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Soliton molecules and some novel interaction solutions to the (2+1)-dimensional B-type Kadomtsev–Petviashvili equation

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Abstract

Soliton molecules may exists in both experimental and theotetical aspects. In this work, we investigate the (2+1)-dimensional B-type Kadomtsev–Petviashvili equation, which can be used to describe weakly dispersive waves propagating in the quasi media and fluid mechanics. Soliton molecules are generated by N-soliton solution and a new velocity resonance condition. Furthermore, soliton molecules can become to asymmetric solitons when the distance between two solitons of the molecule is small enough. Based on the N-soliton solution, we obtain some novel interaction solutions which component of soliton molecules, breather waves and lump waves by deal with part of parameters by applying velocity resonance, module resonance and long wave limit method, and the interactions are elastic. Finally, some graphic analysis are discussed to understand the propagation phenomena of these solutions.

Keywords: Soliton molecules, velocity resonance, asymmetric solitons, interaction solution B-type Kadomtsev-Petviashvili equation

(Some figures may appear in colour only in the online journal)

1. Introduction

Soliton molecules, which is constructed from a number of 'atoms' each being a fundamental soliton, have been became one of the most challenging open frontiers of the field [1]. Soliton molecules have been observed experimentally in some fields [2–7]. Investigation on soliton molecules provides a direct route to study the interactions between solitary waves, and the formation and dissociation of soliton molecules are closely related to many subjects [8]. Besides the significance they bring to the fundamental understanding of soliton physics, soliton molecules also present the possibility of transferring optical data surpassing the limitation of binary coding [9]. The soliton is a universal concept applicable to a large class of solitary wave propagation effects that can be observed in most branches of nonlinear science [10–19].

Lump solutions are the rationally localized solutions of the nonlinear evolution equations. Lump solutions appear in many physical phenomena, such as plasma, shallow water-wave, optic media, Bose–Einstein condensate [20, 21]. The nontrivial internal interaction between lumps can represent a model of a strongwave turbulence. Lump solutions can be regarded as the lump-type rogue wave, which is used to describe nonlinear phenomena in oceanography. It becomes a very interesting topic to investigate lump solutions of the nonlinear evolution equations. Lump is a rational function solution and localized in all directions in the space, lump solutions for many integrable equations are obtained and discussed in detail [22–25].

As the particular solutions of nonlinear systems, breathers propagate steadily and localize in either time or space, such as Akhmediev breathers [26] and Kuznetsov–Ma breathers [27, 28]. Akhmediev breathers are periodic in space and localized in time, while Kuznetsov–Ma breathers are periodic in time and localized in space, these solutions for the integrable equations have widely discussed. Another special type of analytic solution is the rogue wave localized in both space and time, and it has peak amplitude usually more than twice of the background wave height [29–33]. Besides, rogue waves always appear from nowhere and disappear without a trace, and they can be written in terms of the rational functions of coordinates.

The interaction phenomenon becomes a very interesting topic of the nonlinear evolution equations. As we know, there will happen collision among different types of solutions, it is mainly containing two kinds of collision, the elastic collision and the non-elastic collision. The collision of lump solutions are so widely discussed [34–36], lump solutions will retain their shapes, amplitudes and velocities after the collision with soliton solutions, which means the collision is completely elastic [37]. On the contrary, there are also other collisions that are completely non-elastic [38]. According to different conditions, the collision will change essentially. The propagation and interaction of the bell-type, kink-type and periodic-depression solitons and the evolution of the shock-wave solutions are investigated in [39]. To the best of our knowledge, soliton molecules interact with lump waves and breather waves have not been studied yet.

