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The Galactic Center Black Hole, Sgr A*, as a Probe of New Gravitational Physics with the Scalaron Fifth Force

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Abstract

The Galactic Center black hole (Sgr A*) provides an ideal laboratory for astronomical tests of new gravitational physics. This work reports that curvature correction ($f(R)$) to quantum vacuum fluctuations naturally yields a Yukawa-type scalar fifth force with potential $\exp(-M_\psi r)/r$, where M_ψ is the mass of the $f(R)$ scalarons. Estimating the UV and IR cutoff scales of vacuum fluctuations, the Yukawa coupling strength is connected to the scalaron field amplitude. Whereas recently constrained Yukawa coupling and range correspond to light scalarons with $M_\psi = (1.37 \times 10^{-21}$–$5.49 \times 10^{-20})$ eV, vacuum fluctuations yield a massive scalaron with $M_\psi = 10^{-16}$ eV. Scalaron-induced periastron shift of stellar orbits near Sgr A* has been studied with respect to the semimajor axis in the range $a = 10$–1000 au. It is found that the scalarons resulting from quantum fluctuations affect the precession of orbits with $a = 128$–$256\ R_s$. The possibility of future constraints on massive scalarons in observations near Sgr A* is discussed. This is a new and independent effort to express a prototype quantum gravity effect in terms of astronomically accessible quantities.

Unified Astronomy Thesaurus concepts: Galactic center (565); Gravitation (661)

1. Introduction

General relativity (GR) is the most successful description of gravity and has passed several independent experimental and observational tests (Will 2001, 2014; Turyshev 2008). It now provides the foundation for cosmological problems. Astronomical tests of the theory particularly involve solar system tests (Will 2014), binary pulsars (Taylor & Weisberg 1982), gravitational-wave binaries (Abbott et al. 2016a, 2016b, 2017), the Galactic Center (see the recent test of gravitational redshift near the black hole at the Galactic Center in Abuter et al. 2018), galaxy clusters (Wojtak et al. 2011), a test of the strong equivalence principle in a triple compact star system (Archibald et al. 2018), and a test involving a strong lensing extragalactic system (Collett et al. 2018). However, in a cosmological setting the application of GR has given rise to the problems of dark matter and dark energy (Zakharov 2018a). We do not have a direct test of the theory in cosmological scales independently of the benchmark ΛCDM model (Wojtak et al. 2011). Also, the theory has seldom been tested in a strong gravitational field (large spacetime curvature) regime (Johannsen 2016). Only very recently have tests of the theory near the Galactic Center black hole appeared (Abuter et al. 2018).

The Galactic Center, harboring Sgr A*, a supermassive black hole of mass $M \approx 4 \times 10^6 M_\odot$, presents an ideal laboratory to test GR through the S-stars belonging to the nuclear star cluster. There have been extensive works on tests of properties of spacetime near Sgr A* using the orbits of those stars encircling the black hole (Fragile & Mathews 2000; Will 2008; Angelil & Saha 2010, 2011; Merritt et al. 2010; Iorio 2011; Borka et al. 2012; Capozziello et al. 2014; Zakharov et al. 2014; Johannsen 2016). Conventional tests of GR involve constraining the spin of the black hole through spin-induced effects (Will 2008; Yu et al. 2016) on these stars and thereby testing the Kerr–Newman geometry—the unique black hole solution of GR. However, a remarkable departure from GR is expected in a quantum theory of gravity near black holes, and this has not hitherto been sufficiently tested. Stellar orbits near the Galactic Center are being continuously monitored (Meyer et al. 2012; Gillessen et al. 2017) and theoretical calculations involving compact orbits (orbital radius of 50–100 au) have been largely performed (Will 2008; Psaltis et al. 2016; Yu et al. 2016; Kalita 2018) to see if there is any detectable deviation from GR (see, for example, Kalita 2018). Several computer simulations for motion of the S-stars near the black hole have also been performed for tests of GR through the upcoming Extremely Large Telescope (ELT) (Do et al. 2017; Grould et al. 2017; Waisberg et al. 2018).

Tests of the full theory of gravity are yet to appear. They require extremely strong gravitational fields (large spacetime curvature and hence compact scales) as well as an extremely large scale (weak fields of cosmological domains). Extremely strong field regimes call for a departure from GR due to quantum gravity corrections (Uzan 2010). On the other hand, weak field tests in cosmology involve assumptions about the nature of dark energy and dark matter, whose fundamental physics is not yet clear (Zakharov 2018a). For example, the recent extragalactic test of GR in the strong lensing system ESO 325-G004 involves specific assumptions about the distribution of dark matter in the lens (Collett et al. 2018).

Although we do not have a fully testable theory of quantum gravity, its phenomenological forms are often found to be helpful to address deep questions in cosmology. For example, the Dvali, Gabadadze, and Porrati (DGP) braneworld scenario (Dvali et al. 2000) and Randall–Sundrum five-dimensional gravity (Randall & Sundrum 1999) take into account accelerated cosmic expansion in the late and early phase of the universe, respectively. Although deduction of the de Sitter (empty accelerating model) phase as a solution in string theory is still an open question (Danielson & Riet 2018) its appearance in a noncommutative version of quantum gravity has been shown as a possibility (Berglund et al. 2019).

There are opportunities to address the dark universe problems both from conservative (looking for violation of the cosmological principle but with no new physics) and orthodox (looking for new physics but respecting the cosmological principle) approaches (Uzan 2010). In one of the orthodox scenarios, gravity undergoes modification which explains the universe without invoking the dark components. Modified gravity theories broadly include two classes: (i) scalar–tensor gravity, where in addition to the spacetime metric, $g_{\mu\nu}$, a universal scalar field φ couples to matter; and (ii) $f(R)$ gravity where the Einstein–Hilbert action $\int R\sqrt{-g}\,d^4x$ gets modified as $\int f(R)\sqrt{-g}\,d^4x$ with $f(R)$ being an analytic function of the Ricci scalar, R. Such alternatives are well motivated by quantum gravity (Uzan 2010) and are not yet fully ruled out. In addition there are large and detailed varieties of modified gravity theories, such as bimetric theories (Rosen 1940; van Dam & Veltman 1970; Zakharov 1970), tensor–vector–scalar theories (Bekenstein 2004), higher-order theories (Brown 1995), Horava–Lifshitz gravity (Horava 2009a, 2009b), Galileon gravity (Nicolis et al. 2009), and extra-dimensional theories (Randall & Sundrum 1999; Dvali et al. 2000, braneworld model; see Clifton et al. 2012 for a review).

One of the phenomena of quantum gravity is the existence of a hypothetical fifth force which manifests as a Yukawa correction $(\exp(-r/\lambda)/r)$ (λ being a scale length) to a Newtonian potential. Recently, properties of such a fifth force have been constrained through 19 yr of orbital data of S-stars with compact orbits near Sgr A* (Hees et al. 2017). The Yukawa limit of $f(R)$ theories has been constrained by Very Large Telescope (VLT) and Keck data for the orbit of S0-2/S-2 (Borka et al. 2013). It is to be noted that there are two nomenclatures for the S-stars: one is from the UCLA Galactic Center group which uses the Keck Telescope, and the other is from the Max Planck Institute for Extraterrestrial Physics (MPE), group which uses the VLT/New Technology Telescope (NTT). In UCLA nomenclature, the S-stars are named as S0-2, S0-38, S0-102, etc. However, in the MPE case they are named as S-2, S-38, S-55, respectively. Recently, Zakharov (2018a) has also reported updates on the constraints of Yukawa interaction near Sgr A* through the kinematics of the S-stars.

