Electromagnetic and gravitomagnetic structure of pions and pion-nucleon scattering

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Abstract. Taking into account PDFs obtained by various Collaborations, the momentum transfer dependence of GPDs of the pion are obtained. The calculated electromagnetic and gravitomagnetic form factors of the pions and nucleons are used for the description of the pion-nucleon elastic scattering in a wide energy and momentum transfer region with minimum fitting parameters.

1. Introduction

The remarkable property of $GPDs$ is that the integration of different momenta of GPDs over x gives us different hadron form factors [1]. The x dependence of $GPDs$ is in most part determined by the standard PDFs, which are obtained by Collaborations from the analysis of the deep-inelastic processes.

Let us modify the original Gaussian ansatz and choose the t-dependence of GPDs in a simple form $\mathcal{H}^q(x,t) = q(x) \, exp[a_+ \, f(x) \, t]$, with $f(x) = (1-x)^2/x^\beta$ [2]. The isotopic invariance can be used to relate the proton and neutron GPDs.

The complex analysis of the corresponding description of the electromagnetic form factors of the proton and neutron by different PDF sets (24 cases) was carried out in [3]. These PDFs include the leading order (LO), next leading order (NLO) and next-next leading order (NNLO) determination of the parton distribution functions. They used the different forms of the x dependence of PDFs. We slightly complicated the form of GPDs in comparison with the equation used in [2], but it is the simplest one as compared to other works (for example [4]).

The hadron form factors will be obtained by integration over x in the whole range $0-1$. Hence, the obtained form factors will be dependent on the x-dependence of the forms of PDF at the ends of the integration region. On the basis of our GPDs with PDFs ABM12 [5] we calculated the hadron form factors by the numerical integration and then by fitting these integral results by the standard dipole form with some additional parameters $F_1(t) = (4m_p - \mu t)/(4m_p - t) \, \tilde{G}_d(t)$ with $\tilde{G}_d(t) = 1/(1 + q/a_1 + q^2/a_2^2 + q^3/a_3^3)^2$ that is slightly different from the standard dipole form on two additional terms with small sizes of coefficients. The matter form factor

$$A(t) = \int_0^1 x \, dx [q_u(x) e^{2\alpha_H f(x)_u/t} + q_d(x) e^{2\alpha_H f_d(x)/t}] \tag{1}$$

is fitted by the simple dipole form $A(t) = \Lambda^4/(\Lambda^2 - t)^2$. These form factors will be used in our model of the proton-proton and proton-antiproton elastic scattering and further in one of the vertices of the pion-nucleon elastic scattering.

2. Hadron form factors and elastic nucleon-nucleon scattering

Both hadron electromagnetic and gravitomagnetic form factors were used in the framework of the high energy generalized structure (HEGS) model of the elastic nucleon-nucleon scattering. This allows us to build a model with a minimum number of fitting parameters [8, 9, 10].

The model is very simple from the viewpoint of the number of fitting parameters and functions. There are no any artificial functions or any cuts which bound the separate parts of the amplitude by some region of momentum transfer. In the framework of the model the description of the experimental data was obtained simultaniously at the large momentum transfer and in the Coulomb-hadron region, using CNI phase [12, 13], in the energy region from $\sqrt{s} = 9$ GeV up to LHC energies. The model gives a very good quantitative description of the latest experimental data at $\sqrt{s} = 13$ TeV [11].

3. GPDs of pion

The pion structure in some sense is simpler than the nucleon structure. In the nucleon there are 3 constituent quarks that can create different configurations, for example, such as "Mercedes star" or a linear structure with a quark on one end and a di-quark on the other. These configurations can lead to different results for hadron interactions, for example, the Odderon-hadron coupling. For a meson we have only two quark states

$$|\pi> = |q\bar{q}> + |q\bar{q}\; q\bar{q}> + |q\bar{q}\; g> \,.....$$

It is needed to note that the standard definition of the pion form factor through the matrix elements of the electromagnetic vector current gives

$$\langle \pi^+(\vec{p'})|V_\mu(0)|\pi^+(\vec{p})\rangle = (p'_\mu + p_\mu)F_\pi(Q^2), \tag{2}$$

with $Q^2 = -q^2$ and $F_\pi(Q^2)$ being the space-like form factor of a pion [14].

