IOP Publishing Physica Scripta
Phys. Scr. **95** (2020) 045301 (8pp) https://doi.org/10.1088/1402-4896/ab6796

A study on the ground and excited states of hypernuclei

S Pal[1,2], R Ghosh[1], B Chakrabarti[3] and A Bhattacharya[1]

[1] Department of Physics, Jadavpur University, Kolkata 700032, India
[2] Department of Physics, Basanti Devi College, Kolkata, India
[3] Department of Physics, Jogamaya Devi College, Kolkata, India

E-mail: ballari_chakrabarti@yahoo.co.in

Received 3 July 2019, revised 20 December 2019
Accepted for publication 3 January 2020
Published 14 February 2020

Abstract
The binding energies of single Λ, double Λ and single Ξ-hypernuclei have been investigated in the framework of non relativistic Schroedinger equation with *Hulthén* potential describing the interaction between the Λ hyperon or Ξ-hyperon and the nuclei core. Both the ground states and the excited states of hypernuclei have been studied. The results obtained are compared with experimental data as well as other theoretical existing predictions. The investigation shows a good agreement with other theoretical predictions and experimental values wherever available. This indicates that the interaction between Λ/Ξ hyperon and the nuclei core described by *Hulthén* potential acts reasonably well.

Keywords: hypernuclei, binding energies, excited states

(Some figures may appear in colour only in the online journal)

1. Introduction

The studies of hypernuclear systems have achieved a great progress both in theory and experiments since their discovery in cosmic ray interaction in nuclei as delayed disintegration of nuclear fragments [1]. Recent experiments on hypernuclei have thrown considerable light on many of their properties [2]. The (K^-, π^-) reaction was the first reaction used for hypernuclei production using emulsion detectors where hyperfragments were identified by their mesonic decays [3]. After the experiment of the $(K^-_{\mathrm{stop}}, \pi^-)$ reaction by Faessler *et al* [4], various hypernuclei were studied using the in-flight (K^-, π^-) reaction in an almost recoilless condition [5–7]. Following the pioneering (K^+, π^+) reaction spectroscopy experiment at the BNLAGS in 1970s [8, 9], excitation spectra have been measured for a wide range of Λ-hypernuclei with superconducting kaon spectrometer (SKS) at the KEK 12 GeV Proton Synchroton (PS) [10–12]. Precision gamma-ray spectroscopy with a Germanium detector array (hyperball) has been performed for p-shell Λ-hypernuclei [13]. High energy electron beams allow electro-production measurements and precision study of hypernuclear structure through the $(e, e'K^+)$ reaction [14]. The identification of well defined ground as well as excited hypernuclei states are also observed [15–18]. Hasegawa *et al* [19] have made a spectroscopic study of ${}^{12}_{\Lambda}$C hypernuclei using SKS and observed two peaks at 2.6 and 6.9 MeV which are interpreted as excited state with ^{11}C core and a Λ hyperon. Formation of double Λ hypernuclei is in focus of strangeness-nuclear interaction since the experimental discovery of ${}^{10}_{\Lambda\Lambda}$Be, ${}^{6}_{\Lambda\Lambda}$He and ${}^{13}_{\Lambda\Lambda}$B [20–22]. Revised values of ${}^{6}_{\Lambda\Lambda}$He binding energies have been obtained in E373(KEK-PS) collaboration [23]. Takahashi *et al* [24] have measured the Λ–Λ interaction energy $\Delta_{\Lambda\Lambda}$ from double hyperfragment event and found that it is weakly attractive whereas Nakazawa *et al* [25] have observed the $\Lambda\Lambda$-hypernuclei via the Ξ hyperon capture at rest reaction in a hybrid emulsion. Recently ${}_{\Lambda\Lambda}$Be has been observed by the J-PARC E07 collaboration in nuclear emulsions tagged by the (K^+, K^-) reaction [26]. Excellent reviews on experimental and theoretical developments in the field of hypernuclei spectroscopy have been presented by Botta *et al* [27], Feliciello *et al* [28], Gal *et al* [29] and Hiyama *et al* [30].

