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Experiments and models about the force between permanent magnets: asymptotic analysis of a difficult problem

Pasquale Onorato[1] and **Massimiliano Malgieri**[2]

[1] Department of Physics, Università degli Studi di Trento, Povo (TN), Italy
[2] Department of Physics, Università degli Studi di Pavia, Pavia, Italy

E-mail: massimiliano.malgieri01@universitadipavia.it

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Abstract

We propose a simple experiment, which allows students to explore quantitatively the magnetic interaction between neodymium cylindrical magnets. The experiment employs a precision digital balance, two screws with known thread pitch and two transparent tubes to measure the repulsive force between two magnets as a function of their distance. Different measurements are performed, focusing on the behavior of the interaction force at short and long distances and the role of the magnets' aspect ratio. We discuss the comparison between theoretical expectations resulting from conceptually simple approximate models and experimental results. The experiments employ inexpensive materials and address a relevant topic in the physics curriculum. Thus, they are appropriate for the undergraduate physics laboratory, for advanced high school students, and in the context of teacher education and in-service training to enhance students' knowledge of magnetism.

Keywords: magnetic force, magnetism, modeling, low-cost experiments

(Some figures may appear in colour only in the online journal)

1. Introduction

Many researches in recent decades have shown how experimental activities are significant sources of knowledge in physics teaching/learning practice, since they offer students the chance to contextualize abstract concepts and to challenge their naïve beliefs [1–3]. In the case of electromagnetic phenomena, the main difficulties encountered by students are clearly identified in the literature [4–7] and in recent years several studies have been published focusing on experimental activities as a means to enhance the students' conceptual understanding [8–15].

In the past, some authors proposed a number of experiments on the magnetic interaction between permanent magnets, e.g. employing magnets supported by a balance [15–20] or placing magnets within a glass tube [20, 21]. In many experiments, a magnetic force proportional to $1/r^2$ was found [20, 21][3], other authors discussed the asymptotic $1/r^4$ behavior [17–19] and a few papers [22] measured the behavior of the interaction force when the distance is smaller than the transverse dimension (radius) of the magnets.

In this paper, we present a simple and handy experiment aimed at exploring quantitatively the magnetic interaction by using low-cost materials. Our aim is to improve the results of previous experiments [17–19] where the authors investigated the relationship between magnetic force with respect to the distance of separation between two identical disc or coaxial cylindrical magnets. The experiment uses a digital precision balance, two screws of known thread pitch and two transparent tubes. This simple setup allows a comparison between the experimental results and the predictions provided by theoretical models of magnetic interaction in the asymptotic regions. Motivating questions posed to students at the beginning of the activity concern studying according to which laws the interaction force between two permanent magnets behaves (a) for very short distances and (b) for very long distances.

The complexity of calculating the force between two cylindrical magnets makes the full theoretical solution substantially inaccessible for students. Even when computed, the analytical expression [17, 22] is expressed as an integral of which a numerical solution is then provided. Such complexity leads us to analyse the system in its asymptotic limits, studying separately the trends at very short and very long distances to see whether intuitive understanding of the behavior can be gained in cases where limits apply. This approach is often followed by scientists when faced with complex problems, and requires students to engage in reflection not only to understand what happens within a certain limit, but in relation to what other quantities or dimensions there are in the problem when the limit is reached. In this sense, it is also useful to discuss what happens for magnets with different aspect ratios, where the relevant dimensional scales may be different from one case to another [22].

The activities described in this paper were tested with 30 students from a laboratory course for perspective physics teachers at the University of Trento. The course is centered on electromagnetism, and is organized in such a way that for most topics in the subject we propose a laboratory activity typically meant to address and discuss some of the most relevant learning difficulties reported in the literature. In the case of this particular experiment, the main motivation was that very little attention is usually given in standard textbooks and courses on electromagnetism to the interaction between permanent magnets [17], whereas quite powerful magnets are nowadays cheaply available for teachers to perform simple and meaningful experiments with their students. Further motivation was provided by the difficulties previous cohorts had shown in understanding intuitively the domain of validity of a dipole approximation, both in earlier versions of the experiment here discussed, and in the analysis on the interaction of electrically charged objects using a balance scale [23, 24]. Both in the case of this experiment, and more generally in our course, we adopt a Predict-Observe-Explain strategy [25] whereby each experiment is preceded by the request for a prediction through one or more questions to investigate the students' initial conceptions, which in some cases (such as in this study) are also used as a pre-test. After the experiment, the students discuss among themselves and with teachers the compatibility between their predictions and

[3] However, the approximate inverse square law interaction fails both when the distance between the magnets increases and when the magnets are in close proximity to each other.

the experimental results. In the case of this study, the students also answer a post-test identical to the pre-test several weeks after the experimental activity, to investigate their long-term learning retention.