In a recent study, this author reported on the testability of gravitational theories near the Galactic Center (Kalita 2018). By considering light deflection near Sgr A* and the periastron shift of S-stars with orbital radii $r = 50$–100 au, it was tested whether the astrometric capability of the upcoming ELT can distinguish scalar–tensor theories and $f(R)$ gravity from GR. It was found that deviation from GR, even for such compact orbits, lies within a few tens of microarcseconds which is only marginally close to the astrometric capabilities of the ELT (10–30 μas) and is overwhelmed by other effects such as stellar perturbations on a periastron shift which goes up to 100 μas yr^{-1} (Merritt et al. 2010; Sadeghian & Will 2011). However, in the case of $f(R)$ gravity it was found that if the scalar modes of gravity that exist in these theories (known as "scalarons") are light enough ($M_\psi \ll 10^{-19}$ eV), it can produce appreciable deviation on light deflection from that predicted by GR.

The "scalarons" manifest as a fifth force with Yukawa correction, $\exp(-M_\psi r)/r$, to Newtonian potential and hence they are synonymous with the quantum gravity phenomenon that has been constrained by Hees et al. (2017). Kalita (2018) interpreted the "scalaron" mode as a "screened black hole hair" and proposed that it can provide a new test of the principle of equivalence.

In the present work, an attempt has been made to demonstrate how the phenomenology of the union of gravity and quantum processes near the black hole naturally gives rise to $f(R)$ theories (particularly theories of type $f(R) \propto R^n$, $n > 1$) and the associated scalar fifth force. The work reports that the curvature corrections to quantum vacuum fluctuation manifests as the scalaron fifth force. The effect of these quantum gravitationally-induced scalarons has been expressed in terms of astronomically measurable quantities of the orbits of stars encircling Sgr A*.

The paper is organized as follows. Section 2 briefly presents the origin of the scalar fifth force in $f(R)$ theories. Section 3 connects the scalaron field $\psi = f'(R)$ with the momenta (ultraviolet and infrared cutoff, $k_{\rm UV}$ and $k_{\rm IR}$) of gravitationally-corrected vacuum fluctuations. It also shows the relation between scalaron mass with the fluctuation momenta. Section 4 discusses the prospects of the scalaron fifth force near Sgr A* by calculating the ratio of the periastron shift of compact orbit stars in the presence of the fifth force to that in GR for different orbital sizes, different ranges, and coupling strengths of the fifth force. Section 5 briefly mentions the possibility of future constraints on the scalar hair represented by the scalarons. Section 6 presents results and discussions. Section 7 concludes.

2. Scalar Fifth Force in $f(R)$ Theories

$f(R)$ theories are geometrical extensions of GR where the usual gravitational action is modified as (in the unit $c = 1$),

$$S = \int \sqrt{-g}\,d^4x\,(16\pi G)^{-1} f(R)$$
$$+(\text{Action due to matter universally coupled to the metric, } g_{\mu\nu}). \tag{1}$$

Starobinsky (1980) used quadratic gravity, $f(R) \propto R^2$ to generate a singularity-free isotropic cosmological model. Thereafter, several authors used $f(R)$ theories to account for the observed cosmic acceleration (Capozziello 2002; Capozziello et al. 2003; Carroll et al. 2004; Hu & Sawicki 2007; Starobinsky 2007) and also for a unification of the early, inflationary expansion and late-time acceleration (Nojiri & Odintsov 2011, 2013). Grib et al. (1994) emphasized higher-order correction to GR action near spacetime singularities. This author advocated for the existence of both positive and negative powers of the Ricci scalar, R, in the gravitational action through a cosmological duality conjecture (Kalita 2016).

This work is based on the idea of the origin of the scalar fifth force in $f(R)$ gravity (particularly of type R^n, $n > 0$) due to gravitational correction to quantum fluctuations. Whereas quantum fluctuations near black holes give rise to the interesting phenomenon of Hawking radiation (Hawking 1974), the addition of gravitational corrections to quantum fluctuations in vacuum meaningfully modify the macroscopic description of gravity (Ruzmaikina & Ruzmaikin 1969; Nojiri & Odintsov 2003).

In order to see the gravitational correction to quantum fluctuation, let us consider the higher-order curvature correction to the Lagrangian of vacuum fluctuation (Ruzmaikina & Ruzmaikin 1969) in de Sitter background with $R_0 = \Lambda \neq 0$,

$$\begin{aligned} L(R) = a\hbar \int k^3 dk \\ + b\hbar\,(R - R_0) \int k\,dk \\ + c\hbar\,(R - R_0)^2 \int k^{-1} dk + \ldots\ldots \end{aligned} \tag{2a}$$

This correction can be achieved by expanding the Lagrangian $L(R)$ of vacuum fluctuation in a Taylor series about the de Sitter vacuum $R = R_0$ and then relating the derivatives $(d^nL/dR^n)_{R=R_0}$ to the wavenumbers of fluctuations, k. The first term is the bare vacuum energy (cosmological constant if it is really the energy of the vacuum!). All other terms are gravitational corrections to the quantum fluctuation expressed in terms of the Ricci scalar R and the numerical constants a, b, c, etc.

Assuming a small cosmological constant for spacetime near a local mass point (such as the black hole; see Section 3 for details) the Lagrangian can be rewritten as

$$L(R) = a\hbar\int k^3 dk + b\hbar R\int kdk + c\hbar R^2\int k^{-1}dk + ... \tag{2b}$$

The Lagrangian in Equation 2(b) was first proposed by Sakharov (1967) and then simplified by Ruzmaikina & Ruzmaikin (1969) to discuss correction in vacuum quantum fluctuations arising out of gravity. The numerical coefficients a, b, c, etc. do not affect the physical considerations as they are of the order of magnitudes of unity (Misner et al. 1973; Sakharov 2000).

For example, the first term, in proper units, is $a(\hbar c)\int k^3 dk$ which is nothing but the bare vacuum energy density ($\hbar c k_0^4$) with k_0 being some ultraviolet cutoff wavenumber (quantum gravity cutoff, $k_0 \approx \lambda_{\rm Pl}^{-1} \approx 10^{33}\,{\rm cm}^{-1}$). Thus $a \approx 1$ (except the factor of 4π which appears in the formal expression of phase-space volume $4\pi k^2 dk$ which is multiplied by $\hbar ck$ to get the energy density). The second term looks like Einstein–Hilbert action $(c^3/16\pi G)\int d^4x\sqrt{-g}R$ with a Lagrangian, $L = (c^3/16\pi G)R = (b\hbar\int kdk)R$. Therefore, $G = c^3/(8\pi b\hbar k_0^2)$. As $k_0 \approx \lambda_{\rm Pl}^{-1} = (c^3/G\hbar)^{1/2}$, $b \approx 1$ (except the factor involving π). It is to be noted that the ultraviolet cutoff k_0 chosen here is only to set the numerical factors, a, b, etc. In Section 3 we will see that the scalaron fifth force results after choosing the unabsorbed scale of vacuum fluctuation momenta, $k_{\rm UV} \propto 1/\lambda_{\rm UV}$, where $\lambda_{\rm UV} \equiv R_S = 2GM/c^2$, the gravitational radius of the black hole. Fluctuations with $\lambda \to \lambda_{\rm Pl}$ ($\ll R_s$) are of no relevance for the description of movement of matter exterior to R_s.

