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A new extraction of the Boer-Mulders function

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Abstract. We extract the valence part of the nucleon Boer-Mulder (BM) function from the new COMPASS data on the unpolarized $cos\phi_h$ and $cos2\phi_h$ asymmetries in SIDIS on deuteron for producing the hadron h and its antiparticle $\bar{h}$ at azimuthal angle ϕ_h. Our results, obtained with only standard assumption of factorization of the x_B (z_h) and transverse momentum dependences, differ significantly from the presently published data on the BM function, obtained using a model assumption of proportionality to the better known Sivers function. This suggests that the published results on the BM function should be reconsidered.

1. Introduction

The Boer-Mulders (BM) function [1] is essential for an understanding of the internal structure of the nucleon. It measures the difference between the number density of quarks polarized parallel and anti-parallel to $(P \times k_\perp)$, where $\mathbf{P}$ is momentum of the nucleon and $\mathbf{k}_\perp$ is transverse momentum of the quark. Due to the scarcity of data, previous attempts to extract the BM function from experiment made the theoretically inconsistent simplifying assumption [2] that for each quark flavor, it is proportional to the better known Sivers function.

Here we present our results obtained in [3] which demonstrate that the new COMPASS data on the unpolarized $\langle \cos\phi_h \rangle$ and $\langle \cos 2\phi_h \rangle$ asymmetries in semi-inclusive deep inelastic scattering (SIDIS) reactions for producing a hadron h and its antiparticle $\bar{h}$ at azimuthal angle ϕ_h, allows an essentially model independent extraction of the BM function.

We study difference asymmetries $A^{h-\bar{h}}$, effectively $A^h - A^{\bar{h}}$, since both for the collinear and transverse momentum dependent (TMD) functions, only the flavor non-singlet valence quark parton densities (PDFs) and fragmentation functions (FFs) play a role and the gluon does not contribute. This is a great advantage explained in [4] and [5]. Moreover, on a deuteron

target an additional simplification occurs that independently of the final hadron, only the sum of the valence-quark TMD functions $Q_V = u_V + d_V$ enters. We use SIDIS COMPASS data on a deuteron target [6] and determine the BM TMD function only for Q_V but without any model assumptions. From our analysis we find also that probably there are significant twist-4 contributions to the $\langle \cos 2\phi_h \rangle$ asymmetry other than the Cahn one.

2. Formalism

The unpolarized TMD functions for Q_V are typically parametrized in a factorial form [7, 8]:

$$f_{Q_V/p}(x_B, k_\perp^2, Q^2) \;=\; Q_V(x_B, Q^2)\, \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle} \tag{1}$$

and

$$D_{h/q_V}(z_h, p_\perp^2, Q^2) = D_{q_V}^h(z_h, Q^2)\, \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}, \tag{2}$$

where $Q_V(x_B, Q^2)$ is the sum of the collinear valence-quark PDFs:

$$Q_V(x_B, Q^2) = u_V(x_B, Q^2) + d_V(x_B, Q^2) \tag{3}$$

and $D_{q_V}^h(z_h, Q^2)$ are the valence-quark collinear FFs:

$$D_{q_V}^h(z_h, Q^2) = D_q^h(z_h, Q^2) - D_{\bar{q}}^h(z_h, Q^2), \tag{4}$$

$\langle k_\perp^2 \rangle$ and $\langle p_\perp^2 \rangle$ are parameters extracted from a study of the multiplicities in unpolarized SIDIS. There is some controversy in the literature about their values. This study allows to distinguish the favored values.

