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A New 3-D Chaotic Jerk System with Four Nonlinear Terms, its Backstepping Synchronization and Circuit Simulation

K. Lamamra[1,*], S. Vaidyanathan[2], W. T. Putra[3], E. Darnila[4], A. Sambas[5], and Mujiarto[5]

[1] Department of Electronics, University of Larbi Ben Mhidi, Oum El Bouaghi, Algeria
[2] Research and Development Centre, Vel Tech University, Avadi, Chennai, India
[3] Department of Mechanical Engineering, Universitas Muhammadiyah Ponorogo, Indonesia
[4] Department of Informatic, Universitas Malikussaleh, Aceh Utara, Indonesia
[5] Department of Mechanical Engineering, Universitas Muhammadiyah Tasikmalaya, Indonesia

*Lamamra.Kheireddine@univ-oeb.dz

Abstract. A new 3-D chaotic jerk system with four nonlinear systems is proposed in this research work. There is great interest in the literature in discovering chaos in mechanical systems. In this work, we find chaos in a 3-D chaotic jerk system, which is a mechanical oscillator with four nonlinear terms. As a control application, we design backstepping based global chaos synchronization for a pair of new chaotic jerk systems. As a circuit application, we design MultiSIM electronic circuit for the new chaotic jerk system. The MultiSIM outputs show good agreement with the MATLAB outputs for the new jerk system.

1. Introduction

Chaos theory deals with nonlinear dynamical systems exhibiting high sensitivity to small changes in initial conditions [1-2]. Mathematically, chaotic systems are characterized by the presence of at least one positive Lyapunov exponent.

Chaotic systems have several applications in science and engineering. Some important applications of chaotic systems can be cited as temperature systems [3], spring-pendulum oscillator [4], motor DC [5], biological snap oscillator [6], radar [7], robotic [8], economic [9], circuits [10-11], etc.

In physics, a jerk ODE can be written as the third order dynamics

$$\frac{d^3x}{dt^3} = \varphi\left(x, \frac{dx}{dt}, \frac{d^2x}{dt^2}\right) \tag{1}$$

In (1), $x(t)$ stands for the displacement, $\frac{dx}{dt}$ the velocity, $\frac{d^2x}{dt^2}$ the acceleration and $\frac{d^3x}{dt^3}$ the *jerk*.

Thus, we call the third order differential equation (1) as the *jerk differential equation*.

In mechanical engineering, jerk differential equations have several applications in oscillatory motion [1-2].

In qualitative analysis, it is convenient to express the third-order ODE (1) in a system form.

For this purpose, we define the following phase variables:

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$$\begin{cases} x_1(t) = x(t) \\ x_2(t) = \dot{x}(t) \\ x_3(t) = \ddot{x}(t) \end{cases} \tag{2}$$

Thus, the state variables in (2) can be viewed as the *displacement*, *velocity* and *acceleration*, respectively. Using these state variables, we can express the jerk differential equation (1) as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = \varphi(x_1, x_2, x_3) \end{cases} \tag{3}$$

Many jerk systems have been reported in the chaos literature [12-17]. Some famous jerk systems in the mechanical engineering are Sprott systems [12], Li system [13], Elsonbaty system [14], Coullet system [15], Vaidyanathan systems [16-17], etc.

In this research paper, we report the finding of a new chaotic jerk system with four nonlinear terms. We describe the phase plots of the jerk system and carry out a rigorous dynamic analysis by finding equilibrium points, Lyapunov exponents, Kaplan-Yorke dimension, symmetry analysis, etc.

As a control application, we derive new results for the backstepping based synchronization of the new chaotic jerk system with itself. Synchronization of chaotic systems deals with the control problem of finding suitable feedback control laws so as to asymptotically synchronize the respective trajectories of a pair of chaotic systems called as master and slave systems. We use backstepping control method for achieving global chaos synchronization of the new chaotic jerk system with itself. Backstepping control method is a recursive procedure used for stabilizing nonlinear dynamical systems [1-2].

Section 2 describes the new chaotic jerk system, its phase plots and Lyapunov exponents. Section 3 describes the backstepping-based adaptive synchronization of the new chaotic jerk system with itself. Furthermore, an electronic circuit realization of the new chaotic system is presented in detail in Section 4. The circuit experimental results of the new chaotic jerk system in Section 4 agreement with the numerical simulations via MATLAB obtained in Section 2. Section 5 draws the main conclusions.

