Radiative decays of heavy quarkonia into two vector mesons. Spin properties of amplitudes and angular distributions of linearly polarized photons.

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Abstract. Angular distributions of photons with definite linear polarizations relative to the plane spanned by the momentum of initial electron and the total momentum of the V_1V_1 pair in the reaction $e^+e^- \to V \to \gamma X(J^P) \to \gamma V_1V_1$ ($V = J/\psi, \psi', ..., \Upsilon(1S), \Upsilon, ...$ and $V_1 = \rho, \omega, \phi, ...$) are calculated. It is shown that the sign of asymmetry of the distributions of photons polarized in the above plane and orthogonal to it correlates with the signature $P(-1)^J$ of the X resonance with given spin-parity $J^P = 0^\pm, 1^\pm, 2^\pm$ so it may help to establish this quantum number in a way which does not depend on the specific model of the $X(J^P) \to V_1V_1$ amplitude.

1. Introduction

Radiative decays of heavy quarkonia from ψ or Υ families into pair of identical vector mesons composed of light quarks are of interest as possible sources of glueballs. The known example is the decay $J/\psi \to \gamma\phi\phi$ recently studied by BESIII Collab. [1]. The partial wave analysis was fulfilled aimed at determining the properties of the resonance decaying into $\phi\phi$ pair [1]. The $\phi\phi$ state produced in πN collisions was considered as the signal of the glueball production [2]. Spin-parity of the $X \to \phi\phi$ resonances are reported to be $J^P = 0^+, 0^-, 2^+$ [1, 3]. However, the unit spin cannot be excluded for all states with masses $m_X < m_{\eta_c}$ because the $\gamma\gamma$ decay mode ruling out the state $J = 1$ is seen only in case of the resonance $f_2(2300)$. Note that the assignment $J^{PC} = 1^{-+}$ forbidden in the $q\bar{q}$ model is explicitly exotic while the $J^{PC} = 1^{++}$ is allowed in this model. So, establishing the parity of resonances with given spin is crucial for revealing their nature.

The $\phi\phi$ mode could be useful for the determination of the parity of resonances decaying to the above final state by using the analog of the Yang test [4] by measuring the distribution over the angle between the decay planes spanned by the momenta of K^+, K^- mesons from the decay $\phi \to K^+K^-$ [5]. However, in the Yang test, the decay planes of e^+e^- pairs from the decay chain $\pi^0 \to \gamma\gamma$, $\gamma \to e^+e^-$, are clearly distinguishable while ϕ mesons in the decay $X \to \phi\phi$ are relatively slow, hence there is strong interference between the amplitudes $M(X \to \phi\phi \to K^+_{p_1}K^-_{p_2}K^+_{p_3}K^-_{p_4})$ with the permutations of the kaon momenta $p_1 \leftrightarrow p_3$, $p_2 \leftrightarrow p_4$

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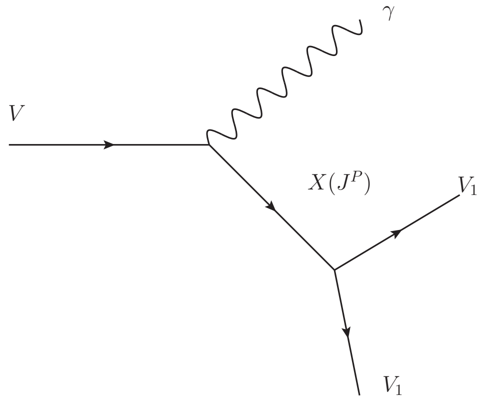

Figure 1. The dominant contribution to the $V \to \gamma V_1 V_1$ decay amplitude

so the K^+K^- decay plane cannot be attributed to the specific ϕ meson. Possible way to overcome this difficulty based on the idea to determine the parity of the $\phi\phi$ state by making the direction-polarization correlation measurements with the linearly polarized photons in the reaction $e^+e^- \to J/\psi \to \gamma X \to \gamma\phi\phi$ was suggested in Ref. [5].

