

Testing for gravitational preferred directions with galaxy and lensing surveys

Miguel Aparicio Resco and Antonio L. Maroto

Departamento de Física Teórica
and Instituto de Física de Partículas y del Cosmos IPARCOS,
Universidad Complutense de Madrid, 28040 Madrid, Spain

E-mail: migueapa@ucm.es, maroto@ucm.es

Received August 21, 2019

Revised November 12, 2019

Accepted January 19, 2020

Published February 13, 2020

Abstract. We analyze the sensitivity of galaxy and weak-lensing surveys to detect preferred directions in the gravitational interaction. We consider general theories of gravity involving additional vector degrees of freedom with non-vanishing spatial components in the background. We use a model-independent parametrization of the perturbations equations in terms of four effective parameters, namely, the standard effective Newton constant G_{eff} and slip parameter γ for scalar modes and two new parameters μ_Q and μ_h for vector and tensor modes respectively, which are required when preferred directions are present. We obtain the expressions for the multipole galaxy power spectrum in redshift space and for the weak-lensing shear, convergence and rotation spectra in the presence of preferred directions. By performing a Fisher matrix forecast analysis, we estimate the sensitivity of a future Euclid-like survey to detect this kind of modification of gravity. We finally compare with the effects induced by violations of statistical isotropy in the primordial power spectrum and identify the observables which could discriminate between them.

Keywords: modified gravity, cosmological parameters from LSS

ArXiv ePrint: [1907.12285](https://arxiv.org/abs/1907.12285)

Contents

1	Introduction	1
2	Model-independent parametrization of anisotropic modified gravities	4
3	Galaxy power spectrum	5
4	Weak lensing: null geodesics with scalar, vector and tensor perturbations	7
5	The distortion tensor	9
6	Weak-lensing power spectra	11
7	Fisher analysis for the multipole power spectrum	15
7.1	Fiducial cosmology and galaxy redshift survey	16
8	Fisher analysis for the redshift-space power spectrum	18
9	Fisher analysis for the weak lensing power spectrum	21
10	Forecasting primordial anisotropies	24
11	Conclusions	25
A	Clustering covariance matrices	26
B	Lensing covariance matrices	29

1 Introduction

Rotational invariance, as part of the Lorentz group, is one of the underlying symmetries in our current description of the fundamental interactions of nature. The weak Equivalence Principle, which is one of the cornerstones of General Relativity (GR), ensures that Lorentz invariance is respected not only in flat space-time, but also in the presence of gravity, where the symmetry is locally preserved [1–3].

On the other hand, it is also well established from current observations [4] that rotational symmetry is also manifested in a statistical way on the large-scale distribution of matter and radiation in the universe. In the standard inflationary scenario, density perturbations are generated from quantum vacuum fluctuations, so that the isotropy of the primordial spectrum of perturbation reflects the invariance under rotations of the quantum vacuum state [5, 6].

Despite the fact that our current description of interactions seems to be compatible with rotational invariance on a wide range of scales, certain observations seem to suggest the existence of preferred spatial directions on cosmological scales. Thus, anomalies have been detected in the low multipoles of the CMB [4, 7]. They include the alignment of quadrupole, octupole and ecliptic plane, a dipole anomaly in the power spectrum that breaks statistical isotropy and the hemispherical anomaly whose maximum asymmetry is observed in the ecliptic frame. On the other hand, large scale bulk flows have also been detected

with an amplitude which has been claimed to exceed the predictions of standard Λ CDM [8–10]. Although the statistical significance of such anomalies is somewhat limited, and they could be due to systematic effects, they have motivated the search for preferred directions in cosmology.

One of the simplest frameworks to explore the consequences of Lorentz symmetry breaking is the presence of tensor fields acquiring non-vanishing vacuum expectation values. This is indeed the case of the so called Standard Model Extension (SME) [11]. In particular, in the case in which such vacuum expectation value is acquired by a vector field, the first models were proposed by Nambu already in the sixties [12]. Depending on the particular type of vector, this mechanism can induce two kinds of gravitational effects. On one hand, if the vector field is timelike, preferred frame effects would be present. On the other hand, a space-like vacuum expectation value for the vector field will generate preferred directions effects in which we are interested in this paper.

Preferred frame effects have been explored in local gravitational experiments through the so called parametrized post Newtonian parameters (PPN) formalism [2, 13]. In particular, two PPN parameters, α_1 and α_2 , have been restricted by Solar System and pulsar observations. Also, modifications in the gravity wave dispersion relations have been studied in [14]. From a theoretical point of view, theories of gravity such as Horava gravity [15] or Einstein-aether [16] have been shown to generate this kind of preferred frame effects. Also on the cosmological framework, different kinds of vector-tensor theories including temporal background vector fields have been analysed in the context of dark energy [17–23].

Preferred directions effects have been explored in the framework of the anisotropic PPN formalism [24] and bounds from laboratory experiments have been obtained in [25]. The possible cosmological implications have been studied both on the CMB temperature power spectrum [4, 6] and in the matter distribution in [26–29]. In those works, the evolution both of the background and perturbations is assumed to be the standard in Λ CDM and the anisotropy is assumed to be present only in the primordial power spectra. Such anisotropic power spectrum can be generated for instance in models of inflation with vectors [30–32] or higher-spin fields [33]. A different kind of effects would be those associated to the presence of non-comoving fluids singling out a preferred direction as those considered in [34–36].

However, in this work we will focus on a different possibility for the generation of preferred direction effects, i.e. that such directions are built in the theory of gravity itself. As commented before, theories of gravity involving additional vector degrees of freedom have been analyzed in detail in recent years in the case in which the vector field acquires a temporal background. If the background vector field A_μ is spatial, the gravitational dynamics can give rise to a modified evolution of perturbations, thus introducing anisotropies in the corresponding transfer functions. Such modified evolution will however depend on the particular theory under consideration. It is precisely the aim of this work to analyze this kind of effects in a model-independent way from the data that will be provided by future galaxy and weak-lensing surveys. With that purpose, we will consider the effective approach to modified gravity for theories involving vector degrees of freedom developed in [37]. Within the sub-Hubble and quasi-static (QSA) approximations, which are very well suited to galaxy surveys analysis, it is well-known that a very general modification of gravity involving additional scalar degrees of freedom can be described with only two additional parameters: an effective Newton constant $\mu(k, a) = G_{\text{eff}}/G$ and a gravitational slip parameter $\gamma(k, a)$ [38, 39]. In the vector case, when the background vector field is purely temporal, the theory can still be parametrized only with $\mu(k, a)$ and $\gamma(k, a)$ parameters, but this scenario changes when

we have a preferred direction. In this case we need two additional effective parameters (if dark matter vorticity can be neglected as is usually the case) which relate matter density perturbations to vector and tensor metric perturbations. Apart from the standard time a and scale k dependence, those four effective parameters can have an additional $y = \hat{k} \cdot \hat{A}$ dependence on the angle between the wave-vector direction \hat{k} and the preferred direction fixed by the background vector field \hat{A} .

As mentioned above our goal is to analyze the impact of preferred directions effects in galaxy and weak lensing surveys. Galaxy surveys can be classified in two main categories: on one hand we have the spectroscopic surveys, such as BOSS [40] or the future DESI [41] and the spectroscopic Euclid [42] survey which produce high-quality spectra that allow to measure galaxy redshifts with high precision for a pre-selected sample of objects. On the other hand we have photometric surveys, such as DES [43] or the future photometric Euclid survey and LSST [44] which measure higher number of objects with poorer redshift precision and, in addition, obtain object images that allow to make weak lensing maps. Finally, there are also spectro-photometric surveys such as J-PAS [45] which, thanks to the high number of filters, can detect a high number of objects with good photo- z precision. The two main observables that can be extracted from galaxy surveys are, on one hand, the galaxy power spectrum [46, 47] and, on the other, the weak lensing shear and convergence spectra [48–50]. In particular, we will consider an Euclid-like experiment with both spectroscopic and photometric surveys for clustering and lensing forecast analysis respectively.

The redshift-space galaxy power spectrum is the main observable for galaxy clustering [51]. It is sensitive to the growth of structures via the growth factor $D(z) = \delta_m(z)/\delta_m(0)$. In addition, thanks to the Alcock-Paczynski effect [52], the power spectrum is sensitive to the Hubble parameter $H(z)$ and the angular distance $D_A(z)$. Finally, due to the peculiar velocities, the position of galaxies in redshift space are distorted (RSD) [53]. This effect introduces a dependence on the line of sight that involves the growth function $f(z) = d \ln D / d \ln(a)$. For all these reasons, the redshift space power spectrum has a strong dependence on the cosmological model and on the underlying gravitational theory. As a matter of fact, when a preferred direction is present, an additional y dependence is present which can be disentangled from the standard angular dependence induced by the RSD. Also the anisotropic effects generated by the gravity modification could be distinguished from those induced by anisotropic primordial power spectra.

On the other hand, we have the weak lensing effect [54, 55] which is the distortion of the shape of galaxies due to the gravitational perturbations. For scalar perturbations, the possible distortions are the convergence κ , i.e the change in the size of the image, and the shear γ_1 and γ_2 , which modifies the ellipticity of the image. In the standard case, the shear power spectra can be obtained from the convergence power spectrum [56, 57]. Moreover, we have the following relationship between them,

$$P_{\gamma_1} + P_{\gamma_2} = P_{\kappa}, \quad (1.1)$$

where, in principle, convergence and shear can be measured independently [58, 59]. These power spectra give us information about the gravitational perturbations that affect light propagation. When a preferred direction is present, density perturbations can source vector and tensor modes thus affecting the lensing distortion tensor. In this case, a new effect is present which is the rotation ω of the images. This rotation mode is rarely studied in the literature because it is a higher-order effect in the standard Λ CDM cosmology [60]. Also, to measure this rotation effect using weak lensing surveys is not possible because there is

no information about the original orientation of the galaxy image [61]. However, as we will show, the rotation effect can be detected in an indirect way using the new closing relation,

$$P_{\gamma_1} + P_{\gamma_2} = P_{\kappa} + P_{\omega}, \quad (1.2)$$

i.e. independent measurements of P_{κ} , P_{γ_1} and P_{γ_2} will allow to constrain the rotation power spectrum P_{ω} . Moreover, the new P_{ω} cannot be generated by an anisotropic primordial curvature spectrum, so that a violation of the closing relation (1.1) will be a smoking gun for this kind of modifications of gravity.

Besides, we find that the modified convergence power spectrum acquires a line-of-sight dependence which is absent in standard Λ CDM. This has allowed us to construct the convergence multipole power spectrum. Thus a future detection of a non-vanishing multipolar component could be a potential signal of the existence of a gravitational preferred direction.

The paper is organized as follows: in 2 we briefly summarize the results of [37] for the anisotropic modified gravity parametrization. In 3 we analyze the multipole power spectrum of clustering in the presence of an anisotropic vector background and we also study the effects of the anisotropy in the weak lensing signals. In 4 we obtain the null geodesics in the presence of scalar, vector and tensor perturbations. In 5 we calculate the distortion tensor, and in 6 we compute the weak lensing power spectra using the model-independent parametrization. In 7 we present the Fisher matrix analysis for the multipole power spectra case and we obtain the sensitivity for measurements of the effective $\mu(a, k, y)$ parameter. In 8 we compute the Fisher matrix for the redshift space power spectrum of galaxies to compare with the multipole case. In 9 we present the Fisher matrix of the convergence power spectrum and we obtain the sensitivity for the modified gravity parameters. In 10 we apply the Fisher formalism to the case of an anisotropic primordial curvature spectrum. In section 11 we briefly discuss the results and conclusions. Finally in the appendices we calculate the covariance matrices for the galaxy and convergence power spectra in the presence of preferred directions.

2 Model-independent parametrization of anisotropic modified gravities

In this first section we summarize the results of [37] on the model-independent parametrization of modified gravity theories with an additional vector field A_{μ} . Let us thus start by considering a general anisotropic Bianchi I cosmology with scalar (Φ, Ψ) , vector Q_i and tensor h_{ij} perturbations in the longitudinal gauge [62],

$$ds^2 = a^2 \left[-(1 + 2\Psi) d\tau^2 + [(1 - 2\Phi) \Xi_{ij} + h_{ij}] dx^i dx^j - 2 Q_i d\tau dx^i \right], \quad (2.1)$$

where Ξ_{ij} is the Bianchi tensor that reduces to $\Xi_{ij} = \delta_{ij}$ in the isotropic limit and vector perturbation satisfy $k^i Q_i = 0$, whereas for tensors we have $k^i h_{ij} = 0$ and $h_i^i = 0$. Considering that the extra vector field A_{μ} can have both temporal and spatial background components, the equations that relate the different metric and matter perturbations in the sub-Hubble regime and in the quasi-static approximation read in Fourier space,

$$k^2 \Psi \equiv -4\pi G a^2 \rho \mu_{\Psi} \delta(k), \quad (2.2)$$

$$k^2 \Phi \equiv -4\pi G a^2 \rho \mu_{\Phi} \delta(k), \quad (2.3)$$

$$k^2 Q_i \equiv 16\pi G a^2 \rho \mu_Q \mathcal{A}_i \delta(k), \quad (2.4)$$

$$k^2 h_{ij} \equiv -4\pi G a^2 \rho \mu_h \Sigma_{ij} \delta(k), \quad (2.5)$$

where we have neglected the contribution from dark matter vorticity. Here G is the gravitational Newton constant, ρ is the pressureless matter density, $\delta(k)$ is the matter density contrast and,

$$\mathcal{A}_i = \hat{A}_i - y \hat{k}_i, \quad (2.6)$$

$$\Sigma_{ij} = 2 \mathcal{A}_i \mathcal{A}_j - (1 - y^2) (\delta_{ij} - \hat{k}_i \hat{k}_j), \quad (2.7)$$

where hat denotes the corresponding unit vector. These quantities satisfy the following properties,

$$\begin{aligned} \Sigma_{ij} &= \Sigma_{ji}, & \hat{k}^i \Sigma_{ij} &= 0, \\ \Sigma^i_i &= 0, & \hat{k}^i \mathcal{A}_i &= 0, \end{aligned} \quad (2.8)$$

Unlike the case of modified gravities with an additional scalar degree of freedom [39] four parameters $(\mu_\Psi, \mu_\Phi, \mu_Q, \mu_h)$ are needed to describe the most general modification of gravity in the presence of vector and tensor perturbations and an anisotropic background. Notice that in general such parameters are functions of (a, k, y) . In the particular case of an isotropic background $A_i = 0$, we need only two parameters μ_Ψ and μ_Φ which are related to those of the scalar case as $\mu = \mu_\Psi$ and $\gamma = \mu_\Phi / \mu_\Psi$ with

$$\mu = \frac{G_{\text{eff}}}{G}, \quad (2.9)$$

and

$$\gamma = \frac{\Phi}{\Psi}. \quad (2.10)$$

3 Galaxy power spectrum

Now we will analyze how the galaxy power spectrum is modified for the theories of gravity we have just introduced. We will consider for simplicity the case in which the background metric is the standard Robertson-Walker metric of Λ CDM cosmology. In this case, the only effects of the preferred direction come either from the matter power spectrum which can now exhibit an statistical anisotropy $P = P(k, y)$ or from the growth of scalar perturbations. Indeed, let us define the growth factor $D(a)$ normalized as $D(a) = \delta(a)/\delta(0)$ and the corresponding growth function

$$f(z) = \frac{d \ln(D(a))}{d \ln(a)}, \quad (3.1)$$

being $a = 1/(1+z)$ the scale factor, which satisfies

$$\dot{f} + f^2 + \left(2 + \frac{\dot{H}}{H}\right) f - \frac{3}{2} \mu \Omega_m(a) = 0, \quad (3.2)$$

where dots denotes derivative with respect to $\ln a$, $H(a) = H_0 E(a)$ is the Hubble parameter and $\Omega_m(a)$ is the matter density parameter $\Omega_m(a) = \Omega_m a^{-3} \frac{H_0^2}{H^2(a)}$. Notice that the only modification with respect to the standard cosmology is the appearance of the effective parameter $\mu(a, k, y)$ which introduces the scale and direction dependence in the growth evolution. For small anisotropy we can always expand [37],

$$\mu(a, k, y) = \mu_0(a, k) + \mu_2(a, k) y^2 + \mu_4(a, k) y^4 + O(y^6). \quad (3.3)$$

Taking the anisotropic growth into account, the redshift-space linear galaxy power spectrum can be written as [51],

$$P_g(z, k, \hat{\mu}, y) = [(1 + \beta(z, k, y) \hat{\mu}^2) b(z) D(z, k, y)]^2 P(k, y), \quad (3.4)$$

where k is the modulus of the perturbation \vec{k} , and $\hat{\mu} = \hat{k} \cdot \hat{n}$ being \hat{n} the line of sight. Here $P(k, y)$ is the matter power spectrum today which can be related with the matter power spectrum today in Λ CDM model $P_\Lambda(k)$ as $P(k, y)/P_\Lambda(k) = D_\Lambda^2(z_{mat})/D^2(z_{mat}, k, y)$, so that

$$P(k, y) = \exp \left[2 \int_0^{z_{mat}} \frac{f(z', k, y) - f_\Lambda(z')}{1 + z'} dz' \right] P_\Lambda(k), \quad (3.5)$$

where $f_\Lambda(z)$ is the growth function in Λ CDM and we have assumed that for $z > z_{mat}$, $f(z, k, y) = f_\Lambda(z)$. For the sake of concreteness in the forecast analysis we will assume that $z_{mat} = 10$ although the results are not very sensitive to its precise value. As mentioned before, $D(z, k, y)$ is the growth factor, $b(z)$ is the galaxy bias and $\beta(z, k, y) = f(z, k, y)/b(z)$.