2. A New Chaotic Jerk System with Four Nonlinear Terms

In this work, we report a new 3-D jerk system modelled by

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = -az - bx^2 z - c\,|x| + xy^2 - x^3 \end{cases} \tag{4}$$

where $X = (x, y, z)$ is the state and a, b, c are positive constants.

In this paper, we show that the 3-D jerk system (4) is *chaotic* for the parameter values

$$a = 2,\ b = 0.2,\ c = 0.5 \tag{5}$$

Using Wolf's algorithm [18], the Lyapunov exponents of the system (4) for the parameter set $(a, b, c) = (2, 0.2, 0.5)$ and the initial state $X(0) = (0.2, 0.2, 0.2)$ were found as

$$LE_1 = 0.1979,\ LE_2 = 0,\ LE_3 = -2.7849 \tag{6}$$

Thus, the 3-D jerk system (4) is chaotic with a positive Lyapunov exponent, LE_1.

It is noted that the sum of the Lyapunov exponents in (6) is negative.

$$LE_1 + LE_2 + LE_3 = -2.5870 < 0 \tag{7}$$

This shows that the system (4) is dissipative with a strange chaotic attractor.

The Kaplan-Yorke dimension of the jerk system (4) is computed as

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$$D_{KY} = 2 + \frac{LE_1 + LE_2}{|LE_3|} = 2.0711 \tag{8}$$

The equilibrium points of the new chaotic jerk system (4) are easily determined as $E_0 = (0,0,0)$ and $E_1 = (-0.7071, 0, 0)$. These equilibrium points are in a critical case, and Lyapunov's first stability method based on linearization matrices is not successful to analyze the stability behaviour of E_0 and E_1. From the phase plots, we infer that these equilibrium points have unstable behaviour.

Figure 1 shows the Lyapunov exponents of the 3-D chaotic jerk system (4) for the parameter set $(a,b,c) = (2, 0.2, 0.5)$ and initial state $X(0) = (0.2, 0.2, 0.2)$.

Figures 2-4 show the 2-D phase portraits of the hyperchaotic system (1) for the parameter set $(a,b,c) = (2, 0.2, 0.5)$ and initial state $X(0) = (0.2, 0.2, 0.2)$.

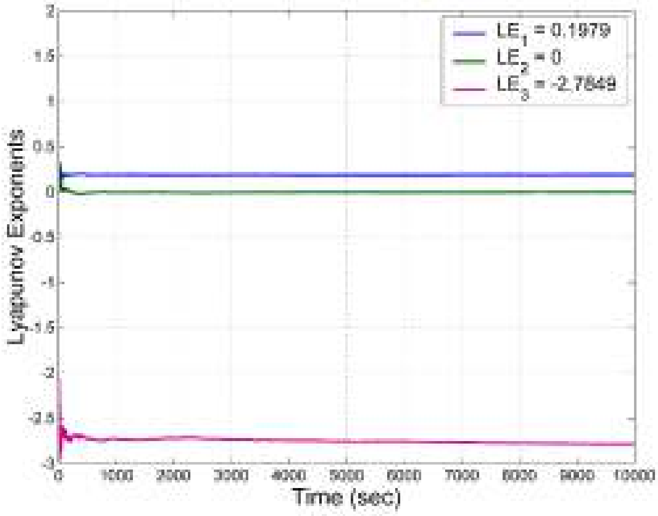

Figure 1. Lyapunov exponents of the new chaotic jerk system (4) for the parameter set $(a,b,c) = (2, 0.2, 0.5)$ and initial state $X(0) = (0.2, 0.2, 0.2)$

