

On the spin correlations of final leptons produced in the high-energy annihilation and two-photon processes $e^+e^- \rightarrow \mu^+\mu^-, \tau^+\tau^-$ and $\gamma\gamma \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-$

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Abstract. The electromagnetic processes of annihilation of (e^+e^-) pairs, into heavy flavor lepton pairs are theoretically studied in the one-photon approximation, using the technique of helicity amplitudes. For the process $e^+e^- \rightarrow \mu^+\mu^-$, it is shown that – in the case of the unpolarized electron and positron – the final muons are also unpolarized but their spins are strongly correlated; the structure of their triplet states is analyzed and explicit expressions for the correlation tensor components are derived. On the other hand, the theoretical study of spin structure for the processes of lepton pair production by photon pairs is also performed: for the two-photon process $\gamma\gamma \rightarrow e^+e^-$, it is found that – quite similarly – in the case of unpolarized photons the spins of final unpolarized particles prove to be strongly correlated, and explicit expressions for the correlation tensor components and relative fractions of singlet and triplet states of the final system are obtained. It is established that in both these processes the spin correlations of final leptons have the purely quantum character, since one of the Bell-type incoherence inequalities for the correlation tensor components is always violated. Analogous analysis can be wholly applied also to the annihilation process $e^+e^- \rightarrow \tau^+\tau^-$ and to the two-photon processes $\gamma\gamma \rightarrow \mu^+\mu^-, \gamma\gamma \rightarrow \tau^+\tau^-$, becoming possible at considerably higher energies.

1. Helicity amplitudes for the annihilation process $e^+e^- \rightarrow \mu^+\mu^-$ and structure of the triplet states of the final $(\mu^+\mu^-)$ system

In the first non-vanishing approximation over the electromagnetic constant $e^2/\hbar c$, the process of conversion of the (e^+e^-) pair into the muon pair is described by the well-known one-photon Feynman diagram [1]. Due to the electromagnetic current conservation, the virtual photon with a time-like momentum transfers the total angular momentum $J = 1$ and negative parity. Since the internal parities of muons μ^+ and μ^- are opposite, the $(\mu^+\mu^-)$ pair is generated in the triplet states (total spin $S = 1$) with $J = 1$ and the orbital angular momenta $L = 0$ and $L = 2$.

The respective helicity amplitudes have the following structure:

$$f_{\Lambda'\Lambda}(\theta, \phi) = R_{\Lambda'\Lambda}(E) d_{\Lambda'\Lambda}^{(1)}(\theta) \exp(i\Lambda\phi), \quad (1)$$

where θ, ϕ are the polar and azimuthal angles of the flight direction of the positive muon (μ^+) in the c.m. frame of the reaction with respect to the initial positron momentum; $d_{\Lambda'\Lambda}^{(1)}(\theta)$ are

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the Wigner functions for $J = 1$; Λ is the difference of helicities of the positron and electron, coinciding with the projections of total spin and total angular momentum of the (e^+e^-) pair onto the direction of positron momentum in the c.m. frame; Λ' is the difference of helicities of the muons μ^+ and μ^- , coinciding with the projection of total angular momentum of the $(\mu^+\mu^-)$ pair onto the direction of momentum of μ^+ in the c.m. frame (see, e.g., [1,2]).

In Eq. (1), $R_{\Lambda'\Lambda}(E) = r_{\Lambda'}^{(\mu)}(E) r_{\Lambda}^{(e)}(E)$ ($\Lambda', \Lambda = +1, 0, -1$) – due to the factorizability of the Born amplitude, and here $r_{+1}^{(\mu)} = r_{-1}^{(\mu)} = r_1^{(\mu)}$, $r_{+1}^{(e)} = r_{-1}^{(e)} = r_1^{(e)}$ – owing to the space parity conservation in the electromagnetic interactions . Further, in accordance with the structure of electromagnetic current for the pairs (e^+e^-) and $(\mu^+\mu^-)$ in the c.m. frame [1], the following relations are valid:

$$r_0^{(\mu)} = \frac{m_\mu}{E} r_1^{(\mu)} = \sqrt{1 - \beta_\mu^2} r_1^{(\mu)}, \quad r_0^{(e)} = \frac{m_e}{E} r_1^{(e)}, \quad (2)$$

where m_μ, m_e are the muon and electron masses and β_μ is the muon velocity in the c.m. frame. Thus, since we always have $E \geq m_\mu \gg m_e$ for the given process, the contribution of (e^+e^-) -states with equal helicities can be neglected, i.e. $R_{\Lambda 0}(E) \approx 0$. Meantime, for $r_1^{(e)}$ and $r_1^{(\mu)}$ the one-photon diagram calculation gives (here e is the electron charge) :

