

Simulation-based optimization of inner layout of a theater considering the effect of pedestrians*

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We propose an extended cellular automaton model based on the floor field. The floor field can be changed accordingly in the presence of pedestrians. Furthermore, the effects of pedestrians with different speeds are distinguished, *i.e.*, still pedestrians result in more increment of the floor field than moving ones. The improved floor field reflects impact of pedestrians as movable obstacles on evacuation process. The presented model was calibrated by comparing with previous studies. It is shown that this model provides a better description of crowd evacuation both qualitatively and quantitatively. Then we investigated crowd evacuation from a middle-size theater. Four possible designs of aisles in the theater are studied and one of them is the actual design in reality. Numerical simulation shows that the actual design of the theater is reasonable. Then we optimize the position of the side exit in order to reduce the evacuation time. It is shown that the utilization of the two exits at bottom is less than that of the side exits. When the position of the side exit is shifted upwards by about 1.6 m, it is found that the evacuation time reaches its minimum.

Keywords: cellular automaton, floor field, crowd evacuation, optimization of pedestrian facilities

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1. Introduction

There are many events held all over the world where a large number of people gather in a rather small area, such as theaters and gymnasiums. In case of emergency, people should be evacuated safely as soon as possible. Meanwhile, it is well known that some interesting self-organized phenomena occur during evacuation, such as arching and clogging at bottlenecks and the faster-is-slower effect.^[1] Therefore, the understanding of crowd evacuation process is very important for both safety and theoretical reasons. Many efforts have been devoted to this issue, including empirical observations, well-controlled experiments, modeling, and simulation.^[1,2] In general, microscopic models allow depicting detailed interaction among individuals, therefore they are more elaborate to mimic collective behaviors. Some commonly-used models include the social force model,^[3] lattice gas (LG) model,^[4] and cellular automata (CA) models.^[5] Both LG and CA models are fully discrete which are rule-based, flexible, and of high computation efficiency.

In reality, usually there are kinds of facilities in public areas with complex inner structures. Hence, it is essential for pedestrians to determine the feasible paths to their destinations. The field-based CA models provide a uniform way to solve this problem. The first field-based CA model is the

floor field cellular automaton (FFCA) model suggested by Burstedde *et al.*^[6] In the FFCA model, there are two types of floor fields: the static floor field and the dynamic floor field. The static floor field is used to specify regions of space which are more attractive, *e.g.*, an exit or other targets. The static floor field contains the global information of inner structures of the building, and a pedestrian can determine where to go according to local field information. More precisely, the gradient of the static floor field suggests the reasonable moving direction for each pedestrian which indicates the shortest way to the target exit. The dynamic floor field can be used to mimic pedestrians' following behaviors, which takes effect locally. It is obvious that the static floor field is changed in the presence of obstacles. In most cases, pedestrian facilities (*e.g.*, walls and tables) are static obstacles. Several algorithms have been put forward to deal with crowd evacuation with obstacles.^[7,8] As a result, pedestrians' route-choice behaviors in complex situations seem to coincide qualitatively with our daily experience. Hereafter, the floor field cellular automaton (FFCA) model has been widely extended to simulate pedestrian traffic in various scenarios.^[9–15] In the context of CA models, usually the movement of pedestrians is updated simultaneously.^[5,6] During evacuation process, conflicts will inevitably occur when two or more pedestrians attempt to enter

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the same site simultaneously. Generally, conflicts are solved by randomly choosing one of the candidates. While in the LG model, the conflicts are excluded since the random sequential updating is adopted. However, the movement of pedestrians are generally believed in a parallel way. Recently, game theory has been used to deal with conflicts among pedestrians.^[16–21] In contrast to a given set of rules in CA models, the local interaction among pedestrians is reflected by the payoff matrix and the possible movement of pedestrians is determined by the related payoffs in game theory. Game theory is also useful for pedestrians to choose their exits during evacuation. In order to get a better understanding of evacuation, some researchers not only performed simulations but also organized experiments on evacuation.^[22–28] Most of experiments are of rather small scale, such as evacuation from a classroom. These empirical results can be used to determine model parameters and testify validation of models in use.

