Unpolarized and spin-dependent DIS structure functions in Double-Logarithmic Approximation

B I Ermolaev[1]**, S I Troyan**[2]

[1] Ioffe Physico-Technical Institute, RAS, 194021 St.Petersburg, Russia
[2] St.Petersburg Institute of Nuclear Physics, NRC KI, 188300 Gatchina, Russia

E-mail: [1] `boris.ermolaev@cern.ch`

Abstract. We demonstrate how to calculate perturbative components of the structure functions F_1 (for unpolarized DIS) and g_1 (spin-dependent DIS) in Double-Logarithmic Approximation, studying separately the cases of fixed and running QCD coupling. We show that as long as only ladder graphs are accounted for (throughout the talk we use the Feynman gauge for virtual gluons) there is no difference at all between F_1 and g_1. However, accounting for contributions of non-ladder graphs brings an essential difference between them. Applying Saddle-Point method to expressions for F_1 and g_1, we obtain their small-x asymptotics. The both asymptotics are of the Regge kind but with different intercepts. The intercept of F_1 proved to be greater than unity, so it is a new contribution to Pomeron. We show the reason for the g_1 intercept should be less than the one of F_1 and thereby argue against using model Pomerons in the spin-dependent processes. Finally, we discuss the applicability region of Regge asymptotics.

1. Introduction

According to QCD factorization, structure functions describing Deeply Inelastic Scattering can be regarded as convolutions of perturbative and non-perturbative contributions. In the present talk we consider the perturbative components only. In particular, we consider perturbative components of the DIS structure functions F_1 and g_1. They can be calculated in fixed orders in the QCD coupling or, alternatively, with total resummation of contributions to all orders in in α_s. The latter can be done only approximately, with selecting most important contributions in every order in α_s and constructing evolution equations accounting for such contributions. The most important contributions are different in different kinematics. Selection of them is expressed in terms of orderings of momenta of virtual partons. They look very simple when the standard Sudakov parametrization[1] is used for momenta $k_i (i = 1, 2, ...)$ of virtual partons:

$$k_i = \alpha_i q' + \beta_i p\prime + k_{i\perp}, \tag{1}$$

where q' and p' are light-cone momenta, each made of the photon momentum q and the initial parton momentum p while $k_{i\perp}$ are the components transversal to the plane formed by p and q. The invariant energy $w = 2pq$ is presumed to be the largest invariant involved. First of all, there is the DGLAP ordering:

$$\beta_1 \sim \beta_2 \sim ... \sim 1, \quad \mu^2 \ll k_{1\perp}^2 \ll k_{2\perp}^2 \ll ... \ll Q^2, \tag{2}$$

where $-Q^2 = q^2$ and μ^2 is a mass scale. For instance, it can be the factorization scale. We keep the standard DGLAP numeration of partonic ladder rungs from the bottom to the top. This ordering means that the DGLAP equations[2, 3, 4, 5] sum logarithms of Q^2 to all orders in α_s and do not account for logs of x. As a result, DGLAP is designed for work in kinematics $x \sim 1$ and $Q^2 \gg \mu^2$ for reactions with both unpolarized and polarized partons, so it describes both F_1 and g_1 at large x. In contrast, the BFKL ordering is

$$1 \gg \beta_1 \gg \beta_2 \gg ..., \quad \mu^2 \sim k_{1\perp}^2 \sim k_{2\perp}^2 \sim .... \tag{3}$$

So, the BFKL equation[6, 7, 8, 9, 10] accounts for logs of x. and does dot deal with logarithms of Q^2. As a result, BFKL is tailored for work in kinematic region of small x and small Q^2. It sums leading logarithms. They contribute to unpolarized processes only, so BFKL contributes to description of F_1 but not g_1. The perturbative series for F_1 in Leading Logarithmic Approximation (LLA) looks as follows:

$$F_1^{LL} = \delta(x - 1) + (1/x) \left[1 + c_1 \alpha_s \ln(1/x) + c_2 (\alpha_s \ln(1/x))^2 + ... \right], \tag{4}$$

where c_r are numerical factors. Alternatively, both logs of Q^2 and logs of x are accounted for when the Double-Logarithmic Ordering[11, 12, 13] is used:

$$1 \gg \beta_1 \gg \beta_2 \gg ..., \quad \mu^2 \sim k_{1\perp}^2/\beta_1 \ll k_{2\perp}^2/\beta_2 \ll .... \tag{5}$$

