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General improved Kudryashov method for exact solutions of nonlinear evolution equations in mathematical physics

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Abstract

This paper is devoted to improving the general Kudryashov method by a new and general auxiliary equation. So, a new method is introduced, which we call 'the general improved Kudryashov method', to produce exact solutions for nonlinear evolution equations arising in mathematical physics. As application examples, exact traveling wave solutions for the combined Korteweg–de Vries and modified Korteweg–de Vries (KdV–mKdV) equation and the (2+1)-dimensional Zakharov–Kuznetsov equation are obtained. These solutions can be classified as solitary and periodic wave solutions. Some of the obtained solutions are graphically sketched.

Keywords: Kudryashov method, nonlinear evolution equations, exact solutions

(Some figures may appear in colour only in the online journal)

1. Introduction

Nonlinear evolution equations play an important role in physics and applied sciences. Searching for exact traveling wave solutions of nonlinear evolution equations in mathematical physics plays a crucial role in miscellaneous fields of science and engineering, such as soliton theory, plasma physics, nonlinear optics, fluid dynamics, biophysics and many others. So, many powerful methods have been introduced, such as the homogeneous balance method [1, 2], the tanh method [3], the tanh-coth method [4], the modified tanh-coth method [5–7], the inverse scattering method [8], Hirota's bilinear method [9], the sine-cosine method [10], the (G'/G) -expansion method [11], the $(G'/G, 1/G)$ -expansion method [12], the Exp-function method [13], F-expansion method [14–16] and so on.

Consider a nonlinear partial differential equation (NPDE) in polynomial form

$$\Omega(\xi, s, u_s, u_\xi, u_{\xi\xi}, u_{\xi\xi\xi}, \ldots) = 0, \tag{1.1}$$

where $(\xi, s) \in \mathbb{R} \times \mathbb{R}_+$ is the independent variable and u is the dependent variable. Applying the variable traveling transformation

$$u(\xi, s) = u(\varpi), \quad \varpi = \xi - \epsilon s, \tag{1.2}$$

we change (1.1) to a nonlinear ordinary differential equation (NODE)

$$\Theta(\varpi, u, u', u'', u''', \ldots) = 0, \tag{1.3}$$

where $' := \frac{\mathrm{d}}{\mathrm{d}\varpi}$. In 2012, Kudryashov [17] proposed his method for finding exact solutions of the NPDE (1.1). He looked for exact solutions taking into account the expression $u(\varpi) = \sum_{i=0}^{x} \alpha_i \chi^i$ where $\chi = \frac{1}{1+\mathrm{e}^{\varpi}}$ which is a solution of the equation $\frac{\mathrm{d}\chi}{\mathrm{d}\varpi} = \chi^2 - \chi$. A modified Kudryashov method

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was introduced by replacing the natural exponential function e^{ϖ} by the general exponential function a^{ϖ} in [18–21]. In these research articles, authors found exact solutions of the NPDE (1.1) by using the expansion $u(\varpi) = \sum_{i=0}^{x} \alpha_i \chi^i$ where $\chi = \frac{1}{1 \pm a^{\varpi}}$ which is a solution of the equation $\frac{d\chi}{d\varpi} = \ln a(\chi^2 - \chi)$. Thereafter, some authors [22–25] applied a general Kudryashov method to explore exact solutions of the NPDE (1.1). They used a rational type expansion $u(\varpi) = \sum_{i=0}^{x} \alpha_i \chi^i / \sum_{j=0}^{x} \beta_j \chi^j$ where $\chi = \frac{1}{1 + Ce^{\varpi}}$ which is a solution of the equation $\frac{d\chi}{d\varpi} = \chi^2 - \chi$. Recently, some authors have discussed the improvement of the Kudryashov method. S M Ege [26, 27] first extend and improved the Kudryashov method by new auxiliary equation. Abdus Salam and Habiba [28] improved the general Kudryashov method given in [22] by introducing the auxiliary equation $\frac{d\chi}{d\varpi} = \sigma\chi^3 - \chi$, $0 \neq \sigma \in \mathbb{R}$. This equation has the general solution $\chi = \frac{\pm 1}{\sqrt{\sigma + Ce^{2\varpi}}}$.

In this paper, we improve the general Kudryashov method given in [22] by a general auxiliary equation $\chi'(\varpi) = \sigma\chi^n(\varpi) - \chi(\varpi)$, $1 < n \in \mathbb{N}$, $0 \neq \sigma \in \mathbb{R}$. So, we present a very straightforward and effective method called general improved Kudryashov method for exact solutions of nonlinear evolution equations. The merits of the introduced method lie in two directions. Firstly, it leads to both the solitary and periodic solutions. Secondly, the solution strategy, by this method and the computing system Mathematica, is of utter simplicity, and can be easily extended to all types of nonlinear evolution equations. Moreover, If $n = 2$ and $n = 3$ our general auxiliary equation reduces to the auxiliary equations used in [17, 22] and [28], respectively. Hence, a novel improved Kudryashov method for finding exact solutions of nonlinear evolution equations in mathematical physics is introduced. As application examples, new sets of exact solutions for the combined KdV–mKdV equation and the (2+1) dimensional Zakharov–Kuznetsov equation are presented.

2. Demonstration of the general improved Kudryashov method

Consider the NPDE (1.1) together with the variable traveling transformation (1.2) and the transformed NODE (1.3). For simplicity, we integrate the NODE (1.3), provided that all terms include derivatives, and set the integration constants to be zero. Subsequently, the transformed NODE (1.3) can be solved by expanding its general solution in the form

$$u(\varpi) = \frac{\sum_{i=0}^{x} \alpha_i \chi^i(\varpi)}{\sum_{j=0}^{y} \beta_j \chi^j(\varpi)}, \tag{2.1}$$

where $\alpha_i, \beta_j (i = 0, 1, \ldots, x, j = 0, 1, \ldots, y)$ are constants to be determined and χ solves the general auxiliary equation

$$\chi'(\varpi) = \sigma\chi^n(\varpi) - \chi(\varpi), \quad 1 < n \in \mathbb{N}, \quad 0 \neq \sigma \in \mathbb{R}. \tag{2.2}$$

Solving equation (2.2), gives a set of general solutions in the form

$$\chi(\varpi) = \begin{cases} \frac{1}{\sqrt[n-1]{\sigma + C\exp[(n-1)\varpi]}}, & n = 2, 4, 6, 8, \ldots, \\ \frac{\pm 1}{\sqrt[n-1]{\sigma + C\exp[(n-1)\varpi]}}, & n = 3, 7, 11, 15, \ldots, \\ \frac{\pm 1}{\sqrt[n-1]{\sigma + C\exp[(n-1)\varpi]}}, \ \frac{\pm i}{\sqrt[n-1]{\sigma + C\exp[(n-1)\varpi]}}, & n = 5, 9, 13, 17, \ldots. \end{cases} \tag{2.3}$$

The positive numbers x and y can be specified by balancing the linear and nonlinear terms of highest order in equation (1.3). Inserting equations (2.1) and (2.2) into equation (1.3), yields an algebraic equation in χ and its powers. Equating the coefficients of the terms that containing the same power for χ to zero, gives an algebraic system of equations in α_i, β_j and ϵ. With the help of the computer symbolic system *Mathematica*, we can obtain α_i, β_j and ϵ. Eventually, by using these values and the solutions (2.3) of equation (2.2), we can produce exact traveling wave solutions of equation (1.1). In fact, equation (2.2) has many solutions unlike the solutions given in equation (2.3). But we only consider the solutions (2.3), because they have interesting and effective properties.

In the following section, our method is applied for $n = 5$ to find exact traveling wave solutions of the combined KdV–mKdV equation and the (2+1)-dimensional Zakharov–Kuznetsov equation.