The effect of pedestrians on evacuation has been recognized from the very beginning. The typical case is that one of exits is blocked due to overcrowding, some evacuees will try to find another farther but less congested exit. Therefore, the density of crowd at each exit can be served as a criterion for evacuees to choose their target exits. This feature has been considered in previous studies. For example, Yue *et al.*^[29,30] proposed the dynamic parameter (DP) model to simulate pedestrian multi-exit evacuation. The dynamic parameters include direction parameter, empty parameter, and cognition parameter which are formulated to instruct the exit selection of pedestrians. The effect of pedestrian density near exits on the evacuation process was taken into account. As mentioned above, the static floor field will be changed by obstacles. However, pedestrians are not treated as obstacles in most cases. In fact, one often faces that he/she is blocked by others in front. Therefore, pedestrians can also be treated as a kind of movable obstacles.^[31] Furthermore, it is evident that still pedestrians have more influence on their followers than moving ones. To the authors' knowledge, the effect of pedestrians with different speeds has been considered less in previous literatures.

In this paper, we propose an extended cellular automaton model based on our previous work.^[31] Because pedestrians are treated as moving obstacles with different speeds, then the static floor fields is no longer static as that in Ref. [5]. At the same time, the confliction factor is introduced to reflect the effect of conflicts among pedestrians who attempt to enter the same cell. The more the confliction factor is, the more difficult pedestrians enter the target cell. Numerical simulations are carried out to validate the presented model. Then we use this model to investigate the optimal design of inner structures and exits in a middle-size theater.

2. Model description

In the proposed model, the space is represented by two-dimensional square grid. The size of each cell is approximately 40 cm × 40 cm which is the typical area occupied by a person in a dense situation.^[5] Each cell can be either empty or occupied by exactly one person or by an obstacle. The desired moving direction of each pedestrian is determined by an improved floor field and his/her actual moving direction also depends on interaction with others or obstacles. It is assumed that pedestrians know exactly their own target exits in the building, then they will move towards their destinations directly without the help of others in front. For simplicity, the dynamic floor field is omitted.

Figure 1(a) shows four possible directions for a person at cell (i, j) , but he/she can keep still. Figure 1(b) shows the transition probabilities from cell (i, j) to its neighbouring cell (i', j') . Notice that, the person at cell (i, j) has the probability $P_{0,0}$ to stay at his/her original position.

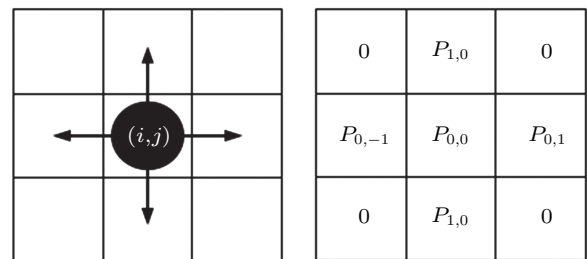


Fig. 1. (a) Possible moving directions for a pedestrians, (b) the corresponding transition probabilities.

2.1. Improved floor field

Usually, once the geometry of the room and the location of exits are determined, each cell is assigned a value of the static floor field which represents its distance to the nearest exit. Several effective algorithms have been suggested to give a feasible path in a room with obstacles.^[7,8] However, pedestrians can also be viewed as movable obstacles.^[31] In this case, the static floor field varies with the movement of pedestrians. Furthermore, moving and still pedestrians are distinguished in this paper, *i.e.*, still pedestrians result in larger increment of the floor field than moving ones. Based on this method suggested by Huang *et al.*,^[8] we take the effect of pedestrians with different speeds into account. The detailed algorithm for the improved floor field is described in the following steps.

Step 0 For each cell (i, j) at all exits, let $f_{i,j} = -2$ and $e_{i,j} = -2$ initially.

Step 1 For each cell (i, j) in a room (excluding the exit cells outside the wall), let $f_{i,j} = 0$ and $e_{i,j} = 0$ if the cell (i, j) is empty. If the cell (i, j) is occupied by an obstacle, $f_{i,j} = -1$ and $e_{i,j} = -1$. If the cell (i, j) is occupied by a moving pedestrian at the last time step, $f_{i,j} = 1$ and $e_{i,j} = 1$. If the cell (i, j) is occupied by a still pedestrian at the last time step, $f_{i,j} = 2$

and $e_{i,j} = 2$. Set $k = 3, l = 3, \alpha = 2$ for still pedestrians, $\beta = 1$ for moving pedestrians.

Step 2.1 Check neighbouring cell (i, j) in forward, backward, left, and right directions of the exit cells, if $f_{i',j'} = 0$, let $f_{i',j'} = k$; if $f_{i',j'} = 1$, let $f_{i',j'} = k + \alpha$; if $f_{i',j'} = 2$, let $f_{i',j'} = k + \beta$.