In this work, we propose to use the photons with linear polarization in another context. For this purpose, we obtain the expressions for the spin structure of the $e^+e^- \to V \to \gamma X(J^P) \to \gamma V_1 V_1$ reaction amplitude ($V = J/\psi, \psi', ..., \Upsilon(1S), \Upsilon, ...$) and $V_1 = \rho, \omega, \phi, ...$) and apply them for finding the angular distributions of photons with the definite linear polarizations relative to the plane spanned by the momentum of initial electron and the total momentum of the V_1V_1 pair. It is shown that the sign of the asymmetry of the distributions of the photons polarized in the above plane and orthogonal to it correlates with the signature $\sigma = P(-1)^J$ of the X resonance with given spin-parity J^P so it may help to establish this quantum number in a way which does not depend on the specific model of the $X(J^P) \to V_1V_1$ transition amplitude. The obtained expressions for the above asymmetry are applied for evaluation of its possible magnitude using available data on the reaction $e^+e^- \to J/\psi \to \gamma\phi\phi$ recently obtained by BESIII Collaboration [1].

2. Amplitudes

Following Ref. [1] the $V \to \gamma V_1 V_1$ decay mechanism shown in Fig. 1 is assumed to be dominant. We start with the expressions for the effective invariant amplitudes $M_{V\to\gamma X(J^P)}$ and $M_{X(J^P)\to V_1V_1}$ for X resonances having spin-parities $J^P = 0^\pm, 1^\pm, 2^\pm$, with the kinematic Lorentz structures parameterized by some coupling constants and then derive the expressions for amplitudes $M^{(J^P)} \equiv M_{V\to\gamma X(J^P)\to\gamma V_1V_1}$ in terms of independent helicity amplitudes $M^{(J^P)}_{\lambda_V,\lambda_\gamma,\lambda_X}$ of the decay $V \to \gamma X(J^P)$ and in terms of the $X(J^P) \to V_1V_1$ decay amplitudes $f^{(J^P)}_{SL}$ with given spin S and orbital angular momentum L of the V_1V_1 state. The details of derivation are given elsewhere [6]. One has

$$
\begin{aligned}
M^{(0^+)} &= \frac{M^{(0^+)}_{1,1,0}(\boldsymbol{\xi e})}{D_{X(0^+)}(m^2_{12})}\left[f^{(0^+)}_{00}\delta_{ab} + f^{(0^+)}_{22} n_{1a}n_{1b}\right]\xi_{1a}\xi_{1b}, \\
M^{(0^-)} &= \frac{iM^{(0^-)}_{1,1,0} f^{(0^-)}_{11}}{D_{X(0^-)}(m^2_{12})}(\boldsymbol{\xi}[\boldsymbol{e}\times\boldsymbol{n}])e_{abc}n_{1c}\xi_{1a}\xi_{2b},
\end{aligned}
$$