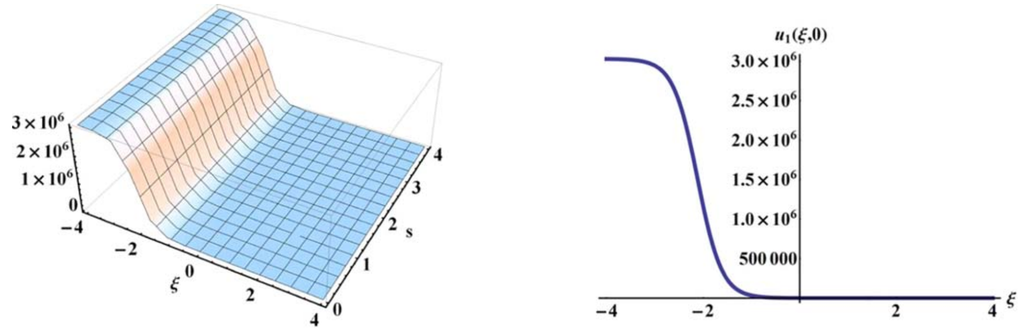
3. Applications

3.1. The combined KdV–MKdV equation

The KdV and mKdV are known as solitary equations which have been extensively researched. The nonlinear terms of the KdV and mKdV equations usually arise simultaneously in many physical problems such as fluid mechanics, quantum field theory and plasma physics. These nonlinear terms construct a nonlinear evolution equation called the combined KdV–mKdV equation

$$\begin{aligned} &u_s + \delta u u_\xi + \rho u^2 u_\xi + u_{\xi\xi\xi} = 0, \\ &(\xi, s) \in \mathbb{R} \times \mathbb{R}_+. \end{aligned} \tag{3.1}$$

Here, we apply the general improved Kudryashov method to investigate the combined KdV–mKdV equation (3.1). By



(a) (b)

Figure 1. (a) 3D plot of the solution $u_1(\xi, s)$. (b) 2D plot of the solution $u_1(\xi, s)$ at $s = 0$. Here, $\sigma = 0.0002$, $\rho = C = 1$, $\delta = 0.002$, $\alpha_0 = 0.005$, $\beta_0 = 0.05$ and $\beta_1 = 0.5$.

using the transformation (1.2), we can convert (3.1) into a NODE

$$-\epsilon u' + \delta u u' + \rho u^2 u' + u''' = 0. \tag{3.2}$$

Integrating the NODE (3.2) and setting the integration constants to be zero, gives

$$-\epsilon u + \frac{\delta}{2}u^2 + \frac{\rho}{3}u^3 + u'' = 0. \tag{3.3}$$

Taking the homogeneous balance between u'' and u^3, we get $x = y + 4$. Let $y = 1$, then $x = 5$. Hence, we can put the traveling wave solution of equation (3.1) in the form

$$u(\varpi) = \frac{\alpha_0 + \alpha_1\chi + \alpha_2\chi^2 + \alpha_3\chi^3 + \alpha_4\chi^4 + \alpha_5\chi^5}{\beta_0 + \beta_1\chi}. \tag{3.4}$$

Substituting equations (3.4) and (2.2) with $n = 5$ into equation (3.3), gives an algebraic equation in χ and its powers. Equating the coefficients of the terms that containing the same power for χ to zero, yields an algebraic system of equations in α_i, $\beta_j (i = 0, 1, 2, 3, 4, 5, j = 0, 1)$ and ϵ (see appendix A). Solving the obtained system by *Mathematica*, we have the following sets of solutions.

Case A.

$$\begin{cases} \alpha_0 = \alpha_0, \ \ \alpha_1 = \frac{36\alpha_0\beta_0^2\beta_1 - 180\alpha_0^2\beta_0\beta_1\delta + 12\alpha_0^2\beta_1(3\beta_0\delta + 2\alpha_0\rho)}{6(\alpha_0^2\rho + \beta_0^2 + \alpha_0\beta_0\delta) - \alpha_0(3\beta_0\delta + 2\alpha_0\rho)}, \\ \alpha_2 = \pm\frac{155\beta_0^3}{102\beta_1^2\sqrt{6\rho}}, \ \ \alpha_3 = \pm\frac{10\beta_0^2}{\beta_1\sqrt{96\rho}}, \ \ \alpha_4 = 0, \ \ \alpha_5 = \pm\frac{\sqrt{96}\beta_1\sigma}{\sqrt{\rho}}, \\ \beta_0 = \beta_0, \ \ \beta_1 = \beta_1, \ \ \epsilon = \frac{\alpha_0(3\beta_0\delta + 2\alpha_0\rho)}{6\beta_0^2}, \end{cases} \tag{3.5}$$

where α_0, β_0 and β_1 are free constants. Substituting the values (3.5) into (3.4) and using (2.3), yields traveling wave solutions of equation (3.1) as follows

$$u_1(\xi, s) = \frac{\sqrt[4]{\sigma + C\exp[4\phi(\xi, s)]}}{\beta_1 \pm \beta_0\sqrt[4]{\sigma + C\exp[4\phi(\xi, s)]}} \times \left[\frac{\alpha_0 + A}{102\beta_1^2\sqrt{6\rho}} \pm \frac{155\beta_0^3}{(\sigma + C\exp[4\phi(\xi, s)])^{\frac{1}{2}}} + \frac{255\beta_1^3}{(\sigma + C\exp[4\phi(\xi, s)])^{\frac{3}{4}}} + \frac{2448\beta_1^3\sigma}{(\sigma + C\exp[4\phi(\xi, s)])^{\frac{5}{4}}}\right], \tag{3.6}$$

$$u_2(\xi, s) = \frac{\sqrt[4]{\sigma + C\exp[4\phi(\xi, s)]}}{\beta_1 \pm i\ \beta_0\sqrt[4]{\sigma + C\exp[4\phi(\xi, s)]}} \times \left[\frac{\alpha_0 + A}{102\beta_1^2\sqrt{6\rho}} \pm i\frac{155\beta_0^3}{(\sigma + C\exp[4\phi(\xi, s)])^{\frac{1}{2}}} \pm i\frac{255\beta_1^3}{(\sigma + C\exp[4\phi(\xi, s)])^{\frac{3}{4}}} \pm i\frac{2448\beta_1^3\sigma}{(\sigma + C\exp[4\phi(\xi, s)])^{\frac{5}{4}}}\right], \tag{3.7}$$

where

$$A = \frac{36\alpha_0\beta_0^2\beta_1 - 180\alpha_0^2\beta_0\beta_1\delta + 12\alpha_0^2\beta_1(3\beta_0\delta + 2\alpha_0\rho)}{6(\alpha_0^2\rho + \beta_0^2 + \alpha_0\beta_0\delta) - \alpha_0(3\beta_0\delta + 2\alpha_0\rho)}$$

and

$$\phi(\xi, s) = \xi - \left[\frac{\alpha_0(3\beta_0\delta + 2\alpha_0\rho)}{6\beta_0^2}\right]s.$$

Figure 1 shows the behaviour of the solution $u_1(\xi, s)$ for $\sigma = 0.0002$, $\rho = C = 1$, $\delta = 0.002$, $\alpha_0 = 0.005$, $\beta_0 = 0.05$ and $\beta_1 = 0.5$.

