Bjorken sum rule with analytic QCD coupling

C Ayala[1], G Cvetič[1], A V Kotikov[2] and B G Shaikhatdenov[2]

[1] Department of Physics, Universidad Técnica Federico Santa María Valparaíso, Chile
[2] Joint Institute for Nuclear Research, 141980 Dubna, Russia

E-mail: `kotikov@theor.jinr.ru`

Abstract. We present details of study of the polarized Bjorken sum rule $\Gamma_1^{p-n}(Q^2)$ carried out recently in [1]. Four QCD coupling versions are considered: perturbative QCD (pQCD) in the $\overline{\rm MS}$ scheme, Analytic Perturbation Theory, and 2δ and 3δ analytic QCD ($\mathcal{A}$QCD) versions. In contrast to pQCD, these QCD variants do not have Landau singularities at low positive Q^2. In general, when 2δ and 3δ QCD coupling is used the fitted curves give the best results.

1. Introduction

The polarized Bjorken sum rule (BSR) $\Gamma_1^{p-n}(Q^2)$ [2, 3] is an important spacelike QCD observable for various reasons. It is a difference of the first moment of the spin-dependent structure functions of proton and neutron, therefore its isovector nature makes it easier to describe it theoretically, in pQCD, than the separate integrals of the two nucleons. Further, high quality experimental results for this quantity, obtained in polarized deep inelastic scattering, are now available in a large range of spacelike squared momenta Q^2: $0.054\ \mathrm{GeV}^2 \leq Q^2 < 5\ \mathrm{GeV}^2$ (see [1] and references therein).

Theoretically, pQCD in $\overline{\rm MS}$ scheme, has been the usual approach to describe such quantities. This approach, however, has the theoretical disadvantage: the running coupling $\alpha_s(Q^2)$ possesses Landau singularities at low positive $Q^2 \lesssim 0.1\ \mathrm{GeV}^2$, and this makes it inconvenient for evaluation of spacelike observables at low Q^2, such as BSR. In recent years, an extension of pQCD couplings to low Q^2, without Landau singularities, called (Fractional) Analytic Perturbation Theory [(F)APT)] [4]-[21] has been applied in the fitting of the theoretical expression to the experimental inelastic contributions to BSR [22]-[25], with good results.

In Ref. [1] (which is an extension of our previous works [26, 27] on BSR) we fitted the theoretical expressions to the experimental BSR results in pQCD, in (F)APT, and two additional extensions of QCD to low Q^2, namely the 2δ [28, 29] and 3δ [30, 31] $\mathcal{A}$QCD. The latter three extensions have the coupling $\mathcal{A}(Q^2)$ which is free of Landau singularities.

2. Bjorken sum rule: theoretical expressions

BSR is defined as the difference between proton and neutron polarized structure functions g_1 integrated over the whole x-Bjorken interval

$$\Gamma_1^{p-n}(Q^2) = \int_0^1 dx \left[g_1^p(x,Q^2) - g_1^n(x,Q^2) \right] \ . \tag{1}$$

Journal of Physics: Conference Series **1435** (2020) 012016 doi:10.1088/1742-6596/1435/1/012016

Based on the various measurements of these and the related structure functions, the inelatic part of the above quantity, $\Gamma_1^{p-n}(Q^2)_{\rm inel.}$, has been extracted at various values of squared momenta Q_j^2 ($0.054\ {\rm GeV}^2 \leq Q_j^2 < 5\ {\rm GeV}^2$).

Theoretically, this quantity can be written as following [2, 3]

$$\Gamma_1^{p-n}(Q^2) = \left|\frac{g_A}{g_V}\right| \frac{1}{6}(1 - \mathcal{D}_{\rm BS}(Q^2)) + \frac{\mu_2(Q^2)}{Q^2} \ , \tag{2}$$

where Q^2-dependence of $\mu_2(Q^2)$ is exactly defined in [32, 33] (see also Ref. [1]).

Here, $|g_A/g_V| = 1.2723 \pm 0.0023$ [34] is the ratio of the nucleon axial charge, $(1 - \mathcal{D}_{\rm BS})$ is the perturbation expansion for the leading-twist (LT) contribution, and μ_2/Q^2 is the higher-twist (HT) contribution. In addition, in Ref. [1] we included the HT term $\mu_6/(Q^2)^2$ but this case is beyond the present report.

