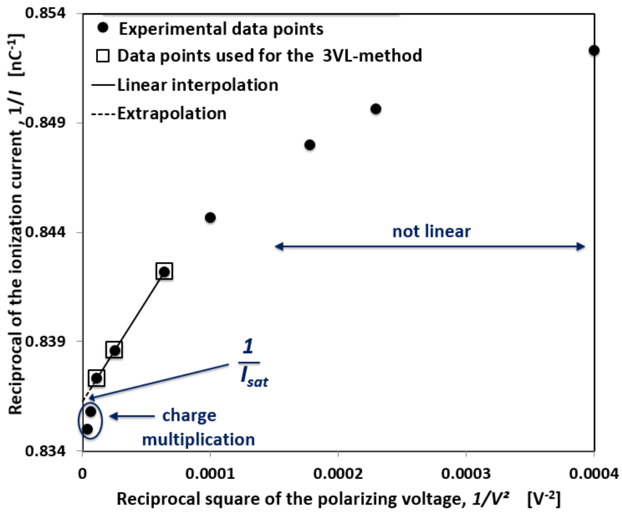

Figure 1. Shows $1/I$ as a function of $1/V^2$ with a non-linear region at low voltage and charge multiplication at high voltage and illustrates the determination of I_{sat} using the 3VL-method based on an extrapolation of a linear fit to three data points measured at three sufficiently high polarizing voltages.

k_s-values. The maximum relative type-A uncertainty (standard deviation of the mean) of the measured current was 0.15%. Similar results have been obtained using a modulated PBS proton beam.

Comparing results presented in figure 2(A) with results obtained in a passively proton beam (figure 3(a) in Palmans *et al* (2006)), a similar behaviour of $1/I$ versus $1/V$ is observed. This confirms that the APSS PBS proton beam can be considered as a continuous beam with respect to recombination. In figure 2(B), experimental data are compared to fits of Niatel's equation (equation (7)) for which recombination coefficients, i.e. A and $B * I_{sat}$ values, have been determined using the Niatel's method. An excellent agreement can be observed between theoretical and experimental k_s-values.

3.1.1.2.Data used for the 3VL-method

Figure 3 presents Jaffé plots obtained in four different beams (open squares). All results have been obtained for clinical dose rates (or dose per pulse). Dashed lines represent theoretical values, resulting from Niatel's equation (figure 3(A)), Jaffé's theory (figure 3(C)) and a combination of Jaffé's theory and Boag's theories (figures 3(B) and (D)). Solid symbols show the three points which will be used to apply the 3VL-method to determine k_s-values (section 3.2). In order not to blur the conclusions with features that are due to type-A uncertainties in the measurements data, these three points are derived from the model fits of each experimental data set. These voltages are 300 V, 200 V and 150 V.

Figure S1 presented in the supplemental material[12] (stacks.iop.org/PMB/65/045015/mmedia) presents the same data as a function of $1/V^2$ showing the same features in a different way.

3.1.1.2.1.Proton beams

The difference between the curves presented in figures 3(A) and (B) is due to the two different modes of radiation and the different time structures of the proton beams. The inverse of the current varies linearly with $1/V^2$ for the continuous proton beam (as shown in figure S1—supplemental material), whereas it varies linearly with $1/V$ for the pulsed PBS proton beam, as predicted to first order by Boag's theories. Figure 3(A) indicates that initial recombination is negligible in this situation.

[12] See supplementary material for plot of $1/I$ against $1/V^2$ obtained in (A) a 100 MeV PBS proton beam at APSS, using the PPC40 (SN 512) ionization chamber; (B) a 96 MeV pulsed PBS proton beam at CAL, using the PPC40 (SN 512) ionization chamber; (C) a 62 MeV/n passively scattered carbon ion beam at INFN-LNS, using a PTW Roos (SN 1684) ionization chamber and (D) a 115 MeV/n scanned carbon ion beam at CNAO, using the PPC40 (SN 512) ionization chamber. A quasi-linear variation between $1/I$ and $1/V^2$ is observed in continuous proton beams (PBS proton beam (figure S1(A))), proving that ion recombination is dominated by volume recombination in this case. For carbon ions (figures S1(C) and (D)), since the LET of the beam increases, the initial recombination contribution is not negligible and the relation between $1/I$ and $1/V^2$ becomes non-linear.