2. Magnet–magnet interaction

It is quite hard to compute the total interaction force between two cylindrical magnets, and the mathematical techniques needed are not accessible to undergraduate students. For two cylindrical magnets with their magnetic dipoles aligned, the force can be computed analytically using elliptic integrals [26]. An analytical expression was discussed in [27]. The expression describes the force between two cylindrical magnets of the same radius R with their dipoles aligned on the same axis z, which is also the central axis of the cylinders, in the assumption of uniform magnetization M. In these conditions, no forces exist along the other axes x and y, while the force acting along z has the form

$$F_z = -8\pi K_d R^2 \int_0^\infty \frac{J_1^2(q)}{q} \sin h(q\tau_1) \sin h(q\tau_2) \mathrm{e}^{-q\varsigma} \mathrm{d}\varsigma, \tag{1}$$

where $\tau_i = d_i/(2R)$, $i = 1, 2$, are the aspect ratios of the two cylinders, which we have assumed to have the same radius R but possibly different heights d; $\zeta = Z/R$ is the scaled distance between the centers of the two cylinders (where Z is the ordinary distance), $J_1(q)$ is a modified Bessel function of the first kind and we have introduced the magnetostatic energy constant $K_d = \mu_0 M^2/2$ for convenience of notation.

Of course, for an expression such as equation (1), any form of intuitive understanding is impeded. Thus, we make two fundamental choices:

(1) We aim at studying experimentally the asymptotic behavior for short and long distances.
(2) We use as a theoretical lens appropriate simplified models, which can help students to gain conceptual understanding of the problem. In particular, we choose the route of electrostatic analogies, i.e. replacing magnets with electric dipoles or more complex charge distributions according the Gilbert model [28].

The Gilbert model assumes that the force between two magnets is due to magnetic charges near the poles repelling or attracting each other in the same manner as the Coulomb force between electric charges. In the Gilbert model, a magnetic H-field is produced by magnetic charges that are 'smeared' around each pole. This model works, even close to the magnet, where the magnetic field depends heavily on the detailed shape and magnetization of the magnet.

Approaching the computation of the force between two cylindrical magnets by using the Gilbert model, we conclude that formally, the field can be expressed as a multipole expansion: a dipole field, plus a quadrupole field, plus an octupole field, etc. Obviously, thinking in terms of asymptotic behavior (multipole expansion) when we analyse the interaction between two cylindrical magnets we must consider three different length scales, the radius R, the height d and the distance between the magnets x.

With this schematization, we can review the interaction for the different distance ranges.

2.1. Contact force and interaction at short distances

When the distance, x, between the magnets is short (i.e. $x \ll$ R and $x \ll d$) the magnitude of the force between two very close 'magnetic surfaces' is given in the case where $d \ll R$ by