Step 2.2 For each cell (i, j) inside the room with $f_{i,j} = k$, check its neighbouring cells in forward, backward, left, and right directions, if $f_{i',j'} = 0$, let $f_{i',j'} = k + 1$; if $f_{i',j'} = 1$, let $f_{i',j'} = k + 1 + \alpha$; if $f_{i',j'} = 2$, let $f_{i',j'} = k + 1 + \beta$.

Step 2.3 If $f_{i,j} \neq 0, 1, 2$ holds for all cells inside the room, then go to Step 3.1, otherwise, $k = k + 1$ and go to Step 2.2.

Step 3.1 Check the neighbouring cells (i, j) in the forward, backward, left, right, and four diagonal directions of the exit cells, if $e_{i',j'} = 0$, let $e_{i',j'} = l$; if $e_{i',j'} = 1$, let $e_{i',j'} = l + \alpha$; if $e_{i',j'} = 2$, let $e_{i',j'} = l + \beta$.

Step 3.2 For each cell (i, j) inside the room with $e_{i,j} = k$, check all its neighbouring cells including the cells in the diagonal directions, if $e_{i',j'} = 0$, let $e_{i',j'} = l + 1$; if $e_{i',j'} = 1$, let $e_{i',j'} = l + 1 + \alpha$; if $e_{i',j'} = 2$, let $e_{i',j'} = l + 1 + \beta$.

Step 3.3 If $e_{i,j} \neq 0, 1, 2$ holds for all cells inside the room, then go to Step 4, otherwise, $l = l + 1$ and go to Step 3.2.

Step 4 For each cell (i, j) , calculate $S_{i,j} = \varepsilon f_{i,j} + (1 - \varepsilon)e_{i,j}$, where $0 \leq \varepsilon \leq 1$.

The transition probability of a pedestrian at cell (i, j) to his/her neighbouring cell (i', j') is determined by the following probability:

$$P_{i',j'} = N \exp(k_S(S_{i',j'} - S_{i,j})) \xi_{i',j'}, \quad (1)$$

where N is a normalization factor to ensure that $\sum_{(i',j')} P_{i',j'} = 1$. $S_{i,j}$ is the value of the floor field at cell (i, j) . $\xi_{i',j'}$ indicates whether the neighbouring cell (i', j') is occupied. $\xi_{i',j'} = 0$ if the cell is occupied by a pedestrian or an obstacle and 1 otherwise. According to Eq. (1), a pedestrian has stronger tendency to move along the gradient of the floor field there.

2.2. Confliction

Since the movement of pedestrians is updated in parallel, so we must handle the conflicts among pedestrians, *i.e.*, more than one pedestrians want to enter the same cell simultaneously. Suppose there are n persons aiming the same cell, and the confliction factor c is introduced which serves as a measure of panic. The larger the confliction factor is, the more panic pedestrians behave. It is generally believed that pedestrians with more competition will lead to less use of space. It is assumed that none can enter the target cell if $r < \min(n \times c, 1)$. Here r is a uniformly distributed random number and $0 \leq r \leq 1$, otherwise one of them will be chosen to enter the target cell according to the following probabilities. For example, $n = 2$. Let $Q = n \times c$. The transition probabilities of the two pedestrians are $P_{i',j'}^1$ and $P_{i',j'}^2$ respectively which are determined by Eq. (1). Then they enter the

same cell with the final probabilities $Q \times P_{i',j'}^1 / (P_{i',j'}^1 + P_{i',j'}^2)$ and $Q \times P_{i',j'}^2 / (P_{i',j'}^1 + P_{i',j'}^2)$, respectively.

2.3. Movement of pedestrians

At each time step, the improved floor field S for all exits are updated with the movement of pedestrians. Then the desired moving direction of each pedestrian is determined by the floor field. If no conflicts, one can enter the target cell. Once a conflict occurs, one of them involved can enter the target cell according to certain probabilities, meanwhile the others keep still. All pedestrians move towards their own exits. Notice that the target exit for a certain pedestrian may be changed due to the varying floor field. Once a pedestrian reaches the exit cell, he/she will be removed from the system immediately. A run of simulation ends when all pedestrians left the room.