As we can see from (3.4), the redshift-space galaxy power spectrum has two different kinds of anisotropic contributions: on one hand the standard contribution from redshift space distortions (RSD) which introduces a quadrupole and hexadecapole in $\hat{\mu}$, and on the other, an extra contribution coming from the y dependence of the growth function. Thus performing a multipole expansion with respect to the line of sight we find,

$$P_g(z, k, \hat{\mu}, y) = \sum_\ell P_\ell(z, k, y) \mathcal{L}_\ell(\hat{\mu}), \quad (3.6)$$

where \mathcal{L}_ℓ are the Legendre polynomials so that

$$P_\ell(z, k, y) = \frac{2\ell + 1}{2} \int_{-1}^1 d\hat{\mu} P_g(z, k, \hat{\mu}, y) \mathcal{L}_\ell(\hat{\mu}). \quad (3.7)$$

obtaining $P_\ell(z, k, y)$ different from zero for $\ell = 0, 2, 4$ i.e. we recover the well-known monopole, quadrupole and hexadecapole contributions but with the new y dependence.

In the particular case in which the modified gravity parameter μ is time independent, i.e. $\mu = \mu(k, y)$ a simple analytical expression for $f(z, k, y)$ can be obtained [63],

$$f(z, k, y) = \xi(\mu(k, y)) f_\Lambda(z), \quad (3.8)$$

being $f_\Lambda(z) = \Omega_m^\gamma(z)$ with $\gamma = 0.55$ [64, 65] and,

$$\xi(\mu) = \frac{1}{4} (\sqrt{1 + 24\mu} - 1). \quad (3.9)$$

In this case, explicit expressions for the multipoles can be obtained. Thus we have

$$P_0(z, k, y) = \left(1 + \frac{2}{3} \xi(\mu) \beta_\Lambda(z) + \frac{1}{5} \xi^2(\mu) \beta_\Lambda^2(z) \right) b^2(z) D_\Lambda^{2\xi(\mu)}(z) P(k, y), \quad (3.10)$$

$$P_2(z, k, y) = \left(\frac{4}{3} \xi(\mu) \beta_\Lambda(z) + \frac{4}{7} \xi^2(\mu) \beta_\Lambda^2(z) \right) b^2(z) D_\Lambda^{2\xi(\mu)}(z) P(k, y), \quad (3.11)$$

$$P_4(z, k, y) = \frac{8}{35} \xi^2(\mu) \beta_\Lambda^2(z) b^2(z) D_\Lambda^{2\xi(\mu)}(z) P(k, y), \quad (3.12)$$

where,

$$\beta_{\Lambda}(z) = \frac{f_{\Lambda}(z)}{b(z)}, \quad (3.13)$$

and

$$f_{\Lambda}(z) = \frac{d \log(D_{\Lambda}(a))}{d \log(a)}. \quad (3.14)$$

The multipole coefficients P_{ℓ} , depend in turn on the angular variable y and therefore could be additionally expanded in a different multipole expansion with respect to y . Alternatively, a bi-polar expansion in $(\hat{\mu}, y)$ [28], could have been performed. However for the Fisher analysis that we will perform in this work, we will directly work with the P_{ℓ} coefficients.

4 Weak lensing: null geodesics with scalar, vector and tensor perturbations

In order to obtain the convergence and shear power spectra for weak lensing in the presence of scalar, vector and tensor perturbations, we start with the Bianchi perturbed metric (2.1), where as in the previous section we have considered for simplicity $\Xi_{ij} \simeq \delta_{ij}$. We will also work in cosmological time t so that the metric reads

$$ds^2 = - (1 + 2\Psi) dt^2 + a(t)^2 [(1 - 2\Phi) \delta_{ij} + h_{ij}] dx^i dx^j - 2 Q_i a(t) dt dx^i, \quad (4.1)$$

For this metric, we are interested in deriving the corresponding null geodesics, satisfying

$$\frac{d^2 x^i}{d\lambda^2} + \Gamma_{\alpha\beta}^i \frac{dx^{\alpha}}{d\lambda} \frac{dx^{\beta}}{d\lambda} = 0. \quad (4.2)$$

We will consider the angular perturbation with respect to the line of sight induced by the metric perturbations. Thus, we define $x^i = \chi \theta^i$ where $\chi = \chi(z)$ is the comoving radial distance and $\theta^i = (\theta^1, \theta^2, 1)$, so that θ^i for $i = 1, 2$ are first order in the gravitational perturbations and $x^3 = \chi$. The goal is to obtain the geodesics (4.2) for $i = 1, 2$,

$$\frac{d^2 x^i}{d\lambda^2} = \frac{d\chi}{d\lambda} \frac{d}{d\lambda} \left(\frac{d\chi}{d\lambda} \frac{d}{d\lambda} (\chi \theta^i) \right), \quad (4.3)$$

where,

$$\frac{d\chi}{d\lambda} = \frac{d\chi}{dt} \frac{dt}{d\lambda}, \quad (4.4)$$

and $\frac{d\chi}{dt} = -\frac{1}{a}$. In order to obtain $\frac{dt}{d\lambda}$ we define $P^{\mu} = \frac{dx^{\mu}}{d\lambda}$, where, for null geodesics,

$$g_{\mu\nu} P^{\mu} P^{\nu} = 0, \quad (4.5)$$

so, at order zero in perturbations, we have,

$$-(P^0)^2 + g_{ij} P^i P^j = 0. \quad (4.6)$$

By defining $p^2 \equiv g_{ij} P^i P^j$ we find,

$$\frac{dt}{d\lambda} = p, \quad (4.7)$$

so that we obtain $\frac{d\chi}{d\lambda} = -\frac{p}{a}$ and, since for $i = 1, 2$, θ^i is first order in perturbations, we can write

$$\frac{d^2 x^i}{d\lambda^2} = -\frac{p}{a} \frac{d}{d\chi} \left(-\frac{p}{a} \frac{d}{d\chi} (\chi \theta^i) \right). \quad (4.8)$$

Thus we only need p to zeroth order, which satisfies $pa \propto \text{const}$ so that,

$$\frac{d^2 x^i}{d\lambda^2} = p^2 \frac{d}{d\chi} \left(\frac{1}{a^2} \frac{d}{d\chi} (\chi \theta^i) \right). \quad (4.9)$$

On the other hand, we have the Christoffel symbol term,

$$\Gamma_{\alpha\beta}^i \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} = \left(\frac{d\chi}{d\lambda} \right)^2 \Gamma_{\alpha\beta}^i \frac{dx^\alpha}{d\chi} \frac{dx^\beta}{d\chi}. \quad (4.10)$$

For the metric (4.1), we have,

$$\Gamma_{00}^i = a^{-2} \Psi_{,i} - a^{-1} (H Q_i + Q_{i,0}), \quad (4.11)$$

$$\Gamma_{j0}^i = \delta_{ij} (H - \Phi_{,0}) - a^{-1} Q_{[i,j]} + \frac{1}{2} h_{ij,0}, \quad (4.12)$$

$$\Gamma_{jk}^i = \Phi_{,i} \delta_{jk} - \Phi_{,k} \delta_{ij} - \Phi_{,j} \delta_{ki} + a H Q_i \delta_{jk} + \frac{1}{2} (h_{ij,k} + h_{ik,j} - h_{jk,i}), \quad (4.13)$$

where a comma denotes derivative with respect to the coordinates (t, x^1, x^2, x^3) and $H = \frac{1}{a} \frac{da}{dt}$ is the Hubble parameter. Let us analyze the different terms of equation (4.10):

- $\alpha = \beta = 0$: in this case we only have the term $\Gamma_{00}^i \left(\frac{dt}{d\chi} \right)^2$, and $\frac{dt}{d\chi} = -a$ to zeroth order, so that we obtain,

$$\Gamma_{00}^i \left(\frac{dt}{d\chi} \right)^2 = \Psi_{,i} - a [H Q_i + Q_{i,0}]. \quad (4.14)$$

- $\alpha = j, \beta = 0$ (and the symmetric case): now we have $\Gamma_{j0}^i \frac{dt}{d\chi} \frac{dx^j}{d\chi}$. For $j = 1, 2$ the derivative $\frac{dx^j}{d\chi}$ is first order in perturbations, so that in this case Γ_{j0}^i must be order zero. However, when $j = 3$, we have $\frac{dx^3}{d\chi} = 1$ then Γ_{30}^i has to be first order in perturbations. Taking all the terms into account we obtain,

$$\Gamma_{j0}^i \frac{dt}{d\chi} \frac{dx^j}{d\chi} = -a H \frac{d}{d\chi} (\chi \theta^i) + Q_{[i,3]} - \frac{1}{2} a h_{i3,0}. \quad (4.15)$$

Notice that since we also have Γ_{0j}^i , the term (4.15) contributes twice to the final expression.

- $\alpha = j, \beta = k$: finally we have $\Gamma_{jk}^i \frac{dx^j}{d\chi} \frac{dx^k}{d\chi}$, because x^j is order one when $j \neq 3$ and Γ_{jk}^i is always order one, the only term that contributes corresponds to $j = k = 3$ ($i = 1, 2$),

$$\Gamma_{33}^i \left(\frac{dx^3}{d\chi} \right)^2 = \Phi_{,i} + a H Q_i + h_{i3,3} - \frac{1}{2} h_{33,i}. \quad (4.16)$$

As we can see in the previous analysis, $\Gamma_{\alpha\beta}^i \frac{dx^\alpha}{d\chi} \frac{dx^\beta}{d\chi}$ is first order in perturbations, so that the prefactor $(\frac{d\chi}{d\lambda})^2$ in (4.10) must be of zeroth order. Finally, equation (4.10) becomes,

$$\Gamma_{\alpha\beta}^i \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} = \left(\frac{p}{a}\right)^2 \left[(\Phi + \Psi)_{,i} - 2aH \frac{d}{d\chi}(\chi\theta^i) + 2Q_{[i,3]} + h_{i3,3} - \frac{1}{2}h_{33,i} - a(Q_i + h_{i3})_{,0} \right]. \quad (4.17)$$

If we expand (4.9), and taking into account that $\frac{d}{d\chi} = -a^2 H \frac{d}{da}$, we find,

$$\frac{d^2 x^i}{d\lambda^2} = \left(\frac{p}{a}\right)^2 \left[\frac{d^2(\chi\theta^i)}{d\chi^2} + 2aH \frac{d}{d\chi}(\chi\theta^i) \right]. \quad (4.18)$$

Thus, using (4.17) and (4.18) we can obtain from the geodesic equation (4.2),

$$\frac{d^2}{d\chi^2}(\chi\theta^i) = -(\Phi + \Psi)_{,i} - 2Q_{[i,3]} - h_{i3,3} + \frac{1}{2}h_{33,i} + a(Q_i + h_{i3})_{,0}. \quad (4.19)$$

At this point we apply the quasi-static approximation (QSA) and the sub-Hubble regime in which we can neglect the time derivatives of perturbations with respect to the spatial derivatives,

$$\frac{d^2}{d\chi^2}(\chi\theta^i) = -(\Phi + \Psi)_{,i} - 2Q_{[i,3]} - h_{i3,3} + \frac{1}{2}h_{33,i}. \quad (4.20)$$

It will be useful to define the source term of equation (4.20) as,

$$Y_i \equiv -\left(\Phi + \Psi + Q_3 + \frac{1}{2}h_{33}\right)_{,i} - (Q_i + h_{i3})_{,3}, \quad (4.21)$$

In the following section we will proceed with the integration of equation (4.20) and the definition of the distortion tensor.

5 The distortion tensor

By integrating twice equation (4.20) we obtain,

$$\theta_i^S = \frac{1}{\chi} \int_0^\chi d\chi'' \int_0^{\chi''} d\chi' Y_i(\chi'\vec{\theta}) + \text{const.} \quad (5.1)$$

Since the integrand is just a function of χ' , we can integrate over χ'' and fix the integration constant as the initial angle θ_i ,

$$\theta_i^S = \theta_i + \int_0^\chi d\chi' Y_i(\chi'\vec{\theta}) \left(1 - \frac{\chi'}{\chi}\right). \quad (5.2)$$

Now, we define the distortion tensor as,

$$\psi_{ij} \equiv \frac{\partial \theta_i^S}{\partial \theta_j} - \delta_{ij}. \quad (5.3)$$

By using $\frac{\partial}{\partial \theta_j} = \frac{\partial x^k}{\partial \theta_j} \frac{\partial}{\partial x^k} = \chi \frac{\partial}{\partial x^j}$ we obtain,

$$\psi_{ij} = \int_0^\chi d\chi' \chi' Y_{i,j} \left(1 - \frac{\chi'}{\chi}\right), \quad (5.4)$$

where $\psi_{ij} = \psi_{ij}(\chi, \vec{\theta})$. We want to integrate over χ to project onto the two-dimensional (θ_1, θ_2) plane. In general the survey contains a distribution of galaxies $W(\chi)$, which is normalized as $\int_0^{\chi_\infty} d\chi W(\chi) = 1$, where $\chi_\infty = \lim_{z \rightarrow \infty} \chi(z)$ so that the projected distortion tensor is,

$$\psi_{ij}(\vec{\theta}) = \int_0^{\chi_\infty} d\chi W(\chi) \int_0^\chi d\chi' \chi' Y_{i,j} \left(1 - \frac{\chi'}{\chi}\right). \quad (5.5)$$

By changing the order of integration, we can obtain,

$$\psi_{ij}(\vec{\theta}) = \int_0^{\chi_\infty} d\chi \chi g(\chi) Y_{i,j}(\chi, \vec{\theta}), \quad (5.6)$$

where we have defined,

$$g(\chi) \equiv \int_\chi^{\chi_\infty} d\chi' \left(1 - \frac{\chi'}{\chi}\right) W(\chi'). \quad (5.7)$$

As we have seen, $i = 1, 2$ so that ψ_{ij} is a 2×2 matrix. This matrix is non-symmetric in general as we can see in (5.6),

$$\psi_{ij} \equiv \begin{pmatrix} -\kappa - \gamma_1 & -\gamma_2 - \omega \\ -\gamma_2 + \omega & -\kappa + \gamma_1 \end{pmatrix}$$

Thus, the convergence and shear parameters are,

$$\kappa = -\frac{\psi_{11} + \psi_{22}}{2}, \quad (5.8)$$

$$\gamma_1 = -\frac{\psi_{11} - \psi_{22}}{2}, \quad (5.9)$$

$$\gamma_2 = -\frac{\psi_{12} + \psi_{21}}{2}, \quad (5.10)$$

whereas the rotation parameter corresponds to

$$\omega = -\frac{\psi_{12} - \psi_{21}}{2}, \quad (5.11)$$

Now, we use equation (4.21) into (5.6) so that

$$\psi_{ij}(\vec{\theta}) = - \int_0^{\chi_\infty} d\chi \chi g(\chi) \left(\Phi + \Psi + Q_3 + \frac{1}{2} h_{33} \right)_{,ij} - \int_0^{\chi_\infty} d\chi \chi g(\chi) (Q_i + h_{i3})_{,3j}, \quad (5.12)$$

As we can see from the previous equation, the vector and tensor perturbations generate the rotation effect in the distortion tensor [61]. Since $x^3 = \chi$, we can integrate by parts the second integral to obtain,

$$\begin{aligned} \int_0^{\chi_\infty} d\chi \chi g(\chi) (Q_i + h_{i3})_{,3j} &= \left[\chi g(\chi) (Q_i + h_{i3})_{,j} \right]_0^{\chi_\infty} \\ &\quad - \int_0^{\chi_\infty} d\chi \left(g + \chi \frac{dg}{d\chi} \right) (Q_i + h_{i3})_{,j}, \end{aligned} \quad (5.13)$$

so that the distortion tensor becomes,

$$\psi_{ij}(\vec{\theta}) = - \int_0^{\chi_\infty} d\chi \chi g(\chi) \left[\left(\Phi + \Psi + Q_3 + \frac{1}{2} h_{33} \right)_{,ij} - \frac{1}{\chi} \left(1 + \frac{\chi}{g} \frac{dg}{d\chi} \right) (Q_i + h_{i3})_{,j} \right]. \quad (5.14)$$

Now, we want to go to the Fourier space of $\vec{\theta}$ so that we define,

$$\tilde{\psi}_{ij}(\vec{\ell}) = \int d^2\theta e^{-i\vec{\ell}\cdot\vec{\theta}} \psi_{ij}(\vec{\theta}). \quad (5.15)$$

Taking into account that,

$$\frac{\partial}{\partial x^i} = \frac{1}{\chi} \frac{\partial}{\partial \theta^i}, \quad (5.16)$$

the Fourier transform of the distortion matrix is,

$$\tilde{\psi}_{ij}(\vec{\ell}) = \int_0^{\chi_\infty} d\chi \frac{g(\chi)}{\chi} \left[\ell_i \ell_j \left(\tilde{\Phi} + \tilde{\Psi} + \tilde{Q}_3 + \frac{1}{2} \tilde{h}_{33} \right) + i \ell_j \left(1 + \frac{\chi}{g} \frac{dg}{d\chi} \right) (\tilde{Q}_i + \tilde{h}_{i3}) \right], \quad (5.17)$$

The power spectrum of this distortion matrix is the weak-lensing observable. In the following section we will relate vector and tensor perturbations to the matter density perturbations using (2.2)–(2.5), so that we can obtain the final weak-lensing power spectrum in terms of the matter power spectrum.

