Effect of Mean Flow on Acoustic Wave Propagation in a Duct with a Periodic Array of Helmholtz Resonators *

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Sound propagation properties of a duct system with Helmholtz resonators (HRs) are affected by mean flow. Previous studies have tended to focus on the effects of mean flows on acoustic response of a duct system with a finite number of HRs. Employing an empirical impedance model, we present a modified transfer matrix method for studying the effect of mean flow on the complex band structure of an air duct system with an infinite periodic array of HRs. The efficiency of the modified transfer matrix is demonstrated by comparison between an example of transmission response calculation for a finite single HR loaded duct and the finite element simulation result calculated using the COMSOL software. Numerical results are presented to analyze the effect of mean flow on the band structure and transmission loss of the sound wave in the duct system. It is hoped that this study will provide theoretical guidance for acoustic wave propagation of HR silencer in the presence of mean flow.

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Helmholtz resonators (HRs) are often used in noise control in air duct systems due to their structural simplicity and tunable characteristics. Sound propagation properties of a duct with HRs can be affected by a grazing flow. Previous studies have shown that the end correction of the neck and acoustic impedance of an HR will change when a flow passes through the neck.[1−4] Researchers have paid a great deal of attention to effect of a flow on acoustic properties of duct systems loaded with HRs, due to the significance of flow effect in many engineering applications.[5,6]

Experimental and empirical investigations of a mean flow on transmission loss of a single HR have been studied for a long time. Meyer *et al.* investigated the effect of a flow on the acoustic impedance of an HR. Their experiments showed that the resonance frequency shifts to a higher frequency with the rising flow speed.[1] Anderson *et al.* measured the acoustic impedance of a single side branch HR in a circular flow duct. The result showed that the fundamental frequency increases and the neck end correction decreases as the flow speed rises.[2] Hersh *et al.* demonstrated that the acoustic resistance of an HR grows linearly as the flow speed increases.[3] Kooi and Sarin proposed an empirical formula with a flow resistance term, which represents the change of resistance due to the grazing flow.[4] Cummings proposed a one-dimensional (1D) time domain numerical approach for analyzing the flow effect on the acoustic resistances of HRs.[7] Seo *et al.* proposed a hybrid acoustic impedance model of HR based on the existing empirical formulas with considering both the high sound pressure and the grazing mean flow.[8] Shi *et al.* studied the effect of flow on a semi-active Helmholtz resonator for duct sound.[9] Shin proposed a theoretical method for analyzing the effect of a flow on Helmholtz resonators.[10]

These investigations have mainly studied the effect of a mean flow on duct systems with a finite number of HRs, while there are significantly fewer studies on acoustic wave propagation properties of the correspondence infinite system in the presence of a mean flow. This study aims to investigate the influence of a mean flow on the band structures of a duct system with an infinite periodic array of HRs.

In this Letter, a modified transfer matrix method, which uses the transfer matrix method and the empirical formulas proposed by Cummings for the acoustic impedance of HRs in a duct with mean flow, is presented for computing the band structure of a duct with a periodic array of HRs in the presence of a mean flow. The acoustic bandgap of an infinite periodic array of HRs with varying Mach number is calculated. The efficiency of the proposed method is demonstrated on the comparison between an example of transmission response calculation for a finite single HR loaded duct and the finite element simulation result calculated using the COMSOL software. Numerical results are presented for analyzing the effect of mean flow on the band structure and transmission loss of the sound wave in the duct system.

Figure 1 presents schematic diagrams of the present model, an air-filled square duct with a periodic

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array of HRs in the presence of the mean flow. The mean flow with a Mach number M ($M = U/c_0$, U is flow velocity and c_0 is acoustic speed) passes through the duct. The dimensions of the model are governed by the lattice constant $a = 110$ cm, the side length $D = 6$ cm of the squared cross section of the duct, the height $l = 8.05$ cm and the diameter $D_{\rm n} = 4.044$ cm of the neck of each HR, and the height $h = 24.42$ cm and the diameter $D_{\rm c} = 15.32$ cm of the cavity of each HR.

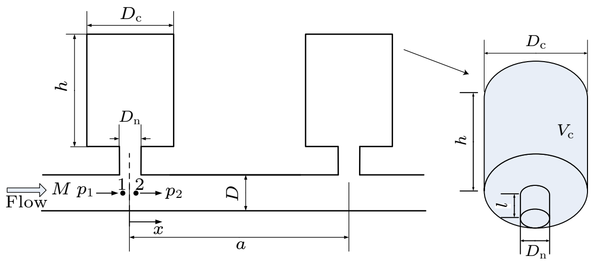

Fig. 1. Schematic of a periodic Helmholtz resonator.

