A Letters Journal Exploring the Frontiers of Physics

January 2020

EPL, **129** (2020) 11001
doi: 10.1209/0295-5075/129/11001

www.epljournal.org

Dirac CP violation as a window to mass hierarchy in neutrino oscillation

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received 12 November 2019; accepted in final form 21 January 2020
published online 14 February 2020

PACS 14.60.Pq – Neutrino mass and mixing
PACS 11.30.Er – Charge conjugation, parity, time reversal, and other discrete symmetries
PACS 12.15.Ff – Quark and lepton masses and mixing

Abstract – Unresolved neutrino problems of mass hierarchy and CP violation have a more interesting interplay in neutrino phenomenology. Differing from the discrete symmetry approach, this issue is pursued from the perspective of neutrino oscillation with the matter effect. The contributions from δ_{CP} and Δm_{31}^2 are entangled in a complex manner when the mass effect is considered. After investigating the role of δ_{CP} in $A_{CP}^{(m)}$, it is found that at some determined values of δ_{CP}, neutrino mass hierarchy can be determined at the same time. By introducing a statistical quantity λ, the requirement of distinguishable mass hierarchy and its confidence level can be investigated. The set of the values of δ_{CP} forms a window for addressing mass hierarchy that is calculated for MINOS, NOνA and DUNE.

Introduction. – Neutrino oscillation has opened up an interesting window into physics beyond the standard model. As a quantum mechanics effect, it can be described by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix between weak eigenstates and mass eigenstates. Even though all mixing angles, including close to zero θ_{13}, in PMNS have been measured by solar, atmosphere, and reactor neutrino experiments, some fundamental problems, such as neutrino mass order, CP violation in neutrino oscillation, and Dirac/Majorana fermions still remained unsolved [1–4]. These problems are not independent and some pioneering studies were made for the combined effects from neutrino experiments [5,6] as well as theoretical work like weak-basis invariance [7] and quark-lepton complementarity [8,9]. More popular works focused on the issues from the discrete symmetry approach [10–15]. The observed pattern of neutrino mixing is believed to encode some underlying flavour symmetry such as A_4, S_4, T' and D_n that exists at a high-energy scale and breaks at a lower scale to the residual symmetry of the leptonic sector [16–21]. This approach predicts two large mixing angles, one of which is close to zero θ_{13}, and one is an unmeasured Dirac CP violation. Mass hierarchy (MH) is another problem that is often treated as an input in the discrete symmetry approach. Different results are obtained for normal hierarchy (NH) and inverted hierarchy (IN). With the additional help of the chosen models such as GUT or seesaw mechanics, the neutrino mass matrix can be generated and the mass order predicted [22–24]. In this paper, we study the interplay of neutrino CP violation and mass hierarchy in long-baseline neutrino oscillations without any additional symmetry assumptions. We point out that the asymmetry A_{CP}^m is an important tool for the study of the combined effects of Dirac CP violation and MH in long-baseline experiments. Based on the results of the neutrino experiments performed to date, we show a Dirac CP violation value window in which mass hierarchy can be determined simultaneously in MINOS, NOνA and DUNE.

The explanation of neutrino oscillation requires different mass eigenvalues. However, oscillation probability only depends on the absolute value of Δm_{31}^2 unless a non-vanishing CP violation δ_{CP} is obtained. Thus, MH is coupled to the CP violation in the framework of the standard neutrino model. The coupling effect is suppressed by the small quantities $\alpha \equiv \Delta m_{21}^2/\Delta m_{31}^2$ and θ_{13}. A small contribution of CP violation requires more experiments with long-baseline, intense beams and high energy resolution in the future. When the matter effect is considered, the situation becomes more complex. Dirac CP violation

