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Research on the looping pendulum phenomenon

Zhou Yu-bo[1,2], **Yang Ming-duo**[3], **Zhang Min**[1,2], **Fan Dai-he**[1,2], **Liu Qi-jun**[1,2], **Chang Xiang-hui**[1,2], **Jia Xin-yan**[1,2] **and Wei Yun**[1,2]

[1] National Demonstration Center For Experimental Physics Education (Southwest Jiaotong University), Chengdu, Sichuan 611756, People's Republic of China
[2] School of Physical Science and Technology, Southwest Jiaotong University, Chengdu 611756, People's Republic of China
[3] School of Materials, Sun Yat-sen University, Guangzhou 510006, People's Republic of China

E-mail: dhfan@swjtu.edu.cn

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Abstract

The phenomenon of the looping pendulum proposed by the International Young Physicists' Tournament in 2019 is investigated experimentally and theoretically. By dividing the looping pendulum phenomenon into two physical stages, and establishing physical models, the effects of mass ratio, thread length, initial release angle, and friction coefficient on the dropping height, velocity of the heavy load, and trajectory of the light load in the phenomenon are numerically simulated. Furthermore, experiments are carried out to verify the theoretical predictions. The experimental results are consistent with the results of the theoretical analysis.

Supplementary material for this article is available online

Keywords: looping pendulum, trajectory, descending height, velocity

(Some figures may appear in colour only in the online journal)

1. Introduction

One of the problems addressed at the 32nd International Young Physicists' Tournament (IYPT) in 2019 is the 'looping pendulum' phenomenon [1, 2]. Its content is as follows: connect two loads, one heavy and one light, with a string over a horizontal rod and lift up the

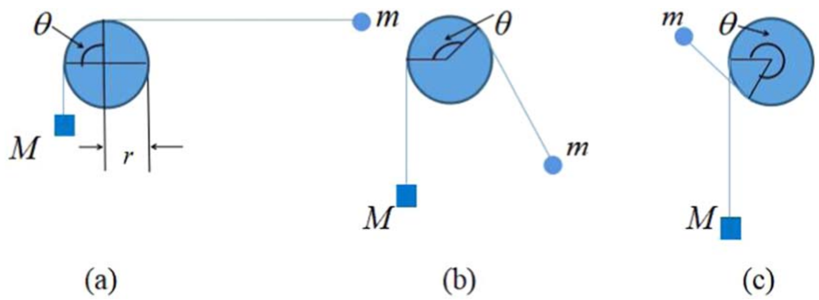

Figure 1. Sketch of an overview of the looping pendulum phenomenon, where r means the radius of the horizontal rod and M means the heavy load, m represents the light load and θ represents the ratio between the length of the thread wound around the horizontal rod and the radius of the horizontal rod.

heavy load by pulling down the light one. Release the light load and it will sweep around the rod, keeping the heavy load from falling to the ground. Investigate this phenomenon.

An overview of the looping pendulum phenomenon is described by figure 1. Two loads, one heavy (M) and one light (m), are connected by a light thread and then cross a horizontal rod (assuming the radius of the rod is r). Initially, the light load is used to pull the heavy load up to just touch the rod (as shown in figure 1(a)). Then release the light load, and the heavy one will drop while the light one will sweep around the horizontal rod (as shown in figure 1(b)). We define θ as the ratio between the length of the thread wound around the horizontal rod and the radius of the horizontal rod. So the length of the thread wound around the horizontal rod is θr .When the heavy one stops dropping, the light one will continue to sweep around the rod (as shown in figure 1(c)), for the light one having a certain speed. Finally, when the thread at the light end is completely wound around the rod, the light one stops moving.

There are some reports interpreting the looping pendulum phenomenon. For example, Steve Spangler Science found that when the heavy-to-light ratio of the two ends of the thread is 14:1, some special experimental phenomena will happen [3]. Nicholas B. Tufillaro investigated a system similar to the looping pendulum system where the friction is zero. The author carefully analyzes the trajectory of the light object by computer calculation and analytical mechanics [4]. However, up to now there is no particular report on how the relevant physical parameters affect the looping pendulum phenomenon.

In this paper we divided the looping pendulum phenomenon into two specific stages, which we then investigated by experimental and theoretical methods. The first stage is from the beginning to the end of the dropping process of the heavy load after releasing the light load. The second stage is from the moment that the heavy load stops dropping to the end where the light load is completely wound around the horizontal rod. In the first step, we design relevant experiments to study the effects of the ratio *M/m*, between the masses, the thread length, and the initial angle of releasing the light load in the looping pendulum phenomenon. The experimental videos are analyzed by the Tracker [5] software and with the help of mobile phone software PHYPHOX [6]. By establishing a simplified physical model and solving it numerically, the effects of the ratio *M/m*, between the masses, the thread length, the initial releasing angle of the light load, the friction coefficient between the thread and the rod, and the radius of the horizontal rod are analyzed theoretically. We found that the results of theoretical analyses are consistent with the experimental results.

