IOP Publishing Physica Scripta

Phys. Scr. 95 (2020) 045605 (6pp) https://doi.org/10.1088/1402-4896/ab5ffc

Electron acoustic waves in atmospheric magnetized plasma

H R Pakzad, K Javidan[1] and P Eslami

Department of Physics, Ferdowsi University of Mashhad, 91775-1436 Mashhad, Iran

E-mail: ttaranomm83@yahoo.com, Javidan@um.ac.ir and eslami@um.ac.ir

Received 14 October 2019, revised 14 November 2019
Accepted for publication 9 December 2019
Published 13 February 2020

Abstract

Propagation of electron acoustic solitary waves is studied in magnetized plasmas consisting of hot and cold electrons and stationary ions in the presence of varying magnetic field. We show that a space dependent magnetic field adds a dissipative term in the wave equation of motion without presenting known dissipation sources. Localized waves propagating in such space dependent magnetic field are governed by the modified KdV-Burgers equation. The numerical results show that, this phenomenon leads to the formation of oscillatory backward moving shock waves in the plasma. Characters of shock profile, according to the plasma parameters are discussed.

Keywords: electron-acoustic waves, solitary waves, modified KdV-burgers equation, varying magnetic field

(Some figures may appear in colour only in the online journal)

1. Introduction

Nonlinear phenomena and their effects in physics have received considerable attentions in the past few decades, especially in the formation and propagation of solitary waves. The study of nonlinear localized waves in plasmas as a reach nonlinear media is an attractive subject in theoretical physics as well as experimental and laboratory purpose. There are several important phenomena in space environments and astrophysical situations which can be understood only through nonlinear analysis, such as the cusp region of the terrestrial magnetosphere [1, 2], geomagnetic tail [3] and description of dayside auroral acceleration region [4, 5], beside experimental applications [6–10]. Electron acoustic (EA) waves is a special kind of plasma wave fluctuations which may occur in media with two distinct electron populations referred to cold and hot electrons. The propagation of EA solitary waves (EASWs) in different plasma systems has been studied by several authors in unmagnetized two electron plasmas [11–13] as well as in magnetized plasmas [14–18]. Large amplitude ion and EASWs in unmagnetized plasmas also have been investigated by Lakhina *et al* [19]. The nonlinear propagation of the EA waves in magnetized plasma was considered by Dubouloz *et al* [14]. They have indicated that the electric field spectrum produced by an EASW is not significantly modified at the presence of a constant magnetic field. The properties of obliquely propagating EASWs in magnetized plasmas have been studied by Mace and Hellberg [17]. They showed that negative potential EASWs corresponding to compression of the cold electron density can be created in such media. Mamun *et al* [18] studied properties of obliquely propagating EAWs in magnetized plasmas. Their model supports EAWs with a positive potential, which corresponds to a hole (hump) in the cold (hot) electron number density. Ergun *et al* [20, 21] observed that BEN bursts in the dayside auroral zone have three-dimensional wave structure by including the magnetic field effects. The external magnetic field and the wave obliqueness are found to change the properties of the EA waves significantly. In all mentioned researches, external magnetic field has been considered as a constant vector throughout the medium, but we know that in a realistic situation, magnetic field is not a constant vector at all. In this work, we have tried to treat the problem using numerical solutions, beside an analytical evaluation using the small amplitude perturbation technique. Such situation can be present more realistic propagation of EASW in the earth atmosphere, where the magnetic field clearly has spatial variations [22].

Outlines of this paper are as follows: the basic dynamical equations governing our plasma model is presented in the section 2. Modified Korteweg-de Vries-Burgers (mKdV-B) equation is derived for nonlinear propagation of EA waves in

[1] Author to whom any correspondence should be addressed.

the section 3. Localized solution of mKdV-B equation is discussed and time evolution of the solution in the medium is analyzed in section 4. The last section is devoted to some concluding remarks.