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can provide a CP violation source in the leptonic sector. This yields the difference between neutrino oscillation and anti-neutrino oscillation. The asymmetry of matter and anti-matter distribution in the Earth also provides a CP asymmetric interaction mechanism. δ_{CP} only controls a part of CP violation in the matter effect. Even when the Dirac CP violation vanishes, neutrino oscillation and its CP process are not equal. For $\nu_\mu \to \nu_e$ with the matter effect, Δm^2_{31} and θ_{13} are replaced by their effective values [25,26], introducing a complex dependence on the MH parameter Δm^2_{31}. Matter effect makes CP violation entangled with neutrino mass order. In this paper, we start from the relation between the Dirac CP violation and neutrino MH. By analyzing the roles of δ_{CP} and Δm^2_{31} in neutrino oscillation with the matter effect, it is found that CP violation is related to the problem of MH, helping us obtain an in-depth understanding of the role of Dirac CP violation and explore a new approach for addressing unknown MH.

The rest of the paper is organized as follows: in the next section, neutrino mixing and oscillation are briefly reviewed. The role of the Dirac CP violation in neutrino oscillation is analyzed in the third section. By comparing with vacuum oscillation, the complex entanglement between δ_{CP} and Δm^2_{31} in the Earth is obtained. The contribution of δ_{CP} to the matter effect is extracted. Next, in the fourth section, the asymmetry $A^{(m)}_{CP}$ error is derived in terms of the error transfer formula. The contributions from the different physical quantities are analyzed. To analyse MH, a statistical quantity λ is proposed. It is defined as the difference of $A^{(m)}_{CP}$ between normal hierarchy (NH) and inverted hierarchy (IH) over the standard deviation. For a determined δ_{CP}, the value of λ represents the confidence of MH. Some numerical results are given in the fifth section. The requirement of addressing MH for the detector parameters, beam energy E and distance L, is calculated with unknown δ_{CP}. We also analyze the range of δ_{CP} to provide an estimate of the neutrino mass order in MINOS, NOνA, and DUME. Finally, a summary is given in the last section.

Review of neutrino oscillation. – As a quantum effect, a neutrino weak eigenstate, denoted by ν_α ($\alpha = e, \mu, \tau$) is a linear combination of mass eigenstates ν_i ($i = 1, 2, 3$) with different mass eigenvalues that can be expressed by a unitary rotation [1,27]

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U^*_{\alpha i} |\nu_i\rangle. \tag{1}$$

For 3 flavor neutrinos, $U_{\alpha i}$ is the PMNS matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathrm{diag}(e^{i\frac{\alpha_1}{2}}, e^{i\frac{\alpha_2}{2}}, 1),$$

parameterized by 3 mixing angles θ_{ij}, and 3 leptonic CP violation phases $\delta_{CP}, \alpha_1, \alpha_2$. Currently, unlike the 3 measured mixing angles, the 3 leptonic CP violation phases remain unknown. Among the 3 phases, α_1 and α_2 only exist for Majorana neutrinos that involve slight differences in some phenomena such as $0\nu\beta\beta$ decay. Another phase, δ_{CP}, often called the Dirac CP violation, exists not only for the Dirac neutrinos but also for the Majorana neutrinos. This phase leads to more applications in particle physics and cosmology. Considering neutrino oscillation from flavor ν_α to ν_β at the distance of L with energy E, the oscillation probability is

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= \Big| \sum_i U^*_{\alpha i} U_{\beta i} e^{-i m_i^2 L/(2E)} \Big|^2 \\ &= \delta_{\alpha\beta} - 4 \sum_{i>j} \mathrm{Re}(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \\ &\quad \times \sin^2\left(\frac{\Delta m^2_{ij} L}{4E} \right) \\ &\quad + 2 \sum_{i>j} \mathrm{Im}(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \\ &\quad \times \sin\left(\frac{\Delta m^2_{ij} L}{2E} \right), \end{aligned} \tag{2}$$

with $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$. For ν_μ to ν_e, the CP violation effect is doubly suppressed by the small value of α and near zero θ_{13}. The CP violation contribution can be enhanced in long-baseline experiments in which the matter effect must be considered. When a neutrino travels through the Earth, the electrons in matter will interact with the neutrino. Due to a lack of positron interaction, asymmetric contributions for neutrino oscillations and anti-neutrino oscillations give rise to an enhanced difference. Replacing Δm^2_{31} and θ_{13} in eq. (2) by their matter-dependent values [26], we obtain

