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Dynamic manipulation of probe pulse and coherent generation of beating signals based on tunneling-induced inference in triangular quantum dot molecules*

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We investigate the dynamic propagation of a probe field via the tunneling-induced interference effect in a triple model of quantum dot molecules. By theoretical analysis and numerical simulation, we find that the number of transparency window relate to the energy splitting and the group velocity of probe field can be effectively controlled by the tunneling coupling intensity. In addition, in the process of light storage and retrieval, when the excited states have no energy splitting in the storage stage but opposite values of the energy splitting in the retrieval stage, the beating signals can be generated.

Keywords: coherent optical effects, electromagnetically induced transparency, triple quantum dots

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1. Introduction

During the past decade, the techniques of controlling light propagation have been paid much attention for its scientific merits. One effective method for achieving the velocity manipulation of light pulse is based on electromagnetically induced transparency (EIT), which can eliminate the absorption of a weak probe field at the resonant frequency via inducing atomic coherence by a strong coupling field.[1–3] Using the EIT technique, one can obtain the slowdown of light by changing the intensity of the coupling field.[4–6] Typically, people have termed "dark-state polaritons" (DSP) defined as form-stable coupled excitations of field and atom, which can explain the propagation of quantum field in EIT medium.[7,8] Subsequently, researchers have demonstrated experimentally that the storage and read-out processes are observed in cold sodium atoms[9] and the solid-state materials.[5,10] Recently, Wang *et al.* experimentally obtained the storage and retrieval of 2D Airy wavepackets in a doped solid driven by EIT.[11] In addition, it is worth noting that after storage procedure, the beating signals are generated based on the quantum interference effect between two weak probe fields,[12] a weak probe field and a control field.[13] Especially, Bao *et al.* obtained the beating signals by modulating the detunings of two coupling fields in the retrieval stage,[14] and by controlling a microwave field in the retrieval stage.[15]

A quantum dot (QD) is a semiconductor nanostructure that restrains the behavior of the conduction band electrons and valence band holes in the three-dimensional space.[16,17] The particle size of QD is generally between 1 nm and 10 nm. So, the electrons and holes in QD occupy the discrete energy level states due to the three-dimensional quantum confinement. By comparing with atomic system, QD has many merits, such as large electric-dipole moments, high nonlinear optical coefficients, controllable energy levels spacing, and ease of integration. While, quantum dot molecules (QDMs) can be formed by two or more closely spaced dots coupled via tunneling effect using the self-assembled dot growth method.[18] Double quantum dots (DQDs) have been deeply investigated in both theories and experiments, because the induced quantum interference[19–21] can be achieved by the tunneling effect of electrons between the dots with an external electric field.[22–24] Some investigators have exploited the DQDs to obtain EIT,[25,26] coherent population transfer,[27–29] optical bistability,[30–32] narrowing of fluorescence spectrum,[33] and the enhancement of Kerr nonlinearity.[34] Recently, triple quantum dots (TQDs) have also been paid much attention, because TQDs have been achieved in some experimental studies[35–38] and they have possessed multi-level structure and more tunable extra parameters. Subsequently, some theoretical works have utilized the TQDs to realize multiple transparency windows,[39,40] the enhanced Kerr nonlinearity,[41,42]

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and tunneling-assisted coherent population transfer and creation of coherent superposition states,[43,44] as well as controlling the Goos–Hänchen shift.[45]

In this paper, we investigate the pulse propagation dynamics in triangular quantum dot molecules which exhibit the electron tunneling coupling between three QDs in the presence of the externally applied voltages. We first deduced theoretically the expression of probe field susceptibility and further obtained the expression of probe group velocity. Then, by the numerical simulation, we found that the number of transparency window relates to the energy slitting and we utilized the dressed state representation to explain the result. Using the Bloch–Maxwell equations, it is found that the tunneling intensity of coupling field has important effects on the probe group velocity in the absence of the energy splitting. On this basis, a method to generate the beating signals (a series of maxima and minima in intensity) is designed. In detail, the probe field could be transformed into the spin coherence by turning off the tunneling coupling intensity without the energy splitting and then we retrieve it after a short storage time by turning on the tunneling coupling intensity with the opposite energy splitting. The retrieved probe field possesses two different optical components with time-dependent phase and leads to the beating signals occurrence owing to the alternate constructive and destructive interferences.