3. Simulation and discussion

3.1. Model calibration

Numerical simulations are performed to calibrate the presented model before we investigate the crowd evacuation from a theater in use. Simulation results are compared with those in previous literatures, *e.g.*, Kirchner *et al.*^[9] They introduced a friction parameter μ in the FFCA model which can be interpreted as a kind of an internal local pressure between the pedestrians, especially in regions of high density. The parameter controls the probability that the movement of all pedestrians involved in a conflict is denied at one time step. It is obvious that here the confliction factor c is similar to the friction parameter, but their effects are not exactly the same. In this paper, the probability for a pedestrian to enter a cell depends on both the confliction factor and the number of pedestrians involved in a conflict (see Subsection 2.2). Simulation on the same case as in Kirchner *et al.*^[9] are carried out. Roughly, we can use the simple relation to estimate the panic of pedestrians, *i.e.*, $\mu = 3c$. Numerical results show good agreement with those in Kirchner in the case of small c , see Fig. 2. However, when c is large (*e.g.*, $c > 0.2$), the evacuation times T in this paper is less than those in Kirchner. This result can be explained as follows: when pedestrians try to leave from the small exit of one cell, 3-person conflict may reduce to 2-person conflict at the next time step, thus the probability for a pedestrian to enter the exit cell increases accordingly in this paper. Therefore, the evacuation time reduces due to considering the effect of pedestrians.

Figure 3 shows the effect of the asymmetric distribution of pedestrians on evacuation. As shown in Fig. 3(a), initially pedestrians are gathering at the upper-left corner of the room. Comparing Fig. 3(b) with Fig. 3(c), it is easy to find the qualitative difference between with and without considering the effect of pedestrians. According to the floor field incorporated

with the effect of pedestrians, the flow pattern seems more realistic. As a result, the utilization of the exit can be enhanced since pedestrians upstream tend to find paths with less congestion.

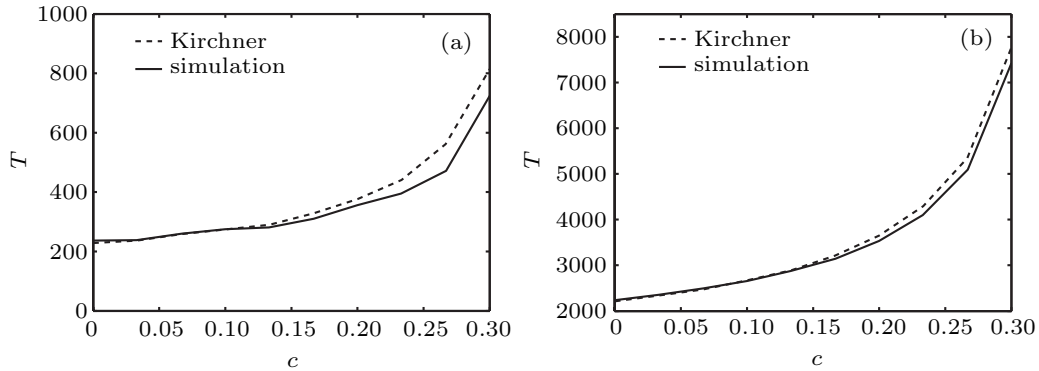


Fig. 2. Average evacuation times T against the conflation factor c for (a) $\rho = 0.03$ and (b) $\rho = 0.3$.

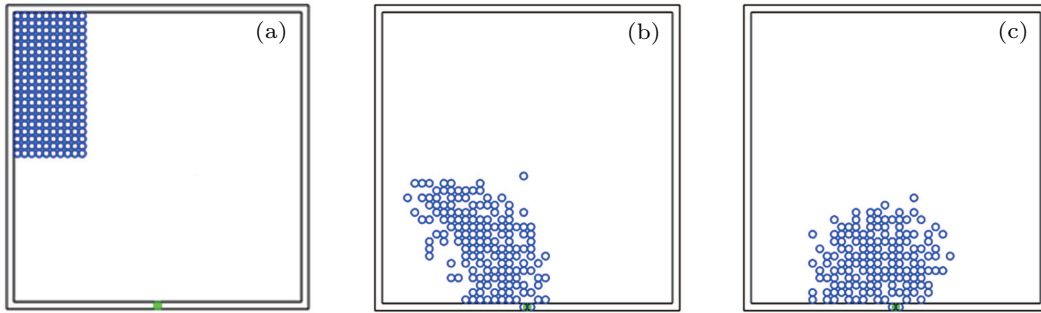


Fig. 3. (a) Initial distribution and (b) distribution at $t = 30$ s without considering the effect of pedestrians, (c) distribution at $t = 30$ s with considering the effect of pedestrians.