Assuming the acoustic wave propagates in a low-frequency range where the higher-order harmonic components of the wave in the duct can be ignored, the sound pressure p approximately satisfies the one-dimensional equation[11]

$$\frac{\partial^2 p}{\partial x^2} + k^2 p = 0. \tag{1}$$

Considering the two connection points 1 and 2 in the nth unit cell as shown in Fig. 1, the transfer matrix relation in terms of the classical state variables can be given by[12]

$$\begin{pmatrix} p_1 \\ u_1 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ Z_{\rm H}^{-1} & 1 \end{bmatrix} \begin{pmatrix} p_2 \\ u_2 \end{pmatrix}, \tag{2}$$

where p_1 and u_1 are the sound pressure and volume velocity at point 1, respectively, p_2 and u_2 are the corresponding physical quantities at point 2. $Z_{\rm H}$ is the acoustic impedance of the HR.

$$Z_{\rm H} = R_{\rm H} + j\Big(\omega {\rm M_H} - \frac{1}{\omega {\rm C_H}}\Big),$$
$$M_{\rm H} = \frac{\rho_0 l_{\rm eff}}{S_{\rm n}}, \quad {\rm C_H} = \frac{V_{\rm c}}{\rho_0 c_0^2}, \tag{3}$$

where $R_{\rm H}$ is the acoustic resistance, $M_{\rm H}$ and $C_{\rm H}$ are the acoustic mass and acoustic capacitance, $l_{\rm eff}$ and $S_{\rm n}$ is the effective length and the cross-sectional area of the neck, $V_{\rm c}$ is the volume of cavity, ρ_0 and c_0 are the density and acoustic speed of air.

In the absence of the mean flow, the acoustic resistance of HR and the effective length of the neck $l_{\rm eff}$ can be expressed as

$$R_{\rm H} = \frac{\rho_0 c_0 k^2}{2\pi}, \quad l_{\rm eff} = l + \delta, \tag{4}$$

where $\delta = 0.48\sqrt{S_{\rm n}}$ is the end correction of the neck without the mean flow.

In the presence of the mean flow, an additional resistance term $R_{\rm f}$ should be included in the acoustic impedance model of HR. Many previous studies measured the acoustic impedance of a single orifice under a grazing flow and related the acoustic impedance to the reciprocal Strouhal number ($St = f \cdot D_{\rm n} c_0/M$).[13,14] In these experiments, the boundary layer is very thin. Several later results have shown that the boundary layer thickness affects the impedance of the orifice.

Goldman and Panton measured the thickness of the boundary layer close to an orifice and showed that a young turbulent boundary layer was present.[15] In their study, the friction velocity U^* of the inner boundary layer was used to replace the free stream velocity in the Strouhal number. Later, Cummings proposed a more properly empirical formula based on the experimental results.[7]

In this study, we take the orifice diameter $D_{\rm n} = 4.044$ cm and the side length $D = 6$ cm, which implies that the thickness of the boundary layer is less than 3 cm. Hence, the boundary layer is thin. Thus the mean flow velocity should be applied in the empirical impedance model proposed by Cummings,[7]

$$\frac{R_{\rm f} c_0}{f D_{\rm n}} = \Big[12.25\Big(\frac{l}{D_{\rm n}}\Big)^{-0.32} - 2.44\Big]\Big(\frac{M c_0}{f D_{\rm n}}\Big) - 3.2. \tag{5}$$

The total acoustic resistance R and the effective length of the neck, $l_{\rm eff}$, can be expressed as

$${\rm R} = R_{\rm H} + R_{\rm f}, \quad l_{\rm eff} = l + \delta_{\rm f}, \tag{6}$$

$$\frac{\delta}{\delta_{\rm f}} = \Big(1 + 0.6\frac{l}{D_{\rm n}}\Big)\exp\Big[-\frac{\frac{Mc_0}{fl} - 0.12\frac{D_{\rm n}}{l}}{0.25 + \frac{l}{D_{\rm n}}}\Big] - 0.6\frac{l}{D_{\rm n}},$$
$$\Big(\frac{Mc_0}{fl} \geqslant \frac{D_{\rm n}}{l}\Big), \tag{7}$$

$$\frac{\delta}{\delta_{\rm f}} = 1, \quad \Big(\frac{Mc_0}{fl} \leqslant \frac{D_{\rm n}}{l}\Big), \tag{8}$$

where $\delta_{\rm f}$ is the end correction of the neck with the mean flow.