The experimental results triggered a good impetus to the theoretical studies of hypernuclei physics. A number of theoretical models describing the hyperon-core interaction have been suggested. Λ hyperon is a distinguishable strange baryon and is not blocked by Pauli exclusion principle. Hence it can stay in the same level with nucleons in nuclei and provide important information regarding the ΛN interaction. Millener *et al* [31] have studied the density dependence and non-locality of Λ-nucleus potential by fitting data on the level spectra of hypernuclei. They have also given the interpretation of the p-shell hypernuclei data in terms of shell model calculation. Λ-hypernuclei magnetic moments have been estimated in a relativistic model by Gattone *et al* [32]. Dover *et al* [33] have estimated the binding energy of multi-Λ hypernuclei and have extended the Bethe–Weizsacker mass formula to strange hadronic matter and concluded that the multi-Λ hypernuclei are not stable for $\Lambda\Lambda \longrightarrow \Xi N$ conversion [34]. Hungerford *et al* [35] have analysed ground and excited state binding energies of hypernuclei to obtain a parametrization of the effective ΛN interaction using Woods–Saxon potential. Kolesnikov *et al* [36] have estimated the binding energies of three-four-and five-particle ground and excited states of hypernuclei. Rijken *et al* [37] have also studied the hypernuclei spectroscopy using a soft core-hyperon nucleon potential as Woods–Saxon potential. Halderson [38] investigated the binding energies of several hypernuclei with different potentials using first order Brueckner—Hartree calculations. Khan *et al* [39] have studied the excited states of double Λ hypernuclei by hyperspherical symmetrical approach. Nogga *et al* [40] have investigated the differences of the Λ separation energies of ${}^{4}_{\Lambda}$H and ${}^{4}_{\Lambda}$He to probe the YN interaction models. Energy levels of double Λ-hypernuclei having mass number A from 7 to 10 have been predicted on the basis of four body cluster structure model by Hiyama *et al* [41]. Xiang *et al* [42] have calculated the binding energy per baryon of the Λ-hypernuclei systematically using the relativistic mean field theory. Shoeb *et al* [43] have investigated energies of degenerate spin flip doublet $\frac{3}{2}^{+}, \frac{5}{2}^{+}$ of ${}^{10}_{\Lambda}$Be and of 2^{+} of ${}^{10}_{\Lambda\Lambda}$Be in α-cluster model. Gaitanos *et al* [44] have used Giessen–Boltzmann–Uehling–Uhlenbeck transport model with relativistic mean field approximation and calculated the double Λ-hypernuclei production cross-section in relativistic heavy-ion collisions. Buyukcizmeci *et al* [45] have investigated the formation of hypernuclei in the spectator region of peripheral relativistic ion collisions which have broad distribution in masses and isospin. They have used the simple liquid-drop model parametrization of binding energy of hypernuclei. The Λ-hypernuclei binding energy has been calculated using the auxiliary field diffusion Monte Carlo method by Lonardoni *et al* [46]. Bhowmick *et al* [47] have studied the shape of light nuclei and Λ hypernuclei in relativistic mean field approach. They have discussed FSU Gold parametrization for this purpose and investigated the deformation in a Λ hypernucleus in the first excited state. Botvina *et al* [48] have studied the production of hypernuclei in relativistic ion collision by hyperon capture using statistical disintegration and suggested that binding energy of hypernuclei can be effectively evaluated from different isotopes of hypernuclei. Cui *et al* [49] have investigated p and d shell of ${}^{13}_{\Lambda}$C and ${}^{21}_{\Lambda}$Ne Λ hypernuclei with Skyrme type NΛ interaction. Ground states of single Λ-hypernuclei have been probed by Pal *et al* [50] in the frame work of non relativistic Schroedinger equation with *Hulthén* potential describing the interaction between the hyperon and the nuclei core for a wide range of hypernuclei from light to heavy sector. Analytical form of binding energies of excited states of 1p,1d,1f and 1g shells of number of single hypernuclei have been studied by Nejad *et al* [51] using Woods–Saxon potential.

In the present work we have studied the binding energies of first excited state Λ-hypernuclei along with their ground states. The current work also focuses our investigation on the binding energies of the excited state along with the ground state of double Λ-hypernuclei, ground state of single Ξ-hypernuclei in the framework of the non-relativistic Schroedinger equation considering *Hulthén* type interaction between the hyperons and the nuclei core. Results are compared with experimental findings wherever available as well as with other theoretical estimates.

2. The model

The presence of hyperon in finite nuclear system gives us opportunity to study baryon–baryon interaction in deep perspective. There are several mechanisms of hypernuclei production. Hypernuclei can be produced in heavy ion collisions [52]. It can also be produced by weak and electromagnetic interaction [53]. Hypernuclei are well produced through (K^{-}, π^{-}) and (π^{+}, K^{+}) reactions [54]. A large number of hypernuclei are produced by hyperon capture by nuclear residues [55].The strangeness exchange process $K^{-} + n \rightarrow \Lambda + \pi^{-}$, associated production $\pi^{+} + n \rightarrow \Lambda + K^{+}$ and the electroproduction reaction $(e, e'K^{+})$ are useful channel for the production of hypernuclei. The strangeness exchange process is used primarily due to the high cross-section of direct production. The resonance mechanism [56] is also an important method where the target nucleon is excited above the strangeness production threshold and finally it decays into hyperon and anti-kaon. Replacing one or two neutrons by hyperons like Λ or Ξ is one of the mechanisms for the production of hypernuclei. Being Λ hyperon the lightest particle in the hyperon family, it can stay long enough in contact with the nuclear core. To find out the binding energies of the hyperons inside the nuclei we have solved the non-relativistic Schroedinger equation for the system. It can be represented as:

$$\left[-\frac{\hbar^2}{2m_r}\nabla^2 + V(r)\right]\Psi(\vec{r}) = E\Psi(\vec{r}), \tag{1}$$

where E is the energy eigenvalue, m_r is the reduced mass of the system, $m_r = m_{\rm core}m_\Lambda/(m_{\rm core} + m_\Lambda)$, $m_{\rm core}$ is the mass of the nuclear core and m_Λ is the mass of the Λ hyperon. $V(r)$ represents the interaction between the nuclei core and the hyperon which we have considered as the *Hulthén* type of screened potential. *Hulthén* potential [57] is one of the short range potentials which is widely used in nuclear, particle, atomic and solid state physics [58–61]. *Hulthén* potential is very useful in describing the bound states as well as continuum of interacting particles both in relativistic and non relativistic approach. The potential is defined as [57]

$$V_H = -\frac{V_0}{[\exp(\lambda r) - 1]}, \tag{2}$$

where V_0 is the strength parameter and represented by $V_0 = \alpha_{\text{eff}}\lambda$, where α_{eff} is a constant and λ is the screening parameter. It may be mentioned that *Hulthén* potential is valid for small values of screening parameter and breaks down with high values of it.

The analytical form of the wave function corresponding to *Hulthén* potential is obtained as [62]

$$\begin{aligned}\Phi_n^H = &\frac{\lambda^{3/2}}{2\sqrt{\pi}}\frac{\Gamma(n+2K_n+1)}{(n+1)!\Gamma(2K_n+1)}\\ &\times [2K_n(n+K_n+1)(n+2K_n+1)]^{1/2}\\ &\times (\lambda r)^{-1}(1-\exp(-\lambda r))(\exp(-\lambda K_n r))\\ &\times {}_2F_1[-n, n+2K_n+2, 2K_n+1; \exp(-\lambda r)], \end{aligned} \quad (3)$$

where $n = 0, 1, 2, 3, \ldots$ .... etc, $K_n = \left(-\frac{m_r E_n}{\lambda^2}\right)^{1/2}$, which is a dimensionless quantity relating eigen energy and the reduced mass m_r. ${}_2F_1[-n, n+2K_n+2, 2K_n+1; \exp(-\lambda r)]$ is the generalised Hypergeometric function. The mass of the nuclear core with single hyperon or double hyperons can be represented as $m_{\text{core}} = Zm_p + (N-1)m_n$ or $m_{\text{core}} = Zm_p + (N-2)m_n$ where Z is the proton number, N is the neutron number. The masses of the proton (m_p) and the neutron (m_n) are taken as 0.938272 GeV and 0.939 565 GeV respectively [63]. The energy eigenvalues are expressed as [62]:

$$E_n = -\frac{\lambda^2}{m_r}\left(\frac{\nu^2-(n+1)^2}{2(n+1)}\right)^2, \quad (4)$$

where $\nu^2 = \frac{m_r V_0}{\lambda^2}$, which is also a dimensionless quantity.