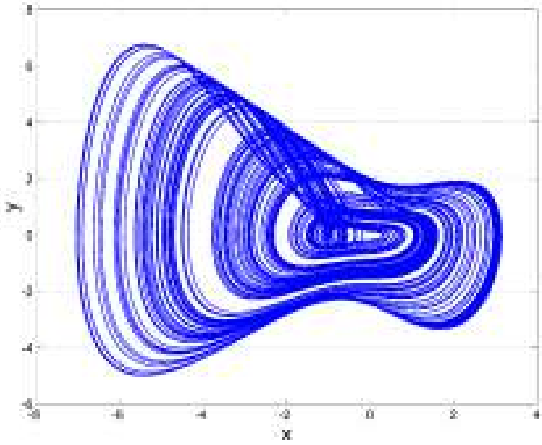

Figure 2. MATLAB plot showing the 2-D phase portrait of the new chaotic jerk system (4) in the $(x, y)-$plane for $(a,b,c) = (2, 0.2, 0.5)$ and $X(0) = (0.2, 0.2, 0.2)$

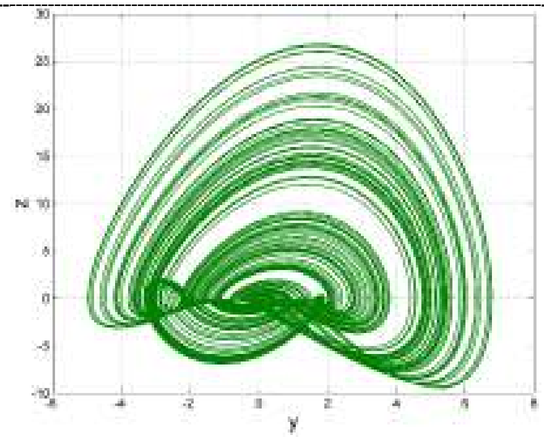

Figure 3. MATLAB plot showing the 2-D phase portrait of the new chaotic jerk system (4) in the $(y, z)-$plane for $(a,b,c) = (2, 0.2, 0.5)$ and $X(0) = (0.2, 0.2, 0.2)$

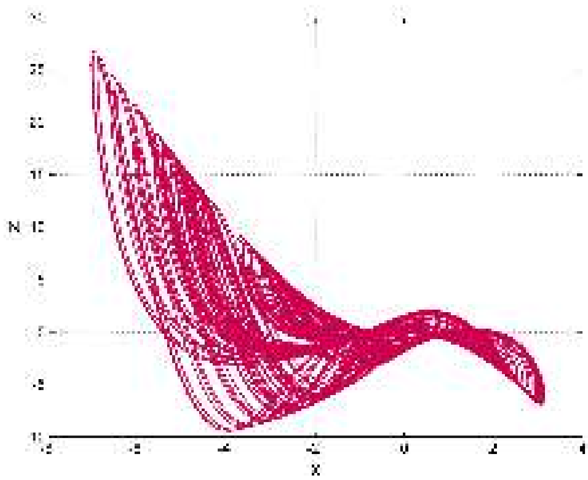

Figure 4. MATLAB plot showing the 2-D phase portrait of the new chaotic jerk system (4) in the $(x, z)-$plane for $(a, b, c) = (2, 0.2, 0.5)$ and $X(0) = (0.2, 0.2, 0.2)$

3. Backstepping-Based Global Chaos Synchronization of the Chaotic Jerk Systems

In this section, we use backstepping control method to achieve global chaos synchronization of the new chaotic jerk systems.

As the master system for the synchronization, we consider the new chaotic jerk system

$$\begin{cases} \dot{x}_1 = y_1 \\ \dot{y}_1 = z_1 \\ \dot{z}_1 = -az_1 - bx_1^2 z_1 - c\,|\,x_1\,| + x_1 y_1^2 - x_1^3 \end{cases} \tag{9}$$

In (9), x_1, y_1, z_1 are the states and a, b, c are positive, constant, parameters.

As the slave system for the synchronization, we consider the new chaotic jerk system

$$\begin{cases} \dot{x}_2 = y_2 \\ \dot{y}_2 = z_2 \\ \dot{z}_2 = -az_2 - bx_2^2 z_2 - c\,|\,x_2\,| + x_2 y_2^2 - x_2^3 + u \end{cases} \tag{10}$$

In (10), x_2, y_2, z_2 are the states and u is a backstepping control to be designed.