$$r_1^{(\mu)}(E) = r_1^{(e)}(E) = \frac{|e|}{\sqrt{2E}}. \quad (3)$$

Taking into account Eqs. (1)–(3), it is clear that, in the cases of total polarization of both the positron and electron along the positron momentum in the c.m. frame and in the direction being antiparallel to the positron momentum, the $(\mu^+\mu^-)$ system is produced, respectively, in the triplet states of the following form [3] :

$$|\Psi\rangle^{(+1)} = \frac{\sqrt{2}}{\sqrt{2 - \beta_\mu^2 \sin^2 \theta}} \left(\frac{1 + \cos \theta}{2} | + 1 \rangle - \sqrt{1 - \beta_\mu^2} \frac{\sin \theta}{\sqrt{2}} | 0 \rangle + \frac{1 - \cos \theta}{2} | - 1 \rangle \right). \quad (4)$$

$$|\Psi\rangle^{(-1)} = \frac{\sqrt{2}}{\sqrt{2 - \beta_\mu^2 \sin^2 \theta}} \left(\frac{1 - \cos \theta}{2} | + 1 \rangle + \sqrt{1 - \beta_\mu^2} \frac{\sin \theta}{\sqrt{2}} | 0 \rangle + \frac{1 + \cos \theta}{2} | - 1 \rangle \right). \quad (5)$$

Here β_μ is the velocity of each of the muons – just as in Eq. (2), and $| + 1 \rangle, | - 1 \rangle, | 0 \rangle$ are the states with the projection of total spin of the $(\mu^+\mu^-)$ pair onto the direction of momentum of μ^+ in the c.m. frame, equaling $+1, -1$ and 0 , respectively.

2. Correlation tensor of the $(\mu^+\mu^-)$ pair and violation of “classical” incoherence inequalities

If the positron and electron are not polarized, then, since $r_0^{(e)} \approx 0$, the final state of the $(\mu^+\mu^-)$ pair represents an incoherent mixture of spin states $|\Psi\rangle^{(+1)}$ (4) and $|\Psi\rangle^{(-1)}$ (5), each of them being realized with the relative probability of $1/2$.

The components of the correlation tensor for two particles with spin $1/2$ are defined as: $T_{ik} = \langle \hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)} \rangle$ ($i, k \rightarrow \{1, 2, 3\} \rightarrow \{x, y, z\}$; the symbol $\langle \dots \rangle$ denotes the averaging over the quantum ensemble). For the final $(\mu^+\mu^-)$ pair under consideration, the axis z is directed along the momentum of μ^+ in the c.m. frame of the reaction $e^+e^- \rightarrow \mu^+\mu^-$, and the axis y – along the normal to the reaction plane.

It is easy to see that, in the case of unpolarized primary positron and electron, the produced muons μ^+ and μ^- are also unpolarized but their spins are correlated, and, in particular, the diagonal components of the correlation tensor have the following form [3] :

$$T_{xx}^{(\mu^+\mu^-)} = \frac{(2 - \beta_\mu^2) \sin^2 \theta}{2 - \beta_\mu^2 \sin^2 \theta}, \quad T_{yy}^{(\mu^+\mu^-)} = -\frac{\beta_\mu^2 \sin^2 \theta}{2 - \beta_\mu^2 \sin^2 \theta}, \quad T_{zz}^{(\mu^+\mu^-)} = \frac{2 \cos^2 \theta + \beta_\mu^2 \sin^2 \theta}{2 - \beta_\mu^2 \sin^2 \theta}, \quad (6)$$

Just as it should hold for triplet states, the “trace” of the correlation tensor is equal to unity: $T_{xx}^{(\mu^+\mu^-)} + T_{yy}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = 1$.

Regarding the components (6) of the correlation tensor, we observe here the violation of the “classical” incoherence inequalities (for incoherent mixtures of factorizable states of two particles with spin 1/2, the modulus of sum of any two (and three) diagonal components of the correlation tensor cannot exceed unity [4,5]). Indeed, according to Eqs. (6), one of the incoherence inequalities is always violated at $\theta \neq 0$ [3] :

$$T_{xx}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = 1 - T_{yy}^{(\mu^+\mu^-)} = \frac{2}{2 - \beta_\mu^2 \sin^2 \theta} > 1. \quad (7)$$

Thus, we see that the spin correlations of muons in the process $e^+e^- \rightarrow \mu^+\mu^-$ have the strongly pronounced quantum character.