2. Basic equations

We consider homogeneous plasmas consisting of a cold electron fluid, hot electrons obeying a Maxwellian distribution and stationary ions in the presence of a space dependent external magnetic field $\vec{B} = B(r)\hat{z}$. The nonlinear dynamics of EASWs is extracted from the continuity and motion equations for cold electrons, in addition to the Poisson's equation [23] as:

$$\begin{aligned} &\frac{\partial n_c}{\partial t} + \vec{\nabla}.(n_c\vec{u_c}) = 0\\ &\frac{\partial u_c}{\partial t} + (\vec{u_c}.\vec{\nabla})\vec{u_c} = a\vec{\nabla}\varphi - b(\vec{u_c}\times\hat{z})\\ &\nabla^2\varphi = \frac{1}{a}n_c + n_h - \left(1+\frac{1}{a}\right). \end{aligned} \tag{1}$$

In the above equations, n_c (n_h) is the cold (hot) electron number density normalized by its equilibrium values n_{c0} (n_{h0}).u_c is the cold electron fluid velocity normalized bythe phase speed of EA ($C_e = (k_B T_h/am_e)^{1/2}$) in which$k_B$ is the Boltzmann's constant,e is the electron charge, m_e electron mass and $a = n_{h0}/n_{c0}$. $b(r) = (eB(r)/mc)/\omega_{pc}$ is the cold electron cyclotron frequency normalized by the cold electron plasma frequency ω_{pc} and φ is the electrostatic wave potential normalized by $k_B T_h/e$. The time and space variables are in units of the cold electron plasma period ω_{pc}^{-1} and the hot electron Debye radius λ_{Dh}, respectively. The basic set of equation (1) can be expanded as:

$$\begin{aligned} &\frac{\partial n_c}{\partial t} + \frac{\partial}{\partial x}(n_c u_{cx}) + \frac{\partial}{\partial y}(n_c u_{cy}) + \frac{\partial}{\partial z}(n_c u_{cz}) = 0\\ &\frac{\partial u_{cx}}{\partial t} + \left(u_{cx}\frac{\partial}{\partial x} + u_{cy}\frac{\partial}{\partial y} + u_{cz}\frac{\partial}{\partial z}\right)u_{cx} = a\frac{\partial\varphi}{\partial x} - bu_{cy}\\ &\frac{\partial u_{cy}}{\partial t} + \left(u_{cx}\frac{\partial}{\partial x} + u_{cy}\frac{\partial}{\partial y} + u_{cz}\frac{\partial}{\partial z}\right)u_{cy} = a\frac{\partial\varphi}{\partial y} + bu_{cx}\\ &\frac{\partial u_{cz}}{\partial t} + \left(u_{cx}\frac{\partial}{\partial x} + u_{cy}\frac{\partial}{\partial y} + u_{cz}\frac{\partial}{\partial z}\right)u_{cz} = a\frac{\partial\varphi}{\partial z}\\ &\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = \frac{1}{a}n_c + n_h - \left(1+\frac{1}{a}\right). \end{aligned} \tag{2}$$

As mentioned before, the Maxwellian distribution for hot electrons is considered as follows:

$$n_h = \mathrm{e}^{\varphi}. \tag{3}$$

3. Reductive perturbation method

In order to study the EASWs in the plasma model under consideration, we construct a weakly nonlinear theory of the electrostatic waves with small but finite amplitude which leads to a scaling of the independent variables through the stretched coordinates $\xi = \varepsilon^{\frac{1}{2}}(l_x x + l_y y + l_z z - \lambda t)$ and $\tau = \varepsilon^{\frac{3}{2}}t$, where ε is a small dimensionless parameter measuring the weakness of the dispersion and nonlinearity. Parameters l_x, l_y and l_z are directional cosines of the wave vector $\boldsymbol{k}$ along the x, y and z axes, respectively, so that $l_x^2 + l_y^2 + l_z^2 = 1$. The λ is unknown phase velocity which will be determined later. In the above transformation λ is normalized by C_e. We also expand n_c, u_{cx} , u_{cy} , u_{cz} and φ in power series of ε as follows:

$$\left[\begin{aligned} n_c &= 1 + \varepsilon n_{1c} + \varepsilon^2 n_{2c} + ...\\ u_{cx} &= \varepsilon^{3/2}u_{1cx} + \varepsilon^2 u_{2cx} + ...\\ u_{cy} &= \varepsilon^{3/2}u_{1cy} + \varepsilon^2 u_{2cy} + ...\\ u_{cz} &= \varepsilon u_{1cz} + \varepsilon^2 u_{2cz} + ...\\ \varphi &= \varepsilon\varphi_1 + \varepsilon^2\varphi_2 + ... \end{aligned}\right. \tag{4}$$