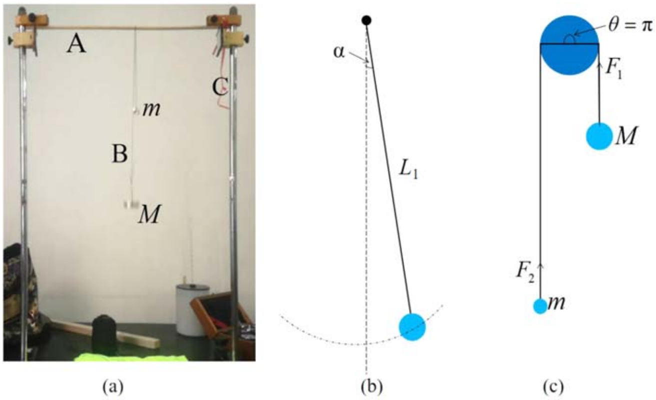

Figure 2. (a) Experimental arrangement: A means the horizontal wooden rod and B means the light cotton thread. C is the equipment used to fix the horizontal rod and m,M represents the light load and the heavy load, respectively. (b) Experimental scheme for measuring the value of g, where L_1 is the pendulum length and $\alpha < 5°$. (c) Experimental scheme for measuring the value of μ.

2. Experimental study

Figure 2(a) shows the entire experimental scheme. Firstly we measured the gravitational constant g as well as the sliding friction coefficient μ between the thread and the horizontal rod under the current experimental condition.

2.1. Measuring the value of gravitational constant g

A simple pendulum method is used to measure the value of g, as shown in figure 2(b). The measured pendulum length is $L_1 = (0.691 \pm 0.001)\,\mathrm{m}$ and the average total time for a 50 time period is 50 $T = 83.45$ s. So the average time period for the pendulum is $T = (1.669 \pm 0.034)$ s. By using the formula $g = 4\pi^2/T^2$ the gravitational constant g can be calculated out as $g = (9.793 \pm 0.034)\mathrm{m/s^2}$.

2.2. Measuring the sliding friction coefficient μ

We measured the sliding friction coefficient μ between the cotton thread and the horizontal rod. As is shown in figure 2(c), one heavy load and one light load are connected by a cotton thread and then cross a wooden rod. In this experiment, both light and heavy loads are vertically suspended at the ends of the horizontal rod, so $\theta = \pi$. By releasing the light load, the heavy load will fall down. We define the force on the heavy load's side as F_1 and on the light load's side as F_2 . As the heavy load falls down, F_1 and F_2 satisfy the following equation [7] (a detailed derivation of this equation is shown in the appendix):

$$F_1 = F_2 e^{\mu\theta}. \tag{1}$$

We can also define the dropping height of the heavy load as h , and the mass ratio of the heavy and light load at both ends of the thread is $\gamma = M/m$ (M means the mass of the heavy one and m means the mass of the light one). By setting the initial release angle $\theta = \pi$ (i.e. both light and heavy loads are vertically suspended at the ends of the horizontal rod), then releasing the light load, the light load will rise vertically and the heavy load will fall vertically. During this process, the value of θ will remain unchanged and the acceleration of the

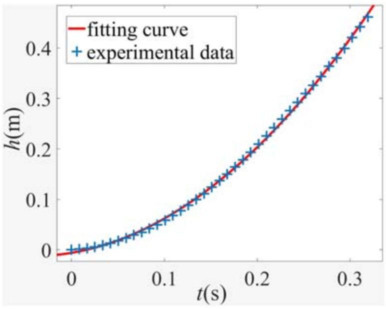

Figure 3. Experimental data and fitting diagram of the heavy load's dropping height.

heavy load can be expressed as follows (a detailed derivation of this equation can be found in section 3):

$$\frac{d^2h}{dt^2} = \frac{Mg - mge^{\mu\theta}}{M + me^{\mu\theta}} = \frac{\gamma g - ge^{\mu\theta}}{\gamma + e^{\mu\theta}} = g\frac{M - me^{\mu\pi}}{M + me^{\mu\pi}}. \tag{2}$$

We chose a nut with a mass of 4.6g (m) as the light load and a small iron ball with a mass of 72.2g (M) as the heavy load (all masses in this paper are measured by the electronic balance, of which the precision is 0.1g). Then the heavy load was connected to the light load by a light cotton thread with the length of (0.700 ± 0.002) m in the experiment. $g = 9.793$ m s^{-2} is taken in the experiments and a horizontal wooden rod with the radius of (4.000 ± 0.002) mm was chosen. Then we recorded the video of the heavy load during the falling process by a 120 frames per second camera. The recorded video is imported into the Tracker software for analysis. The relationship between the dropping height of the heavy load and dropping time is shown in figure 3 ('+'). The red solid line in figure 3 is the fitted line using equation (2).

From figure 3, it can be seen that the experimental points are well fitted by equation (2), which proves the validity of equation (2). At the same time, the friction coefficient between the horizontal rod and the thread can be fitted out as $\mu = 0.26 \pm 0.01$ under the current experimental condition.

2.3. Study of different variables on the looping pendulum phenomenon

2.3.1. Effects of different mass ratios γ. We investigated the effects of γ on the looping pendulum phenomenon. In this experiment, a wooden rod with a radius of $r = 4.000$ mm was chosen as the horizontal rod. We used the iron nut as the light load, and the mass of it was fixed to $m = 9.2$ g. Then it was fixed at one end of the light cotton thread with the thread length of $L = (0.553 \pm 0.001)$ m. The other end of the thread could be fixed with different numbers of iron gaskets to simulate the heavy load (shown as the M in figure 2). So the value of γ can be changed by increasing or reducing the number of gaskets.

