

# Gauge-invariant TMD factorization for Drell-Yan Helicity Structure Functions

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**Abstract.** In framework of the Parton Reggeization Approach, we obtain gauge invariant hadronic tensor  $W^{\mu\nu}$  for Drell-Yan process in Transverse Momentum Dependent factorization. We show that use of Fadin-Sherman effective vertex, which describes annihilation of Reggeized quark-antiquark into virtual photon, leads to new relations between Helicity Structure Functions  $F_{UU}^{(1,2,\cos\phi,\cos 2\phi)}$  and Parton Distribution Functions ( $f_1^q(x, \mathbf{q}_T), h_1^{\perp q}(x, \mathbf{q}_T)$ ) in unpolarized proton-proton collisions. We estimate numerically relative contributions of parton number density function  $f_1^q(x, \mathbf{q}_T)$  and Boer-Mulders function  $h_1^{\perp q}(x, \mathbf{q}_T)$  in the Helicity Structure Functions  $F_{UU}^{(1)}$  and  $F_{UU}^{(\cos 2\phi)}$  at the kinematical conditions of the future collider NICA.

## 1. Introduction

In this talk we continue our study of the role of QED gauge invariance in description of Helicity Structure Functions (HSFs) for Drell-Yan process in unpolarized proton-proton collisions. Contrary to the conventional Transverse Momentum Dependent (TMD) Parton Model (PM), usually connected with Collins-Soper-Sterman (CSS) formalism [1], we take into account off-shell initial-state effects and parton transverse momenta in a gauge-invariant way [2, 3]. Our calculations of lepton pair transverse-momentum ( $q_T$ ) spectra and angular coefficients in the framework of Parton Reggeization Approach (PRA) (see Ref. [4] and references therein) demonstrate good agreement with experimental data at different values of collision energy and  $q_T$  [5]. In Ref. [2] we have shown for the first time that unpolarized TMD Parton Distribution Function (PDF)  $f_1^q(x, \mathbf{q}_T)$ , which is the parton number density, contributes to the HSF  $F_{UU}^{(\cos 2\phi)}$  even at very small values of  $q_T$ . Here, we obtain new relations between HSFs  $F_{UU}^{(1,2,\cos\phi,\cos 2\phi)}$  and TMD PDFs  $f_1^q(x, \mathbf{q}_T)$  and  $h_1^{\perp q}(x, \mathbf{q}_T)$  (Boer-Mulders function) in the PRA.

## 2. Helicity Structure Functions for Drell-Yan process

In the notation of Ref. [6] the differential cross-section of the process of production of the lepton pair ( $l^+l^-$ ) with transverse momentum ( $q_T$ ), squared invariant mass ( $Q^2 = q^2$ ) in the collision of two non-polarized hadrons with center-of-mass energy ( $\sqrt{S}$ ) can be written as:

$$\frac{d\sigma}{dx_A dx_B d^2\mathbf{q}_T d\Omega} = \frac{\alpha^2}{4Q^2} \left[ F_{UU}^{(1)} \cdot (1 + \cos^2 \theta) + F_{UU}^{(2)} \cdot (1 - \cos^2 \theta) + F_{UU}^{(\cos \phi)} \cdot \sin(2\theta) \cos \phi + F_{UU}^{(\cos 2\phi)} \cdot \sin^2 \theta \cos(2\phi) \right], \quad (1)$$



were angles  $\theta$  and  $\phi$  define the direction of momentum of  $l^+$  in the Collins-Soper frame [7],  $F_{UU}^{(1,2,\dots)}(x_A, x_B, q_T)$  are HSFs,  $x_{A,B} = Qe^{\pm y}/\sqrt{S}$ ,  $y$  is the rapidity of lepton pair.