$$
\begin{aligned}
M^{(1^+)} &= \frac{f_{22}^{(1^+)}}{D_{X(1^+)}(m_{12}^2)}\left\{M_{1,1,0}^{(1^+)}(\boldsymbol{\xi}[\boldsymbol{e}\times\boldsymbol{n}])n_c - M_{0,1,1}^{(1^+)}(\boldsymbol{\xi n})[\boldsymbol{e}\times\boldsymbol{n}]_c\right\}\times \\
&\quad (e_{cad}n_{1b}+e_{cbd}n_{1a})n_{1d}\xi_{1a}\xi_{1b}, \\
M^{(1^-)} &= \frac{f_{11}^{(1^-)}}{D_{X(1^-)}(m_{12}^2)}\left\{M_{1,1,0}^{(1^-)}(\boldsymbol{\xi e})[\boldsymbol{n}\times\boldsymbol{n}_1]_c - M_{0,1,1}^{(1^-)}(\boldsymbol{\xi n})[\boldsymbol{e}\times\boldsymbol{n}_1]_c\right\}e_{abc}\xi_{1a}\xi_{1b}, \\
M^{(2^+)} &= \left\{\frac{1}{2}\left(\sqrt{6}M_{1,1,0}^{(2^+)} - M_{-1,1,2}^{(2^+)}\right)(\boldsymbol{\xi}\cdot\boldsymbol{e})n_in_j - M_{-1,1,2}^{(2^+)}\xi_{\perp i}e_j - \sqrt{2}M_{0,1,1}^{(2^+)}(\boldsymbol{\xi}\cdot\boldsymbol{n})e_in_j\right\}\times \\
&\quad \left[f_{20}^{(2^+)}\delta_{ka}\delta_{lb} + f_{02}^{(2^+)}\delta_{ab}n_{1k}n_{1l} + f_{22}^{(2^+)}(n_{1a}\delta_{kb}+n_{1b}\delta_{ka})n_{1l} + f_{24}^{(2^+)}n_{1a}n_{1b}n_{1k}n_{1l}\right]\times \\
&\quad \frac{\Pi_{ij,kl}\xi_{1a}\xi_{2b}}{D_{X(2^+)}(m_{12}^2)}, \\
M^{(2^-)} &= i\left\{\sqrt{2}M_{0,1,1}^{(2^-)}(\boldsymbol{\xi n})[\boldsymbol{n}\times\boldsymbol{e}]_in_j + \sqrt{\frac{3}{2}}M_{1,1,0}^{(2^-)}(\boldsymbol{n}[\boldsymbol{\xi}_\perp\times\boldsymbol{e}])n_in_j + \frac{1}{2}M_{-1,1,2}^{(2^-)}(\xi_{\perp i}[\boldsymbol{n}\times\boldsymbol{e}]_j + \right. \\
&\quad \left.[\boldsymbol{n}\times\boldsymbol{\xi}]_ie_j)\right\}\left(f_{11}^{(2^-)}e_{kab} + f_{13}^{(2^-)}e_{abc}n_{1c}n_{1k}\right)\frac{\Pi_{ij,kl}n_{1l}}{D_{X(2^+)}(m_{12}^2)}\times\xi_{1a}\xi_{2b}, \qquad (1)
\end{aligned}
$$

where $\boldsymbol{\xi}$, $\boldsymbol{\xi}_{1,2}$, are the polarization three-vectors of initial V, final V_1, V_1 resonances in their rest frames, and $\boldsymbol{e}$ is the polarization three-vector of the photon; $\boldsymbol{\xi}_\perp = \boldsymbol{\xi} - \boldsymbol{n}(\boldsymbol{\xi n})$. $\boldsymbol{n}$ is the unit vector in the photon direction of motion in the V rest frame while $\boldsymbol{n}_1$ is analogous quantity for one of the final V_1 mesons in their center-of-mass system. The quantity $D_{X(J^P)}(m_{12}^2)$ in these expressions where m_{12} stands for the invariant mass of the V_1V_1 state is the inverse propagator of the $X(J^P)$ resonance,

$$
D_{X(J^P)}(m_{12}^2) = m_{X(J^P)}^2 - m_{12}^2 - im_{12}[\Gamma_{X(J^P)\to V_1V_1}(m_{12}) + \sum\Gamma_{X(J^P)\to \mathrm{non}V_1V_1}(m_{12})].
$$