Although higher-order terms of the curvature become important in black hole spacetime, here it is restricted to the R^2 term only to see the minimal deviation from GR. The classical gravitational action then becomes

$$S = (16\pi G)^{-1}\int d^4x\sqrt{-g}f(R), \tag{3}$$

where $f(R) = L(R) = \sum_{n\geqslant 0} C_n(k)R^n$. Except the prefactor $C_n(k)$, this is similar to the Lagrangian in $f(R)$ theories. In proper units this action contains all the three fundamental constants $\hbar$, c, G together, and therefore is representative of a quantum theory of gravity. Following the metric formalism where the action given in (3) is varied with respect to the metric only and then made stationary, the Euler–Lagrange equation in vacuum can be obtained as

$$\psi R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu\psi + g_{\mu\nu}\nabla^\alpha\nabla_\alpha\psi = 0. \tag{4}$$

These theories are equivalent to scalar–tensor theories (Wands 1994) with ψ being identified as the scalar field

$$\psi = f'(R). \tag{5}$$

It is seen that the scalar field is governed by detailed features of the quantum fluctuation spectrum encapsulated by the functions $C_n(k)$.

Considerations of linear perturbations of the scalar field $\psi = \psi_0(1 + \delta_\psi)$ (where $\psi_0 = f'(R_0)$, R_0 being the background curvature) show that the field propagates as a massive Klein–Gordon mode (see Kalita 2018 for details of such calculations)

$$(\nabla^\alpha\nabla_\alpha - M_\psi^2)\delta_\psi = 0. \tag{6}$$

This is known as the “scalaron” degree of freedom.

Here

$$M_\psi = \sqrt{(1/3)\left(\frac{\psi_0}{\psi_0'} - R_0\right)} \tag{7}$$

is the mass of the “scalaron.” One can adopt a spatially regular, Yukawa-type solution for perturbation amplitude, δ_ψ in spherically symmetric and static perturbations as (Navaro & Acoleyen 2007; Hod 2012, 2017; Herdeiro & Radu 2014),

$$\delta_\psi = \frac{2GM}{3\psi_0 r}\exp(-M_\psi r). \tag{8}$$

Here M represents the mass of the black hole around which the fluctuations are considered.

Recently, Hees et al. (2017) constrained the range (λ) and strength (α) of a Yukawa-type fifth force through 19 yr of orbital data (by using the Keck Telescope) of two short period stars, S0-2/S-2 and S0-38/S-38 around the Galactic Center. The scale of the fifth force (range) with potential $V(r) \sim \alpha\exp(-r/\lambda)$ has been restricted as $\lambda = (150\text{–}1000)$ au depending upon the strength. If scalarons represent the fifth force on such scales, their mass for this scale is about $M_\psi \approx 10^{-21}\text{–}10^{-20}$ eV (see Section 3).

From the first order metric fluctuation, $\tilde{g}_{\mu\nu} = \psi_0(\eta_{\mu\nu}^{\rm background} + h_{\mu\nu})$ (where $\eta_{\mu\nu}^{\rm background}$ is the background metric), one can express the parameterized post-Newtonian (PPN) approximation to the gravitational field near the black hole as

$$g_{\mu\nu} = \frac{\tilde{g}_{\mu\nu}}{\psi} = \frac{\eta_{\mu\nu}^{\rm background} + h_{\mu\nu}}{1 + \delta_\psi}. \tag{9}$$

A suitable expansion is considered around the de Sitter background with $\eta_{\mu\nu}^{\rm background} = \eta_{\mu\nu}^d$ so that

$$g_{\mu\nu} \approx (\eta_{\mu\nu}^d + h_{\mu\nu})(1 - \delta_\psi), \tag{10}$$

where $\eta_{00}^d = -\left(1 - \frac{\Lambda r^2}{3}\right)$, $h_{00} = \frac{2GM}{\psi_0 r}$, and $h_{ij} = \frac{2GM}{\psi_0 r}\delta_{ij}$.

Therefore, the exterior metric to the first order in perturbation (ignoring the quantity, $h\delta_\psi$) is

$$ds^2 = \left(-1 + \frac{2G_{\rm eff}M}{r} + \frac{\Lambda r^2}{3}\right)dt^2 + \left(1 + \frac{2G_{\rm eff}M}{r}\gamma + \frac{\Lambda r^2}{3}\right)d\boldsymbol{X}^2, \tag{11}$$

with γ being the Eddington–Robertson parameter (PPN)

$$\gamma = \frac{1 - \frac{1}{3}\exp(-M_\psi r) - \frac{\Lambda r^2}{9}\exp(-M_\psi r)}{1 + \frac{1}{3}\exp(-M_\psi r) - \frac{\Lambda r^2}{9}\exp(-M_\psi r)}, \tag{12}$$

$G_{\rm eff}$ is the modified Newton's "constant" defined by

$$G_{\rm eff} = \frac{G}{\psi_0}\left(1 + \frac{1}{3}\exp(-M_\psi r) - \frac{\Lambda r^2}{9}\exp(-M_\psi r)\right) \tag{13}$$

and Λ is the cosmological constant. This generates a Yukawa-type modification to the Newtonian gravitational potential with a de Sitter correction,

$$V(r) = -\frac{GM}{\psi_0 r}\left(1 + \exp(-M_\psi r)\left(\frac{3 - \Lambda r^2}{9}\right)\right) \tag{14}$$

which resembles the fifth force of Nature. The appearance of the three fundamental constants of Nature (G, c, h) along with the cosmological constant in Equations (11)–(14) clearly is a symptom of quantum gravity processes.

The fifth force, therefore, appears as a quantum gravity correction in the metric which spoils its smooth Schwarzschild structure. The term $(G/9\psi_0)(3 - \Lambda r^2)\exp(-M_\psi r)$ in Equation (14) and hence in the metric (11) represents quantum gravity fluctuation of the metric. Therefore, subjecting curvature modification of GR to an observational test is not only equivalent to constraining the fifth force but also to putting quantum gravity effects into tests.

3. Connection between the Field ψ_0 and Quantum Fluctuation Momenta (k)

In this work, the effect of the scalar field ψ_0 on the periastron shift of stellar orbits near the Galactic Center black hole is explored. Orbital dynamics is an important probe of new fields (in this case the scalaron, ψ_0; Dicke 1965). As the scalar field is related to the momenta of quantum vacuum fluctuations, the goal is to express such microscopic effects in terms of astronomically measurable quantities related to orbits of the stars near the black hole. The dependence of the macroscopic field, ψ_0, on quantum fluctuations (k) is shown below.