In the standard TMD PM approach [1, 6], based on a simple  $q\bar{q}$ -annihilation picture of the Drell-Yan process, the hadronic tensor is decomposed as follows:

$$W_{\mu\nu} = W_{\mu\nu}^{(\text{TMD})} + Y_{\mu\nu}, \quad (2)$$

where

$$W_{\mu\nu}^{(\text{TMD})} = \sum_{q,\bar{q}} \frac{e_q^2}{N_c} \text{tr} [\gamma_\mu \Phi_q(q_1, P_1) \otimes_T \gamma_\nu \bar{\Phi}_{\bar{q}}(q_2, P_2)], \quad (3)$$

and  $f_1(\mathbf{q}_{T1}) \otimes_T f_2(\mathbf{q}_{T2}) = \int d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2} \delta(\mathbf{q}_T - \mathbf{q}_{T1} - \mathbf{q}_{T2}) f_1(\mathbf{q}_{T1}) f_2(\mathbf{q}_{T2})$ . Four-momenta of quark( $q_1$ ) and anti-quark( $q_2$ ) are parametrized as  $q_{1,2}^\mu = P_{1,2}^\mu x_{1,2} + q_{T1,2}^\mu$ . The first term in Eq. (3) is a contribution of a leading power in  $q_T/Q$  to a hadronic tensor, while all subleading contributions are supposed to be included to the  $Y_{\mu\nu}$ -term, which is calculated in collinear approximation of PM using perturbative QCD.

Due to a large boost between hadron rest frame and hadronic center-of-mass frame, only terms proportional to  $n_-^\mu = 2P_1^\mu/\sqrt{S}$  contribute to the correlation function of quark fields  $\Phi_q(q_1, P_1)$  at leading power, and it's Dirac structure can be parametrized as follows:

$$\Phi_q(q_1, P_1) = \frac{1}{2} \left[ \hat{n}_- f_1^q(x_1, q_{T1}) + i\sigma^{i-} \gamma_5 \frac{\epsilon_T^{ij} q_{T1}^j}{\Lambda} h_1^{\perp q}(x_1, q_{T1}) \right], \quad (4)$$

where,  $f_1^q(x_1, q_{T1})$  is a number-density TMD PDF,  $h_1^{\perp q}(x_1, q_{T1})$  is a Boer-Mulders function,  $\Lambda$  is a scale of non-perturbative intrinsic transverse momentum of partons inside a hadron, which is typically taken to be  $\Lambda \sim M$ , and analogous decomposition holds for  $\bar{\Phi}_{\bar{q}}$ .

The full hadronic tensor should satisfy Ward identity of QED:

$$q^\mu W_{\mu\nu} = 0,$$

however, it is easy to verify, that for the first term in Eq. (3):  $q^\mu W_{\mu\nu}^{(\text{TMD})} = \mathcal{O}(q_T/Q)$ , and the gauge-invariance can't be restored by some  $\mathcal{O}(q_T/Q)$  power-corrections from  $Y_{\mu\nu}$ , because this quantity is typically computed in Collinear Parton Model and therefore one has  $q^\mu Y_{\mu\nu} = 0$ .

### 3. Parton Reggeization Approach

The gauge invariance of the hadronic tensor holds because apart from the  $t$ -channel  $q\bar{q}$ -annihilation diagram, there exist other contributions to  $p + p \rightarrow \gamma^* + X$ -amplitude, where photon is interacting directly with constituents of the colliding protons and beam-remnants. This contributions are beyond the scope of conventional PM, but one can try to analyze them and look for the limit when contributions of this kind also factorize, leading to some PM-like interpretation, independent on the details of above-mentioned interactions.

In fact, such factorization is well-known in the small- $x$  physics. It is proven in the Leading and Next-to-Leading Logarithmic Approximation w.r.t. resummation of  $\log(1/x)$  in QCD [8, 9], that in the Multi-Regge limit  $Q^2, q_T^2 \ll S$  the universal vertex (Fadin-Sherman vertex) of production of virtual photon in an annihilation of Reggeized quark and antiquark factorizes-out from the amplitude:

$$\Gamma_\mu(q_1, q_2) = \gamma_\mu - \hat{q}_1 \frac{n_\mu^-}{q_2} - \hat{q}_2 \frac{n_\mu^+}{q_1}, \quad (5)$$

where  $n_\pm^\mu = 2P_2^\mu/\sqrt{S}$ . The vertex (5) satisfies the Ward identity  $(q_1 + q_2)^\mu \Gamma_\mu(q_1, q_2) = 0$ .