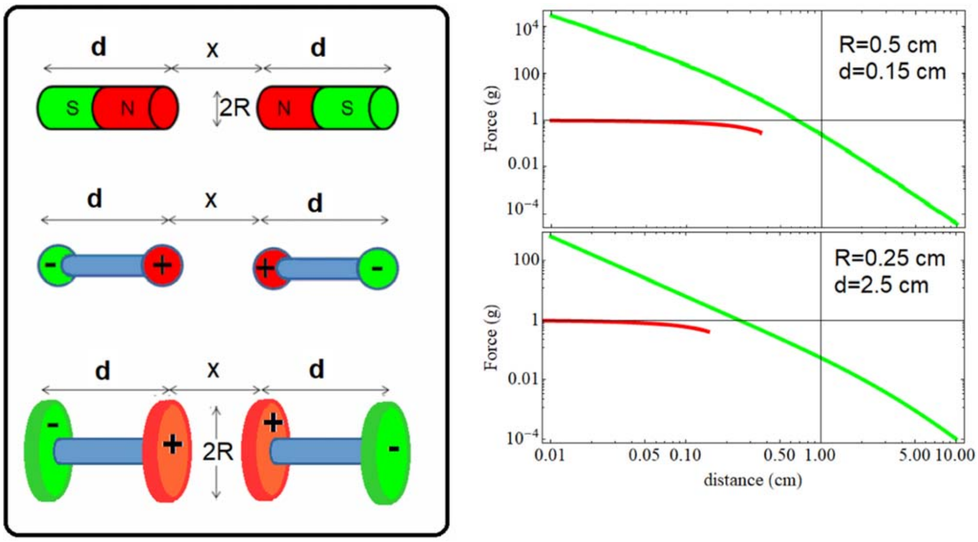


Figure 1. Left: Two models used in the Gilbert approximation, the charged disk approximation (when $x \ll 2R$) and the pointlike charge approximation ($x \gg 2R$). Right: Force versus distance calculated according the Gilbert model. Red line: multipole expansion near the contact. Green line: dipole approximation according to equation (4) with the asymptotic power-law behavior corresponding to the dipole–dipole interaction. This asymptotic regime is reached for short flat magnets (top panel).

$$F_{\text{stick}}^{\infty} = \frac{1}{2\pi\mu_0}\left(\frac{m^2}{d^2}\right)\frac{1}{R^2}, \tag{2}$$

where m is the magnetic moment. The force equation (2) is written in analogy with the force acting between the plates of a capacitor, where a charge $Q = m/d$ is uniformly smeared on each plate:

$$F_{\text{stick}}^{\text{electric}} = \frac{1}{2\pi\varepsilon_0}\left(\frac{Q^2}{R^2}\right).$$

In a more general case, the contact force is reduced for the case of large radii with respect to the length of the magnet [29]:

$$F_{\text{stick}}^{R} \approx F_{\text{stick}}^{\infty}\frac{d}{\sqrt{R^2 + d^2}}.$$

The behavior of the force in proximity of the contact can be obtained using multipole expansion and it is obtained (for $x \ll R$ and $x \ll d$):

$$F_{x<R}(x) \approx F_{\text{stick}}^{R}\left(1 - \frac{x}{\alpha R}\right). \tag{3}$$

This trend is similar to the one of two uniformly charged disks at very short distance (see figure 1 bottom).

2.2. Intermediate and long-distance region

In the intermediate distance region, a well-known expression can be derived from Gilbert's electrostatic model, assuming four interacting pointlike charges [20]:

$$F(x) = F_{\rm stick}^{\infty} \frac{d^2 + R^2}{d^2} R^2 \left(\frac{1}{x^2} + \frac{1}{(x + 2d)^2} - \frac{2}{(x + d)^2} \right)$$
$$\equiv A \left(\frac{1}{x^2} + \frac{1}{(x + 2d)^2} - \frac{2}{(x + d)^2} \right). \tag{4}$$

The same expression can also be derived from low-order expansion of the Bessel functions in equation (1) according to their definitory power series [22]. In the limit of very long distances, equation (4) provides the $1/r^4$ behavior, which is typical of dipole–dipole interactions, but with different constants depending on the magnets' aspect ratio:

Long Magnets From equation (4) in the limit of long distances $\gg d$, and for $d \gg R$:

$$F_\infty(x) = F_{\rm stick}^{\infty} \frac{6d^2R^2}{x^4}. \tag{4b}$$

Flat Magnets From equation (4) in the limit of long distances $x \gg R$, and for $R \gg d$:

$$F_\infty(x) = F_{\rm stick}^{\infty} \frac{6R^4}{x^4}. \tag{4c}$$

3. Experimental procedure and data analysis

An electronic digital balance (with an accuracy of 0.01 g), a transparent tube and a simple screw, which we previously calibrated by measuring the thread pitch (0.076 ± 0.001 cm), were used in this experiment to explore the magnetic force as a function of distance between two identical neodymium cylindrical magnets, as shown in figure 2. One of the magnets was taped onto the balance pan, and the other was fixed on the end of a diamagnetic screw, approximately 15 cm long. A support frame held the screw suspended above the precision electronic scale.