The scalar field, ψ, to linear order in R is expressed as

$$\psi = \hbar c\int_{k_{\rm IR}}^{k_{\rm UV}} kdk + 2\hbar c R\int_{k_{\rm IR}}^{k_{\rm UV}} \frac{dk}{k} + O(R^2) \tag{15}$$

where $k_{\rm UV}$ and $k_{\rm IR}$ are the UV and infrared cutoff of the curvature fluctuations (see below). Therefore,

$$\psi_0 \approx \frac{\hbar c}{2}(k_{\rm UV}^2 - k_{\rm IR}^2) + 2\hbar c R_0 \ln\left(\frac{k_{\rm UV}}{k_{\rm IR}}\right). \tag{16}$$

This gives

$$\psi_0' = 2\hbar c \ln\left(\frac{k_{\rm UV}}{k_{\rm IR}}\right). \tag{17}$$

The mass of the scalaron (see (7)) then takes the form (using Equations (16) and (17))

$$M_\psi = \sqrt{(1/3)\left(\frac{\psi_0}{\psi_0'} - R_0\right)} = \sqrt{\frac{1}{3}\left(\frac{1}{4}\frac{k_{\rm UV}^2 - k_{\rm IR}^2}{\ln\left(\frac{k_{\rm UV}}{k_{\rm IR}}\right)} + R_0 - R_0\right)} = \sqrt{\frac{k_{\rm UV}^2 - k_{\rm IR}^2}{12\ln\left(\frac{k_{\rm UV}}{k_{\rm IR}}\right)}}. \tag{18}$$

It clearly shows how quantum fluctuations ($k_{\rm UV}$, $k_{\rm IR}$) enter the description. They represent two length scales of the theory. The UV cutoff is chosen as the unabsorbed mode ($\lambda_{\rm UV} = R_S = 2GM/c^2$),

$$k_{\rm UV} = 2\pi/R_S = \pi c^2/GM \approx 80.28 \text{ au}^{-1}, \tag{19}$$

where $R_S = 2\,GM/c^2$ is the Schwarzschild radius of the black hole. The infrared cutoff $k_{\rm IR}$ has a natural interpretation. In cosmology, infrared length scales of vacuum fluctuations are calculated from the idea of horizons and associated thermodynamics (Gibbons & Hawking 1977; Padmanabhan 2004). Such scales correspond to irreducible vacuum noise related to the minimum temperature of the fluctuations (Padmanabhan 2010) and can be computed as follows.

If $\lambda_{\rm IR}$ is the infrared length scale, the thermal energy density of vacuum fluctuations is given by

$$\rho_{\rm thermal} = a_B T^4 = hc/\lambda_{\rm IR}^4, \tag{20}$$

where a_B is the thermodynamic radiation constant and T is the temperature. Therefore, the infrared cutoff is expressed as

$$k_{\rm IR} = \frac{2\pi}{\lambda_{\rm IR}} = 2\pi\left(\frac{\rho_{\rm IR}}{hc}\right)^{1/4} = 2\pi\left(\frac{a_B}{hc}\right)^{1/4}T. \tag{21}$$

The irreducible temperature due to vacuum fluctuations is chosen as the temperature of the Hawking radiation of the black hole which is given by $T = \kappa/2\pi$, with $\kappa = \hbar c^3/4\pi k_B GM$ being the surface gravity of the black hole (Hawking 1974). For the Galactic Center black hole, $M = 4 \times 10^6 M_\odot$ and the temperature becomes

$$T \approx 1.5 \times 10^{-14}\text{ K}. \tag{22}$$

Therefore, the infrared cutoff scale comes out as

$$k_{\rm IR} \approx 3.51 \text{ au}^{-1}. \tag{23}$$

Using (19) and (23) in Equation (18) the scalaron mass is found as

$$M_\psi = 13.08 \text{ au}^{-1} = 1.07 \times 10^{-16}\text{ eV}. \tag{24a}$$

Scalarons can form bound states near black holes with the criterion that the Compton wavelength becomes equal to or greater than the Schwarzschild horizon $R_s = 2GM/c^2$ for the black hole (East & Pretorius 2017; Kalita 2017, 2018). The mass bound $M_\psi < 10^{-18}$ eV is consistent with this criterion. The scalarons found in Equation 24(a) are massive freely-propagating scalarons that act as an additional gravitational mode of the fifth force. The mass calculated above is two orders of magnitude higher than that required for formation of bound states and is six to seven orders of magnitude higher than the upper bound on graviton mass provided by the LIGO–Virgo Collaboration through the observations of black hole binaries (Abbott et al. 2016, 2017).

The idea of scalar particles with a mass range $(10^{-22}\text{–}10^{-3})$ eV as dark matter has been with us nearly two decades (Hu et al. 2000; Peebles 2000; Lesgourgues et al. 2002; see Hui et al. 2017, 2019 for recent discussions). Recent measurements on the shadow of the supermassive black hole of M87 carried out by the Event Horizon Telescope (EHT) have been used to rule out some mass ranges of such scalar dark matter (Cunha et al. 2019; Davoudiasl & Denton 2019). If the scalarons predicted here are found to affect the stellar orbits near the Galactic Center black hole, this will complement the existing independent constraints.

In addition to the scalaron mass (M_ψ), the field amplitude ψ_0 can also be computed from the values of the two cutoff momenta scales. In Equation (16) the field amplitude has a dimension of (energy/length). Multiplying Equation (16) by the geometric mean of the two length scales ($\lambda_{\rm UV} = 2\pi/k_{\rm UV}$ and $\lambda_{\rm IR} = 2\pi/k_{\rm IR}$), $\sqrt{\lambda_{\rm UV}\,\lambda_{\rm IR}}$, and then dividing by $k_B T$ (with T given by Equation (22)) one obtains the dimensionless field amplitude as

$$\tilde{\psi}_0 = \frac{\psi_0\sqrt{\lambda_{\rm UV}\,\lambda_{\rm IR}}}{k_B T} = \frac{\sqrt{\lambda_{\rm UV}\,\lambda_{\rm IR}}}{k_B T}\frac{\hbar c}{2}(k_{\rm UV}^2 - k_{\rm IR}^2) + \frac{\sqrt{\lambda_{\rm UV}\,\lambda_{\rm IR}}}{k_B T} 2\hbar c \Lambda \ln\left(\frac{k_{\rm UV}}{k_{\rm IR}}\right), \tag{24b}$$

where $R_0 = \Lambda$ (de Sitter space around which fluctuations are considered). The dimensionless quantity $\sqrt{\lambda_{\rm UV}\,\lambda_{\rm IR}}\,\Lambda$ is at least 20 orders of magnitude smaller than the first term due to the smallness of the cosmological constant, and hence in practice does not contribute to the field amplitude. With $\sqrt{\lambda_{\rm UV}\,\lambda_{\rm IR}} \approx 0.373$ au and $k_B T \approx 1.29 \times 10^{-18}$ eV, the field amplitude then becomes