In Ref. [2], we have proposed to modify the definition of  $W_{\mu\nu}^{(TMD)} \rightarrow W_{\mu\nu}^{(PRA)}$ , by changing  $\gamma_\mu \rightarrow \Gamma_\mu(q_1, q_2)$  as it is required in the Regge limit [4, 5]:

$$W_{\mu\nu}^{(PRA)} = \sum_{q, \bar{q}} \frac{e_q^2}{N_c} \text{tr} \left[ \Gamma_\mu(q_1, q_2) \Phi_q(q_1, P_1) \otimes_T \Gamma_\nu(q_1, q_2) \bar{\Phi}_{\bar{q}}(q_2, P_2) \right]. \quad (6)$$

HSFs are obtained by the projection of a hadronic tensor (6) on the photon states with different polarizations (see formulas (24) – (31) in Ref. [5] for details). Finally, we find

$$F_{UU}^{(1)} = \sum_{q, \bar{q}} e_q^2 \left[ f_1^q(x_1, \mathbf{q}_{T1}) \otimes_T f_1^{\bar{q}}(x_2, \mathbf{q}_{T2}) \frac{2Q^2 + \mathbf{q}_T^2}{2(Q^2 + \mathbf{q}_T^2)} + h_1^{\perp q}(x_1, \mathbf{q}_{T1}) \otimes_T h_1^{\perp \bar{q}}(x_2, \mathbf{q}_{T2}) \frac{2(\mathbf{q}_T \mathbf{q}_{T1})(\mathbf{q}_T \mathbf{q}_{T2}) - \mathbf{q}_T^2(\mathbf{q}_{T1} \mathbf{q}_{T2})}{2\Lambda^2(Q^2 + \mathbf{q}_T^2)} \right], \quad (7)$$

$$F_{UU}^{(2)} = \sum_{q, \bar{q}} e_q^2 \left[ f_1^q(x_1, \mathbf{q}_{T1}) \otimes_T f_1^{\bar{q}}(x_2, \mathbf{q}_{T2}) \frac{(\mathbf{q}_{T1} - \mathbf{q}_{T2})^2}{Q^2 + \mathbf{q}_T^2} + h_1^{\perp q}(x_1, \mathbf{q}_{T1}) \otimes_T h_1^{\perp \bar{q}}(x_2, \mathbf{q}_{T2}) \frac{2\mathbf{q}_{T1}^2 \mathbf{q}_{T2}^2 - (\mathbf{q}_{T1} \mathbf{q}_{T2})(\mathbf{q}_{T1}^2 + \mathbf{q}_{T2}^2)}{2\Lambda^2(Q^2 + \mathbf{q}_T^2)} \right], \quad (8)$$

$$F_{UU}^{(\cos \phi)} = \sum_{q, \bar{q}} \frac{e_q^2}{Q^2 + \mathbf{q}_T^2} \sqrt{\frac{Q^2}{\mathbf{q}_T^2}} \left[ f_1^q(x_1, \mathbf{q}_{T1}) \otimes_T f_1^{\bar{q}}(x_2, \mathbf{q}_{T2}) (\mathbf{q}_{T1}^2 - \mathbf{q}_{T2}^2) - h_1^{\perp q}(x_1, \mathbf{q}_{T1}) \otimes_T h_1^{\perp \bar{q}}(x_2, \mathbf{q}_{T2}) (\mathbf{q}_{T1}^2 - \mathbf{q}_{T2}^2) \frac{(\mathbf{q}_{T1} \mathbf{q}_{T2})}{\Lambda^2} \right], \quad (9)$$