The binding energies of hypernuclei can be expressed as:

$$E_{\text{BE}} = \int |\Phi_n^H|^2 V_H \,\mathrm{d}^3 r. \quad (5)$$

Thus the final expression of binding energy for ground state $(n = 1)$ has been obtained as

$$\begin{aligned}E_{\text{BE}} = &-4\pi V_0\Bigg[\frac{\lambda^{1/2}}{4\pi^{1/2}}\\ &\frac{\Gamma(2+2K_1)}{\Gamma(2K_1+1)}(2K_1)(2+K_1)(2+2K_1)^{1/2}\Bigg]^2\\ &\times \int [\exp(-\lambda(1+2K_1)r) - \exp(-2\lambda(1+K_1)r)]\\ &\times {}_2F_1[-1,\ 3+2K_1, 2K_1+1, \exp(-\lambda r)]^2\,\mathrm{d}r \end{aligned}\quad (6)$$

and the binding energy for the first Λ excited state $(n = 2)$, it is found to be

$$\begin{aligned}E_{\text{BE}} = &-4\pi V_0\Bigg[\frac{\lambda^{1/2}}{12\pi^{1/2}}\\ &\frac{\Gamma(3+2K_2)}{\Gamma(2K_2+1)}(2K_2)(3+K_2)(3+2K_2)^{1/2}\Bigg]^2\\ &\times \int \exp(-\lambda(1+2K_2)r) - \exp(-2\lambda(1+K_2)r)\\ &\times {}_2F_1[-2, 4+2K_2, 2K_2+1, \exp(-\lambda r)]^2\,\mathrm{d}r. \end{aligned}\quad (7)$$

To estimate the binding energies we have considered the radius of the hypernucleus as $r_H = r_\Lambda + r_n$, where r_Λ is the radius of the Λ hyperon and r_n is the radius of the core nucleus in the hypernuclei. r_n has been estimated using the relation $r_n = r_0 A^{1/3}$ where r_0 is nuclear radius parameter, A is the nucleon number of the core nuclei and expressed as $A = Z + (N-1)$ for single hypernuclei and $A = Z + (N-2)$ for double hypernuclei. The radius r_n of the core nucleus has been estimated with $r_0 = 1.2$ fm [64]. Radius of Λ particle is taken as $r_\Lambda = 5\ \text{GeV}^{-1}$ [65] which is supposed to be the typical hadron radius. α_{eff} has wide range of values and we have considered $\alpha_{\text{eff}} \simeq (V_0/\lambda) = 0.6$ [66] for estimating the binding energies E_{BE} $(-\text{B}_\Lambda)$ of ground state of various double Λ-hypernuclei and single Λ and Ξ-hypernuclei using relation (6). The first excited state of several Λ-hypernuclei and double Λ-hypernuclei have also been estimated using relation (7).

We have estimated the screening parameter λ for different hypernuclei in different sectors such as medium (core nucleons in p-shell, or in s–d shell at maximum) and heavy by fitting the experimental values of binding energies of hypernuclei in the equation (6) or (7) for ground and first excited state respectively. We use the experimental value of binding energy of ${}^{15}_{\Xi^-}\text{C} = 16.0 \pm 4.71$ MeV [67] in the medium sector for ground state of Ξ-hypernuclei and have found the value of screening parameter λ as 0.135 1 GeV. Using this λ we have estimated the binding energies of several Ξ-hypernuclei for medium sector and those have been exhibited in table 1. Taking the values of the screening parameter λ as 0.105 and 0.098 GeV [50] for medium and medium-heavy to heavy single Λ-hypernuclei respectively, the ground state binding energies of several Λ-hypernuclei have been calculated and also displayed in table 1. Several Λ-hypernuclei data exhibited in this table have been taken from our previous work [50] for comparison. Again for double Λ-hypernuclei we have fitted the experimental binding energy of ${}^{10}_{\Lambda\Lambda}\text{Be} = 14.94 \pm 0.13$ MeV [68] in the medium sector and have found the value of λ as 0.208 1 GeV. As no experimental values of double Λ-hypernuclei for medium-heavy to heavy sector are available we have fitted the value of ${}^{13}_{\Lambda\Lambda}\text{B} = 23.30 \pm 0.7$ MeV [68], the last heaviest available value of the medium sector for the estimation of the binding energies. We have found λ as 0.1952 GeV and all these binding energy values have been displayed in table 2. By fitting the binding energy values of ${}^{16}_{\Lambda}\text{O} = 2.5 \pm 0.5$ MeV [9] and ${}^{32}_{\Lambda}\text{S} = 8.0 \pm 0.5$ MeV [9] for medium and medium-heavy to heavy sector for the first Λ excited state binding energies in equation (7), screening parameter $\lambda = 0.056$ and 0.0097 GeV have been found out. With these λ other first excited state binding energies of Λ-hypernuclei have been calculated and have been displayed in table 3. As no experimental data of double-Λ-hypernuclei excited state are available we have chosen the best possible values of $\lambda = 0.047$ and 0.03 GeV for first excited state binding energies estimation for medium and heavy sector respectively and those have been exhibited in table 4. All the results in the above tables are compared with corresponding experimental values wherever available as well as other theoretical works [69–77], the references have been

Table 1. Estimated ground state binding energies ($-B_{\Xi^-}$) and ($-B_\Lambda$) of Λ/Ξ-hypernuclei in medium and medium-heavy to heavy sectors.