The LT term has the canonical part $\mathcal{D}_{\rm BS}(Q^2)$ whose perturbation expansion in $a \equiv \alpha_s/\pi$ is known up to $\rm N^3LO$ ($\sim a^4$)

$$\mathcal{D}_{\rm BS}(Q^2)_{\rm pt} \quad = \quad a(\mu^2) + d_1(k)a(\mu^2)^2 + d_2(k)a(\mu^2)^3 + d_3(k)a(\mu^2)^4 + \mathcal{O}(a^5), \tag{3}$$

where the renormalization scale has a general value $\mu^2 = kQ^2$ ($0 < k \sim 1$). The NLO, $\rm N^2LO$ and $\rm N^3LO$ coefficients d_j at $k = 1$ ($j = 1, 2, 3$) were obtained in [35, 36, 37], respectively. The expressions for the coefficients d_j at any k values are obtained on the basis of the the renormalization group equation (RGE). In the considered range of momentum transfer $0 < Q^2 < 5\ {\rm GeV}^2$, we assumed in Ref. [1] that the effective number of active quark flavors is $N_f = 3$, and therefore only the nonsinglet (NS) contributions appear.

In those versions of $\mathcal{A}$QCD where the coupling is a holomorphic function $\mathcal{A}(Q^2)$ with nonperturbative contributions, the power expansion (3) becomes a nonpower expansion where a^n get replaced by $\mathcal{A}_n$ ($\mathcal{A}_n \neq \mathcal{A}^n$)

$$\mathcal{D}_{\rm BS}(Q^2)_{\mathcal{A}\rm QCD} \quad = \quad \mathcal{A}(\mu^2) + d_1(k)\mathcal{A}_2(\mu^2) + d_2(k)\mathcal{A}_3(\mu^2) + d_3(k)\mathcal{A}_4(\mu^2) + \mathcal{O}(\mathcal{A}_5). \tag{4}$$

The construction of the power analogs $\mathcal{A}_n$ of a^n were obtained in Refs. [38]-[40]. These expressions can be done in close analogy with the RGE in the perturbation theory. The couplings $\mathcal{A}_n(Q^2)$ can be obtained once the coupling $\mathcal{A}(Q^2)$ is known. The construction of $\mathcal{A}(Q^2)$ coupling is summarized in Appendix C of [1] for all used variants: (F)APT, 2δ and 3δ $\mathcal{A}$QCD (see also the recent papers [41, 42] and references therein).

3. Numerical fits

In the numerical fits, we considered in Ref. [1] the following parameters to be fitted in the expression (2): (i) the renormalization scale parameter $k \equiv \mu^2/Q^2$ of the LT contribution (3) [for pQCD] or (4) [for $\mathcal{A}$QCD]; (ii) the initial values $\mu_2(Q^2_{\rm in})$ (where $Q^2_{\rm in} = 1\ {\rm GeV}^2$). The experimental data for the inelastic contribution to BSR were taken from [**?**];[1] they are in the momentum interval $0.054\ {\rm GeV}^2 \leq Q_j^2 < 5\ {\rm GeV}^2$. The fits were performed by the least squares method, taking into account the statistical uncertainties $\sigma_{j,{\rm stat}}$ in the data points Q_j^2, which were considered to be independent of each other. These uncertainties $\sigma_{j,{\rm stat}}$ are in general significantly smaller than the systematic uncertainties $\sigma_{j,{\rm sys}}$. The latter are strongly correlated, and we considered them as completely correlated. The uncertainties in the values of the extracted parameters $k \equiv \mu^2/Q^2$ and $\mu_2(Q^2_{\rm in})$ are then due to statistical (small) and systematic (larger) uncertainties of the data. They can be found in Tables in [1]. We refer to Ref. [1] (see its Appendix D) on how we obtained the uncertainties of the extracted values of the fit parameters.

[1] For detailed information on the experimental data, see Ref. [1] and references therein.