$$F_{UU}^{(\cos 2\phi)} = \sum_{q, \bar{q}} e_q^2 \left[ f_1^q(x_1, \mathbf{q}_{T1}) \otimes_T f_1^{\bar{q}}(x_2, \mathbf{q}_{T2}) \frac{\mathbf{q}_T^2}{2(Q^2 + \mathbf{q}_T^2)} + h_1^{\perp q}(x_1, \mathbf{q}_{T1}) \otimes_T h_1^{\perp \bar{q}}(x_2, \mathbf{q}_{T2}) \frac{2Q^2 + \mathbf{q}_T^2}{2(Q^2 + \mathbf{q}_T^2)} \frac{2(\mathbf{q}_T \mathbf{q}_{T1})(\mathbf{q}_T \mathbf{q}_{T2}) - \mathbf{q}_T^2(\mathbf{q}_{T1} \mathbf{q}_{T2})}{\Lambda^2 \mathbf{q}_T^2} \right]. \quad (10)$$

In the limit  $(\frac{q_T}{Q}) \ll 1$ , which is the region of applicability of TMD PM, one recovers well known results

$$F_{UU}^{(1)} = \sum_{q, \bar{q}} e_q^2 [f_1^q(x_1, \mathbf{q}_{T1}) \otimes_T f_1^{\bar{q}}(x_2, \mathbf{q}_{T2})] + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right),$$

$$F_{UU}^{(2)} \simeq \mathcal{O}\left(\frac{q_T^2}{Q^2}\right),$$

$$F_{UU}^{(\cos \phi)} \simeq \mathcal{O}\left(\frac{q_T}{Q}\right),$$

$$F_{UU}^{(\cos 2\phi)} = \sum_{q, \bar{q}} e_q^2 h_1^{\perp q}(x_1, \mathbf{q}_{T1}) \otimes_T h_1^{\perp \bar{q}}(x_2, \mathbf{q}_{T2}) \left[ \frac{2(\mathbf{q}_T \mathbf{q}_{T1})(\mathbf{q}_T \mathbf{q}_{T2}) - \mathbf{q}_T^2(\mathbf{q}_{T1} \mathbf{q}_{T2})}{\Lambda^2 \mathbf{q}_T^2} \right] + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right).$$

Eqs. (7)-(10) are just the same Eq. (7) from our Ref. [2] rewritten in terms of TMD PDFs with the same normalization as in Eq. (4) of the present contribution. The latter is more conventional in the TMD community. Conventional TMD PDFs are related with TMD PDFs of PRA as  $f_1^q(x, t, \mu^2) = \Phi_q(x, t, \mu^2)/(\pi\sqrt{S}x)$ , since in PRA we include the flux-factor for initial-state partons  $1/(2Sx_1x_2) = 1/(2Q_T^2)$  into the cross-section formula and put  $x_{1,2} = Q_T e^{\pm y}/\sqrt{S}$ , where  $Q_T = \sqrt{\mathbf{q}_T^2 + Q^2}$ .

One observes, that the contributions of number-density TMD PDF to all structure functions except  $F_{UU}^{(1)}$  are  $\mathcal{O}(q_T^2/Q^2)$ , as it should be, according to the TMD PM analysis, so that the only leading-power contribution to  $F_{UU}^{(\cos 2\phi)}$  comes from the convolution of two Boer-Mulders functions. However, the Boer-Mulders TMD PDF is expected to be significantly smaller than number-density TMD PDF and its effects are observable only at nonzero  $q_T$ . Taking into account, that values of  $Q^2$  in the existing and planned experiments, such as COMPASS [10] and NICA SPD [11] lie in a ballpark of 10 GeV, the power-suppressed corrections could be important for the extraction of Boer-Mulders TMD PDF, especially in the transition region  $q_T \sim Q$ .