The magnets were then aligned in such a way to have a repulsive interaction, and a plexiglass cylinder[4], slightly wider than the magnets, was used to maintain the alignment during the experiment. Data measurements were performed in the interval of 0–6 cm separating the two magnets. Distance measurements were performed by counting the number of turns of the screw, starting from the point at which the two magnets are almost in contact.

The setup described above was used to carry out two series of measurements for three different pairs of magnets. In each series, the contact distance, x, was varied from 50–1 mm. At the end of each series, the setup was disassembled and then reassembled again to prevent systematic errors. The reliability of the results of the magnetic dipole moment of the disc magnet was tested by performing the experiment in the same conditions for five trials.

The repulsive forces obtained are shown in figures 3–5, where the graphs report the magnitude of magnetic force versus distance of separation between the two magnets in log-log plot scale.

[4] We employed two different plexiglass tubes. The wider tube needed for thin disks has internal diameter $D \approx 10$ mm and height $h \approx 7.5$ cm; the narrow tube has internal diameter $D \approx 5.7$ mm and height $h \approx 10$ cm.

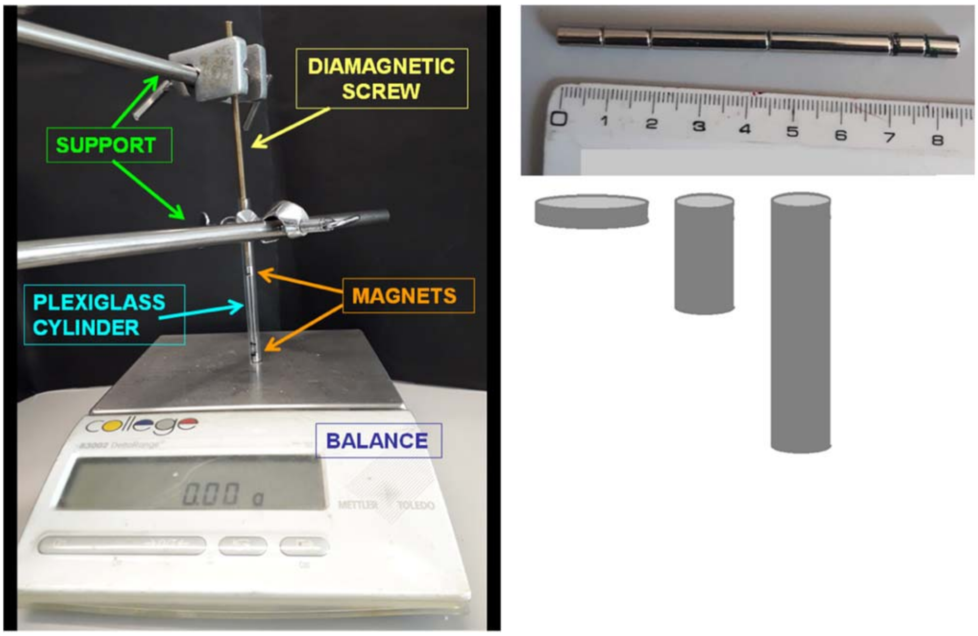


Figure 2. Experimental setup, showing the digital balance. Right: The cylindrical magnets. The bottom magnet was taped onto the balance pan and in principle in this situation it could affect the measurements of the electronic digital balance. In order to make sure that such effect is negligible, we compared the force measurements performed with the magnets at fixed distance in two different cases: taping the bottom magnet onto the balance pan and placing the bottom magnet on a non-magnetic cylinder 4.5 cm in height. Force measured in the two cases differs by less than 0.5%, showing a negligible magnetic interference.

We show the measured force for three different aspect ratios, thin disks ($h_1 = 1.55 \pm 0.05$ mm and $2R_1 = 9.90 \pm 0.05$ mm) $\tau_i = 0.16$, short cylinders ($h_2 = 10.15 \pm 0.05$ mm and $2R_2 = 4.85 \pm 0.05$ mm), $\tau_2 = 2.09$ and a long cylinder ($h_2 = 25.35 \pm 0.05$ mm and $2R_2 = 4.85 \pm 0.05$ mm), $\tau_2 = 5.23$.

