Hadron polarization in strong magnetic field

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Abstract. The magnetic dipole polarizabilities and hyperpolarizabilities of the vector ρ^0 and $\rho^\pm$ have been calculated. The new characteristic of the QCD vaccum in the strong magnetic field called tensor polarizability has been introduced, which is related to the tensor polarization. The contribution of the dipole magnetic polarizabilities to the tensor polarization of the vector mesons in the external abelian magnetic field has been explored.

1. Introduction
The presence of an abelian magnetic field of a hadron scale creates a kind of anisotropy in the vacuum of quantum chromodynamics. By a strong magnetic field we mean fields that can affect the quark currents inside hadrons.This leads to the appearance of the higher order magnetic polarizabilities and hyperpolarizabilities and to a deviation of the energy square from the linear field dependence corresponding to Landau levels. In our previous simulations in the lattice $SU(3)$ gauge theory without dynamical quarks we have observed this non-linear behavior at $eB \sim 0.2 - 0.4$ GeV^2 [1]. The magnetic field effect on the hadronic medium was explored in theoretical models [2, 3, 4, 5, 6, 7, 8] and in lattice quantum chromodynamics [9, 10, 11, 12, 13, 14, 15].

The observed response of a strongly interacting medium to a strong magnetic field arises as a result of the interaction of QED and QCD. Spin interactions play a very important role, since the energy of vector mesons strongly depends on the relative orientation of the spins of the quarks that form the meson and determine the values of polarizability and hyperpolarizability. Therefore, the energy of a vector meson in a strong magnetic field depends on its polarization. The vector ρ mesons decay into virtual photons produce the dilepton asymmetries [16] relative to the direction of emission of the virtual photon. The dilepton asymmetries can allow to investigate the various channels to which particles decay. Also these physical quantities allow to explore the physical properties and evolution of a quark-gluon plasma [17]. The tensor polarizabilities and polarization make possible to appreciate the values of the dilepton asymmetries, so below we can explore the contribution of the dipole magnetic polarizabilities to the tensor polarization of the vector mesons.

2. Magnetic polarizabilities of the $\rho^{\pm}$ mesons

According to the conservation of parity for the spin projection $s_z = 0$, the square of energy receives corrections from the non-linear terms of even powers in the magnetic field. For spin projections $s_z = +1$ and $s_z = -1$ the squared energy contains terms of both even and odd powers in the field. We found that the contributions of these terms depend on the cjnsidered interval of magnetic fields. So, for the considered lattices the correction of the third power term to the lowest energy sub-level with $qs_z = +1$ is not larger than 20% and compatible with errors at $eB \in [0, 1.2]\,\text{GeV}^2$. This can be seen from Fig. 9 presented in our previous work [1]. Similarly, a correction of the fourth power does not give a significant contribution to the energy squared for the case $s_z = 0$.

At strong magnetic field the energy squared reveals non-linear behavior depending on the magnetic field value. To take this phenomena into account we include the terms with the magnetic dipole polarizability and hyperpolarizabilities into consideration. It follows from the parity conservation that the energy squared for the spin projection $s_z = 0$ can contains only terms of even powers in a field, while for the spin projections $s_z = -1$ and $s_z = +1$ the terms both even and odd powers in a field contribute. In our previous work we have found that the contributions of these terms depend on the field interval considered [1].

So, at $eB \in [0, 1.2]\,\text{GeV}^2$ one can find the magnetic dipole polarizability β_m for the for the spin projection $s_z = 0$ from the following relation

$$E^2_{s_z=0} = |eB| + m^2 - 4\pi m \beta_m (eB)^2, \tag{1}$$

where m are the particle mass at zero magnetic field, eB is the magnetic field in GeV^2. The values of the β_m and m were found as the fit parameters from the lattice data using formula (1). At Fig. 1 we depict the energy squared versus the magnetic field for the lattice volume 18^4, the lattice spacings 0.105 fm and 0.115 fm and different pion masses. The values of the β_m are shown in Table 1.
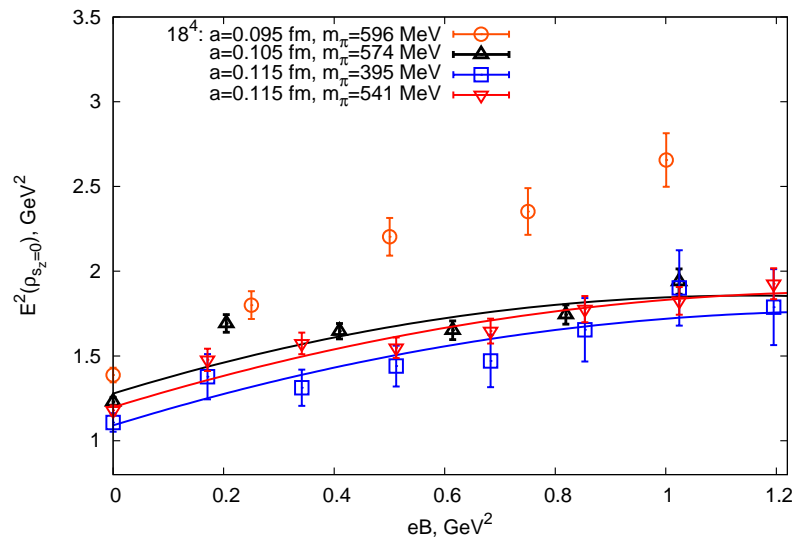