3.2. Effect of the conflation factor

We further investigated the effect of the conflation factor on evacuation from a room with a single exit. The size of the room is 42×41 cells where both the length and width of each cell are 0.4 m. There are no obstacles in the room. Initially pedestrians are randomly distributed. The densities of pedestrians are taken as 0.05, 0.1, 0.15, 0.2, 0.25, and 0.3. The exit width w is set as 1, 2, 3, and 4 cells. The conflation factors are set as 0, 0.1, 0.2, and 0.3 which correspond to ideal, normal, critical, and panic states respectively. The data points in Figs. 4 and 5 are computed according to the statistics of 100

simulation runs.

As shown in Fig. 4, the average evacuation time increases approximately in a linear way with the density. The smaller the exit width is, the faster the evacuation time increases with the density. In the case of panic (see Fig. 4(b)), the evacuation time is significantly larger than that in the case of normal state (see Fig. 4(a)) when the exit width is only one cells. However, there are no distinct differences between panic and normal states when the exit width is large enough. These results coincides with our daily experience. It indicates that the presented model can provide a realistic description of evacuation process.

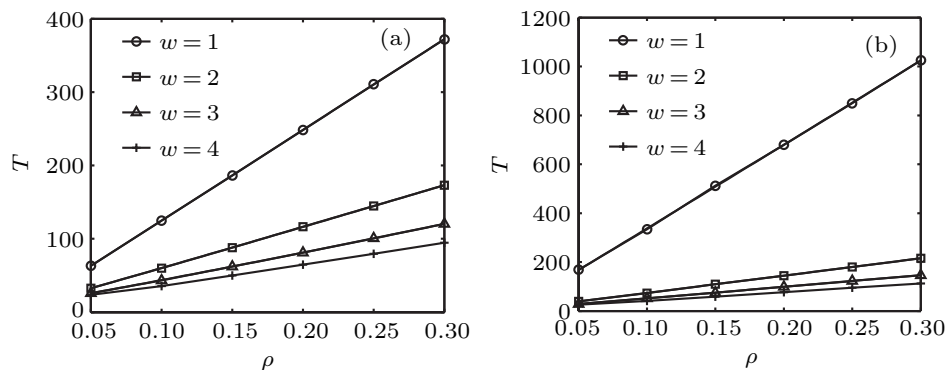


Fig. 4. Average evacuation times T against the density ρ under different exit widths: (a) $c = 0.1$, (b) $c = 0.3$.

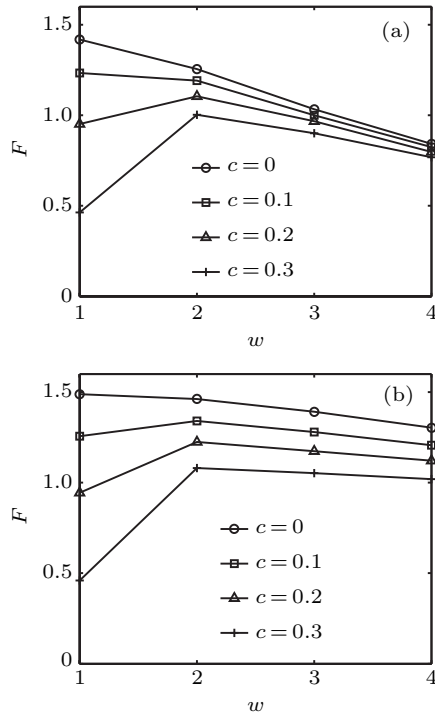


Fig. 5. Specific flow F against the exit width w under different conflict factors: (a) $\rho = 0.05$, (b) $\rho = 0.2$.

Then we studied the utilization rate of the exit. The specific flow F is defined as

$$F = \frac{N}{T \times w}, \quad (2)$$

where N is the number of pedestrians in the room. In Fig. 5(a), the number of pedestrians are not large enough to form seri-

ous congestion near the exit, most of pedestrians can leave the room fluently. Even in this case, the largest confliction factor reduces the utilization of exit significantly. When the density is equal to 0.2 or larger, pedestrians gather at the exit and congestion occurs. Thus the evacuation processes are similar when the density is large enough. Therefore, the specific flow does not change obviously in this density range. In the ideal state, the specific flow decreases monotonously with the exit width. In other cases, the specific flow usually increases first and then decreases gradually. The specific flow reaches its maximum when the exit width is about 2 cells (*i.e.*, 0.8 m). This is just the width of the door widely used in reality. It is shown that the presented model can give reasonable results compared to practical designs.

