

Hadron multiplicity calculation: A configurational entropy approach to the saturation scale in QCD

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Abstract – This paper investigates the configurational entropic content of hadron-nucleus collisions. Hadron multiplicities and Au nuclei are employed to compute the critical points of the configurational entropy as a function of the saturation scale in deep inelastic scatterings, in QCD. The results match phenomenological data to the precision of 0.39%.

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Introduction. – The concept of the configurational entropy (CE) was introduced properly to find the correct value of the experimental parameters and data in the framework of QCD and Color Glass Condensate approximation [1]. Such a technique nowadays can straightforwardly be utilized as a trustworthy apparatus in the study of different channels of hot nuclear interaction [2]. This approach has been most frequently used to study the stability of nuclear systems at high excitations, as well as the dominance of quantum resonances that correspond to critical points of the CE. One can also use the critical points of the CE, underlying the evolution of interactions. The critical points of the CE have already been extensively investigated in QCD and particle physics. In fact, scalar and tensor mesons [3–8], glueballs [9], charmonium and bottomonium [10], the quark-gluon plasma [11], baryons [12] and several other systems in QCD have been scrutinized [13], using AdS/QCD and information entropy. The CE, founded in refs. [14–16], emulating the Shannon information entropy, was also studied in refs. [17–20]. In addition, it was used in order to predict the relative stability of physical configurations in refs. [21–23], for AdS black holes and their quantum portrait as Bose-Einstein graviton condensates. Skyrmions, emulating magnetic structures, were scrutinized in ref. [24]. Other physical aspects of the CE were explored in refs. [25–27].

The CE approach to the Color Glass Condensate (CGC) has spectacular results in the high-energy regime, matching, corroborating and predicting phenomenological and

experimental data [1,2,13,28–30]. At such regime, the parton saturation can be observed. This is caused by strong coherent gluon fields [31]. The LHC opens the possibility to explore QCD, in order to deeply investigate and study several new experimental results in the high-density parton mode. General features of the inelastic nucleus-nucleus or/and proton-proton collisions at LHC energy range can be well described in the frame of CGC. To describe the data from RHIC, the calculation at the low x regime can be considered. One of the main features of the computations involves the multiplicities of partons. As it has been suggested in ref. [31], the above-mentioned features do not vary substantially from the initial channel of the interaction up to its final state. It can thus be interpreted as a local parton hadron duality, in other words, the entropy conservation. Obtaining these features of the interaction can enlighten more deeply the mechanism and the dynamics of the collision [1,2]. It is interesting to use, instead of the energy density of any spatially localized system a quantity such as the reaction cross-section, which is also spatially localized, defining the nuclear CE. In such a case, the Fourier transform of the reaction probability allows to derive the critical points of the nuclear CE [1,2,29,30]. Another possibility, to be employed here, is to compute the CE using the hadronic multiplicity as the localized function, for fixed values of the rapidity.

In this context, the critical points of the CE, underlying some system in QCD, can benchmark the existing experimental data and also find the most convenient parameters that can describe several nuclear phenomena. Using the CE concept we provide the saturation scale parameter,

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which predicts the value of the hadron multiplicity at LHC, based on the CGC approach.

The present paper is presented as follows. In the second section, we present the general formalism of nucleus-nucleus or/and hadron-nucleus collisions. The third section is devoted to giving some details of the hadron multiplicities and on the influence of higher-order corrections and the effects of the coupling constants on the obtained results. We present the results and how the critical point of the CE determines an important physical parameter that defines the saturation scale in the case of deep inelastic scatterings. We then summarize our results in the conclusion.

Nucleus–nucleus collisions in the frame of the Glauber approach. – The high parton density, so-called a parton saturation, in the frame of QCD is used in order to study the hadron production at the high-energy regime for nucleus-nucleus central collisions. The results based on the Glauber approach describe such main characteristics of the inelastic events as the hadron multiplicity in nucleus-nucleus, proton-nucleus and proton-proton collisions at LHC energies on the energy in the wide range of the measurements. The Glauber approach allows to reconstruct the geometry of the collision as well as estimate the hadron multiplicity and the inelastic cross-section of the nucleus-nucleus interaction at the certain values of centrality and rapidity. One can suggest that the interaction in the final state does not change significantly the multiplicities of partons, which may arise as a result of the entropy conservation or a local parton duality. So, hadron production also can give us some useful information about the reaction dynamics.