#### 4. Numerical results

In Ref. [2] we have performed a numerical analysis with the help of Eq. (6) and a realistic model for number-density TMD PDF, based on the Kimber-Martin-Ryskin formula [12] and MSTW-2008 [13] set of collinear PDFs. This TMD PDF allowed us to reproduce the observed  $q_T$ -spectra of Drell-Yan pairs from several low-energy experiments and polarization observables measured by NuSea Collaboration [14], see Refs. [2,5] for more details.

To analyze relative contributions of parton number density TMD PDF and Boer-Mulders TMD PDF, we suggest that they are proportional to each-other

$$h_1^{\perp q}(x, \mathbf{q}_T) = \alpha_{BM} \times f_1^q(x, \mathbf{q}_T), \quad (11)$$

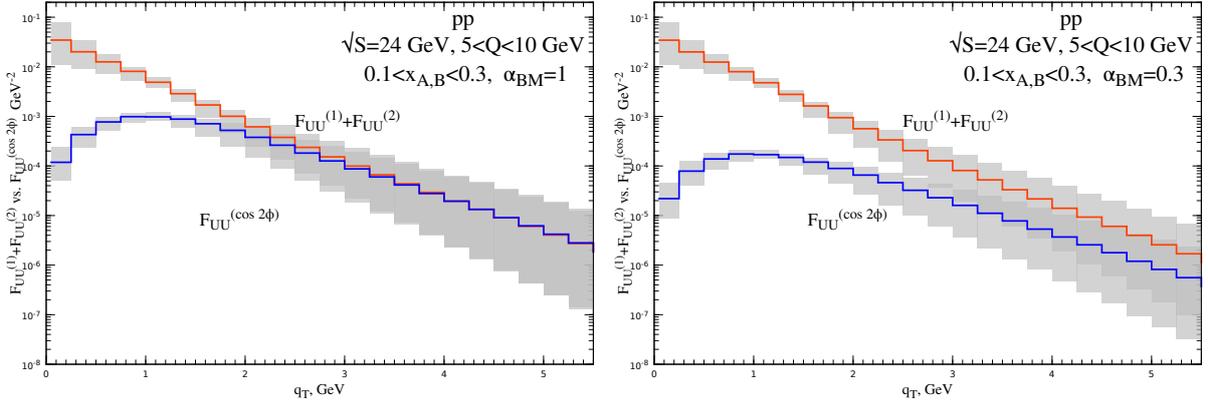
where parameter  $\alpha_{BM}$  does not depend on  $x$  and  $q_T$ . The following relation between structure functions is required for the positivity of angular distribution (1):  $F_{UU}^{(\cos 2\phi)} \leq F_{UU}^{(1)} + F_{UU}^{(2)}$ , which in turn leads to a constraint  $\alpha_{BM} \leq 1$ , see Fig. 1. The latter argument essentially depends on the assumption, that Boer-Mulders function has the same high- $q_T$  tail as the number density. If Boer-Mulders function is softer, larger contribution of it is allowed, which can not be accounted for by simple model (11).

As one can see from Fig. 2, for  $\alpha_{BM} = 1$  the contribution of Boer-Mulders TMD PDF in  $F_{UU}^{(\cos 2\phi)}$  dominates even in considered kinematic conditions of NICA collider. However with decrease of  $\alpha_{BM}$  this contribution decreases rapidly, as  $\alpha_{BM}^2$  and already at the  $\alpha_{BM} = 0.3$  contributions from Boer-Mulders effect and from parton density PDF are equal, see the Fig. 3.