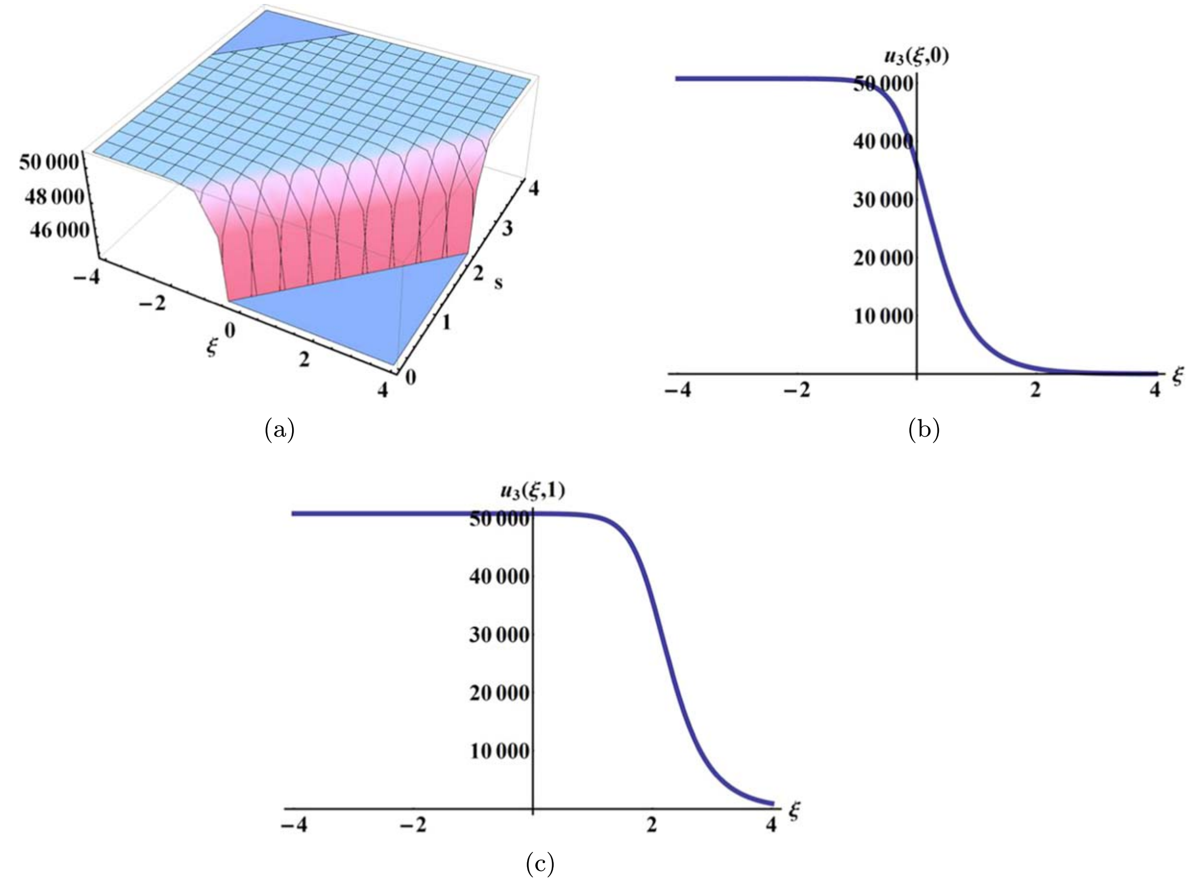


Figure 2. (a) 3D plot of the solution $u_3(\xi, s)$. (b) and (c) 2D plots of the solution $u_3(\xi, s)$ at $s = 0$ and $s = 1$, respectively. Here, $\rho = 0.0001$, $\sigma = C = 1$, $\delta = 6.7$, $\alpha_4 = 0.02$, $\beta_0 = 1.5$ and $\beta_1 = 0.002$.

Case B.

$$\begin{cases} \alpha_0 = \frac{\beta_0\sqrt{9\delta^2 - 384\rho} - 3\beta_0\delta}{8\rho}, \quad \alpha_1 = \frac{\beta_1\sqrt{3\delta^2 - 128\rho} - 3\beta_1\delta}{8\rho}, \\ \alpha_2 = \frac{\alpha_4(813\beta_0^2\sigma^2 - 8\alpha_4^2\rho)}{120\beta_1^2\sigma^2}, \quad \alpha_3 = \frac{-306\alpha_4\beta_0}{72\beta_1}, \quad \alpha_4 = \alpha_4, \quad \alpha_5 = 0, \\ \beta_0 = \beta_0, \quad \beta_1 = \beta_1, \quad \epsilon = \frac{\delta(3\delta - \sqrt{9\delta^2 - 384\rho}) - 64\rho}{32\rho}, \end{cases} \tag{3.8}$$

where α_4, β_0 and β_1 are free constants.

Substituting the values (3.8) into (3.4) and using (2.3), yields traveling wave solutions of equation (3.1) as follows

$$\begin{aligned} u_3(\xi, s) = &\frac{\sqrt[4]{\sigma + C\exp[4\phi(\xi, s)]}}{\beta_1 \pm \beta_0\sqrt[4]{\sigma + C\exp[4\phi(\xi, s)]}} \\ &\times \left[\frac{\beta_0\sqrt{9\delta^2 - 384\rho} - 3\beta_0\delta}{8\rho}\right. \\ &\pm \frac{\beta_1\sqrt{3\delta^2 - 128\rho} - 3\beta_1\delta}{8\rho(\sigma + C\exp[4\phi(\xi, s)])^{\frac{1}{4}}} \\ &+ \frac{\alpha_4(813\beta_0^2\sigma^2 - 8\alpha_4^2\rho)}{120\beta_1^2\sigma^2(\sigma + C\exp[4\phi(\xi, s)])^{\frac{1}{2}}} \\ &\pm \frac{306\alpha_4\beta_0}{72\beta_1(\sigma + C\exp[4\phi(\xi, s)])^{\frac{3}{4}}} \\ &\left.+ \frac{\alpha_4}{(\sigma + C\exp[4\phi(\xi, s)])}\right], \end{aligned} \tag{3.9}$$

$$\begin{aligned} u_4(\xi, s) = &\frac{\sqrt[4]{\sigma + C\exp[4\phi(\xi, s)]}}{\beta_1 \pm i\ \beta_0\sqrt[4]{\sigma + C\exp[4\phi(\xi, s)]}} \\ &\times \left[\frac{\beta_0\sqrt{9\delta^2 - 384\rho} - 3\beta_0\delta}{8\rho}\right. \\ &\pm i\frac{\beta_1\sqrt{3\delta^2 - 128\rho} - 3\beta_1\delta}{8\rho(\sigma + C\exp[4\phi(\xi, s)])^{\frac{1}{4}}} \\ &+ \frac{\alpha_4(813\beta_0^2\sigma^2 - 8\alpha_4^2\rho)}{120\beta_1^2\sigma^2(\sigma + C\exp[4\phi(\xi, s)])^{\frac{1}{2}}} \\ &\pm i\frac{306\alpha_4\beta_0}{72\beta_1(\sigma + C\exp[4\phi(\xi, s)])^{\frac{3}{4}}} \\ &\left.+ \frac{\alpha_4}{(\sigma + C\exp[4\phi(\xi, s)])}\right], \end{aligned} \tag{3.10}$$

where

$$\phi(\xi, s) = \xi - \left[\frac{\delta(3\delta - \sqrt{9\delta^2 - 384\rho}) - 64\rho}{32\rho}\right]s.$$

Figure 2 shows the behaviour of the solution $u_3(\xi, s)$ for $\rho = 0.0001$, $\sigma = C = 1$, $\delta = 6.7$, $\alpha_4 = 0.02$, $\beta_0 = 1.5$ and $\beta_1 = 0.002$.

Case C.

$$\begin{cases} \alpha_0 = \alpha_0, \quad \alpha_1 = \frac{\beta_1(-3\delta + \sqrt{9\delta^2 - 528\rho})}{8\rho}, \quad \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0, \\ \beta_0 = -\frac{\alpha_0}{66}(3\delta + \sqrt{9\delta^2 - 528\rho}), \quad \beta_1 = \beta_1, \quad \epsilon = \frac{-3\delta^2 + \delta\sqrt{9\delta^2 - 528\rho} - 88\rho}{32\rho}, \end{cases} \tag{3.11}$$

where α_0 and β_1 are free constants.

Substituting the values (3.11) into (3.4) and using (2.3), yields traveling wave solutions of equation (3.1) as follows

$$u_5(\xi, s) = \frac{528\alpha_0\rho(\sigma + C\exp[4\phi(\xi, s)])^{\frac{1}{4}} \pm 66\beta_1(-3\delta + \sqrt{9\delta^2 - 528\rho})}{-8\alpha_0\rho(3\delta + \sqrt{9\delta^2 - 528\rho})(\sigma + C\exp[4\phi(\xi, s)])^{\frac{1}{4}} \pm 528\beta_1\rho}, \tag{3.12}$$

$$u_6(\xi, s) = \frac{528\alpha_0\rho(\sigma + C\exp[4\phi(\xi, s)])^{\frac{1}{4}} \pm 66\, i\, \beta_1(-3\delta + \sqrt{9\delta^2 - 528\rho})}{-8\alpha_0\rho(3\delta + \sqrt{9\delta^2 - 528\rho})(\sigma + C\exp[4\phi(\xi, s)])^{\frac{1}{4}} \pm 528\, i\, \beta_1\rho}, \tag{3.13}$$

where

$$\phi(\xi, s) = \xi - \left[\frac{-3\delta^2 + \delta\sqrt{9\delta^2 - 528\rho} - 88\rho}{32\rho}\right]s.$$

Figure 3 shows the behaviour of the solution $u_5(\xi, s)$ for $\rho = 0.0001$, $\sigma = C = 1$, $\delta = 2.5$, $\alpha_0 = -0.005$, and $\beta_1 = -0.002$.

