

# Search for spiral structures in heavy ion physics and astrometry

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**Abstract.** The appearance of helical structures in astrophysics and heavy-ion collisions (HIC) physics is discussed and the interplay between these two branches of physics is investigated. As far as in HIC the data are available in momentum space only, the assumption that the Milky Way has a spiral structure in the velocity space is checked. We find the spiral structures in phase and coordinate spaces from the rotation curve of our Galaxy include contribution of different parts of Galaxy, as thin disk, bulge and spherically symmetric Dark Matter Halo (Navarro-Frenk-White density profile). The helical structures in coordinate and velocity space are found and compared. An algorithm for searching helical structures has been developed and tested on logarithmic spiral with some random background.

## 1. Introduction

The structure of spiral galaxies is known to be one of the major astrophysical problems. Interest in this issue is caused also by the fact that spiral structures can appear as a result of the heavy-ion collisions, when they can be observed only in momentum (phase) space. This is the major difference from the galactic scale, where only the structures in a coordinate space can be observed and, in principle, compared with a phase space.

In the paper [1] self-similar discrete structures in particle physics are discussed and a parallel is drawn with a logarithmic spiral, as an example of such a structure, which can manifest itself in particular in what the galaxy looks like in phase space.

In the paper [2] is discussed application of the method, based on the use of the Fourier expansion of azimuthal distributions of produced particles in Heavy Ion Physics for astrometric data, obtained by GAIA mission. Gaia is a mission to chart a three-dimensional map the Milky Way and the Local Group, in the process revealing the composition, formation and evolution of the Galaxy.

One of the motivations is the similarity of structures in heavy ion physics and in galaxy physics. One example is the emergence of the so-called "small galaxies" - vortex layers formed as a result of collisions of heavy ions [3].

## 2. Building the hodograph of the Milky Way in coordinate and phase spaces

To consider the galaxy in the momentum space, we construct a hodograph based on Milky Way rotation curve [4, 5]. This requires the angular velocity of rotation of the velocity vector, which can be obtained by constructing the inverse function  $R(V)$ . If we know this function, the



angular velocity of rotation of the velocity vector is determined by  $\frac{v}{R(v)}$ .  $\phi$  is the initial phase. The components of the hodograph in phase space:

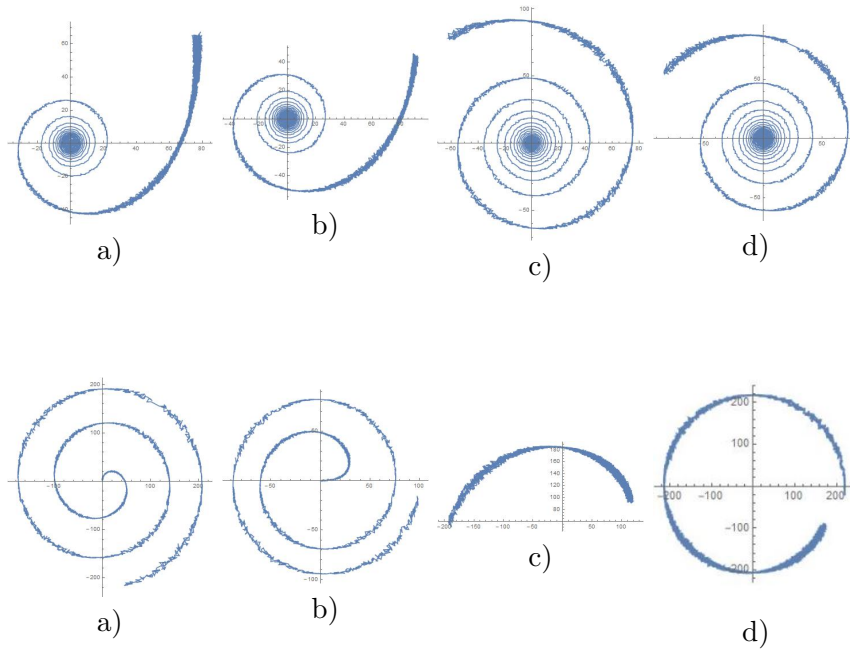
$$\xi[v] = v \cos\left(\frac{v * T}{R(v)} + \phi\right); \quad \eta[v] = v \sin\left(\frac{v * T}{R(v)} + \phi\right). \quad (1)$$