Recently, Lou [40] introduced a new possible mechanism, the velocity resonant, to form soliton molecules of a fluid model of the (1+1)-dimensional fifth order KdV, SK and KK cases, and we extend the method to soliton molecules of (2+1)-dimensional systems. In this work, we will consider the following (2+1)-dimensional B-type Kadomtsev–Petviashvili (BKP) equation

$$\begin{aligned}&u_t + u_{xxxxx} - 5(u_{xxy} + \int u_{yy}\mathrm{d}x)\\&+ 15(u_x u_{xx} + u u_{xxx} - u u_y - u_x \int u_y \mathrm{d}x)\\&+ 45u^2 u_x = 0,\end{aligned} \tag{1}$$

where $u = u\ (x,\ y,\ t)$ is a analytic function with scaled spatial coordinates $(x,\ y)$ and temporal coordinate t, the subscripts mean partial derivatives, and $\int$ is integration operator. The BKP equation, as a subclass of the KP hierarchy, can be used to describe weakly dispersive waves propagating in the quasi media and fluid mechanics. Multiple soliton solutions, rational solutions, periodic solutions and rouge wave solutions were investigated [41–43].

The bilinear form of equation (1) have been given

$$(D_x^6 - 5D_x^3 D_y - 5D_y^2 + D_x D_t) f \cdot f = 0, \tag{2}$$

under the dependent variable transformation:

$$u = 2(\ln f)_{xx}, \tag{3}$$

where D is the Hirotas bilinear differential operator, and $f = f\,(x,\ y,\ t)$ is a real function of variables $x,\ y$ and t. Based on the Hirotas bilinear theory, equation (1) admits N-soliton solutions as follows:

$$f = \sum_{\rho=0,1} \exp\left(\sum_{j=1}^{(N)} \rho_i \eta_i + \sum_{1\leqslant j<i\leqslant N}^{N} \rho_i \rho_j A_{ij}\right), \tag{4}$$

with

$$\begin{aligned}\eta_i &= k_i x + p_i y + w_i t + \phi_i,\ w_i\\&= \frac{-k_i^6 + 5k_i^3 p_i + 5p_i^2}{k_i},\\ \mathrm{e}^{A_{ij}} &= \frac{a_{ij} + k_i k_j (k_i - k_j)[k_i^4 k_j - 2k_i^3 k_j^2 + k_j^2 p_i - k_i k_j (k_j^3 + 2p_i - 2p_j) + b_{ij}]}{a_{ij} + k_i k_j (k_i - k_j)[k_i^4 k_j - 2k_i^3 k_j^2 + k_j^2 p_i - k_i k_j (k_j^3 + 2p_i - 2p_j) + b_{ij}]},\\ a_{ij} &= k_i p_j - k_j p_i,\ b_{ij} = k_i^2(2k_j^3 - p_j),\ (i, j = 1, 2,\cdots,N),\end{aligned}$$

and the sum taken all the possible conbinations of $\rho_i = 0,\ 1$ $(i = 1,\ 2,\ \cdots,\ N)$, where $k_i,\ p_i$ $(i = 1,\ 2,\ \cdots,\ N)$ are free parameters.

The present paper is organized as follows. In section 2, we aim to introduce a new velocity resonant condition firstly, then soliton molecules are obtained based on N-soliton formula with applying the velocity resonant condition, and further we explore their fascinating dynamical behaviors. In section 3, partial parameters are handled with velocity resonant condition and long wave limit method, the interaction solutions include soliton molecules, lump waves and breather waves are derived. We present general restrictions on interaction solutions to generate m soliton molecules, s breather waves and r lump waves. In the last section, we give the conclusions of this paper.