3.2. The (2+1)-dimensional modified Zakharov–Kuznetsov equation

In [29], Schamel constructed the (2+1)-dimensional modified Zakharov–Kuznetsov equation:

$$u_s + \mu u^2 u_\xi + \nu(u_{\xi\zeta\zeta} + u_{\xi\xi\xi}) = 0, \quad (\xi, \zeta, s) \in \mathbb{R}^2 \times \mathbb{R}_+, \tag{3.14}$$

which portrays the ion-acoustic waves inside a cold-ion plasma when the behavior of the electrons does not isothermal during their passway of the wave. According to the transformation:

$$u(\xi, \zeta, s) = u(\varpi), \quad \varpi = \xi + \zeta - \epsilon s, \tag{3.15}$$

Equation (3.14) can be converted into a NODE

$$-\epsilon u' + \mu u^2 u' + 2\nu u''' = 0. \tag{3.16}$$

Integrating the NODE (3.16) and putting the constants of integration to be zero, yields

$$-\epsilon u + \frac{\mu}{3}u^3 + 2\nu u'' = 0. \tag{3.17}$$

Balancing u^3 with u'' gives $x = 5$ and $y = 1$. Therefore, we can set the traveling wave solution of equation (3.14) in the form

$$u(\varpi) = \frac{\alpha_0 + \alpha_1\chi + \alpha_2\chi^2 + \alpha_3\chi^3 + \alpha_4\chi^4 + \alpha_5\chi^5}{\beta_0 + \beta_1\chi}, \tag{3.18}$$

substituting equations (3.18) and (2.2) with $n = 5$ into equation (3.17), gets an algebraic equation in χ and its powers. Equating the coefficients of the terms that containing the same power for χ to zero, gives an algebraic system of equations in α_i, $\beta_j (i = 0, 1, 2, 3, 4, 5, j = 0, 1)$ and ϵ (see appendix B). Solving this system by *Mathematica*, we have the following sets of solutions.

Case I.

$$\begin{cases} \alpha_0 = 0, \ \alpha_1 = \alpha_1, \ \alpha_2 = \alpha_3 = \alpha_4 = 0, \ \alpha_5 = -2\alpha_1\sigma, \\ \beta_0 = 0, \ \beta_1 = \pm\frac{\alpha_1 i}{4}\sqrt{\frac{\mu}{\nu}}, \ \epsilon = -16\nu, \end{cases} \tag{3.19}$$

where α_1 is a nonzero free parameter.

Substituting the values (3.19) into (3.18) and using (2.3), gets traveling wave solution of equation (3.14) as follows

$$u_1(\xi, \zeta, s) = \pm\frac{4i\,(1 - 2\sigma)\sqrt{\frac{\nu}{\mu}}}{\sigma + C\exp[4(\xi + \zeta + 16\nu s)]}. \tag{3.20}$$

Figure 4 shows the behaviour of the solution $u_1(\xi, \zeta, s)$ for $\mu = 0.1$, $\nu = -1$, $\sigma = 0.25$ and $C = 1$.

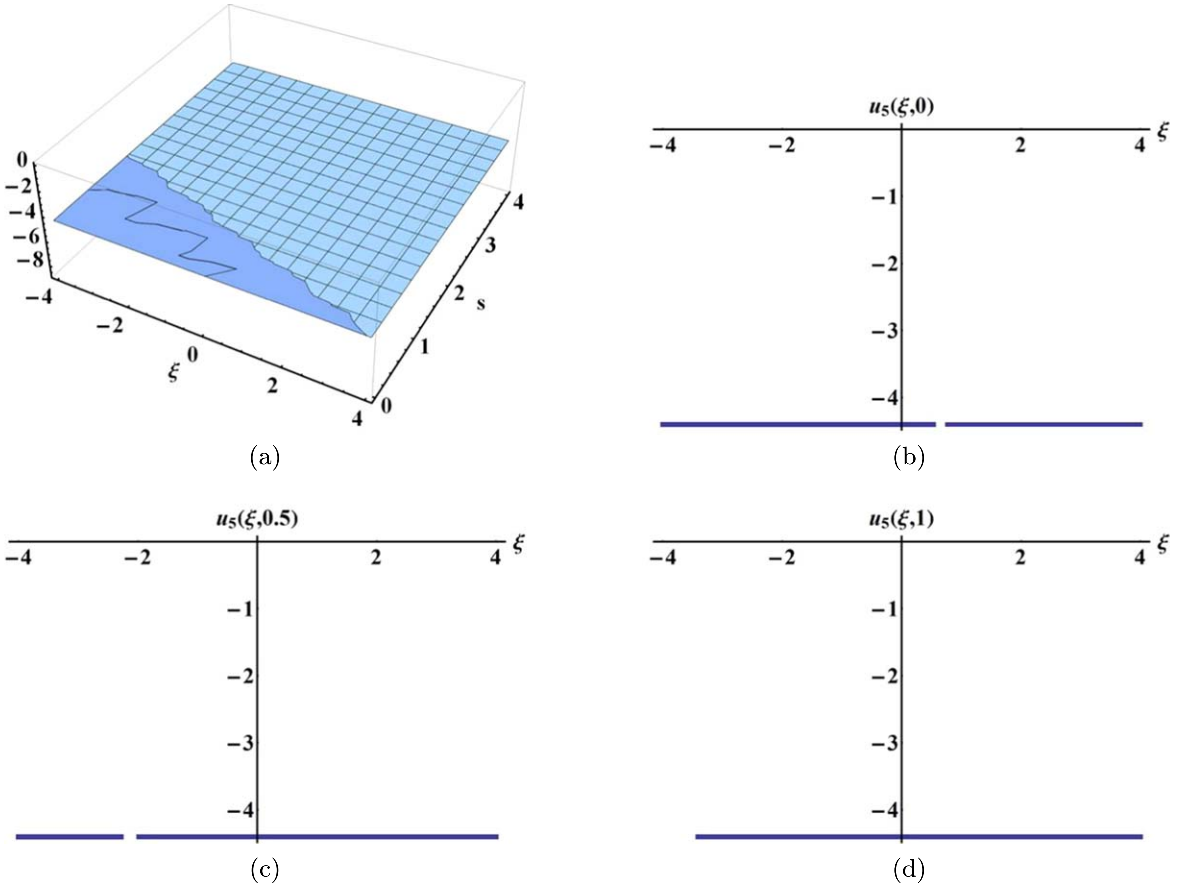


Figure 3. (a) 3D plot of the solution $u_5(\xi,\, s)$. (b)–(d) 2D plots of the solution $u_5(\xi,\, s)$ *at* $s = 0$, $s = 0.5$ and $s = 1$, respectively. Here, $\rho = 0.0001$, $\sigma = C = 1$, $\delta = 2.5$, $\alpha_0 = -0.005$, and $\beta_1 = -0.002$.

Case II.

$$\begin{cases} \alpha_0 = \pm 4i\ \beta_0\sqrt{\frac{3\nu}{\mu}},\ \alpha_1 = \pm 4i\ \beta_1\sqrt{\frac{3\nu}{\mu}},\ \ \alpha_2 = \alpha_3 = 0,\ \alpha_4 = \pm 8i\ \beta_0\sigma\sqrt{\frac{3\nu}{\mu}},\\ \alpha_5 = \pm 8i\ \beta_1\sigma\sqrt{\frac{3\nu}{\mu}},\ \beta_0 = \beta_0,\ \beta_1 = \beta_1,\ \ \epsilon = -16\nu, \end{cases} \tag{3.21}$$

where β_1 and β_2 are free parameters.

Substituting the values (3.21) into (3.18) and using (2.3), gets traveling wave solution of equation (3.14) as follows

$$u_2(\xi,\,\zeta,\,s) = \pm 4i\ \sqrt{\frac{3\nu}{\mu}} \times \left(1 + \frac{4\sigma}{\sigma + C\exp[4(\xi + \zeta + 16\nu s)]}\right). \tag{3.22}$$

Figure 5 shows the behaviour of the solution $u_1(\xi,\,\zeta,\,s)$ for $\mu = C = 1$ and $\nu = \sigma = -1$.