2. Atomic model and relevant equations

We consider the setup of the TQDs as composed of three QDs with different band structures and a triangular arrangement as shown in Fig. 1(a). The QDs are formed with a thin barrier of GaAs/AlGaAs, so the tunneling effect between the dots can be created and controlled by the gate voltage. Figure 1(b) shows the level configuration of the TQDs. In this system, the ground level $|1\rangle$ has no excitation, the direct exciton level $|4\rangle$ has an electron–hole pair in QD1, and the two indirect exciton levels $|2\rangle$ and $|3\rangle$ have the hole in QD1, as well as the electron in QD$_i$ ($i=2,3$), respectively. When the gate electrode is applying, the tunneling effect between the QDs can take place. The ground level $|1\rangle$ is coupled with the direct exciton level $|4\rangle$ by a weak probe field in the direction of $\hat{z}$. The weak probe field is a time- and space-dependent electric field and can be described by

$$\hat{E}_{\rm p}(z,t)=\hat{\varepsilon}_{\rm p}\sqrt{\frac{\hbar\omega_{\rm p}}{2\varepsilon_0 V}}f(z,t)\,{\rm e}^{-{\rm i}\omega_{\rm p}t+{\rm i}k_{\rm p}z}, \tag{1}$$

where $\hat{\varepsilon}_{\rm p}$, $\omega_{\rm p}$, $k_{\rm p}$, V, and $f(z,t)$ correspond to the polarization vector, the carrier frequency, the wave number, the quantization volume, and the dimensionless pulse envelope, respectively. Using the rotating-wave and electric-dipole approximation, with the assumption of $\hbar=1$, the interaction Hamiltonian can be obtained

$$\begin{aligned}H_{\rm I}=&(\delta_{\rm p}-\omega_{42})|2\rangle\langle 2|+(\delta_{\rm p}-\omega_{43})|3\rangle\langle 3|+\delta_{\rm p}|4\rangle\langle 4|\\&-(\Omega_{\rm p}|1\rangle\langle 4|+T_2|2\rangle\langle 4|+T_3|3\rangle\langle 4|+{\rm h.c.}),\end{aligned} \tag{2}$$

where $\Omega_{\rm p}=g_{\rm p}E_{\rm p}$ is Rabi frequency of the probe and $g_{\rm p}=\sqrt{\frac{\omega_{\rm p}}{2\hbar\varepsilon_0 V}}\hat{\varepsilon}_{\rm p}\cdot d_{14}$ is the coupling constant of the probe as well as $d_{14}=d_{41}^*$ is the dipole moment for the transition $|1\rangle\longleftrightarrow|4\rangle$. T_2 and T_3 denote the intensities of tunneling coupling, which relate to the barrier properties and the external electric field. The probe detuning is defined as $\delta_{\rm p}=\omega_{41}-\omega_{\rm p}$, where ω_{41} is the transition frequency from $|4\rangle$ to $|1\rangle$. In addition, ω_{42} and ω_{43} express the energy splitting of the excited states, which depend on the effective confinement potential controlled by the external electric field.

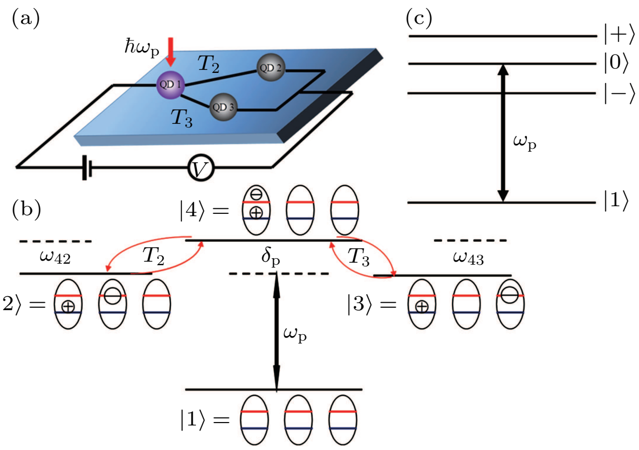

Fig. 1. (a) The diagram of a setup of the triangular TQDs. The probe field with central frequency $\omega_{\rm p}$ transmits the QD1. V denotes a bias voltage. (b) The scheme of the level configuration for the TQDs. (c) The dressed state representation for the two tunneling couplings T_2 and T_3.