Journal of Physics: Conference Series **1435** (2020) 012016 doi:10.1088/1742-6596/1435/1/012016

Using specific values of the renormalization scale parameter $k \equiv \mu^2/Q^2$ may allow us to incorporate in our evaluations (4) at least a part of the contributions of higher orders of the series. It is expected that $k \sim 1$, and usually it is taken in the literature in the range $1/2 < k < 2$, sometimes $1/4 < k < 4$. In the considered work, after replacing the powers of the pQCD coupling by their analytic counterparts, cf. Eqs. (3)-(4), a spacelike observable depends usually weakly on the contributions of higher orders. Still, in order not to miss the possibly relevant influence of higher orders, we decided in Ref. [1] to increase the range of possible k values to: $1/16 < k < 16$.

We show below the results of the fits of the inelastic contributions to the expression (2), in Sec. 3.1. In Sec. 3.2 we present the corresponding fits for the case when the HT term has a mass parameter (see Eq. (5) below).[2]

In each case, the fits were performed for four variants of QCD: in pQCD (in $\overline{\text{MS}}$), in (F)APT (in $\overline{\text{MS}}$), and in 2δ [28, 29] and 3δ $\mathcal{A}$QCD [30, 31]. Each of these fits is performed by excluding the data points with $Q^2 < Q^2_{\text{min}}$, where $Q^2_{\text{min}} = 0.268$ or 0.66 GeV^2. In $\overline{\text{MS}}$ pQCD and in 2δ and 3δ $\mathcal{A}$QCD the pQCD running coupling $a(Q^2)$ is determined by the requirement $\pi a(M_Z^2; \overline{\text{MS}}) = 0.1185$ [43, 34], and in (F)APT we use $\overline{\Lambda}_3 = 0.45$ GeV.

3.1. The basic case

In Figs. 1(a),(b), we present the curves in the mentioned four QCD variants, for $Q^2_{\text{min}} = 0.66$ and 0.268 GeV^2, respectively. The corresponding results and various fit quality parameters χ^2/d.o.f

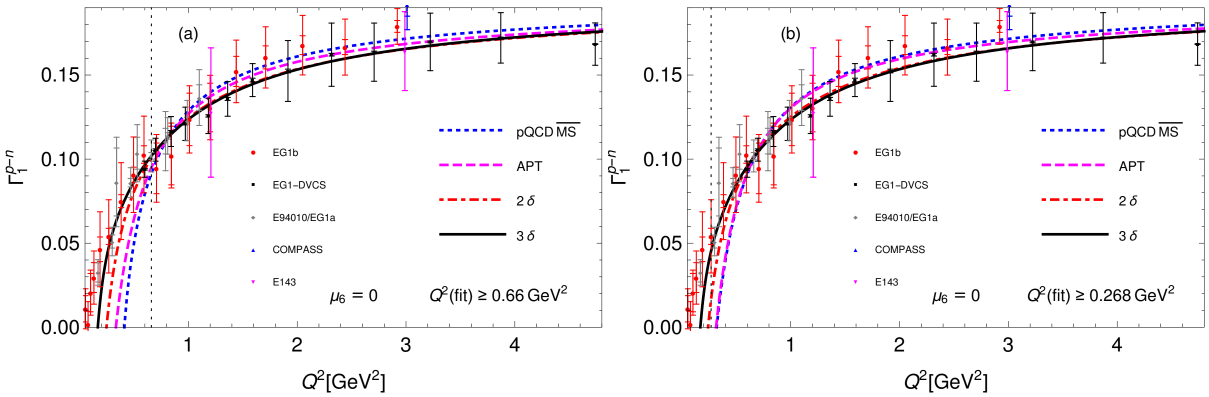

Figure 1. (color online): Fits of the expression (2) to the experimental data for $\Gamma_1^{p-n}(Q^2)_{\text{inel.}}$, in four different QCD variants, where in the fits: (a) $Q^2 \geq 0.66$ GeV^2; (b) $Q^2 \geq 0.268$ GeV^2. The respective lower bound of the fitting interval, $Q^2_{\text{min}} = 0.66$ or 0.268 GeV^2, is included as the thin dotted vertical line.

are given in Ref. [1].