Using Eq. (1), one can write the full amplitude as the sum of these expressions and study the coherence properties of these contributions to the $V\to\gamma V_1V_1$ distribution over m_{12} and photon solid angle. The direct calculation of the final probability distribution shows that, when summed over polarizations of the final V_1 mesons but keeping fixed their direction of motion, the vanishing are all interference terms with opposite space parities. The nonvanishing are (0^+-2^+), (0^--2^-), (1^--2^-), and (1^+-2^+). Among them, the (0^+-2^+) and (0^--2^-) interference terms are proportional to $n_{1k}n_{1l}-\delta_{kl}/3$. They vanish after integration over $\boldsymbol{n}_1$ because of relation $\langle n_{1i}n_{1j}\rangle = \frac{1}{3}\delta_{ij}$. The (1^+-2^+) interference term, after summation over V_1 meson polarizations, is proportional to $(\boldsymbol{e}\cdot\boldsymbol{n}_1)n_kn_{1l}-(\boldsymbol{n}\cdot\boldsymbol{n}_1)e_kn_{1l}$ which, after integration over $\boldsymbol{n}_1$ reduces to $e_kn_l-e_nn_k$ and also vanishes after multiplication by the tensor $\Pi_{ij,kl}$ which is symmetric in both pairs of indices, ij and kl. The (1^--2^-) interference term contains two structures, $[\boldsymbol{n}\times\boldsymbol{n}_1]_kn_{1l}$ and $[\boldsymbol{e}\times\boldsymbol{n}_1]_kn_{1l}$. After integration over $\boldsymbol{n}_1$ they reduce to, respectively, $\epsilon_{akl}n_a$ and $\epsilon_{akl}e_a$ and vanish being multiplied by the tensor $\Pi_{ij,kl}$ symmetric in kl. So, after summation over polarizations and integration over solid angle of V_1 mesons, the probability distribution $d\Gamma/d\Omega_\gamma dm_{12}$ is represented as incoherent sum of contributions with the different quantum numbers.

3. Angular distributions for linearly polarized photons

Angular distribution of photons,

$$
\frac{d\Gamma_{V\to\gamma X\to\gamma V_1V_1}}{d\Omega_\gamma} = \frac{1}{\pi}\int_{4m_{V_1}^2}^{m_V^2}\frac{m_{12}\Gamma_{X\to V_1V_1}(m_{12}^2)}{|D_X(m_{12}^2)|^2}\frac{d\Gamma_{V\to\gamma X}}{d\Omega_\gamma}dm_{12}^2 \equiv \left\langle\frac{d\Gamma_{V\to\gamma X}}{d\Omega_\gamma}\right\rangle, \qquad (2)
$$

is expressed through the average of the modulus square of independent $V \to \gamma X$ helicity amplitudes,

$$a^{(J^P)}_{\lambda_V,\lambda_\gamma,\lambda_X} \equiv \frac{1}{\pi}\int_{4m^2_{V_1}}^{m^2_V} \frac{m_{12}\Gamma_{X(J^P)\to V_1V_1}(m^2_{12})}{|D_{X(J^P)}(m^2_{12})|^2}\left|M^{(J^P)}_{\lambda_V,\lambda_\gamma,\lambda_X}\right|^2 |\boldsymbol{k}|dm^2_{12}, \tag{3}$$

$D_{X(J^P)}(m^2_{12})$ is the inverse propagator of the X resonance, m_{12} is invariant mass of the V_1V_1 state. Specification of coordinate system in the reaction $e^+e^- \to V \to \gamma V_1V_1$ is as follows. Electrons move along z axis, the plane spanned by the momenta $(\boldsymbol{p}_{e^-},\boldsymbol{k})$ is the xz plane, so that the photon direction is $\boldsymbol{n} = \boldsymbol{k}/|\boldsymbol{k}| = (\sin\theta_\gamma, 0, \cos\theta_\gamma)$. The vectors of two linear polarization states in this system are chosen in the form $\boldsymbol{e}_1 = (0,1,0)$, $\boldsymbol{e}_2 = (-\cos\theta_\gamma, 0, \sin\theta_\gamma)$ so that $\boldsymbol{e}_1$ is orthogonal to the $(\boldsymbol{p}_{e^-},\boldsymbol{k})$ plane, $\boldsymbol{e}_2$ lies in this plane. Since V meson is produced in e^+e^- annihilation, the summation rule over its polarizations is $\sum_{\lambda_V=\pm1}\xi_i^{(\lambda_V)}\xi_j^{(\lambda_V)} = \delta_{ij}-\delta_{i3}\delta_{j3}$. Angular distributions