3.3. Case-study: inner layout of a theater

Evacuation from a room with inner facilities (*e.g.*, classroom) has been investigated by many researchers.^[10,23,24] In this paper, we investigated crowd evacuation from a middle-size theater which contains 900 seats. As shown in Fig. 6(a), it is the actual design of the theater in reality which is simplified slightly due to the discrete nature of CA models. The theater has four exits: two side exits at middle and two exits at bottom. There is one horizontal aisle at middle and two symmetrical vertical aisles. Each exit faces a certain aisle. Initially, it is assumed that the theater is full of people. The values in the following tables are obtained according to the statistics of 200 simulation runs.

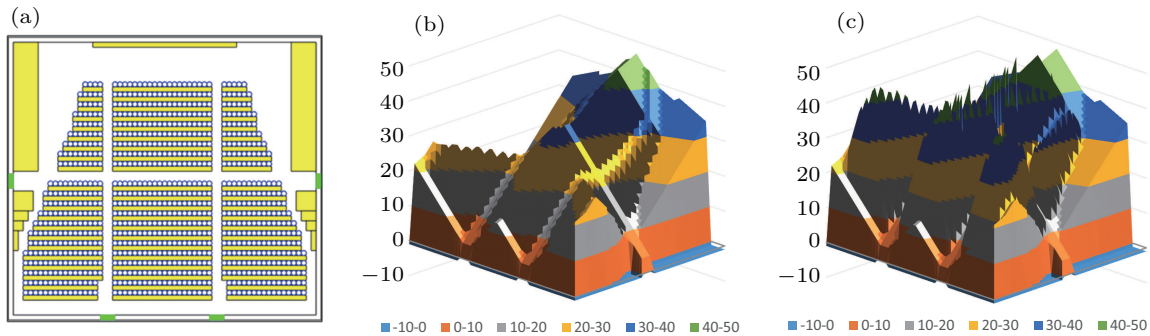


Fig. 6. (a) Sketch of a theater, (b) floor field without considering the effect of pedestrians, (c) floor field with considering the effect of pedestrians.

The floor field in Fig. 6(b) is generated by the method in Ref. [8] which only considers the inner structures and exits of the theater. When we consider the effect of pedestrians, the floor field changes considerably, see Fig. 6(c). It is shown there is a peak of floor field in the middle of each row. That means persons in that position usually takes longer time to leave the theater. It coincides with our daily experiences.

3.3.1. The arrangement of aisles

It is obvious that the arrangement of aisles plays a key role in crowd evacuation. For comparison, we also consider

the other three setups of aisles, see Fig. 7. The number of aisles increases from Case 1 to Case 3. The original design of the theater in Fig. 6(a) is named as Case 0.

As shown in Table 1, the average evacuation time increases with the confliction factor in each case. The evacuation time in Case 1 reaches the maximum since there is only one aisle. Only fewer persons choose the bottom exits. However, when the number of aisle is more than one, there are no significant difference between them. And it is shown that the original design is reasonable.

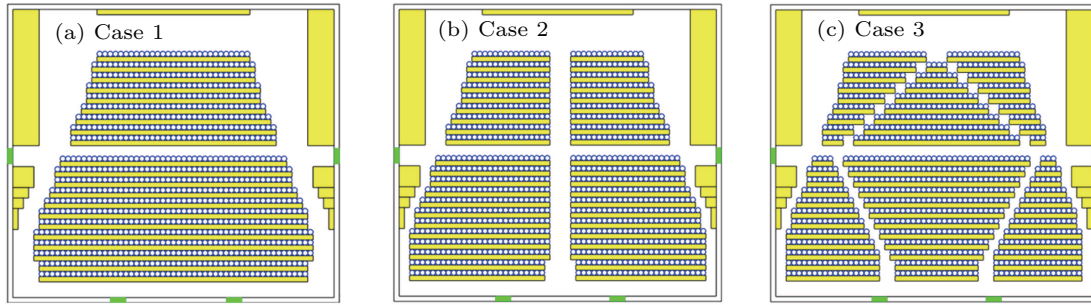
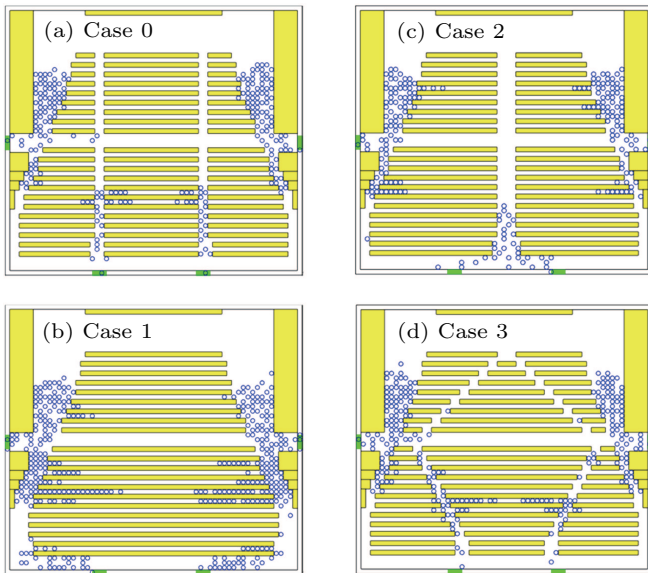