$$\Delta m^2_{31} \to \Delta_m m^2_{31} = \Delta m^2_{31} \sqrt{(1-A)^2 - 4A\sin^2(\theta_{13})}, \tag{3}$$

$$\sin(2\theta_{13}) \to \sin(2\theta_{13})_m = \frac{\sin(2\theta_{13})}{\sqrt{(1-A)^2 - 4A\sin^2(\theta_{13})}}, \tag{4}$$

with $A = 2\sqrt{2} G_F N_e E / \Delta m^2_{31}$ and electron number density of the Earth N_e, and we have the $\nu_\mu \to \nu_e$ oscillation probability in the Earth, up to $\mathcal{O}^2(\alpha, \theta_{13})$, as

$$\begin{aligned} P^{(m)}(\nu_\mu \to \nu_e) &= c_0^{(m)} + \alpha c_{1s}^{(m)} \sin(\delta_{CP}) + \alpha c_{1c}^{(m)} \cos(\delta_{CP}) \\ &\quad + \alpha^2 c_2^{(m)} + \dots, \end{aligned} \tag{5}$$

with

$$c_0^{(m)} = \sin^2(\theta_{23}) \sin^2(2\theta_{13}) \frac{\sin^2[(1-A)\Delta]}{(1-A)^2}, \tag{6}$$

$$\begin{aligned} c_{1s}^{(m)} &= -\sin(2\theta_{13}) \sin(2\theta_{12}) \sin(2\theta_{23}) \cos(\theta_{13}) \\ &\quad \times \frac{\sin[(1-A)\Delta] \sin[A\Delta]}{A(1-A)} \sin(\Delta), \end{aligned} \tag{7}$$

$$c_{1c}^{(m)} = \sin(2\theta_{13})\sin(2\theta_{12})\sin(2\theta_{23})\cos(\theta_{13}) \times\frac{\sin[(1-A)\Delta]\sin[A\Delta]}{A(1-A)}\cos(\Delta), \quad (8)$$

$$c_2^{(m)} = \cos^2(\theta_{23})\sin^2(2\theta_{12})\frac{\sin^2[A\Delta]}{A^2}. \quad (9)$$

In the above, the CP violation δ_{CP} is coupled to the matter effect correction A in a complex from.

Dirac CP violation in $A_{CP}^{(m)}$ with matter effect. – CP violation in PMNS involves a difference between the neutrino oscillation and anti-neutrino oscillation. A more sensitive CP violation observable is the asymmetry between $P(\nu_\mu - \nu_e)$ and $P(\bar{\nu}_\mu - \bar{\nu}_e)$. In vacuum, the asymmetry is given by

$$\begin{aligned} A_{CP} &\equiv P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu - \bar{\nu}_e) \\ &= -J_{CP}\left\{\sin\left(\frac{\Delta m_{21}^2 L}{2E}\right) + \sin\left(\frac{\Delta m_{32}^2 L}{2E}\right)\right. \\ &\quad \left. + \sin\left(\frac{\Delta m_{13}^2 L}{2E}\right)\right\}, \end{aligned}$$

with weak basis invariant

$$J_{CP} = \frac{1}{8}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\sin\delta_{CP}.$$

When δ_{CP} vanishes, the neutrino oscillation probability $P(\nu_\mu \to \nu_e)$ is equal to its CP corresponding $P(\bar{\nu}_\mu \to \bar{\nu}_e)$, and A_{CP} vanishes. However, this is not true when the matter effect is considered. The corrections from the matter effect can be seen more clearly by expanding eq. (3),

$$\Delta m_{31}^2 \to \left\{(1-A) - \frac{2As_{13}^2}{(1-A)}\right\}\Delta m_{31}^2 + \mathcal{O}(s_{13}^3).$$