6 Weak-lensing power spectra

The power spectrum of the distortion tensor is defined in the following way,

$$P_{ijlm}^\psi(\vec{\ell}) \equiv \frac{1}{(2\pi)^2} \int d^2\ell' \langle \tilde{\psi}_{ij}(\vec{\ell}) \tilde{\psi}_{lm}^*(\vec{\ell}') \rangle. \quad (6.1)$$

Using expressions (2.2)–(2.5), we can obtain the power spectrum (6.1) as a function of the matter power spectrum,

$$\langle \delta(a, \vec{k}) \delta^*(a, \vec{k}') \rangle = (2\pi)^3 \delta^3(\vec{k} - \vec{k}') P(a, \vec{k}). \quad (6.2)$$

With that purpose it is first necessary to relate the Fourier transforms in the $\vec{\theta}$ and \vec{x} variables. Let us thus denote with a bar the Fourier transform in \vec{x} at a given time i.e.

$$\bar{f}(\vec{k}) \equiv \int d^3x e^{-i\vec{k}\cdot\vec{x}} f(\vec{x}). \quad (6.3)$$

Thus, we can write

$$\bar{f}(k_3, \vec{\ell}) = \int d\chi \int \chi^2 d^2\theta e^{-i\vec{\ell}\cdot\vec{\theta}} e^{-ik_3\chi} f(\vec{x}), \quad (6.4)$$

where we have used $\ell_i = \chi k_i$ for $i = 1, 2$ so that using the definition of the Fourier transform in $\vec{\theta}$ in (5.15) we obtain,

$$\bar{f}(k_3, \vec{\ell}) = \int d\chi \chi^2 e^{-ik_3\chi} \tilde{f}(\chi, \vec{\ell}). \quad (6.5)$$

By performing the inverse transform in k_3 we get,

$$\tilde{f}(\chi, \vec{\ell}) = \frac{1}{2\pi\chi^2} \int dk_3 e^{ik_3\chi} \bar{f}(k_3, \vec{\ell}). \quad (6.6)$$

In order to obtain the power spectrum in (6.1), we rewrite equation (5.17) for $\tilde{\psi}_{ij}(\vec{\ell})$ in the following compact way,

$$\tilde{\psi}(\vec{\ell}) = \int_0^{\chi_\infty} d\chi \sum_{\alpha} C_{\alpha}(\chi, \vec{\ell}) \tilde{f}_{\alpha}(\chi, \vec{\ell}), \quad (6.7)$$

where we have omitted the indices, f_{α} are the different metric perturbations and C_{α} the corresponding coefficients. Using this expression we obtain,

$$P^{\psi}(\vec{\ell}) = \frac{1}{(2\pi)^2} \int d^2\ell' \int_0^{\chi_\infty} d\chi \int_0^{\chi_\infty} d\chi' \sum_{\alpha, \beta} C_{\alpha}(\chi, \vec{\ell}) C_{\beta}^*(\chi', \vec{\ell}') \langle \tilde{f}_{\alpha}(\chi, \vec{\ell}) \tilde{f}_{\beta}^*(\chi', \vec{\ell}') \rangle, \quad (6.8)$$

and using (6.6) in $\langle \tilde{f}_{\alpha}(\chi, \vec{\ell}) \tilde{f}_{\beta}^*(\chi', \vec{\ell}') \rangle$, we obtain

$$\langle \tilde{f}_{\alpha}(\chi, \vec{\ell}) \tilde{f}_{\beta}^*(\chi', \vec{\ell}') \rangle = \frac{1}{2\pi\chi^2} \frac{1}{2\pi\chi'^2} \int dk_3 e^{ik_3\chi} \int dk'_3 e^{-ik'_3\chi'} \langle \bar{f}_{\alpha}(k_3, \vec{\ell}) \bar{f}_{\beta}^*(k'_3, \vec{\ell}') \rangle. \quad (6.9)$$

As we can see from (2.2)–(2.5), metric perturbations \bar{f} can be related to the density perturbations according to the following generic form,

$$\bar{f}_{\alpha}(k_3, \vec{\ell}) = B_{\alpha}(\vec{k}) \delta(\vec{k}), \quad (6.10)$$

where $k_i = \ell_i/\chi$ for $i = 1, 2$, so that, formally we obtain,

$$\langle \bar{f}_{\alpha}(k_3, \vec{\ell}) \bar{f}_{\beta}^*(k'_3, \vec{\ell}') \rangle = B_{\alpha}(\vec{k}) B_{\beta}^*(\vec{k}') \langle \delta(\vec{k}) \delta^*(\vec{k}') \rangle. \quad (6.11)$$

Using equation (6.2) and considering,

$$\delta^3(\vec{k} - \vec{k}') = \delta^2\left(\frac{\vec{\ell}}{\chi} - \frac{\vec{\ell}'}{\chi'}\right) \delta(k_3 - k'_3), \quad (6.12)$$

we obtain,

$$\langle \tilde{f}_{\alpha}(\chi, \vec{\ell}) \tilde{f}_{\beta}^*(\chi', \vec{\ell}') \rangle = \frac{2\pi}{\chi^2 \chi'^2} \delta^2\left(\frac{\vec{\ell}}{\chi} - \frac{\vec{\ell}'}{\chi'}\right) \int dk_3 e^{ik_3(\chi - \chi')} B_{\alpha}(\vec{k}) B_{\beta}^*(\vec{k}') P(\vec{k}). \quad (6.13)$$

For small distortion angles θ , we can consider $k_3 \ll k_1, k_2$ so that, $\vec{k} \simeq \vec{\ell}/\chi$ and accordingly,

$$\langle \tilde{f}_{\alpha}(\chi, \vec{\ell}) \tilde{f}_{\beta}^*(\chi', \vec{\ell}') \rangle = \frac{(2\pi)^2}{\chi^2 \chi'^2} \delta^2\left(\frac{\vec{\ell}}{\chi} - \frac{\vec{\ell}'}{\chi'}\right) \delta(\chi - \chi') B_{\alpha}\left(\frac{\vec{\ell}}{\chi}\right) B_{\beta}^*\left(\frac{\vec{\ell}'}{\chi'}\right) P\left(\frac{\vec{\ell}}{\chi}\right). \quad (6.14)$$

Using this expression in (6.8) and writing $\delta^2\left(\frac{1}{\chi}(\vec{\ell} - \vec{\ell}')\right) = \chi^2 \delta^2(\vec{\ell} - \vec{\ell}')$, we obtain

$$P^{\psi}(\vec{\ell}) = \int_0^{\chi_\infty} \frac{1}{\chi^2} P\left(\frac{\vec{\ell}}{\chi}\right) \left[\sum_{\alpha, \beta} C_{\alpha}(\chi, \vec{\ell}) C_{\beta}^*(\chi, \vec{\ell}) B_{\alpha}\left(\frac{\vec{\ell}}{\chi}\right) B_{\beta}^*\left(\frac{\vec{\ell}}{\chi}\right) \right] d\chi. \quad (6.15)$$

Finally changing from χ to the redshift variable $z = 1/(1+a)$, including explicitly the time dependence of the matter power spectrum through the growth factor $D^2(z)$ and using $[4\pi G a^2 \rho]^2 = \frac{9H_0^4}{4} \Omega_m^2 (1+z)^2$ we get,

$$P_{ijlm}^\psi(\vec{\ell}) = \frac{9H_0^4 \Omega_m^2}{4} \int_0^\infty dz \frac{(1+z)^2}{H(z)} g^2(z) \frac{\kappa_i \ell_j \kappa_l^* \ell_m}{\ell^4} D^2(z) P\left(\frac{\vec{\ell}}{\chi(z)}\right), \quad (6.16)$$

where

$$\kappa_i \equiv \ell_i \alpha - i \left(1 + \frac{\chi g'}{\chi' g}\right) v_i, \quad (6.17)$$

$$\alpha \equiv \mu_\Psi(1+\gamma) - 4\mu_Q \mathcal{A}_3 + \frac{1}{2} \mu_h \Sigma_{33}, \quad (6.18)$$

$$v_i \equiv 4\mu_Q \mathcal{A}_i - \mu_h \Sigma_{i3}, \quad (6.19)$$

being $\gamma \equiv \frac{\mu_\Phi}{\mu_\Psi}$, a prime denotes derivative with respect to redshift and

$$g(z) = \int_z^\infty \left(1 - \frac{\chi(z)}{\chi(z')}\right) n(z') dz', \quad (6.20)$$

with $n(z)dz = W(\chi)d\chi$ and $n(z)$ the galaxy density function as a function of redshift.

Now we can use expressions (5.8)–(5.11) to construct the power spectra for convergence, shear and rotation,

$$P_\kappa = \frac{1}{4} \left(P_{1111}^\psi + P_{2222}^\psi + P_{1122}^\psi + P_{2211}^\psi \right), \quad (6.21)$$

$$P_{\gamma_1} = \frac{1}{4} \left(P_{1111}^\psi + P_{2222}^\psi - P_{1122}^\psi - P_{2211}^\psi \right), \quad (6.22)$$

$$P_{\gamma_2} = \frac{1}{4} \left(P_{1212}^\psi + P_{2121}^\psi + P_{1221}^\psi + P_{2112}^\psi \right), \quad (6.23)$$

$$P_\omega = \frac{1}{4} \left(P_{1212}^\psi + P_{2121}^\psi - P_{1221}^\psi - P_{2112}^\psi \right), \quad (6.24)$$

These expressions can be written in a more compact fashion by introducing the following variables. We define $\ell_1 \equiv \ell \Upsilon$ and $\ell_2 \equiv \ell \sqrt{1 - \Upsilon^2}$ where,

$$\Upsilon \equiv \frac{\hat{A}^i \ell_i}{\ell \sqrt{1 - \hat{A}_3^2}}. \quad (6.25)$$

Considering the small-angle approximation $k_3 \ll k_1, k_2$, the conditions $\hat{k}^i Q_i = 0$ and $\hat{k}^i h_{ij} = 0$ imply,

$$\ell^i v_i = 0. \quad (6.26)$$

Using this expression we can write v_2 as a function of v_1 and then we relate it with $v^2 \equiv v_1^2 + v_2^2$,

$$v_1^2 = (1 - \Upsilon^2) v^2. \quad (6.27)$$

Finally using (6.25) and (6.26) in the expressions of the power spectra (6.21)–(6.24) we obtain,

$$P_\kappa = P_\alpha, \quad (6.28)$$

$$P_{\gamma_1} = (1 - 2\Upsilon^2)^2 P_\alpha + 4\Upsilon^2 (1 - \Upsilon^2) P_v, \quad (6.29)$$

$$P_{\gamma_2} = 4\Upsilon^2 (1 - \Upsilon^2)^2 P_\alpha + (1 - 2\Upsilon^2)^2 P_v, \quad (6.30)$$

$$P_\omega = P_v, \quad (6.31)$$

where P_α and P_v are,

$$P_\alpha = \frac{9 H_0^4 \Omega_m^2}{4} \int_0^\infty dz \frac{(1+z)^2}{H(z)} g^2(z) \frac{\alpha^2}{4} D^2(z) P\left(\frac{\ell}{\chi(z)}\right), \quad (6.32)$$

$$P_v = \frac{9 H_0^4 \Omega_m^2}{4} \int_0^\infty dz \frac{(1+z)^2}{H(z)} g^2(z) \left(1 + \frac{\chi g'}{\chi' g}\right)^2 \frac{v^2}{4 \ell^2} D^2(z) P\left(\frac{\ell}{\chi(z)}\right). \quad (6.33)$$

As we can see from equations (6.28)–(6.31), we have the following closing relation,

$$P_{\gamma_1} + P_{\gamma_2} = P_\kappa + P_\omega, \quad (6.34)$$

This is a useful relation since it allows to determine the rotation power spectrum, which is not directly measurable in lensing surveys, from shear and convergence measurements.

We can use the expressions of \mathcal{A}_i and Σ_{ij} considering $\hat{k}_3 \ll 1$, and the definition of Υ , to obtain expressions for α and v^2 ,

$$\alpha = \mu_\Psi (1 + \gamma) - 4 \xi \mu_Q + (2 \xi^2 + (1 - \xi^2) \Upsilon^2 - 1) \frac{\mu_h}{2}, \quad (6.35)$$

$$v^2 = (4 \mu_Q - 2 \xi \mu_h)^2 (1 - \xi^2) (1 - \Upsilon^2), \quad (6.36)$$

being $\xi \equiv \hat{A}_3$ with $-1 \leq \xi \leq 1$. Since \hat{A}_3 is the projection of \hat{A} along the line of sight, we can perform a multipole expansion of α^2 and v^2 above, using the Legendre polynomials in ξ . Thus for $\alpha^2 = \sum_{r=0}^4 M_\alpha^r P_r(\xi)$ we have

$$M_\alpha^0 = \frac{1}{20} f_1^2 + \frac{1}{6} f_1 f_2 + \frac{1}{4} f_2^2 + \frac{16}{3} \mu_Q^2, \quad (6.37)$$

$$M_\alpha^1 = -4 \left(\frac{3}{5} f_1 + f_2 \right) \mu_Q, \quad (6.38)$$

$$M_\alpha^2 = \left(\frac{1}{7} f_1 + \frac{1}{3} f_2 \right) f_1 + \frac{32}{3} \mu_Q^2 \quad (6.39)$$

$$M_\alpha^3 = -\frac{8}{5} f_1 \mu_Q, \quad (6.40)$$

$$M_\alpha^4 = \frac{2}{35} f_1^2 \quad (6.41)$$

where $f_1 \equiv (2 - \Upsilon^2) \mu_h$ and $f_2 \equiv 2 \mu_\Psi (1 + \gamma) - (1 - \Upsilon^2) \mu_h$. On the other hand for $v^2 = \sum_{r=0}^4 M_v^r P_r(\xi)$

$$M_v^0 = \frac{8}{15} \mu_h^2 + \frac{32}{3} \mu_Q^2, \quad (6.42)$$

$$M_v^1 = -M_v^3 = -\frac{32}{5} \mu_Q \mu_h, \quad (6.43)$$

$$M_v^2 = \frac{8}{21} \mu_h^2 - \frac{32}{3} \mu_Q^2, \quad (6.44)$$

$$M_v^4 = -\frac{32}{35} \mu_h^2. \quad (6.45)$$

With these definitions we obtain,

$$P_\alpha^r = \frac{9 H_0^4 \Omega_m^2}{4} \int_0^\infty dz \frac{(1+z)^2}{H(z)} g(z)^2 \frac{M_\alpha^r}{4} D(z)^2 P\left(\frac{\ell}{\chi(z)}\right), \quad r = 0, 1, 2, 3, 4 \quad (6.46)$$

$$P_v^r = \frac{9 H_0^4 \Omega_m^2}{4} \int_0^\infty dz \frac{(1+z)^2}{H(z)} g(z)^2 \left(1 + \frac{\chi g'}{\chi' g}\right)^2 \frac{(1 - \Upsilon^2) M_v^r}{4 \ell^2} D(z)^2 P\left(\frac{\ell}{\chi(z)}\right), \quad r = 0, 1, 2, 3, 4 \quad (6.47)$$

Finally, if we want to analyze the weak lensing signal at different redshift bins, we define the following window functions,

$$g_i(z) = \int_z^\infty \left(1 - \frac{\chi(z)}{\chi(z')}\right) n_i(z') dz', \quad (6.48)$$

where we consider a galaxy density function of the form,

$$n(z) = \frac{3}{2 z_p^3} z^2 e^{-(z/z_p)^{3/2}}, \quad (6.49)$$

being $z_p = z_{\text{mean}}/\sqrt{2}$ and z_{mean} the survey mean redshift. Then, for each bin we have the following galaxy distribution function, where we have take into account the photometric redshift error σ_i in the corresponding bin,

$$n_i(z) \propto \int_{\bar{z}_{i-1}}^{\bar{z}_i} n(z') e^{-\frac{(z'-z)^2}{2\sigma_i^2}} dz', \quad (6.50)$$

where $\sigma_i = \delta z (1 + z_i)$, \bar{z}_i is the upper limit of the i -bin and $n_i(z)$ is normalized to one.