The synchronization error between the jerk systems (9) and (10) can be defined as follows:

$$\begin{cases} e_x = x_2 - x_1 \\ e_y = y_2 - y_1 \\ e_z = z_2 - z_1 \end{cases} \tag{11}$$

We find the error dynamics as follows:

$$\begin{cases} \dot{e}_x = e_y \\ \dot{e}_y = e_z \\ \dot{e}_z = -ae_z - b\left(x_2^2 z_2 - x_1^2 z_1\right) - c(|x_2| - |x_1|) + x_2 y_2^2 - x_1 y_1^2 - x_2^2 + x_1^3 + u \end{cases} \tag{12}$$

Using active backstepping control, we establish a key result of this section.

Theorem 1. The master and slave chaotic jerk systems represented by (9) and (10) are globally and asymptotically synchronized by means of the active backstepping controller given by

$$u = -3e_x - 5e_y - (3-a)e_z + b\left(x_2^2 z_2 - x_1^2 z_1\right) + c\left(|x_2| - |x_1|\right) - x_2 y_2^2 + x_1 y_1^2 + x_2^3 - x_1^3 - K\sigma_z \tag{13}$$

where $\sigma_z = 2e_x + 2e_y + e_z$ and $K > 0$ is a gain constant.

Proof. The result is proved via backstepping control method, which is a recursive procedure in Lyapunov stability theory [1]. We start with the Lyapunov function

$$V_1(\sigma_x) = 0.5\ \sigma_x^2 \tag{14}$$

where $\sigma_x = e_x$.

Differentiating V_1 along the error dynamics (12), we get

$$\dot{V}_1 = \sigma_x \dot{\sigma}_x = e_x e_y = -e_x^2 + e_x\left(e_x + e_y\right) \tag{15}$$

We define

$$\sigma_y = e_x + e_y \tag{16}$$

Using Eq. (16), we can simplify Eq. (15) as

$$\dot{V}_1 = -\sigma_x^2 + \sigma_x \sigma_y \tag{17}$$

Next, we define the Lyapunov function

$$V_2\left(\sigma_x, \sigma_y\right) = V_1(\sigma_x) + 0.5\sigma_y^2 = 0.5\left(\sigma_x^2 + \sigma_y^2\right) \tag{18}$$

Differentiating V_2 along the error dynamics (12), we get

$$\dot{V}_2 = -\sigma_x^2 - \sigma_y^2 + \sigma_y\left(2e_x + 2e_y + e_z\right) \tag{19}$$

We define

$$\sigma_z = 2e_x + 2e_y + e_z \tag{20}$$

Using (20), we can express (19) as

$$\dot{V}_2 = -\sigma_x^2 - \sigma_y^2 + \sigma_x \sigma_y \tag{21}$$

Finally, we define the quadratic Lyapunov function

$$V(\sigma_x, \sigma_y, \sigma_z) = 0.5\left(\sigma_x^2 + \sigma_y^2 + \sigma_z^2\right) \tag{22}$$

It is evident that V is a positive definite function on R^3.

Differentiating V along the error dynamics (14) and (16), we get

$$\dot{V} = -\sigma_x^2 - \sigma_y^2 - \sigma_z^2 + \sigma_z T \tag{23}$$

where

$$T = \sigma_z + \sigma_y + \dot{\sigma}_z = \sigma_z + \sigma_y + \left(2\dot{e}_x + 2\dot{e}_y + \dot{e}_z\right) \tag{24}$$

A simple calculation shows that

$$T = 3e_x + 5e_y + (3-a)e_z - b\left(x_2^2 z_2 - x_1^2 z_1\right) - c\left(|x_2| - |x_1|\right) + x_2 y_2^2 - x_1 y_1^2 - x_2^3 + x_1^3 + u \tag{25}$$

Substituting the value of u from Eq. (13) into Eq. (25), we obtain

$$T = -K\sigma_z \tag{26}$$

Substituting the value of T from Eq. (26) into Eq. (23), we get

$$\dot{V} = -\sigma_x^2 - \sigma_y^2 - (1+K)\sigma_z^2 \tag{27}$$

Thus, by Lyapunov stability theory, we conclude that $(\sigma_x(t), \sigma_y(t), \sigma_z(t)) \to (0,0,0)$ as $t \to \infty$. Hence, it follows that $(e_x(t), e_y(t), e_z(t)) \to (0,0,0)$ as $t \to \infty$. This completes the proof. ■

For numerical plots, we take the constants (a,b,c) as in the chaotic case, *viz.* $a = 2$, $b = 0.2$ and $c = 0.5$. We take the gain as $K = 12$.