$$\frac{d\Gamma^{(J^P)}}{d\Omega_\gamma} \equiv \frac{d\Gamma_{V\to\gamma X(J^P)\to\gamma V_1V_1}}{d\Omega_\gamma}$$

normalized to partial widths look as follows:

$$\begin{aligned}
\frac{d\Gamma^{(0^+)}}{d\Omega_\gamma} &= Ba^{(0^+)}_{1,1,0}\left\{\begin{array}{c} 1,\ \boldsymbol{e}=\boldsymbol{e}_1,\\ \cos^2\theta_\gamma,\ \boldsymbol{e}=\boldsymbol{e}_2. \end{array}\right. ,\\
\frac{d\Gamma^{(0^-)}}{d\Omega_\gamma} &= Ba^{(0^-)}_{1,1,0}\left\{\begin{array}{c} \cos^2\theta_\gamma,\ \boldsymbol{e}=\boldsymbol{e}_1,\\ 1,\ \boldsymbol{e}=\boldsymbol{e}_2. \end{array}\right. ,\\
\frac{d\Gamma^{(1^+)}}{d\Omega_\gamma} &= B\left\{\begin{array}{c} a^{(1^+)}_{1,1,0}\cos^2\theta_\gamma + a^{(1^+)}_{0,1,1}\sin^2\theta_\gamma,\ \boldsymbol{e}=\boldsymbol{e}_1,\\ a^{(1^+)}_{1,1,0} + a^{(1^+)}_{0,1,1}\sin^2\theta_\gamma,\ \boldsymbol{e}=\boldsymbol{e}_2. \end{array}\right. ,\\
\frac{d\Gamma^{(1^-)}}{d\Omega_\gamma} &= B\left\{\begin{array}{c} a^{(1^-)}_{1,1,0} + a^{(1^-)}_{0,1,1}\sin^2\theta_\gamma,\ \boldsymbol{e}=\boldsymbol{e}_1,\\ a^{(1^-)}_{1,1,0}\cos^2\theta_\gamma + a^{(1^-)}_{0,1,1}\sin^2\theta_\gamma,\ \boldsymbol{e}=\boldsymbol{e}_2. \end{array}\right. ,\\
\frac{d\Gamma^{(2^+)}}{d\Omega_\gamma} &= B\left[a^{(2^+)}_{0,1,1}\sin^2\theta_\gamma + \frac{1}{2}a^{(2^+)}_{-1,1,2}(1+\cos^2\theta_\gamma) + a^{(2^+)}_{1,1,0}\times\left\{\begin{array}{c} 1,\ \boldsymbol{e}=\boldsymbol{e}_1,\\ \cos^2\theta_\gamma,\ \boldsymbol{e}=\boldsymbol{e}_2 \end{array}\right.\right],\\
\frac{d\Gamma^{(2^-)}_1}{d\Omega_\gamma} &= B\left[a^{(2^-)}_{0,1,1}\sin^2\theta_\gamma + \left(a^{(2^-)}_{1,1,0} + \frac{1}{4}a^{(2^-)}_{-1,1,2}\right)\cos^2\theta_\gamma + \frac{1}{4}a^{(2^-)}_{-1,1,2}(2+\cos^2\theta_\gamma)\right],\\
\frac{d\Gamma^{(2^-)}_2}{d\Omega_\gamma} &= B\left[a^{(2^-)}_{0,1,1}\sin^2\theta_\gamma + a^{(2^-)}_{1,1,0} + \frac{1}{4}a^{(2^-)}_{-1,1,2} + \frac{1}{4}a^{(2^-)}_{-1,1,2}(2+\cos^2\theta_\gamma-\sin^2\theta_\gamma)\right];
\end{aligned} \tag{4}$$