We wish to point out that the approach of ($\overline{\text{MS}}$) QCD in the case of $Q^2_{\text{min}} = 0.268$ GeV^2 is, in principle, not applicable. This is so because the corresponding coupling $a(Q^2)$ has a Landau branching point at $Q^2_{\text{branch}} = 0.371$ GeV^2, which makes the running coefficient $\mu_2(Q^2)$ in Eq. (2) undefined at $Q^2 \leq 0.371$ GeV^2. Nonetheless, in order to be able to present a curve, we applied in the fitting case $Q^2_{\text{min}} = 0.268$ GeV^2 in the $\overline{\text{MS}}$ pQCD approach the replacement in the running coefficient $\mu_2(Q^2)$: $\mu_2(Q^2) \mapsto \mu_2(kQ^2)$. It solves the problem because $k > 1.383$ (see the corresponding Tables in [1]). In other approaches (APT and $\mathcal{A}$QCD's) this is not necessary, as there are no Landau singularities.

As we can deduce from Figs. 1, the best results are obtained in 3δ $\mathcal{A}$QCD. Moreover, we can observe in the results (see Tables in [1]) that the values of the HT parameter $\mu_2(Q^2_{\text{in}})$ are

[2] The fits with twist-6 term, with the elastic contribution and for two different ansätze for BSR at very low Q^2 can be found in Ref. [1].

in the analytic variants of QCD smaller (in absolute value) than in pQCD, this reduction being especially strong in the 3δ QCD variant. It has been noted in the literature that in pQCD there is a duality between the order of truncation of the LT series and the HT contribution [22]-[25], [44]-[52]: HT contribution often significantly decreases with the inclusion of higher orders in the LT part. This effect and the related ambiguity become stronger in the ranges where the perturbation theory becomes questionable (for example, at the large and low values of the Bjorken variable x, as it was shown in Refs. [53, 54], respectively). It has been observed that the HT contribution is smaller, but also more stable (under the inclusion of more terms in the LT), in QCD variants with infrared modifications of the coupling (various modifications lead to quite similar results [55]). The latter probably incorporate a part of the HT contributions (which are rather cumbersome [56]) into (formally) the LT contribution for small x range at moderately small Q^2 values ($\lesssim 1$ GeV^2) (see Refs. [57]-[61] for the DIS structure function F_2).

3.2. The "massive" case

For comparison, we performed a similar fit, but now with a "massive" higher-twist term instead of the truncated OPE expression (2)

$$\Gamma_1^{p-n,\mathrm{m}}(Q^2;M^2) = \left|\frac{g_A}{g_V}\right|\frac{1}{6}(1-\mathcal{D}_{\mathrm{BS}}(Q^2)) + \frac{\mu_2(Q^2)}{(Q^2+M^2)}\ , \tag{5}$$

where the squared mass M^2 in the denominator of the HT part[3] is taken to be constant (not running)[4]. The resulting curves, for $Q^2_{\mathrm{min}} = 0.268$ GeV^2, are given in Figs. 2(a),(b), at the higher Q^2 and the lower $Q^2 < 1$ GeV^2 momenta, respectively. Numerically, the behavior at low