Sector	Element	Our work in MeV	Expt. Value in MeV	Other works in MeV
Medium	$^{11}_{\Xi^-}$B	13.140	9.2 ± 2.2 [67]	
	$^{12}_{\Xi^-}$B	13.973		
	$^{12}_{\Xi^-}$Be	13.975		
	$^{13}_{\Xi^-}$C	14.720	18.1 ± 3.2 [67]	
	$^{14}_{\Xi^-}$N	15.419		
	$^{17}_{\Xi^-}$O	15.471	16.0 ± 5.5 [67]	
	$^{28}_{\Xi^-}$Al	20.192	23.2 ± 6.8 [67]	
	$^{30}_{\Xi^-}$Mg	20.344		
Medium	$_\Lambda{}^{10}$Be	9.671	9.11 ± 0.22 [76]	
	$_\Lambda{}^{13}$B	11.41		
	$_\Lambda{}^{12}$C	10.901	10.80 ± 0.18 [75]	
	$_\Lambda{}^{13}$C	11.447	11.69 ± 0.12 [76]	11.590 [63]
	$_\Lambda{}^{14}$N	11.914	12.17 ± 0.00 [7]	
	$_\Lambda{}^{16}$O	12.645	12.50 ± 0.35 [9]	12.5031 [55]
	$_\Lambda{}^{16}$N	12.967		
	$_\Lambda{}^{19}$F	13.536		14.77 [74]
Medium-heavy to heavy	$_\Lambda{}^{23}$Na	16.341		15.362 [44]
	$_\Lambda{}^{25}$Mg	16.632		16.014 [44]
	$_\Lambda{}^{26}$Mg	16.882		17.648 [44]
Medium-heavy to heavy	$_\Lambda{}^{26}$Al	16.882		16.601 [44]
	$_\Lambda{}^{27}$Al	16.979		17.872 [44]
	$_\Lambda{}^{28}$Al	17.197		
	$_\Lambda{}^{28}$Si	17.240	16.00 ± 0.29 [9]	17.872 [44]
	$_\Lambda{}^{29}$Si	17.261		18.897 [44]
	$_\Lambda{}^{40}$Ca	18.493	18.70 ± 1.10 [9]	18.7022 [55]
	$_\Lambda{}^{51}$V	18.964	19.90 ± 1.00 [9]	22.91 [27]
	$_\Lambda{}^{56}$Fe	19.279	21.00 ± 1.50 [77]	
	$_\Lambda{}^{89}$Y	19.928	22.10 ± 1.60 [9]	21.83 [61]
	$_\Lambda{}^{139}$La	20.433	23.8 ± 1.00 [10]	27.42 [27]
	$_\Lambda{}^{208}$Pb	20.624	26.5 ± 0.5 [10]	27.35 [27]

Table 2. Estimated ground state binding energies ($-B_{\Lambda\Lambda}$) of double Λ-hypernuclei in the medium and medium-heavy to heavy sectors.

Sector	Element	Our work in MeV	Expt. Value in MeV	Other works in MeV
Medium	$_{\Lambda\Lambda}{}^{11}$Be	16.923	17.53 ± 0.11 [29]	18.23 [41]
	$_{\Lambda\Lambda}{}^{12}$Be	18.671	22.48 ± 1.21 [68]	
	$_{\Lambda\Lambda}{}^{12}$B	18.705	20.60 ± 0.74 [68]	20.85 [68]
	$_{\Lambda\Lambda}{}^{13}$B	20.092	23.30 ± 0.7 [68]	23.21 [68]
	$_{\Lambda\Lambda}{}^{16}$O	23.365		
Medium-heavy to heavy	$_{\Lambda\Lambda}{}^{28}$Si	33.408		
	$_{\Lambda\Lambda}{}^{32}$S	34.435		
	$_{\Lambda\Lambda}{}^{40}$Ca	35.848		
	$_{\Lambda\Lambda}{}^{51}$V	37.197		
	$_{\Lambda\Lambda}{}^{56}$Fe	37.557		
	$_{\Lambda\Lambda}{}^{56}$Ni	37.556		
	$_{\Lambda\Lambda}{}^{83}$Mo	38.951		
	$_{\Lambda\Lambda}{}^{89}$Y	39.197		
	$_{\Lambda\Lambda}{}^{139}$La	40.036		
	$_{\Lambda\Lambda}{}^{208}$Pb	40.753		

displayed in the tables . Binding energies are found to be in reasonably good agreement with the experimental values wherever available. It may be mentioned that the screening parameter represents the shielding effect due to the charges. The variation of screening parameter may be due to the space charge which affects the screening. The interaction potential between the core and the hyperon may change the screening parameter due to associated light $q\bar{q}$ pairs out of vacuum.

Table 3. Estimated binding energies ($-B_\Lambda$) of Λ-hypernuclei for first Λ-excited state for medium and medium-heavy to heavy sector.

Sector	Element	Our work in MeV	Expt.value in MeV	Other works in MeV
Medium	${}_{\Lambda}{}^{10}Be$	1.7872		
	${}_{\Lambda}{}^{13}B$	2.2425		
	${}_{\Lambda}{}^{13}C$	2.24	0.8 ± 0.3 [29]	
	${}_{\Lambda}{}^{16}N$	2.53	2.84 ± 0.16 [29]	
	${}_{\Lambda}{}^{19}F$	2.7247		4.465 [47]
Medium-heavy to heavy	${}_{\Lambda}{}^{23}Na$	7.902		6.921 [47]
	${}_{\Lambda}{}^{25}Mg$	8.494		8.039 [47]
	${}_{\Lambda}{}^{26}Mg$	8.689		9.30 [47]
	${}_{\Lambda}{}^{26}Al$	8.597		12.103 [47]
	${}_{\Lambda}{}^{27}Al$	8.619		9.07 [47]
	${}_{\Lambda}{}^{28}Si$	8.543	7.0 ± 0.2 [10]	7.49 [64]
				8.099 [71]
				6.039 [72]
				7.68 [51]
	${}_{\Lambda}{}^{29}Si$	8.290		9.182 [44]
	${}_{\Lambda}{}^{40}Ca$	8.138	11.0 ± 0.5 [29]	8.780 [64]
				8.78 [64]
				8.718 [72]
				9.60 [70]
				11.06 [51]
	${}_{\Lambda}{}^{51}V$	9.090	11.90 ± 0.17 [12]	11.05 [70]
	${}_{\Lambda}{}^{89}Y$	12.834	16.1 ± 0.3 [10]	14.23 [70]
	${}_{\Lambda}{}^{139}La$	13.016	20.1 ± 0.4 [10]	
	${}_{\Lambda}{}^{208}Pb$	14.429	21.3 ± 0.7 [10]	18.60 [70]

Table 4. Estimated binding energies ($-B_{\Lambda\Lambda}$) of double Λ-hypernuclei for first excited state in the medium and heavy sectors.