We have focused on the zero-skewness limit, where GPDs have a probability-density interpretation in the longitudinal Bjorken x and the transverse impact-parameter distributions,

$$\mathcal{H}^q(x,t) \; = q(x) \; e^{2a_H f(x)_q \; t}; \quad \mathcal{E}^q(x,t) \; = q(x)(1-x)^\gamma \; e^{2a_E \; f(x)_q \; t}; \tag{3}$$

where $f_q(x) = \frac{(1-x)^{2+\epsilon_u}}{(x_0+x)^m}$.

Hence, the obtained form factors will be dependent on the x-dependence of the forms of PDF at the ends of the integration region. Some PDFs have the polynomial form of x with different power. Some other have the exponential dependence of x. As a result, the behavior of PDFs, when $x \to 0$ or $x \to 1$, can impact the form of the calculated form factors.

Different Collaborations have determined the PDF sets from the inelastic processes only in some region of x, which is only approximated to $x = 0$ and $x = 1$. Also, there is a serious problem in determining the main ingredient of GPDs of a pion - the form of the parton distribution functions. The predictions based on the perturbative QCD and the calculations using the Dyson-Schwinger equation lead to $(1-x)^2$ at $x \to 1$. However, the constituent quark model and calculation in the framework of the Nambu-Jona-Lasino model lead to the linear behavior $(1-x)_{x\to1}$. Several next-to-leading order (NLO) analyses of the Drell-Yan data show that the valence distribution turned out to be rather hard at high momentum fraction x , typically showing only a linear or slightly faster falloff. Correspondingly, there are many different forms of the PDF of a pion.

We examine many of them and keep two PDFs leading to approximately the same results: one is (Mezrag et al. (2016) [15]) and other is [16]

On the basis of our GPDs with PDFs we have calculated the pion form factors by the numerical integration and then by fitting these integral results by the standard monopole
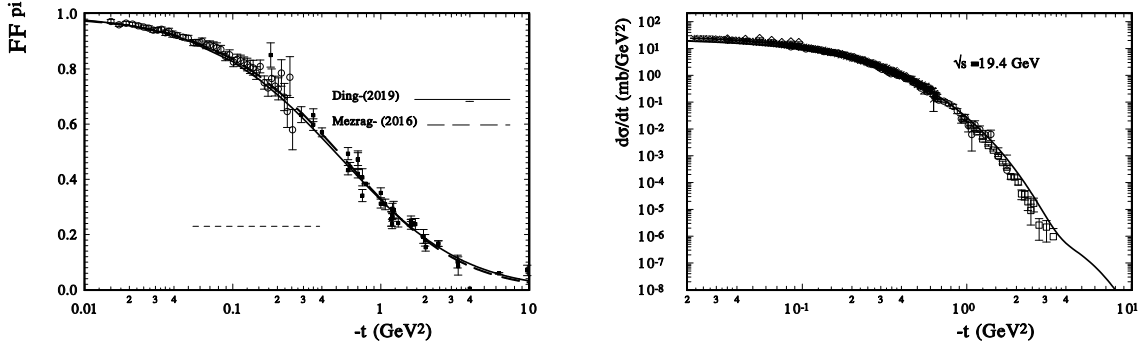
Figure 1. a) [left] The electromagnetic form factor of the π-meson (hard and dashed curves our calculations with PDF [15] and and [16], respectively; the circles and squares - the experimental data [17]

b) [right] The differential cross sections of the elastic scattering of $\pi^+ p$ at $\sqrt{s} = 19.4$ GeV (curves - our model calculations, circles, quires, triangles up and triangles down - [18]

form and obtained $\Lambda_\pi^2 = 0.55$. In Fig.1a, the comparison of our calculation with the existing experimental data of the pion form factor are presented. It is seen that the difference between the calculations of our two chosen PDFs is small.