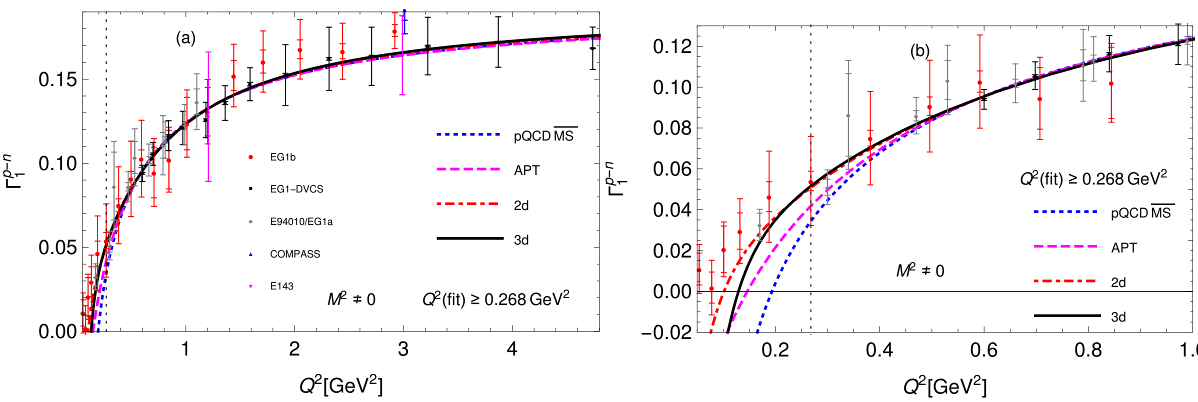

Figure 2. (color online): As Fig. 1, but for the fit of the expression (5) with "massive" higher-twist term was used.

Q^2 in the "massive" case is significantly influenced by the Q^2-dependence of $\mu_2(Q^2)$. In the $\overline{\mathrm{MS}}$ pQCD case, as before, we replaced in the HT running parameter $\mu_2(Q^2)$ the scale Q^2 by the renormalization scale kQ^2, in order to artificially avoid the problem of Landau singularities in the pQCD coupling $a(Q^2)$. Comparing Figs. 2 with the corresponding "non-massive" case Figs. 1, we see that the results and extrapolations are better in the "massive" approach, especially in the cases of $\mathcal{A}$QCD.

[3] Similar HT expressions were used in the analyses of BSR in [62]-[64] where the LT contribution was evaluated with the "Massive" Perturbation Theory (MPT) [65, 66].
[4] The results with Q^2-dependent parameterization of the dynamical effective gluon mass [67, 68] can been found in Ref. [1].

4. Summary

Experimental results for the polarized Bjorken sum rule (BSR) $\Gamma_1^{p-n}(Q^2)$ were fitted in Ref. [1], for various ranges of Q^2, with theoretical expressions using QCD couplings obtained in four different approaches: perturbative QCD (pQCD) in $\overline{\rm MS}$ scheme; (Fractional) Analytic Perturbation Theory [(F)APT]; Two-delta $\mathcal{A}$QCD (2δ); and Three-delta lattice-motivated $\mathcal{A}$QCD (3δ). The QCD running coupling $\mathcal{A}(Q^2)$ in the latter three QCD variants does not have Landau singularities, in contrast to the pQCD coupling $a(Q^2)$.

The fits were presented here for the ranges $Q^2_{\rm min} \leq Q^2 \leq 3$ GeV^2, where $Q^2_{\rm min} = 0.66$ and 0.268 GeV^2. In general, the best curves were obtained when 2δ or 3δ-couplings were used. The quality of the fitted curves, in the range of the fit and in the extrapolated ranges of Q^2, in general did not depend significantly on $Q^2_{\rm min}$ of the fit (see Tables in Ref. [1]). Even better results were obtained when "massive" HT term was used and the QCD coupling was either from (F)APT or 2δ or 3δ $\mathcal{A}$QCD.

When the range of fit had $Q^2_{\rm min} = 0.268$ GeV^2, the pQCD $\overline{\rm MS}$ coupling approach worked and gave acceptable results only if the renormalization scale of the coupling was changed $Q^2 \mapsto kQ^2$, in the HT coefficient μ_2, in order to avoid the problem of the Landau singularities.

The results in Ref. [1] (see Tables in [1]) can be interpreted as an additional indication of the following important property: the evaluation of the LT contribution of spacelike low-Q^2 QCD observables such as inelastic BSR, in QCD variants 2δ and in particular 3δ $\mathcal{A}$QCD [both have infrared finite and holomorphic coupling $\mathcal{A}(Q^2)$], appear to resum effectively a large part of the perturbative contribution of the observables, and leads to reduced extracted values of the HT term. This property was noted earlier, for different observables, in Refs. [69, 30]. In this context, it appears to be important that in 2δ and 3δ $\mathcal{A}$QCD the coupling practically merges with the underlying pQCD coupling $a(Q^2)$ at higher values of $Q^2 \gg \Lambda^2_{\rm QCD}$. This property is not shared by the (F)APT holomorphic coupling where the LT series contains parts of the HT contribution.

