Representation of a robotic manipulator mechanical subsystem as a component circuit with vector links

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Abstract. This paper discusses the formalized representation of a mechanical subsystem of a robotic manipulator as a component circuit with vector links for the purposes of dynamics problems. Set-theoretical definition of a component circuit is represented. A matrix-topological interpretation of the components of solid body and kinematic node is discussed. Component models can be used for mechanical systems modeling and simulation systems software develipment.

1. Introduction
The methodology for design of various mechatronic devices and systems relies on information technology and computer simulation. A wide range of tools for design and research purposes in this area is available through ADAMS [1-5], SolidWorks and MatLab applications for mechanical systems simulation – such as MatLab/Simscape Multibody [6-8] and MatLab/SimMechanics [9-11]. There is integration between them. Original software is also being developed [12, 13]. In Russia most famous simulation software solutions are Universal Mechanism [14, 15], MVTU [16], SimInTech [17, 18] as a means to offer a substitution to imported products.

Suggested method for automated simulation of heterogeneous devices and systems is based on the component circuit method. This method is used in simulation of devices and systems that are classified as heterogeneous, characterized by the diverse physical nature of their elements, which is typical of mechatronic systems. An object's initial formalized representation for the purposes of simulation is a component circuit, which is a collection of components and links between them that is based on its structural model. Further development of the component circuit method was driven by the development of the visual simulation technology.

One important aspect of the formalized representation of mechanical systems is that the links between their components are defined by not one, but several variables. In component circuit method, such links are known as vector-type links. Similarly, MatLab/Simulink make use of blocks with vector signals. This paper aims to create a methodology for simulation of a robot's mechanical subsystem for the purposes of dynamics problems. In this paper, we focus on the class of robotic manipulators.

2. General description of the component circuit method
This method is designed to facilitate simulation of physically heterogeneous devices and systems with their initial information represented in the form of a component circuit, i. e. in the form of a structural model. The main structural entity of the component circuit method is a multi-pole component with an arbitrary number and type of links that are incidental to topological coordinates – numbers of nodes *n*

and branches $n_в$, and potential V_n and stream $V_{nв}$ type link variables. The difference between potential and stream-type variables is that, when a numerical component circuit model is constructed, nodal topological laws of conservation are automatically created for stream variables. Correspondence will be denoted with the following sign: «→». The component circuit method provides for four types of links S:

- energy-type links that correspond to a pair of topological coordinates (n, $n_в$) and a pair of dual variables $S^e \rightarrow (n, n_в) \rightarrow (V_n, V_{nв})$;
- information-type links $S^i \rightarrow n \rightarrow V_n$, that correspond to one topological coordinate (node number) and one link variable (potential-type);
- scalar-type links that include energy and information links;
- vector-type links S^v, which are a collection of scalar-type links: $S^v \rightarrow \mathbf{n} \rightarrow \left(n_1^e, n_2^e, \dots, n_{ke}^e, n_1^i, n_2^i, \dots, n_{ki}^i\right) \rightarrow \left(S_1^e, S_2^e, \dots, S_{ke}^e, S_2^i, \dots, S_{ki}^i\right)$, where ke, ki – are quantity of energy-type links and information-type links.

These link types enable the component circuit method to deliver component decomposition and a formalized representation of simulated objects as component circuits that are similar in their graphical representation to the language of (electrical, kinematic) block diagrams and algorithm diagrams.
The mathematical model of a component that describes the process of a specific element's operation is an equation or a set of algebraic and differential equations in relation to its link variables $\mathbf{V}_n$, $\mathbf{V}_{nв}$ and, if necessary, additional internal variables $\mathbf{V}_I$. The mathematical model of a component circuit in general is a set of equations that includes models of its components, equations for its backbone node and nodal topological laws of conservation for its stream variables (equal-zero sum of stream variables for links of components that are incidental to the component circuit node). A solution of a component circuit model is a vector $\mathbf{V}$ of all component circuit variables, including the internal component variables. The component circuit method is implemented in the circuit simulation system MARS (russian abbreviation for Simulation and Automated Calculation of Systems).