The initial conditions of the master jerk system (9) are chosen as

$$x_1(0) = 5.2,\ \ y_1(0) = 1.4,\ \ z_1(0) = 6.9 \tag{28}$$

The initial conditions of the slave jerk system (10) are chosen as

$$x_2(0) = 2.8,\ y_2(0) = 7.3,\ z_2(0) = 1.4 \tag{29}$$

Figures 5-8 show the global chaos synchronization of the chaotic jerk systems (9) and (10).

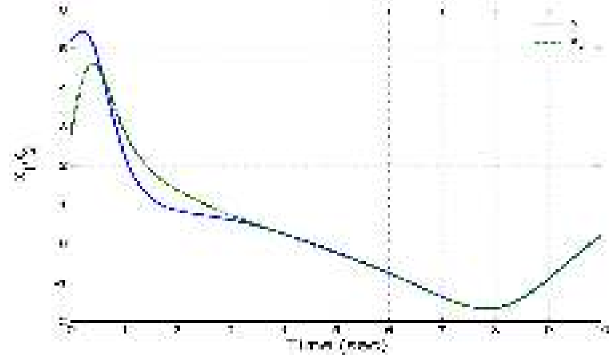

Figure 5. Complete synchronization of the states x_1 and x_2 of the chaotic jerk systems (9) and (10)

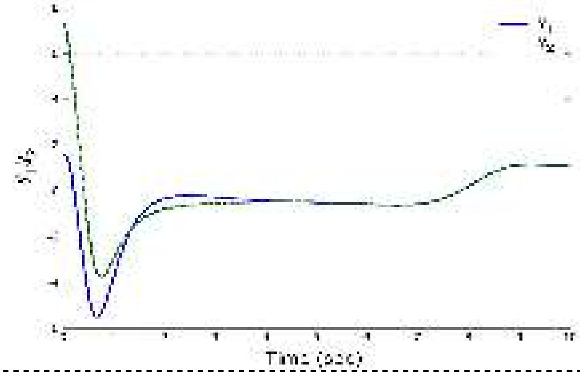

Figure 6. Complete synchronization of the states y_1 and y_2 of the chaotic jerk systems (9) and (10)

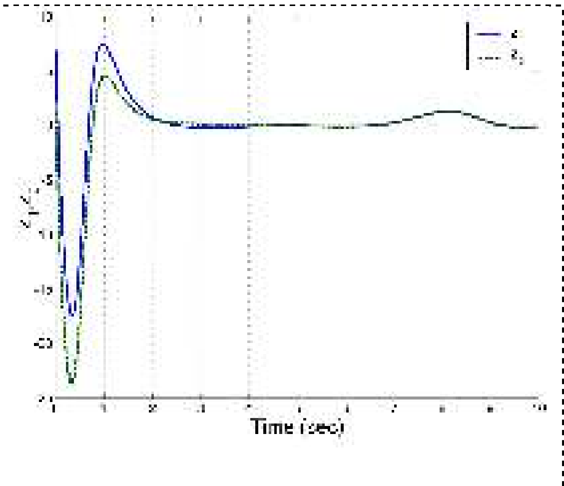

Figure 7. Complete synchronization of the states z_1 and z_2 of the chaotic jerk systems (9) and (10)

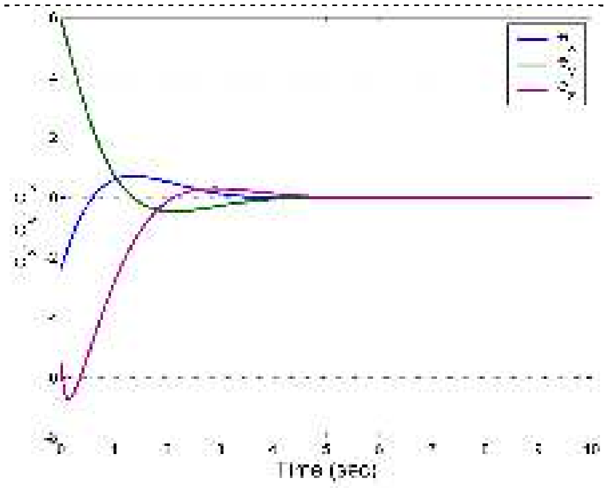

Figure 8. Time-history of the synchronization errors between the chaotic jerk systems (9) and (10)

4. Circuit Implementation of the New Chaotic Jerk System

This section presents the MultiSIM design of an analog circuit in order to approve the theoretical models and confirm the applicability of the mathematical model. The schematic diagram of Figure 9 has been simulated in MultiSIM software. The electronic circuit involves only common available components such as resistors, capacitors, operational amplifiers, multipliers and diodes.