3.1. Thin disks

The first example of graphs from five trials of data measurements of magnetic force as a function of distance between disc magnets is shown in figure 3.

Starting from equation (3), we found the curve that has the best fit to the series of data points corresponding to the first measurements at very short distance ($x < R$) (red line in figure 3). Thus, we obtain $F^R_{\rm stick} = 420 \pm 20$ g; $\alpha R = 0.50 \pm 0.05$ cm and $F^\infty_{\rm stick} \approx F^R_{\rm stick}\frac{R}{d} = 1360 \pm 70$ g.

As the distance increases $x \gg d$ the force varies inversely as the fourth power of the distance between the two magnets with a power exponent $n = 4$. Thus, according equation (4*c*),

$$F_\infty(x) = F^\infty_{\rm stick}\frac{6R^4}{x^4} = \frac{B}{x^4},$$

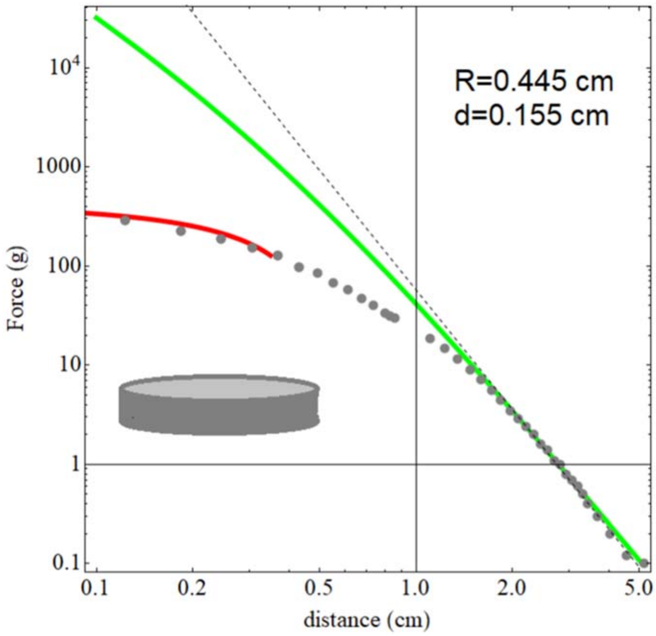


Figure 3. Force versus contact distance for thin disks ($h_1 = 1.55 \pm 0.05$ mm and $2R_1 = 9.90 \pm 0.05$ mm). Gray circles: measured values; continuous red line: multipole expansion at short distances; continuous green line: pointlike dipole approximation equation (4); dashed line: asymptotic point dipole model equation (4*c*).

we found the value of B, so that the curve has the best fit to the series of data points corresponding to distances between 2 and 5 cm. The best-fit value is $B = 514 \pm 4$ g cm^2 with coefficient of determination, ρ squared, $\rho^2 = 0.999\,22$. It follows that we can compare the extrapolated values $F_{\text{stick}}^{\infty} = \frac{B}{6R^4} \approx 1370 \pm 80$ g from the long-distance fit and $F_{\text{stick}}^{\infty} = 1360 \pm 70$ g from the short-distance fit and we have very good agreement between the two values.

3.2. Long cylinders

The second example of graphs from trials of data measurements of magnetic force as a function of distance between two long cylindrical magnets is shown in figure 4.

Starting from equation (3), we found the curve that has the best fit to the series of data points corresponding to the first measures at very short distance ($x < R$ and $x < d$) (red line in figure 4). Thus, we obtain $F_{\text{stick}}^{R} = 595 \pm 10$ g and $\alpha R = 0.38 \pm 0.02$ cm with the coefficient of determination $\rho^2 = 0.999\,86$.

As the distance increases and $x \gg d$ the force varies according to equation (4), while the asymptotic dipole/dipole model of interaction, equation (4*b*), is not reached with our experimental setup.

