

Kerr-Newman black hole as spinning particle

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Abstract. The Kerr-Newman Black Hole (BH) solution has many remarkable properties, which allow us to identify it with a model of the consistent with gravity electron. We consider regularized version of this solution, and show that it gives new important effect – the gravitationally induced Wilson line, which may play very important role in the physics of spin.

1. Introduction

Black Holes (BH) are classical solutions created by gravitational effect of frame-dragging (FD)[1]. In particle physics this effect leads to two new field models which result in unification of the particle physics with gravity:

A) solutions with topologically non-trivial space-time structure, which changes effective scale of gravitational interaction;

B) emergence of the gravitationally induced Wilson loop leading to quantization of the solutions.

The effects A) works in the famous Kerr-Newman (KN) solution, which is the known solution for a charged and rotating black hole, while the effect B) appears in the regularized KN solution.

In 1968, Carter noticed that KN solution has gyromagnetic ratio $g = 2$ just the same as that of the Dirac electron [2], which gave rise to study of the electron model based on the KN solution, see [2, 3, 4, 5, 6, 7] and so on.

The KN model of electron is consistent with gravity by nature, and it is interesting, how the known insuperable contradictions between gravity and quantum physics is solved in the KN solution.

In this paper, and in the series of previous works [8, 9], we show that the reason of contradiction lies in the delusion about weakness of gravity, and in underestimation of the role of spin in gravitational interactions.

The standard reasoning says [10]: “... a length called the Schwarzschild radius, l_s , ... results in the formation of a black hole,... we have

$$l_c = \frac{\hbar}{mc} \quad (1)$$

and

$$l_s = \frac{Gm}{c^2}. \quad (2)$$



These two lengths become equal when m is the Planck mass. And when this happens, they both equal the Planck length...". These arguments become invalid in the Kerr black hole.

Gravitational field of the Kerr solution becomes strong at the Kerr singular ring, which has the radius $a = \frac{J}{mc}$, where J is angular momentum of the Kerr solution. For parameters of the spinning particles one has to use $J \approx \hbar$, which gives for characteristic length of the Kerr solution

$$l_c = l_{Kerr} = a \approx \frac{\hbar}{mc}, \quad (3)$$

equal to the reduced Compton wave length, and the given by John C Baez arguments on the exclusive role of the Planck length do not work for the spinning Kerr gravity.

Kerr singular ring is the branch line of the KN space into two sheets. In fact, the KN solution with parameters of elementary particles is not black hole, because (in the dimensionless units) the typical spinning particles have $a \gg m$, which is condition for disappearance of the black hole horizons. The Kerr singular ring creates two-sheeted space-time with naked singular ring – not covered by horizon, and the special mechanism of regularization is necessary.

2. Basic relations determining structure of the Kerr-Newman solution

In the Kerr-Schild coordinates, metric of the KN solutions is [3]

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \quad (4)$$

where $\eta_{\mu\nu}$ is metric of an auxiliary Minkowski space M^4 , (signature $(-+++)$), and H is the scalar function which for the KN solution takes the form

$$H_{KN} = \frac{mr - e^2/2}{r^2 + a^2 \cos^2 \theta}, \quad (5)$$

and the KN vector potential is

$$A_\mu = \frac{-er}{(r^2 + a^2 \cos^2 \theta)} k_\mu, \quad (6)$$

where

$$k_\mu dx^\mu = dr - dt - a \sin^2 \theta d\phi_K, \quad (7)$$

and the Kerr angular coordinates are related to Cartesian coordinates as follows

$$\begin{aligned} x + iy &= (r + ia) \exp\{i\phi_K\} \sin \theta, \\ z &= r \cos \theta, \quad \rho = r - t, \end{aligned} \quad (8)$$

Vector field k_μ , ($k_\mu k^\mu = 0$), forms Principal Null Congruence of the Kerr solution [3]. Congruence $k^\mu(x)$ forms a vortex of the null field which propagates analytically from negative sheet of Kerr metric, $r < 0$, to positive one, $r > 0$, where the ingoing field k_μ^- turns into outgoing k_μ^+ . In equatorial plane, $\cos \theta = 0$, lines of the Kerr congruence focus on singular ring $r = 0$.

The metrics on the ingoing and outgoing (\pm)-sheets become different $g_{\mu\nu}^\pm = \eta_{\mu\nu} + 2Hk_\mu^\pm k_\nu^\pm$, and there is a freedom in choice of the k_μ^+ or k_μ^- for the "basic" sheet of space [1]. We take as the "basic" the outgoing field k_μ^+ , that corresponds to the classical physical picture with the retarded electromagnetic field, and we chose as positive rotation the anticlockwise direction of the Kerr congruence near the Kerr singular ring.