During collisions of high-energy nucleons, the track nucleons are assumed to go into a straight line, due to a small scattering angle as well as the small radius for the nucleon-nucleon collision. Nucleons in interaction can be classified as participants, N_{part} , and the spectator, N_{spect} , respectively for nucleons which undergo at least one inelastic interaction and the non-interacted nucleons. For a nucleus with mass number A , then $N_{\text{part}} = A - N_{\text{spect}}$, which in the case of nucleus- A –nucleus- B collision depends on the impact parameter b in the following form:

$$\begin{aligned} N_{\text{part}}^{AB}(b) &= \int d^2s n_{\text{part}}^{AB}(\mathbf{b}, \mathbf{s}) \\ &= A \int d^2s T_A(\mathbf{s}) \left\{ 1 - [1 - \sigma_{in} T_B(\mathbf{b} - \mathbf{s})]^B \right\} \\ &\quad + B \int d^2s T_B(\mathbf{b} - \mathbf{s}) \left\{ 1 - [1 - \sigma_{in} T_A(\mathbf{s})]^A \right\}, \quad (1) \end{aligned}$$

where $T_A(\mathbf{s}) = \int_{-\infty}^{\infty} dz \rho_A(z, \mathbf{s})$ is the nuclear thickness function, normalized by $\int d^2s T_A(\mathbf{s}) = 1$. The value of σ_{in} represents the inelastic cross-section for the proton-proton interaction.

In the case of proton-nucleus (pA) collision, with the assumption of a point-like size of incident proton, one can

put $B = 1$ in (1) and derive the expression for the number of participants and its average value in the form:

$$N_{\text{part}}^{pA}(b) = A \sigma_{in} T_A(b) + \left\{ 1 - P_0^{pA}(b) \right\}, \quad (2)$$

$$\langle N_{\text{part}}^{pA} \rangle = \frac{\int b N_{\text{part}}^{pA}(b)}{\int b [1 - P_0(b)]} = A \frac{\sigma_{in}}{\sigma_{pA}} + 1, \quad (3)$$

where $P_0^{pA}(b)$ is the probability that no collision occurs between a proton and a nucleus at given impact parameter b , and eq. (1).

One of the main characteristics of the interaction is the multiplicity of charged particles N_{ch} , which can be obtained from the number of participants, $N_{\text{part}}(b)$. Indeed, one can obtain the form for the actual multiplicity, which fluctuates around its mean value, $(2\pi a \langle N_{\text{ch}}(b) \rangle)^{-1/2} e^{-\frac{[N_{\text{ch}} - \langle N_{\text{ch}}(b) \rangle]^2}{2a \langle N_{\text{ch}}(b) \rangle}}$. The coefficient a fixes the width of such fluctuation.

Let us introduce the unintegrated gluon distribution $\phi(x, k_t^2)$ which describes the probability to find a gluon with a given x and transverse momentum k_t inside the nucleus A . The main expression that can be used in order to obtain the inclusive production cross-section from ref. [31] is

$$E \frac{d\sigma}{d^3p} = \frac{3\pi}{2p_t^2} \int^{p_t} dk_t^2 \alpha_s \varphi_{A_1}(x_1, k_t^2) \varphi_{A_2}(x_2, (p - k)_t^2), \quad (4)$$

where $\varphi_{A_1, A_2}(x, k_t^2)$, for $x_{1,2} = (p_t/\sqrt{s}) \exp(\mp y)$, y denoting the rapidity, is the gluon distribution of a nucleus, being s the center-of-mass energy, involving two nuclei, A_1 and A_2 . Integrating eq. (4) over p_t yields the multiplicity distribution,

$$\frac{dN}{dy} = \frac{1}{\sigma} \int d^2p_t E \frac{d\sigma}{d^3p}, \quad (5)$$

with σ being the inelastic cross-section. The saturation scale, Q_s , of deep inelastic scattering, reads