The correction factor of Δm_{31}^2 in the first term is slightly less than 1, reducing the oscillation probability $P^{(m)}(\nu_\mu \to \nu_e)$ sensitivity to MH. For the anti-neutrino oscillation, the probability can be obtained by changing $A \to -A$ and $\delta_{CP} \to -\delta_{CP}$. Expanding $A_{CP}^{(m)}$ in terms of the small A ($\sim 10^{-4}$)

$$\begin{aligned} A_{CP}^{(m)} &\equiv P^{(m)}(\nu_\mu \to \nu_e) - P^{(m)}(\bar{\nu}_\mu \to \bar{\nu}_e) \\ &= \Delta c_0^{(m)} + \alpha\Delta c_{1s}^{(m)}\sin(\delta_{CP}) \\ &\quad +\alpha\Delta c_{1c}^{(m)}\cos(\delta_{CP}) + \mathcal{O}^3(\alpha, \theta_{13}, A), \end{aligned} \quad (10)$$

with

$$\begin{aligned} \Delta c_0^{(m)} &= 4\sin^2(\theta_{23})\sin^2(2\theta_{13})\sin(\Delta) \\ &\quad \times\left\{\sin(\Delta) - \Delta\cos(\Delta)\right\}A, \\ \Delta c_{1s}^{(m)} &= -2\Delta\sin(2\theta_{13})\sin(2\theta_{12})\sin(2\theta_{23}) \\ &\quad \times\cos(\theta_{13})\sin^2(\Delta), \\ \Delta c_{1c}^{(m)} &= 2\Delta\sin(2\theta_{13})\sin(2\theta_{12})\sin(2\theta_{23})\cos(\theta_{13})\cos(\Delta) \\ &\quad \times\left[\sin(\Delta) - \Delta\cos(\Delta)\right]A. \end{aligned}$$

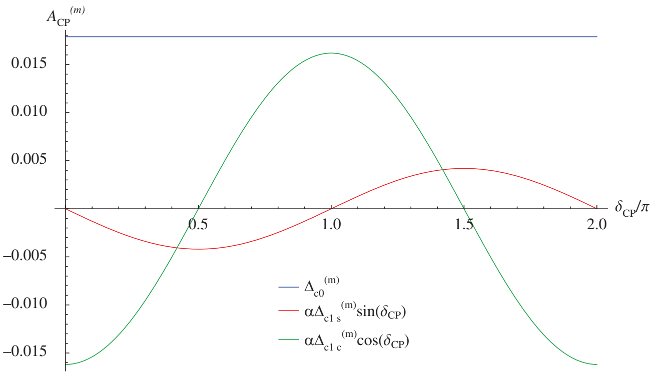


Fig. 1: $A_{CP}^{(m)}$ coefficients *vs.* δ_{CP} at $L = 1500$ km and $E = 1.7$ GeV.

When $\delta_{CP} = 0$, only $\Delta c_{1s}^{(m)}$ vanishes. Other two non-vanishing terms $\Delta c_0^{(m)}$ and $\Delta c_{1c}^{(m)}$ stem from the matter effect. Even more remarkably, CP violation is entangled with MH by A and Δ in the order of α (see fig. 1 for details). Due to the dependence not only on the absolute value of Δm_{13}^2 but also on its sign, $A_{CP}^{(m)}$ provides an approach for addressing the neutrino MH problem by entanglement with δ_{CP}.