As the considered asymmetries involve a product of the BM parton density and the Collins FF, $\Delta^N D_{h/u_V\uparrow}(z_h, p_\perp, Q^2)$, one requires parametrizations of the BM function and transverse momentum dependent Collins function as well. These have a form similar to Eqs. (1) and (2):

$$\Delta f_{BM}^{Q_V}(x_B, k_\perp, Q^2) = \Delta f_{BM}^{Q_V}(x_B, Q^2)\, \sqrt{2e}\, \frac{k_\perp}{M_{BM}}\, \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_{BM}}}{\pi \langle k_\perp^2 \rangle}, \tag{5}$$

with

$$\Delta f_{BM}^{Q_V}(x_B, Q^2) = 2\, \mathcal{N}_{BM}^{Q_V}(x_B)\, Q_V(x_B, Q^2). \tag{6}$$

Here the $\mathcal{N}_{BM}^{Q_V}(x_B)$ is an unknown function and M_{BM}, or equivalently $\langle k_\perp^2 \rangle_{BM}$:

$$\langle k_\perp^2 \rangle_{BM} = \frac{\langle k_\perp^2 \rangle\, M_{BM}^2}{\langle k_\perp^2 \rangle + M_{BM}^2}, \tag{7}$$

is an unknown parameter. Analogously, the Collins FF

$$\Delta^N D_{h/u_V\uparrow}(z_h, p_\perp, Q^2) = \Delta^N D_{h/u_V\uparrow}(z_h, Q^2)\, \sqrt{2e}\, \frac{p_\perp}{M_C}\, \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle_C}}{\pi \langle p_\perp^2 \rangle}, \tag{8}$$

where

$$\Delta^N D_{h/u_V\uparrow}(z_h, Q^2) = 2\,\mathcal{N}_C^{h/u_V}(z_h)\, D^h_{u_V}(z_h, Q^2)\,. \tag{9}$$

The quantities $\mathcal{N}_C^{h/u_V}(z_h)$ and M_C, or equivalently $\langle p_\perp^2\rangle_C$:

$$\langle p_\perp^2\rangle_C = \frac{\langle p_\perp^2\rangle\, M_C^2}{\langle p_\perp^2\rangle + M_C^2}\,, \tag{10}$$

are known from studies of the azimuthal correlations of pion-pion, pion-kaon and kaon-kaon pairs produced in e^+e^- annihilation: $e^+e^- \to h_1h_2 + X$ and the $\sin(\phi_h + \phi_S)$ asymmetry in polarized SIDIS [9–11].

Besides the BM-Collins contributions to the $\langle\cos\phi_h\rangle$ and $\langle\cos 2\phi_h\rangle$ unpolarized asymmetries, there exists also a contribution known as the Cahn effect [12], which involves only the collinear unpolarized PDFs and FFs.

As mentioned, we form the type of difference asymmetries $A_{UU}^{J,\,h^+-h^-}$ advocated in [13] from the corresponding usual asymmetries $A_{UU}^{J,\,h^+}$ and $A_{UU}^{J,\,h^-}$ for positive and negative charged hadron production measured in COMPASS [6] via the relation [14]:

$$A_{UU}^{J,\,h^+-h^-} = \frac{1}{1-r}\left(A_{UU}^{J,\,h^+} - r A_{UU}^{J,\,h^-}\right), \qquad J = \cos\phi_h,\ \ \cos 2\phi_h. \tag{11}$$

Here r is the ratio of the unpolarized $x_{_B}$-dependent SIDIS cross sections for production of negative and positive hadrons $r = \sigma^{h^-}(x_{_B})/\sigma^{h^+}(x_{_B})$ measured in the same kinematics [14]. In practice we construct the difference asymmetries using smooth fits to the data on the usual asymmetries and to the ratio r. For $\langle Q^2\rangle(x_{_B})$ we perform a linear interpolation of the COMPASS data points. These difference asymmetries are related to the theoretical functions via:

$$A_{UU}^{\cos\phi_h,h-\bar h} = \sqrt{\frac{\langle k_\perp^2\rangle}{\langle Q^2\rangle(x_B)}}\,\left\{\mathcal{N}_{BM}^{Q_V}(x_{_B})\,\mathcal{C}^h_{BM} + \mathcal{C}^h_{Cahn}\right\}, \tag{12}$$

$$A_{UU}^{\cos 2\phi_h,h-\bar h} = \left\{\mathcal{N}_{BM}^{Q_V}(x_{_B})\,\hat{\mathcal{C}}^h_{BM} + \frac{\langle k_\perp^2\rangle}{\langle Q^2\rangle(x_B)}\,\hat{\mathcal{C}}^h_{Cahn}\right\}, \tag{13}$$

