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Radiative corrections to QCD SR for meson distribution amplitudes up to $O(\alpha_s^2\beta_0)$

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Abstract. We obtain QCD radiative corrections to distribution amplitudes of π and ρ mesons within QCD sum rules. To this end, we calculate correlators of two different composite vertices at order $O(\alpha_s^2\beta_0)$, where β_0 is the first coefficient in the expansion of QCD β-function.

1. Introduction

An important problem in the description of hard processes is to calculate distribution amplitudes (DA) $\varphi_M(x)$ of light mesons $M = \pi$, ρ, etc. with the help of QCD sum rules (SR). These distributions of partons by the fraction xp of an hadron momentum p appear naturally as a consequence of "factorization theorems" for hard exclusive processes [1, 2]. In the factorization approach, DAs accumulate information about long-distance dynamics of partons in hadrons. For helicity-zero vector mesons $M_\parallel = \rho_\parallel,\ \rho'_\parallel, \ldots$ and for axial mesons $M_A = \pi, a_1^\parallel, \ldots$, the twist-2 DAs are defined by projecting currents $V(z) = d(z)\hat{z}u(0)$ and $A(z) = d(z)\hat{z}\gamma_5 u(0)$ on a meson state $|M\rangle$:

$$\langle 0|\bar{d}(z)\hat{z}(-i\gamma_5)u(0)|M_{\parallel(A)}(p)\rangle\Big|_{z^2=0} = f_{M_{\parallel(A)}}(zp)\int_0^1 dx\, e^{ix(zp)}\ \varphi_{M_{\parallel(A)}}(x). \tag{1}$$

The corresponding definition for a helicity-one state, $M_\perp^{\lambda=\pm1} = \rho_\perp,\ b_{1\perp}\ldots$, is as follows:

$$\langle 0|\bar{d}(z)\sigma_{\mu\nu}u(0)|M_\perp^\lambda(p)\rangle\Big|_{z^2=0} = if^T_{M_\perp}\left(\varepsilon_\mu^{(\lambda)}(p)p_\nu - \varepsilon_\nu^{(\lambda)}(p)p_\mu\right)\int_0^1 dx\, e^{ix(zp)}\ \varphi_{M_\perp}(x). \tag{2}$$

These DAs and their moments can be extracted from a correlator Π of the currents $J(z) = V(z), A(z)$, or $T_\mu(z) = \bar{d}(z)\sigma_{\mu\alpha}z^\alpha u(0)$ [3–6]—for example, for the local current $T^\mu_{(n)}(y) \equiv \bar{d}(y)\sigma^{\mu\alpha}z_\alpha\,(z\nabla)^n\,u(y)$, we have

$$i\int dy\, e^{ipy}\langle 0|\mathrm{T}\left\{T^{\mu+}_{(0)}(y)T^\mu_{(n)}(0)\right\}|0\rangle = -2i^n\,(zp)^{n+2}\,\Pi_{(n0)}(p^2)\,. \tag{3}$$

The key element to calculate the inverse Mellin image of $\Pi_{(n0)}$, $\hat{M}_{(n\to x)}\Pi_{(n0)} = \Pi(x;p^2)$, at order $O(\alpha_s^2\beta_0)$ was obtained in [7] and will be considered in the next section. The borelized correlator $\hat{\mathbf{B}}_{(M^2)}\Pi(x;p^2)$ directly determines the *pQCD content* $\Delta\varphi_M$ of the corresponding DA φ_M within QCD SRs. Further, we discuss the impact and main features of these radiative corrections.

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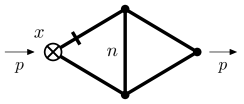
2. Two-loop correlators with composite vertices

In order to calculate the required contributions of the order $O(\alpha_s^2\beta_0)$, we need to evaluate the two-loop diagram of kite topology with one of its external vertices being composite:



$$= \int \frac{d^D k_1\, d^D k_2\, \delta\left[x-(zk_1)/(zp)\right]}{k_1^2 k_2^2 (k_1-p)^2 (k_2-p)^2 \left[(k_1-k_2)^2\right]^n} = \frac{(-)^{1+n}\pi^D}{(-p^2)^{n+4-D}} G(n;x;D), \quad (4)$$

where z is a lightlike vector and the tick on a line denotes the Dirac δ-function that accompanies the composite vertex in the integrand. Even more general kite correlator of two composite vertices (as well as the Mellin moments of it) was considered in [7] with the propagators raised to arbitrary powers. The α representation of such two-loop correlator can be evaluated directly as a hypergeometric integral. In the general case, the result of integration is expressed in terms of a hypergeometric series in two variables—the Kampé de Fériet functions. A chain of reductions to simpler functions can be found for some special cases. In particular, the correlator (4) amounts to a generalized hypergeometric function:

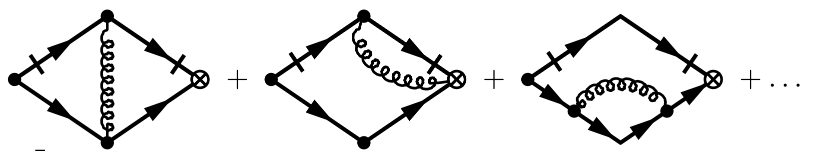
$$\begin{aligned} G(n;x;D) = &-\frac{\Gamma^2(-\dot{n})\Gamma(1+\dot{n}-\lambda)\Gamma(\lambda)}{\Gamma(n)\Gamma(1-\dot{n})\Gamma(\lambda-\dot{n})}(x\bar{x})^{\lambda-1} \\ &\times\left\{\frac{\Gamma(n)\Gamma^2(\dot{n})\Gamma^3(1-\dot{n})\Gamma(\lambda-\dot{n})}{\Gamma^2(\lambda)\Gamma(2\dot{n})\Gamma(1-2\dot{n})\Gamma(-\dot{n})} + \hat{\mathbf{S}}\left[x^{-\dot{n}}\,{}_3F_2\left(\left.\begin{matrix}1,\ \lambda,\ -\dot{n}\\ 1-\dot{n},\ \lambda-\dot{n}\end{matrix}\right| x\right)\right]\right\}, \end{aligned} \quad (5)$$

where $\lambda = D/2-1$, $\dot{n} = n-\lambda$, $\hat{\mathbf{S}}T(x) = T(x)+T(\bar{x})$, and $\bar{x} = 1-x$. The integral of $G(n;x;D)$ over x coincides with the well-known results in [8–10] after some transformations of ${}_3F_2(1)$. The Laurent expansions of Eq. (5) near even and odd D can be obtained with the help of the standard algorithms and results (see [11] and references therein).

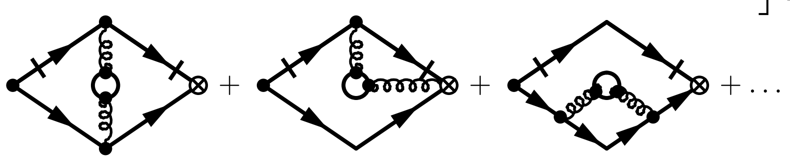
3. $\langle \boldsymbol{AA}\rangle$ and $\langle \boldsymbol{VV}\rangle$ correlators. Distribution amplitudes of $\boldsymbol{\pi}$ and $\boldsymbol{\rho}_{\|}$

In what follows, we use the notations $a_s = \alpha_s(\mu^2)/(4\pi)$, $\beta_0 = \frac{11}{3}C_A - \frac{2}{3}n_f$ for the first coefficient of β-function, and M^2 is a parameter of the Borel transform $\hat{\mathbf{B}}_{(M^2)}$ applied to correlators $\Pi(p^2)$ in the framework of QCD SR (e.g., see [12]). To take into account NLO corrections and a part of N^{2}LO corrections that are proportional to β_0, we have to deal with the following diagrams:

$$i\Delta\varphi_M^{(1)}(M^2;x) = \hat{\mathbf{B}}_{(M^2)}\left[\ \cdots\ \right]$$



$$i\Delta\varphi_M^{(2)}(M^2;x) = -\frac{3\beta_0}{2n_f}\hat{\mathbf{B}}_{(M^2)}\left[\ \cdots\ \right]$$



3.1. $\langle V(A)\, V(A)\rangle$ correlator at orders up to $O(\alpha_s^2\beta_0)$