Mass hierarchy in $A_{CP}^{(m)}$. – The latest global fit results have been given for the NH case and IH case, respectively [28–30] (see table 1 for details). If there is a gap of $A_{CP}^{(m)}$ between NH and IH, the neutrino MH can be determined. In the formation of the gap, it is important to consider the error band of $A_{CP}^{(m)}$. This error band is spanned by all parameters deviation. When the error band of NH overlaps the error band of IH, MH cannot be determined. By contrast, when the error bands of NH and IH are distinguishable, we can obtain the information regarding MH. Using the error transfer formula, the standard deviation of $A_{CP}^{(m)}$ becomes

$$\delta A_{CP}^{(m)} = \sqrt{\sum_q F_q^2(\delta q)^2}, \quad (11)$$

with parameters q taking $\theta_{12}, \theta_{13}, \theta_{23}, \Delta m_{21}^2, \Delta m_{31}^2$, beam energy E, oscillation distance L, and also unknown δ_{CP}. The coefficient F_q is $F_q = \partial A_{CP}^{(m)}/\partial q$. Now, let us analyze the role of δ_{CP} in eq. (11). Dirac CP violation affects the deviation of $A_{CP}^{(m)}$ in two ways: coefficient F_q and deviation δq. In $F_{\delta_{CP}}$, the contribution from $\Delta c_1^{(m)}c$ is suppressed by the small A, so that $F_{\delta_{CP}}$ is dominated by the $\Delta c_{1s}^{(m)}$ term. Notice that because the contribution from the $\Delta c_{1s}^{(m)}$ term is proportional to $\cos(\delta_{CP})$, $F_{\delta_{CP}}$ is vanishing at the maximal CP violation. The other F_q also depends on the unknown value of δ_{CP}, but by contrast mainly contributing from the $\Delta c_{1s}^{(m)}$ term at the maximal CP violation. In addition to the F_q coefficient in eq. (11), all of the parameter measurement accuracy δq

Table 1: Best-fit values with the standard derivation for three flavour neutrino oscillation parameters from [29]. (For other global fit results, see [28,30].)

Parameter	NH (1σ range)	IH (1σ range)
$\sin^2\theta_{12}/10^{-1}$	2.81–3.14	2.81–3.14
$\sin^2\theta_{23}/10^{-2}$	4.10–4.46	5.67–6.05
$\sin^2\theta_{13}/10^{-2}$	2.08–2.22	2.07–2.24
$\Delta m^2_{21}/10^{-5}$ (eV)^2	7.52 ± 0.18	7.52 ± 0.18
$\Delta m^2_{32}/10^{-3}$ (eV)^2	2.444 ± 0.034	-2.55 ± 0.04

together determines the error of $A^{(m)}_{CP}$. Notice that the deviation of Dirac CP violation plays an important role, and even dominates $\delta(A^{(m)}_{CP})$ in some cases. Before further discussing the error space of $A^{(m)}_{CP}$, we must focus on the deviation of the Dirac CP violation $\delta(\delta_{CP})$. Being δ_{CP} in a circle $(0, 2\pi)$, its deviation must be limited to a finite range when it is determined by neutrino experiments. It is recommended that a minimal limit of its standard deviation should be less than $\pi/4$ in [31], corresponding to 2σ C.L. on a half-cycle. The statistical condition of δ_{CP} gives a minimal distinguishing limit at its value domain. Now, the next issue is to study the effect of the δ_{CP} value on the gap in the making. With the help of the statistical condition, fig. 2 shows that a gap appears with increasing source-detector distance L due to the accumulation of asymmetric contributions from the interactions with the matter effect. Furthermore, the gap can be described precisely by the number of statistical deviations λ, defined as

$$\lambda = \frac{\left|A^{(m),NH}_{CP} - A^{(m),IH}_{CP}\right|}{\delta(A^{(m)}_{CP})|_{NH} + \delta(A^{(m)}_{CP})|_{IH}}. \tag{12}$$