Through a best-fit procedure in the range of data points between 0.7 and 5 cm we found the value of $A = F_{\text{stick}}^{\infty}\frac{d^2+R^2}{d^2}R^2 = 37 \pm 2$ g cm^2, with coefficient of determination $\rho^2 = 0.999\,89$. It follows that we can compare the extrapolated values $F_{\text{stick}}^{\infty} = A\frac{d^2}{(d^2+R^2)R^2} \rightarrow F_{\text{stick}}^{\infty} \approx 580 \pm 40$ g, from the long-distance fit and

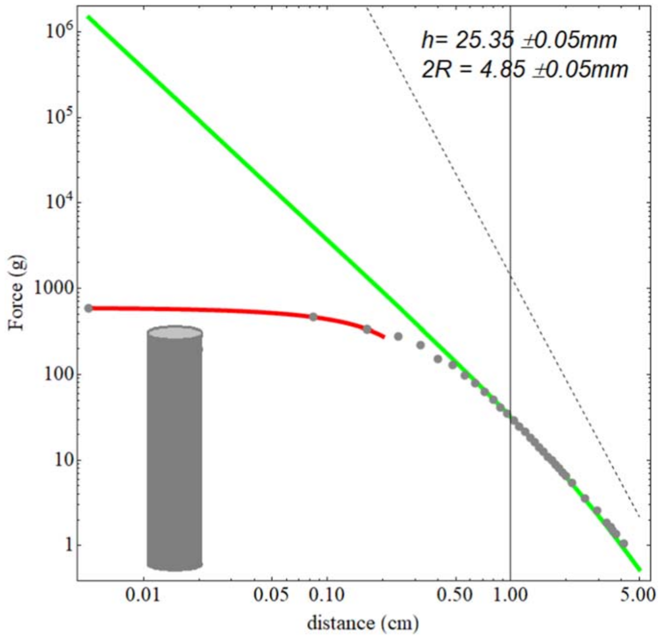


Figure 4. Force versus contact distance for thin disks ($h_2 = 25.35 \pm 0.05$ mm *and* $2R_2 = 4.85 \pm 0.05$ mm). Gray circles: measured values; continuous red line: multipole expansion at short distances; continuous green line: pointlike dipole approximation equation (4); dashed line: asymptotic point dipole model equation (4*b*).

$F_{\text{stick}}^{R} = 595 \pm 10$ g from the short-distance fit, observing that the two results overlap significantly.

3.3. Short cylinders

The third example of graphs from trials of data measurements of magnetic force as a function of distance between two short cylindrical magnets is shown in figure 6.

Starting from equation (3), we found the curve that has the best fit to the series of data points corresponding to the first measures at very short distance ($x < R$ and $x < d$) (red line in figure 4). Thus, we obtain $F_{\text{stick}}^{R} = 450 \pm 10$ g and $\alpha R = 0.40 \pm 0.02$ cm. As the distance increases $x \gg d$ the force varies according to equation (4) while the asymptotic dipole/dipole model of interaction, with the force of $1/r^4$, is not reached.

Through a best-fit procedure in the range of data points between 0.5 and 4 cm, we found the value of $A = F_{\text{stick}}^{\infty}\frac{d^2+R^2}{d^2}R^2 = 26.7 \pm 0.3$ g cm^2, with coefficient of determination $\rho^2 = 0.9992$. Thus, we can compare the extrapolated values $F_{\text{stick}}^{\infty} = A\frac{d^2}{(d^2+R^2)R^2} \rightarrow F_{\text{stick}}^{\infty} \approx 410 \pm 80$ g, from the long-distance fit and $F_{\text{stick}}^{R} = 450 \pm 10$ g from the short-distance fit, observing again that the two results overlap significantly.

The agreement between the parameters obtained in each experiment for the short and intermediate/long distance separately shows the effectiveness of both the experiment and theoretical approach.