Now we write equation set (2) in different powers of ε separately. From the lowest order of ε in the continuity equation, the z component of the momentum equation and Poisson's equation we have: $n_{1c} = \frac{-al_z^2}{\lambda^2}\varphi_1$, $u_{1cz} = \frac{-al_z}{\lambda}\varphi_1$ and $\lambda = l_z$. One can write the lowest order of the x and y components of the momentum equation as:

$$u_{1cx} = \frac{-al_y}{b}\frac{\partial\varphi_1}{\partial\xi},\ u_{1cy} = \frac{al_x}{b}\frac{\partial\varphi_1}{\partial\xi} \tag{5}$$

which are the x and y components of the cold electron drift, arising due to the balance between the electric and Lorentz forces respectively. We can also obtain the next higher order of ε in x and y components of the momentum equation as follows:

$$\begin{aligned} u_{2cx} &= \frac{-al_x\lambda}{b}\frac{\partial}{\partial\xi}\left(\frac{1}{b}\frac{\partial\varphi_1}{\partial\xi}\right),\\ u_{2cy} &= \frac{-al_y\lambda}{b}\frac{\partial}{\partial\xi}\left(\frac{1}{b}\frac{\partial\varphi_1}{\partial\xi}\right). \end{aligned} \tag{6}$$

To the next higher order in ε, from the continuity equation, the z-component of the momentum equation and Poisson's equation, we obtain:

$$\left|\begin{aligned} &\frac{\partial n_{1c}}{\partial\tau} - \lambda\frac{\partial n_{2c}}{\partial\xi} + l_x\frac{\partial u_{2cx}}{\partial\xi} + l_y\frac{\partial u_{2cy}}{\partial\xi}\\ &\quad + l_z\frac{\partial}{\partial\xi}(n_{1c}u_{1cz} + u_{2cz}) = 0\\ &\frac{\partial u_{1cz}}{\partial\tau} - \lambda\frac{\partial u_{2cz}}{\partial\xi} + u_{1cz}l_z\frac{\partial u_{1cz}}{\partial\xi}\\ &\quad - al_z\frac{\partial\varphi_2}{\partial\xi} = 0\\ &\frac{\partial^2\varphi_1}{\partial\xi^2} - \frac{n_{2c}}{a} - \varphi_2 - \frac{1}{2}{\varphi_1}^2 = 0 \end{aligned}\right. . \tag{7}$$

Finally, from (5)–(7), the following nonlinear equation yields:

$$\frac{\partial\varphi_1}{\partial\tau} - \frac{l_z}{2}(1 + 3a)\varphi_1\frac{\partial\varphi_1}{\partial\xi} + \frac{l_z}{2}\frac{\partial^3\varphi_1}{\partial\xi^3} + \frac{l_z}{2}(1 - l_z^2)\frac{\partial}{\partial\xi}\left[\frac{1}{b}\frac{\partial}{\partial\xi}\left(\frac{1}{b}\frac{\partial\varphi_1}{\partial\xi}\right)\right] = 0. \quad (8)$$

Propagation of EASWs in non-uniform magnetized plasma with hot and cold electrons and stationary ions is fully evaluated by the equation (8). In fact, the above equation is derived for the magnetized plasma, affected by a varying magnetic field which has been introduced for the first time. In the equation (8), 'b' is a function of the variable magnetic field and plays important role in characteristics of wave propagation in the medium. The fourth term in the above equation gives us a new kind of dissipation and obviously it depends on the variation of magnetic field in space. For greater values of dissipative term, strength of shock waves becomes dominant and, in such cases, energy of solitary wave reduces due to radiation. It can be found that equation (8) changes into the KdV equation [16, 18, 23], when the magnetic field is a constant vector. In other words, solitary wave solutions in plasmas with space dependent magnetic field are not valid [23–27].