$$\tilde{\psi}_0 \approx 1217.05. \tag{24c}$$

Comparing Equation (14) with the standard expression of the Yukawa potential $V(r) = -(GM/r)\alpha\exp(-r/\lambda)$ (Hees et al. 2017), one can write the Yukawa coupling (strength of the coupling) parameter $\alpha\,(r)$ as a function of position as

$$\alpha(r) = \frac{3 - \Lambda r^2}{9\tilde{\psi}_0}. \tag{25a}$$

The upper bound on the cosmological constant through physics near the local mass density appeared several years ago through the model of graviton mass related to the cosmological constant (Tajmar 2006). In such models the dark energy density is shown to appear due to graviton mass, which is dependent on local mass density. The cosmological constant was bounded in the Milky Way as $\Lambda \leqslant 6.29 \times 10^{-52}\ {\rm cm^{-2}} = 1.41 \times 10^{-25}\ {\rm au^{-2}}$. Very recently, Zakharov (2018b) has given the upper bound on the cosmological constant as

$$\Lambda \approx (10^{-41}\text{–}10^{-39})\ {\rm cm^{-2}} = (2.25 \times 10^{-15}\text{–}2.25 \times 10^{-13})\ {\rm au^{-2}} \tag{25b}$$

through its effect on the periastron shift of S-2 that will be visible through the astrometric capabilities of the Keck Telescope, the GRAVITY detector on board the VLT and the upcoming Thirty Meter Telescope (TMT). Therefore, the quantity Λr^2 for the scales of the orbits of stars near Sgr A* (r of the order of a few hundred to a few thousand au) is quite small compared to 3 and hence the Yukawa coupling becomes (from Equation 25(a))

$$\alpha \approx \frac{1}{3\tilde{\psi}_0}. \tag{26}$$

Available bounds on the range of the Yukawa fifth force (λ) and its coupling (α) (Borka et al. 2013; Hees et al. 2017; Zakharov 2018a) along with the scalaron mass ($M_\psi = h/\lambda c$) and the field amplitude ($\tilde{\psi}_0$) are displayed in Table 1.

Table 1
Parameters of the Scalaron Fifth Force According to Currently Available Observational Bounds

λ (au)	α	$\tilde{\psi}_0$	M_ψ(eV)
150	10^{-2}	33.33	5.49×10^{-20} (6.70×10^{-3} au^{-1})
3000	0.33	1.01	2.75×10^{-21} (3.36×10^{-4} au^{-1})
6000	1	0.33	1.37×10^{-21} (1.67×10^{-4} au^{-1})

The scalaron field with mass represented by Equation 24(a) is result of quantum fluctuations near the black hole. The scales corresponding to $k_{\rm UV}$ and $k_{\rm IR}$ are $\lambda_{\rm UV} = 0.078$ au and $\lambda_{\rm IR} = 1.789$ au, respectively, both being several decades above the Planck scale $\lambda_{\rm Pl} \approx 10^{-47}$ au. The thermal energy $k_B T \approx 1.29 \times 10^{-18}$ eV (where T is given by Equation (22)) of scalarons satisfy the inequality $k_B T < M_\psi$ thereby signifying nonrelativistic (cold) scalarons. The fifth force represented by these cold scalarons is the manifestation of quantum gravity fluctuations on large scales. The periastron shift of compact stellar orbits near Sgr A* with the theoretically predicted mass of the scalaron (Equation 24(a)) and the field amplitude $\tilde{\psi}_0$ (Equation 24(c)) is studied along with that produced by the observationally inferred bounds on M_ψ and $\tilde{\psi}_0$ displayed in Table 1. It is discussed in Section 4.

4. Prospects Near the Galactic Center

Earlier works on constraining the Yukawa-type fifth force through the orbits of S-stars near the Galactic Center black hole (Borka et al. 2013; Hees et al. 2017) have enhanced the potential of the Galactic Center in understanding gravity. VLT and NTT have been used earlier for S0-2/S-2 (Borka et al. 2013). Very recently, the Keck Telescope's data has been used for S0-2/S-2 and S0-38/S-38 to put a limit on the range (λ) and strength (α) of the fifth force (Hees et al. 2017).

In this section, the periastron shift of stellar orbits near the Galactic Center black hole in the presence of the scalar fifth force is compared with that in general relativity. The periastron shift for the fifth force ($\dot{\theta}_{\rm prec}^{\rm ff}$) and the contribution to it from the first post-Newtonian (1 PN) effect of GR ($\dot{\theta}_{\rm prec}^{\rm GR}$) are studied.

The rate of periastron advance in the presence of the Yukawa fifth force with a range λ has been given by Li et al. (2014) to constrain the fifth force in the solar system. Very recently, an analytic expression for the periastron shift of orbits for the Yukawa force having a range (scale length) much greater than the semimajor axis (a) of orbiting test particles has also been given by De Laurentis et al. (2018). The periastron shift formula deduced by the latter approach is valid only for stellar systems where $\lambda \approx 1000$ au but a is of the order of only few astronomical units (see the toy models of De Laurentis et al. 2018). But the former approach does not assume such a condition. Therefore, this work has adopted the Li et al. (2014) approach to estimate the periastron shift. The general formula is expressed as

$$\dot{\theta}_{\rm prec}^{\rm ff} = \alpha \frac{na}{e\lambda}\sqrt{1-e^2}\exp\left(-\frac{a}{\lambda}\right)I_1(ae/\lambda), \tag{27}$$

where α is the strength of the coupling of the fifth force, e is the eccentricity of the Keplerian orbit and $I_1(x)$ is the modified Bessel function of the first kind with index 1 (can be found in Arfken & Weber 2005 or in Wolfram Mathematica software). In the present case, $\alpha = 1/3\tilde{\psi}_0$. The function $I_1(x)$ takes the series form

$I_1(x) = 1 + \frac{x}{2} + \frac{x^3}{16} + \frac{x^5}{384} + ...$ if the argument $x < 1$ (here $x = ae/\lambda$). But it can be generally studied for the input parameters a, e, and λ (here it is done in the Mathematica environment). $n = 2\pi/P$ (P being the period in the orbit) is the mean Keplerian motion with $P = 2\pi\sqrt{a^3/GM}$ being the orbital period.

In the presence of the fifth force the gravitational constant gets modified according to Equation (13) (ignoring the de Sitter term, Λr^2) as $G \rightarrow G_{\rm eff} \approx \frac{G}{\tilde{\psi}_0}\left(1 + \frac{1}{3}\exp(-M_\psi a)\right)$ where r is replaced by the semimajor axis, a. Also the range of the fifth force, λ, is the inverse of M_ψ (in natural units of $\hbar = 1 = c$). Therefore, these considerations give

$$\dot{\theta}_{\rm prec}^{\rm ff} = \frac{1}{3\tilde{\psi}_0}\sqrt{\frac{GM}{a^3}}\sqrt{\tilde{\psi}_0^{-1} + \frac{\tilde{\psi}_0^{-1}}{3}\exp(-M_\psi a)} \times \frac{M_\psi a}{e}\sqrt{1-e^2}\exp(-M_\psi a)I_1(ae/\lambda). \tag{28}$$

The factor $\sqrt{\tilde{\psi}_0^{-1} + \frac{\tilde{\psi}_0^{-1}}{3}\exp(-M_\psi a)}$ appears due to the effect of the modified gravitational constant in the formula for the orbital period P.