Fig. 7. Three different designs of the theater.

Table 1. Average evacuation times in all cases with typical conflict factors.

Case No	$c = 0$	$c = 0.1$	$c = 0.2$	$c = 0.3$
0	78.64 s	83.75 s	89.40 s	96.88 s
1	87.25 s	93.81 s	100.47 s	110.27 s
2	78.54 s	83.64 s	89.38 s	97.42 s
3	79.30 s	84.74 s	90.46 s	98.48 s

The snapshots of pedestrian distribution at $t = 60$ s are given in Fig. 8 which provide an intuitional description of evacuation process. In Case 1, considerable persons have not left the row and entered the aisle. In contrary, most of persons have entered the aisles in other cases. The average distance from seat to exit in Case 1 is a little longer than that in other cases. These differences lead to the longest evacuation time in Case 1.

Fig. 8. Distributions of pedestrian in each case at $t = 60$ s.

3.3.2. The optimal position of two side exits

As we have known, when the number of aisles is more than one and they are distributed properly, there are only negligible difference of the evacuation time between these cases. In order to reduce the evacuation time, it is natural to find the best position of exits. In the original design (Case 0), the horizontal aisle faces the side exits. Such a design is symmetric

and pretty. But the position of exits may not the best one from the viewpoint of crowd evacuation. According to the snapshots of pedestrian distribution at $t = 90$ s (not shown in Subsection 3.1), there are still someone waiting to leave from the side exits, however, all those who chose the bottom exits have left the room. It is indicated that the side exit should be moved upward to cover a bit fewer persons. In general, when persons leave from each exit simultaneously, the evacuation time reaches its minimum.

For simplicity, we adopt the arrangement of aisle in Case 0 and hold the position of the two bottom exits unchanged. Then we only shift the position of the two side exits, the position offset of the side exit is represented by d . And d is set to 1, 2, 3, 4, 5, 10, 15, and 20 cells, respectively. In Case 0, $d = 0$. Numerical results are shown in Table 2. It is found that $d = 4$ (*i.e.*, 1.6 m) is the best one of the eight choices. The evacuation time reduces considerably in comparison to Case 0. When $d > 4$, the evacuation time begins to increase although it is still less than that in Case 0. When $d = 20$, the evacuation time approaches or even exceeds that in Case 0.

Table 2. Average evacuation times with different offsets and typical conflict factors.

Offset	$c = 0$	$c = 0.1$	$c = 0.2$	$c = 0.3$
0	78.64 s	83.75 s	89.40 s	96.88 s
1	77.56 s	82.32 s	88.28 s	96.52 s
2	75.04 s	80.22 s	85.96 s	93.63 s
3	73.73 s	78.69 s	83.94 s	91.20 s
4	73.30 s	78.03 s	83.21 s	90.40 s
5	73.51 s	78.55 s	83.64 s	90.86 s
10	74.61 s	79.70 s	85.15 s	93.44 s
15	75.76 s	80.57 s	86.26 s	96.81 s
20	78.95 s	84.15 s	90.32 s	101.75 s

From the snapshots of evacuation process with different offset d in Fig. 9, it is found that the number of persons choosing the bottom exits increases with moving the side exits up. And the congestion in the lower part of the theater becomes serious. However, the number of persons leaving the theater from the side exits is still considerable more than that from the bottom ones. When $d = 4$, the times for the last person to leave from each exit are nearly the same. It is also indicated that $d = 4$ is the best choice.