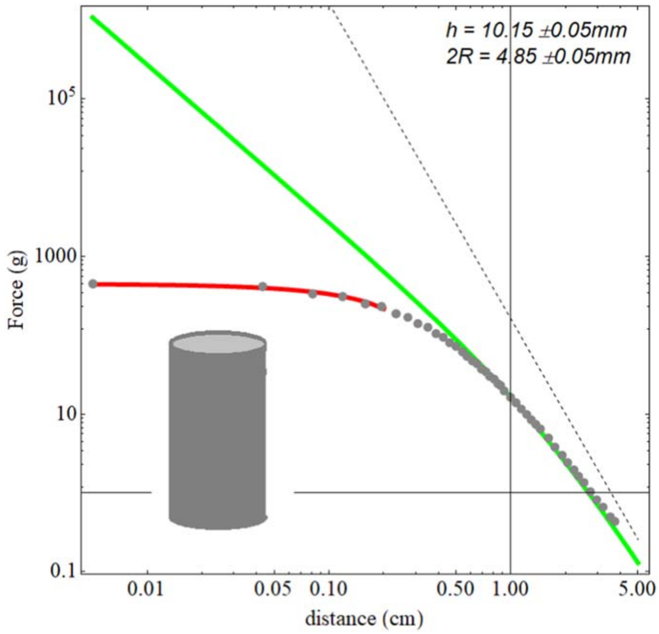


Figure 5. Force versus contact distance for short cylinders ($h_2 = 10.15 \pm 0.05$ mm *and* $2R_2 = 4.85 \pm 0.05$ mm). Gray circles: measured values; continuous red line: multipole expansion at short distances; continuous green line: pointlike dipole approximation equation (4); dashed line: asymptotic point dipole model $1/r^4$ as in equations (4*b*) and (4*c*).

4. Educational results

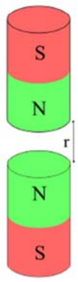
In the prediction/pre-test phase, the item reported in Figure 6 was given to the students.

The most striking result of this pre-test was that a majority of the students (16/30) showed the alternative idea that the point-charge model is valid in this case for long distances and the dipole model for short distances. The correct answer *c.* was chosen by one third of the students (10/30). Other distractors were not very relevant. The full results, along with those of the delayed post-test, are reported in table 1.

At the end of the activity, judging from discussion and the written reports of the student groups, all students, or at least all groups, seemed to have understood the asymptotic behavior of magnetic forces. A large part of the discussion with the groups involved understanding how to determine *a priori* the respective domains of validity of the point-charge model and dipole model.

About two months after the activity, the students answered the same item in the context of a delayed post-test, which also contained other questions related to various topics and experiments performed in the course. This time, the majority of students (17/26 or 65%) provided the correct answer for the behavior of the magnetic force with distance. However, the aforementioned misconception showed a certain degree of robustness, reappearing in almost one third (8/26) of the responses in the delayed post-test.

Question. Two cylindrical magnets are aligned as shown in the figure below, with r indicating the distance between their nearest ends.



Concerning the behaviour of their interaction force as r varies, which of the following statements you think is correct? (pick only one)

a. The force is proportional to $\frac{1}{r^2}$, similar to the one between two point charges, for any value of r.
b. The force is proportional to $\frac{1}{r^4}$ like in the case of dipole-dipole interaction, for short distances, and proportional to $\frac{1}{r^2}$, as in the case of point charges, for long distances.
c. The force is proportional to $\frac{1}{r^4}$ as in the case of dipole-dipole interaction, for long distances; while for short distances, as the magnets are almost in contact, it reaches an upper value.
d. The force is proportional to $\frac{1}{r^4}$, similar to the one between two dipoles, for any value of r.

Figure 6. Item concerning magnet–magnet interaction force, which was given to students both as a pre-test and delayed post-test.

Table 1. Students' answers to the item reported in figure 6.

Answer	Pre-test (N = 30)	Delayed post-test (N = 26)
a.	2	0
b.	16	8
c.	10	17
d.	2	1

5. Conclusions

In this paper, we proposed a simple experiment meant to validate simplified and asymptotic models exploring quantitatively the magnetic force between permanent magnets, a paradigmatic case where the asymptotic analysis of a difficult problem is effective. We note that the true asymptotic regime for long distances can only be reached for thin disc magnets, while for magnets of different aspect ratio the expression equation (4) gives a reasonably good approximation for intermediate distances, as reported by other authors. The experiment employs simple and inexpensive materials, addresses a relevant topic in the physics curriculum, and allows students to practice not so common laboratory skills.