The extracted parameters in the HT contribution, including those in the "massive" case, are especially reduced in 3δ $\mathcal{A}$QCD (see Tables in [1]). This suggests the possibility that the true HT contribution is small and that the result with 3δ $\mathcal{A}$QCD LT gives, through fitting, an extracted value which is a good approximation to this true value of the HT contribution. Numerically, the significantly reduced extracted value in 3δ $\mathcal{A}$QCD is probably partly related with the fact that 3δ $\mathcal{A}$QCD differs from both 2δ and (F)APT $\mathcal{A}$QCD variants in that its coupling goes to zero in the deep infrared regime, $\mathcal{A}^{(3\delta)}(Q^2) \sim Q^2 \to 0$. Recall that the latter property is suggested by the large-volume lattice calculations of the dressing functions of the Landau-gauge gluon and ghost propagators at low Q^2 values (see [70]-[72] and references therein).

Acknowledgments

This work was supported by FONDECYT Postdoctoral Grant No. 3170116 (C.A.) and by FONDECYT Regular Grant No. 1180344 (G.C. and C.A.).

[1] Ayala C, Cvetič G, Kotikov A V and Shaikhatdenov B G 2018 *Eur. Phys. J.* C **78** 1002
[2] Bjorken J D 1966 *Phys. Rev.* **148** 1467
[3] Bjorken J D 1970 *Phys. Rev.* D **1** 1376
[4] Shirkov D V and Solovtsov I L 1996 *JINR Rapid Commun.* **2[76]** 5
[5] Shirkov D V and Solovtsov I L 1997 *Phys. Rev. Lett.* **79** 1209
[6] Solovtsov I L and Shirkov D V 1998 *Phys. Lett.* B **442** 344
[7] Milton K A and Solovtsov I L 1997 *Phys. Rev.* D **55** 5295
[8] Shirkov D V 2001 *Theor. Math. Phys.* **127** 409
[9] Shirkov D V 2001 *Eur. Phys. J.* C **22** 331
[10] Nesterenko A V 2000 *Phys. Rev.* D **62** 094028
[11] Nesterenko A V 2001 *Phys. Rev.* D **64** 116009
[12] Nesterenko A V 2003 *Int. J. Mod. Phys.* A **18** 5475
[13] Aguilar A C *et al.* 2005 *J. Phys.* G **31** 997
[14] Bakulev A P *et al.* 2005 *Phys. Rev.* D **72** 074014