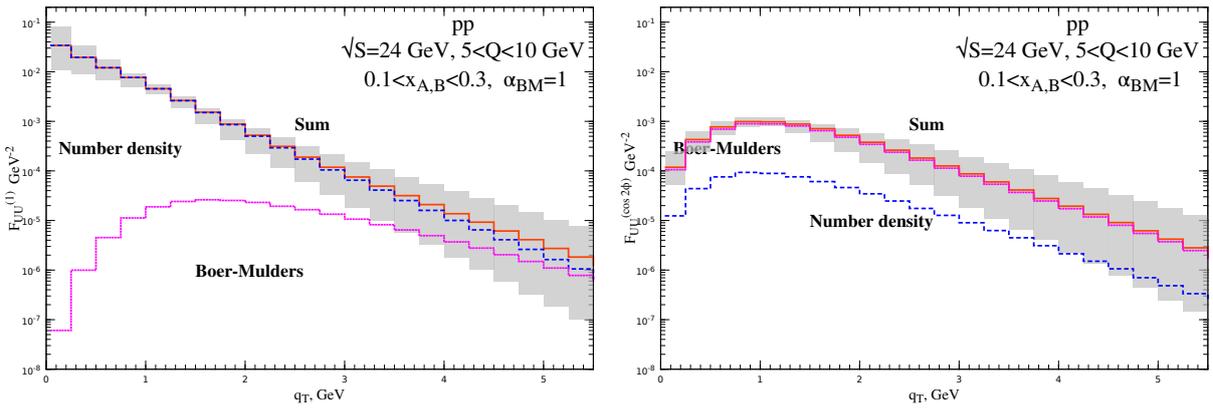
Such a way, at the energy range of future collider NICA, as well as in the COMPASS++/AMBER experiment at CERN, when possible values of  $Q$  are not large enough, contributions from Boer-Mulders PDF and power-suppressed terms, which restore QED gauge invariance, in the HSF  $F_{UU}^{(\cos 2\phi)}$  may be of the same order. If all the TMD PDFs have similar  $q_T$  dependence the task of extraction of Boer-Mulders PDF from experimental data become considerably difficult.

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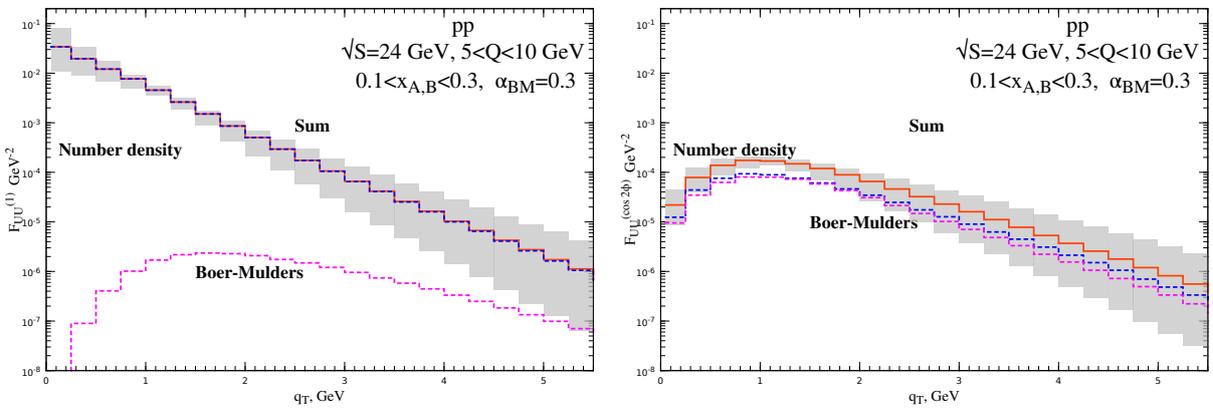
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**Figure 1.** The sum  $F_{UU}^{(1)} + F_{UU}^{(2)}$  versus  $F_{UU}^{(\cos 2\phi)}$  at the  $\alpha_{BM} = 1$  (left panel) and  $\alpha_{BM} = 0.3$  (right panel).



**Figure 2.** HSFs  $F_{UU}^{(1)}$  (left panel) and  $F_{UU}^{(\cos 2\phi)}$  (right panel) at the  $\alpha_{BM} = 1$  as functions of  $q_T$ .



**Figure 3.** HSFs  $F_{UU}^{(1)}$  (left panel) and  $F_{UU}^{(\cos 2\phi)}$  (right panel) at the  $\alpha_{BM} = 0.3$  as functions of  $q_T$ .

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