This ordering makes possible to account for logs of x and Q^2 in Double-Logarithmic Approximation (DLA) for both unpolarized and spin-dependent processes and therefore both F_1 and g_1 can be calculated in DLA. The DL perturbative series for both g_1 and F_1 looks as follows:

$$\begin{aligned}
F_1^{DL} &= \delta(x - 1) + c_1' \alpha_s \ln^2(1/x) + c_2' (\alpha_s \ln^2(1/x))^2 + ... , \\
g_1^{DL} &= \delta(x - 1) + \tilde{c}_1 \alpha_s \ln^2(1/x) + \tilde{c}_2 (\alpha_s \ln^2(1/x))^2 + ... ,
\end{aligned} \tag{6}$$

where c_r' and $\tilde{c}_r$ are numerical factors. The overall factor $1/x$ in Eq. (4) is huge at small x, so the DL contribution F_1^{DL} of Eq. (6) looks negligibly small compared to F_1^{LL}. In the present talk we demonstrate that this impression is false.

2. Calculating the structure functions F_1 and g_1 in DLA

In order to calculate F_1 and g_1 in DLA F_1 and g_1 in DLA we construct and solve Infra-red Evolution Equations (IREEs). This method was suggested by L.N. Lipatov. The basic idea is to introduce an infra-red cut-off μ in the transverse momentum space and trace evolution with respect to μ. The key-stone idea here is factorization of DL contributions of partons with minimal transverse momenta, which was proved by V.N. Gribov in the QED context. History and details of application of the method to DIS can be found in Ref. [14]. It is convenient to begin with calculating amplitudes of elastic Compton scattering off a quark and a gluon, which we denote A_q and A_g respectively, and obtain $F_1^{(q,g)}$ and $g_1^{(q,g)}$ from them with Optical theorem:

$$\begin{aligned}
F_1^q &= \frac{1}{2\pi} \Im A_q^{(+)}, &\quad F_1^g &= \frac{1}{2\pi} \Im A_g^{(+)}, \\
g_1^q &= \frac{1}{2\pi} \Im A_q^{(-)}, &\quad g_1^g &= \frac{1}{2\pi} \Im A_g^{(-)},
\end{aligned} \tag{7}$$

where the signature amplitudes $A_q^{(\pm)}$ and $A_g^{(\pm)}$ defined as follows:

$$A_q^{(\pm)}(w, Q^2) = A_q(w, Q^2) \pm A_q(-w, Q^2), \quad A_g^{(\pm)}(w, Q^2) = A_g(w, Q^2) \pm A_g(-w, Q^2). \quad (8)$$

It is convenient to express A_q and A_g through the Mellin transform:

$$A_{q,g}^{(\pm)}(\frac{w}{\mu^2}, \frac{Q^2}{\mu^2}) = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} \left(\frac{w}{\mu^2}\right)^{\omega} \xi^{(\pm)}(\omega) F_{q,g}^{(\pm)}(\omega, \frac{Q^2}{\mu^2}) = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} e^{\omega\rho} \xi^{(\pm)}(\omega) F_{q,g}^{(\pm)}(\omega, y), \quad (9)$$

where we have introduced the signature factor $\xi^{(\pm)}(\omega) = -\left(e^{-\imath\omega} \pm 1\right)/2$ and the logarithmic variables ρ, y (using the standard notation $w = 2pq$):

$$\rho = \ln(w/\mu^2), \quad y = \ln(Q^2/\mu^2). \quad (10)$$

In what follows we will address $F_q^{(\pm)}, F_g^{(\pm)}$ as Mellin amplitudes and will use the same form of the Mellin transform for other amplitudes as well. Constructing IREEs for the Compton amplitudes $F_q^{(\pm)}$ and $F_q^{(\pm)}$ is identical, so we keep generic notations F_q and F_g for them without the signature superscripts. Technology of composing and solving IREEs in detail can be found in Ref. [14, 15]. As the first step, we construct IREEs involving $F_{q,g}$ and auxiliary amplitudes $h_{rr'}$. They are related to the parton-parton amplitudes $f_{rr'}$: $h_{rr'} = \frac{1}{8\pi^2} f_{rr'}$, with $r, r' = q, g$. So, we obtain the following IREEs:

$$\begin{aligned}
[\partial/\partial y + \omega] F_q(\omega, y) &= F_q(\omega, y)h_{qq}(\omega) + F_g(\omega, y)h_{gq}(\omega), \\
[\partial/\partial y + \omega] F_g(\omega, y) &= F_q(\omega, y)h_{qg}(\omega) + F_g(\omega, y)h_{gg}(\omega),
\end{aligned} \quad (11)$$