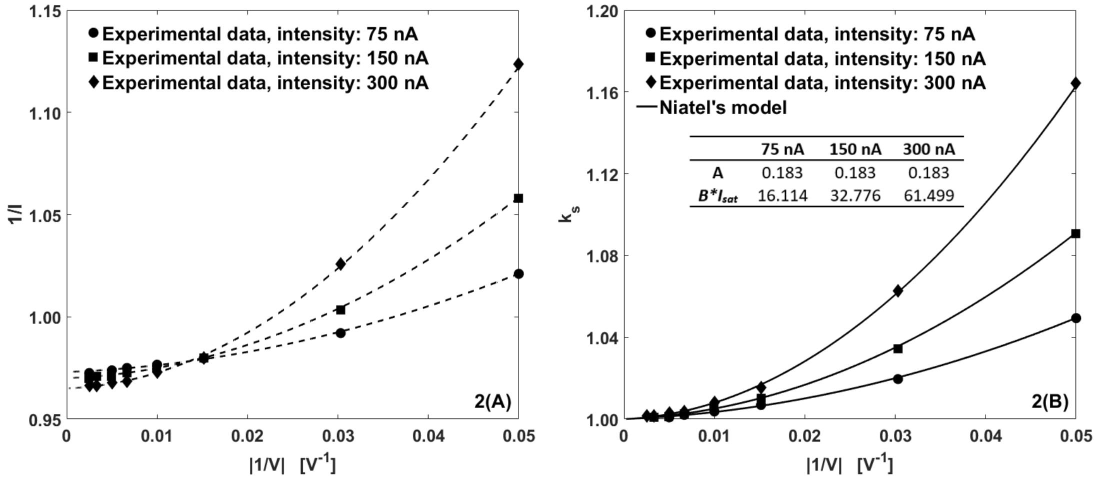

Figure 2. Data obtained using the IBA-PPC40 (SN 512) ionization chamber for three different beam intensities in a 100 MeV APSS proton beams: (A) Jaffé plot and (B) ion recombination correction factor against $1/V$ from experiment using the 3VL-method using 300 V, 200 V and 150 V (symbols) and Niatel's equation (equation (7)) fitted to the experimental data using the Niatel's method (continuous lines).