$$Q_s^2(x) = Q_0^2 \left(\frac{x_0}{x} \right)^\lambda, \quad (6)$$

where the value of $\lambda = 0.288 \pm 0.03$. Denoting with \mathcal{E} the collision energy, then the energy and rapidity depend on the saturation scale as

$$Q_s^2(A, y, \mathcal{E}) = Q_0^2(A, \mathcal{E}_0) \left(\frac{\mathcal{E}}{\mathcal{E}_0} \right)^\lambda e^{\lambda y}, \quad (7)$$

where

$$\lambda \equiv \frac{d \log(Q_s^2(x)/\Lambda_{\text{QCD}}^2)}{d \log(1/x)} \approx 0.252. \quad (8)$$

Integration of (7) yields [31]

$$Q_s^2(\mathcal{E}) = \Lambda_{\text{QCD}}^2 e^{\sqrt{2\delta \log\left(\frac{\mathcal{E}}{\mathcal{E}_0}\right) + \log^2\left[Q_s^2\left(\frac{\mathcal{E}_0}{\Lambda_{\text{QCD}}^2}\right)\right]}}. \quad (9)$$

In eq. (9), $Q_s^2(\mathcal{E}_0)$ denotes the saturation scale, characterized by the energy \mathcal{E}_0 , the parameter $\Lambda_{\text{QCD}}^2 = 0.04 \text{ GeV}^2$ and $\delta = \lambda \log(Q_{s0}^2/\Lambda_{\text{QCD}}^2)$ [31].

At high-energy range, the expression that links such concepts as the energy, rapidity, and atomic number dependence on hadron multiplicity reads [31]

$$\rho(s, y, \lambda) = N_{\text{part}} \left(\frac{s}{s_0} \right)^{\frac{\lambda}{2}} e^{-\lambda|y|} \left[\log \left(\frac{Q_s^2}{\Lambda_{\text{QCD}}^2} \right) - \lambda|y| \right] \times \left[1 + \lambda|y| \left(1 - \frac{Q_s}{\sqrt{s}} e^{(1+\lambda/2)|y|} \right)^4 \right]. \quad (10)$$

Equation (10) reasonably predicts the experimentally observed hadron multiplicity at RHIC [31], describing the energy dependence of the charged multiplicity in central Au-Au collisions, at $\sqrt{s} = 130$ GeV and $\lambda = 0.25$ [31]. In the next section the CE will be computed, for the hadron multiplicity as the localized function¹. We will show that this precise value $\lambda = 0.253$, see eq. (8), corresponds to a global minimum of the CE, for central collisions, then corroborating experimental values.

Hadron multiplicity and configurational entropy.

– We consider the hadron multiplicity as a convenient tool for the introduction of a concept such as conditional entropy since it gives us information about mechanism of nucleus-nucleus collisions. The decomposition of the Fourier function related to the hadron multiplicity into various weighted components leads to the definition of the normalized structure factor via the collective coordinates [6]. The modal fraction of CE defines the correlation probability distribution to the power spectrum associated with the hadron multiplicity, which enlarges the information content of the nuclear system. The Shannon entropy approach translates the behavior of the nuclear system through its spatial outlines. It reflects the material content of a complex system via an information source that is available for the given localized configuration. Thus, the main task is to give a correct interpretation of the whole information that can be extracted from CE in order to evaluate various quantitative characteristics of the localized spatial systems.

First, remember that the CE concept involves localized functions [14,16]. Therefore, we compute the Fourier transform of the energy-weighted correlation for the corresponding multiplicity distribution at the LHC, in the CGC approach. This can be implemented by using eq. (10) as the localized function to be employed, for fixed rapidities:

$$\rho(k, \lambda) = \frac{1}{2\pi} \int_{\mathbb{R}} \rho(s, \lambda) e^{iks} ds. \quad (11)$$

Therefore, the modal fraction reads

$$f_{\rho(k, \lambda)} = \frac{|\rho(k, \lambda)|^2}{\int_{\mathbb{R}} |\rho(k, \lambda)|^2 dk}. \quad (12)$$

¹For fixed, but arbitrary, rapidities, as a function of the parameter λ .