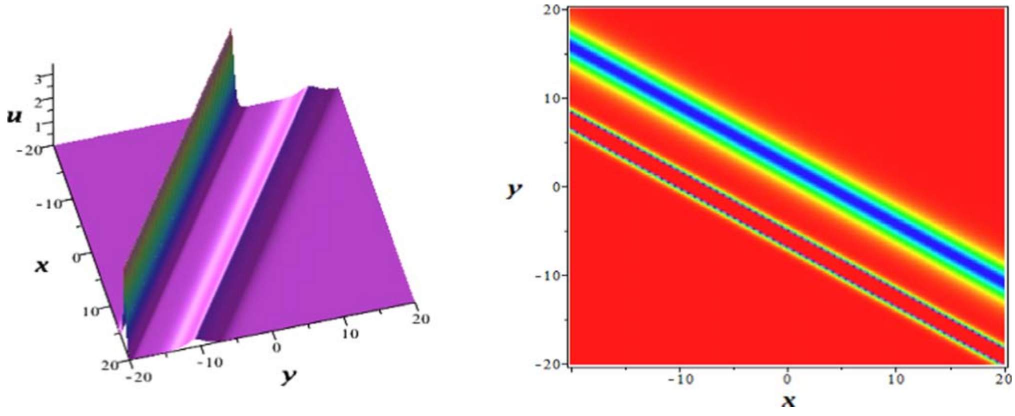
2. Soliton molecules

To find nonsingular analytical resonant excitations from equation (4) we apply a novel type of resonant conditions $(k_i \neq \pm k_j,\ p_i \neq \pm p_j)$, the velocity resonance,

$$\frac{k_i}{k_j} = \frac{p_i}{p_j} = \frac{w_i}{w_j}. \tag{5}$$

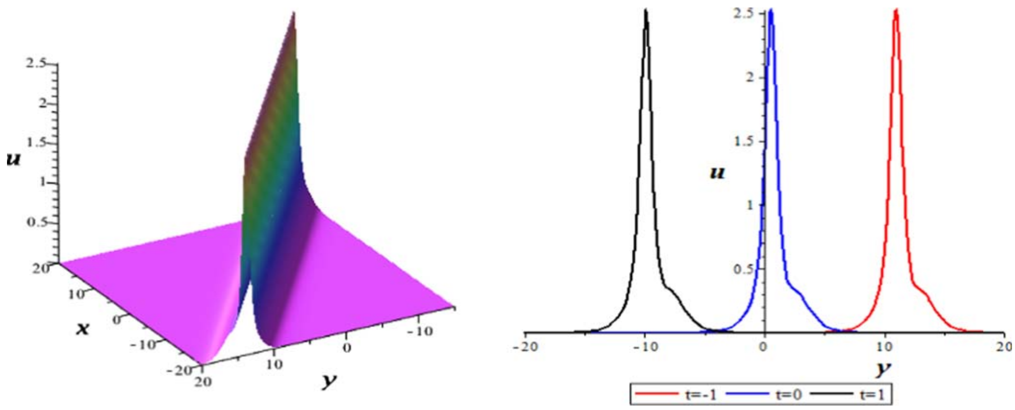
Then we can obtain following expression

$$k_i = \sqrt{\frac{-k_j^3 + 5p_j}{k_j}},\ p_i = \frac{p_j}{k_j}\sqrt{\frac{-k_j^3 + 5p_j}{k_j}}, \tag{6}$$



(a) (b)

Figure 1. Soliton molecule structure for BKP equation with parameter selections (8) at $t = 0$. (a) 3-dimension plot. (b) Density plot.



(a) (b)

Figure 2. Asymmetric soliton for the BKP equation with parameter selections (8) at $t = 0$ except $\phi_2 = -\frac{13}{3}$. (a) 3-dimension plot. (b) 2-dimension plot when $x = 0$ at $t = -1$, $t = 0$, $t = 1$.

and

$$k_i = -\sqrt{\frac{-k_j^3 + 5p_j}{k_j}}, \, p_i = -\frac{p_j}{k_j}\sqrt{\frac{-k_j^3 + 5p_j}{k_j}}. \tag{7}$$

For $N = 2$ in equation (4), two-soliton solution exhibits one soliton molection under the resonance condition. Figure 1 displays the molecule structure with paramreter selections

$$\begin{aligned} k_1 &= -\frac{4}{5}, \, k_2 = \frac{7\sqrt{14}}{10}, \, p_1 = -\frac{6}{5}, \\ p_2 &= \frac{21\sqrt{14}}{20}, \, \phi_1 = 0, \, \phi_2 = 30. \end{aligned} \tag{8}$$

If we change values of ϕ_1 and ϕ_2, the distance between two solitons of the molecule will change repectively. When two solitons interact with each other, the soliton molecule will become one asymmetric soliton solution. Here we change $\phi_2 = -\frac{9}{2}$ in equation (8), see figure 2.

