

Constituent quark mass and spin in NRCQM model

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Abstract. We show that the success of the Nonrelativistic Constituent Quark Model can be associated with symmetrization, which manifests itself in the exact 3:1 ratio between the initial constituent quark mass $M_q = m_\Xi/3 = 441$ MeV and the parameter of quark interaction $\Delta M_\Delta = (m_\Delta - m_N)/2 = 147$ MeV. The interconnection of these parameters with the electron mass, its symmetry, the masses of the fundamental fields and the QED radiative correction is considered. The discreteness with the parameter m_e can be used in the study of gravitation and dark matter problems.

1. Introduction

According to [1], R. Feynman noted that "non-relativistic quark model is correct as it explains so much data. It is for theorists to explain why". The Nonrelativistic Constituent Quark Model (NRCQM) [2-6] plays an important role in the development of the Standard Model (SM) – a theory of all interactions (except gravitation) with its components [7]:

$$SU(3)_{col} \otimes SU(2)_L \otimes U(1)_Y \quad (1)$$

Y. Nambu noted that in the Standard Model "a) the unification of forces is only partially realized; and b) there are too many input parameters, especially concerning the masses, which are not explained" [8]. Empirical relations in the particle masses were noticed by many authors, including Y. Nambu [9], who found the relations $m_N = m_\mu + 6m_\pi$ and $m_\Lambda = 8m_\pi$. G. Wick, R. Sternheimer and P. Kropotkin noted [10-12] a stable character of intervals, which later served as the main parameter of the NRCQM model: the baryon constituent quark mass (M_q) equal to 1/3 of the initial baryon mass $3M_q = 1350$ MeV = m_Ξ (+ on Fig. 1) and the meson constituent quark mass (M_q'') equal to 1/2 of the vector meson mass ($m_\omega/2 = 391$ MeV or $m_\rho/2 = 387$ MeV).

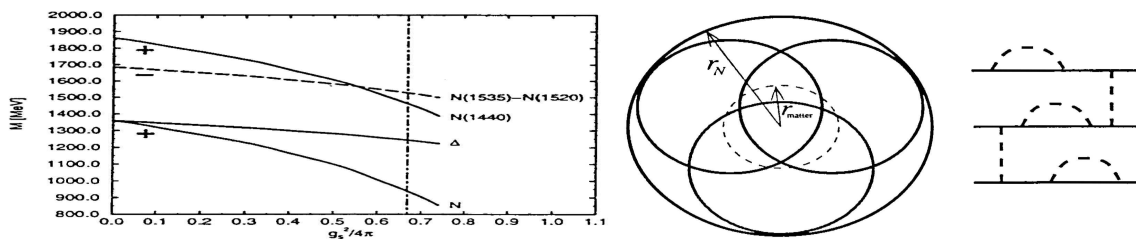


Figure 1. *Left:* Calculation of baryon mass as a function of interaction strength within Goldstone Boson Exchange NRCQM; initial mass 1350 MeV = 3×450 MeV is marked + [5].

Right: Schematic representation of the nucleon structure [5]: a larger radius r_N and a smaller r_{matter} correspond to the nucleon size and the space of baryon matter (shift $-m_e$ in CODATA relations [6-7]).



2. Discreteness in NRCQM parameters

Two main parameters of the NRCQM model: 1) the initial constituent quark mass $M_q=441\text{ MeV}=(m_\Xi=1321\text{ MeV})/3$ and 2) the parameter of residual quark interaction $\Delta M_\Delta=147\text{ MeV}$, derived as half a nucleon Δ -excitation $m_\Delta - m_N=294\text{ MeV}$ (shown on the vertical line in the center of Fig. 1 [5]) together with the muon mass m_μ and three pion parameters (m_π , the pion β -decay parameter $f_\pi=130.7(4)\text{ MeV}$ and the doubled energy of the pion β -decay $2(\delta m_\pi - m_e)=8.166\text{ MeV}$, close to $8.176\text{ MeV}=16m_e = \delta$), form empirical ratios [13-15]:

$$1 : 13 : 16 : 17 : 18 : (48 = 3 \times 16) : (54 = 3 \times 18) = \delta : m_\mu : f_\pi : m_\pi : \Delta M_\Delta : M_q'' : M_q. \quad (2)$$