With these definitions, the convergence, shear and rotation multipole power spectra are,

$$P_{\kappa ij}^r(\ell, \Upsilon) = \frac{9 H_0^4 \Omega_m^2}{4} \int_0^\infty dz \frac{(1+z)^2}{H(z)} g_i(z) g_j(z) \frac{M_\alpha^r}{4} D(z)^2 P\left(\frac{\ell}{\chi(z)}\right), \quad (6.51)$$

$$P_\omega^r(\ell, \Upsilon) = \frac{9 H_0^4 \Omega_m^2}{4} \int_0^\infty dz \frac{(1+z)^2}{H(z)} g_i(z) g_j(z) \times \left(1 + \frac{\chi g'_i}{\chi' g_i}\right) \left(1 + \frac{\chi g'_j}{\chi' g_j}\right) \frac{(1 - \Upsilon^2) M_v^r}{4 \ell^2} D(z)^2 P\left(\frac{\ell}{\chi(z)}\right), \quad (6.52)$$

$$P_{\gamma_1 ij}^r(\ell, \Upsilon) = (1 - 2\Upsilon^2)^2 P_{\kappa ij}^r(\ell, \Upsilon) + 4\Upsilon^2 (1 - \Upsilon^2) P_\omega^r(\ell, \Upsilon), \quad (6.53)$$

$$P_{\gamma_2 ij}^r(\ell, \Upsilon) = 4\Upsilon^2 (1 - \Upsilon^2) P_{\kappa ij}^r(\ell, \Upsilon) + (1 - 2\Upsilon^2)^2 P_\omega^r(\ell, \Upsilon). \quad (6.54)$$

7 Fisher analysis for the multipole power spectrum

Considering a set of cosmological parameters $\{p_\alpha\}$, the Fisher matrix for the multipole power spectrum (3.7) can be written as [66],

$$F_{\alpha\beta} = \sum_{n,n'} \sum_{\ell,\ell'} \left. \frac{\partial P_\ell(\vec{k}_n)}{\partial p_\alpha} \right|_f C_{\ell\ell'}^{-1}(\vec{k}_n, \vec{k}_{n'}) \left. \frac{\partial P_{\ell'}(\vec{k}_{n'})}{\partial p_\beta} \right|_f, \quad (7.1)$$

where sub-index f denotes that the corresponding quantity is evaluated on the fiducial model, \vec{k}_n are the discrete modes and $C_{\ell\ell'}(\vec{k}_n, \vec{k}_{n'})$ is the covariance matrix. In appendix A the explicit calculation of the covariance matrix for the anisotropic power spectrum assuming gaussian fields can be found. In each redshift bin this expression reads,

$$F_{\alpha\beta}(z) = \frac{V(z)}{8\pi^2} \int_{k_{\min}}^{k_{\max}} \int_{-1}^1 k^2 dk dy \sum_{\ell, \ell'} \left. \frac{\partial P_\ell(k, z, y)}{\partial p_\alpha} \right|_f C_{\ell\ell'}^{-1}(z, k) \left. \frac{\partial P_{\ell'}(k, z, y)}{\partial p_\beta} \right|_f, \quad (7.2)$$

$$C_{\ell\ell'}(z, k) = \frac{(2\ell+1)(2\ell'+1)}{2} \int_{-1}^1 d\hat{\mu} \mathcal{L}_\ell(\hat{\mu}) \mathcal{L}_{\ell'}(\hat{\mu}) \left[P_g(z, k, \hat{\mu})|_f e^{-k^2 \hat{\mu}^2 \sigma_r^2} + \frac{1}{n(z)} \right]^2, \quad (7.3)$$

where we have included the effect of redshift errors [51] in the power spectrum through the $e^{-k^2 \hat{\mu}^2 \sigma_r^2}$ factor, where $\sigma_r = (\delta z (1+z))/H(z)$ with δz the redshift error. Notice that we assume that systematics are under control so that we do not include additional free parameters such as non-gaussian shot noise. Here $n(z)$ is the mean galaxy density and $V(z)$ the volume of the bin z . We consider a flat fiducial model,

$$V(z) = \frac{4\pi f_{\text{sky}}}{3} [\chi^3(z + \Delta z/2) - \chi^3(z - \Delta z/2)], \quad (7.4)$$

being f_{sky} the fraction of the sky, Δz the width of the bin and $\chi(z)$ the comoving radial distance,

$$\chi(z) = H_0^{-1} \int_0^z \frac{dz'}{E(z')}. \quad (7.5)$$

As expected, this Fisher matrix reduces to the isotropic case when $P_\ell(k, z, y) = P_\ell(k, z)$. Finally, we need to know the values for k_{\min} and k_{\max} in each bin. k_{\min} can be fixed to $0.007 h/\text{Mpc}$ [67], and we obtain $k_{\max} = k_{\max}(z_a)$ by imposing that $\sigma^2(z_a, \pi/2 k_{\max}(z_a)) = 0.35$ so that we only consider modes in the linear regime. Thus, the amplitude of the fluctuations at a scale R at redshift z is given by

$$\sigma^2(z, R) = D^2(z) \int \frac{k'^2 dk'}{2\pi^2} P(k') |\hat{W}(R, k')|^2, \quad (7.6)$$

where we have used a top-hat filter $\hat{W}(R, k)$, defined by

$$\hat{W}(R, k) = \frac{3}{k^3 R^3} [\sin(kR) - kR \cos(kR)]. \quad (7.7)$$

7.1 Fiducial cosmology and galaxy redshift survey

The fiducial cosmology we consider is given by $\Omega_c h^2 = 0.121$, $\Omega_b h^2 = 0.0226$, $\Omega_\nu h^2 = 0.00064$, $n_s = 0.96$, $h = 0.68$, $H_0^{-1} = 2997.9 \text{ Mpc/h}$ (in units $c = 1$), $\Omega_k = 0$ and $\sigma_8 = 0.82$ in the standard ΛCDM model. For this cosmology,

$$E(z) = \sqrt{\Omega_m (1+z)^3 + (1 - \Omega_m)}. \quad (7.8)$$

The growth function follows equation (3.8) and the growth factor is,

$$D_\Lambda(z) = \exp \left[\int_0^{N(z)} f_\Lambda(N') dN' \right], \quad (7.9)$$

z	k_{max}	$n \times 10^{-3}$	$\delta\mu_0/\mu_0(\%)$	$\delta\mu_2/\mu_0(\%)$
0.6	0.195	3.56	1.77	3.88
0.8	0.225	2.42	1.08	1.99
1.0	0.260	1.81	0.73	1.06
1.2	0.299	1.44	0.66	0.77
1.4	0.343	0.99	0.62	0.59
1.8	0.447	0.33	0.65	0.57

Table 1. Redshift bins, k_{max} values in h/Mpc units, galaxy densities in $(\text{h/Mpc})^3$ units and relative errors for μ_0 and μ_2 for an Euclid-like survey. We compare $\delta\mu_2$ with respect to μ_0 because the fiducial value of μ_2 is zero.

z	k_{max}	$n \times 10^{-3}$	$\delta\mu_0/\mu_0(\%)$	$\delta\mu_2/\mu_0(\%)$	$\delta\mu_4/\mu_0(\%)$
0.6	0.195	3.56	2.20	13.6	15.2
0.8	0.225	2.42	1.27	6.96	7.78
1.0	0.260	1.81	0.81	3.71	4.15
1.2	0.299	1.44	0.71	2.69	3.01
1.4	0.343	0.99	0.66	2.06	2.30
1.8	0.447	0.33	0.67	1.99	2.22

Table 2. The same as in table I but including μ_4 as an additional independent parameter in the Fisher analysis.

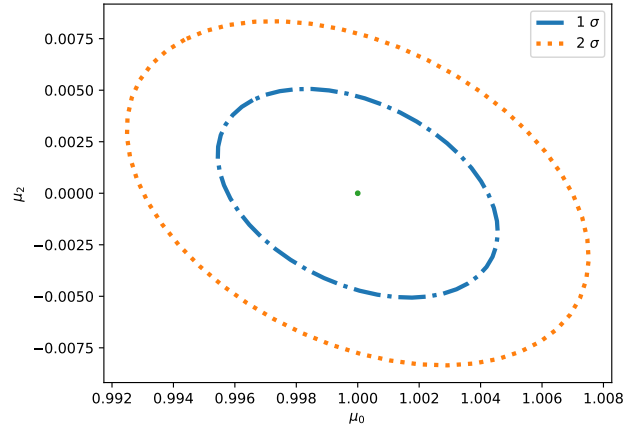


Figure 1. Marginalized 1σ and 2σ regions for μ_0 and μ_2 for an Euclid-like survey from the multipole power spectrum information.

being $N(z) = -\ln(1+z)$. For the fiducial cosmology we obtain the present linear matter power spectrum $P(k)$ from CLASS [68]. Notice that the differences between linear and non-linear (HALOFIT) matter power spectra vary in the range 10–30% for the k_{max} considered. So that we do not expect important changes in our estimated errors. For the bias, we use a fiducial value of the form [42],

$$b(z) = \sqrt{1+z}. \quad (7.10)$$

Finally, we will limit ourselves to constant modified gravity parameters so that $\mu = \mu_0 + \mu_2 y^2 + \mu_4 y^4$, and since we use Λ CDM as fiducial cosmology, we will take $[\mu_0, \mu_2, \mu_4]_r = [1, 0, 0]$.

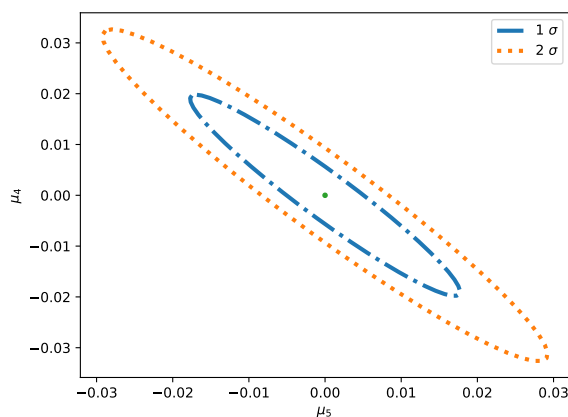


Figure 2. Marginalized 1σ and 2σ regions for μ_2 and μ_4 for an Euclid-like survey from the multipole power spectrum information.

The inputs we need to compute $F_{\alpha\beta}$ are therefore redshift bins and the galaxy densities for each bin which can be found in table 1 and 2 for an spectroscopic Euclid-like galaxy redshift survey. The fraction of the sky is $f_{\text{sky}} = 0.364$ corresponding to 15000 deg^2 and the redshift error is $\delta z = 0.001$. We have taken the same redshift bins as in [67] where the bin size is $\Delta z = 0.2$ for all the bins except for the last one with $\Delta z = 0.6$. The quoted values correspond to the center of the bin, so that for instance, the first bin would be $0.5 < z < 0.7$.

First of all, we consider as independent parameters in each bin $[b\sigma_8, \mu_0, \mu_2]$ and we present the marginalized errors for μ_0 and μ_2 in table 1. In figure 1 we plot the 1-sigma and 2-sigma contours summing all the information in the whole redshift range. In such a case we obtain errors for μ_0 and μ_2 of order 1 %.

Then, we add the parameter μ_4 in each bin and we present the marginalized errors for μ_0 , μ_2 and μ_4 in table 2. In figure 2 we plot the 1- σ and 2- σ contours for μ_2 and μ_4 summing all the information in the full redshift range. As we can see, if we add a y^4 dependence, the errors for μ_2 increase in a factor 3 – 4 but the errors for μ_0 remain the same. Errors for μ_4 are slightly larger than for μ_2 .

8 Fisher analysis for the redshift-space power spectrum

An alternative way to perform the Fisher analysis consists in using the redshift-space power spectrum (3.4) rather than the multipoles considered in the previous section. This, in fact, allows to take into account the Alcock-Paczynski effect [52] so that we can write

$$P_g(z, k_r, \hat{\mu}_r, \xi) = \frac{D_{Ar}^2 E}{D_A^2 E_r} \left[(1 + \beta_\Lambda \xi \hat{\mu}^2) b(z) D_\Lambda^\xi \right]^2 P(k), \quad (8.1)$$

where as mentioned before, the r sub-index denotes that the corresponding quantity is evaluated on the fiducial cosmology. The dependence $k = k(k_r)$ and $\hat{\mu} = \hat{\mu}(\hat{\mu}_r)$ are given by,

$$k = Q k_r, \quad (8.2)$$

$$\hat{\mu} = \frac{E \hat{\mu}_r}{E_r Q}, \quad (8.3)$$

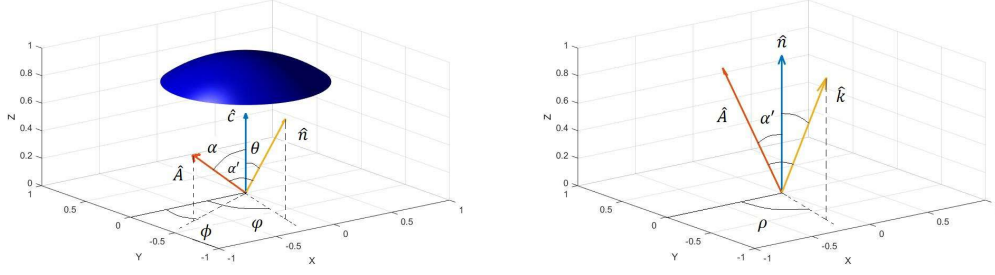


Figure 3. From left to right, reference frame for the Fisher analysis of the redshift space power spectrum, and auxiliary reference frame to calculate the integral in \vec{k} .

with

$$Q = \frac{\sqrt{E^2 \chi^2 \hat{\mu}_r^2 - E_r^2 \chi_r^2 (\hat{\mu}_r^2 - 1)}}{E_r \chi}. \quad (8.4)$$

The $\xi = \xi(\mu)$ parameter follows equation (3.9) and μ equation (3.3). D_A is the angular distance which, in a flat Universe, satisfies $D_A = (1+z)^{-1} \chi(z)$.

Thus, considering a set of cosmological parameters $\{p_\alpha\}$, the corresponding Fisher matrix for clustering at a given redshift bin centered at z_a and for a solid angle of the survey centered at the line of sight \hat{n} is,

$$dF_{ij} = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \left. \frac{\partial \log P_g}{\partial p_i} \right|_f \left. \frac{\partial \log P_g}{\partial p_j} \right|_f \left[\frac{\bar{n} P_g}{1 + \bar{n} P_g} \right]_f dV_s,$$

where,

$$dV_s = V_z d\varphi d\theta \sin \theta, \quad (8.5)$$

and

$$V_z = \frac{1}{3} [\chi^3(z_a + \Delta z/2) - \chi^3(z_a - \Delta z/2)], \quad (8.6)$$

with $\hat{n}(\theta, \varphi)$ where φ and θ are the azimuthal and polar angles in the axes frame on the left panel of figure 3.