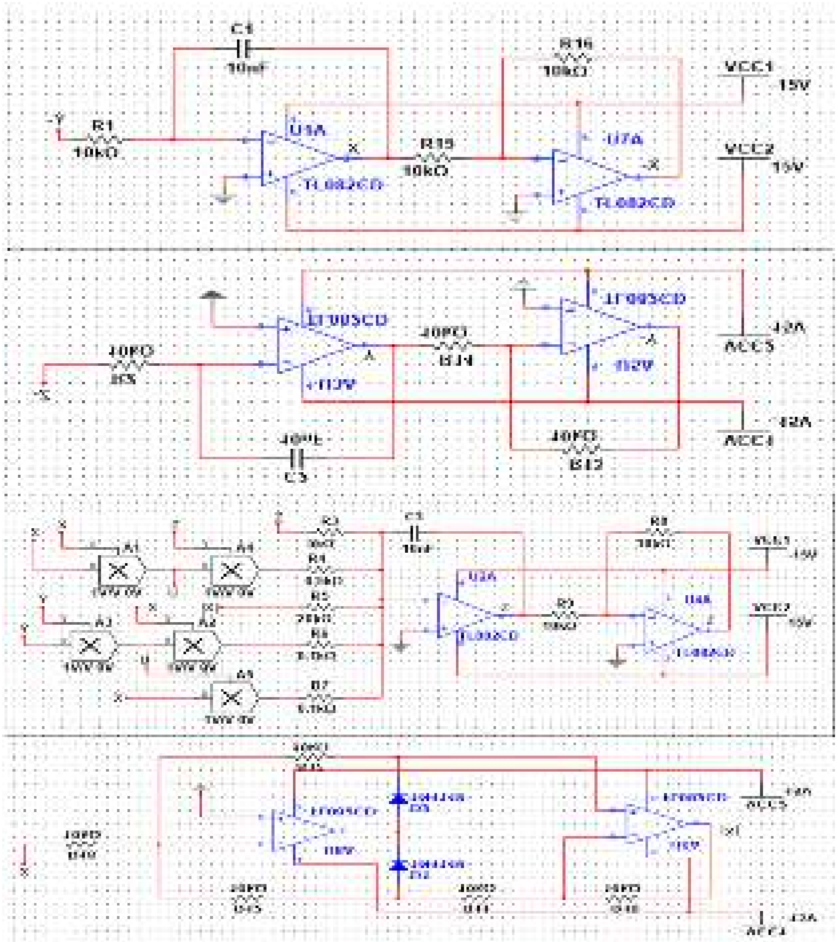

Figure 9 Circuit design for the new chaotic jerk system (4)

By applying Kirchhoff electrical circuit laws yield the following circuit equations:

$$\begin{cases} \dot{x} = \dfrac{1}{C_1 R_1} y \\ \dot{y} = \dfrac{1}{C_2 R_2} z \\ \dot{z} = -\dfrac{1}{C_3 R_3} z - \dfrac{1}{100 C_3 R_4} x^2 z - \dfrac{1}{C_3 R_5} |x| + \dfrac{1}{100 C_3 R_6} x y^2 - \dfrac{1}{100 C_3 R_7} x^3 \end{cases} \tag{30}$$

Here, x, y, z are the voltages across the capacitors C_1, C_2, and C_3, respectively.

We choose the values of electronic component as: $R_3 = 5$ kΩ, $R_4 = 0.5$ kΩ, $R_5 = 20$ kΩ, $R_6 = R_7 = 0.1$ kΩ, $R_1 = R_2 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = R_{15} = R_{16} = R_{17} = R_{18} = 100$ kΩ, $C_1 = C_2 = C_3 = 10$

nF. Multisim results are reported in Figures 10-12. Clearly, the MultiSIM results show good agreement with the numerical simulations shown in Figures 2-4.