The discreteness with a period $\delta = 16m_e$ is seen in the sum distribution of the differences between the all masses of particles (Fig. 2). The positions of both first maxima at 17 MeV and 48 MeV coincide with 2δ and 6δ . The stable intervals 445 MeV and 462 MeV are close to $M_q=441\text{ MeV}$, and the intervals 3940 MeV and 3959 MeV are close to $9M_q=3969\text{ MeV}$ [13-15]. This discreteness is also illustrated by the two-dimensional representation shown in Fig. 3 [10,13-15], where the large period $f_\pi = M_q''/3 = 16\delta = 16 \times 16m_e$ ($N=16$ in units of $16m_e$) is used along the horizontal axis, and the residues are plotted along the vertical axis. Intervals equal to f_π , m_{π^\pm} and ΔM_Δ ($N=16, 17, 18$) are visible as lines with different slopes. This means that intervals close (or multiple) to M_q (parallel lines in Fig. 3) are common in the mass spectrum.

The initial baryon mass of $3M_q$ ($N=3 \times 54$) is a starting value of the evolution of the baryon mass to the nucleon mass of $m_N=940\text{ MeV}=m_\mu + M_q'' + M_q$, $N=115=13+48+54$. The ratio between M_q and ΔM_Δ is exactly $3/1=441\text{ MeV}/147\text{ MeV}$. The line connecting the nucleon with the Δ -baryon (interval $2\Delta M_\Delta$) and with the Σ -hyperon is parallel to upper line going to $m_\Xi = 3M_q$ (slope of 147 MeV). The accuracy of NRCQM calculations is within 10–15 MeV [5,6]. The proximity in the NRCQM calculations of two parameters: $(m_\Delta - m_N)/2=147\text{ MeV}$ and the mass difference between strange- and d -quarks $m_\Sigma - m_\Delta \approx 150\text{ MeV}$ corresponds to SU(6) flavor-spin symmetry [7]. The parameter M_q also manifests itself in radial excitations: $(m_{J/\Psi(1S)}=3097\text{ MeV})/M_q=7.02$. The masses of the d -quark $m_d=4.67(40)\text{ MeV}$ and the c -quark $m_c=1275(25)\text{ MeV}$ [7] are close to $9m_e=4.599\text{ MeV}$ and $9m_\pi=1255\text{ MeV}$ [13,14]. The discussed discreteness with parameter $\delta = 16m_e$ is consistent with the conclusion of R. Frosch on periodicity with a period of $3m_e$ in particle masses [16].

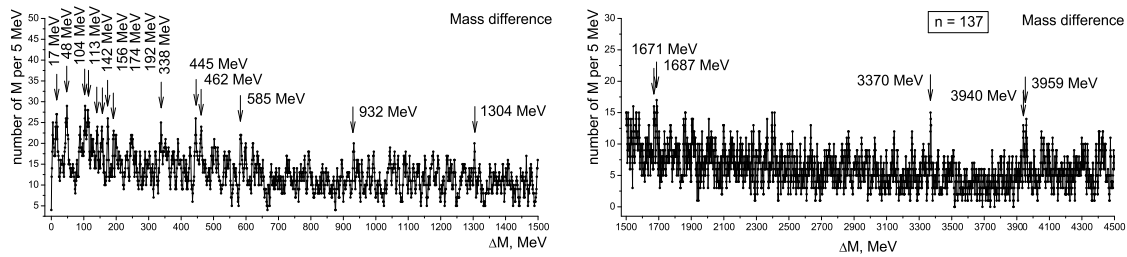


Figure 2. Distribution of mass differences ΔM in regions 0–1500 MeV and 1500 MeV–4500 MeV. Maxima at $\Delta M=17\text{ MeV}$ and 48 MeV coincide with 2δ and 6δ . Maximum at $\Delta M=142\text{ MeV}$, the doublet at 1671–1687 MeV and the maximum at 3370 MeV are close to integers $k=1, 12$ and 24 of the m_{π^\pm} . Doublets at 445 MeV–462 MeV and 3940 MeV–3959 MeV are close to $M_q=441\text{ MeV}$ and $9M_q=3969\text{ MeV}$.