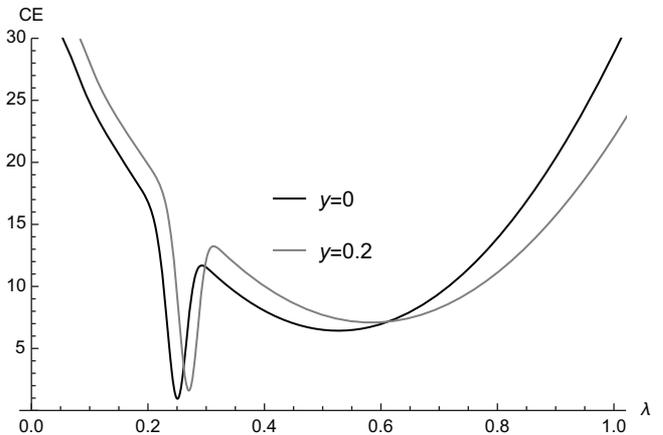


Fig. 1: Configurational entropy (CE) as a function of the saturation scale λ , for rapidity $y = 0$ (black line) and $y = 0.2$ (gray line). For $y = 0$, the CE global minimum occurs at $\lambda = 0.253$, whereas for $y = 0.2$, the CE global minimum is at $\lambda = 0.271$.

Using the corresponding formula for the CE [14], one can get the following:

$$\text{CE}(\lambda) = - \int_{\mathbb{R}} f_{\rho(k, \lambda)} \log f_{\rho(k, \lambda)} dk. \quad (13)$$

The CE can be computed via eqs. (11)–(13) for the hadron multiplicity distribution at the LHC based on the CGC approach [31], using eq. (10). This is implemented numerically, into the plots in fig. 1.

The results obtained for CE, for the multiplicity distribution, show an excellent agreement for the predicted saturation scale $\lambda \approx 0.25$. It is worth emphasizing that the value $y = 0$, adopted in ref. [31] for the rapidity, for the Au nucleus at a fixed energy of $\mathcal{E}_0 = 130$ GeV, corresponds to the cut of 0–6% of most central collisions. Therefore, in the plots of fig. 1, the only one to be compared to the literature will be $y = 0$, being the other plot, regarding $y = 0.2$, shown just for the sake of completeness, as there is no related experimental data for $y = 0.2$ in the literature, up to our knowledge. We will discuss more about it later.

Numerically calculated by eqs. (11)–(13), using eq. (13), the nuclear CE is then plotted in fig. 1.

The CE has a global minimum at $\lambda = 0.253$ for $y = 0$, whereas for $y = 0.2$, the CE global minimum is at $\lambda = 0.271$. These minima occur after a sharp decrement of the CE into a valley of more stable configurations. For $y = 0$, this sharp valley of the CE is in the range $0.2 \lesssim \lambda \lesssim 0.299$. For $y = 0.2$, the valley of the CE appears in the range $0.23 \lesssim \lambda \lesssim 0.32$. These results for $y = 0$ match the expression (8) and the ones in ref. [31], involving central Au-Au collisions, at $\lambda = 0.252$, within 0.39%, for $y = 0$, fixing the most stable configuration attained by the nuclear system. Using the concept of the Shannon information entropy [15], one can figure out the critical points of the CE, that are global in the range analyzed, and thus

establish the natural selection of the saturation scale λ . It corresponds to the point where the nuclear system is more stable, from the informational point of view of the CE. Besides, the only physically acceptable value adopted for the rapidity is $y = 0$ [31], for the Au nucleus, since the value of the saturation scale for $y = 0$ was implemented in ref. [31]. However, it is interesting to realize that the CE for $y = 0$, at the minimum $\lambda = 0.253$, is 0.92, whereas the CE for $y = 0.2$, at the minimum $\lambda = 0.271$, is 1.51. Since the value of the CE at the absolute minimum is lower for $y = 0$, it means that the set of modes constituting the nuclear system has a more stable configuration for $y = 0$. We can show numerically that the higher the rapidity, the higher the CE.

Therefore our results yield a framework in nuclear physics that corroborates the predicted values of the saturation scale λ , for any value of the rapidity, as a global minimum of the CE. Hence, one can assert that the global minimum of the CE complies with the most dominant state of the nuclear configuration, involving central Au-Au collisions.