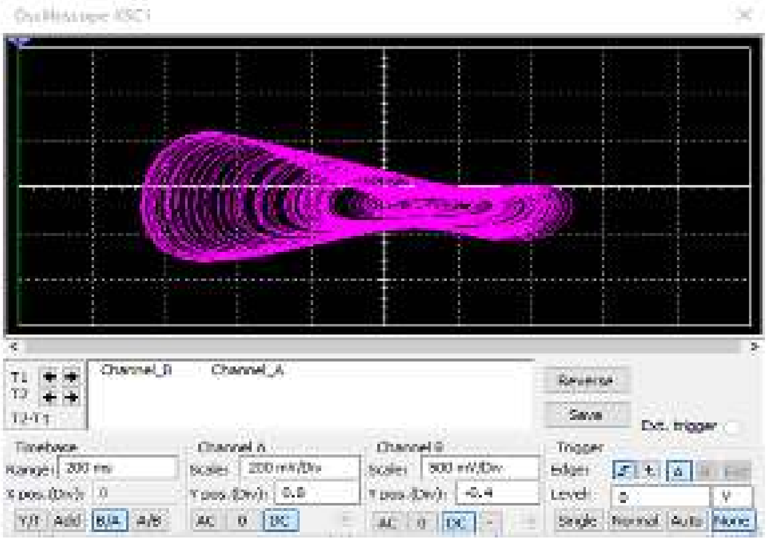

Figure 10. MultiSIM output of the new chaotic jerk system in $(x, y)-$plane

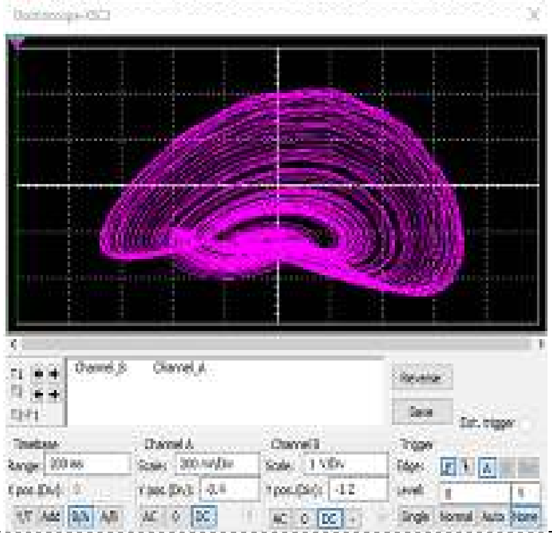

Figure 11. MultiSIM output of the new chaotic jerk system in $(y, z)-$plane

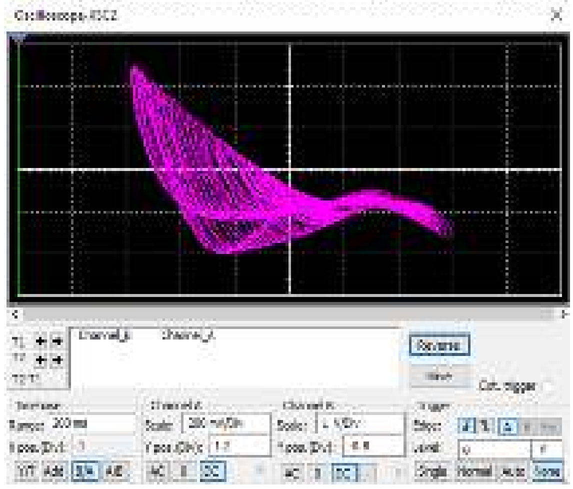

Figure 12. MultiSIM output of the new chaotic jerk system in $(x, z)-$plane

5. Conclusions

A new 3-D chaotic jerk system with four nonlinear systems was proposed in this study. We reported chaos in a 3-D mechanical chaotic jerk system with four nonlinear terms. As a control application, we designed active backstepping based global chaos synchronization for a pair of new chaotic jerk systems. As a circuit application, we designed MultiSIM electronic circuit for the new chaotic jerk system and showed that the MultiSIM outputs are in good agreement with the MATLAB outputs for the new jerk

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