Sector	Element	Our work in MeV	Other works in MeV
Medium	${}_{\Lambda\Lambda}{}^{10}Be$	13.429	14.80 [73]
	${}_{\Lambda\Lambda}{}^{13}B$	16.791	20.71 [73]
	${}_{\Lambda\Lambda}{}^{16}O$	19.43	—
Heavy	${}_{\Lambda\Lambda}{}^{28}Si$	29.204	—
	${}_{\Lambda\Lambda}{}^{32}S$	30.448	—
	${}_{\Lambda\Lambda}{}^{40}Ca$	32.885	—
	${}_{\Lambda\Lambda}{}^{51}V$	32.994	—
	${}_{\Lambda\Lambda}{}^{56}Fe$	33.265	—
	${}_{\Lambda\Lambda}{}^{56}Ni$	33.27	—
	${}_{\Lambda\Lambda}{}^{83}Mo$	34.292	—
	${}_{\Lambda\Lambda}{}^{89}Y$	34.381	—
	${}_{\Lambda\Lambda}{}^{139}La$	35.067	—
	${}_{\Lambda\Lambda}{}^{208}Pb$	35.146	—

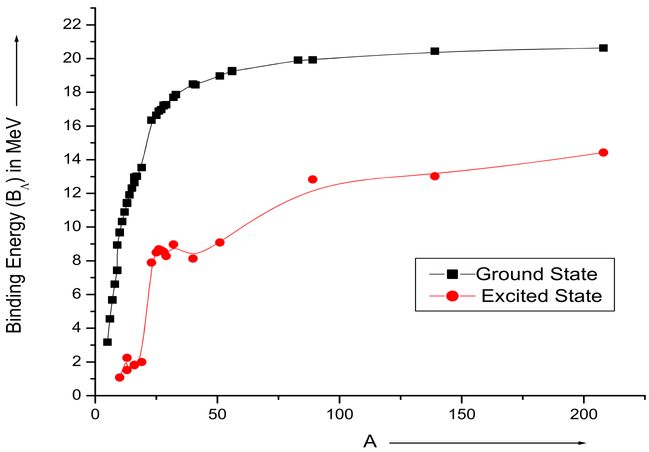

Figure 1. (B_Λ) versus mass no.(A) for the ground as well as excited states of single Λ-hypernuclei.

Lattice calculations proposed such screening effect [78] . Apart from the valence quark the lattice sea quarks induce a screening effect on potential when the inter-quark distance increases [79]. Increase in particle concentration also affects the screening length.

3. Discussions and conclusions

In the current work we have investigated the binding energies of first excited state of single Λ and $\Lambda\Lambda$-hypernuclei along with the ground state binding energies of double $\Lambda\Lambda$, single Λ and Ξ-hypernuclei with the *Hulthén* potential as interaction potential. The variation of the binding energies of ground and excited state of single Λ-hypernuclei with A, the mass number has been displayed in figure 1. We have included some results from our previous work [50] to draw the graph along with the new hypernuclei like ${}^{19}_{\Lambda}F$, ${}^{23}_{\Lambda}Na$, ${}^{25}_{\Lambda}Mg$, ${}^{26}_{\Lambda}Al$, ${}^{27}_{\Lambda}Al$, ${}^{29}_{\Lambda}Si$. Figure 2 describes the variation of the binding energy/nucleon for single Λ-hypernuclei. It is interesting to observe from figure 2 that the maximum binding energy/nucleon is found to be around ${}_{\Lambda}{}^{9}B$ and rapidly decreases with A. Binding energy per nucleon is observed to be much less than the usual nucleus. It may be

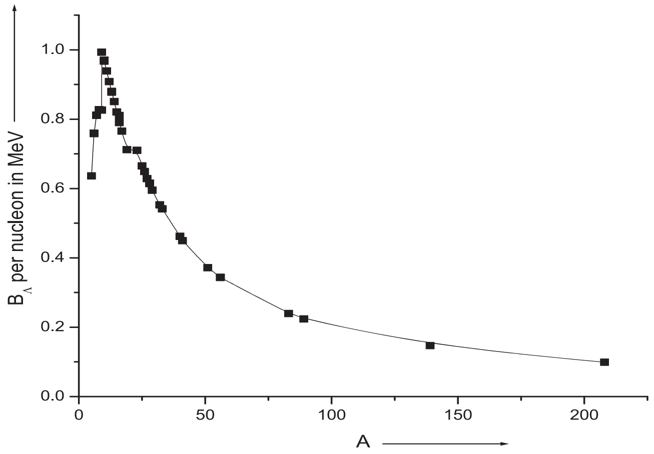

Figure 2. (B_Λ) per nucleon versus mass no.(A) for the ground states of single Λ-hypernuclei.