M.H. Mac Gregor noticed [17] that many interval between the masses of hadrons containing charmed quark are close to the pion mass $m_\pi^\pm=139.6\text{ MeV}$ (3 values coincide within 2 MeV):

$$D^{*\pm} - D^\pm = (140.7 \pm 0.3)\text{ MeV} \approx m_\pi \quad (3)$$

$$D_s^{*\pm} - D_s^\pm = (143.8 \pm 1.9)\text{ MeV} \approx m_\pi \quad (4)$$

$$\chi_{C2}(1P) - \chi_{C0}(1P) = (141.5 \pm 0.3)\text{ MeV} \approx m_\pi. \quad (5)$$

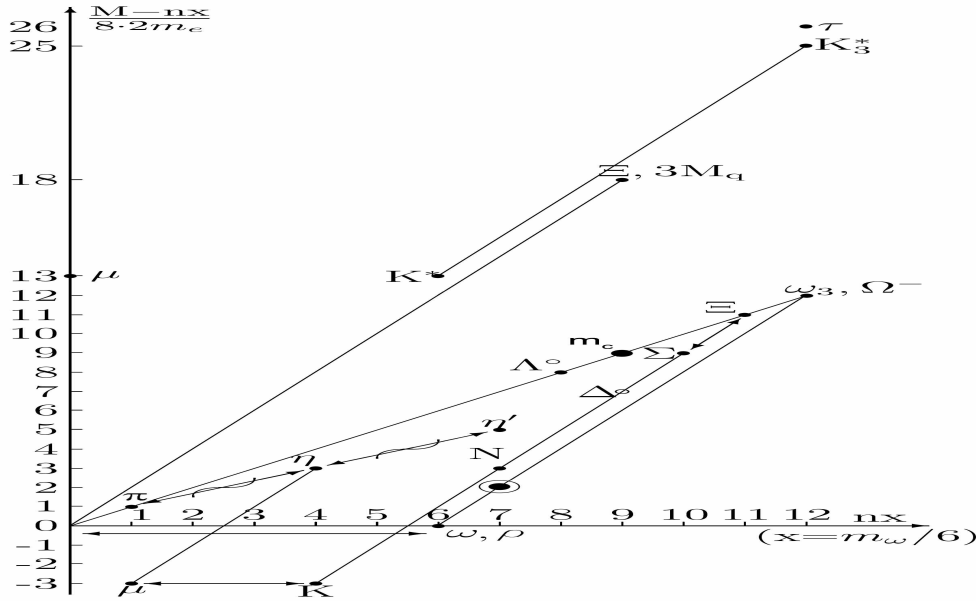


Figure 3. Evolution of the baryon mass from $3M_q$ to nucleon mass M_N is shown in two-dimensional presentation: values in horizontal direction are in units of $16 \cdot 16m_e = f_\pi = 130.7 \text{ MeV}$, remainders $M_i - n(16 \cdot 16m_e)$ are plotted along vertical axis in $16m_e$. Nucleon mass in nuclear medium (circled point) is close to the sum $\Delta M_\Delta + 6f_\pi$. Parameters, $f_\pi = M_q''/3 = M_w/3L$ and $\Delta M_\Delta = M_q/3 = M_Z/3L$, are connected with the fundamental fields and with CODATA period $\delta = 16m_e$ ($N=16$ and $N=18$, see text). Three different slopes correspond to three pion parameters: $f_\pi = 16\delta$, $m_{\pi^\pm} = 17\delta$ and $\Delta M_\Delta = 18\delta$. Line with the slope of $m_\pi = 140 \text{ MeV} = f_\pi + \delta$ ($N=16+1$) goes through Λ -, Ξ -, Ω -hyperons and the charmed quark $m_c = 9m_\pi$. Lines with ω - and K^* -mesons correspond to stable mass intervals $\Delta M = 2M_q = 6\Delta M_\Delta$, $\Delta J=2$. Mass of the τ lepton is close to $2m_\mu + 2M_q''$ [14,15].