To examine the dynamical evolution of the probe field, the following density matrix equations can be derived:

$$\begin{aligned}\dot\rho_{11}=&\Gamma_{41}\rho_{44}+\Gamma_{21}\rho_{22}+\Gamma_{31}\rho_{33}-{\rm i}\Omega_{\rm p}\rho_{14}+{\rm i}\Omega_{\rm p}^*\rho_{41},\\ \dot\rho_{22}=&-\Gamma_{21}\rho_{22}+{\rm i}T_2(\rho_{42}-\rho_{24}),\\ \dot\rho_{33}=&-\Gamma_{31}\rho_{33}+{\rm i}T_3(\rho_{43}-\rho_{34}),\\ \dot\rho_{21}=&-[{\rm i}(\delta_{\rm p}-\omega_{42})-\gamma_2]\rho_{21}+{\rm i}T_2\rho_{41}-{\rm i}\Omega_{\rm p}\rho_{24},\\ \dot\rho_{23}=&-[{\rm i}(\omega_{42}-\omega_{43})-(\gamma_2+\gamma_3)]\rho_{23}+{\rm i}T_2\rho_{43}-{\rm i}T_3\rho_{24},\\ \dot\rho_{31}=&-[{\rm i}(\delta_{\rm p}-\omega_{43})-\gamma_3]\rho_{31}+{\rm i}T_3\rho_{41}-{\rm i}\Omega_{\rm p}\rho_{34},\\ \dot\rho_{41}=&-({\rm i}\delta_{\rm p}-\gamma_4)\rho_{41}+{\rm i}T_2\rho_{21}\\&+{\rm i}T_3\rho_{31}-{\rm i}\Omega_{\rm p}(\rho_{44}-\rho_{11}),\\ \dot\rho_{42}=&-[{\rm i}\omega_{42}-(\gamma_2+\gamma_4)]\rho_{42}+{\rm i}\Omega_{\rm p}\rho_{12}\\&+{\rm i}T_3\rho_{32}+{\rm i}T_2(\rho_{22}-\rho_{44}),\\ \dot\rho_{43}=&-[{\rm i}\omega_{43}-(\gamma_3+\gamma_4)]\rho_{43}+{\rm i}\Omega_{\rm p}\rho_{13}\\&+{\rm i}T_2\rho_{23}+{\rm i}T_3(\rho_{33}-\rho_{44})\end{aligned} \tag{3}$$

constrained by $\rho_{11}+\rho_{22}+\rho_{33}+\rho_{44}=1$ and $\rho_{ij}=\rho_{ji}^*$, $\gamma_i=\frac{1}{2}\Gamma_{i1}+\gamma_{i1}^{\rm d}$ ($i=2,3,4$) denote the types of effective decay rate, Γ_{i1} is the radiative decay rate from level $|i\rangle$ to $|1\rangle$, and $\gamma_{i1}^{\rm d}$ is the pure dephasing decay rates.

In the limit of a weak field, the steady-state solutions of ρ_{14} can be obtained, furthermore the probe susceptibility can be derived as

$$\chi_{\mathrm{p}}=\frac{N|d_{14}|^2}{\hbar\varepsilon_0}\frac{1}{(\delta_{\mathrm{p}}-\mathrm{i}\gamma_4)-\frac{T_2^2}{(\delta_{\mathrm{p}}-\omega_{42}-\mathrm{i}\gamma_2)}-\frac{T_3^2}{(\delta_{\mathrm{p}}-\omega_{43}-\mathrm{i}\gamma_3)}},\tag{4}$$

where N is the quantum dot density. In order to investigate the dynamic evolution of a probe field in these TQDs, the wave propagation equation in the slowly varying envelope approximation can be given

$$c\frac{\partial E_{\mathrm{p}}(z,t)}{\partial z}+\frac{\partial E_{\mathrm{p}}(z,t)}{\partial t}=\frac{\mathrm{i}\hbar\kappa^2}{d_{14}}\rho_{14}(z,t)\tag{5}$$

with $\kappa^2=c\gamma_4\alpha$ and $\alpha=Nd_{14}^2\omega_{\mathrm{p}}/2\hbar\varepsilon_0c\gamma_4$ being the propagation constant and c being light velocity in vacuum. Then, in the local retarded frame for $\tau=t-z/c$ and $\xi=z$, equation (5) can be rewritten as

$$\frac{\partial E_{\mathrm{p}}(\xi,\tau)}{\partial\xi}=\frac{\mathrm{i}\hbar\kappa^2}{d_{14}}\rho_{14}(\xi,\tau).\tag{6}$$