Conclusions. – Based on the CGC theory, the parton saturation results are used in order to calculate the dependence of the hadron multiplicity on the rapidity, the energy and the saturation scale. Employing eqs. (11)–(13), the global minima of CE were computed for different values of rapidity. To compare with the literature, for the value $y = 0$ the resulting saturation scale $\lambda = 0.253$ matches phenomenological data within 0.39% of precision. The minimum of the CE predicts the predominant nuclear states, providing the natural set of the observables and show an excellent agreement not only with theoretical and phenomenological predictions but also with experimental data.

From the systematic analysis and detailed calculations, we conclude that in the framework of the CE, the hadron multiplicity distribution dependence of the predicted saturation scale at $\lambda \approx 0.25$ provides an excellent description of the observed phenomenological data. Such calculations were obtained taking into account the fixed value of the rapidity $y = 0$ for the central nucleus-nucleus collisions at the energy $\mathcal{E}_0 = 130$ GeV, which is shown in fig. 1 and which has been considered as the most appropriate data for the given value of rapidity in ref. [31]. The calculation at $y = 0$ was compared with the appropriate system for $y = 0.2$, showing that the value of the CE for the absolute minimum at $y = 0$ reflects the more stable configuration of the nuclear system.

From the calculation, we found that the CE displays minima at $\lambda = 0.253$ for $y = 0$, and at $\lambda = 0.271$ for $y = 0.2$. The first result ($y = 0$) is in agreement with the data observed in ref. [31], for the central nucleus-nucleus collisions and there is no experimental result yet for the second one ($y = 0.2$), in the literature. It should be noted that the critical points of the CE can be observed as the most predicted choice for the experimentally obtained

value of the saturation scale λ . Thus, the minima on the calculation curve reflect the stability of the localized nuclear system.

One can study other types of nuclear configurations, with other field theoretical effects and other wave functions, as the ones proposed in refs. [32–37]. It is our aim to implement also the CE in such a context.

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REFERENCES

- [1] KARAPETYAN G. S., *Phys. Lett. B*, **786** (2018) 418 (arXiv:1807.04540 [nucl-th]).
- [2] KARAPETYAN G. S., *Phys. Lett. B*, **781** (2018) 201 (arXiv:1802.09105 [hep-ph]).
- [3] FERREIRA L. F. and DA ROCHA R., *Phys. Rev. D*, **99** (2019) 086001 (arXiv:1902.04534 [hep-th]).
- [4] BERNARDINI A. E. and DA ROCHA R., *Phys. Rev. D*, **98** (2018) 126011 (arXiv:1809.10055 [hep-th]).
- [5] BRAGA N. R. F., FERREIRA L. F. and DA ROCHA R., *Phys. Lett. B*, **787** (2018) 16 (arXiv:1808.10499 [hep-ph]).
- [6] BERNARDINI A. E. and DA ROCHA R., *Phys. Lett. B*, **762** (2016) 107 (arXiv:1605.00294 [hep-th]).
- [7] BARBOSA-CENDEJAS N., CARTAS-FUENTEVILLA R., HERRERA-AGUILAR A., MORA-LUNA R. R. and DA ROCHA R., *Phys. Lett. B*, **782** (2018) 607 (arXiv:1805.04485 [hep-th]).
- [8] FERREIRA L. F. and DA ROCHA R., arXiv:1907.11809 [hep-th].
- [9] BERNARDINI A. E., BRAGA N. R. F. and DA ROCHA R., *Phys. Lett. B*, **765** (2017) 81 (arXiv:1609.01258 [hep-th]).
- [10] BRAGA N. R. F. and DA ROCHA R., *Phys. Lett. B*, **776** (2018) 78 (arXiv:1710.07383 [hep-th]).
- [11] GONCALVES DA SILVA A. and DA ROCHA R., *Phys. Lett. B*, **774** (2017) 98 (arXiv:1706.01482 [hep-th]).
- [12] COLANGELO P. and LOPARCO F., *Phys. Lett. B*, **788** (2019) 500 (arXiv:1811.05272 [hep-ph]).
- [13] MA C. W. and MA Y. G., *Prog. Part. Nucl. Phys.*, **99** (2018) 120 (arXiv:1801.02192 [nucl-th]).
- [14] GLEISER M. and STAMATOPOULOS N., *Phys. Rev. D*, **86** (2012) 045004 (arXiv:1205.3061 [hep-th]).
- [15] GLEISER M. and STAMATOPOULOS N., *Phys. Lett. B*, **713** (2012) 304.
- [16] GLEISER M. and SOWINSKI D., *Phys. Lett. B*, **727** (2013) 272 (arXiv:1307.0530 [hep-th]).
- [17] GLEISER M., STEPHENS M. and SOWINSKI D., *Phys. Rev. D*, **97** (2018) 096007 (arXiv:1803.08550 [hep-th]).
- [18] GLEISER M. and SOWINSKI D., *Phys. Lett. B*, **747** (2015) 125 (arXiv:1501.06800 [hep-th]).
- [19] GLEISER M. and GRAHAM N., *Phys. Rev. D*, **89** (2014) 083502 (arXiv:1401.6225 [hep-th]).
- [20] GLEISER M. and JIANG N., *Phys. Rev. D*, **92** (2015) 044046 (arXiv:1506.05722 [hep-th]).
- [21] FERNANDES-SILVA A., FERREIRA-MARTINS A. J. and DA ROCHA R., *Phys. Lett. B*, **791** (2019) 323 (arXiv:1901.07492 [hep-th]).