Case III.

$$\begin{cases} \alpha_0 = \frac{\beta_1(9\mu^2 - 23\nu)}{\nu\sqrt{2\nu^2 - 1}},\ \alpha_1 = 0,\ \ \alpha_2 = \pm\frac{3\mu\nu}{5\beta_1\sqrt{2\nu^2 - 1} + 37\nu},\ \alpha_3 = 0,\\ \alpha_4 = \pm\frac{3\mu(\nu^2 + \sigma^2)}{2\beta_1\sqrt{2\nu^2 - 1}},\ \alpha_5 = \beta_0 = 0,\ \ \beta_1 = \beta_1,\ \epsilon = \frac{-4\mu^2 + \mu(27\mu^2 - 74\nu) - 33\nu}{\sqrt{2\nu^2 - 1}}, \end{cases} \tag{3.23}$$

where β_1 is free parameters.

Substituting the values (3.23) into (3.18) and using (2.3), gets traveling wave solution of equation (3.14) as follows

$$u_3(\xi,\,\zeta,\,s) = \pm\frac{(\sigma + C\exp[4\psi(\xi,\,\zeta,\,s)])^{\frac{1}{4}}}{\beta_1}\left[\frac{\beta_1(9\mu^2 - 23\nu)}{\nu\sqrt{2\nu^2 - 1}} \pm \frac{3\mu\nu}{(5\beta_1\sqrt{2\nu^2 - 1}\ + 37\nu)(\sigma + C\exp[4\psi(\xi,\,\zeta,\,s)])^{\frac{1}{2}}} \pm \frac{3\mu(\nu^2 + \sigma^2)}{(5\beta_1\sqrt{2\nu^2 - 1}\,)(\sigma + C\exp[4\psi(\xi,\,\zeta,\,s)])}\right], \tag{3.24}$$

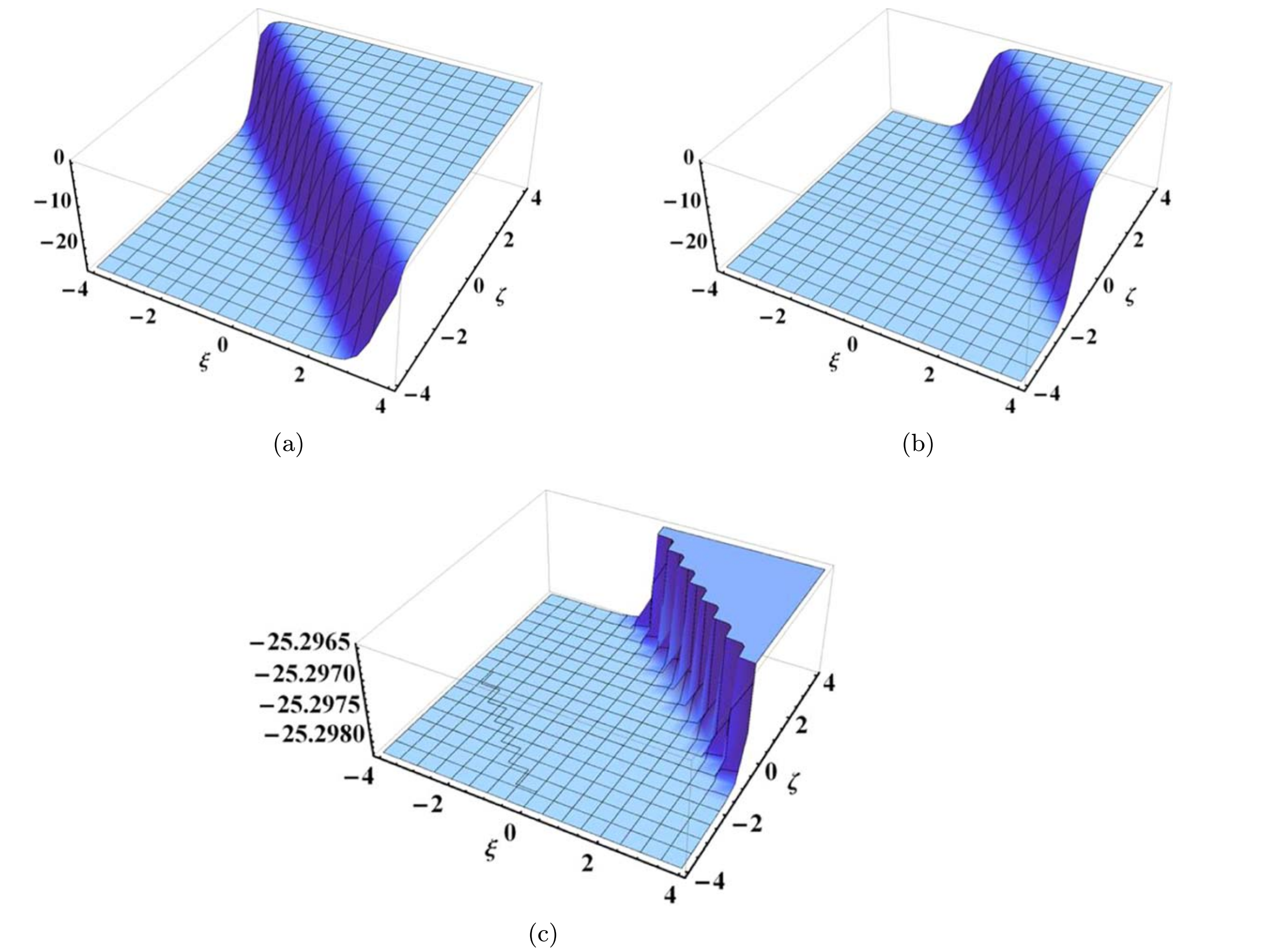


Figure 4. (a)–(c) 3D plots of the solution $u_1(\xi, \zeta, s)$ at $s = 0$, 0.2 and 0.4, respectively. Here, $\mu = 0.1$, $\nu = -1$, $\sigma = 0.25$ and $C = 1$.

where

$$\psi(\xi, \zeta, s) = \xi + \zeta - \left(\frac{-4\mu^2 + \mu(27\mu^2 - 74\nu) - 33\nu}{\sqrt{2\nu^2 - 1}}\right)s.$$

Figure 6 shows the behaviour of the solution $u_3(\xi, \zeta, s)$ for $\mu = \nu = C = 1$, $\sigma = 2$ and $\beta_1 = -0.0002$.

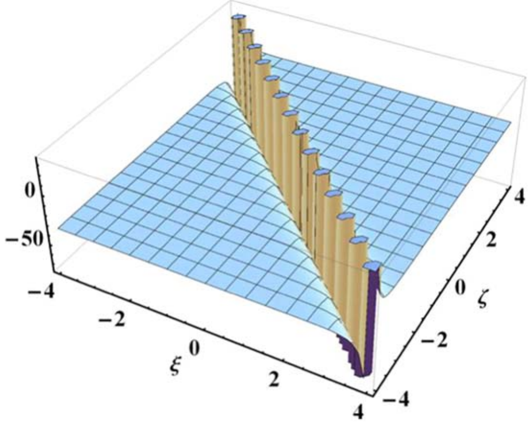
4. Conclusive remarks

This section is devoted to some conclusive remarks.