Solving Eqs. (11), we express $F_{q,g}$ in terms of auxiliary amplitudes $h_{rr'}$ which can be found by the same method. Explicit expressions for them can be found in [14, 15]. Substituting them in solutions of Eqs. (11) allows us to arrive at explicit expressions for $F_{q,g}$ and then obtain F_1 and g_1 in DLA.

3. Small-x asymptotics of F_1 and g_1

Pushing $x \to 0$ and applying Saddle-Point method to the expressions for F_1 and g_1, we arrive at their small-x asymptotics. They both are of the Regge kind, though with different stationary points $\omega_0^{(\pm)}$:

$$g_1 \sim \frac{\Pi\left(\omega_0^{(-)}\right)}{\ln^{3/2}(1/x)} x^{-\omega_0^{(-)}} \left(\frac{Q^2}{\mu^2}\right)^{\omega_0^{(-)}/2}, \qquad F_1 \sim \frac{\Pi\left(\omega_0^{(+)}\right)}{\ln^{3/2}(1/x)} x^{-\omega_0^{(+)}} \left(\frac{Q^2}{\mu^2}\right)^{\omega_0^{(+)}/2}, \quad (12)$$

where we again introduced the signature notations $(\pm)$. Explicit expressions of the factors $\Pi(\omega_0^{(\pm)})$ depend on the type of QCD factorization (see Refs. [14, 15] for detail). In Regge theory, $\omega_0^{(\pm)}$ are called intercepts. They control the x-dependence of the structure functions. Intercept $\omega_0^{(-)}$ of g_1 was calculated in Ref. [16] and $\omega_0^{(+)}$ was obtained in Ref. [15]. When the running α_s effects are accounted for, the intercepts are:

$$\omega_0^{(-)} = 0.86, \qquad \omega_0^{(+)} = 1.07. \quad (13)$$

It is interesting to notice that the intercept $\omega_0^{(-)}$ is in good agreement with the result $\omega_0^{(-)} = 0.88 \pm 0.17$ obtained in Ref. [17] by extrapolating the HERA data to the region of

$x \to 0$. Intercept $\omega_0^{(+)} > 1$, so this Reggeon is a new contribution to Pomeron. Throughout the talk we will address it as DL Pomeron. Despite its value is pretty close to the NLO BFKL Pomeron intercept, DL and BFKL Pomerons have nothing in common: BFKL equation sums leading logarithms whereas IREEs of Eq. (11) deal with double logarithms.

Another interesting observation is that the intercepts $\omega_0^{(+)}$ and $\omega_0^{(-)}$ coincide as long as DL contributions of non-ladder Feynman graphs[1] are neglected. In this case $\omega_0^{(+)} = \omega_0^{(-)} = 1.25$. Non-ladder graphs contributions diminish them both but their impact on $\omega_0^{(-)}$ is greater than on $\omega_0^{(+)}$. Effect of difference in such impacts was first noticed in Ref. [18] in the QED context.

Eq.(12) manifests that Regge asymptotics of are represented by simple and elegant expressions in contrast to the parent amplitudes/structure functions. However, the asymptotics should be used within their applicability regions. Keeping a general notation F for F_1 and g_1 and denoting $\widetilde{F}$ their asymptotics, we introduce their ratio R as follows:

$$R(x, Q^2) = \widetilde{F}(x, Q^2)/F(x, Q^2). \tag{14}$$

Obviously, the asymptotics reliably represent their parent structure functions when $R \approx 1$. Let us fix $Q^2 = 10 \text{ GeV}^2$ and study the x-dependence of R. Numerical calculations yield that $R > 0.9$ at $x < x_{max}$, with