4. Numerical results and discussion

In this section, effects of variable magnetic field on the features of the EASWs are explained. Equation (8) can be written with more details as follows:

$$\frac{\partial\varphi_1}{\partial\tau} + A\varphi_1\frac{\partial\varphi_1}{\partial\xi} + B\frac{\partial^3\varphi_1}{\partial\xi^3} + C\frac{\partial^2\varphi_1}{\partial\xi^2} + D\frac{\partial\varphi_1}{\partial\xi} = 0, \quad (9)$$

where the coefficients are:

$$A = \frac{-l_z}{2}\{1 + 3a\},$$
$$B = \frac{l_z}{2}\left\{1 + \frac{1 - l_z^2}{b^2}\right\},$$
$$C = \frac{l_z}{2}(1 - l_z^2)\left(-\frac{3}{b^3}\right)\frac{\partial b}{\partial\xi},$$
$$D = \frac{l_z}{2}(1 - l_z^2)\left[\frac{3}{b^4}\left(\frac{\partial b}{\partial\xi}\right)^2 - \frac{1}{b^3}\frac{\partial^2 b}{\partial\xi^2}\right]. \quad (10)$$

The parameter $b(\xi)$ in the above equation is related to the variable magnetic field. Variation of $b(\xi)$ acts as an important parameter for creating shock wave profiles.

Equation (9) is called the modified KdV-Burgers (mKdVB) equation while $C(\xi)$ and $D(\xi)$ are dissipation coefficients. In a uniform magnetic field ($C = D = 0$), evolution equation is a usual KdV equation [18, 23]. It means that stable solitary waves in uniform magnetic fields move without any dispersion. But our derived equation shows that in a non-uniform magnetic field, solitary profiles emerge backward propagating waves (which are called shock waves) while interacting with varying magnetic field. Using equations (9) and (10) we can estimate characteristics of these

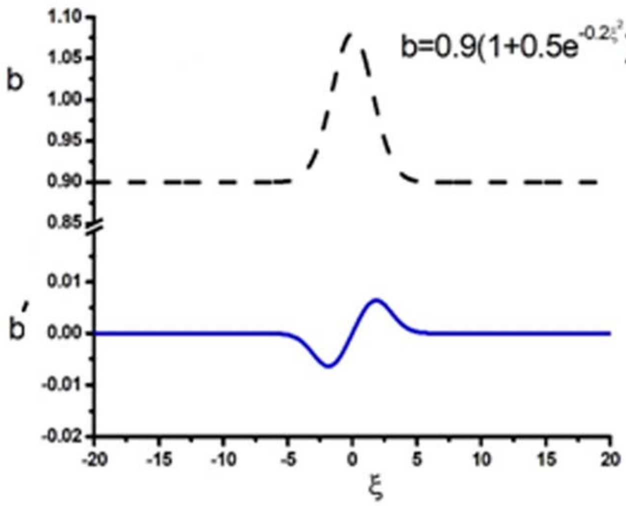

Figure 1. Magnetic field $\boldsymbol{b}(\boldsymbol{\xi})$and its derivative $\frac{\partial \boldsymbol{b}}{\partial \boldsymbol{\xi}}$ as functions of $\boldsymbol{\xi}$.

waves. According to (10), dissipation coefficients (C and D) are proportional to the $\frac{1}{b^3}$ and $\frac{1}{b^4}$. This means that smaller magnetic fields create larger dissipation effects. It may be also noted that, maximum value of C and D coefficients respect to the cosine direction is occurred for $l_z = \frac{\sqrt{3}}{3}$, thus the amplitude of shock wave becomes larger when l_z goes toward the value $l_z = 0.577$.

It is interesting, if we study behavior of EASW after interacting with the variable magnetic field. We can expect that such shock structures are observed in plasmas with space dependent magnetic fields, like what happens in atmospheric plasmas. For this reason, we consider a localized Gaussian perturbation for magnetic field as:

$$b = b_0(1 + b_m e^{-\alpha\xi^2}), \quad (11)$$

where b_0 is the background magnetic field and $b_0 b_m$ defines strength of magnetic field perturbation. Indeed, in real situations (like atmospheric plasmas) variation of magnetic field is very close to the Gaussian function (11). At positions far from the varying magnetic field, the magnetic field is constant ($b = b_0$). Figure 1 demonstrates magnetic field $b(\xi) = 0.9(1 + 0.5e^{-0.2\xi^2})$ and its derivative ($b' = \frac{\partial b}{\partial\xi}$) respect to ξ.

The figure 1 clearly shows that dissipation term acts only in the region of magnetic field perturbation. This means that outside of the perturbation region, dissipative terms are absent and therefore (9) reduces to the well-known KdV equation. Thus, we can choose the solution of the KdV equation as initial condition for numerical simulation of propagated wave in the magnetic field perturbation.