Since we are interested in summing all the angular information, we have to integrate over the angles φ and θ but taking into account that $\left. \frac{\partial \log P_g}{\partial p_\alpha} \right|_f$ may depend on these angles. Thus, we integrate a spherical cap that encloses a fraction f_{sky} of the sky,

$$F_{ij} = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \int_0^{2\pi} d\varphi \int_0^{\arccos(1-2f_{\text{sky}})} \sin \theta d\theta \left. \frac{\partial \log P_g}{\partial p_i} \right|_f \left. \frac{\partial \log P_g}{\partial p_j} \right|_f \left[\frac{\bar{n} P_g}{1 + \bar{n} P_g} \right]_f V_z. \quad (8.7)$$

The only angular dependences we have are $\hat{\mu} = \hat{k} \cdot \hat{n}$ and $y = \hat{k} \cdot \hat{A}$. It is useful to keep $\hat{\mu}$ as an integration variable, so that we have to relate y with $\hat{\mu}$. With the choice of axes of figure 3, we find that,

$$y = \sin \alpha' \sqrt{1 - \hat{\mu}^2} \cos \rho + \cos \alpha' \hat{\mu}, \quad (8.8)$$

and,

$$\cos \alpha' = \sin \alpha \sin \theta \cos(\varphi - \phi) + \cos \alpha \cos \theta, \quad (8.9)$$

being $\hat{A}(\alpha, \phi)$ with α and ϕ the polar and azimuthal angles in the axes frame in figure 3 left, where the Z axis is chosen in the direction of the center of the survey \vec{c} . Thus, $y =$

$y(\alpha, \phi, \hat{\mu}, \rho, \theta, \varphi)$ so that we have the following integration variables $[k, \hat{\mu}, \rho, \theta, \varphi]$. Finally, we have chosen as independent parameters for the Fisher matrix in each bin: $[E, b\sigma_8, \mu_0, \mu_2, \mu_4]$. For these parameters the derivatives are,

$$\begin{aligned} \left. \frac{\partial \log P_g}{\partial E} \right|_f &= 1 + \frac{2 \Delta z_a}{E^2 H_0 \chi(z_a)} + \frac{4\beta_\Lambda \hat{\mu}^2 (1 - \hat{\mu}^2)}{1 + \beta_\Lambda \hat{\mu}^2} \left(\frac{1}{E} - \frac{\Delta z_a}{E^2 H_0 \chi(z_a)} \right), \\ \left. \frac{\partial \log P_g}{\partial (b\sigma_8)} \right|_f &= \frac{2}{b\sigma_8}, \end{aligned} \quad (8.10)$$

$$\left. \frac{\partial \log P_g}{\partial \mu_0} \right|_f = \frac{6}{5} \left[\log D_\Lambda + \int_{\bar{z}_{i-1}}^{\bar{z}_i} \frac{f_\Lambda(z')}{1+z'} dz' + \frac{\beta_\Lambda \hat{\mu}^2}{1 + \beta_\Lambda \hat{\mu}^2} \right], \quad (8.11)$$

$$\left. \frac{\partial \log P_g}{\partial \mu_2} \right|_f = \left. \frac{\partial \log P_g}{\partial \mu_0} \right|_f y^2, \quad (8.12)$$

$$\left. \frac{\partial \log P_g}{\partial \mu_4} \right|_f = \left. \frac{\partial \log P_g}{\partial \mu_0} \right|_f y^4. \quad (8.13)$$

As we can see, the only angular dependence appear in the derivatives respect to μ_2 and μ_4 which involve even powers of y . Thus, we can extract this dependence and define the following function,

$$\begin{aligned} f_{ij}^y(\hat{\mu}, \alpha, \phi) &= \int_0^{2\pi} d\varphi \int_0^{\arccos(1-2f_{\text{sky}})} \sin \theta d\theta \int_0^{2\pi} d\rho (\delta_{1i} + \delta_{2i} + \delta_{3i} + y^2 \delta_{4i} + y^4 \delta_{5i}) \\ &\quad (\delta_{1j} + \delta_{2j} + \delta_{3j} + y^2 \delta_{4j} + y^4 \delta_{5j}), \end{aligned} \quad (8.14)$$

where $y = y(\alpha, \phi, \hat{\mu}, \rho, \theta, \varphi)$ and $i, j = E, b\sigma_8, \mu_0, \mu_2, \mu_4 = 1, 2, 3, 4, 5$. Notice that for $i, j = 1, 2, 3$ we have $f_{ij}^y = 8\pi^2 f_{\text{sky}}$, and we recover the isotropic case for the Fisher matrix. Finally, the Fisher matrix for the redshift-space power spectrum in the presence of a preferred direction pointing in the (α, ϕ) direction can be written as,

$$\begin{aligned} F_{ij}(z_a, \alpha, \phi) &= \frac{V_z(z_a)}{16\pi^3} \int_{-1}^1 d\hat{\mu} \int_{k_{\min}}^{k_{\max}} k^2 \left. \frac{\partial \log P_g(z_a, \hat{\mu}, k)}{\partial p_i} \right|_f \left. \frac{\partial \log P_g(z_a, \hat{\mu}, k)}{\partial p_j} \right|_f \\ &\quad \times f_{ij}^y(\hat{\mu}, \alpha, \phi) \left[\frac{\bar{n} \hat{P}_g(z_a, \hat{\mu}, k)}{1 + \bar{n} \hat{P}_g(z_a, \hat{\mu}, k)} \right] \Big|_f dk, \end{aligned} \quad (8.15)$$

being $\hat{P}_g = P_g e^{-k^2 \hat{\mu}^2 \sigma_r^2}$ and in this expression,

$$\left. \frac{\partial \log P_g}{\partial p_4} \right|_f = \left. \frac{\partial \log P_g}{\partial p_5} \right|_f = \left. \frac{\partial \log P_g}{\partial \mu_0} \right|_f. \quad (8.16)$$

The values for k_{\min} and k_{\max} are the same as in the previous section.

Notice that the final Fisher matrix (8.15) depends on the angles (α, ϕ) . We could have considered them as additional cosmological parameters p_i and obtain an extended Fisher matrix. However, since we are considering an isotropic fiducial model, the corresponding entries would be identically zero. Instead, we will study that dependence of the errors on the orientation of the vector \hat{A} . Thus, we find that errors are maximized for $\alpha = 0$, i.e. when the

z	k_{\max}	$n \times 10^{-3}$	$\delta\mu_0/\mu_0(\%)$	$\delta\mu_2/\mu_0(\%)$	$\delta\mu_4/\mu_0(\%)$
0.6	0.195	3.56	2.60	14.8	17.1
0.8	0.225	2.42	1.47	7.39	8.68
1.0	0.260	1.81	0.99	4.03	4.67
1.2	0.299	1.44	0.92	3.03	3.48
1.4	0.343	0.99	0.89	2.40	2.74
1.8	0.447	0.33	0.87	2.30	2.61

Table 3. Redshift bins, k_{\max} values in h/Mpc units, galaxy densities in $(\text{h/Mpc})^3$ units and relative errors for μ_0 , μ_2 and μ_4 for an Euclid-like survey using the redshift space power spectrum with $\alpha = 0$. We compare $\delta\mu_2$ and $\delta\mu_4$ with respect to μ_0 because their fiducial values are zero.

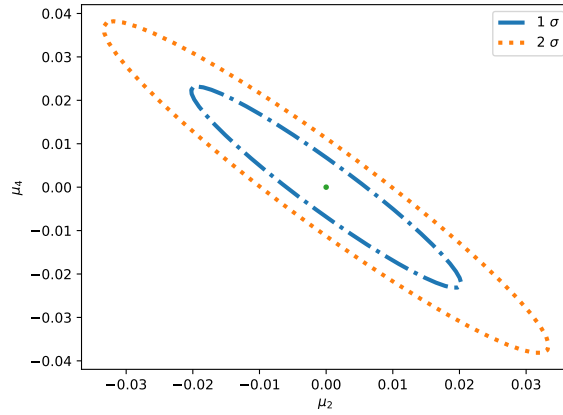


Figure 4. Marginalized 1σ and 2σ regions for μ_2 and μ_4 using the information of the redshift space power spectrum and considering $\alpha = 0$ for an Euclid-like survey.

preferred direction points towards the center of the survey, for any value of ϕ , whereas they are minimized for $\alpha = \pi/2$ for any value of ϕ . Notice that in any case errors vary at most in a 10% of their values. We use the same fiducial cosmology as in the previous section for and Euclid-like survey. Results are summarized in table 3 and in figure 4 we plot the $1\text{-}\sigma$ and $2\text{-}\sigma$ contours for μ_2 and μ_4 summing all the information in each bin.

As we can see, with this method we obtain slightly larger errors for μ_0 , μ_2 and μ_4 than in the previous section.

9 Fisher analysis for the weak lensing power spectrum

In this section we extend in a simple way the Fisher matrix formalism for the weak lensing convergence power spectrum in the presence of a preferred direction. To do that, we have to analyze the multipole power spectrum for the convergence (6.51) and sum over all the multipoles r and $\vec{\ell}$. The Fisher matrix is of the following form,

$$F_{\alpha\beta} = \sum_{\vec{\ell}_a, \vec{\ell}_b} \sum_{rr'} \frac{\partial P_{\kappa ij}^r(\vec{\ell}_a)}{\partial p_\alpha} \left[\text{Cov}_{jmn}^{rr'}(\vec{\ell}_a, \vec{\ell}_b) \right]^{-1} \frac{\partial P_{\kappa mn}^{r'}(\vec{\ell}_b)}{\partial p_\beta}, \quad (9.1)$$

where we are summing in indexes i , j , m and n . We obtain the covariance matrix as an extension of the covariance matrix in the isotropic space in appendix B assuming once again

that the fields are gaussian. The corresponding Fisher matrix reads,

$$F_{\alpha\beta} = f_{\text{sky}} \int_{-1}^1 \frac{d\Upsilon}{\pi \sqrt{1-\Upsilon^2}} \sum_r \sum_\ell \Delta \ln \ell \frac{(2\ell+1)\ell}{2(2r+1)} \left. \frac{\partial P_{\kappa ij}^r}{\partial p_\alpha} \right|_f C_{jm}^{-1} \left. \frac{\partial P_{\kappa mn}^r}{\partial p_\beta} \right|_f C_{ni}^{-1}, \quad (9.2)$$

where the sub-index f denotes that the corresponding quantity is evaluated on the fiducial model and,

$$C_{ij} = P_{\kappa ij} + \frac{\gamma_{\text{int}}^2}{\hat{n}_i} \delta_{ij}, \quad (9.3)$$

being $\gamma_{\text{int}} = 0.22$ the intrinsic ellipticity [69], \hat{n}_i the galaxies per steradian in the i -th bin,

$$\hat{n}_i = n_\theta \frac{\int_{\bar{z}_{i-1}}^{\bar{z}_i} n(z) dz}{\int_0^\infty n(z) dz}, \quad (9.4)$$

where n_θ is the areal galaxy density. We sum in ℓ with $\Delta \ln \ell = 0.1$ from $\ell_{\text{min}} = 5$ to ℓ_{max} , where ℓ_{max} values are obtained following [67] and can be found in table 4. For the multipole power spectrum we use the following expression,

$$P_{\kappa ij}^r(\ell, \Upsilon) = \frac{1}{4} \sum_a P_{ij}(z_a, \ell) M_\alpha^r(\Upsilon). \quad (9.5)$$

where

$$P_{ij}(z_a, \ell) = \frac{9 H_0^3 \Omega_m^2}{4} \frac{(1+z_a)^2}{E_a} \Delta z_a g_i(z_a) g_j(z_a) D_a^2 P\left(\frac{\ell}{\chi(z_a)}\right), \quad (9.6)$$

Regarding the parameters θ_α , it can be proved that in each redshift bin, the power spectrum depends on four independent parameters, $[E_a, \mu_{\Psi a}(1+\gamma_a)D_a, \mu_{Q a}, \mu_{h a}]$ where the sub-index a denotes different redshift bins. In this section, in order to get errors for γ_a we will assumed that $\mu_{\Psi a}$ and D_a are fixed to their fiducial values. Thus, we have a total Fisher matrix of size $4n \times 4n$, being n the total number of z bins.

For the sake of simplicity, we will consider that the modified gravity parameters are isotropic and scale invariant, i.e. $\gamma_a = \gamma(z_a)$, $\mu_{Q a} = \mu_Q(z_a)$ and $\mu_{h a} = \mu_h(z_a)$. The (non-vanishing) derivatives, which are evaluated in Λ CDM as fiducial model, are,

$$\left. \frac{\partial P_{\kappa ij}^0}{\partial \gamma_a} \right|_f = P_{ij}(z_a), \quad (9.7)$$

$$\left. \frac{\partial P_{\kappa ij}^1}{\partial \mu_{Q a}} \right|_f = -4 P_{ij}(z_a), \quad (9.8)$$

$$\left. \frac{\partial P_{\kappa ij}^0}{\partial \mu_{h a}} \right|_f = \frac{2}{3} (2 - \Upsilon^2) P_{ij}(z_a), \quad (9.9)$$

$$\left. \frac{\partial P_{\kappa ij}^2}{\partial \mu_{h a}} \right|_f = \frac{4}{3} (2 - \Upsilon^2) P_{ij}(z_a), \quad (9.10)$$

$$\left. \frac{\partial P_{\kappa ij}^0}{\partial E_a} \right|_f = -\frac{P_{ij}(z_a)}{E_a} + \sum_b \frac{1}{g_i(z_b)} \frac{\partial g_i(z_b)}{\partial E_a} P_{ij}(z_b) + \sum_b \frac{1}{g_j(z_b)} \frac{\partial g_j(z_b)}{\partial E_a} P_{ij}(z_b), \quad (9.11)$$

z	ℓ_{\max}	$\delta\gamma/\gamma(\%)$	$\delta\mu_Q/\mu(\%)$	$\delta\mu_h/\mu(\%)$
0.6	311	6.75	0.64	1.48
0.8	385	5.43	1.45	3.04
1.0	515	9.27	2.71	4.73
1.2	609	20.6	6.70	9.18
1.4	760	70.5	23.0	24.9
1.8	959	816	250	324

Table 4. Redshift bins, ℓ_{\max} values and relative errors for γ , μ_Q and μ_h for the Euclid forecast. We compare μ_Q and μ_h with μ because the fiducial values of μ_Q and μ_h are zero.

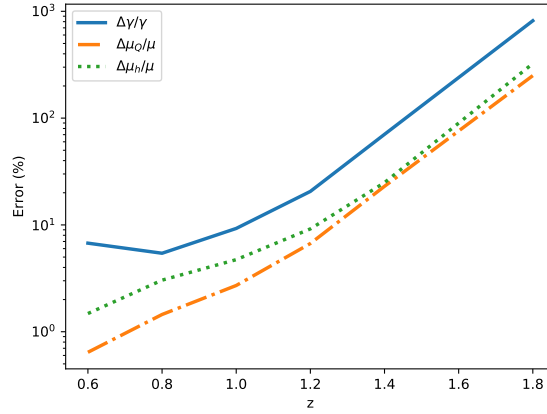


Figure 5. Relative errors for γ , μ_Q and μ_h using weak lensing information for an Euclid-like survey.

with,

$$\frac{\partial g_i(z_b)}{\partial E_a} = \frac{\Delta z_a}{H_0 E_a^2} \left[-\hat{\theta}(z_a - z_b) \chi(z_b) \int_{z_a}^{\infty} \frac{n_i(z')}{\chi(z')^2} dz' + \theta(z_b - z_a) \int_{z_b}^{\infty} \left(1 - \frac{\chi(z_b)}{\chi(z')} \right) \frac{n_i(z')}{\chi(z')} dz' \right], \quad (9.12)$$

where we have discretized the integration of $E(z)^{-1}$ in $\chi(z)$ for the different bins, and the step functions $\theta(z)$ and $\hat{\theta}(x)$ are defined so that $\hat{\theta}(0) = 0$ and $\theta(0) = 1$. We consider a photometric Euclid-like weak lensing survey with a fraction of the sky $f_{\text{sky}} = 0.364$, $z_{\text{mean}} = 0.9$ and $n_{\theta} = 35$ galaxies per square arc minute with $\delta z = 0.05$. We summarize the results in table 4 and in figure 5.

Finally, if we further assume that γ , μ_Q and μ_h are just constants, we can sum the information in all redshift bins. We plot the corresponding 1- σ and 2- σ contours for μ_Q and μ_h in figure 6.

