NLO radiative corrections to the Drell–Yan process at LHC Run3

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Abstract. NLO electroweak and QCD radiative corrections to the Drell-Yan process were calculated. The numerical estimations for forward-backward asymmetry in $\mu^+\mu^-$ pair production performed in various rapidity ranges and different PDF sets for center-of-mass energy $\sqrt{S} = 7$ TeV are in agreement with CERN Large Hadron Collider (LHC) Run1 experimental data. To simulate the detector acceptance we used the standard CMS detector cuts. We present predictions for radiatively corrected differential cross section and forward-backward asymmetry at CMS LHC Run3 with energy $\sqrt{S} = 14$ TeV.

1. Introduction

Despite the fact that the Standard Model (SM) keeps for oneself the status of consistent and experimentally confirmed theory, the search of New Physics (NP) manifestations is continued. One of powerful tool in the modern experiments at LHC from this point of view is the investigation of Drell-Yan lepton-pair production:

$$pp \to l^+l^-X \tag{1}$$

at large invariant mass M of lepton pair ($M \geq 1$ TeV).

Common convolution formula for Born and contribution of additional virtual particle (V-contribution) to cross section of the Drell-Yan process looks as

$$\sigma_V^H = \frac{1}{3}\int d^3\Gamma \sum_{q=u,d,s,c,b} \theta_K\theta_M\theta_D[f_q^A(x_1,Q^2)f_{\bar{q}}^B(x_2,Q^2)\sigma_V^{q\bar{q}} + f_{\bar{q}}^A(x_1,Q^2)f_q^B(x_2,Q^2)\sigma_V^{\bar{q}q}], \tag{2}$$

where index $V = \{0, \mathrm{BSE}, \mathrm{LV}, \mathrm{HV}, \mathrm{Box}, \mathrm{fin}\}$, $\mathrm{Box} = \{\gamma\gamma, \gamma Z, ZZ, WW\}$ denotes the contribution of radiative corrections. θ_K, θ_M, θ_D are kinematical factors, and $d^3\Gamma$ is the phase space of dilepton.

Born cross section is given by

$$\sigma_0^{q\bar{q}} = \frac{2\pi\alpha^2}{s^2}\sum_{i,j=\gamma,Z} D^i D^{j*}(b_+^{i,j}t^2 + b_-^{i,j}u^2), \tag{3}$$

where

$$b^{i,j}_{\pm} = \lambda^{i,j}_{q^+}\lambda^{i,j}_{l^+} \pm \lambda^{i,j}_{q^-}\lambda^{i,j}_{l^-}, \qquad \lambda^{i,j}_{f+} = v^i_f v^j_f + a^i_f a^j_f, \qquad \lambda^{i,j}_{f-} = v^i_f a^j_f + a^i_f v^j_f$$

are combination of coupling constants:

$$v^{\gamma}_f = -Q_f, \quad a^{\gamma}_f = 0, \quad v^W_f = a^W_f = \frac{1}{2\sqrt{2}s_W}, \quad v^Z_f = \frac{I^3_f - 2Q_f s^2_W}{2s_W c_W}, \quad a^Z_f = \frac{I^3_f}{2s_W c_W}$$

and

$$D^{js} = \frac{1}{s - m^2_j + im_j\Gamma_j} \tag{4}$$

is the propagator for j-boson depending on its mass and width.

The expression for asymmetry is as follows

$$A_{\rm FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}, \tag{5}$$

where σ_F (in case of $\cos\theta^* > 0$) is the "forward" cross section and σ_B ($\cos\theta^* < 0$) is the "backward" one, θ^* is the emission angle of the negatively-charged particle related to the quark momentum in the center-of-mass frame of the dilepton system. Asymmetry is considered within the Collins–Soper frame [1].

2. NLO radiative corrections

To obtain the NLO electroweak (EWK) corrections, the contribution of various types of Feynman diagrams must be considered. The EWK Boson Self Energies contribution is given by expression

$$\sigma^{q\bar{q}}_{\rm BSE} = -\frac{4\alpha^2\pi}{s^2}\left[\sum_{i,j=\gamma,Z} \Pi^i D^i D^{j*} \sum_{\chi=+,-} (\lambda^{\gamma,j}_{q_\chi}\lambda^{Z,j}_{l_\chi} + \lambda^{Z,j}_{q_\chi}\lambda^{\gamma,j}_{l_\chi})(t^2 + \chi u^2)\right], \tag{6}$$

which is connected with the renormalized γ-, Z- and γZ-self energies as $\Pi^Z = \hat{\Sigma}^Z/(s - m^2_Z)$, $\Pi^\gamma = \hat{\Sigma}^\gamma/s$, $\Pi^{\gamma Z} = \hat{\Sigma}^{\gamma Z}/s$.