$B = 1/(8\pi m_V)^2$. Asymmetry in the angular distributions of the linearly polarized photons in the reaction $e^+e^- \to V \to \gamma X(J^P) \to \gamma V_1V_1$

$$\begin{aligned}
A^{(J^P)}(\theta_\gamma) &= \left(\left\langle\frac{d\Gamma^{(1)}_{V\to\gamma X(J^P)}}{d\Omega_\gamma}\right\rangle - \left\langle\frac{d\Gamma^{(2)}_{V\to\gamma X(J^P)}}{d\Omega_\gamma}\right\rangle\right)\times\\
&\quad\left(\left\langle\frac{d\Gamma^{(1)}_{V\to\gamma X(J^P)}}{d\Omega_\gamma}\right\rangle + \left\langle\frac{d\Gamma^{(2)}_{V\to\gamma X(J^P)}}{d\Omega_\gamma}\right\rangle\right)^{-1}
\end{aligned} \tag{5}$$

is calculated to be

$$A^{(0^\pm)}(\theta_\gamma) = \pm\frac{\sin^2\theta_\gamma}{1+\cos^2\theta_\gamma},$$

$$
\begin{aligned}
A^{(1^\mp)}(\theta_\gamma) &= \pm a^{(1^\mp)}_{1,1,0}\sin^2\theta_\gamma\left[a^{(1^\mp)}_{1,1,0}(1+\cos^2\theta_\gamma)+2a^{(1^\mp)}_{0,1,1}\sin^2\theta_\gamma\right]^{-1},\\
A^{(2^\pm)}(\theta_\gamma) &= \pm a^{(2^\pm)}_{1,1,0}\sin^2\theta_\gamma\left[\left(a^{(2^\pm)}_{1,1,0}+a^{(2^\pm)}_{-1,1,2}\right)(1+\cos^2\theta_\gamma)+2a^{(2^\pm)}_{0,1,1}\sin^2\theta_\gamma\right]^{-1}. \qquad (6)
\end{aligned}
$$

One can see that the form of expressions for equal spins J but different signatures $\sigma = P(-1)^J$ looks the same, but the sign of asymmetry coincides with the signature of the resonance X. All dynamical details of the $X(J^P) \to V_1V_1$ transition are encoded in the quantities $a^{(J^P)}_{\lambda_V,\lambda_\gamma,\lambda_X}$ (3).

4. Asymmetry estimates using available data

Let us estimate the possible magnitude of asymmetry using the recent data [1] on disentangling the $X(J^P)$ resonance contributions to the $e^+e^- \to J/\psi \to \gamma X(J^P) \to \gamma\phi\phi$ reaction amplitude. The authors of Ref. [1] restricted their analysis by the resonances with quantum numbers $0^\pm$ and 2^+. In case of the spin zero resonances the asymmetry shown with the dotted and dash-dotted lines in Fig. 2 does not depend on the dynamical details. Further treatment requires the knowledge of the magnitudes of coupling constants parameterizing invariant amplitudes. They can be found in Ref. [6]. Since the 0^+ and 0^- contributions do not interfere, the asymmetry due to the combined spin zero component is

$$
A^{(0^+ + 0^-)}(\theta_\gamma) = \frac{\sin^2\theta_\gamma}{1+\cos^2\theta_\gamma}\times\frac{N^{0^+}-N^{0^-}}{N^{0^+}+N^{0^-}} = -\frac{0.84\sin^2\theta_\gamma}{1+\cos^2\theta_\gamma}. \qquad (7)
$$