Appearance of the maximum at 142 MeV in ΔM -distribution (Fig. 2) together with the doublet at $\Delta M = 1671\text{--}1687 \text{ MeV}$ (close to $12m_\pi = m_\Omega$, see Fig. 3) and the maximum at $\Delta M = 3370 \text{ MeV} = 24m_\pi$ corresponds to the stability of intervals close to m_π ($N=17$).

Another maximum in Fig. 3 at 104–112 MeV corresponds to maxima at 107 MeV (close to $m_\mu = 105.66 \text{ MeV}$) in two separate ΔM -distribution for all unflavored particles and for all particles containing b -quarks (see Table 4 in [13]). In Table 1 these values m_μ and m_π ($N=13, 17$ of the period $\delta = 16m_e$) are compared with numbers of fermions N^{ferm} in the central field.

Table 1. Comparison of numbers of fermions in the central field (top line, N^{ferm}) with numbers N in presentation of particle masses as $m_i = N \cdot \delta$ and ratios between masses and parameters close to $\alpha/2\pi$. Boxed are a hole configuration in the $1p$ shell and a valence configuration.

N^{ferm}	N=1	N=13	16·13-1=L	N=16	N=17	16·17+1	n=18	N=3·16·18
Mass	δ	m_μ	f_π	m_{π^\pm}	ΔM_Δ	M_q		
Parameters	m_e/M_q	m_μ/M_Z	f_π/M_H'		$\Delta M_\Delta/M_{H^0}$			
Ratio	$\alpha/2\pi$	$115.87 \cdot 10^{-5}$	$114 \cdot 10^{-5}$		$117 \cdot 10^{-5}$			
Comments		hole in $1p$	filled shells	valence				

3. The unique role of CODATA relations

It was noticed [10,13] that the nucleon mass difference $\delta m_N = m_n - m_p = 1293$ keV and electron mass correspond to stable nuclear energy intervals (or related values). The stable interval $161 \text{ keV} = \delta m_N / 8$ in nuclei with $Z \approx 50$ (see Fig. 5 in [13]) was connected with the one-pion exchange dynamics [18,19]. The discreteness with the same parameter 161 keV in nuclei with $Z=20-28$ is presented in Table 1, where the intervals that are multiples of 161 keV , are exactly in the ratio $1:2:3:4:5:8:9:12:16$. The universal character of parameters $161 \text{ keV} = \delta m_N / 8$ and $170 \text{ keV} = m_e / 3$ (see Figs. 6,7 in [13], Fig. 2 in [15]) is in agreement with the analysis of nucleon masses themselves.

Table 2. Discreteness with a period of 161 keV in excitations of nuclei around $Z=20$ and $Z=28$.

2 top sections: The values of E^* in the magic ^{41}Ca and neighboring nuclei together with the number of proton pairs $(Z-14)/2$ and neutrons in $1d_{3/2}$ subshell. Double-boxed are $^{37,38,43}\text{S}$ excitations, multiples of $161 \text{ keV} = \delta m_N / 8$.

3 bottom sections: Excitations in nuclei with 3 holes in the double-magic ^{56}Ni are compared with multiples of $161 \text{ keV} = \Delta^{TF} = \delta m_N / 8$ (boxed). Double-boxed are excitations in ^{53}Co (a configuration of proton hole and 2 neutron holes) and in Ni isotopes with 3 holes and 3 particles over ^{56}Ni .