3. Numerical calculation and discussion

We have investigated the steady optical response of TQDs and given the imaginary and real parts of the probe susceptibility χ_{p} as a function of the probe detuning δ_{p} as shown in Fig. 2. It is clear that the numbers of transparency windows relate to the energy splitting ω_{42} and ω_{43}. In the case of $\omega_{42}\neq\omega_{43}$, a pair of EIT windows appears between three absorption peaks and they are accompanying by steep normal dispersions at two different frequencies. However, for $\omega_{42}=\omega_{43}$ the only one EIT window arises in the probe absorption spectrum. The above phenomenon can be well explained in the dressed-state picture of T_2 and T_3. When $\omega_{42}=-\omega_{43}=\omega$ and $T_2=T_3=T$, the three dressed states can be written as

$$|+\rangle=\frac{\omega-\sqrt{2T^2+\omega^2}}{2\sqrt{2T^2+\omega^2}}|2\rangle+\frac{-\omega-\sqrt{2T^2+\omega^2}}{2\sqrt{2T^2+\omega^2}}|3\rangle+\frac{2T}{2\sqrt{2T^2+\omega^2}}|4\rangle,\tag{7}$$

$$|0\rangle=\frac{-T}{\sqrt{2T^2+\omega^2}}|2\rangle+\frac{T}{\sqrt{2T^2+\omega^2}}|3\rangle+\frac{\omega}{2\sqrt{2T^2+\omega^2}}|4\rangle,\tag{8}$$

$$|-\rangle=\frac{\omega+\sqrt{2T^2+\omega^2}}{2\sqrt{2T^2+\omega^2}}|2\rangle+\frac{-\omega+\sqrt{2T^2+\omega^2}}{2\sqrt{2T^2+\omega^2}}|3\rangle+\frac{2T}{2\sqrt{2T^2+\omega^2}}|4\rangle\tag{9}$$

with the eigenvalues of the dressed levels being $\lambda_+=\omega+\sqrt{2T^2+\omega^2}$, $\lambda_0=\omega$, $\lambda_-=\omega-\sqrt{2T^2+\omega^2}$. From Eqs. (7)–(9), the three dressed states are all comprised of the bare states $|2\rangle$, $|3\rangle$, and $|4\rangle$, and the three absorptive peaks are generated by the transitions from levels $|+\rangle$, $|0\rangle$, $|-\rangle$ to the level $|1\rangle$. Therefore, the quantum destructive interference among the three dipole-allowed transition leads to form two EIT windows.

While, when $\omega_{42}=\omega_{43}=\omega$ and $T_2=T_3=T$, the expressions of the dressed states are

$$|+\rangle=\frac{\omega-\sqrt{8T^2+\omega^2}}{2\sqrt{8T^2+\omega^2-\omega\sqrt{8T^2+\omega^2}}}|2\rangle+\frac{\omega-\sqrt{8T^2+\omega^2}}{2\sqrt{8T^2+\omega^2-\omega\sqrt{8T^2+\omega^2}}}|3\rangle+\frac{4T}{2\sqrt{8T^2+\omega^2-\omega\sqrt{8T^2+\omega^2}}}|4\rangle,\tag{10}$$

$$|0\rangle=-\frac{1}{\sqrt{2}}|2\rangle+\frac{1}{\sqrt{2}}|3\rangle,\tag{11}$$

$$|-\rangle=\frac{\omega+\sqrt{8T^2+\omega^2}}{2\sqrt{8T^2+\omega^2+\omega\sqrt{8T^2+\omega^2}}}|2\rangle+\frac{\omega+\sqrt{8T^2+\omega^2}}{2\sqrt{8T^2+\omega^2+\omega\sqrt{8T^2+\omega^2}}}|3\rangle+\frac{4T}{2\sqrt{8T^2+\omega^2+\omega\sqrt{8T^2+\omega^2}}}|4\rangle.\tag{12}$$