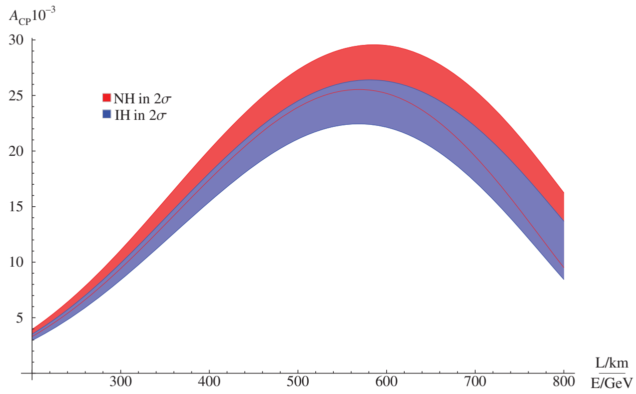
A gap appears corresponding to $\lambda = 1$, which means that NH and IH can be divided as 1σ C.L.. Now, we consider the sensitivity of λ to the global fit results. The expression $A^{(m),IH}_{CP}$ for IH can be obtained by replacing

$$\Delta m^2_{31} \to -\Delta m^2_{31}$$

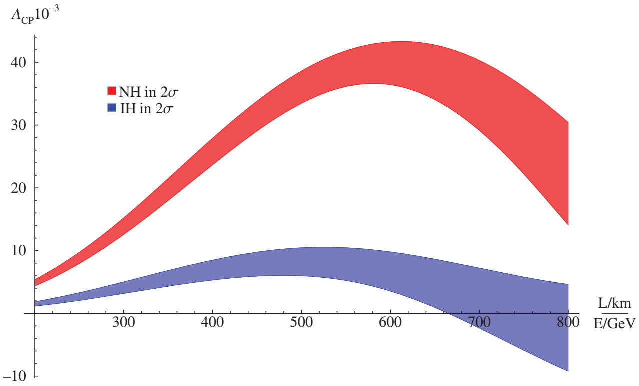
in NH $A^{(m),NH}_{CP}$,

$$\begin{aligned}
&A^{(m),NH}_{CP} - A^{(m),IH}_{CP} = \\
&P^{(m),NH}(\nu_\mu \to \nu_e) \\
&-P^{(m),NH}(\nu_\mu \to \nu_e)|_{A\to -A,\delta_{CP}\to-\delta_{CP}} \\
&-P^{(m),NH}(\nu_\mu \to \nu_e)|_{A\to -A,\Delta\to-\Delta,\alpha\to-\alpha} \\
&+P^{(m),NH}(\nu_\mu \to \nu_e)|_{\delta_{CP}\to-\delta_{CP},\Delta\to-\Delta,\alpha\to-\alpha}.
\end{aligned}$$

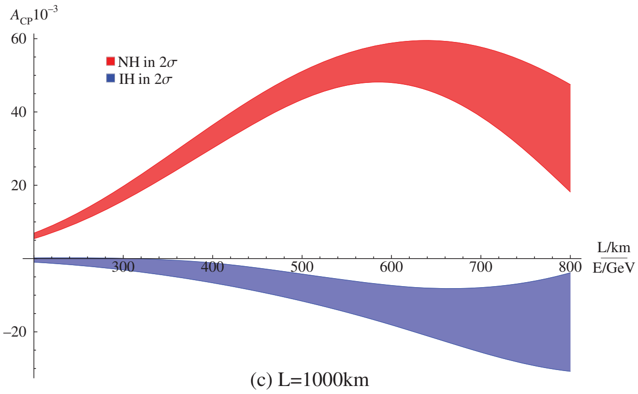
Clearly, if all of the neutrino parameters have the same values for NH and IN, λ vanishes up to $\mathcal{O}^2(\alpha, \theta_{13}, A)$. This means that λ is controlled by the difference of the global fit results for NH and IH, which is the reason why $A^{(m)}_{CP}$ is sensitive to MH. (Only θ_{23} and the MH parameter Δm^2_{32} have different global fit results in table 1.)