3. Component circuit of a robotic manipulator

We shall identify features that are essential for formalized representation and simulation of a mechanical system:

- analysis mode – kinematic and dynamic;
- type of motion – rotational R and translational T.

Segments interconnected by kinematic pairs are the elementary entities of a mechanical subsystem. The operation space for a mechanical system can be described through indicators of presence (or absence) of components of rotational R_x, R_y, R_z and translational motion T_x, T_y, T_z. For example, the operation space for a one-dimensional mechanical circuit can be defined by the relations $P^1 = R_x \cup R_y \cup R_z$ or relations $P^1 = T_x \cup T_y \cup T_z$. The mechanical circuit that corresponds to the planar motion of the mechanism has two translational and one rotational degree of freedom, e. g. in the OYZ plane $P^2 = R_x \cap \left(T_y \cap T_z\right)$. For multidimensional systems with the spatial motion of segments, various options are possible to describe their operation space, e. g. $P^4 = \left(R_x \cap R_y \cap R_z\right) \cap T_x$. Or free motion without any restraining links (6 degrees of freedom) $P^6 = \left(R_x \cap R_y \cap R_z\right) \cap \left(T_x \cup T_y \cup T_z\right)$.

Multidimensional mechanical systems require that the space of relative motion in kinematic pairs should be described as well. For V-class kinematic pairs, that means translational motion along one of the axes or rotation around the axis. For other pairs, combinations of motions are possible. If necessary, they can be reduced to V-class kinematic pairs by means of decomposition.

The mechanical subsystem of the robotic manipulator includes a multi-segment handling mechanism with a gripper. Given the natural decomposition of the mechanical system into segments and kinematic pairs, the mechanical subsystem of the robotic manipulator can be represented as a component circuit C_M:

$$C_M = (\mathbf{SB}, \mathbf{KN}, \mathbf{E}, \mathbf{DC}, \mathbf{N}, \mathbf{B}),$$

where **SB** is a set of solid bodies; **KN** is a set of kinematic nodes (converters) that correspond to kinematic pairs or couplings; **E** is a set of external factors (power drives); **DC** is a set of dynamic components that reflect the physical effects observed during motion (elasticity, friction, loss, backlash); **B**, **N** are sets of links of components (circuit branches) and nodes in the circuit (link couplings).

Let us design a component circuit of a two-segment robotic manipulator shown in figure 1, *a*. In general, a solid body component can have an arbitrary number of links. This paper discusses a solid body with two links, which is typical for a formalized representation of segments of a manipulator. Let us assume that the motion of segments occurs in the XOY plane, and kinematic pairs provide rotations around the OZ axis. The relative motion in the first joint is determined by power drives, and the laws of velocity variation are known. The motion of segments in the second joint is restricted, e. g., by viscous friction. The circuit class is defined as $P^3 = (T_x \cap T_y)R_z$ by the dynamic elements of translational

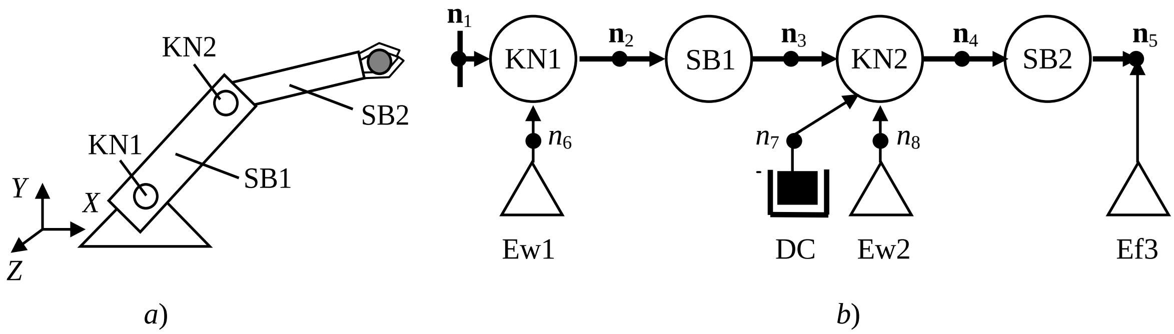

Figure 1. Two-segment robotic manipulator:
a – robotic manipulator model; *b* – component circuit

motion along the *X* and *Y* axes and rotational motion around the *Z* axis with vector links between components. Figure 1, *b* shows the component circuit of the robotic manipulator with vector links. It includes components: solid bodies SB1, SB2, kinematic nodes KN1, KN2, velocity sources Ew1, Ew2, force and moment source Ef3 and dynamic component damper DC.