Certainly, the above consideration can be wholly applied also to the process $e^+e^- \rightarrow \tau^+\tau^-$ – with the replacements $m_\mu \rightarrow m_\tau$, $\beta_\mu \rightarrow \beta_\tau$. At very high energies $E \gg m_\mu$ (m_τ), when $\beta_\mu, \beta_\tau \rightarrow 1$, the diagonal components (6) of the correlation tensor for the final lepton pair take the following values:

$$T_{xx} = \frac{\sin^2 \theta}{1 + \cos^2 \theta}, \quad T_{yy} = -\frac{\sin^2 \theta}{1 + \cos^2 \theta}, \quad T_{zz} = 1, \quad (8)$$

and we see that one of the incoherence inequalities is still violated : $T_{xx} + T_{zz} \geq 1$.

Finally, it should be noted that at very high energies the annihilation processes $e^+e^- \rightarrow \mu^+\mu^-$, $\tau^+\tau^-$ are conditioned not only by the electromagnetic interaction through the virtual photon, but also by the weak interaction of neutral currents through the virtual Z^0 boson [6] (see our paper [3] for the most detailed analysis, and also [7,8]). Here we just remark that the structure of the correlation tensor of the final leptons involving the weak interaction is, on the whole, similar to that for purely electromagnetic annihilation at very high energies (see Eqs. (8)): the diagonal components of the correlation tensor are: $T_{zz} = 1$ and $T_{xx} = -T_{yy}$ (but, of course, the expression for T_{xx} changes [3]), and again one of the incoherence inequalities for the correlation tensor components is violated: $T_{xx} + T_{zz} > 1$ [3] .

3. Spin structure of the two-photon process $\gamma\gamma \rightarrow e^+e^-$ and correlation tensor of the (e^+e^-) pair

Now let us consider another electromagnetic process – the electron-positron pair production by two photons : $\gamma\gamma \rightarrow e^+e^-$.

The spin state of the electron-positron system is described, in the general case, by the two-particle spin density matrix :

$$\hat{\rho}^{(e^-e^+)} = \frac{1}{4} \left[\hat{I}^{(e^-)} \otimes \hat{I}^{(e^+)} + \hat{I}^{(e^-)} \otimes (\hat{\sigma}^{(e^+)} \mathbf{P}^{(e^+)}) + (\hat{\sigma}^{(e^-)} \mathbf{P}^{(e^-)}) \otimes \hat{I}^{(e^+)} + \sum_{i=1}^3 \sum_{k=1}^3 T_{ik} \hat{\sigma}_i^{(e^-)} \otimes \hat{\sigma}_k^{(e^+)} \right], \quad (9)$$

where \hat{I} is the two-row unit matrix, $\mathbf{P}^{(e^-)}$ and $\mathbf{P}^{(e^+)}$ are the polarization vectors of the electron and positron, respectively, T_{ik} – components of the correlation tensor ($T_{ik} = \langle \hat{\sigma}_i^{(e^-)} \otimes \hat{\sigma}_k^{(e^+)} \rangle$, $i, k = \{1, 2, 3\} = \{x, y, z\}$). In the absence of correlations, we have: $T_{ik} = P_i^{(e^-)} P_k^{(e^+)}$.

The process $\gamma\gamma \rightarrow e^+e^-$ is described by two well-known Feynman diagrams [1]. Within the Born approximation, in the case of unpolarized primary photons the final electron and positron prove to be also unpolarized, but their spins are correlated, i.e. in the formula for $\hat{\rho}^{(e^+e^-)}$ (9) we have: $\mathbf{P}^{(e^-)} = \mathbf{P}^{(e^+)} = 0$; meantime, the components of the correlation tensor of the (e^+e^-) pair, generated by unpolarized γ quanta, may be calculated by applying the results of the paper [9]. Finally we obtain the following expressions:

$$T_{zz} = 1 - \frac{2(1 - \beta^2)[\beta^2(1 + \sin^2 \theta) + 1]}{1 + 2\beta^2 \sin^2 \theta - \beta^4 - \beta^4 \sin^4 \theta}, \quad (10)$$

$$T_{xx} = \frac{(1 - \beta^2)[\beta^2(1 + \sin^2 \theta) - 1] + \beta^2 \sin^4 \theta}{1 + 2\beta^2 \sin^2 \theta - \beta^4 - \beta^4 \sin^4 \theta}, \quad T_{yy} = \frac{(1 - \beta^2)[\beta^2(1 + \sin^2 \theta) - 1] - \beta^2 \sin^4 \theta}{1 + 2\beta^2 \sin^2 \theta - \beta^4 - \beta^4 \sin^4 \theta}, \quad (11)$$