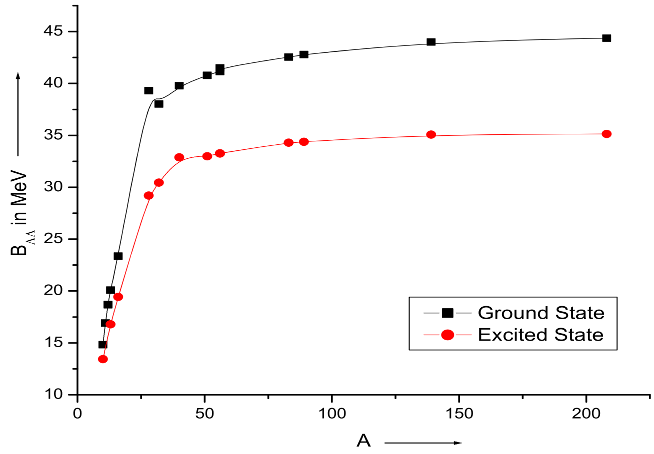

Figure 4. ($B_{\Lambda\Lambda}$) versus mass no.(A) for the ground as well as excited states of double Λ-hypernuclei.

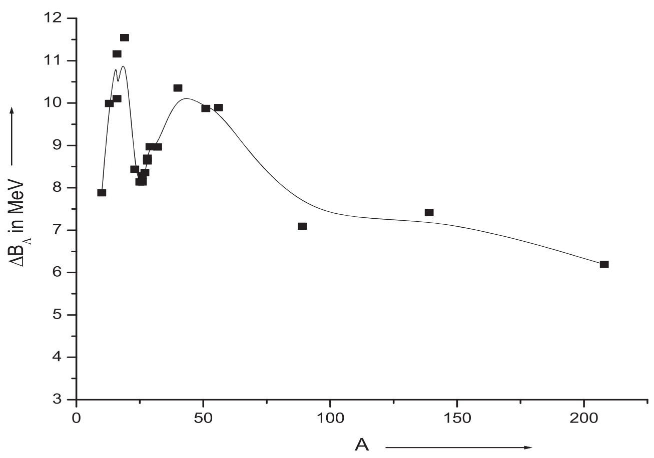

Figure 3. Splitting in binding energy (B_Λ)versus mass no.(A) of single Λ-hypernuclei.

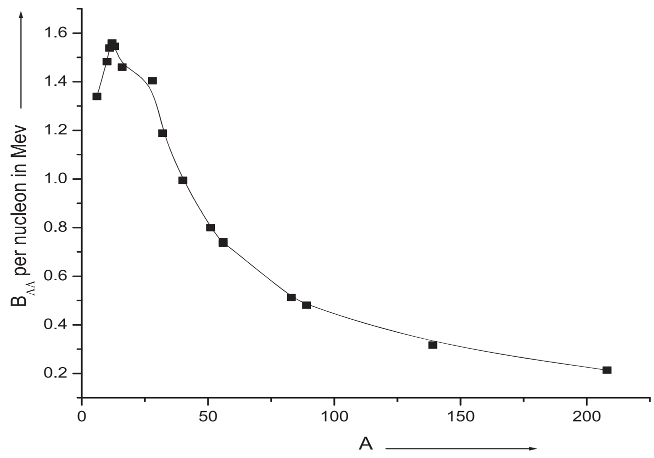

Figure 5. ($B_{\Lambda\Lambda}$) per nucleon versus mass no. (A) for the ground states of double Λ-hypernuclei.

suggested that the impurity of Λ hyperon made the system weakly coupled with lesser binding energy decreasing rapidly with mass number. The variation of energy splitting ΔB_Λ between the ground and excited state of Λ-hypernuclei with mass number A has been plotted in figure 3. It is found that ΔB_Λ consistently increases up to $A \leqslant 20$, attains maximum value at $A = 20$ then decreases and then again increases with a maximum at around $A = 45$ and then shows a gradual decrease. In figure 4 the variation of the binding energies for ground and excited state for double Λ hypernuclei has been plotted with mass number A. The binding energies for both ground and excited states are found to increase sharply up to $A < 40$ with a saturation starting form $A > 50$. Figure 5 shows the variation of the binding energy/nucleon for ΛΛ-hypernuclei with mass number A and it displays the maximum binding at ${}_{\Lambda\Lambda}{}^{13}$B.

Binding energies for single Λ-hypernuclei along with the Ξ-hypernuclei are shown in table 1. Results are found to be in reasonably good agreement with experimental results wherever available and other theoretical works . It may be mentioned that recently ${}^{12}_{\Xi^-}$Be has been observed at J-PARC although no final result is given [80]. Our estimated values of binding energy of ${}^{10}_{\Lambda}$Be, ${}^{12}_{\Lambda}$C, ${}^{13}_{\Lambda}$C, ${}^{14}_{\Lambda}$N, ${}^{16}_{\Lambda}$O compare favourably well with the experiments. In table 2 we have found slightly lower values of binding energies compared to the experiments in the medium sector of double Λ-hypernuclei. In table 3 estimated value of first excited state of ${}^{16}_{\Lambda}$N is in good agreement with the experimental value whereas we have found higher value of that of ${}^{13}_{\Lambda}$C in comparison to the corresponding experiment. It is observed that the binding energies for ${}^{51}_{\Lambda}$V, ${}^{89}_{\Lambda}$Y, ${}^{139}_{\Lambda}$La, ${}^{208}_{\Lambda}$Pb are found to be small compared to the experimental values. In this context it may be pointed out that the uncertainty in the results may be attributed to the uncertainty in the values of the screening parameter and the radius parameter. For double Λ-hypernuclei excited state binding energies the results show reasonable agreement with the other theoretical works available.

It may be pointed out that binding energy of the core nucleus has been neglected while estimating the core nucleus mass. Recently Wang *et al* [81] have furnished the revised mass formula for the atomic mass evolution where the correction term due to ionization is considered. They have estimated the corrections and the uncertainty in the results which have been found to be in KeV order. In the light of their work we have also estimated the corrections for binding energy of the core nucleus which is of the order of KeV. It is small compared to the mass of the core nucleus and in the first approximation it could be neglected. It may be mentioned that we have taken an approximation to find out the radius of hypernuclei by addition of the radius of core and the Λ side by side. However Song *et al* [82] suggested that with the additional degree of freedom of strangeness Λ can penetrate into the interior of the core bringing the change in nuclear radii. The shrinkage effect has also been probed by Hong *et al* [83] in the context of giant monopole resonance of hypernuclei. Some works show that the change in core root mean square radius with mass number A is very small. Tan *et al* [84] have studied the presence of hyperon in bulk properties of nuclei. Hiyama [85] observed that not all nuclei are compressed by Λ injection, nuclei having mass number $A < 11$ is not affected but some are affected by 30%. The definite conclusion is yet to be achieved with reliable data. In our future works we would like to include this shrinkage effect of radii.