We lifted the light load so that the heavy load was pulled up to just contact the rod. In such a situation, the heavy load was kept steady and free to drop. Then, with the help of the mobile phone software PHYPHOX [6], we could guarantee that the thread of the light load's end was as parallel as possible to the horizontal plane (i.e. $\theta = \pi/2$). At this time, we released the light load and recorded the dropping process of the heavy load by a 120 frames per second camera. By importing the videos into Tracker software, the motion of both the light and heavy

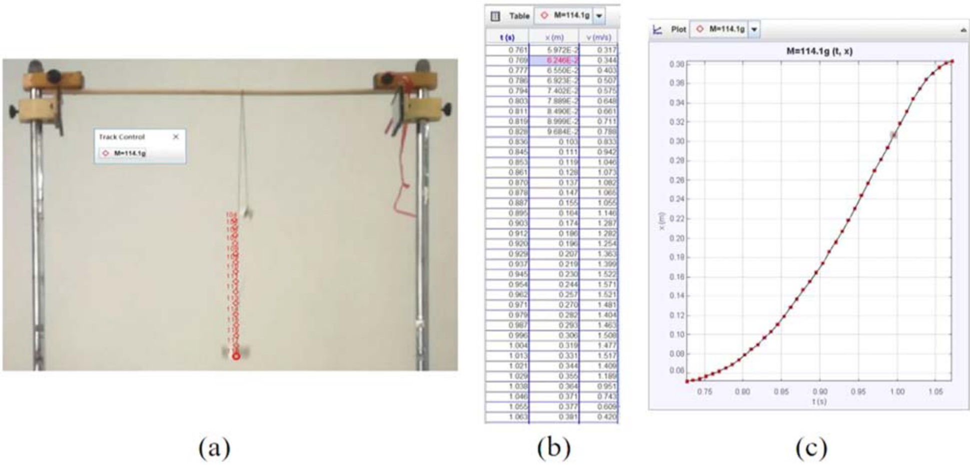

Figure 4. (a) Red dots denote the location of the heavy load in each video frame. (b) Raw data acquired by the Tracker software. (c) Diagram of the relationship of the heavy load's dropping height with the time obtained by the Tracker software.

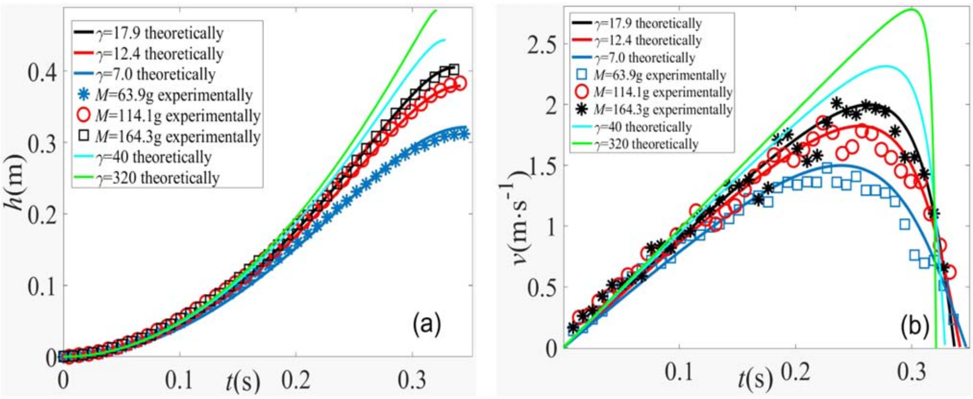

Figure 5. Diagram of the dropping height (a) and dropping velocity (b) with time when changing the values of γ. The black, red, and blue discrete lines represent the cases of $\gamma = 17.9$, $\gamma = 12.4$, and $\gamma = 7.0$, respectively.

load could be analyzed. Part of the original experimental data from Tracker is shown in figure 4.

Finally, the relationship between the dropping height and velocity of the heavy load and the dropping time t can be seen in figure 5. The solid curves are the results of theoretical analysis, which will be described in section 3.3.1.

From figure 5(a), we can conclude that in the first stage, the dropping height of the heavy one will increase nonlinearly with the increase of time for any value of γ. And with the increase of γ, the final dropping height of the heavy one will increase. As can be seen from figure 5(b), for any γ value, with the increase of time, the dropping speed of the heavy one will increase at first and then decrease. However, with the increase of γ, the maximum

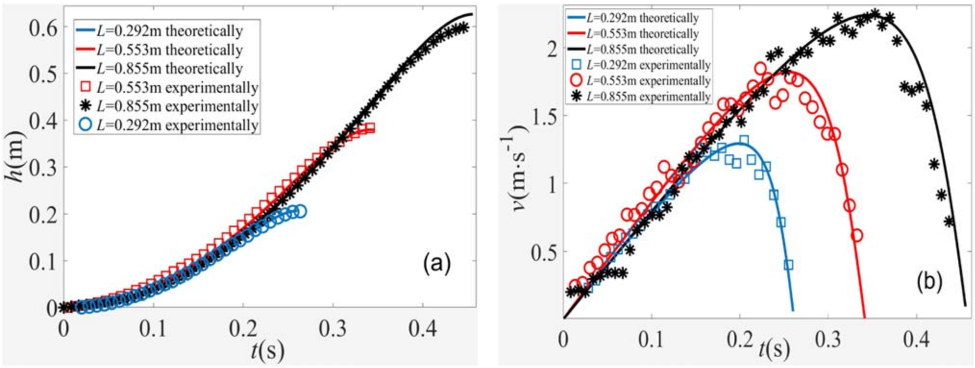

Figure 6. Diagram of the dropping height (a) and dropping velocity (b) with time when changing values of L. The black, red, and blue discrete lines represent the cases of $L = 0.855$ m, $L = 0.553$ m, and $L = 0.292$ m, respectively.

velocity of the dropping load will increase. Moreover, the increase of γ will shorten the time from dropping to stopping, but the decreasing is not obvious.