AZ ($Z-14$)/2 N	^{33}Mg -1 N=20+1	^{32}Si 0 N=20-2	^{37}S 1 $\Delta N=1$	^{38}S 1 $\Delta N=2$	^{33}S 1	^{43}S 1 $\Delta N=7$	^{39}Ar 2 20+1
E^*	159	484	1942	646.2	1292	322	320.7
$2J_o^\pi$	3^-	0^+	7^-	0^+	D	(3^-)	7^-
$2J^\pi$	(7^-)	(3^-)	2^+	3^-	2^+	(7^-)	3^-
$k \frac{\delta m_N}{8}$	161	483	1941	647	1293	322	322
k	1	3	12	4	8	2	2
AZ N	^{41}K 20+2	^{41}Ca 21 (Z-14)/2=3	^{47}Sc 20+5	^{47}Ti 20+5	^{47}V 20+5	^{50}V 20+5	^{51}V 20+5
E^*	980.4	1293.6	1943	807.9	159.37	1294.9	320.2
$2J_o^\pi$	3^+	7^-	7^-	2^+	5^-	3^-	6^+
$2J^\pi$	4^+	7^-	3^-	3^-	7^-	11^-	4^+
$k \frac{\delta m_N}{8}$	971	1293	1941	808	161	1293	322
k	6	8	12	5	1	8	2
AZ	^{53}Ni $2J_o=7^-$	^{58}Ni	^{59}Ni	^{61}Ni	^{63}Ni	^{65}Ni	^{67}Ni
E^*	320(3)	1292*	1456*	1454.2	339.4	1454.8	1289.1
$2J^\pi$	(5^-)	(3^-)	(11^-)	2^+	$3^- - 5^-$	7^-	9^+
$k \frac{\delta m_N}{8}$	322	1293	1454	1454	322	1454	1293
k	2	8	9	9	2	9	8
AZ	^{53}Mn	^{55}Mn	^{57}Mn	^{59}Mn	^{61}Mn	^{63}Mn	^{65}Mn
E^*	378	1289.9	1441.3	2563.1	2573.1	1289.1	1292.1
$2J^\pi$	5^-	3^-	(11^-)	13^-	7^-	$5^- - 11^+$	11^-
$k \frac{\delta m_N}{8}$	322	1293	1454	2586	2586	1293	1283
k	(2)	8	9	(16)	16	8	8
AZ	^{53}Co	^{59}Co	^{69}Cu	^{71}Cu	^{73}Cu	^{75}Cu	^{77}Cu
E^*	646.2	1291.6	1459	2581.7	2585.8	1297.9	1453.3
$2J^\pi$	7^-	3^-	11^-	$3^- - 7^-$	7^-	$3^- - 1, 3^-$	$3^- - 9^-$
$k \frac{\delta m_N}{8}$	647	1293	1454	2586	2586	1293	1454
k	4	8	9	16	16	8	9

The analysis of nucleon masses is based on the exact value of the ratio $m_n/m_e=1838.6836605(11)$ between the neutron and electron masses [20]. The shift of neutron mass from the integer $115 \cdot 16 - 1$ of m_e is $\delta m_n = 161.6491(6)$ keV, which coincides with the interval $D_{ij}=161$ keV in near-magic nuclei and is exactly $1/8$ of the independently determined [20] nucleon mass splitting $\delta m_N=1293.3322(4)$ keV. The ratio between these two shift values coincides with an integer $\delta m_N : \delta m_n = 8.00086(3) \approx 8 \times 1.0001(1)$ and results in relations:

$$m_n = 115 \cdot 16m_e - m_e - \delta m_N/8 \quad (6)$$

$$m_p = 115 \cdot 16m_e - m_e - 9(\delta m_N/8) \quad (7)$$

called "CODATA relations". The ratio of $\delta m_n = \delta m_N/8$ to the pion mass $\delta m_n/m_{\pi^\pm} = 115.86 \cdot 10^{-5}$ is very close to the radiative correction $\alpha/2\pi = 115.96 \cdot 10^{-5}$, which was considered by V. Belokurov and D. Shirkov [21] as a property of physical condensate: "the radiative correction can be considered as the reaction of quantum vacuum on the physical particle in it". The relations between leptons masses, pion parameters, fundamental fields, and QED radiative correction shown in Table 1, demonstrate the symmetry motivated aspects of the SM.

