

Conceptual Understanding And Procedural Knowledge: A Case Study on Learning Mathematics of Fractional Material in Elementary School

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Abstract. Learning theory sees conceptual knowledge as a source of students' procedural knowledge in learning mathematics. This is supported by several previous studies which show that students learn conceptual understanding before procedural knowledge. So the researcher highlights the relationship that might occur between conceptual and procedural knowledge. Whether conceptual knowledge might have a greater influence on procedural knowledge than vice versa. This research is a qualitative research with a case study approach. The results showed that students who succeeded in fraction learning were able to combine conceptual and procedural knowledge well. It can be concluded that students with conceptual and procedural knowledge can develop good knowledge in learning mathematics

1. Introduction

Mathematics is a science that has an important role in human life, where the concepts in mathematics consist of the simplest to the most complex, systematic, logical, and hierarchical. So according to Bruner learning mathematics will succeed if the learning process is directed at understanding the concepts and knowledge of the procedures contained in the material being taught. However, based on research many students have difficulty implementing meaningful understanding of their mathematical conceptual understanding. This is especially so for the learning of fractions [1].

In fraction and decimal material, students need new understanding to understand the relationship of each concept. But the topics of fractions and decimals demand new and extended understanding of units and their relationships [2]. Learning should be able to facilitate students in organizing understanding of concepts they have received as provisions to receive new concept knowledge. In fraction material, it turns out students have difficulty organizing their knowledge. We assessed students' knowledge on fractions using equivalent pretest, immediate and delayed posttest versions, each of which took 30 minutes to complete [3]. This gives an explanation that conceptual understanding and procedural knowledge are very needed by students in solving mathematical problems they face.

Conceptual understanding is not just knowing information. But more than that students can interpret the information into other forms that are more meaningful. The reality of information in the context of learning is related to three things: meaning, experience that results in meaning, and culture that has an



impact on the creation of experience in the meaning process [4]. The meaning process is used to solve more difficult problems. Conceptual understanding of mathematics will help and is the initial capital of students in solving mathematical problems [5]. Conceptual understanding (knowing why) supports understanding mathematical principles that are considered as the product of a process that connects prior knowledge with new knowledge [6].

The main emphasis of mathematics in schools is on procedural knowledge (procedural knowledge) or known as procedural expertise (procedural fluency) [7]. Procedural knowledge is built on the basis of conceptual understanding. Procedural knowledge involves understanding the rules and routines of mathematics while conceptual knowledge involves an understanding of mathematical relationships [8]. Procedural knowledge is a series of steps that must be followed to solve mathematical problems. This knowledge includes knowledge of algorithmic skills, techniques, and methods. Procedural knowledge also includes knowledge about the criteria used to determine when to use various procedures.

So that students' skills in knowing the relationship between conceptual understanding and procedural skills are very important, with the hope that students can understand a concept and the skills to choose steps that will be used to solve mathematical problems. In this study is to see constructivist how a student completes fraction material with a framework of students' conceptual understanding and procedural knowledge.

2. Research Method

On this research, researchers used qualitative with case study approach. This qualitative case study is an approach to research that facilitates exploration of a phenomenon within its context using a variety of data sources [9]. Case study in this study is Plausibility probes: Preliminary studies are used to determine whether further examination is warranted. [10]. The research procedure went through three stages: the preparation, implementation, and data analysis. Subjects in this research were all students of class V at one particular elementary school in Majalengka. The process of research preparation, researchers make indicators of contextual understanding and knowledge procedures, according to Kilpatrick et al [11]: Three indicators were used to measure students Conceptual Understanding in this research. (1) Being able to know the relationship between mathematical concepts. (2) Being able to represent and communicate the idea. (3) Knowing the most suitable representation for specific situations. Three indicators to measure students Procedural Knowledge; (1) Knowing the procedure to solve the problem. (2). Knowing about when and how to use the procedure. (3) Being able to use procedures effectively. Based on these indicators, we formulated a design to find out the students' Conceptual Understanding and Procedural Knowledge levels, namely meaning of fractions, comparison fractions, concepts of addition, subtraction and multiplication with fraction, representation and rules of operations. On implementation there are three fraction problems that should be solved by students:

1. Which of the fractions below is bigger? Why? Explain your answer
 - a. $\frac{4}{5}$ or $\frac{8}{9}$
 - b. $\frac{6}{5}$ or $\frac{7}{6}$
 - c. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$ is the same as $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$, why?
2. What is the solution of addition $\frac{1}{3} + \frac{1}{6} + \frac{1}{12}$
3. Find the fractions represented by letters in the operations below.
 - a. $\frac{9}{10} - b = \frac{4}{5}$
 - b. $1\frac{1}{8} + a = 2\frac{3}{8}$

Lastly, the researcher analysis the students answers to detect and describe the Conceptual Understanding and Procedural Knowledge made by students.

3. Result and Discussion

Conceptual Knowledge and procedural knowledge are knowledge that is needed by students to be able to understand mathematical concepts. Conceptual knowledge is knowledge of math facts and properties that are recognized as being related in some way, Procedural Knowledge is identified as defined as the set of rules and algorithms used to solve math problems [12]. Students with high levels of conceptual

understanding are capable of solving problems that they have never come across before [13]. It is true that if conceptual understanding and procedural knowledge are developed in students, this will have an impact on the development of other conceptual understanding [14]. When students are not able to balance conceptual understanding and procedural knowledge, students do not have full facilities in learning a mathematical topic [15]. So that the relationship between conceptual knowledge and procedural knowledge is very important, therefore students in learning fractions need to be directed to the mastery of conceptual knowledge with procedural knowledge as their provision to learn mathematics.