Assuming Schwarzschild geometry near the Galactic Center black hole, the rate of precession of the periapses of the S-stars (up to 1 PN level only) is expressed as

$$\dot{\theta}_{\rm prec}^{\rm GR} = \frac{6\pi GM}{c^2 a(1-e^2)}\frac{1}{P}. \tag{29}$$

With the mass of the black hole, $M \approx 4.32 \times 10^6 M_\odot$, this gives the general relativistic periastron shift as

$$\dot{\theta}_{\rm prec}^{\rm GR} = \frac{0.736}{(a/1\ {\rm au})^{5/2} \times 5 \times 10^{-4} \times (1-e^2)}(206369)\ {\rm as\ yr}^{-1}. \tag{30}$$

Here $0.736\ {\rm au} = \frac{6\pi GM}{c^2}$, the number 206,369 is the conversion factor from radians to arcseconds, and $P = 2\pi\sqrt{a^3/GM} = 5 \times 10^{-4} \times (a/1\ {\rm au})^{3/2}(1\ {\rm yr})$.

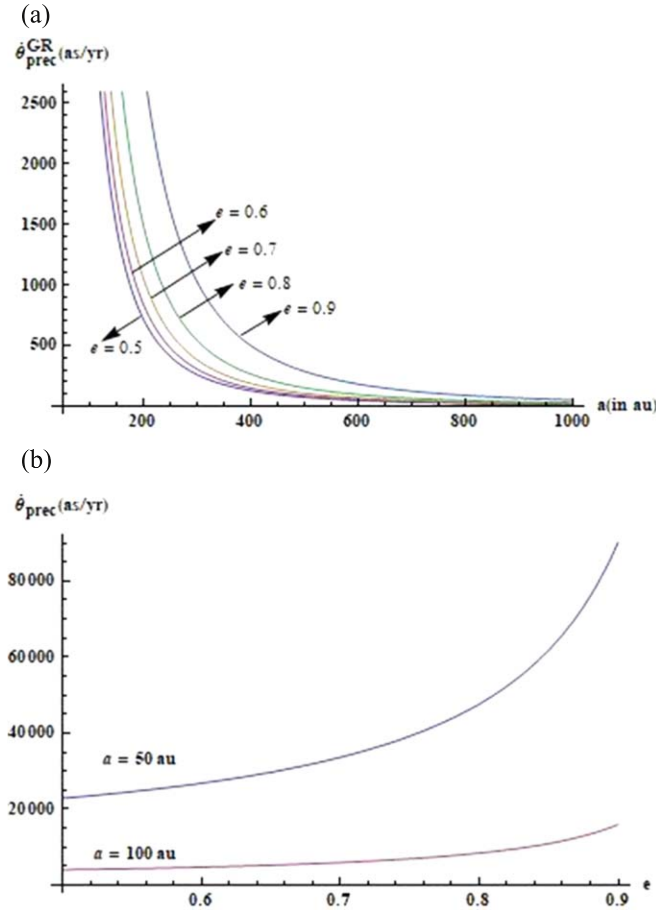

Figure 1. (a) Variation of the rate of the periastron shift in GR for various eccentricities with respect to the semimajor axis within $a = (50\text{–}1000)$ au. (b) Variation of the rate of the periastron shift in GR for two semimajor axes $a = 50$ au and $a = 100$ au with respect to eccentricities.

Similarly the shift of the periastron contributed by the scalaron expressed by Equation (28) becomes

$$\dot{\theta}_{\rm prec}^{\rm ff} = \frac{2\pi \times 2.07 \times 10^3}{3\tilde{\psi}_0}(206369)\frac{1}{(a/1\ {\rm au})^{3/2}} \times \sqrt{\tilde{\psi}_0^{-1} + \frac{\tilde{\psi}_0^{-1}}{3}\exp(-M_\psi a)}\frac{M_\psi a}{e} \times \sqrt{1-e^2}\exp(-M_\psi a)I_1(M_\psi ae)({\rm as\ yr}^{-1}). \tag{31}$$

The variations of general relativistic precession shift (Equation (30)) and that due to the fifth force (Equation (31)) with respect to the semimajor axis in the range $a = (10\text{–}1000)$ au are shown in Figures 1–3 for various eccentricities. The parameters $(\tilde{\psi}_0, M_\psi)_O$ and $(\tilde{\psi}_0, M_\psi)_T$ represent the observationally inferred values and theoretical prediction for the pair $(\tilde{\psi}_0, M_\psi)$, respectively.

Whereas the variation of periastron shift with respect to the semimajor axis is the set of well-known numbers in GR (Figure 1(a)), its variation with eccentricity at specific values of the semimajor axis shown in Figure 1(b) can be used for comparing new effects with those in GR. The values of the semimajor axis as $a = 50$ au and $a = 100$ au are chosen as scales below 100 au and are going to be resolved by upcoming extremely large telescopes such as the TMT (Skidmore 2015).