The contribution to the cross section of vertex diagrams may be obtained by replacing coupling constants at Born cross section to form factors $v^j_f \to \delta F^{jf}_V$, $a^j_f \to \delta F^{jf}_A$. Electroweak form factors in ultrarelativistic limit depend on the Sudakov logarithms (SL). Thus, vertex contribution to cross section, which is the sum of "light" (LV) and "heavy" vertexes (HV), looks like

$$\sigma^{q\bar{q}}_{\rm Ver} = \frac{4\pi\alpha^2}{s^2}{\rm Re}\sum_{i,j=\gamma,Z} D^i D^{j*} \sum_{\chi=+,-} (\lambda^{F^{i,j}}_{q_\chi}\lambda^{i,j}_{l_\chi} + \lambda^{i,j}_{q_\chi}\lambda^{F^{i,j}}_{l_\chi})(t^2 + \chi u^2). \tag{7}$$

The calculation of box (two boson) contribution is more complicate procedure since it demands the integration of 4-point functions with complex masses in unlimited from above kinematical region of invariants. Generally it is given by expression

$$d\sigma_{ZZ} = -\frac{4\alpha^3}{\pi s} d\Gamma_2 {\rm Re}\frac{i}{(2\pi)^2}\int d^4k \sum_{k=\gamma,Z} D^{ks^*}(D^{ZZ} + C^{ZZ}), \tag{8}$$

here D^{ZZ} is the contribution of direct box diagram, and C^{ZZ} is the crossed box (boson legs are crossed in this diagram).

Using equivalent transformation based on the close connection of infrared divergency and SL terms: $D^{ZZ} = (D^{ZZ}_{k\to 0} + D^{ZZ}_{k\to q}) + (D^{ZZ} - D^{ZZ}_{k\to 0} - D^{ZZ}_{k\to q}) = D^{ZZ}_1 + D^{ZZ}_2$, integrating over

4-momentum k and retaining the terms which are proportional to the zero, first and second power of Sudakov logarithms we get the asymptotic expressions [2], [3]:

$$\frac{i}{(2\pi)^2}\int d^4kD_1^{ZZ} \approx -\frac{2}{s}(b_+^{ZZ}t^2 + b_-^{ZZ}u^2)(\frac{\pi^2}{3} + \frac{1}{2}l_{Z,t}^2), \tag{9}$$

$$\frac{i}{(2\pi)^2}\int d^4kD_2^{ZZ} \approx b_-^{ZZ,k}u\ln\frac{s}{|t|} + (b_-^{ZZ,k}\frac{t^2+u^2}{2s} + b_+^{ZZ,k}\frac{t^2}{s})\ln^2\frac{s}{|t|}, \tag{10}$$

The NLO QCD corrections can be obtained from QED case by substitution:

$$Q_q^2\alpha \to \sum_{a=1}^{N^2-1} t^at^a\alpha_s = \frac{N^2-1}{2N}l\alpha_s \to \frac{4}{3}\alpha_s \tag{11}$$

here $2t^a$ – Gell-Mann matrices, $N = 3$.

Finally, we need to consider photon and gluon bremsstrahlung with gluons inverse contribution. An expression for fin-part (sum of virtual and soft photon part and gluon part) is following

$$\sigma_{\text{fin,EWK}}^{q\bar{q}} = \frac{\alpha}{\pi}\delta_{\text{EWK}}\sigma_0^{q\bar{q}},\ \sigma_{\text{fin,QCD}}^{q\bar{q}} = \frac{4}{3}\frac{\alpha_s}{\pi}\delta_{\text{QCD}}\sigma_0^{q\bar{q}}, \tag{12}$$

where

$$\begin{aligned}\delta_{\text{EWK}} &= 2\ln\frac{2\omega}{\sqrt{s}}\left(Q_q^2(\ln\frac{s}{m_q^2}-1) - 2Q_qQ_l\ln\frac{t}{u} + Q_l^2(\ln\frac{s}{m^2}-1)\right) + Q_l^2\left(\frac{3}{2}\ln\frac{s}{m^2} - 2 + \frac{\pi^2}{3}\right) + \\ &+ Q_q^2\left(\frac{3}{2}\ln\frac{s}{m_q^2} - 2 + \frac{\pi^2}{3}\right) - Q_qQ_l\left(\ln\frac{s^2}{tu}\ln\frac{t}{u} + \frac{\pi^2}{3} + \ln^2\frac{t}{u} + 4\text{Li}_2\frac{-t}{u}\right),\end{aligned} \tag{13}$$