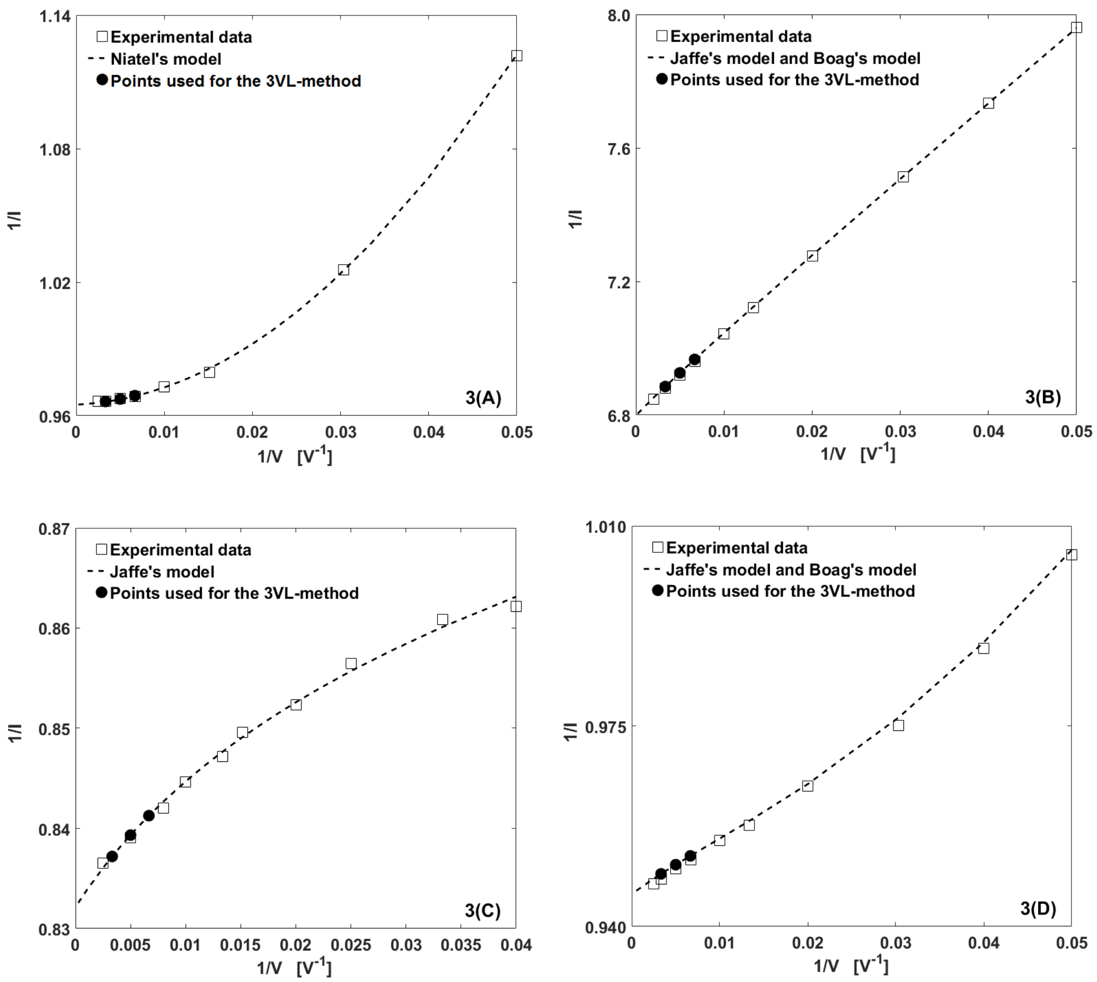

Figure 3. Jaffé plots obtained in (A) a 100 MeV PBS proton beam (300 nA) at APSS, using the PPC40 (SN 512) ionization chamber (same data presented in figure 2(A)); (B) a 96 MeV pulsed PBS proton beam (0.47 pC/pulse) at CAL, using the PPC40 (SN 512) ionization chamber; (C) a 62 MeV/n passively scattered carbon ion beam (2.3 Gy $\min^{-1}$) at INFN-LNS, using a PTW Roos (SN 1684) ionization chamber and (D) a 115 MeV/n scanned carbon ion beam (5e7 ions s^{-1}) at CNAO, using the PPC40 (SN 512) ionization chamber. Dashed lines represent theoretical values detailed in Rossomme *et al* (2016, 2017) and figure 2(A). Open squares represent experimental data points while solid symbols represent theoretical points used for the 3VL-method.

3.1.1.2.2.Ion beams

The difference between the curves presented in figures 3(C) and (D) is due to the different beam intensities of the two carbon ion beams. For the passively scattered carbon ion beam (figure 3(C)), due to the low radiotherapeutic dose rates, volume recombination is negligible, and the total ion recombination is dominated by initial recombination (Jaffé's theory). For the scanned carbon ion beam (figure 3(D)), the instantaneous dose rate is substantially higher and volume recombination must be considered so that Jaffé's theory and Boag's theory must be combined to fit with experimental data over the entirely range of voltages. With respect to recombination, the scanned carbon ion beam must be considered as continuous (linear variation of volume recombination with $1/V^2$) since the radiation pulse duration time is larger than the charge collection time.

3.1.2. Cylindrical ionization chambers

Figure 4 shows a comparison between theoretical models and experimental data on the variation of $1/I$ against $1/V$ or $1/V^2$ determined for a NE2571 cylindrical ionization chamber at different beam intensities in different proton and carbon ion beams. All results have been obtained for clinical dose rates (or dose per pulse). The recombination parameter has been determined using the Niatel's method (i.e. determination of variation of $1/I$ versus $1/V^2$ at different dose rates or dose per pulses). The value of the relative type-A standard uncertainty of the measured current (repeatability) is less than 0.1%, except in passively scattered carbon ion beams for which is 0.5%. No result has been published before.

For the five beams illustrated in figure 4 (solid squares symbols), k_s-values have been determined using the 3VL-method between 400 V and 33 V. I_{sat} has been obtained using 300 V, 200 V and 150 V. Figure 5 shows the variation of k_s with $1/V^2$ for continuous beams and the variation of k_s with $1/V$ for pulsed beam.

3.2. Comparison between the three-voltage linear method, the two-voltage method and theoretical models

3.2.1. Plane-parallel ionization chambers

Table 2 presents ion recombination correction factors determined by different methods, at two operating voltages (100 V and 300 V). To investigate the consistency of the 3VL-method, numerical values are based on $1/I$-value derived from theoretical models (represented by solid circles in figure 3 or S1 (supplemental material)). The third column shows numerical results obtained using theoretical models (i.e. Niatel's model, Jaffé's model or a combination of Boag's model and Jaffé's model). Having obtained experimental data using Roos-type ionization chambers (electrode spacing = 2 mm), the three-voltages used for the 3VL-method are 300 V, 200 V and 150 V, which correspond to an electric field of 1500 V cm^{-1}, 1000 V cm^{-1} and 750 V cm^{-1}, respectively. In table 2, values in round brackets indicate the difference between the method tested and the theoretical model. For the 3VL-method, the regression coefficient (R^2) of the linear interpolation used to determine I_{sat} is given.

As mentioned before, results presented in table 2 have been derived using fitted values to avoid blurring the conclusions with features that are due to type-A uncertainties in the measurement data. Nevertheless, to investigate the influence of type-A uncertainties in the measured data, a comparison between k_s-values determined by the 2V-method and the 3VL-method, based on the raw experimental data (presented in figure 3) has been performed as well. Table 3 shows results obtained in proton beams. Similar results have been obtained in carbon ion beams.

The three voltages used for the 3VL-method are 300 V, 200 V and 150 V. In table 3, values in brackets indicate the difference between the 2V-method and the 3V-linear method. We can observe a difference of maximum 0.1% between these values and the values presented in brackets in table 2, allowing concluding that type-A uncertainty in the measurement data does not influence the 3VL-method.

3.2.2. Cylindrical ionization chambers

Similar to the study made in the previous section for plane-parallel ionization chambers, table 4 shows the comparison between different methods to determine ion recombination correction factors at different voltages: theoretical models, the 2V-method and the 3VL-method. Table 4 is based on the five examples shown in figure 4.

For beams tested in this study, the value of the regression coefficient of the linear interpolation used to derive I_{sat} with the 3VL-method is equal to 1.000 for the passively scattered proton beam, 0.994 for the PBS proton beam, 0.999 for the pulsed PBS proton beam, 0.996 for the passively scattered carbon ion beam and 0.992 for the scanned carbon ion beam.