Figure 1. The energy squared of the $\rho^{\pm}$ meson for the spin $s_z = 0$ versus the magnetic field value for various lattices and pion masses. The solid lines are the fits of the lattice data for the energy of the $\rho^{\pm}$ meson.

V	$m_\pi(\text{MeV})$	$a(\text{fm})$	$\beta_m(\text{GeV}^{-3})$	$\chi^2/d.o.f.$
18^4	574 ± 7	0.105	0.03 ± 0.01	6.90
18^4	395 ± 6	0.115	0.028 ± 0.006	0.53
18^4	541 ± 3	0.115	0.027 ± 0.004	1.25

Table 1. The values of the magnetic polarizability β_m of the $\rho^\pm$ meson for the spin projection $s_z = 0$ for various lattices and several pion masses.

The lattice values of the energy squared of the $\rho^\pm$ with $qs_z = +1$ (which corresponds to ρ^- at $s_z = -1$ and ρ^+ at $s_z = +1$) can be fitted by the following formula

$$E^2_{qs_z=+1} = |eB| - g(eB) + m^2 - 4\pi m \beta_m (eB)^2 \tag{2}$$

at $eB \in [0, 1.2]$ GeV^2. The values of m, g and β_m are the fit parameters. In Table 2 we represent the β_m values. The corresponding lattice data with their fits are shown in Fig.2 for various lattices and pion masses.
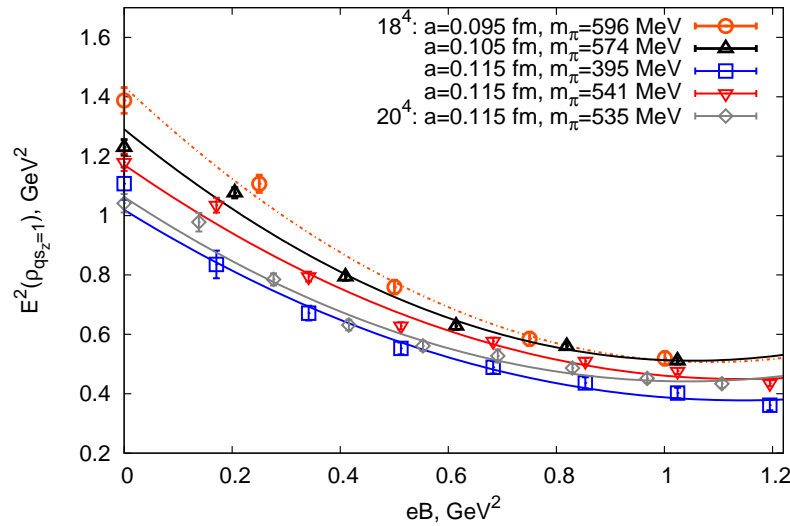

Figure 2. The energy squared of the charged ρ meson for the case $qs_z = +1$ depending on the magnetic field value for various lattice data sets. The solid lines correspond to the fits of the lattice data obtained with use of the formula (2)

For the case $qs_z = -1$ one can find the β_m using the formula

$$E^2_{qs_z=-1} = |eB| + g(eB) + m^2 - 4\pi m \beta_m (eB)^2 \,. \tag{3}$$

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The parity conservation demands the equality of the magnetic dipole polarizabilities for $s_z = +1$ and $s_z = -1$. It also follows from (2) and (3) and was suggested by our previous results [1].