Therefore, we assume that the stationary solution can be expressed as $\varphi_1 = \varphi_1(\chi)$, where $\chi = \xi - u\tau$ and u is the soliton velocity. Stationary solitary wave solution is:

$$\varphi_1 = \varphi_0 \mathrm{sech}^2\left(\frac{\chi}{w}\right), \quad (12)$$

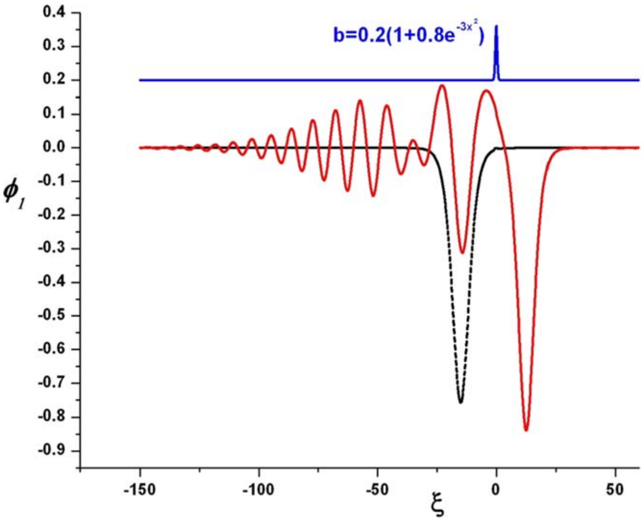

Figure 2. Time evolution of the electron acoustic soliton while interacting with the variable magnetic field $b(\xi) = 0.2(1 + 0.8e^{-3\xi^2})$ with $l_z = 0.95$, $a = 0.5$ and $u = 0.3$. Dashed line presents initial solitary profile and solid line is the solitary wave profile after interaction with the varying magnetic field.

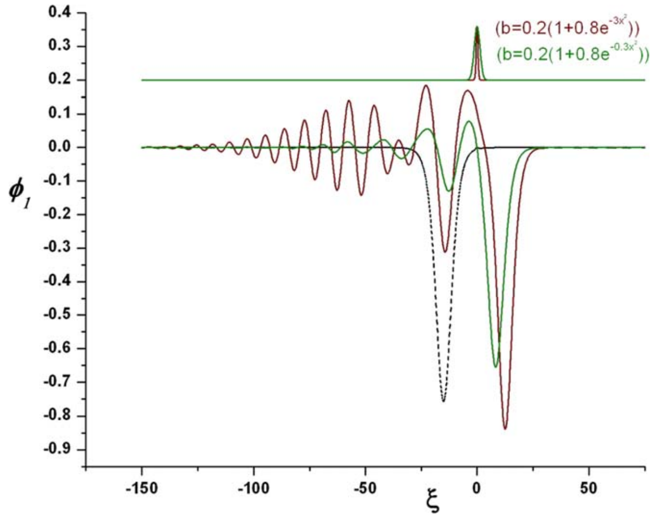

Figure 3. Time evolution of the electron acoustic soliton while interacting with two different magnetic fields $b(\xi) = 0.2(1 + 0.8e^{-3\xi^2})$ and $b(\xi) = 0.2(1 + 0.8e^{-0.3\xi^2})$ with fixed values of $l_z = 0.95$, $a = 0.5$ and $u = 0.3$.

where $\varphi_0 = 3u/A$ is the soliton amplitude and $w = 2\sqrt{B/u}$ defines its width. It is clear that, the initial position of the solitary solution should be located out of perturbation region. We use numerical calculation for simulating the evolution of the initial solution (soliton) in varying magnetic field. The equation (9) has been solved using the fourth order Runge–Kutta method for time derivation and finite difference method for space derivations. The grid spacing has been chosen $\Delta\zeta = 0.001$ and 0.005 (as cross check for numerical stability of solution) and time grid spacing has been taken as $\Delta\tau = 0.0001$.

As C and D coefficients are complicated functions of $b(\xi)$ and its derivatives respect to spatial position, we use some plots to explain effects of varying magnetic field on the behavior of propagated EA waves. Figures present the EASWs at $t = 0$ (far from the location of variable magnetic field) and after the interaction when it has passed from the varying magnetic field.