The activities were tested with 30 undergraduate students from a laboratory course at the University of Trento in less than 1 h. The students preformed the experiment and data analysis

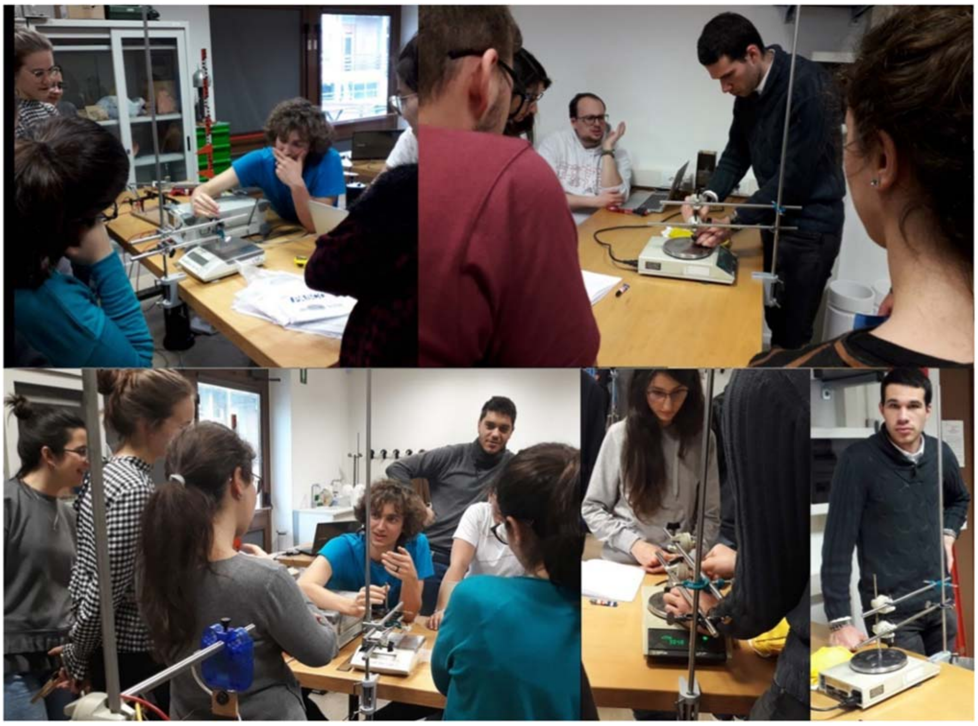


Figure 7. Students busy with the experimental activities.

autonomously, working in large groups (figure 7). The students also answered some questions before, during and after the experimental activities.

From their answers given before the activity, we observe that students, although familiar with the behavior of real magnets, often do not discard the idea that the force may diverge when the magnets are in contact. More generally, they apply formulas using power-law relationships with no concern about the range of validity of each approximation. Finally, the misconception that the magnetic force is always proportional to $1/r^2$ [20, 21] is common among our students. Before the experiments, only one third of our students applied the correct power-law formula for dipole–dipole interaction. In fact, from the pre-test we identified a more general critical point in the students' ideas, concerning the nature and validity of asymptotic behavior. Students are often unable to understand the domain of validity of an approximate model; when they consider a dipole–dipole approximation, they sometimes wonder whether it applies at short or long distances; and are confused by the simultaneous presence of different length scales such as R, d….

After performing the experiment, the students recognized that the force does not diverge when the magnets are put in contact, and carried out a detailed data analysis based on asymptotic behavior. The latter quantitative analysis can produce an effective understanding of the complex relationship between force and distance. About two months after the activity, two thirds of our students could correctly characterize the behavior of the magnetic force between two identical magnets as a function of the distance.

Even though the students had already gone through previous undergraduate lab courses, they were unfamiliar with drawing relationships in log-log or semilogarithmic scales, and appreciated the usefulness of such techniques to transform power laws into linear relationships. The laboratory activity represents an appropriate context to practise such lab competencies, which are explicitly mentioned in the AAPT recommendations for the undergraduate physics laboratory curriculum [30].

In summary, considering the relevance of the topic and the relative straightforwardness of the experimental apparatus and procedure, the theoretical-experimental activities proposed here can offer a valuable contribution to the promotion of the successful conceptual understanding of magnetic phenomena in an appropriate learning environment, and to enhance the students' laboratory skills.

ORCID iDs

Pasquale Onorato https://orcid.org/0000-0002-2110-7582
Massimiliano Malgieri https://orcid.org/0000-0002-9254-2354

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