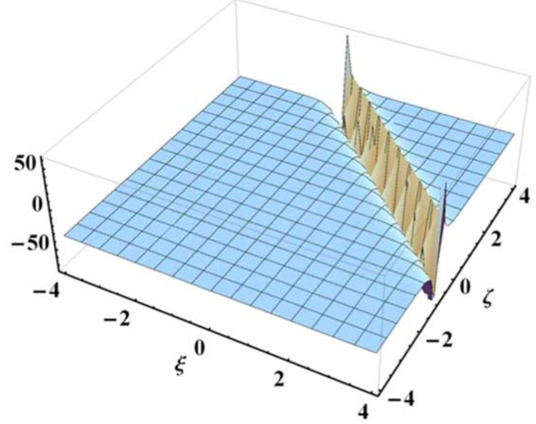
Remark 1. According to the identity $e^\theta = \cosh\theta + \sinh\theta$, our traveling wave solutions, obtained by the general improved Kudryashov method, can be handily converted to solitary wave solutions. For example, the solution $u_1(\xi, s)$ of the combined KdV–mKdV equation can be converted to a solitary wave solution as follows

$$\begin{aligned} u_1^*(\xi, s) &= \frac{\sqrt[4]{\sigma + C(\cosh[4\phi(\xi, s)] + \sinh[4\phi(\xi, s)])}}{\beta_1 \pm \beta_0 \sqrt[4]{\sigma + C(\cosh[4\phi(\xi, s)] + \sinh[4\phi(\xi, s)])}} \\ &\times \left[\frac{\alpha_0 + A}{102\beta_1^2\sqrt{6\rho}}\right. \\ &\pm \frac{155\beta_0^3}{(\sigma + C(\cosh[4\phi(\xi, s)] + \sinh[4\phi(\xi, s)])^{\frac{1}{2}}} \\ &+ \frac{255\beta_1^3}{(\sigma + C(\cosh[4\phi(\xi, s)] + \sinh[4\phi(\xi, s)])^{\frac{3}{4}}} \\ &+ \left.\frac{2448\beta_1^3\sigma}{(\sigma + C(\cosh[4\phi(\xi, s)] + \sinh[4\phi(\xi, s)])^{\frac{5}{4}}}\right], \end{aligned} \tag{4.1}$$

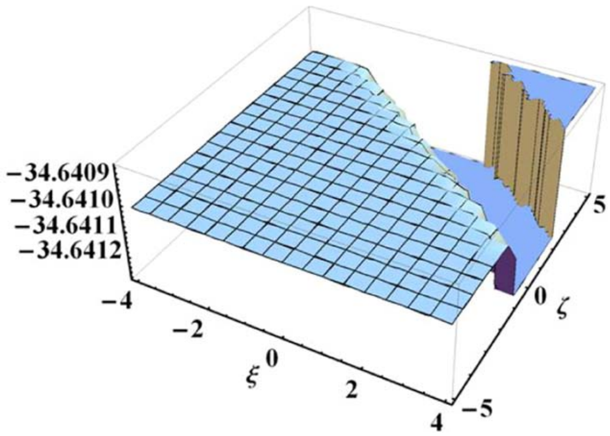
where A and $\phi(\xi, s)$ are given in Case A. Also, the solution $u_1(\xi, \zeta, s)$ of the (2+1)-dimensional modified Zakharov–Kuznetsov equation can be converted to a solitary wave



(a)



(b)



(c)

Figure 5. (a)–(c) 3D plots of the solution $u_2(\xi, \zeta, s)$ at $s = 0$,0.2 and 0.4, respectively. Here, $\mu = C = 1$ and $\nu = \sigma = -1$.

solution as follows

$$u_1^{*}(\xi, \zeta, s) = \pm\frac{4i\,(1-2\sigma)\sqrt{\frac{\nu}{\mu}}}{\sigma + C(\cosh[4(\xi+\zeta+16\nu s)] + \sinh[4(\xi+\zeta+16\nu s)]}. \tag{4.2}$$

Remark 2. Using the identity $e^{i\theta} = \cos\theta + i\sin\theta$, our traveling wave solutions, obtained by the general improved Kudryashov method, can be easily converted to periodic wave solutions. For example, the solution $u_1(\xi, s)$ of the combined KdV–mKdV equation can be converted to a periodic wave solution as follows

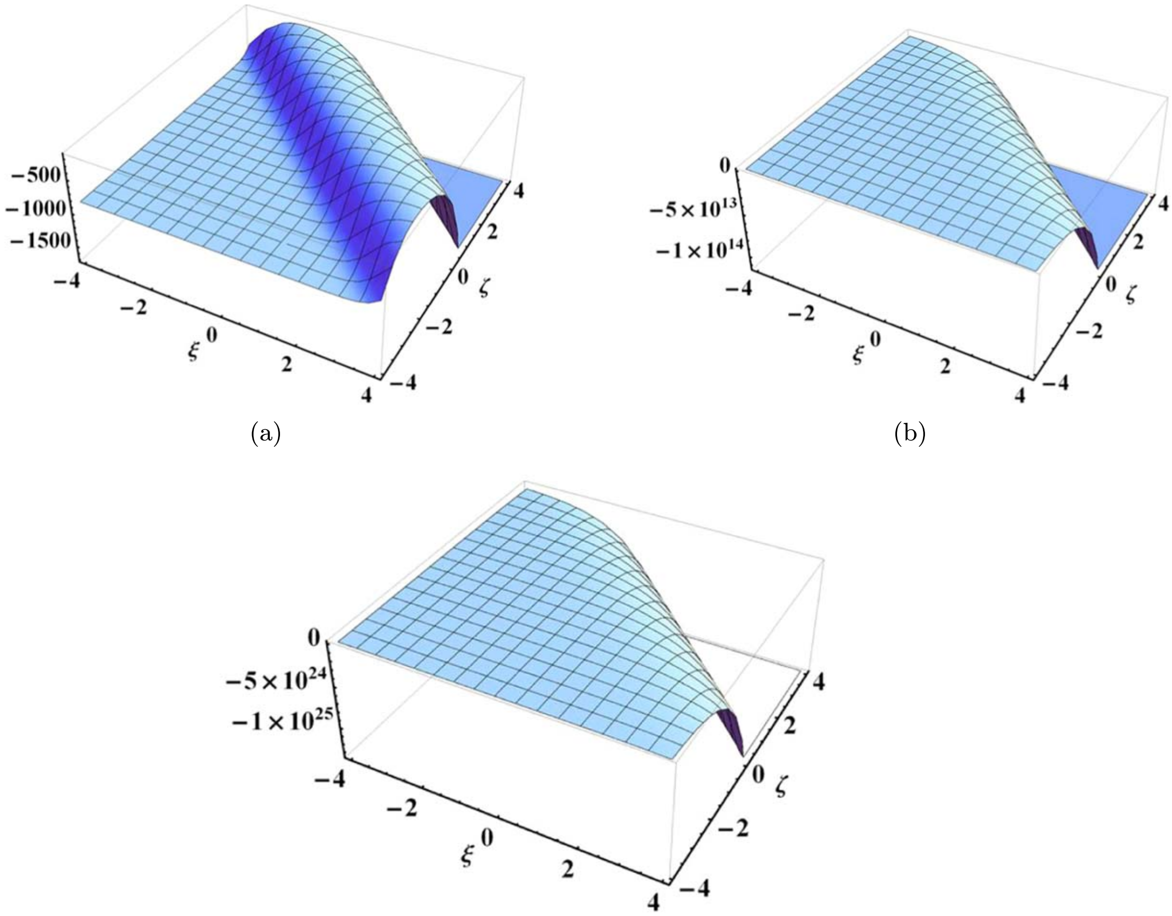
$$u_1^{**}(\xi, s) = \frac{\sqrt[4]{\sigma + C(\cos[4i\phi(\xi, s)] - i\sin[4i\phi(\xi, s)])}}{\beta_1 \pm \beta_0\sqrt[4]{\sigma + C(\cos[4i\phi(\xi, s)] - i\sin[4i\phi(\xi, s)])}}\left[\frac{\alpha_0 + A}{102\beta_1^2\sqrt{6\rho}} \pm \frac{155\beta_0^3}{(\sigma + C(\cos[4i\phi(\xi, s)] - i\sin[4i\phi(\xi, s)])^{\frac{1}{2}}} + \frac{255\beta_1^3}{(\sigma + C(\cos[4i\phi(\xi, s)] - i\sin[4i\phi(\xi, s)])^{\frac{3}{4}}} + \frac{2448\beta_1^3\sigma}{(\sigma + C(\cos[4i\phi(\xi, s)] - i\sin[4i\phi(\xi, s)])^{\frac{5}{4}}}\right], \tag{4.3}$$

where A and $\phi(\xi, s)$ are given in Case A. Also, the solution $u_1(\xi, \zeta, s)$ of the (2+1)-dimensional modified Zakharov–Kuznetsov equation can be converted to a periodic wave solution as follows

$$u_1^{**}(\xi, \zeta, s) = \pm\frac{4i\,(1-2\sigma)\sqrt{\frac{\nu}{\mu}}}{\sigma + C(\cos[4i(\xi+\zeta+16\nu s)] - i\sin[4i(\xi+\zeta+16\nu s)]}. \tag{4.4}$$