We can see that lensing convergence measurements are very sensitive to the dipole term $P_{\kappa ij}^1$, so that errors in μ_Q are much smaller than for γ and μ_h . Notice also that multipoles $r = 3, 4$ do not appear in the derivatives (9.7)–(9.11) since those terms are quadratic in μ_Q and μ_h so that on the fiducial Λ CDM cosmology the corresponding derivatives vanish. For the same reason, the Fisher matrix for the rotation power spectrum also vanishes.

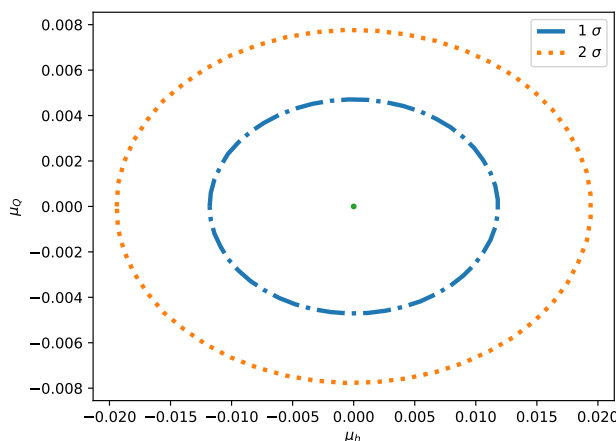


Figure 6. Marginalized 1σ and 2σ regions for μ_Q and μ_h summing the information of the whole redshift range for an Euclid-like survey.

z	k_{\max}	ℓ_{\max}	$n \times 10^{-3}$	$100 \times \delta g_*^C$	$100 \times \delta g_*^L$
0.6	0.195	311	3.56	0.61	4.23
0.8	0.225	385	2.42	0.43	7.36
1.0	0.260	515	1.81	0.32	9.65
1.2	0.299	609	1.44	0.30	16.1
1.4	0.343	760	0.99	0.29	38.4
1.8	0.447	959	0.33	0.29	591

Table 5. Redshift bins, k_{\max} values in h/Mpc units, ℓ_{\max} values, galaxy densities in $(\text{h/Mpc})^3$ units and forecasted absolute errors for g_* from clustering (C) and lensing (L) for an Euclid-like survey.

10 Forecasting primordial anisotropies

So far we have studied the effects of preferred directions in the evolution of density and metric perturbations, but anisotropies could also be present in the primordial curvature power spectrum [6, 26]. In this case, and assuming parity symmetry, the leading effects can be described by a modification of the primordial power spectrum from $P(k)$ to $P'(\vec{k})$ such that,

$$P'(\vec{k}) = (1 + g_* y^2) P(k). \quad (10.1)$$

Assuming a scale-independent modification, g_* is just a dimensionless constant. We can use the Fisher formalism of section 3 and section 8 to forecast the sensitivity with which future surveys could measure the g_* parameter. With that purpose, we consider both, the multipole power spectrum for the matter distribution and the multipole power spectrum for lensing convergence. For clustering we consider the following independent parameters in the Fisher analysis $(f\sigma_{8a}, b\sigma_{8a}, g_{*a})$, whereas for lensing we take (E_a, L_a, g_{*a}) where the sub-index a denotes the different redshift bins and $L_a \equiv \Omega_m D_a \sigma_8$. We summarize the results in table 5 for an Euclid-like survey as in previous sections.

As we can see, we have better precision with the multipole power spectrum of galaxy distribution. If we sum the information of clustering and lensing and in each bin, we obtain and absolute error $\delta g_* = 1.4 \times 10^{-3}$.

Notice that the effects of the g_* terms would be strongly degenerate with the μ_2 parameter. However, the gravitational preferred direction could be discriminated from a primordial anisotropy through the contribution of μ_Q and μ_h to P_ω , since (10.1) does not produce vector or tensor modes which are the ones that generate the lensing rotation effect.

11 Conclusions

In this work we have considered possible observational signatures of modifications of gravity involving preferred spatial directions. In the model-independent approach, these theories can be parametrized in the sub-Hubble regime and using the quasi-static approximation by four effective parameters $\mu(a, k, y)$, $\gamma(a, k, y)$, $\mu_Q(a, k, y)$ and $\mu_h(a, k, y)$. We have analyzed the effects of the existence of a preferred direction in galaxy distribution and weak lensing observations. In the galaxy power spectrum, we find that a new angular dependence on the line of sight appears which is different from the usual effect induced by redshift space distortions. In the lensing case, we have two new effects. On one hand, a dependence on the line of sight is introduced in the shear and convergence power spectra which is absent in the isotropic case. In particular the μ_Q parameter introduces a dipole contribution, whereas the μ_h produces a quadrupole term. On the other hand, images rotation is induced in addition to the standard convergence and shear effects. Thus, we have found a useful relation between the different power spectra,

$$P_{\gamma_1} + P_{\gamma_2} = P_\kappa + P_\omega, \quad (11.1)$$

which shows that even though P_ω cannot be measured directly using weak lensing maps, it can be derived from P_{γ_1} , P_{γ_2} and P_κ .

We have also forecasted the precision with which future surveys will be able to measure the four effective parameters. With that purpose we have extended the standard Fisher matrix approach in order to include the presence of preferred directions. In particular, explicit expressions for the covariance matrices for the multipole galaxy power spectrum and for the convergence power spectrum have been derived in the appendices. For the galaxy power spectrum, we have considered two different approaches. On one hand, we have obtained the Fisher matrix for the power spectrum in redshift space, which allows us to include the Alcock-Paczynski effect. On the other, we computed the corresponding Fisher matrices for the multipole power spectra. In both cases, we obtain that the precision on measurements of the effective Newton constant $\mu = \mu_0 + \mu_2 y^2 + \mu_4 y^4$ for an Euclid-like survey will be around 1% for μ_0 and a few percent for μ_2 and μ_4 (in the case of constant μ_0 , μ_2 and μ_4 parameters and marginalizing over $E(z)$ and $b\sigma_8(z)$ in each redshift bin). Very much as in the clustering case, the lensing forecast indicates that the γ parameter could be measured with a few percent precision, whereas μ_Q and μ_h parameters could be determined with precision around 1% (again in the constant case and marginalizing over $E(z)$).

Notice that in this analysis, systematic effects are assumed to be controlled, so that no additional parameters such as non-gaussian shot noise, multiplicative shear, etc are introduced. We do not expect such isotropic systematics to affect in an important way the determination of preferred direction effects, although it would be interesting to explore the possible degeneracy with other anisotropic systematics.

The effects of gravitational preferred directions could, in principle, be degenerate with anisotropies in the primordial power spectrum. In fact, clustering information alone would not be able to discriminate between them. However, this degeneracy could be broken since

gravitational preferred directions can generate vector and tensor perturbations which unlike primordial anisotropies induce new anisotropic features in the lensing signal. Thus, a dipole or quadrupole component of the lensing convergence power spectrum or a violation of the consistency relation (1.1) will be the smoking guns of this kind of models.

Acknowledgments

M.A.R acknowledges support from UCM predoctoral grant. This work has been supported by the MINECO (Spain) project FIS2016-78859-P(AEI/FEDER, UE).

A Clustering covariance matrices

In this appendix we will calculate the covariance matrix for an anisotropic matter power spectrum. To do that, we start by reviewing the standard calculation in the isotropic case [70–73] and then we will extend it to include a preferred direction. Let us thus start with the estimator for the matter power spectrum in the isotropic case,

$$\hat{P}(k_i) = V_f \int_{k_i} \frac{d^3 \vec{k}}{V_s(k_i)} \delta(\vec{k}) \delta(-\vec{k}), \quad (\text{A.1})$$

where $V_f = (2\pi)^3/V$, being V the volume of the survey and $\int_{k_i} d^3 \vec{k} = V_s(k_i) = 4\pi k_i^2 dk_i$ the volume of the k_i bin. Thus if we use,

$$\langle \delta(\vec{k}_1) \delta(\vec{k}_2) \rangle = \delta_D(\vec{k}_1 + \vec{k}_2) P(\vec{k}_1), \quad (\text{A.2})$$

we can prove that $\langle \hat{P}(k_i) \rangle = P(k_i)$, where $\delta_D(0) = 1/V_f$. Now, we want to calculate the covariance matrix, defined as

$$C(k_i, k_j) = \langle \hat{P}(k_i) \hat{P}(k_j) \rangle - P(k_i) P(k_j). \quad (\text{A.3})$$

We consider only the gaussian case, so that

$$\begin{aligned} \langle F(s_1) F(s_2) F(s_3) F(s_4) \rangle &= \langle F(s_1) F(s_2) \rangle \langle F(s_3) F(s_4) \rangle \\ &+ \langle F(s_1) F(s_3) \rangle \langle F(s_2) F(s_4) \rangle \\ &+ \langle F(s_1) F(s_4) \rangle \langle F(s_2) F(s_3) \rangle. \end{aligned} \quad (\text{A.4})$$

Using this relation we obtain,

$$C(k_i, k_j) = \frac{V_f^2 \delta_D(0)}{V_s(k_i) V_s(k_j)} \int_{k_i} d^3 \vec{k} \int_{k_j} d^3 \vec{k}' [\delta_D(\vec{k} + \vec{k}') + \delta_D(\vec{k} - \vec{k}')] P(k)^2, \quad (\text{A.5})$$

where we have used the property $\delta_D^2(x) = \delta_D(0) \delta_D(x)$, and $P(\vec{k}) = P(k)$ being $k = |\vec{k}|$. We consider that $P(k)$ is constant in the integral of the k_i -bin, so that we can extract it from the integral as $P(k_i)$. Thus, the integrals in k and k' become,

$$\int_{k_i} d^3 \vec{k} \int_{k_j} d^3 \vec{k}' [\delta_D(\vec{k} + \vec{k}') + \delta_D(\vec{k} - \vec{k}')] = 2 V_s(k_j) \delta_{ij}. \quad (\text{A.6})$$

Finally if we take into account the shot noise, the observable galaxy density contrast becomes,

$$\delta^{\text{obs}}(k) = \delta(k) + \epsilon(k), \quad (\text{A.7})$$

where $\epsilon(k)$ is a random gaussian variable with $\langle \epsilon(k) \rangle = 0$ and $\langle \epsilon(k) \epsilon(k') \rangle = \delta_D(k - k') / \bar{n}$ with \bar{n} the average galaxy number density. Then the observable power spectrum becomes $P^{\text{obs}}(k_i) = P(k_i) + \frac{1}{\bar{n}}$ and we obtain,

$$C(k_i, k_j) = \delta_{ij} \frac{2V_f}{V_s(k_i)} \left[P(k_i) + \frac{1}{\bar{n}} \right]^2. \quad (\text{A.8})$$

Now we want to extend this procedure for the case in which we have an anisotropic power spectrum depending not only on the full wavevector \vec{k} , but also on its orientation with respect to the line of sight \hat{n} . In particular, we have a power spectrum $P(\vec{k}, \hat{\mu})$ where $\hat{\mu} = \hat{k} \cdot \hat{n}$. We can decompose this power spectrum in the form,

$$P(\vec{k}, \hat{\mu}) = \sum_{\ell} P_{\ell}(\vec{k}) \mathcal{L}_{\ell}(\hat{\mu}), \quad (\text{A.9})$$

being $\mathcal{L}_{\ell}(\hat{\mu})$ the Legendre polynomials,

$$P_{\ell}(\vec{k}) = \frac{2\ell + 1}{2} \int_{-1}^1 d\hat{\mu} P(\vec{k}, \hat{\mu}) \mathcal{L}_{\ell}(\hat{\mu}). \quad (\text{A.10})$$

We define the estimator for this multipole power spectrum in the following way,

$$\hat{P}_{\ell}(\vec{k}_i) = V_f \int_{\vec{k}_i} \frac{d^3 \vec{k}}{V_s(\vec{k}_i)} \frac{2\ell + 1}{2} \int_{-1}^1 d\hat{\mu} \delta(\vec{k}, \hat{\mu}) \delta(-\vec{k}, \hat{\mu}) \mathcal{L}_{\ell}(\hat{\mu}), \quad (\text{A.11})$$

where in general $\int_{\vec{k}_i} d^3 \vec{k} = V_s(\vec{k}_i) = k_i^2 dk_i dy_i d\phi_i$, being $y_i = \cos \theta_i$. If we consider,

$$\langle \delta(\vec{k}_1, \hat{\mu}) \delta(\vec{k}_2, \hat{\mu}) \rangle = \delta_D(\vec{k}_1 + \vec{k}_2) P(\vec{k}_1, \hat{\mu}), \quad (\text{A.12})$$

we can prove that $\langle \hat{P}_{\ell}(\vec{k}_i) \rangle = P_{\ell}(\vec{k}_i)$. With this estimator we can calculate the covariance matrix,

$$C_{\ell\ell'}(\vec{k}_i, \vec{k}_j) = \langle \hat{P}_{\ell}(\vec{k}_i) \hat{P}_{\ell'}(\vec{k}_j) \rangle - P_{\ell}(\vec{k}_i) P_{\ell'}(\vec{k}_j). \quad (\text{A.13})$$

As in the isotropic case, we consider only gaussian perturbations satisfying (A.4), so that

$$\begin{aligned} C_{\ell\ell'}(\vec{k}_i, \vec{k}_j) = & V_f^2 \frac{(2\ell + 1)(2\ell' + 1)}{4} \int_{\vec{k}_i} \frac{d^3 \vec{k}}{V_s(\vec{k}_i)} \int_{\vec{k}_j} \frac{d^3 \vec{k}'}{V_s(\vec{k}_j)} \int_{-1}^1 d\hat{\mu} \int_{-1}^1 d\hat{\mu}' \mathcal{L}_{\ell}(\hat{\mu}) \mathcal{L}_{\ell'}(\hat{\mu}') \\ & \times \left[\langle \delta(\vec{k}, \hat{\mu}) \delta(\vec{k}', \hat{\mu}') \rangle \langle \delta(-\vec{k}, \hat{\mu}) \delta(-\vec{k}', \hat{\mu}') \rangle \right. \\ & \left. + \langle \delta(\vec{k}, \hat{\mu}) \delta(-\vec{k}', \hat{\mu}') \rangle \langle \delta(-\vec{k}, \hat{\mu}) \delta(\vec{k}', \hat{\mu}') \rangle \right]. \end{aligned} \quad (\text{A.14})$$

At this stage we use the distant observer approximation in which we assume that the integrand of (A.14) is non negligible only when $\hat{\mu}' \simeq \hat{\mu}$, then we obtain,

$$C_{\ell\ell'}(\vec{k}_i, \vec{k}_j) \simeq V_f^2 \frac{(2\ell+1)(2\ell'+1)}{2} \int_{\vec{k}_i} \frac{d^3\vec{k}}{V_s(\vec{k}_i)} \int_{\vec{k}_j} \frac{d^3\vec{k}'}{V_s(\vec{k}_j)} \int_{-1}^1 d\hat{\mu} \mathcal{L}_\ell(\hat{\mu}) \mathcal{L}_{\ell'}(\hat{\mu}) \\ \times \left[\langle \delta(\vec{k}, \hat{\mu}) \delta(\vec{k}', \hat{\mu}) \rangle \langle \delta(-\vec{k}, \hat{\mu}) \delta(-\vec{k}', \hat{\mu}) \rangle \right. \\ \left. + \langle \delta(\vec{k}, \hat{\mu}) \delta(-\vec{k}', \hat{\mu}) \rangle \langle \delta(-\vec{k}, \hat{\mu}) \delta(\vec{k}', \hat{\mu}) \rangle \right]. \quad (\text{A.15})$$

Using (A.12) and taking into account once again that $\delta_D^2(x) = \delta_D(0)\delta_D(x)$ and $\delta_D(0) = 1/V_f$, we obtain,

$$C_{\ell\ell'}(\vec{k}_i, \vec{k}_j) \simeq V_f \frac{(2\ell+1)(2\ell'+1)}{2} \int_{\vec{k}_i} \frac{d^3\vec{k}}{V_s(\vec{k}_i)} \int_{\vec{k}_j} \frac{d^3\vec{k}'}{V_s(\vec{k}_j)} \\ \times \int_{-1}^1 d\hat{\mu} \mathcal{L}_\ell(\hat{\mu}) \mathcal{L}_{\ell'}(\hat{\mu}) \left[\delta_D(\vec{k} + \vec{k}') + \delta_D(\vec{k} - \vec{k}') \right] P(\vec{k}, \hat{\mu}) P(-\vec{k}, \hat{\mu}). \quad (\text{A.16})$$