4. Discussion

The 3VL-method consists of using three high voltages for which $1/I$ vary linearly with $1/V$ or $1/V^2$ to determine I_{sat} and then k_s-values. While in such a region any method that relies on linearity can be used, such as a two-voltage method or a linear fit to multiple data points, the 3VL-method is especially useful when recombination needs to be determined for many dose points for which the radiation conditions are not very different. Examples of such cases are lateral and depth dose distributions or dose array distributions within a complex field. In such cases,

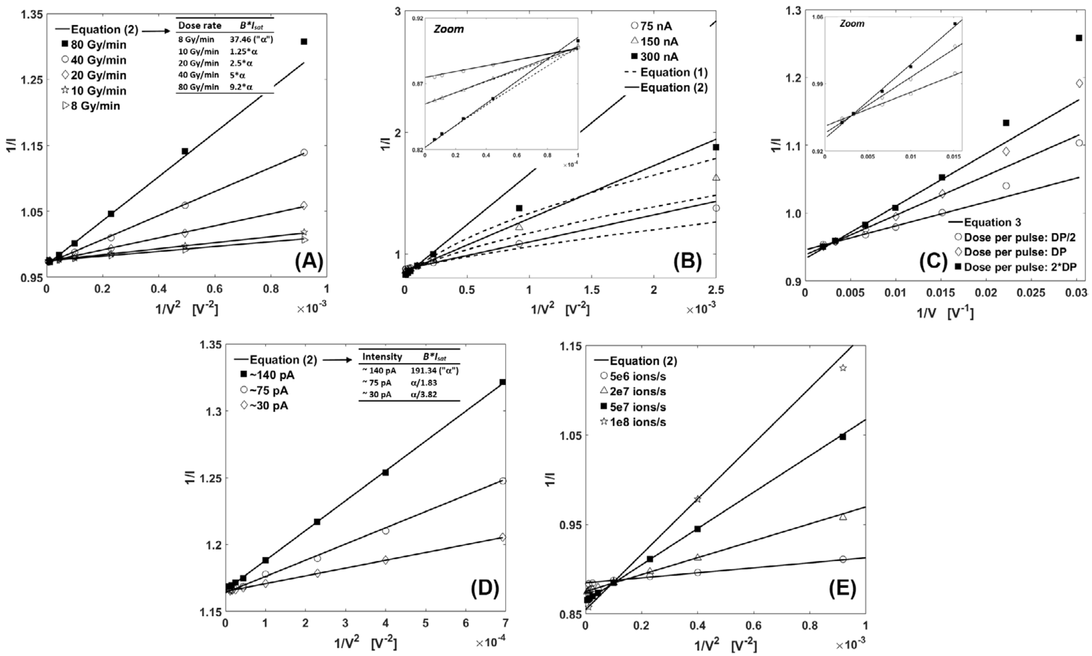

Figure 4. $1/I$ plotted against $1/V^2$ or $1/V$ in continuous or pulsed beams, respectively. Results have been obtained using the NE2571 (SN 1940) cylindrical ionization chamber in (A) a 62 MeV passively scattered proton beam (CAL); (B) a 100 MeV PBS proton beam (APPS); (C) a 96.17 MeV pulsed PBS proton beam (CAL); (D) a 62 MeV/n passively scattered carbon ion beam (INFN-LNS) and (E) a 115 MeV/n scanned carbon ion beam (HIT). Symbols represent experimental data while curves represent theoretical values derived from Boag's models (equations (1), (2) or (3)). For continous beams, continuous and dashed curves are obtained using equations (1) and (2), respectively.

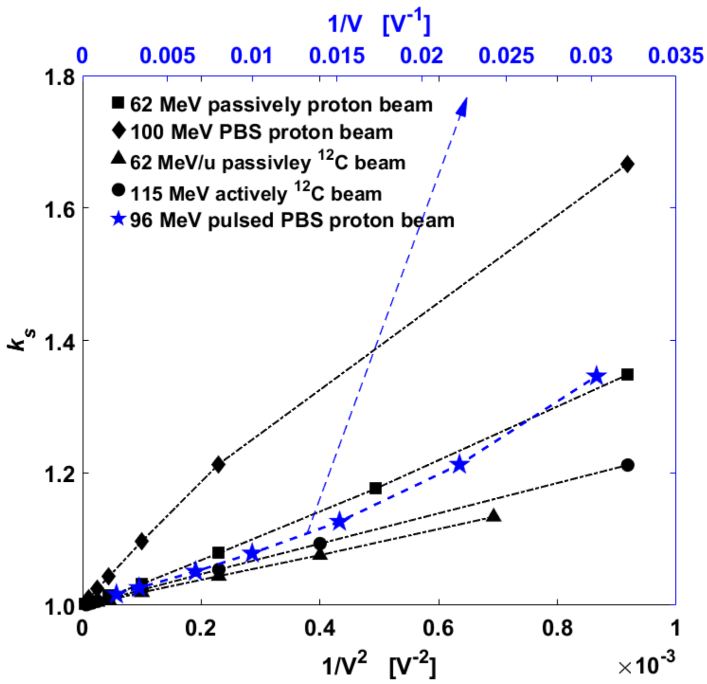

Figure 5. k_s-values as a function of $1/V^2$ or $1/V$ for continuous or pulsed beams, respectively. These values have been obtained using the 3VL-method. Black axes (lower horizontal axis) correspond to results obtained in continuous beams while blue axe (upper horizontal axis as shown by the dashed arrow) corresponds to results obtained in the pulsed PBS proton beam.