$$x_{max} = 10^{-6}. \tag{15}$$

Nevertheless, it is well-known that in practice the Regge asymptotics have been used at $x \gg x_{max}$. Doing so leads to artificial increase of the intercepts. Indeed, let us assume that the model Pomeron x^{-a} is used at $x = x_1 = 10^{-4}$. It is supposed to represent F and therefore $x_1^a \approx F$. On the other hand, Eq. (15) states $(x_{max})^{\omega_0^{(+)}}$. Equating these expressions, we arrive at

$$x_1^a \approx (x_{max})^{\omega_0^{(+)}}, \tag{16}$$

which leads to

$$a \approx \frac{3}{2}\, \omega_0^{(+)} \approx 1.6, \tag{17}$$

which means that the model Pomeron is hard. Applying the same reasoning to the spin-dependent Reggeon in Eq. (12), with the intercept $\omega_0^{(-)} < 1$, makes easy to arrive at a Reggeon with the "intercept"> 1 and obtain thereby a fictitious "spin-dependent Pomeron". Therefore, using the asymptotics outside their applicability region inevitably leads to introducing Pomeron(s), often hard ones, for both unpolarized and spin-dependent DIS, though without theoretical grounds.

4. Conclusion

In the present talk we have demonstrated how to calculate the structure functions F_1 and g_1 in DLA and how to calculate their small-x asymptotics. It turned out that the both asymptotics are of the Regge form but their intercepts are different. They coincide when only the ladder Feynman graphs are accounted for but impact of double logarithms from non-ladder graphs brings different contributions to these intercepts. As a result, the intercept $\omega_0^{(-)}$ of the g_1 asymptotics in Eq. (12) is less than unity while the intercept $\omega_0^{(+)}$ of F_1-asymptotics is a bit

---

[1] the terms "ladder" and "non-ladder" contributions are gauge-dependent. We use them in regard of the Feynman gauge.

greater than unity and therefore this Reggeon is a new contribution to Pomeron although it has nothing in common to the BFKL Pomeron.

We also fixed in Eq. (15) the maximal value of x where the small-x asymptotics of g_1 and F_1 can be used instead of the parent structure functions. It drives us to conclude that widespread substitution of scattering amplitudes or structure functions by their small-x/high-energy asymptotics at experimentally available energies is groundless.

Acknowledgments

References

[1] Sudakov V V 1956 *Sov. Phys. JETP* **3** 65
[2] Altarelli G and Parisi G 1977 *Nucl. Phys.* B **126** 297
[3] Gribov V N and Lipatov L N 1972 *Sov. J. Nucl. Phys.* **15** 438
[4] Lipatov L N 1972 *Sov. J. Nucl. Phys.* **20** 95
[5] Dokshitzer Yu L 1977 *Sov. Phys. JETP* **46** 641
[6] Kuraev E A, Lipatov L N and Fadin V S 1976 *Sov. Phys. JETP* **44** 443
[7] Kuraev E A, Lipatov L N and Fadin V S 1977 *Sov. Phys. JETP* **45** 199
[8] Balitsky I I and Lipatov L N 1978 *Sov. J. Nucl. Phys.* **28** 822
[9] Fadin V S and Lipatov L N 1998 *Phys. Lett.* B **429** 127
[10] Camici G and Ciafaloni M 1998 *Phys. Lett.* B **430** 349
[11] Gorshkov V G, Gribov V N, Frolov G V and Lipatov L N 1967 *Yad. Fiz.* **6** 129
[12] Gorshkov V G, Gribov V N, Frolov G V and Lipatov L N 1967 *Yad. Fiz.* **6** 361
[13] Gorshkov V G 1973 *Uspekhi Fiz. Nauk* **110** 45
[14] Ermolaev B I, Greco M and Troyan S I 2010 *Riv. Nuovo Cim.* **33** 57
[15] Ermolaev B I and Troyan S I 2018 *Eur. Phys. J.* C **78** 204
[16] Ermolaev B I, Greco M and Troyan S I 2004 *Phys. Lett.* B **579** 321
[17] Kochelev N I, Lipka K, Nowak W G, Vento V and Vinnikov A V 2003 *Phys. Rev.* D **67** 074014
[18] Gorshkov V G, Lipatov L N and Nesterov M M 1969 *Yad. Fiz.* **9** 1221