The contribution of order $O(a_s)$ was obtained first in [12] and leads to a visible correction that is especially significant near the endpoints. Here, we recalculate it in arbitrary covariant gauge:

$$\Delta\varphi^{(0+1)}_{M_{\|(A)}}(M^2;x) = \hat{\mathbf{B}}_{(M^2)}\Pi^{V(A)}_{\text{LO+NLO}}(p^2) = \frac{N_c}{2\pi^2}x\bar{x}\left\{1 + a_s C_F\left[5 - \frac{\pi^2}{3} + \ln^2\left(\frac{\bar{x}}{x}\right)\right]\right\}. \quad (6)$$

At β_0N^{2}LO, we obtain the following expression involving dependence on $L_{\rm B} = \ln\left(\frac{M^2}{\mu^2}e^{-\gamma_{\rm E}}\right)$:

$$\Delta\varphi^{(2)}_{M_{\parallel(A)}}(M^2;x) = \frac{N_c}{4\pi^2}a_s^2 C_F\beta_0\hat{\mathbf{S}}\Big\{-x\bar{x}\Big[10\,\mathrm{Li}_3(x) - 2\ln x\,\mathrm{Li}_2(x) + \ln^2 x\ln\bar{x} - \frac{5}{6}\ln^2\left(\frac{\bar{x}}{x}\right) - \frac{1}{3}\ln^3 x + \frac{5\pi^2}{18} - \frac{2\pi^2}{3}\ln x - \frac{7}{6}\Big] - 2x\left[\mathrm{Li}_2(x) - \frac{\pi^2}{6} - \frac{3}{4}\ln^2 x + \left(\frac{31}{12} - L_{\rm B}\right)\ln x\right]\Big\}, \quad (7)$$

3.2. Perturbative content of twist-2 DA for π and $\rho_\parallel$ mesons
Important characteristics of $\Delta\varphi_M$ of DA are the norm $\langle x^0\rangle_M$ and inverse moment $\langle x^{-1}\rangle_M$:

$$\langle x^n\rangle_M \equiv \int_0^1 dx\, x^n\Delta\varphi_M(x),\ \langle x^0\rangle_{M_{\parallel(A)}} = \frac{N_c}{12\pi^2}\left[1 + a_s C_F 3 + a_s^2\beta_0 C_F 3\left(\frac{11}{2} - 4\zeta_3 - L_{\rm B}\right)\right], \quad (8)$$

$$\langle x^{-1}\rangle_{M_{\parallel(A)}} = \frac{N_c}{4\pi^2}\left\{1 + a_s C_F 5 + a_s^2\beta_0 C_F\left[\frac{7}{18} - \frac{5}{3}\zeta_3 + \frac{31}{108}\pi^2 - \frac{\pi^2}{9}L_{\rm B}\right]\right\}. \quad (9)$$

The $\langle x^0\rangle_M$ in (8) coincides with the corresponding part of the Adler D-function, as expected. The impact of $O(a_s^2\beta_0)$ contribution to $\Delta\varphi_M$ looks especially significant for intermediate values of x, see Fig. 1 (left panel), while in the vicinity of endpoints it is less important, which is reflected by a minor contribution to $\langle x^{-1}\rangle_{M_{\parallel(A)}}$ in (9).

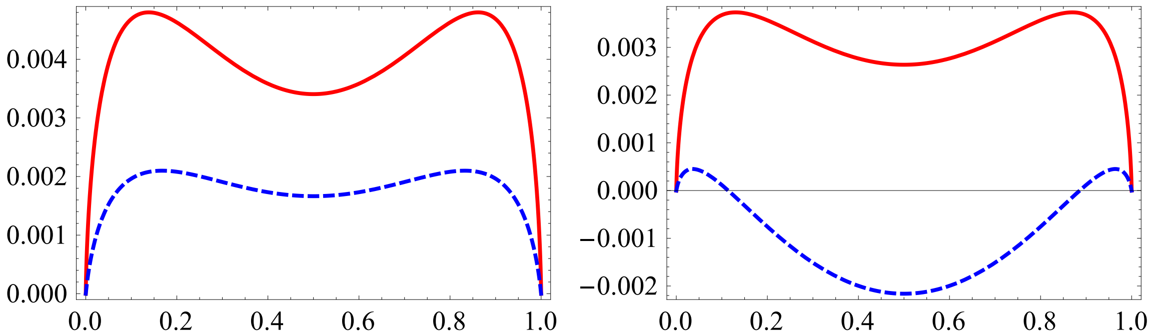


Figure 1. Comparison of **NLO** (——) and **β_0N^{2}LO** (– – –) contributions to DAs: (left panel) pseudoscalar or longitudinally polarized vector mesons, Eqs. (6) and (7); (right panel) transversally polarized vector meson, Eqs. (10) and (11). All curves are for the case of $L_{\rm B} = \ln(M^2/\mu^2) - \gamma_{\rm E} = 0$ and $\alpha_s(\mu^2 = 1\ \mathrm{GeV}^2) \approx 0.494$.