Table 2. Ion recombination correction factors determined by different methods at two voltages (100 V and 300 V) using Roos-type ionization chambers. Values in brackets indicate the difference between the method tested and the theoretical model. For clarity, differences are indicated until 0.01%.

	Voltage	Theoretical model	2V-method $V_1/V_2 = 3$	2V-method $V_1/V_2 = 2$	3VL-method $1/I$ versus $1/V$	3VL-method $1/I$ versus $1/V^2$
Proton beams						
					$R^2 = 0.995$	$R^2 = 0.999$
PBS	100 V	1.0080	1.0067[a] (−0.12%)	1.0068[a] (−0.12%)	1.0094 (0.14%)	1.0075 (−0.05%)
	300 V	1.0013	1.0008[a] (−0.05%)	1.0009[a] (−0.04%)	1.0027 (0.14%)	1.0008 (−0.05%)
					$R^2 = 1.000$	$R^2 = 0.990$
Pulsed PBS	100 V	1.0362	1.0380[b] (0.17%)	1.0342[b] (−0.20%)	1.0354 (−0.08%)	1.0269 (−0.90%)
	300 V	1.0127	1.0132[b] (0.05%)	1.0123[b] (−0.04%)	1.0119 (−0.08%)	1.0036 (−0.90%)
Carbon ion beams						
					$R^2 = 0.999$	$R^2 = 0.985$
Passively scattered	100 V	1.0151	1.0021[a] (−1.28%)	1.0031[a] (−1.18%)	1.0138 (−0.13%)	1.0104 (−0.47%)
	300 V	1.0061	1.0011[a] (−0.50%)	1.0016[a] (−0.45%)	1.0048 (−0.13%)	1.0014 (−0.47%)
					$R^2 = 1.000$	$R^2 = 0.989$
Scanned	100 V	1.0103	1.0028[a] (−0.74%)	1.0033[a] (−0.69%)	1.0098 (−0.04%)	1.0075 (−0.28%)
	300 V	1.0037	1.0008[a] (−0.29%)	1.0011[a] (−0.26%)	1.0033 (−0.04%)	1.0010 (−0.28%)

[a] Values obtained using equation (4).
[b] Values obtained using equation (5).

the method offers with a minimum of measurements sufficient information to determine the recombination correction and to verify in the same if the linearity condition is fulfilled for each dose point. Comparing the response of a plane-parallel ionization chamber (figures 3 and S1) and the response of a cylindrical ionization chamber (figure 4) in the same beam types, there are notable differences in the behaviour of the ionization current as a function of the polarizing voltage.

Table 3. Ion recombination correction factors for plane-parallel ionization chambers determined by the 2V-method and the 3VL-method at two different voltages in proton beams.

		2V-method	3VL-method	
PBS	100 V	1.0068[a]	$1/I$ versus $1/V^2$	1.0073 (0.05%)
	300 V	1.0008[a]		1.0006 (−0.03%)
Pulsed PBS	100 V	1.0363[b]	$1/I$ versus $1/V$	1.0361 (−0.02%)
	300 V	1.0135[b]		1.0120 (−0.14%)

[a] Values obtained using equation (4).
[b] Values obtained using equation (5).

4.1. Plane-parallel ionization chambers

In pulsed proton beams, since initial and volume recombination varies linearly with $1/V$ (figure 3(B)), the 3VL-method is based on a linear interpolation of $1/I$ against $1/V$. This high-voltage range should be chosen with care avoiding charge multiplication in the ionization chamber (see section 4.3).

In passively scattered carbon ion beams (figure 3(C)), since the contribution of volume recombination is negligible, recombination is described by Jaffé's model. Even if the first-order term of a series expansion around $1/V = 0$ of Jaffé's relation in terms of $1/V$ is not valid for the entire voltage range, it is valid at high voltages. Consequently, the 3VL-method is also based on a linear variation of $1/I$ against $1/V$.

For scanned light-ion beams, both recombination mechanisms must be considered. Since total recombination is dominated by the initial recombination process at high voltage, the 3VL-method is also based on a linear variation of $1/I$ against $1/V$ (figures 3(D) and 4(D)).

For low LET beams which can be considered as continuous with respect to recombination (i.e. scattered proton beams or PBS proton beams) and for which volume recombination is small, the 3VL-method can use the variation of $1/I$ with $1/V$ or $1/V^2$ since initial recombination contribution is small, as we can observe in figures 3(A) and 4(A).