The corresponding eigenvalues of the dressed levels are $\lambda_+=\omega+\sqrt{8T^2+\omega^2}/2$, $\lambda_0=0$, $\lambda_-=\omega-\sqrt{8T^2+\omega^2}/2$. From Eq. (11) it is shown that the dressed state $|0\rangle$ is the coherent superposition state of the levels $|2\rangle$ and $|3\rangle$, and it is independent of the excitation level $|4\rangle$. According to the selection rule, the transitions $|2\rangle\to|1\rangle$ and $|3\rangle\to|1\rangle$ are dipole-forbidden, so the electric dipole moment of the dressed state $|0\rangle$ and level $|1\rangle$ is zero. Hence, the two absorptive peaks correspond to the dressed-state transition pathways $|+\rangle\to|1\rangle$ and $|-\rangle\to|1\rangle$, which lead to the only one transparency window appearing due to the quantum destructive interference in the two transition pathways (see Fig. 2(b)).

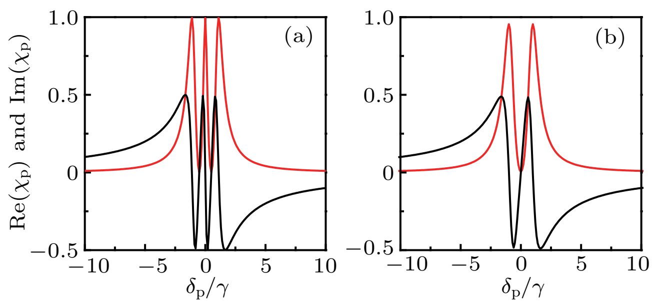

Fig. 2. The real part (black line) and imaginary part (red line) of probe susceptibilities for $\omega_{42}=-\omega_{43}=0.5\gamma$ in panel (a) and for $\omega_{42}=\omega_{43}=0$ in panel (b). Relevant parameters are $\gamma_4=\gamma=10$ μeV, $\gamma_2=\gamma_3=10^{-3}\gamma$, $T_2=T_3=0.7\gamma$.

In the transparency window region, there are steep varieties of the dispersion as shown in Fig. 2, which will make the light slow. The group velocity of the probe pulse can be defined as

$$\upsilon_{\mathrm{g}}=\frac{c}{\left[1+\frac{1}{2}\mathrm{Re}(\chi)+\frac{\omega_{\mathrm{p}}}{2}\frac{\partial\,\mathrm{Re}(\chi)}{\partial\delta_{\mathrm{p}}}\right]},\tag{13}$$

which is further derived as

$$\upsilon_g = \frac{T_2^2 + T_3^2}{\alpha \gamma_4} \tag{14}$$

in the case of $\delta_p = \omega_{42} = \omega_{43} = 0$, meanwhile the time delay for a medium of length ξ is given by

$$\tau = \frac{\xi}{\upsilon_g} = \alpha \xi \frac{\gamma_4}{T_2^2 + T_3^2}. \tag{15}$$

From Eq. (14), it can be found that υ_g depends on the tunneling coupling intensity. Figure 3 shows the magnitude square of the probe pulse at different penetration lengths as a function of time delay. The system parameters can be taken as $\gamma = 10$ μeV, $d_{14} = 10^{-16}$ esu·cm, $N = 10^{18}$ cm^{-3}, and $\omega_p = \omega_{41}$ at resonance, with these parameters we can obtain $\alpha = 6.7 \times 10^7$ cm^{-1}.[46–48] As we can see that the group velocity of probe pulse could significantly reduce by changing the tunneling coupling intensities. It can be found that the group velocity of probe field $\upsilon_g = 27.32$ m/s with the time delay $\Delta\tau = 16\gamma^{-1}$ for $T_2 = T_3 = 0.7\gamma$, and $\upsilon_g = 97.20$ m/s with the time delay $\Delta\tau = 4.4\gamma^{-1}$ for $T_2 = T_3 = 1.35\gamma$ if $\alpha\xi = 16$. As we know that the width of transparency window becomes wider and the dispersions slope gently when the tunneling coupling intensities become strong. In the following, we design a scheme to realized stored light and dynamically control the beating signals based on the tunneling-induced quantum interference in the three quantum dots. In order to explain the scheme, we used the definition of DSP (the dark state polariton) as a coherent mixture of the quantum field and spin coherence. According to Refs. [7,8],