An important example of varying magnetic field is the magnetic field of the earth atmosphere. The geomagnetic field varies on a range of scales at different position of the atmosphere [22]. Three components of earth magnetic field, usually assigned as X (northerly intensity), Y (easterly intensity) and Z (vertical intensity, positive downwards). Measurements show that, horizontal magnetic field near the north pole are small (0–1000 nT). It raises up to 40 000 nT at the equator and then decreases to 20 000 nT at the south pole. Variation of horizontal magnetic field has almost a bell-shaped pattern. Also there exists some high intensity spots of magnetic field in the atmosphere. For example total intensity of the earth magnetic field rapidly jump to 65 000 nT in -60(latitude):+120(altitude) while it is about 25 000 nT in -60:0. The best curve for description these variations is the Gaussian function as we have used as the magnetic field perturbations in following calculations. Indeed, it is more realistic perturbation. However, it is possible to find other shapes of varying magnetic field in other physical situations. Please note that our calculations is independent of the selected function for the varying magnetic field.

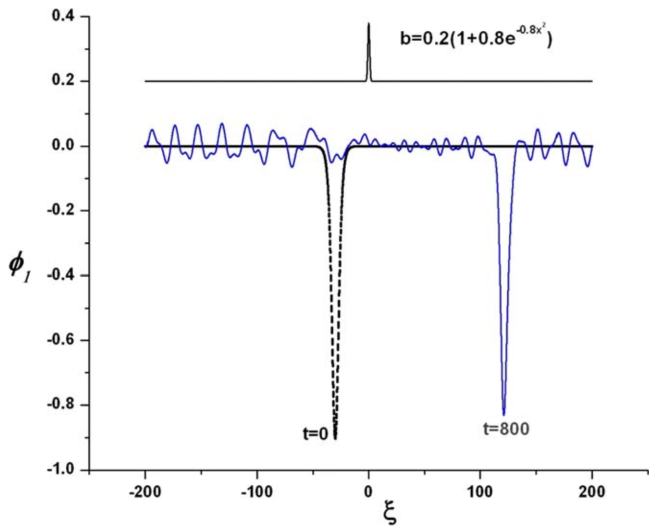

Figure 4. Time evolution of the electron acoustic soliton while interacting with the variable magnetic field $b(\xi) = 0.2(1 + 0.8e^{-0.8\xi^2})$ with $l_z = 0.95$, $a = 0.6$ and $u = 0.4$.

Figure 2 shows EASWs evolution before and after interacting with the varying magnetic field $b(\xi) = 0.2(1 + 0.8e^{-3\xi^2})$ with directional cosine $l_z = 0.95$ at $\xi = -15$ (initial position) and $\xi = +12$ (after the interaction). Initial soliton moves toward the perturbation with initial speed $u = 0.3$. We expect to find a solitary wave after passing through the perturbation, with different amplitude and width in comparison with initial solitary wave. According to the figure 2, the EA wave radiates some amount of its energy during their traveling through the varying magnetic field which is appeared like an oscillatory shock wave. It is important to note that the shock wave starts to propagate from the

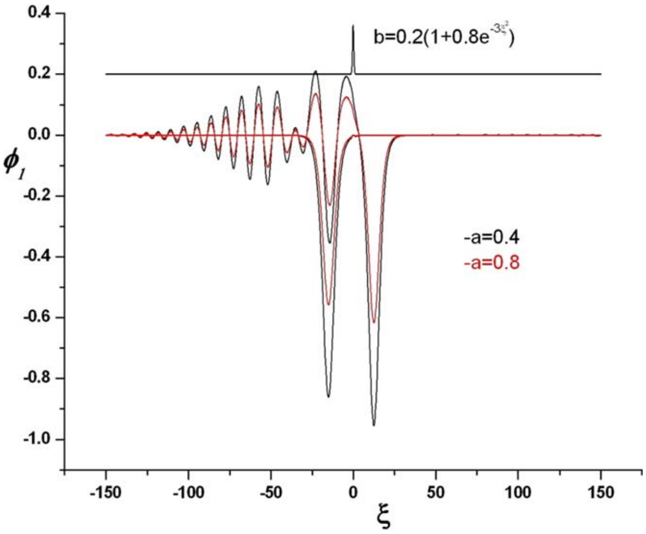

Figure 5. Time evolution of the electron acoustic soliton while interacting with the variable magnetic field $b(\xi) = 0.2(1 + 0.8\mathrm{e}^{-3\xi^2})$ with $l_z = 0.95$, $u = 0.4$ and different values of a.

location of field perturbation. This means that shock structures in the plasma is created by the magnetic field perturbations. In other word, non-uniform magnetic field can act as a new source of dissipation and creates backward moving oscillatory shock waves in magnetized plasmas.