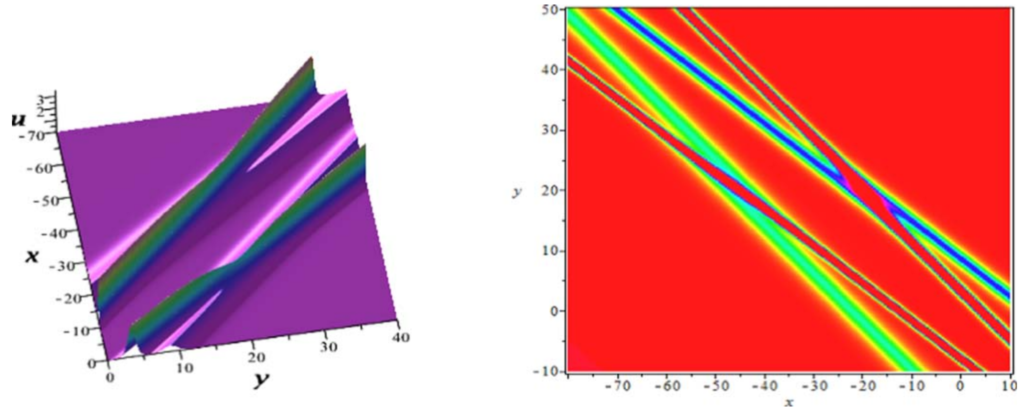
Two soliton molecules can be generated from four-soliton, k_1, p_1, w_1 and k_2, p_2, w_2 satisfy equation (5), k_3, p_3, w_3 and k_4, p_4, w_4 satisfy equation (5) at the same time. Figure 3 displays the elastic interaction property for BKP equation with $N = 4$ and with parameter selections

$$\begin{aligned} &k_1 = -1, \, k_2 = \sqrt{7}, \, k_3 = -\frac{4}{5}, \, k_4 = -\frac{\sqrt{561}}{10}, \\ &p_1 = -\frac{8}{5}, \, p_2 = \frac{8\sqrt{7}}{5}, \, p_3 = -1, \\ &p_4 = -\frac{\sqrt{561}}{8}, \, \phi_1 = 10, \, \phi_2 = 20, \, \phi_3 = -10, \, \phi_4 = 5. \end{aligned} \tag{9}$$

As we can see in figure 3, height of wave peaks and velocities are not changed except for phase after collision.

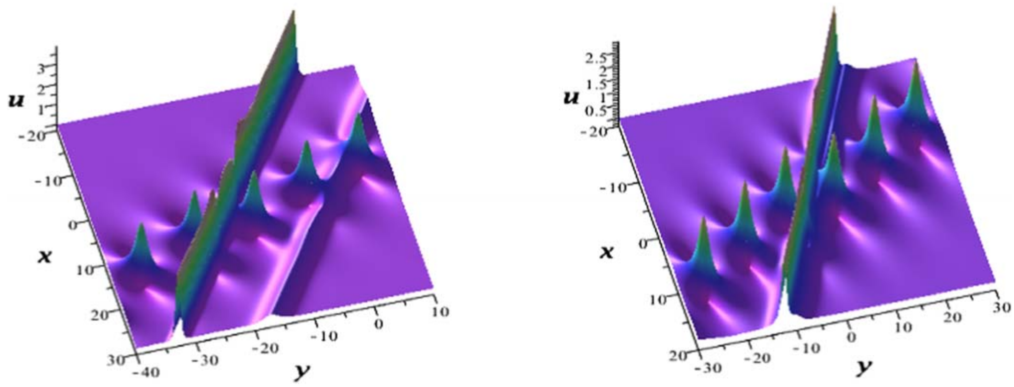
3. Some novel interaction solutions

In this section, some novel interaction solutions have been investigated which are soliton molcelue interacting with breather wave and lump wave. To our knowledge, soliton