where $\langle Q^2\rangle(x_{_B})$ is some mean value of Q^2 for each $x_{_B}$-bin and the coefficients $\mathcal{C}_{_{BM}}$, $\mathcal{C}_{Cahn}$, $\hat{\mathcal{C}}_{_{BM}}$ and $\hat{\mathcal{C}}_{Cahn}$ are dimensionless constants given by integrals over various products of the unpolarized or Collins FFs and, crucially, whose values depend on the parameters $\langle k_\perp^2\rangle$, $\langle p_\perp^2\rangle$, $M_{_{BM}}$ and $M_{_C}$. For a finite range of integration over P_T^2, corresponding to the experimental kinematics, $a \le P_T^2 \le b$, they are given by the expressions obtained in [3]:

$$\mathcal{C}^h_{Cahn} = -2\,\frac{\int dz_h\, z_h\,[D^h_{q_V}(z_h)]S_1(a,b;\langle P_T^2\rangle)/(\eta + z_h^2)^{1/2}}{\int dz_h\,[D^h_{q_V}(z_h)]\,S_0(a,b;\langle P_T^2\rangle)} \tag{14}$$

$$\mathcal{C}^h_{BM} = 4e\,K\,\frac{\langle p_\perp^2\rangle}{\int dz_h\,[D^h_{q_V}(z_h)]\,S_0(a,b;\langle P_T^2\rangle)}\int dz_h\,\left[\Delta^N D^h_{q_V\uparrow}(z_h)\right]\,\big[\,z_h^2\lambda_{_{BM}}\,S_3(a,b,\langle P_T^2\rangle_{_{BM}}) + (\eta\lambda_{_C} - z_h^2\lambda_{_{BM}})\,S_1(a,b;\langle P_T^2\rangle_{_{BM}})\big]\,/(z_h^2\lambda_{_{BM}} + \eta\lambda_{_C})^{3/2} \tag{15}$$

$$\hat{\mathcal{C}}^h_{Cahn} = \frac{2\int dz_h\,\left(z_h^2/[\eta + z_h^2]\right)\,[D^h_{q_V}(z_h)]\,S_2(a,b;\langle P_T^2\rangle)}{\int dz_h\,[D^h_{q_V}(z_h)]\,S_0(a,b;\langle P_T^2\rangle)} \tag{16}$$

$$\hat{\mathcal{C}}^h_{_{BM}} = -2e\,K\,\langle p_\perp^2\rangle\,\frac{\int dz_h\,[z_h\,\Delta^N D^h_{q_V\uparrow}(z_h)]/(z_h^2\lambda_{_{BM}} + \eta\lambda_{_C})\,S_2(a,b;\langle P_T^2\rangle_{_{BM}})}{\int dz_h\,[D^h_{q_V}(z_h)]\,S_0(a,b;\langle P_T^2\rangle)} \tag{17}$$

where

$$\begin{aligned}
\langle P_T^2\rangle &= \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle, \quad \langle P_T^2\rangle_{BM} = \langle p_\perp^2\rangle_C + z_h^2\langle k_\perp^2\rangle_{BM}\,, \\
S_n(a,b;\tau) &= \int_a^b dP_T^2\, P_T^n e^{-P_T^2/\tau}/\tau^{1+n/2}, \quad \tau = \langle P_T^2\rangle \text{ or } \langle P_T^2\rangle_{BM} \\
\eta &= \frac{\langle p_\perp^2\rangle}{\langle k_\perp^2\rangle}, \quad \lambda_C = \frac{M_C^2}{\langle p_\perp^2\rangle + M_C^2}, \quad \lambda_{BM} = \frac{M_{BM}^2}{\langle k_\perp^2\rangle + M_{BM}^2}, \quad K = \frac{\lambda_{BM}^2\lambda_C^2}{M_{BM}M_C}, \quad (18)
\end{aligned}$$