Remark 3. This paper improved the general Kudryashov method [22] by a general auxiliary equation

$$\chi'(\varpi) = \sigma\chi^n(\varpi) - \chi(\varpi),$$
$$1 < n \in \mathbb{N},\ 0 \neq \sigma \in \mathbb{R}, \tag{4.5}$$



(a) (b) (c)

Figure 6. (a)–(c) 3D plots of the solution $u_3(\xi,\zeta,s)$ *at* $s = 0$,0.5 and 1, respectively. Here, $\mu = \nu = C = 1$, $\sigma = 2$ and $\beta_1 = -0.0002$.

which has numerous general solutions depend on the natural number n, see (2.3). If $n = 2$, equation (4.5) reduces to the auxiliary equation $\chi'(\varpi) = \sigma\chi^2(\varpi) - \chi(\varpi)$ which used in [17, 22] together with the expansion (2.1) to produce a families of exact solutions for some nonlinear evolution equations. Also, for $n = 3$, equation (4.5) reduces to the auxiliary equation $\chi'(\varpi) = \sigma\chi^3(\varpi) - \chi(\varpi)$ which used in [28] together with the expansion (2.1) to produce a family of exact solutions for some nonlinear evolution equations. In our work, we just used equation (4.5) for $n = 5$. But one can use it for many values of the natural number n and obtain new pairwise disjoint sets of exact solutions. Moreover, we apply our method for the combined KdV–mKdV equation and the (2+1)-dimensional Zakharov–Kuznetsov equation as application examples. But, in fact, one can use it to solve many nonlinear evolution equations arising in mathematical physics, such as Hirota-Satsuma coupled KdV, Sawada-Kotera, Zhiber–Shabat and KdV–Burgers equations.

5. Conclusion

In this paper, the general Kudryashov method is improved by a novel auxiliary equation. So, a new method to constructing exact solutions for nonlinear evolution equations is introduced. This method is called the general improved Kudryashov method. The main merit of the general improved Kudryashov method over the others lies in the fact that it uses a particularly straightforward and effective algorithm to obtain exact solutions for a large number of nonlinear evolution equations. Also, a great variety of exact solution can be derived easily on choosing the parameters that appeared. We apply this method to explore exact traveling wave solution for the combined KdV–mKdV equation and the (2+1)-dimensional Zakharov–Kuznetsov equation. These solutions include solitary and periodic wave solutions. Also, some of the obtained solutions are graphically sketched.

Besides that, our approach generalizes some previous approaches. It is based on improving the general Kudryashov method [22] by the general auxiliary equation (2.2) which has various general solutions depend on the natural number n. If $n = 2$ and3our approach reduces to the approaches presented in [17, 22] and [28], respectively. In our work, we just used our general auxiliary equation for $n = 5$. But one can use it for many values of the natural number n and obtain new pairwise disjoint sets of exact solutions. In our point of view, there is no any demerit in applying our approach to finding exact solutions for nonlinear evolution equations.

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Appendix A

The system of algebraic equations in α_i, β_j ($i = 0, 1, 2, 3, 4, 5, j = 0, 1$) for the combined KdV–mKdV equation

χ^0: $-2\alpha_1\beta_0^2 + \alpha_0(2\beta_0\beta_1 + \alpha_0\beta_0\delta - 2\beta_0^2\epsilon + \alpha_0^2\rho) = 0,$

χ^1: $-8\alpha_2\beta_0^2 + \alpha_1(2\beta_0\beta_1 + 2\alpha_0\beta_0\delta - 2\beta_0^2\epsilon + 3\alpha_0^2\rho) + \alpha_0\beta_1(2\beta_1 + \alpha_0\beta_1\delta + 4\beta_0\epsilon) = 0,$

χ^2: $\alpha_0(3\alpha_1^2\rho - 2\beta_1^2\epsilon) + \alpha_1(\alpha_1\beta_0\delta + 2\alpha_0\beta_1\delta + 4\beta_0\beta_1\epsilon) + \alpha_2(2\alpha_0\beta_0\delta - 6\beta_0\beta_1 - 2\beta_0^2\epsilon + 3\alpha_0^2\rho) - 18\alpha_3\beta_0^2 = 0,$

χ^3: $-32\alpha_4\beta_0^2 + \alpha_3(2\alpha_0\beta_0\delta - 22\beta_0\beta_1 - 2\beta_0^2\epsilon + 3\alpha_0^2\rho) + \alpha_2(2\beta_1^2 + 2\alpha_1\beta_0\delta + 2\alpha_0\beta_1\delta - 4\beta_0\beta_1\epsilon + 6\alpha_0\alpha_1\rho) + \alpha_1(\alpha_1\beta_1\delta - 2\beta_1^2\epsilon + \alpha_1^2\rho) = 0,$

χ^4: $-50\alpha_5\beta_0^2 + \alpha_4(46\beta_0\beta_1 + 2\alpha_0\beta_0\delta - 2\beta_0^2\epsilon + 3\alpha_0^2\rho) + \alpha_3(2\alpha_1\beta_0\delta - 8\beta_1^2 + 2\alpha_0\beta_1\delta + 4\beta_0\beta_1\epsilon + 6\alpha_0\alpha_1\rho) + \alpha_2(\alpha_2\beta_0\delta + 2\alpha_1\beta_1\delta - 2\beta_1^2\epsilon + 3\alpha_0\alpha_2\rho + 3\alpha_1^2\rho) + 10\beta_0\sigma(\alpha_1\beta_0 - \alpha_0\beta_1) = 0,$

χ^5: $\alpha_5(2\alpha_0\beta_0\delta - 78\beta_0\beta_1 - 2\beta_0^2\epsilon + 3\alpha_0^2\rho) + \alpha_4(2\alpha_1\beta_0\delta - 18\beta_1^2 + 2\alpha_0\beta_1\delta + 4\beta_0\beta_1\epsilon + 6\alpha_0\alpha_1\rho) + \alpha_3(2\alpha_2\beta_0\delta + 2\alpha_1\beta_1\delta + 3\alpha_1^2\rho - 2\beta_1^2\epsilon + 6\alpha_0\alpha_2\rho) + \alpha_2(\alpha_2\beta_1\delta + 3\alpha_1\alpha_2\rho + 24\beta_0^2\sigma) + 6\beta_1\sigma(\alpha_1\beta_0 - \alpha_0\beta_1) = 0,$

χ^6: $\alpha_5(-32\beta_1^2 + 2\alpha_1\beta_0\delta - 4\beta_0\beta_1\delta + 6\alpha_0\alpha_1\rho + 2\alpha_0\beta_1\delta) + \alpha_4(2\alpha_2\beta_0\delta + 2\alpha_1\beta_1\delta - 2\beta_1^2\epsilon + 3\alpha_1^2\rho + 6\alpha_0\alpha_2\rho) + \alpha_3(\alpha_3\beta_0\delta + 2\alpha_2\beta_1\delta + 6\alpha_1\alpha_2\rho + 3\alpha_0\alpha_3\rho + 42\beta_0^2\sigma) + \alpha_2(30\beta_0\beta_1\sigma + \alpha_2^2\rho) = 0,$

χ^7: $\alpha_5(2\alpha_2\beta_0\delta + 2\alpha_1\beta_1\delta - 2\beta_1^2\epsilon + 3\alpha_1^2\rho + 6\alpha_0\alpha_2\rho) + \alpha_4(2\alpha_3\beta_0\delta + 2\alpha_2\beta_1\delta + 6\alpha_1\alpha_2\rho + 6\alpha_0\alpha_3\rho + 64\beta_0^2\sigma) + \alpha_3(\alpha_3\beta_1\delta + 3\alpha_2^2\rho + 3\alpha_1\alpha_3\rho + 62\beta_0\beta_1\sigma) + 10\alpha_2\beta_1^2\sigma = 0,$

χ^8: $\alpha_5(2\alpha_3\beta_0\delta + 2\alpha_2\beta_1\delta + 6\alpha_1\alpha_2\rho + 6\alpha_0\alpha_3\rho + 90\beta_0^2\sigma) + \alpha_4(\alpha_4\beta_0\delta + 2\alpha_3\beta_1\delta + 3\alpha_2^2\rho + 3\alpha_0\alpha_4\rho + 6\alpha_1\alpha_3\rho + 102\beta_0\beta_1\sigma) + \alpha_3(3\alpha_2\alpha_3\rho + 24\beta_1^2\sigma) = 0,$