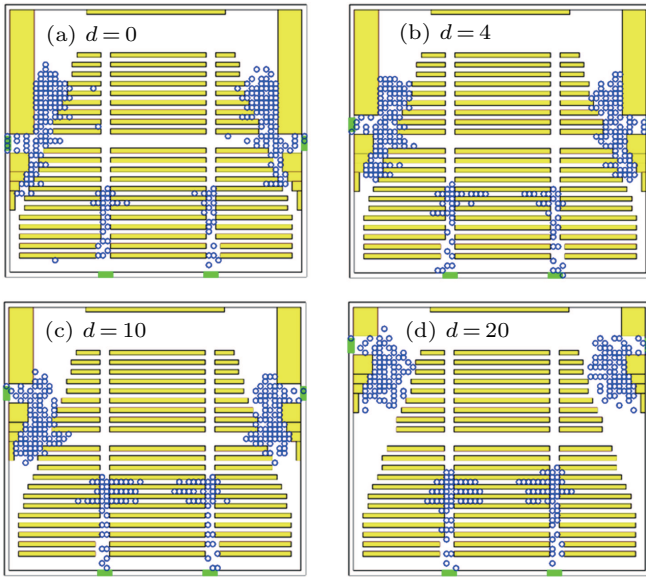


Fig. 9. Distributions of pedestrian with different offsets at $t = 50$ s.

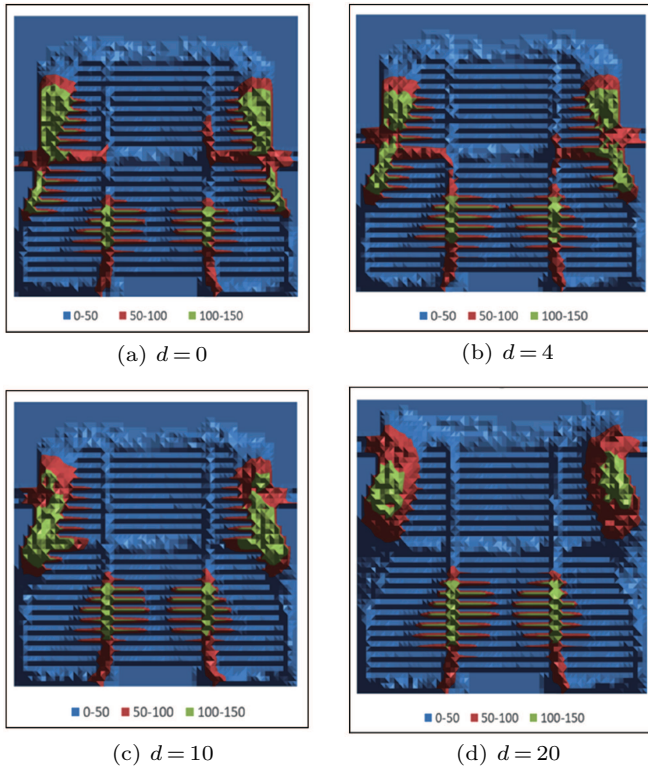


Fig. 10. Thermodynamic diagrams with different offsets d .

Figure 10 gives the thermodynamic diagrams which count the total time τ of each cell occupied by pedestrians during evacuation. At each time step, the value of τ in a cell will be added one if the cell is occupied. It is easy to find where the congested areas are in the theater. When $d > 4$, the congested areas near the side exits begin to shrink, and those in the middle of lower vertical aisles begin to enlarge. In the narrow aisles, uni-directional pedestrian flows are interrupted by the inflows from both sides. Therefore, the congestion in the vertical aisles near the bottom exits becomes serious. Finally,

it will take longer time for persons to leave from the bottom exits.

4. Conclusion and perspectives

In this paper, we proposed an extended FFCA model in which the effect of pedestrians is considered. Furthermore, still and moving pedestrians are distinguished. The effect of pedestrians on the floor field is helpful for pedestrians to find a feasible route with less congestion. Then we applied this model to investigate crowd evacuation from a middle-size theater. Our aim is to find the optimal design of the inner structures and exits. It is found that the actual design of the theater is reasonable but not the best one for crowd evacuation. Numerical results show that the side exits should be shifted upwards by four cells. However, it is only a preliminary study of simulation-based optimization on real buildings. It is believed that both numerical simulation and artificial intelligence algorithms may provide a satisfied solution.

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