2.3.2. Effects of different thread lengths L. The influence of the thread length L on the looping pendulum phenomenon is also measured experimentally. Taking $M = 114.1$g and $m = 9.2$ g (i.e. $\gamma = 12.4$), $\theta = \pi/2$ and the radius of the horizontal rod $r = 4.000$ mm. By changing the length of thread L, the relationship between the dropping height and the velocity of the heavy load with the time is measured by the same measuring method mentioned above. The experimental results are shown in figure 6. The solid curves are the results of theoretical analysis, which will be introduced in section 3.3.2.

From figure 6(a), it can be seen that in the first stage, with the increase of time, the dropping height of the heavy load will increase for any value of L. With the increase of L, the maximum dropping height of the heavy load will also increase. As can be seen from figure 6(b), for any value of L, with the increase of time, the dropping speed of the heavy load will increase at first and then decrease. However, with the increase of L, the maximum dropping velocity will also increase in the dropping process. At the same time, the time from the beginning to the end of the dropping process will also increase obviously.

2.3.3. Effects of different initial release angles. The effects of the initial release angle θ on the looping pendulum phenomenon are also investigated in the experiments. Taking $M = 114.1$g, $m = 9.2$g ($\gamma = 12.4$), $r = 4.000$ mm, $L = 0.553$ m, we can change the initial release angle θ of the light load with the help of the PHYPHOX software. The relationship between the dropping height and the velocity of the heavy load with time under different initial release angles can be obtained. The discrete curves in figure 7 show the experimental results, and the solid curves show the theoretical results, which will be introduced in section 3.3.3.

It can be seen from figure 7(a) that, for any value of θ, with the increase of time, the maximum dropping height of the heavy load will increase. But the increasing is not obvious. It can also be seen from figure 7(b) that, for any value of θ, with the increase of time, the speed of the dropping load will increase firstly and then decrease. At the same time, with the increase of θ, the maximum dropping speed has a tendency of increasing overall. In addition, the increasing of θ will also increase the time of the heavy load from dropping to stopping.

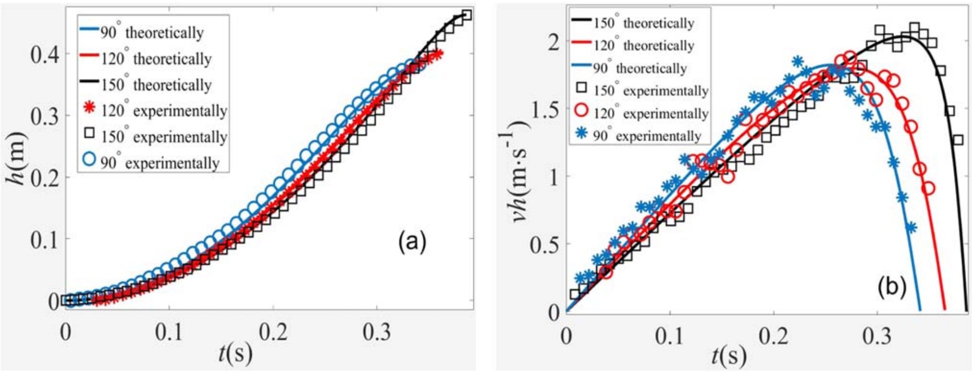

Figure 7. Diagram of the dropping height (a) and dropping velocity (b) with time when changing values of θ. The black, red, and blue discrete lines indicate the cases of the initial release angles of 150°, 120°, and 90°, respectively.

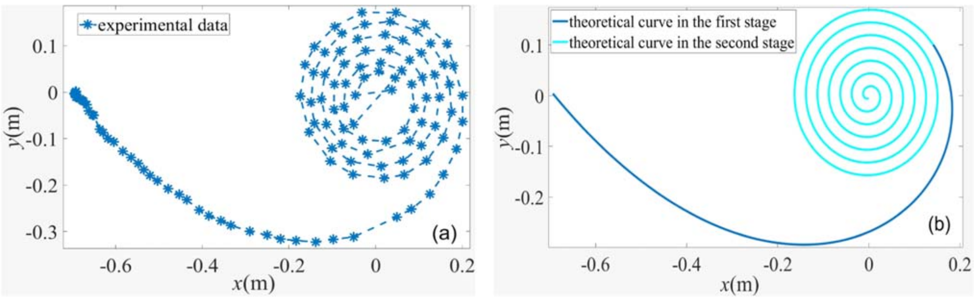

Figure 8. The experimental points (a) and theoretical simulation (b) of the entire looping pendulum process.

2.4. Study on the trajectory of the light load

Finally, the trajectory of the light load in the looping pendulum phenomenon is also investigated experimentally. The experimental parameters are set to $M = 72.2$ g, $m = 4.6$ g (i.e. $\gamma = 15.7$), $L = 0.700$ m, and initial release angle $\theta = \pi/2$, $\mu = 0.26$, $r = 4.000$ mm. By using the Tracker software to analyze the motion of the light load, the trajectory of the light load is shown in figure 8(a). The theoretical curves in figure 8(b) will be introduced in section 3.3.4.