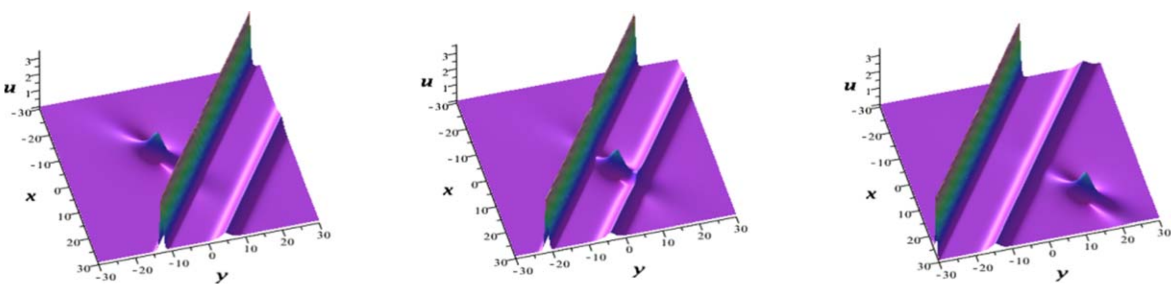
(a) (b)

Figure 3. Two-soliton molecules with parameter selection (9) at $t = 0$. (a) 3-dimension plot. (b) Density plot.



(a) (b)

Figure 4. (a) Interaction of a soliton molecule and a breather wave for BKP equation with parameter (10) at $t = 0$. (b) Interaction of one asymmetric soliton and one breather wave for BKP equation with the parameter selections (10) at $t = 0$ except $\phi_2 = -3$.



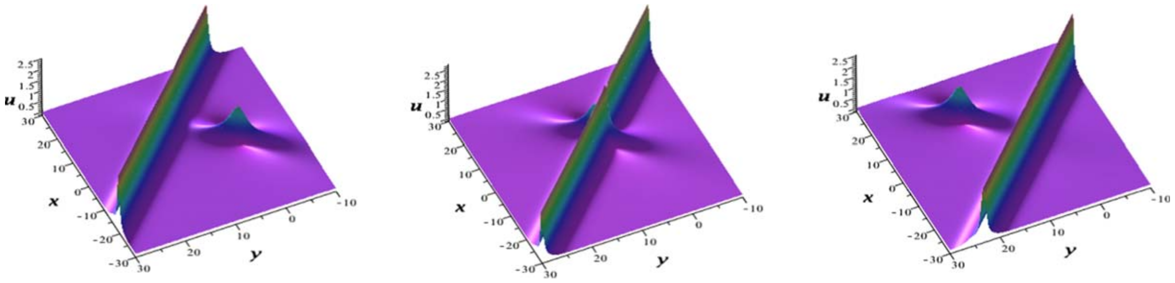
(a) $t = -\frac{1}{2}$ (b) $t = 0$ (c) $t = \frac{4}{3}$

Figure 5. Interaction of one soliton molecule and one lump wave for BKP equation with parameter (11).

molecules interact with lump waves and breather waves have not been studied yet. We study the interaction solutions by velocity resonance, module resonance and long-wave limit method.

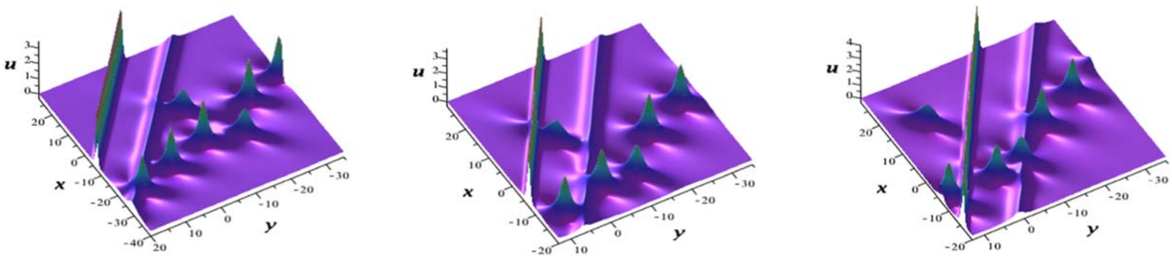
Interaction solution of one soliton molecule and breather wave can be generated by four-soliton. Furthermore, k_i, p_i and w_i ($i = 1, 2$) should satisfy velocity resonance condition (5), η_3 and η_4 should satisfy module resonance condition $\eta_3 = \bar{\eta}_4$. For instance, taking parameters as follows