and

$$\delta_{\text{QCD}} = 2\ln\frac{2\omega}{\sqrt{s}}(\ln\frac{s}{m_q^2} - 1) + \frac{3}{2}\ln\frac{s}{m_q^2} - 2 + \frac{\pi^2}{3}. \tag{14}$$

It is necessary to rebuild all of the cross sections to completely differential form

$$\sigma_C \to \sigma_C^{(3)} \equiv \frac{d^3\sigma_C}{dMdyd\psi}, \tag{15}$$

where y is dilepton rapidity, ψ – cosine of angle between $\vec{P}_A$ and $\vec{k}_1$.

For non-radiative part the transition to differential form can be done using the Jackobian J_N:

$$J_N = \frac{D(x_1,x_2,t)}{D(M,y,\psi)} = \frac{4M^3e^{2y}}{S[1+\psi+(1-\psi)e^{2y}]^2}. \tag{16}$$

The radiative Jackobian can be introduced in the following way:

$$J_R^{(3)} = \frac{4Me^{2y}}{S}\frac{(v+M^2)(z_1+M^2)(u_1+M^2)}{[(1+\psi)(z_1+M^2)+(1-\psi)e^{2y}(u_1+M^2)]^2}, \tag{17}$$

using substitution $z_1 = 2p_1p,\ u_1 = 2p_2p,\ z = 2k_1p, v = 2k_2p$, where p is the 4-momenta of real photon or gluon.

To solve Quark Mass Singularity (QS) problem in $\overline{MS}$-scheme [4, 5] the collinear logarithm terms are adsorbed into PDFs depending on the factorization scale, M_{SC}. The part to be subtracted is

$$\sigma_{QS} = \frac{1}{3}\int d^3\Gamma \int_0^{1-\frac{2\omega}{M}} d\eta \sum_{q=u,d,s,c,b} [(f_q(x_1,Q^2)\Delta\bar{q}(x_2,\eta)+ \\ + \Delta q(x_1,\eta) f_{\bar{q}}(x_2,Q^2))\sigma_0^{q\bar{q}} + (q \leftrightarrow \bar{q})]\theta_K\theta_M\theta_D, \tag{18}$$

$$\Delta q(x,\eta) = C_{\rm RC}\left[\frac{1}{\eta} f_q(\frac{x}{\eta}, M_{SC}^2)\theta(\eta - x) - f_q(x, M_{SC}^2)\right]\frac{1+\eta^2}{1-\eta}\left(\ln\frac{M_{SC}^2}{m_q^2(1-\eta)^2} - 1\right) \tag{19}$$

where for $C_{\rm QED}$, $C_{\rm QCD}$ multipliers the following expressions are valid

$$C_{\rm QED} = \frac{\alpha}{2\pi}Q_q^2,\ C_{\rm QCD} = \frac{4}{3}\frac{\alpha_s}{2\pi}.$$

For inverse gluon emission (IGE) the result of QS-term subtraction is trivial:

$$\sigma_{\rm IGE} - \sigma_{\rm IGE,QS} = \sigma_{\rm IGE}(m_q \to M_{SC}) \tag{20}$$

3. Discussion of numerical results. Code READY

The scale of radiative corrections and their effect on the observables of Drell-Yan processes will be discussed using FORTRAN program READY [6]. In READY we used the standard PDG set of SM input electroweak parameters with opportunity to choose one of two versions: PDG'08 [7] or PDG'16 [8]. Also there are five active flavors of quarks in proton, and their masses are regulators of the collinear singularity. In addition it is possible to choose one of PDF sets: CTEQ, CT10, or MMHT14 (with the choice $Q = M_{SC} = M$).

To take into account the features of the CMS experiment, the following restrictions were placed in the program:

- Restriction on the detected lepton angle $-\zeta^* \le \zeta \le \zeta^*$ and on the rapidity $|y(l)| \le y(l)^*$: for CMS detector the cut values of ζ^* and $y(l)^*$ are determined as $y(l) = 2.4$.
- The standard CMS restriction to $p_T(l)$: $p_T(l) \ge 20$ GeV.
- The "bare" setup for muons identification requirements (no smearing, no recombination of muon and photon).