The downward shift of $-m_e = -3 \cdot (m_e/3)$ in CODATA relations for both nucleon masses can be assigned to the sum effect of three quarks in the nucleon with values of $-m_e/3 = -170$ keV for each of quarks (according to the nucleon scheme given in the right part of Fig. 1). Here, the three constituent quarks overlap within a small central region of radius r_{matter} , which is associated with the baryon number. The nucleon mass as a whole is the result of strong interaction (the gluon quark-dressing effect), which provides the constituent quark mass M_q of about 400 MeV [22]. According to the scheme in Fig. 1, the central region extends on approximately $1/3$ part of the quark, that's $\approx M_q/3 = \Delta M_\Delta = 147$ MeV. This means a presence of two fine structures in the nucleon mass presentation: the first one connected with the electron mass (shift $-m_e/3 = 170$ keV in addition to the main value presented with $N=115$ of the period $16m_e = \delta$), and the second, with the period 161 keV $= \delta m_N/8$ related to the m_{π^\pm} . In Table 3, both fine structure parameters (170 keV, 161 keV) and connected with them (by the factor $\alpha/2\pi$) two pion parameters (Δ -excitation 147 MeV and the pion mass itself 140 MeV) are double-boxed in neighboring columns (n=17 and 18). It was noticed [14] that the value $m_\pi = 140$ MeV $\approx \Lambda_{QCD}$ [13] is simultaneously related (using a radiative correction for a short distance $\alpha_Z/2\pi = 123.4 \cdot 10^{-5}$, $Q=91$ GeV, $\alpha_Z = 1/129$) with parameters M'_H and $m_e/3$ located in the columns n=16 and n=18. This means that the parameter $m_e/3 = 170$ keV in the CODATA relations is connected with both pion parameters and both scalar fields (such involvement of the QED interaction at short and long distances was considered by V. Andreev and V. Gribov as possible color origin).

The stable intervals at $k \times M_q$ ($k=8, 9, 10$) in the sum ΔM -distribution (Fig. 2) are in agreement with the discreteness with $\delta^\circ = 16M_q = 7.06$ GeV observed as the proximity of the ratios $M^{L3}/8=7.25$, $M'_H/16=7.19$, $m_t/24=7.22$, $M_Z/13$ and M_{H° (see Table 3, top).

Table 3. Presentation of particle masses (3 top sections) and nuclear data (bottom) by the expression $n \cdot 16m_e(\alpha/2\pi)^X M$ [13]. Intervals in nuclear binding energies ΔE_B ($X=0, M=1$) and fine structure in E^* and D_{ij} ($X=1-2$) are considered in [23,24].

X	M	n = 1	n = 13	n = 16	n = 17	n = 18	n = 3 × 18
-1	3/2	$m_t = 173.2 \approx 24\delta^\circ$					
GeV	1/2, 1	$16M_q = \delta^\circ$	$M_Z = 91.2$	$M^{L3} = 58, M'_H = 115$		$M_{H^\circ} = 125 \approx 18\delta^\circ$	
0	1	$16m_e = 2m_d - 2m_e$	$m_\mu = 106$	$f_\pi = 130.7$	m_π, Λ_{QCD}	$\Delta M_\Delta = 147$	$m_s = m_\Sigma - m_\Delta, M_q = 441$
MeV	3	NRCQM		$M''_q = m_\rho/2$			
1	1	$16m_e = \delta = 8\varepsilon_\circ$			$k\delta - m_n - m_e =$	$170 = m_e/3$	$m_e = 511$
keV	8,1	CODATA	$\delta m_N = 1293.3$		$= 161.651$		
1	1	$9.5 = \delta' = 8\varepsilon'$	123	152	$\Delta^{TF} = 161$	170 (Sn)	
2, eV	1	$11 = \delta'' = 8\varepsilon''$	143 (As)		749 (Br, Sb)	Neutron	resonances

4. General remarks and conclusions

The accuracy of the integer relations between the electron mass and the masses of many well-known particles (m_μ , M_Z , nucleon masses etc.) demonstrate the possibility to apply this empirical fact to problems of gravitation [24] and dark matter. The CODATA relations open also a unique possibility to combine the results of nuclear data analysis with the results obtained in particle physics. The 3:1 ratio between M_q and $\Delta M_\Delta = m_s$ (at $n=18$ in Table 3) is similar to the ratio between m_e and 170 keV (quark mass shift). This is a common factor in nuclear microscopic models where quasi-particles form S-bosons. The difference in the masses of the electron and neutrino is three times larger than the parameter $m_e/3 = (\alpha/2\pi) \times \Delta M_\Delta = (\alpha/2\pi)^2 \times M_{H^0}$.