4. $\langle TT\rangle$ correlator and perturbative part $\Delta\varphi_{M_\perp}$ of DA for transversal ρ meson

The NLO contribution was derived first in [3] and recalculated by us together with its non-logarithmic part (not shown here):

$$\Delta\varphi^{(0+1)}_{M_\perp}(M^2;x) = \frac{N_c}{2\pi^2}x\bar{x}\left\{1 + a_s C_F\left[6 - \frac{\pi^2}{3} + \ln^2\left(\frac{\bar{x}}{x}\right) + \ln(x\bar{x}) + 2L_{\rm B}\right]\right\} \quad (10)$$

The contribution of the NLO corrections is as moderate as in Eq. (6). The β_0N^{2}LO terms read

$$\Delta\varphi^{(2)}_{M_\perp}(M^2;x) = \frac{N_c}{12\pi^2}a_s^2\beta_0 C_F\hat{\mathbf{S}}\left\{x\bar{x}\left(\frac{\pi^2}{6} - L_{\rm B}^2\right) + x\left[6\left(2-\bar{x}\right)\ln(x) + 19\bar{x}\right]L_{\rm B} + x\bar{x}\Big[-30\mathrm{Li}_3(x) + 6\mathrm{Li}_2(x)\ln(x) + \ln^3(x) + \ln^2(x)\left[2 - 3\ln(\bar{x})\right] + \left(2\pi^2 + 19\right)\ln(x) - 5\ln(x)\ln(\bar{x}) - \frac{5\pi^2}{6} - \frac{193}{12}\Big] - x\left[12\mathrm{Li}_2(x) - 2\pi^2 + 16\ln(x) - 9\ln^2(x)\right]\right\}. \quad (11)$$

In comparison with the LO and NLO terms, the contribution of $\Delta\varphi^{(2)}_{M_\perp}$ is mainly of an opposite sign and comparable in magnitude with the NLO in the middle region of x, see Fig. 1 (right panel).

$$\langle x^0\rangle_{M_\perp} = \frac{N_c}{12\pi^2}\left[1 + a_s C_F\left(\frac{7}{3} + 2L_{\rm B}\right) + a_s^2\beta_0 C_F\left(\frac{\pi^2}{6} - 12\zeta_3 + \frac{383}{36} + 2L_{\rm B} - L_{\rm B}^2\right)\right], \quad (12)$$

$$\langle x^{-1}\rangle_{M_\perp} = \frac{N_c}{4\pi^2}\left[1 + a_s C_F 2\left(2 + L_{\rm B}\right) + a_s^2\beta_0 C_F\left(2\zeta_3 + \frac{19\pi^2}{18} - \frac{493}{36} + \frac{25-2\pi^2}{3}L_{\rm B} - L_{\rm B}^2\right)\right], \quad (13)$$

The $O(a_s^2\beta_0)$ contribution to $\langle x^{-1}\rangle_{M_\perp}$ in (13) is numerically tiny. The norm $\langle x^0\rangle_M$ in Eq. (12) is in agreement with the result in [13] obtained for a correlator of the corresponding local currents $T^{\mu}_{(0)}$. The magnitude of $O(a_s^2\beta_0)$ contribution in the norm (12) is moderate.

5. Conclusions

We briefly analyze the perturbative corrections $\Delta\varphi^{(2)}_M$ to DAs of leading twist for pion and (longitudinally or transversely) polarized light vector mesons at order $O(a_s^2\beta_0)$. To this end, we calculate vector-vector (axial-axial), $\langle V(A)\ V(A)\rangle$, and tensor-tensor $\langle TT\rangle$ correlators with the corresponding composite currents $V(A)$ and T up to the order $O(a_s^2\beta_0)$. The impact of these β_0N^{2}LO corrections is moderate, while the sign of the correction in the transverse case $\Delta\varphi^{(2)}_{M_\perp}$ is opposite to the one of LO and NLO terms.

Acknowledgments

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