Due to the nature and type of variables that define them, links between components in kinematic problems are defined as vector information-type links. The algorithms for interpretation of link variables for the purposes of visual simulation, where the initial information about the component circuit is available as topological information about the drawing of the component circuit created in a graphics editor, were developed and implemented in simulation system MARS. This paper discusses the aspects of topological interpretation of links in the context of the problems of dynamics.

For the purposes of topological interpretation of links, let us consider the component links to be of energy type. Let us assume that linear *v* and circular velocity projections are potential variables, and force *F* and torque *M* projections are stream variables. In this case, the nodal topological law of conservation is a power and torque balance equation (d'Alembert's dynamic equilibrium).

The mathematical models of components discussed in these papers are based on the Euler–Lagrange formalism. In this paper the Newton–Euler formalism is used. Let us consider models of these components in a universal form that makes it possible to account for the different types of motion and for the different modes of operation of the circuit (kinematic, dynamic, etc.).

4. Model of a free solid body

The model of a free solid body SB with two vector links S_1^v and S_2^v (figure 2) describes its motion with six degrees of freedom within the coordinate system $X_1Y_1Z_1$ associated with the solid body, where the origin of the coordinate system matches the center of mass of the solid body. The model's parameters are the mass *m*, the matrix of moments of inertia ***J***, and coordinates of points of external factors $\boldsymbol{\rho}_j$, j = 1, 2.

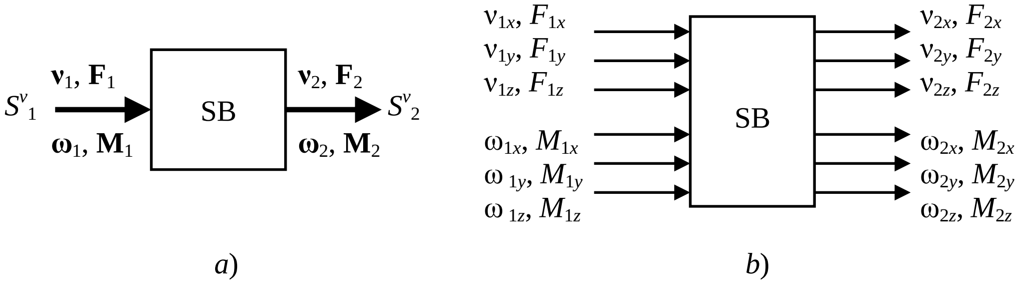

Figure 2. Formalized representation of a solid body component:
a – with vector links; *b* – with scalar links

Component links are incident to sets of variables:

$$S_j^v \to \mathbf{V}_j = (\boldsymbol{\nu}_j, \boldsymbol{\omega}_j, \mathbf{F}_j, \mathbf{M}_j),\ j = 1, 2,$$

where $\boldsymbol{\nu}_j = (\nu_{jx} \quad \nu_{jy} \quad \nu_{jz})^{\mathrm{T}}$, $\boldsymbol{\omega}_j = (\omega_{jx} \quad \omega_{jy} \quad \omega_{jz})^{\mathrm{T}}$ – are projections of the vector of linear and angular velocity on the axes $X_1Y_1Z_1$; $\mathbf{F}_j = (F_{jx} \quad F_{jy} \quad F_{jz})^{\mathrm{T}}$, $\mathbf{M}_j = (M_{jx} \quad M_{jy} \quad M_{jz})^{\mathrm{T}}$ – are projections of force vectors and force moments.