For numerical integration our program uses Monte Carlo routine based on the VEGAS algorithm [9].

We provide calculation of $A_{\rm FB}$ asymmetry with $\sqrt{S} = 7$ TeV collider energy to verify good correlation between theoretical predictions and experimental data. Figure 1 shows that the experimental data from Run1 [10] are in agreement with READY numerical predictions in all rapidity ranges. Thus it is possible to use READY for obtain numerical predictions for Run3 at CMS experiment. Run3 will start from 2021, with $\sqrt{S} = 14$ TeV and luminosity up to 300 fb^{-1}. It is supposed that experimental data will be obtained at the invariant mass region $M < 5.2$ TeV with special attention to $M > 4.6$ TeV mass range.

Relative corrections to the differential cross section are presented at Figure 2. It is expected that total NLO correction value will be slowly increased due to rapid growth of QCD-qq correction in the region of large invariant masses and decreasing of EWK corrections value. It is also expected that the IGE correction will change slightly in this invariant mass area.

Forward-backward asymmetry $A_{\rm FB}$ at discussed invariant mass region will also slightly increase in all rapidity ranges. It is expected that its value will not exceed 0.3 for $|y| < 1$ rapidity cut and 0.6 for other cuts.

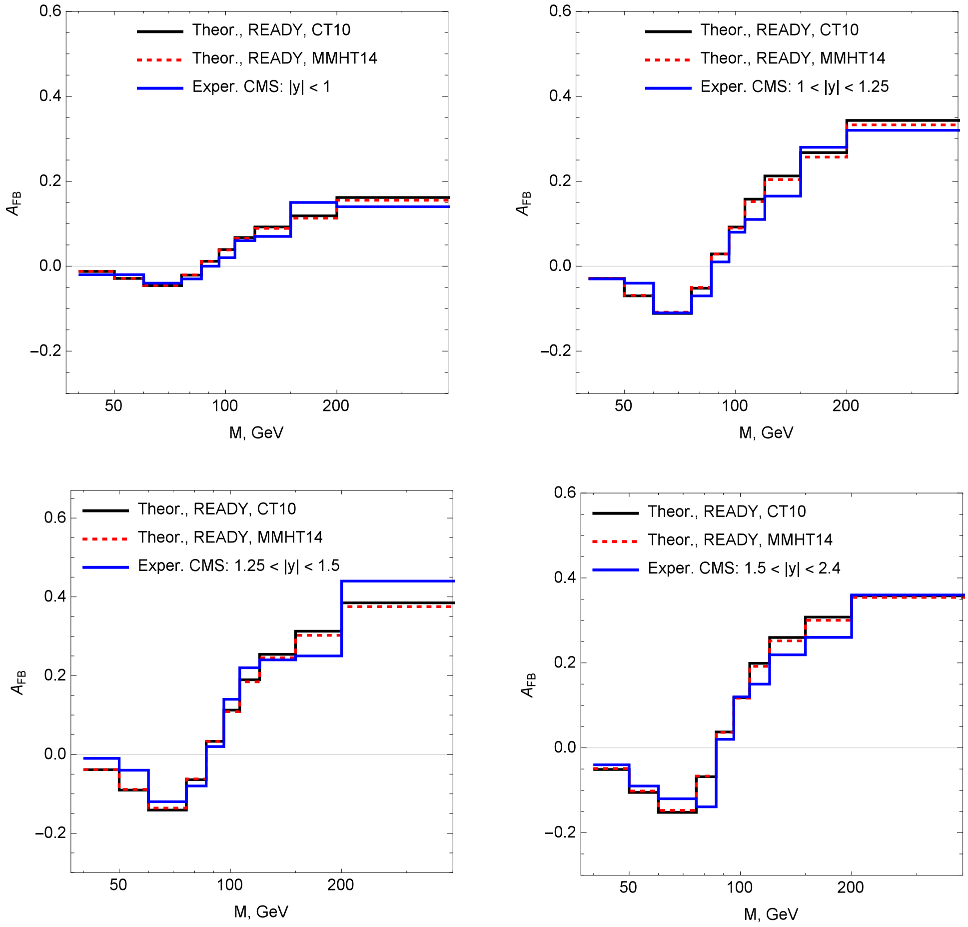

Figure 1. The unfolded $\mu^-\mu^+$ measurements of $A_{\rm FB}$ at the Born level for CMS rapidity cuts at $\sqrt{S} = 7$ TeV, 5 fb^{-1}.