We have extracted the values of screening parameter λ by fitting the experimental values of binding energy for different hypernuclei in the medium and medium-heavy to heavy sector in our formulation. The estimated binding energies show good agreement for some hypernuclei and comparable agreement for some cases with other works and experimental works available. The main source of uncertainty in the results comes from the values of the screening parameter and the radius parameter of hyperon. It may be pertinent to recall that Woods–Saxon potential has been used by a number of workers [35, 37, 38, 51, 64] to study the binding energies of the hypernuclei which describes well the hyperon-core interaction. We have already compared our results with them and the comparison has been displayed in the corresponding tables. In addition the *Hulthén* form of the potential allows one to solve the Schroedinger problem in an analytically exact way to find out the eigen energies. It may help one to develop some feelings about the underline physics. The study of excited state of hypernuclei is of immense importance to understand the YN spectroscopy with the new generation experiments coming for core-hyperon excited states. It is noteworthy to mention the experiment of ALICE collaboration [86] on the lifetime measurement of hypertriton ${}^{3}_{\Lambda}$H and anti-hypertriton which would help one to continue further research in this emerging field. The new experimental updated results from different groups are coming out with main focus on high resolution spectroscopy and decay modes. Future experimental efforts at FAIR, FRIB, J-Lab, J-PARC will suppose to reveal the structure of neutron rich nuclei. The understanding of possible description of hyperon-nuclei potential, the theoretical investigation is of utter importance at this conjecture. Systematic study of hypernuclei spectroscopy with good resolution will improve our knowledge of ΛN interaction.

Acknowledgments

Authors are thankful to University Grants Commission, New Delhi, India for financial assistance.

ORCID iDs

B Chakrabarti https://orcid.org/0000-0003-0455-5751

References

[1] Danysz M and Pniewski J 1953 *Phil. Mag.* **44** 348
[2] Hashimoto O and Tamura H 2006 *Prog. Part. Nucl. Phys.* **57** 564
[3] Juric M *et al* 1972 *Nucl. Phys.* B **47** 36
Juric M *et al* 1973 *Nucl. Phys.* B **52** 1
[4] Faessler M A *et al* 1973 *Phys. Lett.* B **46** 468
[5] Bruckner W 1978 *Phys. Lett.* B **79** 157
[6] Bertini R *et al* 1981 *Nucl. Phys.* A **368** 365
[7] May M *et al* 1981 *Phys. Rev. Lett.* **47** 1106
[8] Milner C *et al* 1985 *Phys. Rev. Lett.* **54** 1237
[9] Chrien R E *et al* 1988 *Nucl. Phys.* A **478** 705c
Pile P H 1989 *Nuovo Cimento* A **102** 413
Pile P H *et al* 1991 *Phys. Rev. Lett.* **66** 2585
[10] Hasegawa T *et al* 1996 *Phys. Rev.* C **53** 1210
[11] Hashimoto O *et al* 1998 *Nucl. Phys.* A **639** 93c
[12] Hotchi H *et al* 2001 *Phys. Rev.* C **64** 044302
[13] Tamura H *et al* 2000 *Phys. Rev. Lett.* **84** 5963
Tamura H *et al* 2013 *Nucl. Phys.* A **914** 99
[14] Miyoshi T *et al* 2003 *Phys. Rev. Lett.* **90** 232502
[15] Ajimura S *et al* 1998 *Nucl. Phys.* A **639** 93
[16] Iodice M *et al* 2007 *Phys. Rev. Lett.* **99** 052501
[17] Agnello M *et al* 2011 *Phys. Lett.* B **698** 219
[18] Nakamura S N *et al* 2013 *Phys. Rev. Lett.* **110** 012502
[19] Hasegawa T *et al* 1995 *Phys. Rev. Lett.* **74** 224
[20] Danysz M *et al* 1963 *Nucl. Phys.* **49** 212
Danysz M *et al* 1963 *Phys. Rev. Lett.* **11** 29
[21] Prowse D J 1966 *Phys. Rev. Lett.* B **17** 782
[22] Aoki S *et al* 1991 *Prog. Theo. Phys.* **85** 1287
[23] Ahn J K *et al* (E373(KEK-PS)Collab.) 2013 *Phys. Rev.* C **88** 014003
[24] Takahashi H *et al* 2001 *Phys. Rev.Lett.* **87** 212502
[25] Nakazawa K *et al* (KEK-E-373) 2010 *Nucl. Phys.* A **835** 207
[26] Ekawa H *et al* (J PARC Expt.) 2019 *Prog. Theo. Expt. Phys.* (**2019** 021D02
[27] Botta E, Bressani T and Garbarino G 2012 *Eur. Phys. J.* A **48** 41
[28] Feliciello A and Nagae T 2015 *Rep. Prog. Phys.* **78** 096301
[29] Gal A, Hungerford E V and Millener D J 2016 *Rev. Mod. Phys.* **88** 035004
[30] Hiyama E and Nakazawa K 2018 *Ann. Rev. Nucl. Part. Sc.* **68** 131
[31] Millener D J *et al* 1988 *Phys. Rev.* C **38** 2700
Millener D J 2010 *Nucl. Phys.* A **835** 11
Millener D J 2012 *Nucl. Phys.* A **881** 298
[32] Gattone O A, Chiapparini M and Izquierdo E D 1991 *Phys. Rev.* C **44** 548
[33] Dover C B *et al* 1991 *Phys. Rev.* C **44** 1905
[34] Dover C B and Gal A 1993 *Nucl. Phys.* A **560** 559
[35] Hungerford E V and Furic M 1999 *Fizika* B **8** 1
Hungerford E V 1994 *Prog. Theor. Phys.* **117** 135
Randemiya S D and Hungerford E V 2007 *Phys. Rev.* C **76** 064308