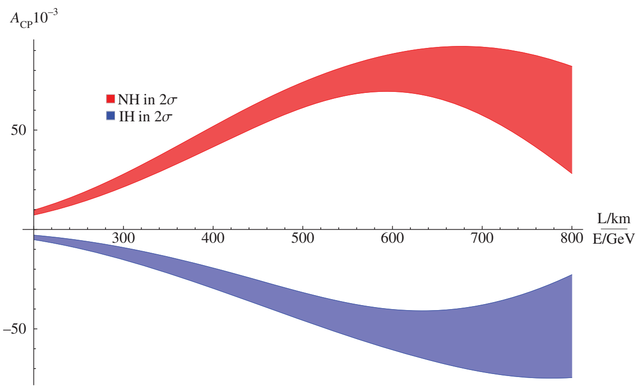
(a) L=50km



(b) L=500km



(c) L=1000km



(d) L=2000km

Fig. 2: A gap of $A^{(m)}_{CP}$ for NH and IH with different lengths of baselines in 2σ C.L. The purple area is the overlay area from NH and IH. The best fit values of the mixing angles and squared-mass difference can be found in table 1.

Table 2: λ values at $\delta_{CP} = 3\pi/2$ and a window range corresponding to $\lambda > 3$ and $\lambda > 5$ at MINOS, NOνA and DUNE.

Exp.	$\lambda\|_{\delta_{CP}=3\pi/2}$	δ_{CP}/π range for $\lambda > 3$	δ_{CP}/π range for $\lambda > 5$
MINOS	3.5	$[0.41, 0.53] \cup [1.41, 1.53]$	–
NOνA	11.1	$[0.18, 0.62] \cup [1.18, 1.62]$	$[0.34, 0.56] \cup [1.34, 1.56]$
DUNE	9.2	$[0.14, 0.70] \cup [1.14, 1.70]$	$[0.29, 0.58] \cup [1.29, 1.58]$

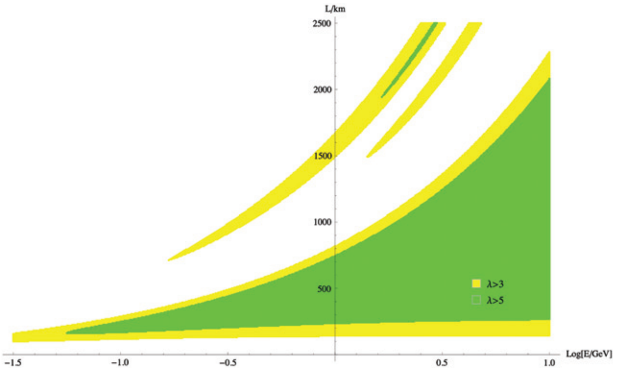


Fig. 3: Parameter space for MH in the plane E-L. The yellow area (green area) indicates the separation of NH and IH in 3σ (5σ) C.L., *i.e.*, $\lambda = 3$ (5).

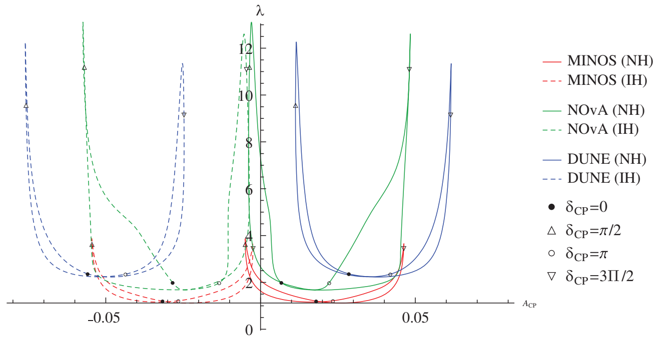


Fig. 4: Values of λ and $A_{CP}^{(m)}$ at NOνA, DUME and MINOS are shown.

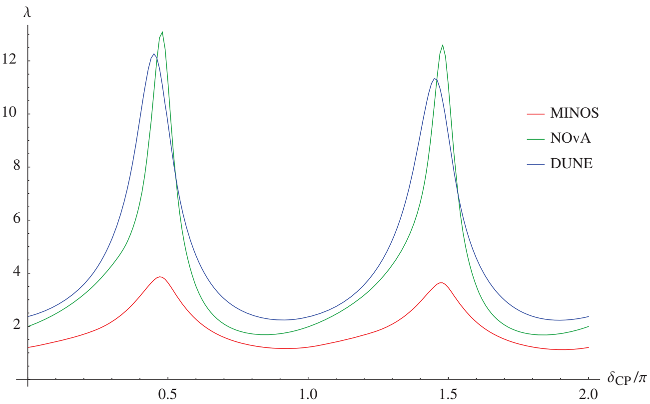


Fig. 5: Dirac CP violation for neutrino MH in NOνA, DUME and MINOS.