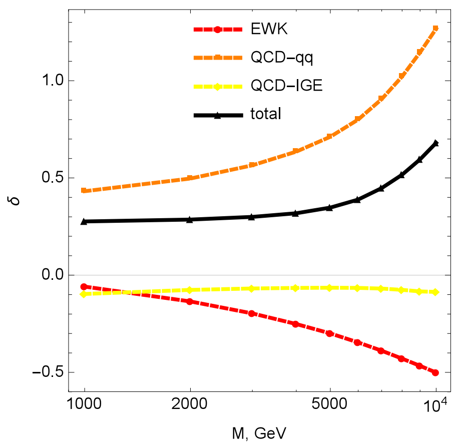

Figure 2. Relative corrections to differential cross section $d\sigma/dM$: $\mu^-\mu^+$ measurements using CT10 PDF set, $\sqrt{S} = 14$ TeV with standard SMC cuts.

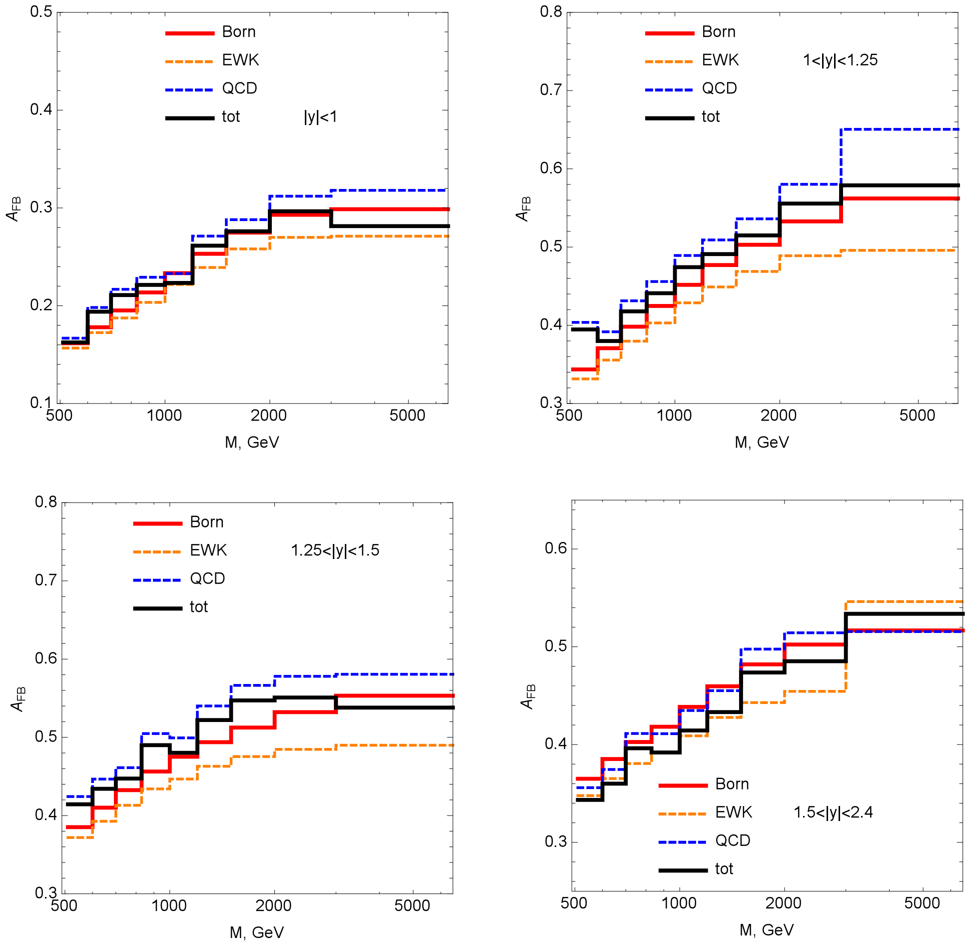

Figure 3. $A_{\rm FB}$ prediction for Run3 of CMS LHC with CT10 PDF set for $\mu^-\mu^+$ measurements at $\sqrt{S} = 14$ TeV for different rapidity cuts.

Acknowledgments
We would like to thank the RDMS CMS group members for the stimulating discussions.

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