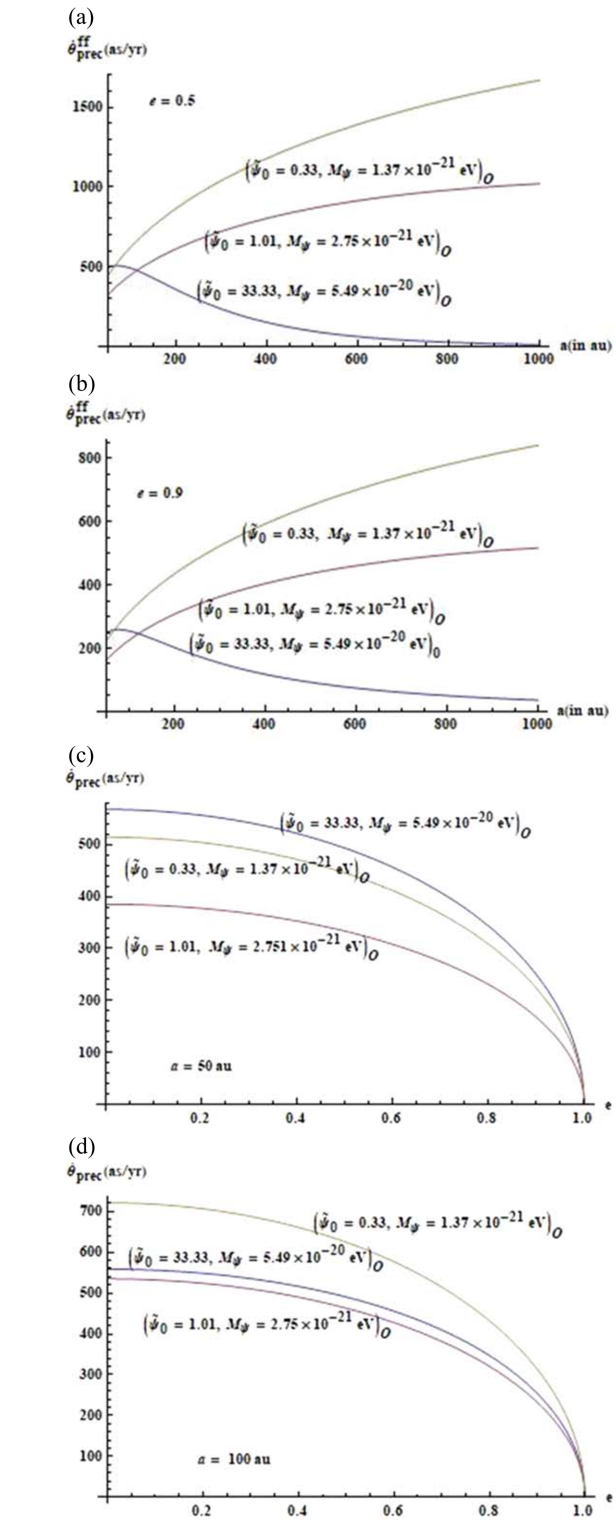

Figure 2. (a) Variation of the rate of the periastron shift at eccentricity $e = 0.5$, with respect to the semimajor axis for various scalaron field strengths and scalaron masses estimated from observational bounds on the Yukawa coupling strength and the scale at which such constraints were put (Hees et al. 2017). (b) Variation of the rate of periastron shift at eccentricity $e = 0.9$, with respect to the semimajor axis for various scalaron field strengths and scalaron masses estimated from observational bounds on the Yukawa coupling strength and the scale at which such constraints were put (Hees et al. 2017). (c) Variation of the periastron shift for various scalarons at 50 au with respect to orbital eccentricities. (d) Variation of the periastron shift for various scalarons at 100 au with respect to orbital eccentricities.

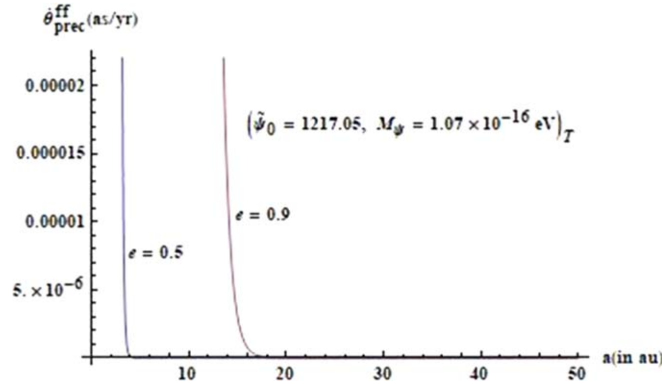

Figure 3. Periastron shift with respect to the semimajor axis for eccentricities, $e = 0.5$ and $e = 0.9$.

From Figures 2(a) and (b) it is seen that if the present constraint on the Yukawa coupling strength (α) reported in Hees et al. (2017) is due to scalaron amplitude $\tilde{\psi}_0$, the high mass scalaron ($M_\psi \approx 10^{-20}$ eV) produces a peak of periastron shift at 500 as yr^{-1} for $e = 0.5$ and at around 250 as yr^{-1} for $e = 0.9$. The peak occurs below $a \ll 200$ au. The effect of low mass scalarons ($M_\psi \approx 10^{-21}$ eV) survives from <100 au to a few thousands of astronomical unit (>1000 au) and with a much larger amplitude of periastron shift—around 800 as yr^{-1} for the lowest mass scalaron (see Figure 2(b)) in $e = 0.9$ and around 1500 as yr^{-1} for the same (see Figure 2(a)) in $e = 0.5$.

The deviation from GR in a given semimajor axis is visible from the pattern of variation of the periastron shift with eccentricity. Figures 2(c) and (d) and their comparisons with Figure 1(b) show that the periastron shift drifts in the opposite direction relative to GR as one increases the eccentricity. For $a = 50$ au, the highest periastron shift is shown by the high mass scalaron, $M_\psi = 5.49 \times 10^{-20}$ eV (Figure 2(c)), and the lowest is shown by the scalaron with $M_\psi = 2.75 \times 10^{-21}$ eV. The amplitudes are 565 as yr^{-1} and 380 as yr^{-1}, respectively. For $a = 100$ au, the highest periastron shift is shown by the low mass scalaron, $M_\psi = 1.37 \times 10^{-21}$ eV (Figure 2(d)), and the lowest is shown by the scalaron with $M_\psi = 2.75 \times 10^{-21}$ eV. The amplitudes are 700 as yr^{-1} and 530 as yr^{-1}, respectively. This gives an indication that as one probes compact orbits, the signature of massive scalarons is likely to appear.

The magnitude of the periastron shift expected from scalarons having the parameters ($\tilde{\psi}_0 = 1217.05$, $M_\psi = 1.07 \times 10^{-16}$ eV) predicted from the consideration of vacuum fluctuations having two cutoff scales ($\lambda_{\rm UV}$, $\lambda_{\rm IR}$), one provided by the Schwarzschild horizon and the other by the irreducible vacuum temperature, is shown in Figure 3.

The effects are prominent for a much smaller semimajor axis, $a = (10–20)$ au, depending on the eccentricity with the magnitude of the periastron shift lying in the range $\dot{\theta}^{\rm ff}_{\rm prec} = (1–10)\ \mu$as yr^{-1}. For a continuous range of eccentricities from $e = 0.1–0.9$, the periastron shift of magnitude (1–10) μas yr^{-1} is obtained only for $a = 10$ au (see Figure 4).

5. Future Constraint on Scalar Hair?

It appears that the scalar fifth force described by the scalarons acts as black hole hair near the Galactic Center (see the discussion of scalar hair near black holes in Kalita 2018 and

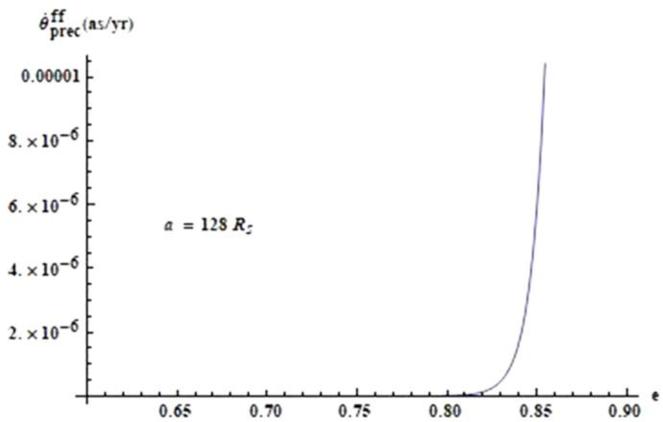

Figure 4. Variation of the periastron shift due to the scalaron fifth force with eccentricity at $a = 10$ au $= 128R_s$.