The components of hodograph in a coordinate space is

$$\xi[v] = v \cos\left(\frac{V(r) * T}{r} + \phi\right); \quad \eta[v] = v \sin\left(\frac{V(r) * T}{r} + \phi\right). \quad (2)$$

The functions  $R(v)$  for the rotation curve of the galaxy, the contribution of dark matter and a flat disk in the galaxy rotation curve were obtained by fitting the rotation curve of the galaxy and its individual components. The inverse function is uniquely determined not for all values of  $v$ . Therefore the function  $R(v)$  is obtained for certain intervals of  $v$ .

Suppose that the Milky Way galaxy rotates clockwise, which can always be achieved by choosing the direction from which we observe it. Then you can see that spiral hodographs in coordinate and phase space twist in different ways.



**Figure 1.** Hodographs of the component of RC and RC of Milky Way in a coordinate (up) and phase(down) spaces: a) Bulge, b) Stellar disk, c) Dark matter, d) Rotation Curve

### 3. Building an algorithm for finding spirals structures

In this part we propose to search for a violation of mirror symmetry on a plane using the statistical method, which is a special case of momentum correlation in high energy physics, the so-called handedness [6, 7]. This method has been applied to the description of heavy ion collisions. In handedness usually left and right triples of vectors are used:

$$H = \frac{\sum(N_L - N_R)}{\sum(N_L + N_R)}. \quad (3)$$

We will consider two-dimensional vectors, which in heavy-ion physics will correspond to their transverse components being the projection of momenta onto a plane perpendicular to the direction of the colliding beams. Such a choice also corresponds to the standard definition of flows in heavy ion physics and may be compared to the earlier handedness analysis in HIC [8].

To search for spiral structures on a selected plane we use an array of complex numbers, where the real part is the coordinate along the X-axis, while the imaginary one - along the Y-axis. To verify the existence of a spiral, we consider the following criterion: if a spiral structure exists, then when turning from a larger to a smaller modulus of a complex number (or vice versa) the angle will be of a definite sign. Necessity to fix the length of the vector distinguishes our situation from considering flows where the length the vector isn't important, but only the azimuthal angle is.

Consider some monotonic function of the angle  $S$ , then a constant counterclockwise rotation (positive angle) and anticlockwise rotation (negative angle) will be match:

$$s = \frac{\sum S_{ij}}{\sum |S_{ij}|} = 1; \quad (4)$$

$$s = \frac{\sum S_{ij}}{\sum |S_{ij}|} = -1; \quad (5)$$

Moreover, the two considered vectors will form the left or right triple together with a normal to the plane. In case of the presence of elements of differently twisted spirals the value will be  $-1 \leq s \leq 1$ .

We pass to the specific definition of  $S_{ij}$ . Consider all pairs of complex numbers  $Z_i$  and  $Z_j$ . and the product of  $Z_i$  by the conjugate  $Z_j^*$ . Its imaginary part corresponds to the phase of the complex number according to Euler's formula:

$$e^{i\phi} = \cos(\phi) + i * \sin(\phi) \quad (6)$$

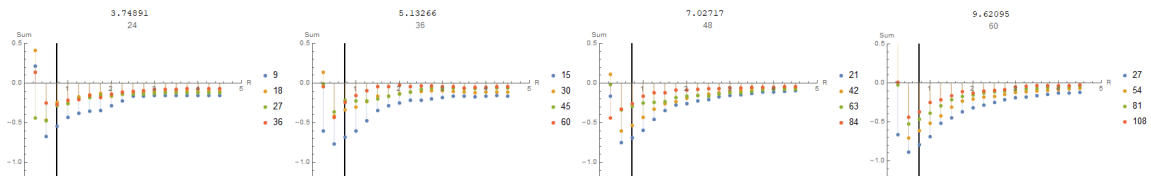
From the expansion in the Taylor series it follows that for small  $\phi$   $\sin(\phi) \approx \phi$  and it is especially clear that the sine sign uniquely determines in which direction the rotation occurs. Thus, as a function for checking helicity, we can use:

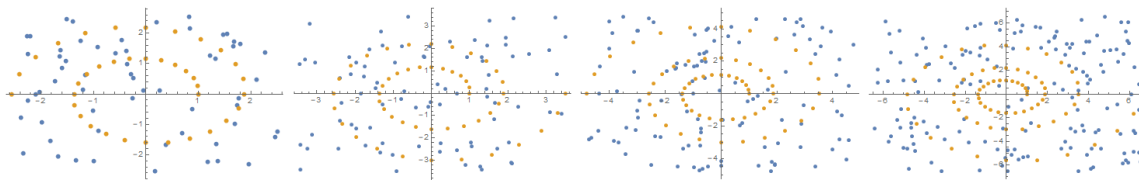
$$S(i, j) = \frac{\text{Im}(Z_i \times Z_j^*)}{|Z_i||Z_j|} \times \Theta(|Z_j| - |Z_i|) \quad (7)$$

To account for each pair only once with a correct sign we use a  $\theta$ -function. To search for spirals with several turns, the following function is suggested:

$$S(i, j) = \frac{\text{Im}(Z_i \times Z_j^*)}{|Z_i||Z_j|} \times \Theta(|Z_j| - |Z_i|) \times \Theta(R - |Z_j - Z_i|) \quad (8)$$

The second  $\theta$ -function is responsible for comparing only those complex numbers that are on the same turn of a spiral. R is the maximum distance between points in pair in one coil.





**Figure 2.** Dependence  $s = \frac{\sum S_{ij}}{\sum |S_{ij}|}$  from the value of  $R$ , which determines the proximity points under consideration for test spirals and random background. In the figures above the are average distance between points on the spiral and number of points are indicated. The number of background points is indicated on the right of each figure.

#### 4. Discussion and Conclusions

The rotation curve of the Milky Way galaxy and the components contributing to this curve for setting initial conditions with moderate random perturbations lead to the formation of spiral structures. Structures in the coordinate space and velocity space are different in the direction of concavity of the spiral. Needless to say, that our model does not pretend to explain the real origins of galaxies spiral structure.

A statistical algorithm for the search for spiral structures is proposed, which is a sort of P-odd momentum correlation (handedness). Procedure is checked for logarithmic spirals on a random background. When choosing a sufficiently small distance between points in a pair, the calculated parameter indicates the existence of a spiral structure. Received the results do not yet allow a clear separation of the signal of the spiral structure and background. This problem appears also in the study of handedness in real experimental data [9] - [12]. In the future, to improve the separation of signals from the background, it is planned to changing improved random number generation methods used in high physics energies.

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#### References

- [1] Rustamov A and Rustamov J N 2016 On the Spiral Structures in Heavy-Ion Collisions *Preprint* arXiv:1602.01812 [hep-ph]
- [2] Zavada P. and Pka . 2017 *J. Phys. Conf. Ser.* **938** 012037
- [3] Baznat M I, Gudima K K, Sorin A S and Teryaev O V 2016 *Phys. Rev. C* **93** 031902
- [4] Pijushpani Bhattacharjee, Soumini Chaudhury, and Susmita Kundu 2014 Rotation Curve of the Milky Way out to 200kpc *Preprint* arxiv:1310.2659v3 [Astro-ph.GA]
- [5] Julio F.Navarro, Carlos S.Frenk, Simon D.M.White 1995 The Structure of Cold Dark Matter Halos *Preprint* astro-ph/9508025
- [6] Efremov A V, Mankiewicz L. and Tornqvist N A 1992 *Phys. Lett. B* **291** 473
- [7] Efremov A V, Mankiewicz L and Tornqvist N A 1992 *Phys. Lett. B* **284** 394
- [8] Teryaev O and Usubov R 2015 *Phys. Rev. C* **92** 014906
- [9] Efremov A V, Ivanshin Y I, Tkachev L G and Zulkarneev R Y 1998 *High energy spin physics* (Protvino) p 579
- [10] Efremov A V, Ivanshin Y I, Tkachev L G and Zulkarneev R Y 1997 *Deep inelastic scattering off polarized targets, Physics with polarized protons at HERA* (Hamburg/Zeuthen)p. 181
- [11] Efremov A, Potashnikova I and Tkachev L 1995 Jet handedness correlation in hadronic  $Z_0$  decays, *Preprint* JINR-E1-95-417
- [12] Efremov A and Tkatchev L 1995 *AIP Conf. Proc.* **343** 821