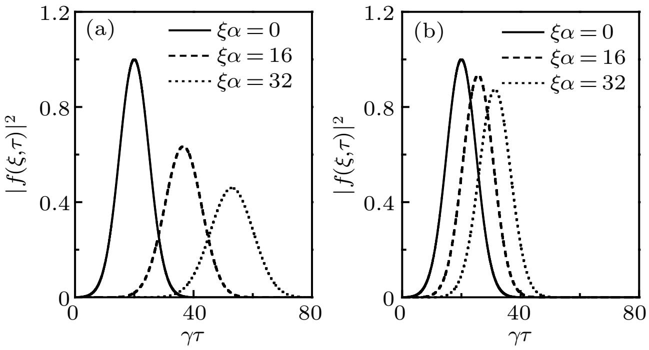

Fig. 3. Pulse dynamics of the probe field at different medium depths ξ as a function of time for $T_2 = T_3 = 0.7\gamma$ in panel (a), and for $T_2 = T_3 = 1.35\gamma$ in panel (b), with $\rho_{11}(0) = 1$, $\rho_{22}(0) = \rho_{33}(0) = \rho_{44}(0) = 0$, $\omega_{42} = \omega_{43} = 0$. Other parameters are the same as those in Fig. 2.

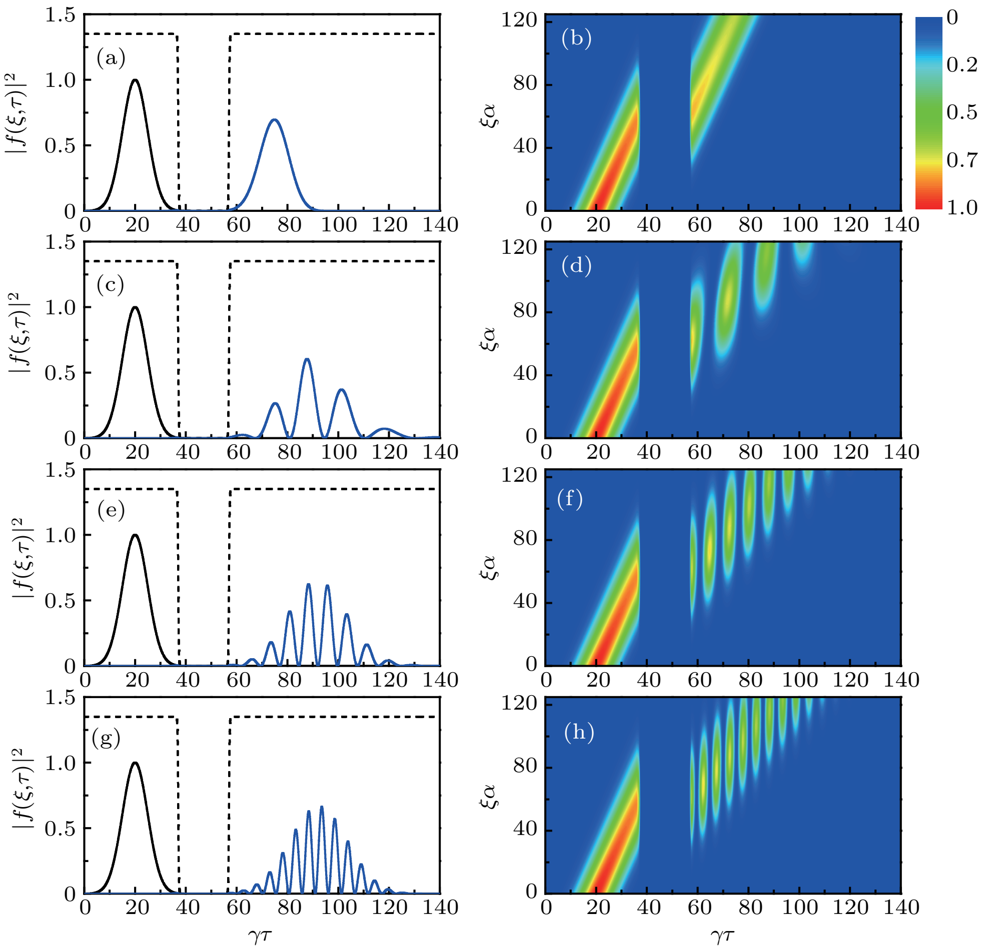

Fig. 4. Pulse dynamics of the probe field inside the TQDs [panels (a), (c), (e), (g)] and the probe pulse at the sample exit [panels (b), (d), (f), (h)] as a function of time for different energy splittings: (a) and (b) $\omega_{42} = -\omega_{43} = 0$; (c) and (d) $\omega_{42} = -\omega_{43} = 0.2\gamma$; (e) and (f) $\omega_{42} = -\omega_{43} = 0.4\gamma$; (g) and (h) $\omega_{42} = -\omega_{43} = 0.6\gamma$; $T_2 = T_3 = 1.35\gamma$. The black solid line is the probe field at the entrance of TQDs and the blue solid line is the probe field at the exit, and the dotted line is the time sequence of the tunneling coupling T_2 and T_3. Other parameters are the same as those in Fig. 2.

equations (3) and (6) can be transformed into

$$\left(\frac{\partial}{\partial t}+c\cos^2\theta\frac{\partial}{\partial z}\right)\Psi(z,t)=0, \tag{16}$$

which indicates a shape preserving propagation of the two-mode DSP

$$\Psi(z,t)=\Psi_a(z,t)+\Psi_b(z,t), \tag{17}$$

$$\Psi_a(z,t)=\beta\cos\theta E_{\mathrm{p}}(z,t)\,\mathrm{e}^{\mathrm{i}\omega_{42}t}-\kappa\sin\theta\cos\phi\rho_{12}, \tag{18}$$

$$\Psi_b(z,t)=\beta\cos\theta E_{\mathrm{p}}(z,t)\,\mathrm{e}^{\mathrm{i}\omega_{43}t}-\kappa\sin\theta\sin\phi\rho_{13} \tag{19}$$

with $\beta=d_{14}/\hbar$, $\tan\theta=\kappa/T'$, $\tan\phi=T_3/T_2$, $T'=\sqrt{T_2^2+T_3^2}$. In the first stage, we turn on the two tunneling couplings T_2 and T_3 to guide the probe field $E_{\mathrm{p}}(z,t)$ into the TQDs with slow speed without energy splitting. In the next stage, we turn off the two tunneling couplings T_2 and T_3 to convert the probe field $E_{\mathrm{p}}(z,t)$ into a pair of spin coherences $\cos\phi\rho_{12}$ and $\sin\phi\rho_{13}$, so the intensity of probe field gets