Comparing numerical k_s-values derived from the theoretical models with those derived with the 3VL-method using $1/I$ plotted against $1/V$ or $1/V^2$, table 2 confirms that, at high voltages (i.e. voltages higher than 100 V), the variation between k_s and $1/V$ can be considered as linear for all beams tested in this study with acceptable accuracy. We have a good agreement between the 3VL-method and theoretical models. The difference between both numerical values is less than 0.15%. However, in PBS proton beams, since ion recombination is dominated by volume recombination (which varies linearly with $1/V^2$), the difference between theoretical results and the 3VL-method is even better using an extrapolation of the linear interpolation of $1/I$ versus $1/V^2$ instead of $1/V$ which can be expected given the continuous character of the beam.

Allowing the evaluation of the quality of linear interpolation, the regression coefficient value (R^2) can be used to know if the second necessary condition for the use of the 3VL-method (i.e. assumption of linearity) is fulfilled. As shown in table 2, in PBS proton beams, a R^2-value equal to 0.995 or 0.999 has been obtained either by using a linear interpolation of $1/I$ against $1/V$ or $1/V^2$, respectively. In both cases, the difference between theoretical results and the 3VL-method is less than 0.15%. For other beams tested in this study, the R^2 coefficient for the linear interpolation of $1/I$ against $1/V^2$ is inferior to 0.990 and leads to a difference between 0.28% and 0.90%, which is significant. Nevertheless, if we consider the variation of $1/I$ against $1/V$, the quality of the linear interpolation improves (R^2 equal to 0.999 or 1.000) giving a k_s-value in agreement with theoretical values.

The k_s-values determined using the 2V-method in table 2 show there is no significant difference between using a voltage ratio of 3 or 2. For all beams tested in this study, the difference between the 2V-method and theoretical models increases with decreasing voltage, due to the increase of the initial recombination contribution. In proton beams, as the contribution of initial recombination is small, the difference between the 2V-method and theoretical models is around 0.20%, similarly to the difference between the 3VL-model (based on a linear interpolation of $1/I$ against $1/V$) and theoretical models. In high LET beams, such as carbon ion beams, since k_s depends on initial and volume recombination, the difference between the theoretical models and the 2V-method increases and the latter is not adequately applicable.

4.2. Cylindrical ionization chambers

In figure 4(D), we can observe that for a cylindrical chamber, k_s varies linearly with $1/V^2$ and we have satisfactory agreement with Boag's model developed for volume recombination. We thus conclude that no significant initial recombination occurs in cylindrical ionization chamber and we did not further consider initial recombination in the current analysis. This is opposed to published data obtained in the same beams with plane-parallel chambers showing that for the latter, initial recombination, in particular columnar recombination, dominates (Rossomme *et al* 2016). In plane-parallel ionization chambers, the electric field lines are parallel with the beam direction and it is well known that the initial recombination effect decreases when the ion tracks are not perpendicular

Table 4. Ion recombination correction factors determined for a NE2571 cylindrical ionization chamber by different methods at different voltages for a NE2571 ionization chamber. Values in round brackets indicate the difference between k_s-values derived using the method tested and the theoretical model (i.e. Boag models). Values in square brackets indicate the difference between the reciprocal of the measured ionization current and the reciprocal of the theoretical ionization current. For clarity, differences are indicated until 0.01%. Due to a lack of experimental data, some results represented by '—' for the 2V-method are missing.

	Voltage	Experimental data	Theoretical model		2V-method		3VL-method
		$1/I$	$1/I$	k_s	k_s $(V_1/V_2 = 3)$	k_s $(V_1/V_2 = 2)$	k_s
Proton beams							
Passively scattered (80 Gy min^{-1})	33 V	1.3075	1.2757 [−2.43%]	1.3165	—	—	1.3482 (−2.35%)
	66 V	1.0462	1.0457 [−0.05%]	1.0791	—	1.0908(−1.07%)[a]	1.0788 (0.03%)
	100 V	1.0009	1.0024 [0.15%]	1.0345	1.0398 (−0.51%)[a]	—	1.0321 (0.23%)
	300 V	0.9734	0.9727 [−0.07%]	1.0038	1.0035 (0.03%)[a]	1.0034(0.04%)[a]	1.0037 (0.01%)
PBS (300 nA)	33 V	1.3718	1.5903 [15.93%]	1.9355	—	—	1.6668 (16.12%)
	66 V	0.9984	1.0138 [1.54%]	1.2339	—	1.1424(8.01%)[a]	1.2131 (1.71%)
	100 V	0.9026	0.9053 [0.31%]	1.1019	1.0695 (3.03%)[a]	—	1.0967 (0.48%)
	200 V	0.8435	0.8425 [−0.12%]	1.0255	1.0235 (0.19%)[a]	—	1.0249 (0.05%)
	300 V	0.8318	0.8309 [−0.10%]	1.0113	1.0108 (0.06%)[a]	1.0109(0.04%)[a]	1.0106 (0.07%)
Pulsed PBS (0.94 pC/pulsed)	33 V	1.2578	1.1675 [−7.19%]	1.2514	—	—	1.3459 (−7.02%)
	66 V	1.0526	1.0502 [−0.23%]	1.1257	—	1.3350(−15.68%)[b]	1.1263 (−0.05%)
	100 V	1.0079	1.0103 [0.24%]	1.0830	1.1900 (−8.99%)[b]	—	1.0785 (0.42%)
	300 V	0.9593	0.9587 [−0.06%]	1.0277	1.0272 (0.04%)[b]	1.0244(0.32%)[b]	1.0264 (0.12%)
Carbon ion beams							
Passively scattered (140 pA)	38 V	1.3212	1.3204 [−0.06%]	1.1325	—	—	1.1335 (−0.09%)
	66 V	1.2170	1.2171 [0.01%]	1.0439	—	—	1.0441 (−0.02%)
	100 V	1.1884	1.1882 [−0.02%]	1.0191	1.0142 (0.49%)[a]	1.0187(0.04%)[a]	1.0195 (−0.04%)
	300 V	1.1686	1.1683 [−0.02%]	1.0021	1.0021 (0.00%)[a]	1.0018(0.03%)[a]	1.0026 (−0.04%)
Scanned (5×10^7 ions s^{-1})	33 V	1.0474	1.0506 [0.31%]	1.2157	—	—	1.2117 (0.33%)
	66 V	0.9111	0.9108 [−0.04%]	1.0539	—	1.0525(0.14%)[a]	1.0540 (−0.01%)
	100 V	0.8846	0.8845 [−0.02%]	1.0235	1.0235 (0.00%)[a]	1.0232(0.03%)[a]	1.0234 (0.01%)
	300 V	0.8664	0.8664 [0.00%]	1.0026	1.0026 (0.00%)[a]	1.0026(0.00%)[a]	1.0024 (0.03%)