and $[D^h_{q_V}]$ and $[\Delta^N D^h_{q_V\uparrow}(z_h)]$ are combinations of the collinear and Collins FFs, respectively [5]:

$$[D^h_{q_V}(z_h,Q^2)] = e_u^2\, D^h_{u_V} + e_d^2\, D^h_{d_V}, \quad (19)$$

$$[\Delta^N D^h_{q_V\uparrow}(z_h,Q^2)] = e_u^2\, \Delta^N D^h_{u_V\uparrow} + e_d^2\, \Delta^N D^h_{d_V\uparrow} \quad (20)$$

3. Extraction of the BM function

The relations (12) and (13) provide 2 independent equations for the extraction of $\mathcal{N}_{BM}(x_B)$. Since the values of $\langle k_\perp^2\rangle$ and $\langle p_\perp^2\rangle$ and also M_{BM} and M_C are still in some dispute, we use in our analysis different sets corresponding to the values of these parameters given in literature. The coefficients $\mathcal{C}_{Cahn}$, $\mathcal{C}_{BM}$, $\hat{\mathcal{C}}_{Cahn}$, and $\hat{\mathcal{C}}_{BM}$ calculated for different sets of $\{\langle k_\perp^2\rangle,\ \langle p_\perp^2\rangle,\ M_{BM},\ M_C\}$ are shown in Table 1.

Table 1. $\mathcal{C}_{Cahn}$, $\mathcal{C}_{BM}$, $\hat{\mathcal{C}}_{Cahn}$ and $\hat{\mathcal{C}}_{BM}$ calculated for different sets of $\langle k_\perp^2\rangle$, $\langle p_\perp^2\rangle$, M_{BM}^2 ($M_{BM}^2 = M_S^2$ assumed) and M_C^2. The parametrizations for the collinear FFs are from AKK'2008 [15], and for Collins functions – for sets I – IV – from [9] and [11], and for set V – from [10] and [11]. The integrations are according to COMPASS kinematics: $0.01 \le P_T^2 \le 1\,GeV^2$ and $0.2 \le z_h \le 0.85$ [6].

SET	$\langle k_\perp^2\rangle$	$\langle p_\perp^2\rangle$	M_{BM}^2	M_C^2	$\mathcal{C}_{Cahn}$	$\mathcal{C}_{BM}$	$\hat{\mathcal{C}}_{Cahn}$	$\hat{\mathcal{C}}_{BM}$
I	0.18	0.20	0.34	0.91	-0.68	2.1	0.31	-0.47
II	0.18	0.20	0.19	0.91	-0.68	1.8	0.31	-0.40
III	0.25	0.20	0.34	0.91	-0.77	1.9	0.38	-0.49
IV	0.25	0.20	0.19	0.91	-0.77	1.4	0.38	-0.39
V	0.57	0.12	0.80	0.28	-1.2	0.89	0.84	-0.50

We found that the 2 independent extractions of $\mathcal{N}_{BM}(x_B)$ from Eqs. (12) and (13) are not completely compatible with each other for any choice of the parameters given in Table 1. The source of the disagreement, we believe, lies in the value of the Cahn contribution $\hat{\mathcal{C}}_{Cahn}$ in

Eq. (13). The point is that this Cahn term is a twist-4 contribution and there are probably other twist-4 contributions which we are not able to calculate.

We think it interesting to obtain an estimate of the missing twist-4 terms. Thus, we have found perfect agreement for the parameter Set I with $\hat{\mathcal{C}}_{Cahn}$ replaced by $\hat{\mathcal{C}}_{Cahn} + \hat{\mathcal{C}}_1$ for the following parameter values:

$$\langle k_\perp^2 \rangle = 0.18, \quad \langle p_\perp^2 \rangle = 0.20, \quad M_{BM}^2 = 0.34, \quad M_C^2 = 0.91, \quad \hat{\mathcal{C}}_1 = -1.16\,, \tag{21}$$

where $\hat{\mathcal{C}}_1$ is a free parameter adjusted to improve the compatibility of the two extractions of $\mathcal{N}_{BM}^{Q_V}$ from Eqs. (12) and (13). This is shown in Fig. 1. The value obtained for $\hat{\mathcal{C}}_{Cahn} + \hat{\mathcal{C}}_1 = -0.85$