As done before, we consider that $P(\vec{k}, \hat{\mu}) \simeq P(\vec{k}_i, \hat{\mu})$ in the integral and also that

$$\int_{\vec{k}_i} d^3\vec{k} \int_{\vec{k}_j} d^3\vec{k}' \delta_D(\vec{k} - \vec{k}') = V_s(\vec{k}_j) \delta_{k_i, k_j} \delta_{y_i, y_j} \delta_{\phi_i, \phi_j}, \quad (\text{A.17})$$

and

$$\int_{\vec{k}_i} d^3\vec{k} \int_{\vec{k}_j} d^3\vec{k}' \delta_D(\vec{k} + \vec{k}') = V_s(\vec{k}_j) \delta_{k_i, k_j} \delta_{y_i, -y_j} \delta_{\phi_i, \phi_j + \pi}. \quad (\text{A.18})$$

Using these expressions we obtain,

$$C_{\ell\ell'}(\vec{k}_i, \vec{k}_j) = \frac{V_f \delta_{k_i, k_j}}{V_s(\vec{k}_i)} [\delta_{y_i, y_j} \delta_{\phi_i, \phi_j} + \delta_{y_i, -y_j} \delta_{\phi_i, \phi_j + \pi}] \\ \times \frac{(2\ell+1)(2\ell'+1)}{2} \int_{-1}^1 d\hat{\mu} \mathcal{L}_\ell(\hat{\mu}) \mathcal{L}_{\ell'}(\hat{\mu}) P(\vec{k}_i, \hat{\mu}) P(-\vec{k}_i, \hat{\mu}). \quad (\text{A.19})$$

If we consider that the fiducial power spectrum is of the form $P(\vec{k}_i, \hat{\mu}) = P(k_i, \hat{\mu})$, and we have only dependence on y^2 then $\delta_{y_i, -y_j} = \delta_{y_i, y_j}$. Also, we can integrate in ϕ_i so that $V_s(\vec{k}_i) = 2\pi k_i^2 dk_i dy_i$, and

$$C_{\ell\ell'}(\vec{k}_i, \vec{k}_j) = \delta_{\vec{k}_i, \vec{k}_j} \frac{2V_f}{V_s(\vec{k}_i)} \frac{(2\ell+1)(2\ell'+1)}{2} \int_{-1}^1 d\hat{\mu} \mathcal{L}_\ell(\hat{\mu}) \mathcal{L}_{\ell'}(\hat{\mu}) P^2(k_i, \hat{\mu}). \quad (\text{A.20})$$

Finally we take into account that the observable matter power spectrum is $P^{\text{obs}}(k_i, \hat{\mu}) = P(k_i, \hat{\mu}) + 1/\bar{n}$ so that the final expression for the covariance matrix we will consider reads

$$C_{\ell\ell'}(\vec{k}_i, \vec{k}_j) = \delta_{\vec{k}_i, \vec{k}_j} \frac{2V_f}{V_s(\vec{k}_i)} \frac{(2\ell+1)(2\ell'+1)}{2} \int_{-1}^1 d\hat{\mu} \mathcal{L}_\ell(\hat{\mu}) \mathcal{L}_{\ell'}(\hat{\mu}) \left[P(k_i, \hat{\mu}) + \frac{1}{\bar{n}} \right]^2. \quad (\text{A.21})$$

B Lensing covariance matrices

In this appendix we will calculate the covariance matrix for an anisotropic lensing convergence power spectra. As in clustering case, we start by reviewing [70–73] the standard isotropic calculation. Let us thus first obtain the estimator for the convergence power spectra in redshift bins i, j ,

$$\hat{P}_{ij}(\ell_a) = A_f \int_{\ell_a} \frac{d^2 \vec{\ell}}{A_s(\ell_a)} \kappa_i(\vec{\ell}) \kappa_j(-\vec{\ell}), \quad (\text{B.1})$$

where $A_f = (2\pi)^2/\Omega$, being $\Omega = 4\pi f_{\text{sky}}$ the total area of the survey and $\int_{\ell_a} d^2 \vec{\ell} = A_s(\ell_a) = 2\pi \ell_a d\ell_a$ the area of the ℓ_a bin. Then if we use,

$$\langle \kappa_i(\vec{\ell}_1) \kappa_j(\vec{\ell}_2) \rangle = \delta_D(\vec{\ell}_1 + \vec{\ell}_2) P_{ij}(\vec{\ell}_1), \quad (\text{B.2})$$

we can prove that $\langle \hat{P}_{ij}(\ell_a) \rangle = P_{ij}(\ell_a)$, where $\delta_D(0) = 1/A_f$. The corresponding covariance matrix is defined by,

$$C_{ij i' j'}(\ell_a, \ell_b) = \langle \hat{P}_{ij}(\ell_a) \hat{P}_{i' j'}(\ell_b) \rangle - P_{ij}(\ell_a) P_{i' j'}(\ell_b). \quad (\text{B.3})$$

As before, we take into account (A.4) for gaussian perturbations so that

$$\begin{aligned} C_{ij i' j'}(\ell_a, \ell_b) &= \frac{A_f^2 \delta_D(0)}{A_s(\ell_a) A_s(\ell_b)} \int_{\ell_a} d^2 \vec{\ell} \int_{\ell_b} d^2 \vec{\ell}' \\ &\times [\delta_D(\vec{\ell} + \vec{\ell}') P_{ii'}(\ell) P_{jj'}(\ell) + \delta_D(\vec{\ell} - \vec{\ell}') P_{ij'}(\ell) P_{ji'}(\ell)], \end{aligned} \quad (\text{B.4})$$

where once more we have used the property $\delta_D^2(x) = \delta_D(0) \delta_D(x)$, and $P_{ij}(\vec{\ell}) = P_{ij}(\ell)$ being $\ell = |\vec{\ell}|$. By assuming that $P_{ij}(\ell)$ is constant within the ℓ_a bin, we can extract it from the integral as $P_{ij}(\ell_a)$. Then, the integrals in ℓ and ℓ' become,

$$\int_{\ell_a} d^2 \vec{\ell} \int_{\ell_b} d^2 \vec{\ell}' \delta_D(\vec{\ell} \pm \vec{\ell}') = A_s(\ell_b) \delta_{ab}. \quad (\text{B.5})$$

Finally if we take into account the intrinsic ellipticity γ_{int} , the observable convergence reads,

$$\kappa_i^{\text{obs}}(\ell) = \kappa_i(\ell) + \gamma_{\text{int}} \epsilon_i(\ell), \quad (\text{B.6})$$

where $\epsilon_i(\ell)$ is a random gaussian variable with $\langle \epsilon_i(\ell) \rangle = 0$ and $\langle \epsilon_i(\ell) \epsilon_j(\ell') \rangle = \delta_D(\ell - \ell') \delta_{ij} / \hat{n}_i$, with \hat{n}_i the areal galaxy density (per steradian) in the redshift bin i . Then the observable power spectrum is,

$$P_{ij}^{\text{obs}}(\ell) = P_{ij}(\ell) + \frac{\gamma_{\text{int}}^2}{\hat{n}_i} \delta_{ij}. \quad (\text{B.7})$$

and the covariance is,

$$C_{ij i' j'}(\ell_a, \ell_b) = \frac{A_f \delta_{ab}}{A_s(\ell_a)} [P_{ii'}^{\text{obs}}(\ell_a) P_{jj'}^{\text{obs}}(\ell_a) + P_{ij'}^{\text{obs}}(\ell_a) P_{ji'}^{\text{obs}}(\ell_a)]. \quad (\text{B.8})$$

If we use the expressions for A_f and $A_s(\ell_a)$ and consider $\ell \simeq (2\ell + 1)/2$ we obtain the known result [73],

$$C_{ijj'j'}(\ell_a, \ell_b) = \frac{\delta_{ab}}{(2\ell_a + 1)f_{\text{sky}}d\ell_a} [P_{ii'}^{\text{obs}}(\ell_a)P_{jj'}^{\text{obs}}(\ell_a) + P_{ij'}^{\text{obs}}(\ell_a)P_{ji'}^{\text{obs}}(\ell_a)]. \quad (\text{B.9})$$

Now we want to obtain the covariance matrix for the case in which the power spectrum depends not only on the full $\vec{\ell}$ vector but also we have a dependence on the observation direction \hat{n} , in particular a polar dependence ($\xi = \hat{n} \cdot \hat{A}$) where \hat{A} is the preferred direction. As in (A.9), we can write

$$P_{ij}(\vec{\ell}, \xi) = \sum_r P_{ij}^r(\vec{\ell}) \mathcal{L}_r(\xi), \quad (\text{B.10})$$

where,

$$P_{ij}^r(\vec{\ell}) = \frac{2r+1}{2} \int_{-1}^1 d\xi P_{ij}(\vec{\ell}, \xi) \mathcal{L}_r(\xi). \quad (\text{B.11})$$

We define the estimator for this multipole power spectrum in the following way,

$$\hat{P}_{ij}^r(\vec{\ell}_a) = A_f \int_{\vec{\ell}_a} \frac{d^2\vec{\ell}}{A_s(\vec{\ell}_a)} \frac{2r+1}{2} \int_{-1}^1 d\xi \kappa_i(\vec{\ell}, \xi) \kappa_j(-\vec{\ell}, \xi) \mathcal{L}_r(\xi), \quad (\text{B.12})$$

being $\int_{\vec{\ell}_a} d^2\vec{\ell} = A_s(\vec{\ell}_a) = \ell_a d\ell_a d\phi_a$, where $\cos \phi_a = \Upsilon_a$ (6.25). If we consider,

$$\langle \kappa_i(\vec{\ell}_1, \xi) \kappa_j(\vec{\ell}_2, \xi) \rangle = \delta_D(\vec{\ell}_1 + \vec{\ell}_2) P_{ij}(\vec{\ell}_1, \xi), \quad (\text{B.13})$$

we can prove that $\langle \hat{P}_{ij}^r(\vec{\ell}_a) \rangle = P_{ij}^r(\vec{\ell}_a)$. With this estimator we can calculate the covariance matrix,

$$C_{ijj'j'}^{rr'}(\vec{\ell}_a, \vec{\ell}_b) = \langle \hat{P}_{ij}^r(\vec{\ell}_a) \hat{P}_{i'j'}^{r'}(\vec{\ell}_b) \rangle - P_{ij}^r(\vec{\ell}_a) P_{i'j'}^{r'}(\vec{\ell}_b). \quad (\text{B.14})$$

As in the previous case, we consider only gaussian perturbations, so that

$$\begin{aligned} C_{ijj'j'}^{rr'}(\vec{\ell}_a, \vec{\ell}_b) &= A_f^2 \frac{(2r+1)(2r'+1)}{4} \int_{\vec{\ell}_a} \frac{d^2\vec{\ell}}{A_s(\vec{\ell}_a)} \int_{\vec{\ell}_b} \frac{d^2\vec{\ell}'}{A_s(\vec{\ell}_b)} \int_{-1}^1 d\xi \int_{-1}^1 d\xi' \mathcal{L}_r(\xi) \mathcal{L}_{r'}(\xi') \\ &\times \left[\langle \kappa_i(\vec{\ell}, \xi) \kappa_{i'}(\vec{\ell}', \xi') \rangle \langle \kappa_j(-\vec{\ell}, \xi) \kappa_{j'}(-\vec{\ell}', \xi') \rangle \right. \\ &\quad \left. + \langle \kappa_i(\vec{\ell}, \xi) \kappa_{j'}(-\vec{\ell}', \xi') \rangle \langle \kappa_j(-\vec{\ell}, \xi) \kappa_{i'}(\vec{\ell}', \xi') \rangle \right]. \quad (\text{B.15}) \end{aligned}$$

At this stage we use once again the distant observer approximation, in which we assume that the integrand of (B.15) is non negligible only when $\xi' \simeq \xi$, then we obtain,

$$\begin{aligned} C_{ijj'j'}^{rr'}(\vec{\ell}_a, \vec{\ell}_b) &\simeq A_f^2 \frac{(2r+1)(2r'+1)}{2} \int_{\vec{\ell}_a} \frac{d^2\vec{\ell}}{A_s(\vec{\ell}_a)} \int_{\vec{\ell}_b} \frac{d^2\vec{\ell}'}{A_s(\vec{\ell}_b)} \int_{-1}^1 d\xi \mathcal{L}_r(\xi) \mathcal{L}_{r'}(\xi) \\ &\times \left[\langle \kappa_i(\vec{\ell}, \xi) \kappa_{i'}(\vec{\ell}', \xi) \rangle \langle \kappa_j(-\vec{\ell}, \xi) \kappa_{j'}(-\vec{\ell}', \xi) \rangle \right. \\ &\quad \left. + \langle \kappa_i(\vec{\ell}, \xi) \kappa_{j'}(-\vec{\ell}', \xi) \rangle \langle \kappa_j(-\vec{\ell}, \xi) \kappa_{i'}(\vec{\ell}', \xi) \rangle \right]. \quad (\text{B.16}) \end{aligned}$$

Using (B.13) and taking into account once more $\delta_D^2(x) = \delta_D(0)\delta_D(x)$ and $\delta_D(0) = 1/A_f$, we finally obtain,

$$C_{ij'j'}^{rr'}(\vec{\ell}_a, \vec{\ell}_b) \approx A_f \frac{(2r+1)(2r'+1)}{2} \int_{\vec{\ell}_a} \frac{d^2\vec{\ell}}{A_s(\vec{\ell}_a)} \int_{\vec{\ell}_b} \frac{d^2\vec{\ell}'}{A_s(\vec{\ell}_b)} \int_{-1}^1 d\xi \mathcal{L}_r(\xi) \mathcal{L}_{r'}(\xi) \\ \times \left[\delta_D(\vec{\ell} + \vec{\ell}') P_{ii'}(\vec{\ell}, \xi) P_{jj'}(-\vec{\ell}, \xi) + \delta_D(\vec{\ell} - \vec{\ell}') P_{ij'}(\vec{\ell}, \xi) P_{ji'}(-\vec{\ell}, \xi) \right], \quad (\text{B.17})$$

As done before, we consider that $P_{ij}(\vec{\ell}, \xi) \simeq P_{ij}(\vec{\ell}_a, \xi)$ in the integral and also that,

$$\int_{\vec{\ell}_a} d^2\vec{\ell} \int_{\vec{\ell}_b} d^2\vec{\ell}' \delta_D(\vec{\ell} - \vec{\ell}') = A_s(\ell_b) \delta_{\ell_a \ell_b} \delta_{\phi_a \phi_b}, \quad (\text{B.18})$$

and

$$\int_{\vec{\ell}_a} d^2\vec{\ell} \int_{\vec{\ell}_b} d^2\vec{\ell}' \delta_D(\vec{\ell} + \vec{\ell}') = A_s(\ell_b) \delta_{\ell_a \ell_b} \delta_{\phi_a \phi_b + \pi}, \quad (\text{B.19})$$

so that we finally obtain,

$$C_{ij'j'}^{rr'}(\vec{\ell}_a, \vec{\ell}_b) = \frac{A_f \delta_{\ell_a \ell_b}}{A_s(\vec{\ell}_a)} \frac{(2r+1)(2r'+1)}{2} \int_{-1}^1 d\xi \mathcal{L}_r(\xi) \mathcal{L}_{r'}(\xi) \\ \times \left[\delta_{\phi_a \phi_b + \pi} P_{ii'}(\vec{\ell}_a, \xi) P_{jj'}(-\vec{\ell}_a, \xi) + \delta_{\phi_a \phi_b} P_{ij'}(\vec{\ell}_a, \xi) P_{ji'}(-\vec{\ell}_a, \xi) \right]. \quad (\text{B.20})$$

Notice that if the only dependence in ϕ_a is in the form of Υ_a^2 , then $\delta_{\phi_a \phi_b} = \delta_{\phi_a \phi_b + \pi} = \delta_{\Upsilon_a \Upsilon_b}$. If we further consider that the fiducial power spectrum is isotropic, we obtain the final result used in the work,

$$C_{ij'j'}^{rr'}(\vec{\ell}_a, \vec{\ell}_b) = \frac{2\pi (2r+1) \delta_{\vec{\ell}_a \vec{\ell}_b} \delta_{rr'}}{f_{\text{sky}} (2\ell_a + 1) d\ell_a d\phi_a} [P_{ii'}^{\text{obs}}(\ell_a) P_{jj'}^{\text{obs}}(\ell_a) + P_{ij'}^{\text{obs}}(\ell_a) P_{ji'}^{\text{obs}}(\ell_a)], \quad (\text{B.21})$$

where $\delta_{\vec{\ell}_a \vec{\ell}_b} = \delta_{\ell_a \ell_b} \delta_{\Upsilon_a \Upsilon_b}$ and we have approximated once more $\ell \simeq (2\ell + 1)/2$.