- [22] CASADIO R. and DA ROCHA R., *Phys. Lett. B*, **763** (2016) 434 (arXiv:1610.01572 [hep-th]).
- [23] BRAGA N. R. F. and DA ROCHA R., *Phys. Lett. B*, **767** (2017) 386 (arXiv:1612.03289 [hep-th]).
- [24] BAZEIA D., MOREIRA D. C. and RODRIGUES E. I. B., *J. Magn. & Magn. Mater.*, **475** (2019) 734 (arXiv:1812.04950 [cond-mat.mes-hall]).
- [25] BERNARDINI A. E. and DA ROCHA R., *Phys. Lett. B*, **796** (2019) 107 (arXiv:1908.04095 [gr-qc]).
- [26] ALVES A., DIAS A. G. and SILVA R., *Braz. J. Phys.*, **47** (2017) 426 (arXiv:1703.02061 [hep-ph]).
- [27] ALVES A., DIAS A. G. and DA SILVA R., *Physica A*, **420** (2015) 1 (arXiv:1408.0827 [hep-ph]).
- [28] KARAPETYAN G. S., *EPL*, **125** (2019) 58001 (arXiv:1901.05349 [hep-ph]).
- [29] KARAPETYAN G. S., *EPL*, **118** (2017) 38001 (arXiv:1705.1061 [hep-ph]).
- [30] KARAPETYAN G. S., *EPL*, **117** (2017) 18001 (arXiv:1612.09564 [hep-ph]).
- [31] KHARZEEV D., LEVIN E. and NARDI M., *Nucl. Phys. A*, **747** (2005) 609.
- [32] BERNARDINI A. E. and DA ROCHA R., *Phys. Lett. B*, **717** (2012) 238 (arXiv:1203.1049 [hep-th]).
- [33] DA ROCHA R. and RODRIGUES W. A. jr., *Mod. Phys. Lett. A*, **21** (2006) 65.
- [34] CORREA R. A. C. and DA ROCHA R., *Eur. Phys. J. C*, **75** (2015) 522 (arXiv:1502.02283 [hep-th]).
- [35] CORREA R. A. C., DANTAS D. M., ALMEIDA C. A. S. and DA ROCHA R., *Phys. Lett. B*, **755** (2016) 358 (arXiv:1601.00076 [hep-th]).
- [36] BAZEIA D., MENEZES R. and DA ROCHA R., *Adv. High Energy Phys.*, **2014** (2014) 276729 (arXiv:1312.3864 [hep-th]).
- [37] DA ROCHA R., BERNARDINI A. E. and HOFF DA SILVA J. M., *JHEP*, **04** (2011) 110 (arXiv:1103.4759 [hep-th]).