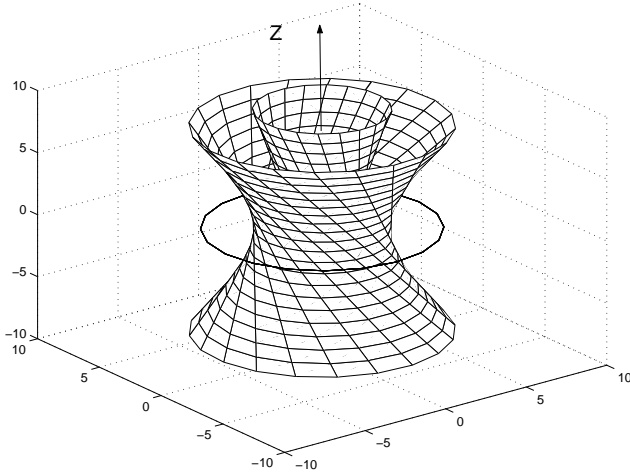


Figure 1. Null lines of the Kerr congruence k^μ form the vortex field which propagates analytically from negative sheet of Kerr metric, $r < 0$, to positive one, $r > 0$.

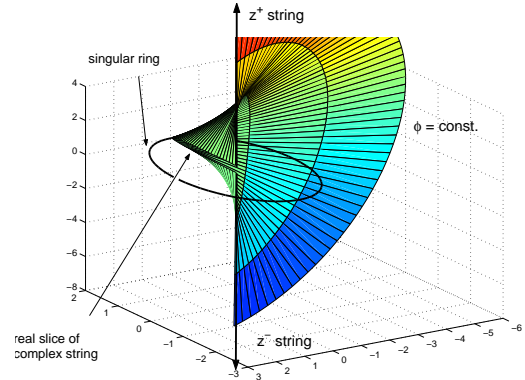


Figure 2. Kerr's coordinate $\phi_K = \text{const.}$ Singular ring drags the KN vector potential, forming the closed Wilson loops along edge border of the bag.

3. Regularized Kerr-Newman black hole as a gravitating bag model [11, 12, 13]

In regularized Kerr-Newman solution, singular region is replaced by a flat core forming interior of the bag model. Flat core is separated from external KN gravity by a Domain Wall, which is built as solution of the Landau-Ginzburg field model of superconductivity. Thus, the flat interior of the bag is to be superconducting [14].

Border of the core is defined unambiguously as the surface where $H_{KN} = 0$, or at

$$r = r_e = e^2/2m, \quad (9)$$

(López, 1986 [5]). The KN metric at this surface becomes flat, $g_{\mu\nu} = \eta_{\mu\nu}$, and the external KN space can be matched with flat interior of the bag, which corresponds to the Gürsey and Gİses class of metrics [15]. As far as r is the Kerr radial coordinate, (8), bag takes the form of an oblate ellipsoid $r = r_e$, see Fig.3. The relations (8) show that bag takes the form of a thin ellipsoidal disk of the thickness $2r_e = e^2/m$ and radius

$$r_c = \sqrt{r_e^2 + a^2}, \quad (10)$$

what is slightly more than the reduced Compton wave length $a = \hbar/2mc$. So that the ratio

$$r_e/a = \alpha \approx 1/137, \quad (11)$$

becomes the fine structure constant, and the ratio $r_e/r_c = \frac{r_e}{a}(1 + r_e^2/a^2)^{-1/2} = \alpha(1 + \alpha^2)^{-1/2}$, which characterizes the degree of oblateness of the bag, is very close to $\alpha \approx 1/137$. The Cartesian distance δ works as cutoff for the KN electromagnetic and gravitational field, see Fig.4.

4. Wilson loop and quantization of angular momentum [11, 13, 16, 18]

From (6) and (7) we obtain that vector-potential of the regularized KN solution takes its maximal value in equatorial plane ($\cos \theta = 0$) at the bag border $r = r_e$,

$$A_\mu^{max} dx^\mu = -\frac{2m}{e}(dr - dt - ad\phi_K). \quad (12)$$

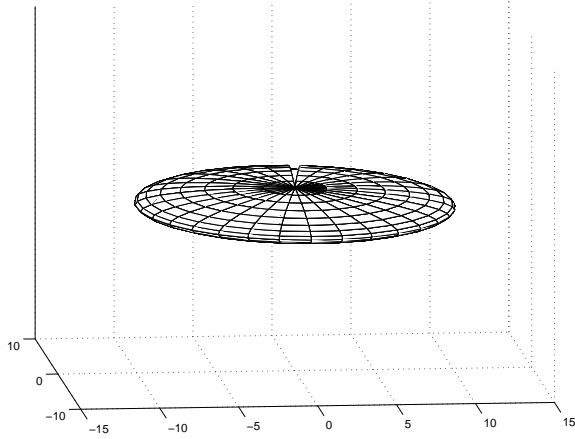


Figure 3. Ellipsoidal shape of the Kerr-Newman bag defined by the boundary surface $r = r_e = e^2/2m$.