The model of motion of a solid body is described using vector-matrix equations:

a) translational motion of the center of solid mass

$$m\left(\frac{d\boldsymbol{\nu}_0}{dt} + \mathbf{G}(\boldsymbol{\omega})\boldsymbol{\nu}_0\right) = \mathbf{F}_1 + \mathbf{F}_2,$$

where m – is the mass; $\boldsymbol{\nu}_0$, $\boldsymbol{\omega}$ – are the translational and the angular velocities of the body; $\mathbf{F}_1, \mathbf{F}_2$– are vectors of forces that act on the links;

$$\mathbf{G}(\mathbf{d}) = \begin{pmatrix} 0 & -d_z & d_y \\ d_z & 0 & -d_x \\ -d_y & d_x & 0 \end{pmatrix},$$

d_x, d_y, d_z – are projections of the vector $\mathbf{d}$ on the axes $X_1Y_1Z_1$; $\mathbf{G}(\boldsymbol{\omega}) = \mathbf{G}(\mathbf{d} = \boldsymbol{\omega})$;

b) rotational motion:

$$\mathbf{J}\left(\frac{d\boldsymbol{\omega}}{dt} + \mathbf{G}(\boldsymbol{\omega})\boldsymbol{\omega}\right) = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{G}(\boldsymbol{\rho}_1)\mathbf{F}_1 + \mathbf{G}(\boldsymbol{\rho}_2)\mathbf{F}_2,$$

where $\mathbf{J}$ – is the matrix of moments of inertia; $\mathbf{M}_1, \mathbf{M}_2$ – are vectors of moments of external force; $\mathbf{G}(\boldsymbol{\rho}_j) = \mathbf{G}(\mathbf{d} = \boldsymbol{\rho}_j)$, $j = 1, 2$;

c) relations for rotational and translational motion variables:

$$\boldsymbol{\nu}_1 = \boldsymbol{\nu}_0 - \mathbf{G}(\boldsymbol{\rho}_1)\boldsymbol{\omega}_1;$$

$$\boldsymbol{\nu}_2 = \boldsymbol{\nu}_0 - \mathbf{G}(\boldsymbol{\rho}_2)\boldsymbol{\omega}_2;$$

$$\boldsymbol{\omega}_2 = \boldsymbol{\omega}_1.$$

5. Kinematic nodes

Solid bodies are linked using kinematic pairs and couplings. In a formalized representation, they correspond to the components identified as kinematic nodes KN (figure 3). A *k*-kind kinematic node delivers *k* degrees of freedom in the relative motion of the solid body. Let us represent each vector link of the KN (figure 3, *a*) as a set of elementary links (figure 3, *b*). The relationship of the physical variables

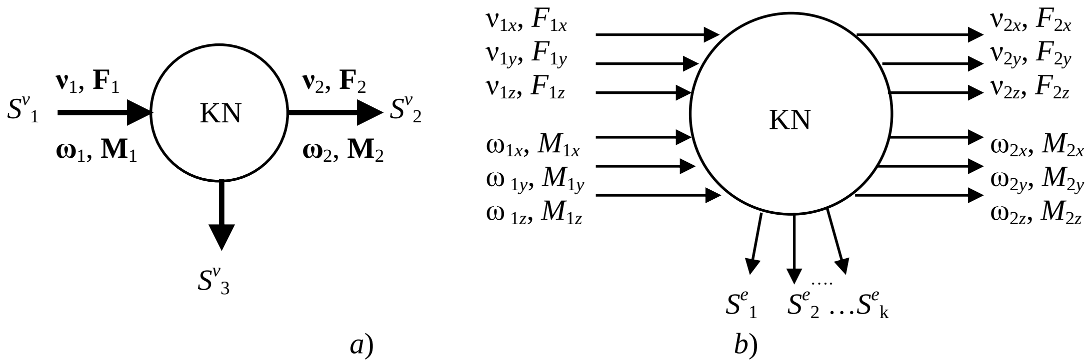

Figure 3. Formalized representation of the kinematic node component:
a – with vector links; *b* – with scalar links

with the local variables of the component is the same as that of the component SB. Vector links S_1^v and S_2^v connect the component KN to the links of solid bodies or other kinematic nodes.