[a] Values obtained using equation (4).

[b] Values obtained using equation (5).

Table 5. Based on the beams tested in this study, summary to identify which method can be used to determine ion recombination correction factors in particle beams for plane-parallel ionization chambers (PPICs) and cylindrical ionization chambers (CylICs) with high accuracy in particle beams. 'Low LET' refers to proton beams while 'high LET' refers to light-ion beams. Note that equation (2) is equivalent to equation (7) in which A-value is equal to 0.

		3VL-method				2V-method		Niatel's method	
		$1/I$ versus $1/V$		$1/I$ versus $1/V^2$				Equation (7)	Equation (2) or (3)
Beam		PPICs	CylICs	PPICs	CylICs	PPICs	CylICs	PPICs	CylICs
low LET	Continuous	Yes	No	Yes	Yes	No	Yes	Yes	Yes
	Pulsed	Yes	Yes	No	No	No	Yes	Yes[a]	Yes
high LET	Continuous	Yes	No	No	Yes	No	Yes	No	Yes
	scanned	Yes	No	No	Yes	No	Yes	No	Yes

[a] For PPICs in low LET pulsed beams, equation (7) must be modified to consider the linear relation between volume recombination and $1/V$ instead of $1/V^2$ (as for continuous beams).

to the electric field, i.e. if the plane-parallel ionization chamber is tilted compared to the ion tracks (Jaffé 1913, Rossomme *et al* 2016). The isotropic orientation of electric field lines in the air cavity of cylindrical ionization chambers explains why we did not observe a substantial initial recombination effect.

For the particle beams tested in this study that should be considered as continuous with respect to recombination (i.e. PBS, scattered or scanned particle beams) we analysed the variation of $1/I$ against $1/V^2$, as recommended by Boag. In figure 4, we observed that this variation is not linear over the entire voltage range used, i.e. from 400 V until 20 V. This non-linearity is illustrated by the comparison between Boag's relation (equation (2)) and experimental data. In all graphs on figure 4, we can observe that lower voltages must be discarded to obtain a good agreement between experimental data and theoretical models. In PBS proton beams (figure 4(B)), a comparison with the full Boag's relation (equation (1)) has been added, which allows having a better (but not perfect) agreement with experimental data at low voltages, as we can see from the dashed lines.

Figure 4(C) shows $1/I$ against $1/V$ obtained in a 96 MeV pulsed PBS proton beam. Similarly, to results obtained in continuous particle beams, we can observe that $1/I$ does not vary linearly with $1/V$ over the entire voltage range showing that Boag's theory for pulsed beams (equation (3)) is not applicable to all k_s-values. Nevertheless, at high voltages, we can observe a good agreement between experimental data and equation (3) meaning that Boag's theory for pulsed beams is applicable at high voltages.

For all beams tested, we can observe the linear behaviour of k_s with $1/V^2$ or $1/V$ until a k_s-value of about 1.15. Consequently, when k_s is less than 1.15, corresponding with the cases tested in this study to a voltage until 100 V, good agreement is found between theoretical models and experimental data obtained in passively scattered and scanned particles beams. Numerical values presented in table 4 also illustrate this agreement. When the voltage decreases, the difference between experimental and theoretical $1/I$-values increases. Comparing k_s-values obtained by the 3VL-method with k_s-values obtained using Boag's models, we can observe that the difference increases when the voltage decreases. However, this difference is similar to the difference between experimental and theoretical $1/I$-values. Differences less than 0.2% between numerical values are due to the statistical uncertainty of the measurements. We can thus conclude that the 3VL-method is in agreement with experimental data and allows determining k_s for cylindrical ionization chambers.