To explain that conceptual knowledge with procedural knowledge is important for students, we describe the results of our study of grade V elementary school students. For the first student (S1) in adding S1 fractions has conceptual knowledge that fractions will have a smaller value if the denominator's value is more large so to compare if there are two different fractions is to look at the numerator. The picture below is the student's work that shows the process of conceptual understanding:

Jawaban

a. $\frac{4}{5}$ atau $\frac{8}{9}$

Sebab : $\frac{4}{5} = \frac{36}{45}$

$\frac{8}{9} = \frac{40}{45}$

nilai yg lebih besar adalah $\frac{8}{9}$

b. $\frac{6}{5}$ atau $\frac{7}{6}$

Sebab : $\frac{6}{5} = \frac{36}{30}$

$\frac{7}{6} = \frac{35}{30}$

nilai yang lebih besar adalah $\frac{6}{5}$

c. bernilai sama

Sebab : $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

$\frac{1}{4} + \frac{1}{2} + \frac{1}{8} + \frac{1}{8} = \frac{6}{8} : \frac{2}{2} = \frac{3}{4}$ nilainya sama

Figure 1. Conceptual Understanding Answers

Based on the answers that students' conceptual understanding is good. This means that students understand the meaning of fractions well, where students are able to explain and solve problems in different contexts and express proportional reasoning well, in this context students can certainly provide a clear explanation of fraction comparisons. The student knows the meaning of the part or whole of the given fraction problem. They have an awareness that the fact that from the basic concepts of the meaning of the part and the whole is division into equal parts. Following is the explanation of the S1 that represents true Conceptual Knowledge about the meaning of parts and whole of fractions::

Researcher: First question: How do you explain that $\frac{3}{4}$ is a part or a whole?

S1: The fraction $\frac{3}{4}$, number "4" indicates the number of equal parts of a whole and is called the denominator. then the number "3" indicates the number of parts of concern or taken from the whole at a given time and is called a numerator.

Researcher: Second question. I want you to explain your friends what meaning the denominator and numerator. Can you tell how you explained?

S1: Firstly I draw a rectangular and then divide it into equal part. All parts show whole and I said this is denominator. The parts that I take or give show the numerator.

Researcher: I wanted to you compare the fractions, How do you compare the fraction?

S1: I found common denominators by expansion. After that I choose which is with smallest numerator smaller than the other fraction, for example $\frac{3}{4}$ is bigger than $\frac{1}{4}$.

Researcher: Okay, are you able to compare fractions without equating the denominator?

S1: It is difficult to compare without finding common denominator because for we take 7 parts from 8 parts or for we take 4 parts from 5 parts. However if we approach the problem from this point of view, there is no difference between these fractions since when we take parts, 1 part is left behind.

The process below is a student palace which shows procedural knowledge:

Handwritten student work showing three math problems with solutions:

$$2. \quad \frac{1}{3} + \frac{1}{6} + \frac{1}{12} = \frac{7}{12}$$

Sebab: $\frac{4}{12} + \frac{2}{12} + \frac{1}{12} = \frac{7}{12}$

$$3. \quad \frac{9}{10} - B = \frac{4}{5} \quad B = \frac{9}{10} - \frac{4}{5} = \frac{9}{10} - \frac{4}{10} = \frac{5}{10}$$

$$4. \quad \frac{B}{8} + a = \frac{23}{8} \quad a = \frac{23}{8} - \frac{11}{8} = \frac{12}{8}$$

Figure 2. Procedural Knowledge Answer

Researcher: We find common denominator for solution adding problems if fractions' denominators are not equal. Why do we find common denominator? Why is it necessary?

S1: common denominator, they represent same whole. Then I can choose which one has big numerator since fraction with bigger numerator is biggest one.

Researcher: When we multiply two positive integer we obtain bigger number than multipliers and sometime we can obtain smaller number than the multipliers. What could be the reason? Do you have any idea?

S1: Because we multiply denominators. It is not finding common denominator like in addition. When we multiply denominators, we make fractions smaller since we divide whole into more parts and obtain smaller pieces. For example at first we divided a whole into 2, now we divide it into 8 parts.

Researcher: I want you to solve this procedural question. Please tell me what you are doing ! What is the result of $(\frac{3}{4} - \frac{5}{6}) * (1\frac{2}{3} - \frac{7}{3})$?

S1: The first I am started to solve from the parenthesis, then since there is no multiplication or division in the parenthesis, subtraction is done. For this subtraction, we firstly find common denominator and which is 24 when 4 and 6 are multiplied. We multiplied 3 by 6, 18 and 4 by 5, 20. Then I am find the common denominator. We subtract them and 18-20 becomes -2. So it is

$-\frac{2}{24}$. If we add this it is $\frac{12}{3}$. When we multiply them, since positive and negative are multiplied, it is negative and the result $-\frac{1}{3}$.

4. Conclusion

Teaching and learning was considered within the framework of the teaching contents suitable for younger pupils (ages 7-11 years old). Many learners never reaching proficiency in fraction arithmetic, this problem starts at Elementary students struggles with fraction knowledge and operations follow them through middle grades, high school, and college [16]. Procedural knowledge of the fraction, firstly must to be understand addition procedure also seems to be essential for acquiring a different type of knowledge of fraction concepts, one that we did not measure in the current study and that has not been given significant empirical attention knowledge of how and why the addition procedure works. Without knowing the procedural knowledge, there is nothing to explain. Based on the research results, conceptual understanding and procedural knowledge are very important to be mastered by students. Because it will have an impact on the mastery of other material in the next mathematics study.

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