3. Theoretical analysis of looping pendulum phenomenon

3.1. Theoretical analysis of stage 1

We analyzed the looping pendulum phenomenon in the two-dimensional plane. Firstly we analyzed the motion of the looping pendulum in the first stage, as shown in figure 1(b), establish the Cartesian coordinate system, and set the central point of the horizontal rod O as the origin of the coordinate. Suppose that M is connected to m by a thread with the length of L. The radius of the horizontal rod is r. Releasing the light load starts the motion of the pendulum. Assuming that at time t, the light load moves to point A as shown in figure 9(b).

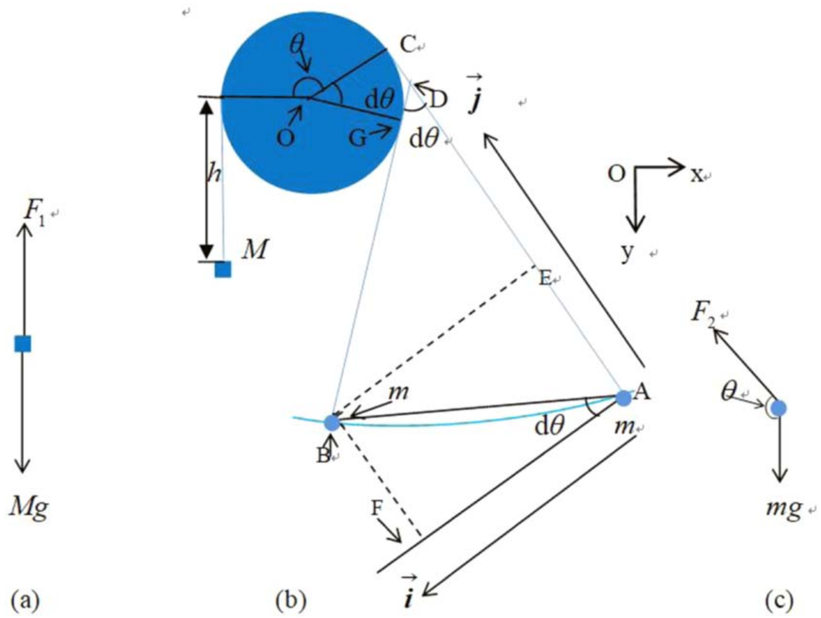

Figure 9. Stage 1, the analysis of the motion of the light load and the force analysis of both the light and heavy loads. Among them, F_1 is the tension along the thread direction of the heavy one; θ is the length of the thread wound around the rod to radius r of the rod; $\mathrm{d}\theta$ is the micro increment of θ when the light load moves from A to B ($\overrightarrow{AB}$ is infinitesimal); $\overrightarrow{i}$ is perpendicular to the thread direction while $\overrightarrow{j}$ is along the thread direction; F_2 is the tension along the thread direction of the light one.

C is the tangent point between the thread and the rod, and the heavy load drops to a height of h. Under such a situation, the length of the thread wound around the horizontal rod can be expressed as θr (i.e. θ increases 2π when the light load passes through each circle of the rod).

Then we established another set of a coordinate system with the origin at the center of the light load (ignoring the distance between the center of the load and the load-thread joint). We define $\overrightarrow{AC}$ as the $\overrightarrow{j}$ direction and the $\overrightarrow{i}$ direction is perpendicular to the $\overrightarrow{j}$ direction. Assuming that the light load moves from A to B in a short period of time of $\mathrm{d}t$ from the moment t (shown as the blue solid line in figure 9(b)). The tangent of the thread to the rod changes to G at this moment). The thread turns $\mathrm{d}\theta$ while the heavy load drops $\mathrm{d}h$. For the convenience of the following analysis, we build BE$//\overrightarrow{i}$ and BF$//\overrightarrow{j}$. Point E is the junction between BE and AC, and BF = AE (as shown in figure 9(b)).

When the light load moves from A to B, its displacement can be indicated as $\overrightarrow{AB}$ ($\overrightarrow{AB}$ is infinitesimal) and its instantaneous velocity at point A can be written as $\frac{\overrightarrow{AB}}{\mathrm{d}t} = \frac{\mathrm{AF}}{\mathrm{d}t}\overrightarrow{i} + \frac{\mathrm{AE}}{\mathrm{d}t}\overrightarrow{j}$.

From figure 9(b), we can know that AC $= (L - \theta r - h)$, BG $= (L - \theta r - h) - dh - rd\theta$, CD $=$ GD $= r\tan(\theta/2)$. $\mathrm{d}\theta$ is so small that we can assume DB$\perp$AB. Under such a situation, we can deduce that AF $=$ (BG $+$ GD)$\sin(\mathrm{d}\theta)$, AE $=$ AC–CD–BD $\cos(\mathrm{d}\theta)$. We can also take the approximation as follows, $\mathrm{d}\theta = \sin(\mathrm{d}\theta) = 2\tan(\mathrm{d}\theta/2)$, $\cos(\mathrm{d}\theta) = 1$. So after neglecting the second order small quantity we can obtain that AF $= (L - \theta r - h)\mathrm{d}\theta$ and AE $= \mathrm{d}h$.