The matter form factor $A_{Gr}^\pi(t)$ is calculated as the second Mellin momentum

$$A_{Gr}^\pi(t) = \int_0^1 x \; dx \; q_\pi(x) e^{2\alpha_\pi f(x)/t} \tag{4}$$

and is fitted by the simple dipole form $A(t) = \Lambda^4/(\Lambda^2 - t)^2$. These form factors will be used in our model of the $\pi^+ p$ and πp elastic scattering. Our calculations of the second momentum of GPDs of a pion are shown that the impact of different PDFs is tangible only at large momentum transfer.

4. Hadron form factors and the elastic pion-nucleon scattering

Let us determine the Born terms of the elastic pion-nucleon scattering amplitude in the same form as we determined the elastic nucleon-nucleon scattering amplitudes. Using both the (electromagnetic and gravitomagnetic) form factors of a pion and a nucleon, we obtain two asymptotic terms:

$$F_{mh}^{Born}(s,t) = \quad h_1 \; G_N(t) \; G_\pi(t) \; F_a(s,t) + h_2 \; A_N(t) \; A_\pi(t) \; F_b(s,t) \tag{5}$$

where $F_a(s,t)$ and $F_b(s,t)$ have the standard Regge form:

$$F_a(s,t) \; = (\hat{s}^{\epsilon_1} \; + \; r_\pi/\sqrt{\hat{s}}) \; e^{B(\hat{s}) \; t}; \quad F_b(s,t) \; = \hat{s}^{\epsilon_1} \; e^{B(\hat{s})/4 \; t},$$

$\hat{s} = s \; e^{-i\pi/2}/s_0$; $s_0 = 1$ GeV^2, and intercept $1 + \Delta_1 = 1.11$ was chosen the same as for the nucleon-nucleon elastic scattering. Hence, at the asymptotic energy we have the universality

of the energy behavior of the elastic hadron scattering amplitudes. The slope of the scattering amplitude having the standard logarithmic dependence on the energy $B(s) = \alpha' \, ln(\hat{s})$ with $\alpha' = 0.24$ GeV^{-2} is again the same size as for the nucleon-nucleon elastic scattering Examining the pion-nucleon elastic scattering at low energies, we take into account the contributions of the non-leading Reggions using the form factors of the pion and nucleon. The final elastic hadron scattering amplitude is obtained after unitarization of the Born term. So, at first, we have to calculate the eikonal phase $\chi(s,b) \;\; = \; -\frac{1}{2\pi} \; \int \; d^2q \; e^{i\vec{b}\cdot\vec{q}} \; F_h^{\mathrm{Born}}(s,q^2)$ and then obtain the final hadron scattering amplitude.

We take into account the experimental data on the $\pi^+ p$ and $\pi^- p$ elastic scattering from $\sqrt{s} = 7.807$ GeV up to the maximum measured at $\sqrt{s} = 25.46$ GeV. The total number of the experimental data $N_{exp.} = 2022$. As in the case of the nucleon scattering, we take into account in the fitting procedure the statistical and systematic errors separately. Only the statistical errors are included in the standard calculations of χ^2. The systematic errors are taken into account as some additional normalization of the experimental data of the separate set. As a result, we have obtained $\chi^2/n_{d.o.f.} = 1.2$. In Fig. 1b the model calculations are compared with the elastic $\pi^+ p$.

5. Conclusion

We have examine the new form of the momentum transfer dependence of GPDs of hadrons to obtain different form factors, including Compton form factors, electromagnetic form factors, transition form factor and gravitomagnetic form factor. Our model of GPDs, based on the analysis of practically all existing experimental data on the electromagnetic form factors of the proton and neutron, leads to a good description of the proton and neutron electromagnetic form factors simultaneously. The chosen form of the momentum transfer dependence of GPDs of the pion (the same as t-dependence of nucleon) allows us to describe the electromagnetic form factor of pions and obtain the pion gravitomagnetic form factors. As a result, the description of different reactions based on the same representation of the hadron structure was obtained. The model opens up a new way to determining the true form of the GPDs and hadrons structure.

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