Here the axis z is aligned along the positron momentum in the c.m. frame, the axis x lies in the reaction plane and the axis y is directed along the normal to the reaction plane; $\beta = \frac{v}{c}$, $1 - \beta^2 = \frac{m_e c^2}{E_+}$, where \mathbf{v} is the positron velocity and E_+ is the positron (or electron) energy in the c.m. frame; θ is the angle between the positron momentum and the momentum of one of the photons in the c.m. frame.

The “trace” of the correlation tensor of the final (e^+e^-) pair is determined by the formula:

$$T = T_{xx} + T_{yy} + T_{zz} = 1 - \frac{4(1 - \beta^2)}{1 + 2\beta^2 \sin^2 \theta - \beta^4 - \beta^4 \sin^4 \theta}. \quad (12)$$

and, respectively, the relative fractions of the triplet and singlet states ($W_t = (T + 3)/4$, $W_s = (1 - T)/4$ [5]) are as follows:

$$W_t = 1 - \frac{1 - \beta^2}{1 + 2\beta^2 \sin^2 \theta - \beta^4 - \beta^4 \sin^4 \theta}, \quad W_s = \frac{1 - \beta^2}{1 + 2\beta^2 \sin^2 \theta - \beta^4 - \beta^4 \sin^4 \theta}. \quad (13)$$

At $\beta \ll 1$ we have: $W_t \approx 0$, $W_s \approx 1$, i.e. the (e^+e^-) pair is generated in the singlet state only. Meantime, if $\beta \approx 1$ and $2\sin^2 \theta - \sin^4 \theta > 1 - \beta^2$, we obtain, on the contrary: $W_t \approx 1$, $W_s \approx 0$ – almost pure triplet state of the pair.

4. Violation of “classical” incoherence inequalities for correlation tensor components in the processes $\gamma\gamma \rightarrow e^+e^-$, $\mu^+\mu^-$, $\tau^+\tau^-$

Let us consider now the particular cases $\theta = 0$ and $\theta = \pi$. According to the general expressions (10), (11) for the correlation tensor components, at $\theta = 0$ and $\theta = \pi$ we have:

$$T_{zz} = 1 - \frac{2(1 + \beta^2)(1 - \beta^2)}{1 - \beta^4} = -1; \quad T_{xx} = T_{yy} = -\frac{1 - \beta^2}{1 + \beta^2}. \quad (14)$$

In doing so, the “trace” of the correlation tensor (12) and the relative fractions of the triplet and singlet states W_t, W_s (13) take the values:

$$T = 1 - \frac{4}{1 + \beta^2} = -\frac{3 - \beta^2}{1 + \beta^2}, \quad W_t = \frac{T + 3}{4} = \frac{\beta^2}{1 + \beta^2}, \quad W_s = \frac{1 - T}{4} = \frac{1}{1 + \beta^2}. \quad (15)$$

At nonrelativistic velocities $W_t \approx 0$, $W_s \approx 1$, in accordance with the general case ; meantime, in the ultrarelativistic limit ($\beta \rightarrow 1$) we have : $W_t = W_s = \frac{1}{2}$.

It should be stressed that in the process $\gamma\gamma \rightarrow e^+e^-$ we observe again – just as in the above-considered reaction $e^+e^- \rightarrow \mu^+\mu^-$ – the violation of the “incoherence” inequalities for diagonal components of the correlation tensor, established previously at the classical level [5] (see Sec. 2). Indeed, for the cases $\theta = 0$ and $\theta = \pi$ we obtain, in particular (since $\beta < 1$) :

$$|T_{zz} + T_{xx}| = |T_{zz} + T_{yy}| = \frac{2}{1 + \beta^2} > 1. \quad (16)$$

Thus, the spin correlations of the final electron and positron in the considered two-photon process also have the strongly pronounced quantum character (see also [10, 11]) – quite similarly to the spin correlations of the final leptons in the processes $e^+e^- \rightarrow \mu^+\mu^-$, $\tau^+\tau^-$.

Finally, one should remark that analogous results hold also for the processes of generation of a muon pair and a tau-lepton pair by two photons : $\gamma\gamma \rightarrow \mu^+\mu^-$, $\gamma\gamma \rightarrow \tau^+\tau^-$, which become possible at considerably higher energies .

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