V	m_π(MeV)	a(fm)	g-factor	β_m(GeV^{-3})	$\chi^2/d.o.f.$
18^4	574 ± 7	0.105	2.48 ± 0.19	-0.049 ± 0.010	2.66
18^4	541 ± 3	0.115	2.26 ± 0.14	-0.041 ± 0.006	2.32
20^4	535 ± 4	0.115	2.19 ± 0.12	-0.044 ± 0.006	1.48
18^4	395 ± 6	0.115	2.12 ± 0.13	-0.039 ± 0.006	1.49

Table 2. The magnetic dipole moment and the magnetic dipole polarizability of the charged ρ meson with $qs_z = +1$ for the lattice spacings 0.105 fm, 0.115 fm, the lattice volume 18^4, various pion masses and for the lattice spacing 0.115 fm, the lattice volume 20^4 and the pion mass $m_\pi = 535(4)$ MeV with their errors and $\chi^2/d.o.f$ values. The results were obtained with the use of 3-parametric fit (2) at $eB \in [0, 1.2]$ shown in Fig. 2.

3. Magnetic polarizability of the ρ^0 meson

When calculating physical observables, it is necessary to take into account different coupling of u and d quarks with an external magnetic field is different. The energy of a neutral ρ meson with a spin projection $s_z = 0$ is expressed as follows:

$$E^2 = m^2 - 4\pi m \beta_m (eB)^2 - 4\pi m \beta_m^{h2}(eB)^4 - 4\pi m \beta_m^{h4}(eB)^6 - 4\pi m \beta_m^{h6}(eB)^8 - ... , \qquad (4)$$

where β_m^{h2}, β_m^{h4} and β_m^{h6} are the magnetic hyperpolarizabilities of higher orders, m is the mass of the meson at zero field.
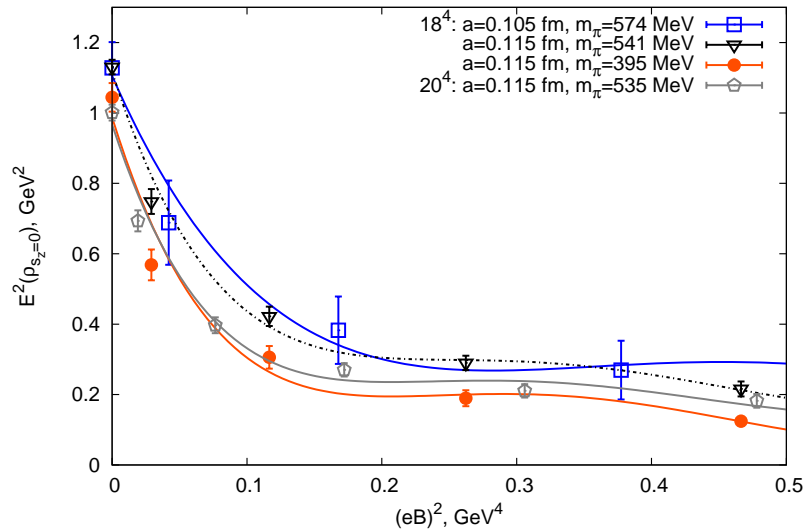

Figure 3. The energy squared of the ρ^0 ($s_z = 0$) ground state versus the field value squared. The data are shown by points for various lattice spacings, pion masses and two lattice volumes 18^4 and 20^4 with the fits obtained using formula (4).

In Fig. 3 the squared energy of the ρ^0 meson with $s_z = 0$ from the squared field for several lattices and pion masses at $(eB)^2 \in [0 : 0.5]$ GeV^4 is shown. It is seen that energy is rapidly falling. The lattice spacing, the lattice volume, and the pion mass also strongly affect the meson energy. From the formula (4) follows the fits, which are lines, which also take into account the terms $\sim (eB)^{10}$ and $\sim (eB)^{12}$. The fits for the lattice with spacing $a = 0.105$ fm were performed

at $(eB)^2 \in [0:1.7]\,\mathrm{GeV}^4$, for the other lattices we use $(eB)^2 \in [0:1.5]\,\mathrm{GeV}^4$. The contribution of the terms of higher degrees is noticeable only at low fields. When restricting the magnetic field, it is not easy to extract magnetic polarizability. Extrapolation to the chiral limit was not carried out, since only qualitative predictions are needed.

The Table 3 shows the magnetic polarizability β_m and hyperpolarizability β_m^{2h} obtained from the fits. The lattice volume V, the lattice spacing a, the pion mass m_π, the interval of fields selected for the fitting procedure and $\chi^2/n.d.o.f.$ are also shown.