$$
\begin{aligned}
&k_1 = -\frac{4}{5},\ k_2 = \frac{7\sqrt{14}}{10},\ p_1 = -\frac{6}{5},\ p_2 = \frac{21\sqrt{14}}{20},\\
&\phi_1 = 0,\ \phi_2 = 30,\ k_3 = \frac{2}{7}(1 - i),\ k_4 = \frac{2}{7}(1 + i),\\
&p_3 = \frac{1}{8}(1 + 4i),\ p_4 = \frac{1}{8}(1 - 4i),\ \phi_3 = 0,\ \phi_4 = 0.
\end{aligned}
\tag{10}
$$



(a) $t=-\frac{1}{2}$ (b) $t=0$ (c) $t=\frac{4}{3}$

Figure 6. Interaction of one asymmetric soliton and one lump wave for BKP equation with parameter selections (11).



(a) $t=-\frac{5}{3}$ (b) $t=-1$ (c) $t=-\frac{8}{17}$

Figure 7. Interaction of one soliton molecule, one lump wave and one breather wave for BKP equation with parameter (13).

Figure 4 displays the interaction between one soliton molecule and one breather wave. Specially, asymmetric soliton can be generated by taking suitable values of ϕ_1, ϕ_2. And the interaction of breathers, asymmetric solitons and soliton molecules are elastic.

The long-wave limit method is a powerful technique to get lump solution, based on the N-soliton solution (4), we can obtain interaction solutions consisting of one soliton molecule and one lump wave. For $N=4$, we taking a long wave limit on k_3, k_4, p_3, p_4 ($\epsilon\to 0$), k_1, k_2, p_1, p_2 satisfy volocity resonance condition, parameters are as follows

$$\begin{aligned}&k_1=-\frac{4}{5},\ k_2=\frac{7\sqrt{14}}{10},\ p_1=-\frac{6}{5},\ p_2=\frac{21\sqrt{14}}{20},\\&\phi_1=20,\ \phi_2=-30,\ k_3=(1+i)\epsilon,\ k_4=(1-i)\epsilon,\\&p_3=-2\epsilon,\ p_4=-2\epsilon,\ \phi_3=i\pi,\ \phi_4=i\pi.\end{aligned}\tag{11}$$

Figures 5 and 6 displayed one lump wave interact with one soliton molecule and one asymmetric soliton respectively. The collision are also elastic, and height of the lump wave does not change before and after the collision.

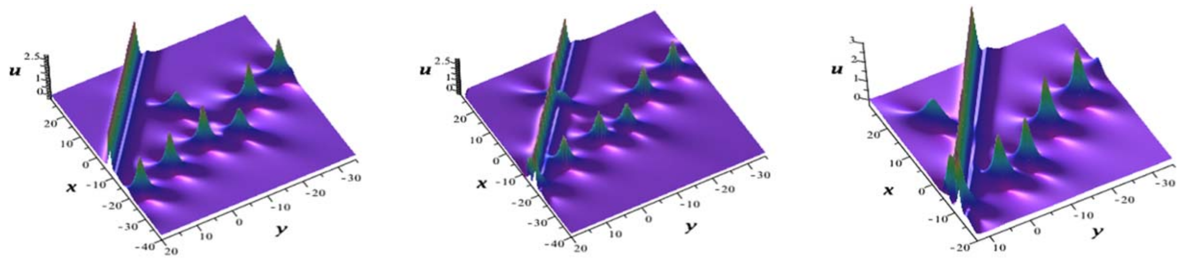
For more general situation, interaction solutions include m soliton molecules, s breather waves and r lump waves can be constrained as follows