references therein). If the present bounds on the Yukawa coupling (α) correspond to the scalaron field, the future observations of the periastron shift of tight stellar orbits can constrain the scenario. The amplitude of the periastron shift lying near a few hundred arcseconds per year (see Figures 2(a)–(d)) is within reach for the facilities such as Keck and the GRAVITY detector on board VLT. Recent observations of the M87 black hole shadow through the EHT have ruled out spin zero bosonic fields with masses of the order of 10^{-21} eV (Cunha et al. 2019; Davoudiasl & Denton 2019). The periastron shift observations near the Galactic Center black hole will complement such tests. The scalarons predicted from quantum vacuum fluctuations are extremely massive ($M_\psi = 10^{-16}$ eV) with a very small Yukawa coupling, $\alpha = 1/3\tilde{\psi}_0 \approx 2.7 \times 10^{-4}$. The coupling strength is two orders of magnitude smaller than that currently probed (Hees et al. 2017). Their effect is visible only for orbits with a semimajor axis around 10–20 au which defines regions within a few hundred times the gravitational radius of Sgr A* ($a = 128$–$256\ R_S$). With the distance to the Galactic Center as $D = 8.2$ kpc, these orbits possess an angular size within $\alpha = a/D \approx (1200$–$2400)\ \mu$as. With an astrometric accuracy of about 10 μas possessed by the GRAVITY detector on board VLT, these orbits and their periastron shift may be determined by future measurements. Observations of tighter orbits with higher eccentricity (see Figures 3 and 4) will be required to constrain scalarons resulting from quantum vacuum fluctuations modeled here. Finding stellar orbits near 10–20 au may prove a formidable challenge for near-infrared (NIR) astrometry that is recently being used to resolve stellar orbits near Sgr A*. However, future radio astronomy carries enough potential to test gravitational physics near Sgr A* through a search for pulsars near it. A search for pulsar–Sgr A* dynamics around 10 au is already under way with the BlackHoleCam and EHT Collaboration (see De Laurentis et al. 2018 and references therein).

6. Results and Discussion

In this work, the scalarons of $f(R)$ type gravity theory resulting from quantum vacuum fluctuations are used as a Yukawa fifth force to calculate the periastron shift of stellar orbits near the Galactic Center black hole. The scalaron field amplitude is related to the Yukawa coupling. Theoretical limits on the scalaron field amplitude and scalaron mass near the black hole are calculated from the consideration of ultraviolet and infrared cutoff scales of vacuum fluctuations. The predicted scalarons have an interesting mass around 10^{-16} eV, which is in between the large mass range of scalar dark matter of 10^{-22}–10^{-3} eV (Hui et al. 2019). The Yukawa coupling is, however, two orders of magnitude smaller than that recently constrained by Keck's orbital data of two stars, S-2 and S-38 (Hees et al. 2017). Implications of these scalarons for future measurements near the Galactic Center black hole are discussed below.

If the present bound on the Yukawa coupling (α) corresponds to that given by scalarons ($\tilde{\psi}_0$), it is found that the low mass scalaron ($M_\psi \approx 10^{-21}$ eV) affects orbital precession to the same order as GR (see Figures 1(a), 2(a) and (b)). Assuming that the general relativistic periastron shift of stellar orbits having a semimajor axis much below 1000 au will be detected by future astrometric measurements of GRAVITY and upcoming ELTs (30–39 class ground based optical NIR telescopes), such scalarons will be ruled out. This will narrow down the possibility of ultralight scalar degrees of freedom being candidates of dark matter and hence complement the recent constraints coming from EHT's shadow measurements.

This will open the window for massive scalarons, $M_\psi \geqslant 10^{-20}$ eV. From numerical calculations it is seen that at $a = 200$ au and $e = 0.5$ (for example), $\dot{\theta}^{\rm ff}_{\rm prec}$ is 50% of the GR value for $M_\psi = 5.49 \times 10^{-20}$ eV. But for $e = 0.9$ at same orbit, $\dot{\theta}^{\rm ff}_{\rm prec}$ goes down to 1% of the GR value. Therefore, with $\dot{\theta}^{\rm GR}_{\rm prec} \sim 10^3$ as yr^{-1} (see Figure 1(a)) the contribution to the periastron shift from the scalar fifth force is about 10 as yr^{-1}, which is within reach of the detectors like GRAVITY (astrometric uncertainty of about 10–30 μas).

It seems apparently challenging to disentangle the effect of massive scalarons from other relativistic effects such as spin, quadrupole, and frame dragging as these effects grow up for compact orbits where massive scalarons become noticeable too (see Figures 2(a) and (b)). But the nature of the periastron shift for the scalaron field shows a marked contrast with general relativistic effects. The Schwarzschild, spin, quadrupole, and frame dragging effects increase with eccentricity as $(1 - e^2)^{-1}$, $(1 - e^2)^{-3/2}$, $(1 - e^2)^{-2}$ and $(1 - e^2)^{-2.5}$, respectively (see Will 2008, for example). But the scalaron effect goes down with eccentricity for $M_\psi = (5.49 \times 10^{-20}$–$10^{-21})$ eV (see Figures 2(c) and (d), and compare with Figure 1(b) where the Schwarzschild contribution is plotted).

Although ultralight scalar particles ($M_\psi \approx 10^{-21}$ eV) were recently constrained by EHT observations, massive scalars are yet to be constrained. The scalarons predicted by gravitationally-corrected quantum vacuum fluctuations near the Galactic Center black hole possess masses of around 10^{-16} eV and a Yukawa coupling of $\alpha \approx 2.7 \times 10^{-4}$. The range of the Yukawa force for such scalarons is of the order of the gravitational radius of the black hole. These are cold scalarons satisfying the condition $k_B T < 10^{-16}$ eV. Observations of highly eccentric stellar orbits ($e \sim 0.9$) with a semimajor axis within a few hundred times the gravitational radius of the black hole will be required to constrain such scalarons through their effect on orbital precession. These orbits have an angular size within 1200–2400 μas and will likely be detected by astrometric facilities such as GRAVITY and upcoming extremely large telescopes. These scalarons contribute $\dot{\theta}^{\rm ff}_{\rm prec} = (1$–$10)\ \mu$as yr^{-1} to the periastron shift. One may look forward to the possibility of ruling out of massive scalarons through future observations of stellar orbits lying very close to Sgr

A*. This heavily relies on the stellar mass function near the black hole (Goddi et al. 2017). But finding pulsars near Sgr A* with orbital radii of the order of a few tens of astronomical units will be potential enough to confirm or rule out the scenario.

7. Conclusion and Forward Look

The Galactic Center black hole is found to carry the potential for unraveling new gravitational physics within a distance of a few hundred times its gravitational radius. Although Yukawa gravity resulting from the modification of general relativity has been used earlier to study orbits of stars near Sgr A*, the connection between Yukawa coupling strength and scalaron field, and that between scalaron mass and quantum fluctuation scales, are new realizations. One can look forward to confirm or rule out such ideas from future measurements, which will help ameliorate the problems of the gravitational sector in the dark universe problem. Future NIR astrometry and radio astronomy of the pulsar–Sgr A* system will let us know the nature of the operation of quantum extensions of GR near black holes.

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