V	$a(\mathrm{fm})$	$m_\pi(\mathrm{MeV})$	$\beta_m(\mathrm{GeV}^{-3})$	$\beta_m^{2h}(\mathrm{GeV}^{-7})$	n	$(eB)^2(\mathrm{GeV}^4)$	$\chi^2/d.o.f.$
18^4	0.105	574 ± 7	0.66 ± 0.16	-2.51 ± 0.98	10	$[0:1.7]$	1.04
18^4	0.115	541 ± 3	0.90 ± 0.16	-5.11 ± 1.59	12	$[0:1.5]$	2.46
20^4	0.115	535 ± 4	0.95 ± 0.15	-5.78 ± 1.60	12	$[0:1.5]$	2.63
18^4	0.115	395 ± 6	0.98 ± 0.30	-5.79 ± 2.74	12	$[0:1.5]$	3.32

Table 3. The value of the magnetic dipole polarizability β_m and the magnetic hyperpolarizability β_m^{2h} for the ρ^0 with spin projection $s_z = 0$ are shown for the various lattice volumes V, lattice spacings a and pion masses m_π. The degree of polynomial n, field range used for the fitting and $\chi^2/d.o.f$ values are represented in columns sixth to eighth correspondingly.
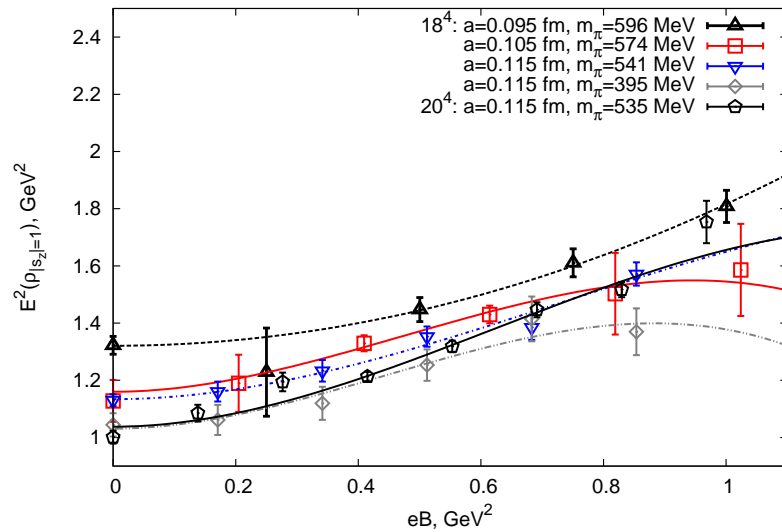

Figure 4. The energy squared of the $\rho^0(|s_z| = 1)$ meson versus magnetic field for various lattice spacings, pion masses, and two lattice volumes 18^4 and 20^4. The lines correspond to the fits of the lattice data obtained using formula (5).

At $eB \in [0:1.2]\,\mathrm{GeV}^2$ for the spin projection $|s_z| = 1$ on the field axis the square of the energy of the neutral vector meson is expressed as follows:

$$E^2 = m^2 - 4\pi m \beta_m (eB)^2 - 4\pi m \beta_m^{h1}(eB)^3\,. \tag{5}$$

In Fig. 4 the energy squared is shown for the ρ^0 meson with the spin projection $|s_z| = 1$, the energies of the neutral vector meson for the $s_z = +1$ and $s_z = -1$ are the same due to the

conservation of C-parity. The magnetic polarizability β_m is obtained from the fit of the lattice data by formula (5) for the lattices with spacings 0.105 fm and 0.115 fm, where m, β_m and β_m^{h1} are the fit parameters. The lattice data for $a = 0.084$ fm and $a = 0.095$ fm are given to check the lattice volume and lattice spacing effects. The β_m values with the errors and other parameters are shown in Table 4. The results agree with each other within the errors

V	a(fm)	m_π(MeV)	β_m(GeV^{-3})	β_m^{1h}(GeV^{-5})	eB, GeV^2	$\chi^2/d.o.f.$
18^4	0.105	574 ± 7	-0.10 ± 0.02	0.07 ± 0.02	$[0:1.1]$	0.47
18^4	0.115	541 ± 3	-0.07 ± 0.02	0.03 ± 0.03	$[0:1.1]$	0.87
20^4	0.115	535 ± 4	-0.10 ± 0.02	0.06 ± 0.03	$[0:1.1]$	1.54
18^4	0.115	395 ± 6	-0.11 ± 0.03	0.08 ± 0.03	$[0:1.1]$	0.65

Table 4. The value of the magnetic dipole polarizability β_m and the magnetic hyperpolarizability β_m^{1h} are shown for ρ^0 with $|s_z| = 1$, for the lattice volumes 18^4 and 20^4, the lattice spacings 0.105 fm and 0.115 fm and various pion masses. The fourth and the last columns contain the intervals of the magnetic field used for the fit and $\chi^2/d.o.f.$ correspondingly.