Numerical results. –

Detecting parameters E and L to MH. Using the global fit results in table 1, we investigate the MH requirement for the neutrino detection parameters. As an unknown parameter, Dirac CP violation takes all its possible values in the $(0, 2\pi)$ range. Its error has been taken to satisfy the statistical condition with a minimal limit of the standard deviation of less than $\pi/4$. In ideal detecting conditions $\delta L = \delta E = 0$, and the parameter space of E and L corresponding to distinguishing MH in 3σ and 5σ C.L. is shown in fig. 3. The minimal distance of the baseline is approximately 100.1 km (162.6 km) in 3σ (5σ) C.L. corresponding to a minimal beam energy of approximately 31.6 MeV (56.2 MeV).

A window to MH in NOνA, DUME and MINOS. Some long-baseline neutrino oscillation experiments have been run or are planned to address the leptonic CP violation and/or neutrino MH problem. With fixed detecting parameters E and L, when taking each δ_{CP} value, we obtain a point in the plane $A_{CP}^{(m)}$-λ. With δ_{CP} taking all its possible values, the line forms a closed loop in fig. 4.

Another more interesting issue is the value of δ_{CP} that can make a distinguishable MH. For a chosen δ_{CP}, when $\lambda > 3$ (5), MH is distinguished in 3σ (5σ) C.L. A set of δ_{CP} values forms a window for addressing the problem of MH. In fig. 5, the window for MH at MINOS, NOνA and DUNE is shown in the plane λ-δ_{CP}. Table 2 lists the λ values of some long-baseline neutrino oscillation experiments at $\delta_{CP} = 3\pi/2$ and the range of Dirac CP violation corresponding to 3σ and 5σ C.L. Here, the actual energy resolution has been considered which tends to increase the error in the denominator of eq. (12) and suppress λ. Due to the sensitivity to energy error in high-energy neutrino beams, the suppressing effect on λ will become more notable. An effective method to enhance the confidence level is to improve the energy resolution of the detectors in the high-energy area. The results in table 2 show that only a narrow window is open near the maximal Dirac CP violation. This arises from the error of the Dirac CP violation. Only at the point of the maximal CP violation does the contribution from the energy resolution vanish. When considering Dirac CP violation and taking the minimal distinguishable error, $\delta[\delta_{CP}] = \pi/4$, a relatively large error dominates the error band of $A_{CP}^{(m)}$. Thus, the window rapidly closes at a range away from $\delta_{CP} = 3\pi/2$ (another maximal CP violation, $\delta_{CP} = \pi/2$ is disfavoured [28]).

Summary. – In summary, the asymmetry $A_{CP}^{(m)}$ in long-baseline experiments provides an important observable for investigating the interplay of Dirac CP violation and MH problem. The two issues are entangled in $A_{CP}^{(m)}$ in the complex form. By analyzing the role of δ_{CP}, we found that MH can be de determined at some values of δ_{CP}. A new statistical quantity λ is defined to describe the confidence level of the MH problem. The window of δ_{CP} has been given in table 2 for the MINOS, NOνA and DUNE experiments. The results show that when Dirac CP violation is determined within the window range, MH can be obtained at the same time. Revealing the entanglement of Dirac CP violation and MH in long-baseline neutrino oscillations helps us understand neutrino physics more clearly in phenomenology.

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The authors would like to thank Dr. Rong Li for useful discussions. Additionally, this work was supported in part by the Fundamental Research Funds for the Central Universities and by NSFC Grant No. 11775165.

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