Based on k_s-values obtained using the 3VL-method we can see that the difference between experimental and Boag's models becomes increasingly important due to the non-linearity between k_s and $1/V^2$ or $1/V$ when the voltage decreases. Based on the linearity condition between both voltages used, the 2V-method must thus be used cautiously for cylindrical ionization chambers in particle beams. In the experimental conditions tested in this study, we can observe in table 4 that, except for the scanned carbon ion beam, the 2V-method cannot be used to determine k_s at 100 V due to the non-linearity of 1/I against $1/V^2$ or $1/V$ between 100 V and 33 V. This condition will thus guide the choice of the voltage ratio used with the 2V-method.

4.3. The three-voltage linear method: the choice of the three voltages

The choice of the three voltages to derive the saturation ionization current is critical and crucial. It must be done with care, since the 3VL-method assumes linearity between $1/I$ and $1/V$ (or $1/V^2$) and the absence of charge multiplication. To satisfy the first condition, although the new approach could work using two voltages (and would then be an alternative two-voltage method in which the operating voltage is not necessarily one of the two voltages), a third voltage is used to confirm the linearity criterion.

If the response of the ionization chamber is influenced by charge multiplication, a full saturation curve has to be acquired once with the ionization chamber used to study to behaviour of $1/I$ against $1/V$ or $1/V^2$ over the entire voltage range, e.g. between 500 V or 300 V and 20 V and determine a linear region to determine I_{sat}.

In presence of charge multiplication effects, the relation between $1/I$ and $1/V$ (or $1/V^2$) becomes non-linear and the determination of k_s using the 3VL-method at high voltages is not accurate. A simple solution is then to determine I_{sat} using a linear interpolation of $1/I$-values at lower voltages (but sufficiently high to be in a linear region) and k_s could then be calculated based on its definition, i.e. $k_s(\mathrm{V}) = I_{sat}/I(V)$, as detailed in (Rossomme *et al*).

5. Conclusions

In this paper, we have investigated a new simple approach to determine ion recombination correction factors (k_s) in light-ion beams: the three-voltage linear method (3VL-method). The novelty of the method is not so much that three voltages are used but that the operating voltage does not have to be one of those. If the dependence of $1/I$ on $1/V$ or $1/V^2$ is known to be linear then two-voltages would be sufficient, which could be compared with other alternative 2V methods that have been proposed for photon and electron beams (Zankowski and Podgorsak 1998, DeBlois *et al* 2000, Carlino *et al* 2018). Such alternative 2V method is distinct from the conventional 2V method recommended in dosimetry protocols given that the operating voltage is not one of the two voltages. The third voltage can give immediate assurance during the measurement and during periodic QA procedures that the relation is indeed linear, which is especially useful when recombination needs to be determined for many dose points for which the radiation conditions are not very different but for which is not *a priori* clear that the linearity condition is fulfilled for all measurement points, such as in the measurement of lateral and depth dose distributions or dose array distributions within a complex field. The aim of this study was to validate this simple method in passively scattered and scanned particle beams for plane-parallel and Farmer-type cylindrical ionization chambers. While the 2V-method is based on linear fits whose coefficients depend on beam modality or voltages used (Weinhous and Meli 1984, Andreo *et al* 2000), the 3VL-method is simple and based on the determination of the saturation current. Comparing k_s-values obtained using the 2V-method and the 3VL-method to theoretical k_s-values (tables 2 and 4), we can observe that the 3VL-method is more accurate than the conventional 2V-method.

The method consists of (1) measuring the ionization current at three high voltages in a linear region, (2) extrapolating the linear interpolation of $1/I$ versus $1/V$ or $1/V^2$ to obtain I_{sat} and (3) determining k_s as the ratio of the saturation current and the ionization current at the polarizing voltage V, $k_s(V) = I_{sat}/I(V)$, except when there is charge multiplication at the operation voltage, which is addressed in separate work (Rossomme *et al*). The use of three (and not two) voltages ensures that the user can verify that measurements are performed in a linear region. Depending on the type of ionization chambers and beam modality tested in this study, table 5 summarizes which method can be used to determine k_s in particle beams with high accuracy.

For the beams used in this study, the 3VL-method gives an accurate determination of k_s at any polarizing voltage provided the three voltages used for the linear interpolation are not too high to avoid charge multiplication. For the ionization chambers tested in this study, the use of the 3VL-method with 300 V, 200 V and 150 V is adequate and gives an accurate determination of k_s. However, if the necessary conditions to determine I_{sat} using the 3VL-method are not satisfied, *i.e.* if the three voltages are not in a linear region, the three voltages have to be chosen in another region.

In this paper, we presented results obtained in proton and carbon ion beams. Nevertheless, based on experimental results obtained in helium and oxygen ion beams presented in Rossomme *et al* (2017), we performed additional analyses allowing extending results presented in this study to such beams.

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