[36] Kolesnikov N N and Tarasov V I 1997 *Russ. Phys. J.* **40** 944
Kolesnikov N N and Kalachev S A 2006 *Phys. Part. Nucl. Lett.* **3** 341
[37] Rijken T A 2001 *Nucl. Phys.* A **691** 322c
Rijken T A, Stokes V G J and Yamamoto Y 1999 *Phys. Rev.* C **59** 21
[38] Halderson D 2000 *Phys. Rev.* C **61** 034001
[39] Khan M A *et al* 2001 *Int. J. Mod. Phys.* E **10** 107
[40] Nogga A, Kamada H and Glockle W 2002 *Phys. Rev. Lett.* **88** 172501
[41] Hiyama E *et al* 2002 *Phys. Rev.* C **66** 024007
[42] Xiang M, Lei L and Zhi N P 2009 *Chin. Phys. Lett.* **26** 072101
[43] Shoeb M *et al* 2009 *Phys. Rev.* C **79** 054321
[44] Gaitanos T, Larinov A B, Lencke H and Mosel U 2012 *Nucl. Phys.* A **881** 240
[45] Buyukcizmeci N, Botvina A S, Pochodzalla J and Bleicher M 2013 *Phys. Rev.* C **88** 014611
[46] Lonardoni D *et al* 2013 *Phys. Rev.* C **87** 041303(R)
[47] Bhowmick B, Bhattacharya A and Gangopadhyay G 2014 *Eur. Phys. J.* A **50** 125
[48] Botvina A S, Buyukcizmeci N, Ergun A, Ogul R, Bleicher M and Pochodzalla J 2016 *Phys. Rev.* C **94** 054615
[49] Cui J W and Zhou X R 2017 *Prog. Theor. Exp. Phys.* **2017** 093D04
[50] Pal S, Ghosh R, Chakrabarti B and Bhattacharya A 2017 *Eur. Phys. J. Plus* **132** 262
[51] Mohammad S, Nejad M and Armat A 2018 *Mod. Phys. Lett.* A **33** 1850022
[52] Botvina A S *et al* 2017 *Phys. Rev.* C **95** 014902
[53] Buyukcizmeci N *et al* 2018 *Phys. Rev.* C **98** 064603
[54] Vander B I S *et al* 2011 *Ann. Phys.* **326** 1085
[55] Alberico W M and Garbarino G 2002 *Phys. Rep.* **369** 1
Tretyakova T Y and Lanskoy D E 2003 *Phys. Atom Nucl.* **66** 1651
[56] Majling L *et al* 1980 *Phys. Lett.* B **92** 256
Wunsch R and Zofka J 1987 *Phys. Lett.* B **193** 7
[57] Hulthen L 1942 *Ark. Mat. Astron. Fys.* **28A** 5
[58] Bhoi J *et al* 2018 *Comm. Theo. Phys.* C **69** 9
[59] Laha U and Bhoi J 2016 *Phys. Atm. Nucl.* **79** 62
[60] Bayrak O and Boztosuu I 2007 *Phys. Scr.* C **76** 92
[61] Agboola D 2009 *Phys. Scr.* **80** 065304
[62] de Lange O L 1991 *Am. J. Phys.* **59** 151
[63] Tanabashi M *et al* (Particle Data Group) 2018 *Phys. Rev.* D **98** 030001
[64] Thakkar K, Majethiya A and Vinodkumar P C 2011 *Proc. DAE Symp. on Nucl. Phys.* **56** 750
[65] Shinozaki T, Oka M and Tekeuchi S 2005 *Phys. Rev.* D **71** 074025
[66] Horvath I, Lichard P, Lietava R, Nogova A and Pisut J 1988 *Phys. Lett.* B **214** 237
[67] Wilkinson D H *et al* 1959 *Phys. Rev. Lett.* **3** 397
Bechdorf A *et al* 1968 *Phys. Lett.* B **26** 174
Mondal A S *et al* 1979 *Nuovo Cimento* A **54** 333
[68] Gal A and Millener D J 2011 *Phys. Lett.* B **701** 342
[69] Bando H, Motoba T and Zofka J 1990 *Int. J. Mod. Phys.* **A5** 4021
[70] Lalazissis G A 1994 *Phys. Rev.* C **49** 1412
[71] Cai C H, Li L, Tan Y H and Ning P Z 2003 *Eur. Phys. Lett.* **64** 448
[72] Koutroulos C G 1991 *J. Phys. G: Nucl. Part. Phys.* **17** 1069
Usmani Q N and Bodmer A R 1999 *Phys. Rev.* C **60** 055215
[73] Yoshiko K E 2018 *Phys. Rev.* C **97** 034324
[74] Umeya A *et al* 2016 *Nucl. Phys.* A **954** 242
[75] Dluzewski P *et al* 1988 *Nucl. Phys.* A **484** 520
[76] Cantwell T *et al* 1974 *Nucl. Phys.* A **236** 445
[77] Hashimoto O *et al* 1989 *Nuovo Cimento* A **102** 679
Akei M *et al* 1989 *Nuovo Cimento* A **102** 457
[78] Born K D 1989 *Phys. Rev.* D **40** 1653
[79] Bali G S 2001 *Phys. Rep.* **343** 1
[80] Nagae T *et al* 2019 *AIP Conf. Proc.* **2130** 020015
[81] Wang M *et al* 2017 *Chin. Phys.* C **41** 030003
[82] Song C Y *et al* 2010 *Int. J. Mod. Phys.* E **19** 2538
[83] Hong L *et al* 2018 *Chin. Phys. Lett.* **35** 062102
[84] Tan Y H *et al* 2004 *Phys. Rev.* C **70** 054306
[85] Hiyama E 2013 *J. Phys.: Conf. Ser.* **463** 012080
[86] ALICE Collab. 2016 *Phys. Lett.* B **754** 360
ALICE Collab. 2019 *Phys. Lett.* B **797** 134905