According to (11), one can find that narrower field perturbation (larger value for α) creates greater values of dissipation coefficients and therefore shock profiles find larger amplitudes. According to the figure 2, one can find that created shock profile only propagated from the location of magnetic field perturbation. This means that trajectory of plasma particles spoil when reach the magnetic field perturbation, but their path is restored into their previous situation when they leave the perturbation area.

Figure 3 shows time evolution of initial solitary solution (same as what we took in the figure 2) while interacting with two different field perturbations $b(\xi) = 0.2(1 + 0.8\mathrm{e}^{-3\xi^2})$ and $b(\xi) = 0.2(1 + 0.8\mathrm{e}^{-0.3\xi^2})$ with fixed values of $l_z = 0.95$, $a = 0.5$ and $u = 0.3$. It is obvious that the shock amplitude and radiated energy become greater when variation of magnetic field perturbation respect to ξ increases. The figure also shows that the velocity of EA soliton increases after passing through the magnetic field perturbation with greater values of α.

Figure 4 shows that the backward moving shock wave is still alive, even when the EASW is far away from the region of perturbation after the interaction.

According to the figures 3 and 4 one can find that effects of changing the strength of magnetic field is the same as changing the width of perturbation. Deflections in the path of plasma particles is related to the changing rate of background magnetic. Therefore, amplitude and width create same effects.

According to (9) and (10), one can find that the ratio of hot/cold electrons (value of the 'a' parameter) controls amplitude and width of initial solitary wave. On the other hand, strength of shock wave depends on the amplitude of initial solitary solution. Therefore, we can say that amplitude of shock waves increases when the population of hot electrons decreases. Effects of parameter 'a' on the evolution of waves in the medium during the interaction with variable magnetic field $b(\xi) = 0.2(1 + 0.8\mathrm{e}^{-3\xi^2})$ have been demonstrated in the figure 5. If the strength of magnetic field increases, the dispersion coefficient (B) become larger. In this situation, the width of solitary waves become larger and thus interaction time of soliton and perturbation increases and thus shock waves find larger amplitude as one can find in the figures 4 and 5.

We can present a preliminary explanation for the dissipation term in the equation (8). It may be noted that we have not considered any dissipative agent (like particle collision, viscosity and …) in our model. Path of plasma particle is determined by the magnetic field. In the region of varying magnetic field some particles experience larger magnetic field and some other move under the influence of lower magnetic field intensity. This means that plasma particles are deflected while traveling through the magnetic field perturbation, and indeed it is the reason of created disorders in the shape of solitary profile.

5. Conclusion and remarks

The present study investigates the behavior of small amplitude EASWs in plasmas containing a cold electron fluid, hot thermal electrons and stationary ions under the influence of a varying magnetic field. We have shown that the descriptive equation for the propagation of the EA wave at the presence of varying magnetic field is the mKdV-Burgers equation instead of the usual KdV equation. In fact, the space dependent magnetic field acts as a new source of dissipation. It is interesting that we have not considered any dissipative sources like particle interaction or temperature effects. Numerical results show that the EASWs radiate some amount of energy during their travelling through the varying magnetic field. Radiated energy emerges as backward moving oscillatory shock profiles. Characteristics of shock waves depend on the strength of magnetic field variation and plasma parameters. We also showed that amplitude of shock profiles increases when the ratio of hot/cold electrons is increased. Our theoretical study confirms the existence of collisionless shocks driven by a laser-produced magnetic piston [28].

Dissipation effects due to varying magnetic field is a noticeable phenomenon which should be considered in magnetized plasmas. It can change the stability conditions of magnetized plasmas. Generating of shock waves in low density magnetized plasmas where dissipative effects are negligible can be explained by this issue. Specifications of generated shock profiles depends on the plasma parameters and characteristics of the environment.

This work should be extended to a large set of studies on plasmas with different constituents and particle distributions to understand the nature of dissipative effects of varying magnetic fields.

ORCID iDs

H R Pakzad https://orcid.org/0000-0001-8514-3155
K Javidan https://orcid.org/0000-0002-7596-4280

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