Now it can be concluded that the instantaneous velocity of the light load at point A is

$$\frac{\overrightarrow{AB}}{\mathrm{d}t} = (L - \theta r - h)\frac{\mathrm{d}\theta}{\mathrm{d}t}\vec{i} + \frac{\mathrm{d}h}{\mathrm{d}t}\vec{j}. \tag{3}$$

According to the literature [8], when $\vec{i}$, $\vec{j}$ change over time t, we can get the acceleration of the light load at point A as

$$\begin{aligned}\vec{a}_{\mathrm{A}} &= \left[-2\frac{\mathrm{d}\theta}{\mathrm{d}t}\frac{\mathrm{d}h}{\mathrm{d}t} - r\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 + (L - \theta r - h)\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\right]\vec{i} \\ &\quad + \left[(L - \theta r - h)\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 + \frac{\mathrm{d}^2h}{\mathrm{d}t^2}\right]\vec{j}.\end{aligned} \tag{4}$$

Then, using the force analysis of the heavy one (as shown in figure 9(a)) we can obtain

$$Mg - F_1 = M\frac{\mathrm{d}^2h}{\mathrm{d}t^2}. \tag{5}$$

By using the force analysis of the light load parallel and perpendicular to the thread direction (as shown in figure 9(c)), we can obtain

$$F_2 - mg\sin\left(\theta - \frac{\pi}{2}\right) = m\left[(L - \theta r - h)\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 + \frac{\mathrm{d}^2h}{\mathrm{d}t^2}\right] \tag{6}$$

$$mg\cos\left(\theta - \frac{\pi}{2}\right) = m\left[-2\frac{\mathrm{d}\theta}{\mathrm{d}t}\frac{\mathrm{d}h}{\mathrm{d}t} - r\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 + (L - \theta r - h)\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\right]. \tag{7}$$

From equations (1), (5)–(7), we can obtain

$$\begin{aligned}\frac{\mathrm{d}^2h}{\mathrm{d}t^2} &= \frac{Mg + me^{\mu\theta}\left[g\cos\theta - (L - \theta r - h)\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2\right]}{M + me^{\mu\theta}} \\ &= \frac{\gamma g + e^{\mu\theta}\left[g\cos\theta - (L - \theta r - h)\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2\right]}{\gamma + e^{\mu\theta}}\end{aligned} \tag{8}$$

and

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = \frac{g\sin\theta + 2\frac{\mathrm{d}h}{\mathrm{d}t}\frac{\mathrm{d}\theta}{\mathrm{d}t} + r\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2}{L - \theta r - h}. \tag{9}$$

3.2. Theoretical analysis of stage 2

At the end of the first stage, the heavy load had stopped dropping. So we could assume that the dropping height of the heavy load is h_0. The length of the thread wound on the rod is h_0r. The angular velocity of the light load sweeping around the rod is $\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega_0$. The length of the thread at the end of the light load is $l_0 = L - h_0 - \theta_0$r. The value of h_0, θ_0, ω_0 can be obtained by solving equations (8) and (9).

Then, we can analyze the second stage of the looping pendulum phenomenon, as shown in figure 1(c). In this stage, the heavy load remains stationary while the light load keeps sweeping around the rod. Under such a situation, the analysis of the motion of the light load can still be indicated by the model shown in figures 9(b)–(c) at any time t. In order to avoid confusion with the analysis of the motion in the first stage, we add subscript 1 to each physical quantity in the analysis of the light load in the second stage, except BG changes to $B_1G_1 = (L - \theta r - h_0) - rd\theta$, and the values of the remaining quantities (such as $A_1\,C_1$, $A_1\,D_1$ etc.) are consistent with the analysis of the first stage. By omitting the second order small quantity, we can obtain $A_1\,F_1 = (L - h_0\text{-}\theta r)\mathrm{d}\theta$ $(l = L - h_0 - \theta r\,)$ and $A_1\,E_1 = 0$. Now it can be concluded that the instantaneous velocity of the light load at point A is

$$\frac{\overrightarrow{A_1B_1}}{\mathrm{d}t} = l\frac{\mathrm{d}\theta}{\mathrm{d}t}\vec{i} + 0\vec{j} \tag{10}$$

and the acceleration of the light load at point A can be calculated as [8]

$$\overrightarrow{a_{\mathrm{A}_1}} = \left[-r\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 + l\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\right]\vec{i} + \left[l\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2\right]\vec{j}. \tag{11}$$

The analysis of the force of the light load along and perpendicular to the thread direction (as shown in figure 9(c)) can be shown as

$$F_{\mathrm{T}} + mg\sin\left(\theta + \frac{\pi}{2}\right) = ml\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}, \tag{12}$$

and

$$mg\cos\left(\theta + \frac{\pi}{2}\right) = m\left[-r\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 + l\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\right], \tag{13}$$

where F_{T} indicates the pull force of the light load along the thread.

By simplifying equations (12) and (13), we can obtain the following differential equations:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = \frac{-g\sin\theta + r\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2}{l}, \tag{14}$$

and

$$F_{\mathrm{T}} = m\left[-g\cos\theta + l\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\right]. \tag{15}$$

Assuming that the distance between the light load and the center of the horizontal rod is $S(\theta)$, we can obtain the following relationship from the geometric relationship and Pythagorean theorem:

$$S^2(\theta) = l^2 + r^2. \tag{16}$$

It can be seen from equation (16) that, once the values of l_0 and θ are determined at the initial time in the second stage, and the value of the rod radius r is determined, the trajectory of the light load in the second stage is determined. The angular velocity ω_0 of the light load sweeping around the rod at the beginning of the second stage will determine the duration time of the second stage.

3.3. Theoretical analysis of different variables

The differential equations (8) and (9) are difficult to obtain the analytic solutions. So we use the MATLAB program to solve them numerically in the first stage as mentioned above.