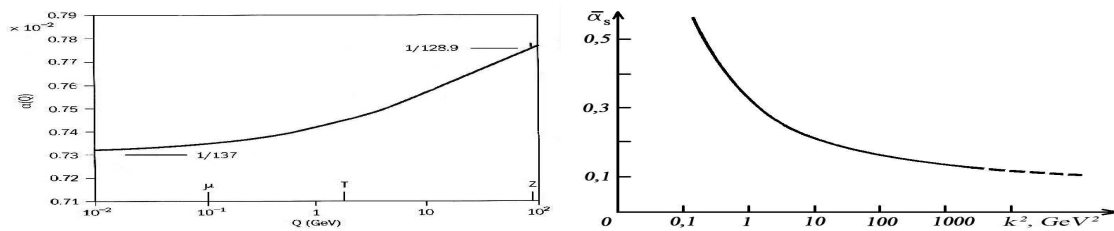


Figure 4. *Left:* Depending of the moment evolution of the QED effective charge (in square). Monotone theoretical dependence is compared with results the α_Z measurements [14,22].

Right: Behaviour of effective coupling α_s in QCD as a function of squared momentum transfer [14,21]. QCD parameter at $M_Z=91$ GeV $\alpha_s=0.1181(11)$ [7,13] is close to $2/17=2\delta/m_\pi=0.1176$.

The exactness of the observed relations containing lepton masses and QED radiative corrections reflects the general origin of these symmetry motivated correlations. The distinguished character of vector fields is supported by observations [14] on the energy dependencies of the SM parameters corresponding to QCD and QED (Fig. 4): the QCD parameter for a short distance $\alpha_Z=1/129$ can be compared with the ratio $M_q/(M_q+8\delta^\circ)=M_q/(2/3)m_t$. Analysis of new data on particle masses could confirm the results.

References

- [1] Close F 2017 *CERN Courier* **53** No 3 31
- [2] Zeldovich Ya B, Sakharov A D 1966 *J Nucl Phys* **4** 395
- [3] Lipkin H J 1989 *Phys Lettr B* **233** 466
- [4] Itoh C *et al.* 1989 *Phys. Rev. D* **40** 3660
- [5] Glozman L A 1998 *Nucl. Phys. A* **629** 121c
- [6] Itoh C <http://meijigakuin.jp/citoh/baryon2018.html>
- [7] Tanabashi M *et al.* 2018 (PDG) *Phys. Rev. D* **98** 030001
- [8] Nambu Y 1998 *Nucl. Phys. A* **629** 3c
- [9] Nambu Y 1952 *Progr. Theor. Phys.* **7** 595
- [10] Sukhoruchkin S I 1972 *Stat. Properties of Nuclei* ed. Garg G (Plenum Press) p 215
- [11] Sternheimer R 1964 *Phys. Rev.* **136** 1364; 1968 **170** 1267
- [12] Kropotkin P 1971 *Field and Matter* (Moscow Univ.) p 106 (in Russian)
- [13] Sukhoruchkin S I 2018 *Nucl. Part. Phys. Proc.* **300-302** 75
- [14] Sukhoruchkin S I 2019 *Nucl. Part. Phys. Proc.* (in press)
- [15] Sukhoruchkin S I 2017 *J. Phys.: Conf. Ser.* **938** 012036
- [16] Frosch R 1991 *Nuovo Cim.* **104** 913
- [17] Mac Gregor M H 1990 *Nuovo Cimento A* **103** 983
- [18] Otsuka T, Suzuki T, Utsuno Y 2008 *Nucl. Phys. A* **805** 127c
- [19] Tanihata I 2010 *Mod. Phys. Lett. A* **25** 1886 and private communication
- [20] Mohr P, Newell D B, Taylor B N 2016 *Rev. Mod. Phys.* **88** 035009
- [21] Belokurov V V, Shirkov D V 1991 *Theory of Particles Interaction* (New-York: AIP)
- [22] Horn T *et al.* 2016 *J. Phys. G: Nucl. Part. Phys.* **43** 073061
- [23] Sukhoruchkin S I 2017 *Discreteness in particle masses* (LAP) ISBN 978-620-2-02258-3
- [24] Sukhoruchkin S I 2016 PoS(ICHEP2016)1198