Hereafter $N^{(0^+)} = 63$ and $N^{(0^-)} = 708$ are the central values of the relative production rates of the scalar and pseudoscalar components of the $\phi\phi$ mass spectrum found in Ref. [6] when fitting the data Ref. [1]. The above estimates were obtained in case of pure contributions with given spin. One can go further and evaluate the asymmetry in case when the contribution of the sum of the $X(0^-)$, $X(0^+)$, $X(2^+)$ resonances is taken into account. One obtains

$$
\begin{aligned}
A(\theta_\gamma) = {}& \sin^2\theta_\gamma\left(N^{(0^+)}-N^{(0^-)}+\frac{\mathcal{N}a^{(2^+)}_{1,1,0}}{12\pi m^2_{J/\psi}}\right)\Bigg\{(1+\cos^2\theta_\gamma)\Bigg[N^{(0^+)}+N^{(0^-)}+\\
& \frac{\mathcal{N}}{12\pi m^2_{J/\psi}}\left(a^{(2^+)}_{1,1,0}+a^{(2^+)}_{-1,1,2}\right)\Bigg]+\frac{\mathcal{N}a^{(2^+)}_{0,1,1}}{6\pi m^2_{J/\psi}}\sin^2\theta_\gamma\Bigg\}^{-1}. \qquad (8)
\end{aligned}
$$

The factor $\mathcal{N}$ in the above expression originates from the fact that only relative production rates $\Gamma_{J/\psi\to\gamma X(J^P)\to\gamma\phi\phi}$ found in Ref. [6] can be extracted from the fits of the BESIII data [1]. Numerical values of parameters necessary for evaluation of the quantities $a^{(2^+)}_{\lambda_{J/\psi},\lambda_\gamma,\lambda_X}$ can be found in Ref. [6].

5. Discussion and conclusion

The question of determining parity of the X resonances with masses below m_{η_c} produced in the decay chain $J/\psi \to \gamma X \to \gamma\phi\phi$ is crucial for establishing their nature [5]. As is shown here, the angular distributions of the photons polarized in the plane spanned by the momenta of the electron and photon in the reaction $e^+e^- \to J/\psi \to \gamma X \to \gamma\phi\phi$ and perpendicular to this plane are different. The signs of asymmetry are model independent and coincide with the signatures $\sigma_X = P_X(-1)^{J_X}$ of the resonance X. The same refers to a wide class of the processes of the type $e^+e^- \to V \to \gamma V_1V_1$.

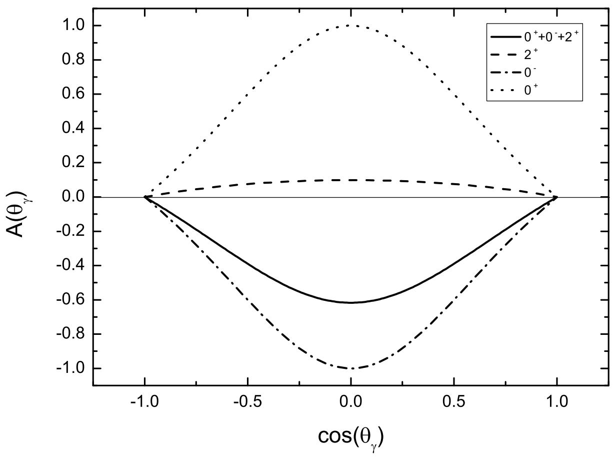

Figure 2. Asymmetry estimated using the data on the reaction $e^+e^- \to J/\psi \to \gamma\phi\phi$ [1]

The method of measurement of the linear polarization of photons by the e^+e^- pair production in the field of heavy nuclei was proposed in Refs. [7, 8], however, with contradictory conclusions. The contradiction was resolved and full theoretical treatment of the method was given in Refs. [9, 10] where it was shown that the electrons prefer to be emitted in the photon polarization plane. In the context of the present work, it seems to be not impossible to detect the e^+e^- pairs which are externally produced by the photons from the decay $V \to \gamma V_1 V_1$. The preferred emission of electrons in the direction orthogonal (parallel) to the plane spanned by the vectors $(\boldsymbol{p}_{e^-}, \boldsymbol{k})$ would point to the linear polarization $\boldsymbol{e}_1$ $(\boldsymbol{e}_2)$, respectively.

All details like expressions of the helicity amplitudes through invariant ones, propagators, partial widths $X(J^P) \to V_1 V_1$ etc. can found in Ref. [11] upon evident replacements.

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