3.3.1. Theoretical analysis of different mass ratios γ. The initial conditions of equations (8) and (9) are as follows: $\frac{dh}{dt} = 0$, $h = 0$, $\frac{d\theta}{dt} = 0$. We take $L = 0.553$ m, $g = 9.793$ m s^{-2}, and initial release angle $\theta = \pi/2$, $\mu = 0.26$, $r = 4.000$ mm. By changing the mass ratios γ, we can obtain the relationship between the dropping height (h) and velocity (v) of the heavy load and the time t using solid lines, as shown in figure 5. We can know that the theoretical simulation calculations are basically consistent with the experimental points, which proves the validity of the above theoretical analysis.

In addition, we can see from the theoretical prediction that, with the increase of γ, the dropping process of the heavy load has a similar tendency compared with that of the free falling motion. If the magnitude of γ is larger than several hundreds, the falling process of the heavy load can be regarded as a free falling process.

3.3.2. Theoretical analysis of different thread lengths L. We fixed the value to $\gamma = 12.4$, $g = 9.793$ m s^{-2}, and initial release angle $\theta = \pi/2$, $\mu = 0.26$, $r = 4.000$ mm. By changing the thread lengths L, we can get the dropping height and velocity of the heavy load with the time, as denoted by the solid line in figure 6. We can see that the theoretical simulation curves and the experimental points are almost consistent.

3.3.3. Theoretical analysis of different initial release angles. Furthermore, we numerically analyze the effect of the initial release angles on the looping pendulum phenomenon, still using the above initial conditions, i.e. $\gamma = 12.4$, $L = 0.553$ m, $g = 9.793$ m s^{-2}, $\mu = 0.26$, and $r = 4.000$ mm. By changing the values of initial release angles, the relationship between the dropping height and velocity of the heavy load with time under different initial release angles can be obtained, as shown by the solid line in figure 7. We can see that the experimental data and theoretical simulation curves are almost consistent, which further proves that the above theoretical analysis is a relatively good one.

3.3.4. Theoretical analysis of different values of friction coefficient between the thread and rod. In fact, the different values of friction coefficient μ between the thread and rod will also affect the phenomenon of the looping pendulum. Using $\gamma = 12.4$, $L = 0.553$ m, $g = 9.793$ m s^{-2}, initial release angle $\theta = \pi/2$, $r = 4.000$ mm to solve equations (8) and (9) through the MATLAB program, we can get the relationship of the dropping height and velocity of the heavy load with time, as denoted by the solid line in figure 10. The black, red, and blue solid lines represent the cases of $\mu = 0$, $\mu = 0.3$, and $\mu = 0.6$, respectively.

It can be seen from figure 10(a) that with the increase of μ, the maximum dropping height of the heavy load will decrease. As can be seen from figure 10(b), for any value of μ, with the increase of time, the velocity of the heavy load will increase at first and then decrease. However, the maximum velocity will also decrease during the dropping process with the increase of μ.

3.3.5. Theoretical analysis of different rod radii. Furthermore, the influence of the radius of the horizontal rod in the looping pendulum phenomenon is analyzed. When taking $\gamma = 12.4$, $L = 0.553$ m, $g = 9.793$ m s^{-2}, initial release angle $\theta = \pi/2$, $\mu = 0.26$, by changing the radius r of the horizontal rod, the relationship of dropping height and velocity of the heavy load with time can be shown by the solid line in figure 11. The solid black, red, and blue lines represent $r = 0$ mm, $r = 10$ mm, and $r = 50$ mm, respectively.

We can conclude from figure 11(a) that with the increase of r, the maximum dropping height will decrease. As can be seen from figure 11(b), for any value of r, with the increase of

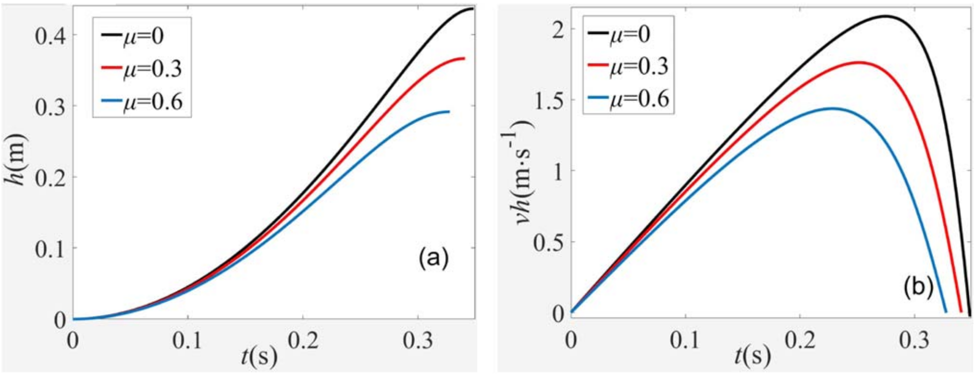

Figure 10. Diagram of the dropping height (a) and dropping velocity (b) with time when changing the values of μ. The solid black, red, and blue lines denote the cases of $\mu = 0$, $\mu = 0.3$, and $\mu = 0.6$, respectively.