According to the Bloch theorem, the Bloch wave vectors q of the acoustic waves propagate along the infinite long duct with a periodic array of HRs are the roots of the determinant,[16]

$$\big|\boldsymbol{T} - {\rm e}^{{\rm j}qa}\boldsymbol{I}\big| = 0, \tag{9}$$

where $\boldsymbol{T}$ is the transfer matrix and $\boldsymbol{I}$ is a 4×4 unit matrix.

The Bloch wave vector q, as a function of ω, constitutes the complex band structure of the duct system. Depending on whether q is real or has an imaginary part, the acoustic wave propagates through the duct (pass bands) or attenuates (stop bands).

For a finite periodic duct system with m cells, the state vectors on two sides can be easily derived

$$\begin{pmatrix} p_{\rm in} \\ u_{\rm in} \end{pmatrix} = \boldsymbol{T}^m \begin{pmatrix} p_{\rm out} \\ u_{\rm out} \end{pmatrix}. \tag{10}$$

Further, if the inlet and the outlet of the periodic duct are perfectly impedance matched, the above equation can be simplified to an explicit formulation,

$$t_{\rm m} = \left| \frac{2\sin(qa)\left(\tau_p^2 - 1\right)\tau_p^m}{\left(1 - \tau_p e^{-{\rm j}qa}\right)^2 - \tau_p^{2\,m}\left(\tau_p - e^{-{\rm j}qa}\right)^2} \right|^2, \tag{11}$$

where τ_p=exp($\pm jka$). Thus the transmission loss (TL) of a finite periodic duct system can be expressed as

$$\mathrm{TL} = 10\log\left(t_{\rm m}^{-1}\right). \tag{12}$$

To evaluate the correctness of the proposed theoretical method, validation is performed on the HR silencer with one cell in the presence of grazing flow. The results are then compared with the three-dimensional finite element method (FEM) simulation results calculated using the COMSOL software. The periodic HR silencer is filled with air whose density ρ_0 and acoustic speed c_0 are 1.224 kg/m^3 and 343 m/s.

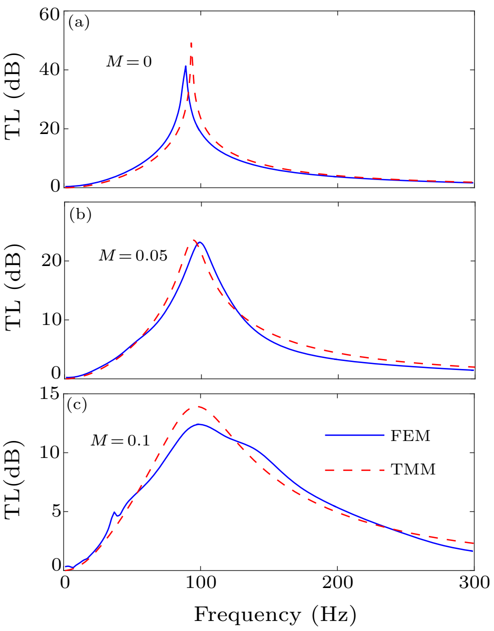

Fig. 2. Transmission Loss of a finite array HR silencer with one cell with varying Mach number.

Figure 2 shows a comparison of the present predictions and the results obtained by FEM simulation with three different Mach numbers, where (a), (b) and (c) denote the results when M equals 0, 0.05 and 0.1, respectively. It can be seen that good agreements are obtained in terms of the transmission loss variation with forcing frequency. Moreover, the predicted results agree well with the experimental measurements presented in the previous work by Selamet and Lee.[17] These studies confirm that the proposed method is able to predict the sound attenuation performance of HR silencer in the presence of mean flow. In addition, the deviations are shown to be more obvious when $M = 0.1$, which may be due to the nonlinear effect being more and more dominant.

Under the aforementioned HR parameters and the flow velocity, the acoustic bandgaps of the periodic HR silencer with different Mach numbers are obtained by using the proposed TMM. Figure 3 denotes the band structure of the periodic HR silencer when Mach number M varied, where the solid line, dotted line and dash-dotted line represent the results of $M = 0$, $M = 0.05$ and $M = 0.1$, respectively.

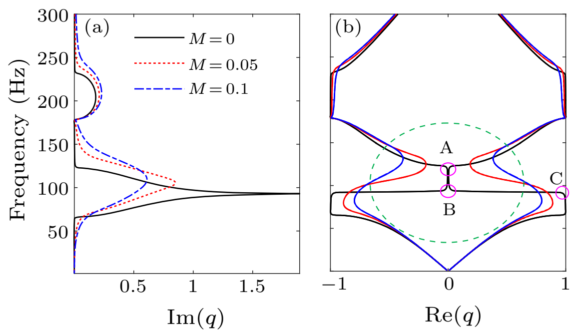

Fig. 3. Band structure of an infinite periodic array of HR silencer with varying Mach number.