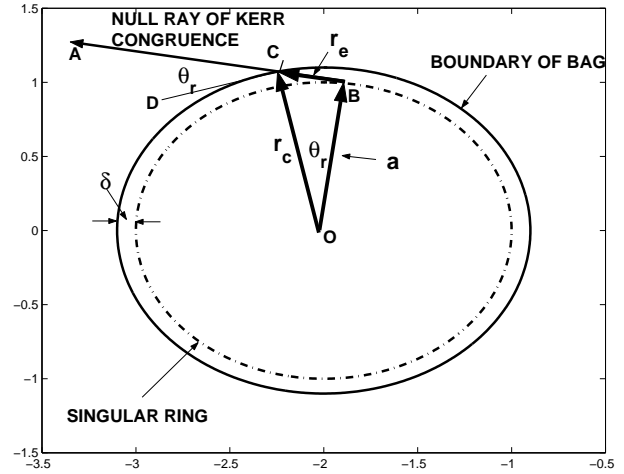


Figure 4. Kinematic relation in equatorial plane: the tangent to singular ring null ray of congruence k_μ crosses border of the bag at the angle $\theta_r \approx \alpha$.

The component A_r is a perfect differential (see for example [3]) and can be ignored. Rest of the potential is tangent to the bag border, and for fixed time, $t = \text{const.}$, it forms closed Wilson loop $W(C) = P \exp e \oint_C A_\mu^{max} dx^\mu$.

Taking the contour C as the closed loop $\phi_K \in [0, 2\pi]$ along border of the bag at $t = \text{const.}$, we obtain following incursion of the potential

$$\delta\phi = e \oint_C A_{\phi_K}^{max} d\phi_K = 4\pi ma = 4\pi J, \quad (13)$$

where the last equality follows from the basic Kerr relation $J = ma$. Definiteness of $W(C)$ requires $\delta\phi = n2\pi$, and we obtain the quantum condition $J = \frac{1}{2}, 1, \frac{3}{2}, \dots$

5. Superconducting core of the regularized KN black hole [14, 18, 19, 20, 21]

To separate classical gravity from quantum interior of the bag we use the Landau-Ginzburg (LG) field model, which is also well known as a phenomenological model of superconductivity. Famous examples of its application is the Nielsen-Olesen string model [22], and the MIT and SLAC bag models [23, 24]. The simplest LG field model has one chiral field Φ , which is interpreted as a complex Higgs field $H = |H| \exp i\chi \equiv \Phi$, [22]. Lagrangian has the form

$$\mathcal{L}_{NO} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\mathcal{D}_\mu\Phi)(\mathcal{D}^\mu\Phi)^* - V(|\Phi|), \quad (14)$$

where $\mathcal{D}_\mu = \nabla_\mu + ieA_\mu$ are $U(1)$ covariant derivatives and $F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}$.

We use the Domain Wall solution which can separate the KN gravity outside the bag from flat space with superconducting vacuum state inside the bag. In particular, $\Phi = 0$ outside the bag, and $\Phi > 0$ in the core. Domain Wall solution gives $V = 0$, inside and outside the bag, and $V > 0$ on the border of bag. Therefore, Domain Wall has ellipsoidal shape. The Higgs field H , concentrated inside the bag, creates there superconducting vacuum state, which interacts with external potential according the equations

$$\partial_\nu \partial^\nu A_\mu^{max} = J_\mu = e|H|^2(\chi_{,\mu} + eA_\mu^{max}). \quad (15)$$

Oscillating phase of the Higgs field $\chi(t, \phi)$ eats the potential A_μ^{max} inside the bag, where $J_\mu = 0$, and there is only the superficial circular current along the bag border. Indeed, situation is more complicated, and the correct description requires the supersymmetric LG model and two Higgs-like fields, [25, 26], for details see [14]. Supersymmetric bag model creates circular stringy structure on the sharp border of the bag and consistent embedding the Dirac equation [8, 12, 27].

6. Conclusion

The over-rotating KN black hole solution brings non-trivial topology in the structure of spinning particles, opening up new possibilities for combining gravity with particle physics.

The regularized KN black hole solution does not contradict quantum structure of the elementary particles, and in the same time it brings a new important effect to particle physics – the appearance of a closed Wilson loop, created by the gravitational effect of frame-dragging.

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