χ^9: $\alpha_5(2\alpha_4\beta_0\delta + 2\alpha_3\beta_1\delta + 3\alpha_2^2\rho + 6\alpha_0\alpha_4\rho + 6\alpha_1\alpha_3\rho + 150\beta_0\beta_1\sigma) + \alpha_4(\alpha_4\beta_1\delta + 6\alpha_2\alpha_3\rho + 3\alpha_1\alpha_4\rho + 42\beta_1^2\sigma) + \alpha_3^3\rho = 0,$

χ^{10}: $\alpha_5(\alpha_5\beta_0\delta + 2\alpha_4\beta_1\delta + 6\alpha_2\alpha_3\rho + 6\alpha_1\alpha_4\rho + 3\alpha_0\alpha_5\rho + 64\beta_1^2\sigma) + 3\alpha_4\rho(\alpha_3^3 + \alpha_2\alpha_4) = 0,$

χ^{11}: $\alpha_5(\alpha_5\beta_1\delta + 3\alpha_3^2\rho + 6\alpha_2\alpha_4\rho + 3\alpha_1\alpha_5\rho) + 3\alpha_3\alpha_4^2\rho = 0,$

χ^{12}: $\alpha_5(6\alpha_3\alpha_4\rho + 3\alpha_2\alpha_5\rho) + \alpha_4^3\rho = 0,$

χ^{13}: $3\alpha_5\rho(\alpha_4^2 + \alpha_3\alpha_5) = 0,$

χ^{14}: $3\alpha_4\alpha_5^2\rho = 0,$

χ^{15}: $\alpha_5^3\rho = 0.$

Appendix B

The system of algebraic equations in α_i, β_j($i = 0, 1, 2, 3, 4, 5, j = 0, 1$) for the (2+1)-dimensional Zakharov–Kuznetsov equation

χ^0: $-3\alpha_0\beta_0^2\epsilon + \alpha_0^3\mu = 0,$

χ^1: $\alpha_1(3\alpha_0^2\mu - 3\beta_0^2\epsilon + 6\beta_0^2\nu) - 6\alpha_0\beta_0\beta_1(\epsilon + \nu) = 0,$

χ^2: $\alpha_2(24\beta_0^2\nu - 3\beta_0^2\epsilon + 3\alpha_0^2\mu) + \alpha_1(6\beta_0\beta_1\nu - 6\beta_0\beta_1\epsilon + 3\alpha_0\alpha_1\mu) + 3\alpha_0\beta_1^2(6\nu - \epsilon) = 0,$

χ^3: $\alpha_3(3\alpha_0^2\mu + 54\beta_0^2\nu - 3\beta_0^2\epsilon) + \alpha_2(18\beta_0\beta_1\nu + 6\alpha_0\alpha_1\mu - 6\beta_0\beta_1\epsilon) + \alpha_1(\alpha_1^2\mu - 3\beta_1^2\epsilon) = 0,$

χ^4: $\alpha_4(3\alpha_0^2\mu - 3\beta_0^2\epsilon + 96\beta_0^2\nu) + \alpha_2(6\alpha_0\alpha_1\mu - 6\beta_0\beta_1\epsilon + 66\beta_0\beta_1\nu) + \alpha_2(3\alpha_1^2\mu - 3\beta_1^2\epsilon + 3\alpha_0\alpha_2\mu + 6\beta_1^2\nu) = 0,$

χ^5: $\alpha_5(3\alpha_0^2\mu - 3\beta_0^2\epsilon + 150\beta_0\nu) + \alpha_4(138\beta_0\beta_1\nu + 6\alpha_0\alpha_1\mu - 6\beta_0\beta_1\epsilon) + \alpha_3(3\alpha_1^2\mu - 3\beta_1^2\epsilon + 6\alpha_0\alpha_2\mu + 24\beta_1^2\nu) + \alpha_1(3\alpha_2^2\mu - 36\beta_0^2\nu\sigma) + 36\alpha_0\beta_0\beta_1\nu\sigma = 0,$

χ^6: $\alpha_5(234\beta_0\beta_1\nu - 6\beta_0\beta_1\epsilon + 6\alpha_0\alpha_1\mu) + \alpha_4(54\beta_1^2\nu - 3\beta_1^2\epsilon + 6\alpha_0\alpha_2\mu + 3\alpha_1^2\mu) + \alpha_3(6\alpha_1\alpha_2\mu + 3\alpha_0\alpha_3\mu) + \alpha_2(\alpha_2\mu + 96\beta_0^2\nu\sigma) + 12\beta_1\nu\sigma(\alpha_0\beta_1 - \alpha_1\beta_0) = 0,$

χ^7: $\alpha_2(6\alpha_0\alpha_2\mu + 3\alpha_1^2\mu - 3\beta_1^2\epsilon + 96\beta_1^2\nu) + 6\alpha_4\mu(\alpha_0\alpha_3 + \alpha_1\alpha_2) + \alpha_3(3\alpha_2^2\mu + 3\alpha_1\alpha_3\mu - 180\beta_0^2\nu\sigma) - 108\alpha_2\beta_0\beta_1\nu\sigma = 0,$

χ^8: $6\alpha_5\mu(\alpha_0\alpha_3 + \alpha_1\alpha_2) + \alpha_4(3\alpha_2^2\mu + 6\alpha_1\alpha_3\mu + 3\alpha_0\alpha_4\mu - 288\beta_0^2\nu\sigma) + \alpha_3(3\alpha_2\alpha_3\mu - 252\beta_0\beta_1\nu\sigma) - 36\alpha_2\beta_1^2\nu\sigma = 0,$

χ^9: $\alpha_5(6\alpha_0\alpha_4\mu - 420\beta_0^2\nu\sigma + 3\alpha_2^2\mu + 6\alpha_1\alpha_3\mu) + \alpha_4(6\alpha_2\alpha_3\mu + 3\alpha_1\alpha_4\mu - 444\beta_0\beta_1\nu\sigma) + \alpha_3(\alpha_3\mu - 96\beta_1^2\nu\sigma) + 30\beta_0\nu\sigma^2(\alpha_1\beta_0 - \alpha_0\beta_1) = 0,$

χ^{10}: $\alpha_5(6\alpha_2\alpha_3\mu + 3\alpha_0\alpha_5\mu - 684\beta_0\beta_1\nu\sigma + 6\alpha_1\alpha_4\mu) + \alpha_4(3\alpha_3^2\mu + 3\alpha_2\alpha_4\mu + 180\beta_1^2\nu\sigma) + \nu\sigma^2(72\alpha_2\beta_0^2 + 18\alpha_1\beta_0\beta_1 - 18\alpha_0\beta_1^2) = 0,$

χ^{11}: $\alpha_5(3\alpha_1\alpha_5\mu - 288\beta_1^2\nu\sigma + 3\alpha_3^2\mu + 6\alpha_2\alpha_4\mu) + \alpha_3(3\alpha_4^2\mu + 126\beta_0^2\nu\sigma^2) + 90\alpha_2\beta_0\beta_1\nu\sigma^2 = 0,$

χ^{12}: $\alpha_5(6\alpha_3\alpha_4\mu + 3\alpha_2\alpha_5\mu) + \alpha_4(192\beta_0^2\nu\sigma^2 + \alpha_4^2\mu) + 3\beta_1\nu\sigma^2(62\alpha_3\beta_0 + 10\alpha_2\beta_1) = 0,$

χ^{13}: $3\alpha_5(\alpha_4^2\mu + \alpha_3\alpha_5\mu + 90\beta_0^2\nu\sigma^2) + 3\beta_1\nu\sigma^2(102\alpha_4\beta_0 + 24\alpha_3\beta_1) = 0,$

χ^{14}: $3\alpha_5(\alpha_4\mu + 150\beta_0\beta_1\nu\sigma^2) + 126\alpha_4\beta_1^2\nu\sigma^2 = 0,$

χ^{15}: $\alpha_5(\alpha_5^2 + 192\beta_1^2\nu\sigma^2) = 0.$

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