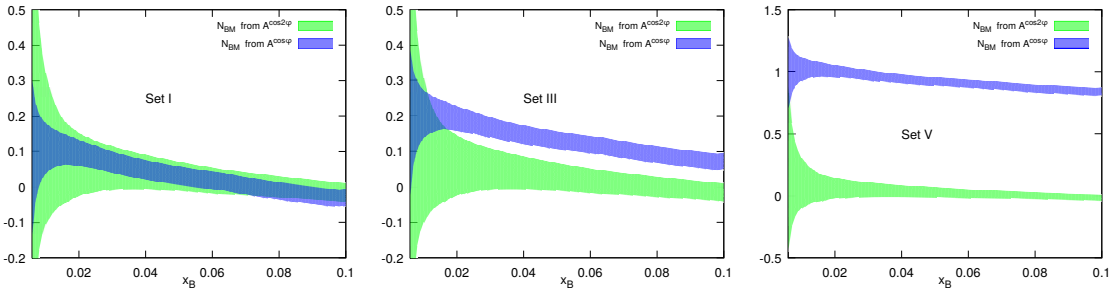

Figure 1. $\mathcal{N}_{BM}^{Q_V}(x_B)$ extracted from the difference asymmetries, Eqs. (12) and (13), using different sets of parameters of Table 1 and $\hat{\mathcal{C}}_{Cahn} + \hat{\mathcal{C}}_1$ instead of $\hat{\mathcal{C}}_{Cahn}$. Plots for Sets. II and IV overlap with those for Sets. I and III, respectively.

suggests that there are other twist-4 contributions in the $A_{UU}^{\cos 2\phi, h-\bar{h}}$ asymmetry.
The analytic expression for the extracted averaged $\mathcal{N}_{BM}^{Q_V}$ for the parameter Set Eq. (21) is:

$$\begin{aligned} \mathcal{N}_{BM}^{Q_V}(x_B) &= N x_B^\alpha (1-x_B)^\beta (1+\gamma x_B), \\ N = 0.475 \pm 0.037, \quad \alpha &= 0.242 \pm 0.022, \quad \beta = 13.3 \pm 1.7, \quad \gamma = -13.7 \pm 0.4\,. \end{aligned} \tag{22}$$

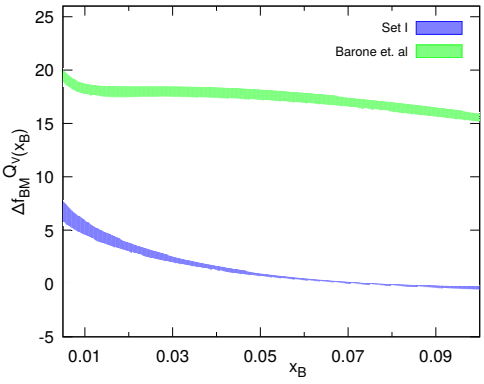

Figure 2. Comparison of $\Delta f_{BM}^{Q_V}$ for Set I, Eq. (21), with the result of Barone et al. [2].

We conclude that the COMPASS data on $A_{UU}^{\cos\phi_h}$ and $A_{UU}^{\cos 2\phi_h}$ strongly favor the parameter values of Set I, Eq. (21) and clearly show that the twist-4 contribution to the $\langle \cos 2\phi_h \rangle$ asymmetry is important and that the Cahn contribution is not the only one.

Our valence BM function $\Delta f_{BM}^{Q_V}(x_B)$ is shown in Fig. 2, where it is compared to $\Delta f_{BM}^{Q_V}(x_B)$ calculated from the BM function published in [2]. We use CTEQ6 parametrization for the collinear PDFs [16]. It is seen that there is a significant difference, suggesting that the extraction of the BM function in [2] is theoretically inconsistent.

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