[15] Bakulev A P *et al.* 2005 *Phys. Rev.* D **72** 074015
[16] Bakulev A P *et al.* 2007 *Phys. Rev.* D **75** 056005
[17] Bakulev A P *et al.* 2010 *JHEP* **1006** 085
[18] Prosperi G M *et al.* 2007 *Prog. Part. Nucl. Phys.* **58** 387
[19] Shirkov D V and Solovtsov I L 2007 *Theor. Math. Phys.* **150** 132
[20] Bakulev A P 2009 *Phys. Part. Nucl.* **40** 715
[21] Stefanis N G 2013 *Phys. Part. Nucl.* **44** 494
[22] Pasechnik R S *et al.* 2008 *Phys. Rev.* D **78** 071902
[23] Pasechnik R S *et al.* 2010 *Phys. Rev.* D **81** 016010
[24] Khandramai V L *et al.* 2012 *Phys. Lett.* B **706** 340
[25] Khandramai V L *et al.* 2013 *Nonlin. Phenom. Complex Syst.* **16** 93
[26] Ayala C *et al.* 2018 *Int. J. Mod. Phys.* A **33** 1850112;
[27] Ayala C *et al.* 2017 *J. Phys. Conf. Ser.* **938** 012055
[28] Ayala C *et al.* 2012 *Phys. Rev.* D **85** 114043
[29] Ayala C and Cvetič G 2015 *Comput. Phys. Commun.* **190** 182
[30] Ayala G *et al.* 2017 *J. Phys.* G **44** 075001
[31] Ayala G *et al.* 2018 *J. Phys.* G **45** 035001
[32] Shuryak E V and Vainshtein A I 1982 *Nucl. Phys.* B **201** 141
[33] Kawamura H *et al.* 1997 *Mod. Phys. Lett.* A **12** 135
[34] Patrignani C *et al.* [Particle Data Group Collaboration] 2016 *Chin. Phys.* C **40** 100001
[35] Gorishnii S G and Larin S A 1986 *Phys. Lett.* B **172** 109
[36] Larin S A and Vermaseren J A M 1991 *Phys. Lett.* B **259** 345
[37] Baikov P A, Chetyrkin K G and Kühn J H 2010 *Phys. Rev. Lett.* **104** 132004
[38] Cvetič G and Valenzuela C 2006 *J. Phys.* G **32** L27
[39] Cvetič G and Valenzuela C 2006 *Phys. Rev.* D **74** 114030
[40] Cvetič G and Kotikov A V 2012 *J. Phys.* G **39** 065005
[41] Cvetič G 2019 *Phys. Rev.* D **99** 014028
[42] Cvetič G 2019 Renormalon-based resummation for QCD observables *Preprint* arXiv:1909.13593 [hep-ph].
[43] Olive K A *et al.* [Particle Data Group Collaboration] 2014 *Chin. Phys.* C **38** 090001
[44] Kataev A L *et al.* 1998 *Phys. Lett.* B **417** 374
[45] Kataev A L *et al.* 1996 *Phys. Lett.* B **388** 179
[46] Kataev A L *et al.* 2000 *Nucl. Phys.* B **573** 405
[47] Kataev A L *et al.* 2003 *Phys. Part. Nucl.* **34** 20
[48] Narison S and Zakharov V I 2009 *Phys. Lett.* B **679** 355
[49] Parente G *et al.* 1994 *Phys. Lett.* B **333** 190
[50] Shaikhatdenov B G *et al.* 2010 *Phys. Rev.* D **81** 034008
[51] Kotikov A V *et al.* 2012 *Phys. Atom. Nucl.* **75** 507
[52] Kotikov A V *et al.* 2015 *J. Phys.* G **42** 095004
[53] Kotikov A V *et al.* 1993 *Z. Phys.* C **58** 465
[54] Krivokhizhin V G and Kotikov A V 2005 *Phys. Atom. Nucl.* **68** 1873
[55] Kotikov A V, Lipatov A V and Zotov N P 2005 *J. Exp. Theor. Phys.* **101** 811
[56] Illarionov A Y *et al.* 2008 *Phys. Part. Nucl.* **39** 307
[57] Cvetič G *et al.* 2009 *Phys. Lett.* B **679** 350
[58] Kotikov A V and Shaikhatdenov B G 2013 *Phys. Part. Nucl.* **44** 543
[59] Kotikov A V and Shaikhatdenov B G 2015 *Phys. Atom. Nucl.* **78** 525
[60] Kotikov A V and Shaikhatdenov B G 2015 *AIP Conf. Proc.* **1606** 159
[61] Kotikov A V and Shaikhatdenov B G 2017 *Phys. Part. Nucl.* **48** 829
[62] Teryaev O 2013 *Nucl. Phys. Proc. Suppl.* **245** 195
[63] Khandramai V L *et al.* 2016 *J. Phys. Conf. Ser.* **678** 012018
[64] Gabdrakhmanov I R *et al.* 2017 *J. Phys. Conf. Ser.* **938** 012046
[65] Shirkov D V 2013 *Phys. Part. Nucl. Lett.* **10** 186
[66] Shirkov D V 2015 *Phys. Part. Nucl. Lett.* **12** 205
[67] Cornwall J M 1982 *Phys. Rev.* D **26** 1453
[68] Aguilar A C, Binosi D and Papavassiliou J 2014 *Phys. Rev.* D **89** 085032
[69] Cvetič G and Villavicencio C 2012 *Phys. Rev.* D **86** 116001
[70] Bogolubsky I L *et al.* 2009 *Phys. Lett.* B **676** 69
[71] Duarte A G *et al.* 2016 *Phys. Rev.* D **94** 014502;
[72] Ilgenfritz E M *et al.* 2006 Gauge-variant propagators and the running coupling from lattice QCD *Preprint* hep-lat/0601027.