$$\begin{aligned}&\frac{k_1}{k_2}=\frac{p_1}{p_2}=\frac{w_1}{w_2},\cdots,\frac{k_{2m-1}}{k_{2m}}=\frac{p_{2m-1}}{p_{2m}}=\frac{w_{2m-1}}{w_{2m}},\\&\eta_{2m+1}=\bar{\eta}_{2m+2},\cdots,\eta_{2m+2s-1}=\bar{\eta}_{2m+2s},\\&k_{2m+2s+1}=\bar{k}_{2m+2s+2},\cdots,k_{2m+2s+2r-1}\\&\quad=\bar{k}_{2m+2s+2r},\ p_{2m+2s+1}=\bar{p}_{2m+2s+2},\cdots,\\&k_{2m+2s+2r-1}=\bar{k}_{2m+2s+2r},\ k_i=K_i\epsilon,\ p_i\\&\quad=P_i\epsilon,\ \epsilon\to 0,\ 2m\\&\quad+2s+1\leqslant i\leqslant 2m+2s+2r.\end{aligned}\tag{12}$$

If we set $N=6$, k_1, k_2, p_1, p_2 satisfy velocity resonant condition, k_5, k_6, p_5, p_6 satisfy module resonance condition, then taking a long-wave limit on k_3, k_4, p_3, $p_4(\epsilon\to 0)$, taking parameters as follows

$$\begin{aligned}&k_1=-\frac{4}{5},\ k_2=\frac{7\sqrt{14}}{10},\ p_1=-\frac{6}{5},\ p_2=\frac{21\sqrt{14}}{20},\\&\phi_1=-20,\ \phi_2=10,\ k_3=(1+i)\epsilon,\\&k_4=(1-i)\epsilon,\ p_3=-2\epsilon,\ p_4=-2\epsilon,\ \phi_3=i\pi,\\&\phi_4=i\pi,\ k_5=\frac{2}{7}(1-i),\ k_6=\frac{2}{7}(1+i),\\&p_5=\frac{1}{8}(1+4i),\ p_6=\frac{1}{8}(1-4i),\ \phi_5=0,\ \phi_6=0,\end{aligned}\tag{13}$$

then we obtain interaction solutions consisting one soliton molecule, one lump wave and one breather wave, see figure 7. The interaction solutions consisting one asymmetric soliton, one lump wave and one breather wave is also obtained, see figure 8. The interaction between these waves is also elastic.



(a) $t=-\frac{4}{3}$ (b) $t=-\frac{9}{11}$ (c) $t=-\frac{8}{17}$

Figure 8. Interaction of one asymmetric soliton, one lump wave and one breather wave for BKP equation with parameter (13) except $\phi_1=-3$, $\phi_2=0$.

4. Conclusions

In this paper, we investigate soliton molecules, asymmetric solitons and interaction solutions of (2+1)-dimensional BKP equation. Based on velocity resonance condition and general N-soliton expression, we obtained soliton molecule by using velocity resonance condition, see figures 1 and 3. When taking suitable values of ϕ, soliton molecule change to asymmetric soliton, see figure 2. By employing velocity resonance condition and module resonance condition on wave numbers, we can get a new interaction solution consisiting soliton molecules and breather waves, see figure 4. Taking a long wave limit on part of parameters and employing resonance condition on others, the new interaction solutions consisitng soliton molecule and lump wave can be obtained, see figures 5 and 6. By using velocity resonance, module resonance and long-wave limit method to different part of wave numbers, interaction solutions consisting soliton moleules, asymmetric solitons, lump waves and breather waves are obtained, see figures 7 and 8. These interaction phenomena may have not been studied. At last, we give the general restrictions to get these new interaction solutions containing m soliton molecules, s breather waves and r lump waves, and their interactions are elastic. The method to construct soliton molecules and these new interaction solutions would be suitable to investigate other models in mathematical physics and engineering.

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