References

- [1] S. Weinberg, *Gravitation and cosmology*, Wiley Interscience, U.S.A. (1972).
- [2] C.M. Will, *Theory and experiment in gravitational physics*, Cambridge University Press, Cambridge U.K. (1993).
- [3] L. Shao and N. Wex, *Tests of gravitational symmetries with radio pulsars*, *Sci. China Phys. Mech. Astron.* **59** (2016) 699501 [[arXiv:1604.03662](#)] [[INSPIRE](#)].
- [4] PLANCK collaboration, *Planck 2015 results. XVI. Isotropy and statistics of the CMB*, *Astron. Astrophys.* **594** (2016) A16 [[arXiv:1506.07135](#)] [[INSPIRE](#)].
- [5] D.H. Lyth and A.R. Liddle, *The primordial density perturbation*, Cambridge University Press, Cambridge U.K. (2009).
- [6] L. Ackerman, S.M. Carroll and M.B. Wise, *Imprints of a primordial preferred direction on the microwave background*, *Phys. Rev. D* **75** (2007) 083502 [Erratum *ibid.* **D 80** (2009) 069901] [[astro-ph/0701357](#)] [[INSPIRE](#)].
- [7] D.J. Schwarz, C.J. Copi, D. Huterer and G.D. Starkman, *CMB anomalies after Planck*, *Class. Quant. Grav.* **33** (2016) 184001 [[arXiv:1510.07929](#)] [[INSPIRE](#)].

- [8] PLANCK collaboration, *Planck intermediate results. XIII. Constraints on peculiar velocities*, *Astron. Astrophys.* **561** (2014) A97 [[arXiv:1303.5090](#)] [[INSPIRE](#)].
- [9] F. Atrio-Barandela et al., *Probing the dark flow signal in WMAP 9-year and Planck cosmic microwave background maps*, *Astrophys. J.* **810** (2015) 143 [[arXiv:1411.4180](#)] [[INSPIRE](#)].
- [10] M.I. Scringour et al., *The 6dF galaxy survey: bulk flows on 50–70 h^{-1} Mpc scales*, *Mon. Not. Roy. Astron. Soc.* **455** (2016) 386 [[arXiv:1511.06930](#)] [[INSPIRE](#)].
- [11] D. Colladay and V.A. Kostelecky, *Lorentz violating extension of the standard model*, *Phys. Rev. D* **58** (1998) 116002 [[hep-ph/9809521](#)] [[INSPIRE](#)].
- [12] Y. Nambu, *Quantum electrodynamics in nonlinear gauge*, *Prog. Theor. Phys. Suppl.* **E 68** (1968) 190 [[INSPIRE](#)].
- [13] Q.G. Bailey and V.A. Kostelecky, *Signals for Lorentz violation in post-Newtonian gravity*, *Phys. Rev. D* **74** (2006) 045001 [[gr-qc/0603030](#)] [[INSPIRE](#)].
- [14] D. Blas and E. Lim, *Phenomenology of theories of gravity without Lorentz invariance: the preferred frame case*, *Int. J. Mod. Phys. D* **23** (2015) 1443009 [[arXiv:1412.4828](#)] [[INSPIRE](#)].
- [15] P. Hořava, *Quantum gravity at a Lifshitz point*, *Phys. Rev. D* **79** (2009) 084008 [[arXiv:0901.3775](#)] [[INSPIRE](#)].
- [16] T. Jacobson and D. Mattingly, *Gravity with a dynamical preferred frame*, *Phys. Rev. D* **64** (2001) 024028 [[gr-qc/0007031](#)] [[INSPIRE](#)].
- [17] C. Armendariz-Picon, *Could dark energy be vector-like?*, *JCAP* **07** (2004) 007 [[astro-ph/0405267](#)] [[INSPIRE](#)].
- [18] C.G. Boehmer and T. Harko, *Dark energy as a massive vector field*, *Eur. Phys. J. C* **50** (2007) 423 [[gr-qc/0701029](#)] [[INSPIRE](#)].
- [19] J. Beltran Jimenez and A.L. Maroto, *A cosmic vector for dark energy*, *Phys. Rev. D* **78** (2008) 063005 [[arXiv:0801.1486](#)] [[INSPIRE](#)].
- [20] J. Beltran Jimenez and A.L. Maroto, *Cosmological electromagnetic fields and dark energy*, *JCAP* **03** (2009) 016 [[arXiv:0811.0566](#)] [[INSPIRE](#)].
- [21] J. Beltran Jimenez and A.L. Maroto, *The electromagnetic dark sector*, *Phys. Lett. B* **686** (2010) 175 [[arXiv:0903.4672](#)] [[INSPIRE](#)].
- [22] J. Beltran Jimenez and L. Heisenberg, *Derivative self-interactions for a massive vector field*, *Phys. Lett. B* **757** (2016) 405 [[arXiv:1602.03410](#)] [[INSPIRE](#)].
- [23] J. Beltran Jimenez and L. Heisenberg, *Generalized multi-Proca fields*, *Phys. Lett. B* **770** (2017) 16 [[arXiv:1610.08960](#)] [[INSPIRE](#)].
- [24] J.K. Nordtvedt, *Anisotropic parametrized post-Newtonian gravitational metric field*, *Phys. Rev. D* **14** (1976) 1511 [[INSPIRE](#)].
- [25] H. Muller et al., *Atom Interferometry tests of the isotropy of post-Newtonian gravity*, *Phys. Rev. Lett.* **100** (2008) 031101 [[arXiv:0710.3768](#)] [[INSPIRE](#)].
- [26] A.R. Pullen and C.M. Hirata, *Non-detection of a statistically anisotropic power spectrum in large-scale structure*, *JCAP* **05** (2010) 027 [[arXiv:1003.0673](#)] [[INSPIRE](#)].
- [27] D. Jeong and M. Kamionkowski, *Clustering fossils from the Early Universe*, *Phys. Rev. Lett.* **108** (2012) 251301 [[arXiv:1203.0302](#)] [[INSPIRE](#)].
- [28] M. Shiraishi, N.S. Sugiyama and T. Okumura, *Polypolar spherical harmonic decomposition of galaxy correlators in redshift space: toward testing cosmic rotational symmetry*, *Phys. Rev. D* **95** (2017) 063508 [[arXiv:1612.02645](#)] [[INSPIRE](#)].

- [29] V. Tansella et al., *Redshift-space distortions from vector perturbations II: anisotropic signal*, *Phys. Rev. D* **98** (2018) 103515 [[arXiv:1807.00731](#)] [[INSPIRE](#)].
- [30] N. Bartolo, S. Matarrese, M. Peloso and A. Ricciardone, *Anisotropic power spectrum and bispectrum in the $f(\phi)F^2$ mechanism*, *Phys. Rev. D* **87** (2013) 023504 [[arXiv:1210.3257](#)] [[INSPIRE](#)].
- [31] N. Bartolo, S. Matarrese, M. Peloso and M. Shiraishi, *Parity-violating CMB correlators with non-decaying statistical anisotropy*, *JCAP* **07** (2015) 039 [[arXiv:1505.02193](#)] [[INSPIRE](#)].
- [32] M.-a. Watanabe, S. Kanno and J. Soda, *The nature of primordial fluctuations from anisotropic inflation*, *Prog. Theor. Phys.* **123** (2010) 1041 [[arXiv:1003.0056](#)] [[INSPIRE](#)].
- [33] N. Bartolo et al., *Detecting higher spin fields through statistical anisotropy in the CMB and galaxy power spectra*, *Phys. Rev. D* **97** (2018) 023503 [[arXiv:1709.05695](#)] [[INSPIRE](#)].
- [34] A.L. Maroto, *Moving dark energy and the CMB dipole*, *JCAP* **05** (2006) 015 [[astro-ph/0512464](#)] [[INSPIRE](#)].
- [35] J. Beltran Jimenez and A.L. Maroto, *Cosmology with moving dark energy and the CMB quadrupole*, *Phys. Rev. D* **76** (2007) 023003 [[astro-ph/0703483](#)] [[INSPIRE](#)].
- [36] J.A.R. Cembranos, A.L. Maroto and H. Villarrubia-Rojo, *Non-comoving Cosmology*, *JCAP* **06** (2019) 041 [[arXiv:1903.11009](#)] [[INSPIRE](#)].
- [37] M.A. Resco and A.L. Maroto, *Parametrizing modified gravities with vector degrees of freedom: anisotropic growth and lensing*, *JCAP* **10** (2018) 014 [[arXiv:1807.04649](#)] [[INSPIRE](#)].
- [38] L. Pogosian, A. Silvestri, K. Koyama and G.-B. Zhao, *How to optimally parametrize deviations from General Relativity in the evolution of cosmological perturbations?*, *Phys. Rev. D* **81** (2010) 104023 [[arXiv:1002.2382](#)] [[INSPIRE](#)].
- [39] A. Silvestri, L. Pogosian and R.V. Buniy, *Practical approach to cosmological perturbations in modified gravity*, *Phys. Rev. D* **87** (2013) 104015 [[arXiv:1302.1193](#)] [[INSPIRE](#)].
- [40] BOSS collaboration, *The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: cosmological analysis of the DR12 galaxy sample*, *Mon. Not. Roy. Astron. Soc.* **470** (2017) 2617 [[arXiv:1607.03155](#)] [[INSPIRE](#)].
- [41] DESI collaboration, *The DESI experiment part I: science, targeting and survey design*, [arXiv:1611.00036](#) [[INSPIRE](#)].
- [42] EUCLID collaboration, *Euclid definition study report*, [arXiv:1110.3193](#) [[INSPIRE](#)].
- [43] DES collaboration, *The dark energy survey*, [astro-ph/0510346](#) [[INSPIRE](#)].
- [44] LSST SCIENCE, LSST PROJECT collaboration, *LSST science book, version 2.0*, [arXiv:0912.0201](#) [[INSPIRE](#)].
- [45] J-PAS collaboration, *J-PAS: the Javalambre-Physics of the Accelerated universe astrophysical survey*, [arXiv:1403.5237](#) [[INSPIRE](#)].
- [46] F. Bernardeau, S. Colombi, E. Gaztanaga and R. Scoccimarro, *Large scale structure of the universe and cosmological perturbation theory*, *Phys. Rept.* **367** (2002) 1 [[astro-ph/0112551](#)] [[INSPIRE](#)].
- [47] B.A. Bassett and R. Hlozek, *Baryon acoustic oscillations*, in *Dark energy*, P. Ruiz-Lapuente ed., Cambridge University Press, Cambridge U.K. (2010), [arXiv:0910.5224](#) [[INSPIRE](#)].
- [48] M. Bartelmann and P. Schneider, *Weak gravitational lensing*, *Phys. Rept.* **340** (2001) 291 [[astro-ph/9912508](#)] [[INSPIRE](#)].
- [49] M. Kilbinger, *Cosmology with cosmic shear observations: a review*, *Rept. Prog. Phys.* **78** (2015) 086901 [[arXiv:1411.0115](#)] [[INSPIRE](#)].

- [50] W. Hu and M. Tegmark, *Weak lensing: prospects for measuring cosmological parameters*, *Astrophys. J.* **514** (1999) L65 [[astro-ph/9811168](#)] [[INSPIRE](#)].
- [51] H.-J. Seo and D.J. Eisenstein, *Probing dark energy with baryonic acoustic oscillations from future large galaxy redshift surveys*, *Astrophys. J.* **598** (2003) 720 [[astro-ph/0307460](#)] [[INSPIRE](#)].
- [52] C. Alcock and B. Paczynski, *An evolution free test for non-zero cosmological constant*, *Nature* **281** (1979) 358 [[INSPIRE](#)].
- [53] L. Samushia, W.J. Percival and A. Raccanelli, *Interpreting large-scale redshift-space distortion measurements*, *Mon. Not. Roy. Astron. Soc.* **420** (2012) 2102 [[arXiv:1102.1014](#)].
- [54] D.J. Bacon, A.R. Refregier and R.S. Ellis, *Detection of weak gravitational lensing by large-scale structure*, *Mon. Not. Roy. Astron. Soc.* **318** (2000) 625 [[astro-ph/0003008](#)] [[INSPIRE](#)].
- [55] N. Kaiser, G. Wilson and G.A. Luppino, *Large scale cosmic shear measurements*, [astro-ph/0003338](#) [[INSPIRE](#)].
- [56] N. Kaiser, *Weak lensing and cosmology*, *Astrophys. J.* **498** (1998) 26 [[astro-ph/9610120](#)] [[INSPIRE](#)].
- [57] W. Hu, *Power spectrum tomography with weak lensing*, *Astrophys. J.* **522** (1999) L21 [[astro-ph/9904153](#)] [[INSPIRE](#)].
- [58] N. Kaiser and G. Squires, *Mapping the dark matter with weak gravitational lensing*, *Astrophys. J.* **404** (1993) 441 [[INSPIRE](#)].
- [59] N. Kaiser, G. Squires and T.J. Broadhurst, *A method for weak lensing observations*, *Astrophys. J.* **449** (1995) 460 [[astro-ph/9411005](#)] [[INSPIRE](#)].
- [60] A. Cooray and W. Hu, *Second order corrections to weak lensing by large scale structure*, *Astrophys. J.* **574** (2002) 19 [[astro-ph/0202411](#)] [[INSPIRE](#)].
- [61] D.B. Thomas, L. Whittaker, S. Camera and M.L. Brown, *Estimating the weak-lensing rotation signal in radio cosmic shear surveys*, *Mon. Not. Roy. Astron. Soc.* **470** (2017) 3131 [[arXiv:1612.01533](#)] [[INSPIRE](#)].
- [62] T.S. Pereira, C. Pitrou and J.-P. Uzan, *Theory of cosmological perturbations in an anisotropic universe*, *JCAP* **09** (2007) 006 [[arXiv:0707.0736](#)] [[INSPIRE](#)].
- [63] M.A. Resco and A.L. Maroto, *Parametrizing growth in dark energy and modified gravity models*, *Phys. Rev. D* **97** (2018) 043518 [[arXiv:1707.08964](#)] [[INSPIRE](#)].
- [64] E.V. Linder, *Cosmic growth history and expansion history*, *Phys. Rev. D* **72** (2005) 043529 [[astro-ph/0507263](#)] [[INSPIRE](#)].
- [65] E.V. Linder and R.N. Cahn, *Parameterized Beyond-Einstein growth*, *Astropart. Phys.* **28** (2007) 481 [[astro-ph/0701317](#)] [[INSPIRE](#)].
- [66] A. Taruya, S. Saito and T. Nishimichi, *Forecasting the cosmological constraints with anisotropic baryon acoustic oscillations from multipole expansion*, *Phys. Rev. D* **83** (2011) 103527 [[arXiv:1101.4723](#)] [[INSPIRE](#)].
- [67] L. Amendola et al., *Model-independent constraints on the cosmological anisotropic stress*, *Phys. Rev. D* **89** (2014) 063538 [[arXiv:1311.4765](#)] [[INSPIRE](#)].
- [68] J. Lesgourgues, *The Cosmic Linear Anisotropy Solving System (CLASS) I: overview*, [arXiv:1104.2932](#) [[INSPIRE](#)].
- [69] S. Hilbert et al., *Intrinsic alignments of galaxies in the illustris simulation*, *Mon. Not. Roy. Astron. Soc.* **468** (2017) 790 [[arXiv:1606.03216](#)] [[INSPIRE](#)].

- [70] K. Yamamoto et al., *A measurement of the quadrupole power spectrum in the clustering of the 2dF QSO Survey*, *Publ. Astron. Soc. Jap.* **58** (2006) 93 [[astro-ph/0505115](#)] [[INSPIRE](#)].
- [71] M. Takada and S. Bridle, *Probing dark energy with cluster counts and cosmic shear power spectra: including the full covariance*, *New J. Phys.* **9** (2007) 446 [[arXiv:0705.0163](#)] [[INSPIRE](#)].
- [72] I. Kayo and M. Takada, *Cosmological parameters from weak lensing power spectrum and bispectrum tomography: including the non-Gaussian errors*, [arXiv:1306.4684](#) [[INSPIRE](#)].
- [73] J. Guzik, B. Jain and M. Takada, *Tests of gravity from imaging and spectroscopic surveys*, *Phys. Rev. D* **81** (2010) 023503 [[arXiv:0906.2221](#)] [[INSPIRE](#)].