The real part of the wave vector q (*real q*) is referred to as the propagation constant. The frequency ranges without any dispersion curves are stop bands (bandgap) and the acoustic wave will attenuate within these bands. In the results presented for the *real q* in Fig. 3, we can observe one intriguing phenomenon: *branch cutoff*. A *branch cutoff* refers to a state whereby the acoustical branch experiences a cutoff, that is, it no longer spans the entire first Brillouin zone, thus creating a partial band gap with respect to the wavenumber (named the wavenumber bandgap).[18] The main reason for emerging of the wavenumber bandgap is due to the acoustic resistance introduced by the flowing fluid, which can be used as a fluid damping, and thus the wavenumber bandgap will be created because of the fluid damping. When M increases from 0, the dispersion curve begins to separate, especially all sharp corners at points A, B and C become rounded and the straight connections become divergent, and the rounding effect gets more pronounced when M reaches a larger value. Hence, the dispersion curve of *real q* even does not occupy every value in the range $[-1, 1]$ so that a wavenumber bandgap is apparent.

The acoustic resistance $R_{\rm f}$ (the fluid damping) and the convergence degree of dispersion curve will increase when the Mach number further increases. Thus, the wavenumber band gap will enlarge, which indicates that the frequency range of the passband increases, and there is a corresponding increase of the acoustic wave that can travel efficiently in a pipe system. Thus the acoustic wave attenuation of the HR silencer will reduce. Furthermore, the effective length of the neck will decrease and the acoustic resistance will increase when Mach number further increase. Further increasing of the acoustic impedance will lead to further decreasing of the attenuation intensity of the HR silencer, besides, the reduction in the effective length of the neck causes the resonance peak to move to higher frequency. Therefore, the combination of the two changes reduces the acoustic wave attenuation performance of the HR silencer in the whole frequency range.

The imaginary part of q (*imag* q) is referred to as attenuation constant and the value denotes the attenuation intensity of the HR silencer. As shown in Fig. 3, the *imag* q will shift to high frequency at resonant peak and the value will decrease when the M increases. The phenomenon indicates that the attenuation intensity reduces and the attenuation band moves to a high frequency. Also, the phenomenon can be explained by the transmission loss (TL) of a finite array HR silencer.

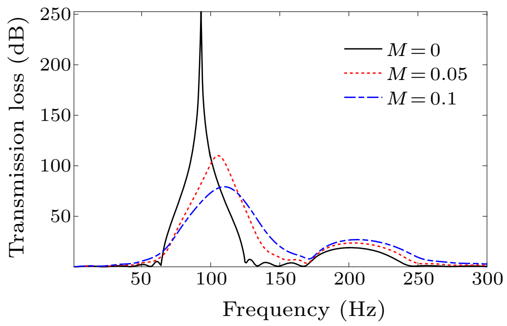

Fig. 4. TLs of a finite array HR silencer with the varying Mach number.

Figure 4 denotes the TLs of a finite array HR silencer with five cells when the Mach number varied, where the solid line, dotted line and dash-dotted line describe the TL when M equals 0, 0.05 and 0.1. As is expected, the resonance peak will shift to high frequency and the attenuation intensity will decrease when M increases. In addition, the location of the BBG is almost kept unchanged. Hence, the results further validate the effect of the mean flow on acoustic wave propagation in the duct system with a periodic array of HRs.

In conclusion, employing an empirical impedance model of HRs, we have presented a modified transfer matrix method for studying the effect of a mean flow on complex band structures of an air duct system with an infinite periodic array of HRs. The efficiency of the proposed method is demonstrated by comparison between an example of transmission response calculation for a finite single HR loaded duct and the finite element simulation result calculated using the COMSOL software. In analysis of the band structure, one intriguing phenomenon emerges due to the presence of acoustic resistance: *branch cutoff*. This refers to a state whereby a dispersion branch does not span the entire first Brillouin zone, thus creating a partial band gap with respect to the wavenumber (named the wavenumber bandgap). The wavenumber bandgap will enlarge due to the increased acoustic resistance when M rises, and the acoustic wave attenuation will reduce and the resonant frequency will shift to a high value. The change trends in band structure are validated by the transmission loss of a finite array HR silencer with five cells. It is hopeful that the present study could provide a basis for investigation of acoustic wave propagation of HR silencers with mean flows.

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