4. Tensor polarizability of the vector mesons

The differential cross section for the dilepton pair production

$$\frac{d\sigma}{dM^2 d\cos\theta} = A(M^2)(1 + B\cos^2\theta), \tag{6}$$

where M^2 is the energy of the lepton pair in their rest system, θ is the angle between the momentum of the virtual photon and the lepton emission direction. The coefficient B is defined by:

$$B = \frac{\gamma_\perp - \gamma_\parallel}{\gamma_\perp + \gamma_\parallel}, \tag{7}$$

where the $\gamma_{\perp,\parallel}$ are transverse and longitudinal polarizations of the virtual intermediate photon.

The cross section (6) depends on the energy of the lepton. At the same time one of the main sources of production of a pair of dileptons is the decay of ρ mesons. We have found that the energy of the neutral ρ meson with the spin projection $s_z = 0$ decreases versus the magnetic field value, while energy for the $s_z = \pm 1$ increases. We also obtain the decreasing energy of the value for the charged ρ meson for case $qs_z = +1$. It is natural to anticipate that particles with lower energy will dominate in non-central heavy-ion collisions, i.e. states with a defined polarization.

The cross section of the emitted dileptons is related to the polarization of the ρ-meson in the following way:

$$\frac{d\sigma}{dM^2 d\cos\theta} = N(M^2)(1 + \frac{1}{4}P_{33}(3\cos^2\theta - 1)), \tag{8}$$

where the form of the third diagonal component of the polarizability tensor P_{33} follows from its definition. One can represent this quantity in terms of the probabilities of the ρ meson to have a spin projection equal to $+1$, -1 and 0 respectively

$$P_{33} = w_{s_z=+1} + w_{s_z=-1} - 2w_{s_z=0}. \tag{9}$$

The value of the P_{33} is related with the asymmetry coefficient B by the following way

$$B = \frac{3P_{33}}{4 - P_{33}}. \tag{10}$$

We introduce some new physical quantity, which can serve as the measure of the magnetic field effect on a vector meson, the tensor polarizability

$$\beta_t = \frac{\beta_{s_z=+1} + \beta_{s_z=-1} - 2\beta_{s_z=0}}{\beta_{s_z=+1} + \beta_{s_z=-1} + \beta_{s_z=0}}, \tag{11}$$

which at high temperature it has to be proportional to the polarization of the vector meson.

In Table 5 we represent the values of the tensor polarizability for the ρ^0-meson for the lattice volumes 18^4 and 20^4, lattice spacings 0.105 fm and 0.115 fm and various pion masses. For the $\rho^\pm$ meson the β_t values are shown in Table 6. The negative values of β_t of the ρ^0-meson reveal that the longitudinal polarization dominate. The dileptons are basically emitted in the plane perpendicular to the field direction. The dominating longitudinal polarization for the soft dileptons was obtained previously in [18]. The formation of soft dileptons was compared with a nonzero component of the conductivity of the strongly interacting matter along direction of the external magnetic field [19].

V	a(fm)	m_π(MeV)	β_t
18^4	0.105	574 ± 7	-3.3 ± 0.6
18^4	0.115	541 ± 3	-2.6 ± 0.2
20^4	0.115	535 ± 4	-2.8 ± 0.3
18^4	0.115	395 ± 6	-2.9 ± 0.5

Table 5. The tensor polarizability β_t of the ρ^0 meson is shown for various lattices and pion masses.

V	a(fm)	m_π(MeV)	β_t
18^4	0.105	574 ± 7	2.3 ± 0.7
18^4	0.115	541 ± 3	2.5 ± 0.5
18^4	0.115	395 ± 6	2.7 ± 0.7

Table 6. The tensor polarizability of the $\rho^\pm$ is represented for lattice volume 18, lattice spacings 0.105 fm and 0.115 fm and various pion masses.

5. Conclusions

In this paper some new interesting statements are represented, such as the fact that dileptons are mainly emitted in the directions perpendicular to the magnetic field axis, and the negative values of β_t suggest the dominating longitudal polarization of the ρ^0-meson. We have found out the convincing result,that the energy of the state with s_z=0 decreases, and the energy of $|s_z|$=1 increases. Also, we have found that the longitudinal polarization of the ρ^0 mesons dominates in the colisions, since low energy is preferable to high. Therefore, the dileptons occuring due to decays of the ρ^0 mesons will be emitted perpendicular to the direction of the magnetic field. Finaly, we have introduced new characteristics of the meson magnetic properties-the tensor polarizability. This quantity has been suggested to be related to the coefficient of asymmetry in the differential cross section for the dilepton production.

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