The link S_1^v is acted on by the variables $\mathbf{V}_1 = (\mathbf{v}_1, \boldsymbol{\omega}_1, \mathbf{F}_1, \mathbf{M}_1)$, that are defined through projections on the coordinate basis e_1; the link S_2^v is acted on by the variables $\mathbf{V}_2 = (\mathbf{v}_2, \boldsymbol{\omega}_2, \mathbf{F}_2, \mathbf{M}_2)$ that are defined through projections on the coordinate basis e_2.

The vector link S_3^v can be used to specify dynamic restrictions for the relative motion, which are defined by the elasticity, friction or backlash in the component KN. The link S_3 can be equivalently represented by the set k of elementary links (figure 3, *b*). The link S_3^v is acted on by the variables of dynamic restrictions $S_3^v \to \mathbf{V}_3 = (\mathbf{v}_3, \boldsymbol{\omega}_3, \mathbf{F}_3, \mathbf{M}_3)$.

The kinematic node rotation model is described by the equations

$$\boldsymbol{\omega}_2 = \mathbf{A}(\boldsymbol{\alpha})\boldsymbol{\omega}_1 + \mathbf{A}_\omega \boldsymbol{\omega}_3;$$

$$\mathbf{M}_2 = \mathbf{A}(\boldsymbol{\alpha})\mathbf{M}_1 + \mathbf{A}_\omega \mathbf{M}_3;$$

$$\mathbf{v}_2 = \mathbf{A}(\boldsymbol{\alpha})\mathrm{v}_1;$$

$$\mathbf{F}_2 = \mathbf{A}(\boldsymbol{\alpha})\mathbf{F}_1;$$

$$\boldsymbol{\omega}_3 = \frac{d}{dt}\boldsymbol{\alpha},$$

where $\mathbf{A}_\omega$ defines the angular position of the vector $\boldsymbol{\omega}_3$ in the basis e_2; $\mathbf{A}(\boldsymbol{\alpha})$ – is the matrix of directional cosines; $\boldsymbol{\alpha}$ – is the angle of rotation.

The equations of the KN translational motion model is described with the following equations

$$\mathbf{v}_2 = \mathbf{v}_1 - \mathbf{G}(\boldsymbol{\rho}_3)\boldsymbol{\omega}_1 + \mathbf{A}_\mathrm{v}\mathbf{v}_3;$$

$$\mathbf{F}_2 = \mathbf{F}_1 + \mathbf{A}_\mathrm{v}\mathbf{F}_3;$$

$$\boldsymbol{\omega}_2 = \boldsymbol{\omega}_1;$$

$$\mathbf{v}_3 = \frac{d}{dt}\boldsymbol{\rho}_3,$$

where $\mathbf{A}_\mathrm{v}$ – is the matrix that defines the angular position $\boldsymbol{\rho}_3$ of the vector $\mathbf{v}_3$ in the basis e_2; $\mathbf{G}(\boldsymbol{\rho}_3)$ – is the skew-symmetric matrix $\mathbf{G}(\boldsymbol{\rho}_3) = \mathbf{G}(\mathbf{d} = \boldsymbol{\rho}_3)$.

The relative motion of segments in the handling mechanism can be subject to dynamic restrictions that are convenient to represents in the form of dynamic components that are defined by elasticity, friction or backlash. These components are connected to the link S_3^v of the component KN. The same applies to sources of external action: sources of velocity or movement or sources of forces.

The initial mathematical models of components are recorded in the local coordinate basis of potential and stream scalar-type link variables and, if necessary, internal variables of the component. The numerical mathematical model of the component circuit in simulation system is constructed in the global coordinate basis. This transition to the global coordinates is connected with decomposition of vector-type links and topological nodes and variables into scalar ones. In simulation system it can be implemented in different ways – during or after the component circuit designing in graphic component circuits editor.

6. Conclusion

The proposed matrix-topological interpretation of mathematical models of solid body and kinematic node components makes it possible to implement a convenient formalized representation of the mechanical subsystem by making use of the natural decomposition of the mechanical system into components and using components with vector-type links.

The proposed method for formalized representation of component models can be used for mechanical systems modeling and software for mechanical systems simulation development.

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