zero. In the final stage, we turn on the two tunneling couplings T_2 and T_3 with the energy splitting $\omega_{42}\neq\omega_{43}$, so the two spin coherences become into slowly propagating DSP described by Eqs. (18) and (19). It is worth to note that the field component of $\Psi_a(z,t)$ and $\Psi_b(z,t)$ gains time-dependent phases $\omega_{42}t$ and $\omega_{43}t$, which lead to produce a series of beating signals due to the alternative constructive and destructive interferences. As shown in Fig. 4, a quantum field $E(z,t)$ evolves more slowly in the TQDs and is turned into spin coherence at the sample center when the tunneling effect is absent ($T_2=T_3=0$) at $t=37\gamma^{-1}$. During a short storage time with $\Delta t=20\gamma^{-1}$, we apply the tunneling couplings T_2 and T_3 to retrieve the quantum field under different energy splittings ω_{42} and ω_{43}, *i.e.*, $\omega_{42}=-\omega_{43}=0$ in panels (a) and (b), $\omega_{42}=-\omega_{43}=0.2\gamma$ in panels (c) and (d), $\omega_{42}=-\omega_{43}=0.4\gamma$ in panels (e) and (f), $\omega_{42}=-\omega_{43}=0.6\gamma$ in panels (g) and (h). We have noticed that the retrieved probe field slowly propagates with a series of maxima and minima (beating signals) in the TQDs when the opposite energy splitting is present.

4. Conclusion

We have theoretically investigated the dynamic propagation of a probe field in triple quantum dot molecules with the tunneling-induced interference effect. Our analytical and numerical results show that when the tunneling detunings are zero, a narrow transparency window with steep normal dispersion appears, which leads the probe field to propagating in the TQD at a reduced group velocity. In addition, we have obtained the dynamic generation of beating signals in an asymmetric procedure of light storage and retrieval. It is notable that the quantum probe field, incident on the TQD, is transformed into the spin coherence in the storage stage and formed the beating signals exhibiting a series of maxima and minima in intensity for opposite energy splittings during the retrieval stage.

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