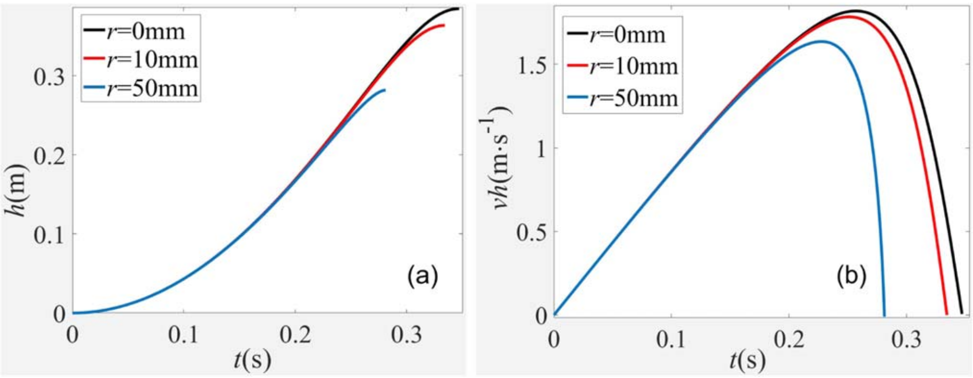

Figure 11. Diagram of the dropping height (a) and dropping velocity (b) with time when changing the values of r. The solid black, red, and blue lines denote $r = 0$ mm, $r = 10$ mm, and $r = 50$ mm, respectively.

time, the dropping velocity of the heavy one will increase at first then decrease. However, with the increase of r, the maximum velocity in the dropping process will decrease.

In summary, through the experiments and corresponding numerical analyses, we analyzed three parameters (i.e. the mass ratio γ, thread length L, and the initial release angle θ of the light load) of the looping pendulum phenomenon. Finally, we predicted the influence of the friction coefficient μ between the thread and rod, and radius r of the horizontal rod on the looping pendulum phenomenon.

3.4. Theoretical analysis of the light load's trajectory

In order to compare the experimental trajectory of the light load (as shown in figure 8(a)), we also give the theoretical numerical analysis results by solving equations (8), (9), (14). We take $\gamma = 15.7$, $L = 0.700$ m, $g = 9.793$ m s^{-2}, initial release angle $\theta = \pi/2$, $\mu = 0.26$, and $r = 4.000$ mm as the initial conditions. The results are shown in figure 8(b).

It can be seen from figure 8 that the tendency of the experimental curve is almost consistent with the theoretical curve, which can prove the validity of the theoretical model for second stage.

4. Conclusion

In this paper, the looping pendulum phenomenon addressed in the 32nd IYPT problems is divided into two stages for experimental and theoretical investigation. It is found that with the increase of the mass ratios γ, thread length L, and initial release angle θ of the light load, the maximum dropping height and the maximum velocity of the heavy load during the dropping process will also increase. However, with the increasing of the friction coefficient μ between the thread and rod or the radius r of the horizontal rod, the maximum dropping height and the maximum dropping velocity of the heavy load will decrease correspondingly. These five factors will also affect the trajectory of the light load in the looping pendulum phenomenon. We believe that our results provide a detailed analyses of the requirements of the IYPT problem.

Note that there are also some errors between the theoretical results and experimental ones. In the experimental study of the looping pendulum phenomenon, the releasing by hand may give an initial velocity to the light load. Recorded video shows that when the light load is initially released, the thread is not straight but bent. The influence of air resistance is not considered in the theoretical analysis. All of these aspects will lead to a small deviation of the experimental data and theoretical analyses. In a further study, we will revise the theoretical model considering the above problems, and then give a better explanation for the looping pendulum phenomenon.

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Appendix

In this appendix, we give the detailed derivation of equation (1), using the methods provided by reference [7].

As is shown in figure 12, when the thread sweeps around the rod, we can first obtain the relationship between the force F in the normal direction at the point of the thread connected the rod, with the tension force T (T_1, T_2) in the thread. The Y axis component of the upward force of the rod on the thread, F, must equal the Y axis downward component of the tension in the thread, $T_1 \sin\theta$. So we can obtain $F = T_1 \sin\theta$ and $T_2 = T_1 \cos\theta$. When the limit of θ goes to zero (the small-angle approximation), we can obtain $\sin(\mathrm{d}\theta) = \mathrm{d}\theta$ and $T_1 = T_2 = T$. Because $F = T\mathrm{d}\theta$, the sliding frictional force over the wrap angle is $\mu F = \mu T\mathrm{d}\theta$, where μ is the coefficient of the sliding friction. The increase in thread tension $\mathrm{d}T$ over the wrap angle $\mathrm{d}\theta$ is the sliding frictional force over $\mathrm{d}\theta$, so we can obtain $\mathrm{d}T = T\mu\mathrm{d}\theta$, $T^{-1}\mathrm{d}T = \mu\mathrm{d}\theta$. By doing the integration for both sides, we can obtain $\int_{T_1}^{T_2} T^{-1}\mathrm{d}T = \int_0^{\theta} \mu\mathrm{d}\theta \left(\text{i.e.} \ln\frac{T_1}{T_2} = \mu\theta\right)$. By taking the index function of both sides, we can obtain $\frac{T_1}{T_2} = e^{\mu\theta}$. Finally, $T_1 = T_2 e^{\mu\theta}$ can be obtained.

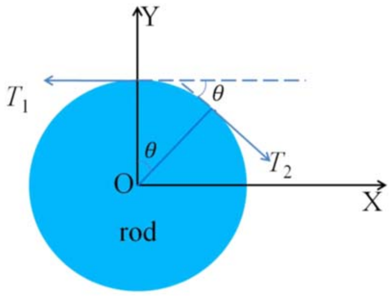

Figure 12. Schematic diagram of the relationship of the thread tension at both ends of the rod.

ORCID iDs

Fan Dai-he https://orcid.org/0000-0003-3009-2631

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