

# Chiral vortical and Chiral torsional effects

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**Abstract.** Corrections to the chiral vortical effect due to the finite volume and the presence of mass are discussed. The possibility to describe the chiral vortical effect (CVE) by an effective gauge field is considered. The new chiral effect arising in the presence of torsion is described.

## 1. Introduction

Chiral vortical effect is the appearance of axial current in fermionic system in the presence of rotation. This effect was predicted for the first time by A. Vilenkin [1]. He found the following expression for the axial current of massless Dirac particles (In the limit of high temperatures):

$$\vec{j}^5 = -\frac{1}{6}\vec{\Omega}T^2 \quad (1)$$

In the presence of chemical potential  $\mu$  the additional term arises in the expression for the axial current:

$$\vec{j}^5 = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2}\right)\vec{\Omega} + \dots \quad (2)$$

There was a hope, that the value of the coefficient in front of the vorticity does not depend on the interactions and can be fixed by the consideration of the axial anomaly. It was shown, however [3, 4], that the higher orders of perturbation theory are able to correct, at least, the coefficient in front of  $T^2$  in Eq. (2).

The CVE belongs to the family of the so - called anomalous transport phenomena. Such phenomena have also their incarnations in the solid state physics [5]. This occurs, in particular, because the electronic system of the discovered recently Weyl and Dirac semimetals simulates relativistic physics and the corresponding excitations at the low energies are described by Dirac equation [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. The other representatives of this family are, for example, the so - called anomalous quantum Hall effect [17], the chiral magnetic effect [18], the scale magnetic effect [19], the rotational Hall effect [20], and Chiral Torsional effect [21]. Recently the presence of the anomalous quantum Hall effect was predicted in the three - dimensional systems: the Weyl semimetals [14, 15] and the topological insulators [22]. At the

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same time the absence of the equilibrium version of the chiral magnetic effect was proved in [23]. It is expected, that both the chiral separation effect and the chiral vortical effect as well as the proposed recently rotational Hall effect [20] are to be observed in the heavy ion collisions [24].

As for the CVE, the correct description of the uniform rotation implies consideration of boundary conditions (for example, in order to keep the causality). Therefore, the influence of the boundary conditions on the CVE has to be investigated.

## 2. Axial current in rotating system with MIT boundary conditions

We consider rotating fermions in the impenetrable infinite cylinder of radius  $R$ . MIT boundary conditions imply the restriction:

$$j^\mu n_\mu = 0 \quad (3)$$

Even in the case of non-interacting fermions the calculation of axial current is a difficult problem. First this is necessary to solve the Dirac equation and then to chose the appropriate solutions using boundary conditions. After that it is possible to calculate the current, which is given by expression:

$$\langle j_z^5(r) \rangle_\beta = \sum_{k,q,\text{sign}(w),j_z} n(w, j_z) \bar{\psi}_{k,q,\text{sign}(w),j_z} \gamma_5 \gamma_3 \psi_{k,q,\text{sign}(w),j_z} \quad (4)$$

$$n(w, j_z) = \frac{\text{sign}(w)}{e^{\beta(w-\mu-\Omega j_z)\text{sign}(w)} + 1} \quad (5)$$

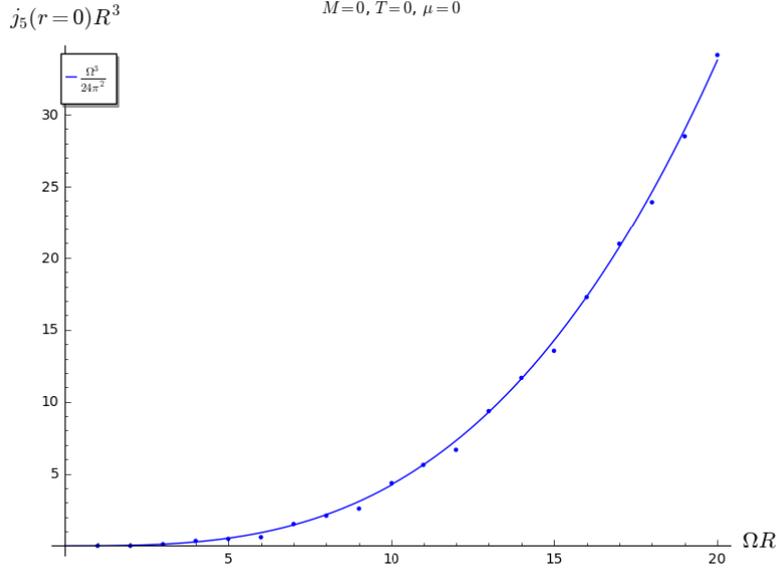
Here the sum is over all quantum numbers  $w$ , which enumerate the eigenstates of energy. By  $\psi_{k,q,\text{sign}(w),j_z}$  we denote the corresponding eigenfunction, which is the 4 - component complex - valued spinor,  $n(w, j_z)$  is Fermi distribution. Correspondingly,  $\bar{\psi} = (\psi^*)^T \gamma^0$  is the conjugated 4 - component spinor. Detailed description of the calculations may be found in [25].

The main conclusions are as follows: the finite volume affects essentially the axial current. In particular, at low enough temperatures the oscillations appear in the dependence of the axial current density on the chemical potential. At any values of temperature the axial current density varies fast when the distance to the rotation axis is changed. The previous results obtained in the infinite volume limit are reproduced in our calculations only in the small vicinity of the rotation axis.

One can also compare the two distinct approaches to the definition of rotation. Rotation may be taken into account via consideration of the system in the rotating referece frame or it may be introduced to the hamiltonian using effective gauge field. At small enough values of temperature (much smaller than  $|\mu - M|$ ) those two approaches give the same value of the axial current density in the infinite volume limit. At the same time the first definition of rotation clearly differs from the second one in case of vanishing chemical potential (i.e. at  $\mu \ll T$ ).

The results obtained using the introduction of rotation via an effective gauge field ensure that the coefficient at the term  $\sim \mu^2$  in Eq. (2) is topologically protected[2, 26] at vanishing temperature. That means that it is not changed when we modify smoothly the system. The introduction of interactions being such a smooth change cannot renormalize, therefore, this coefficient. There is no such a correspondence between the two mentioned approaches at large enough temperatures, which is in accordance with the conclusions of [3, 4] that the corresponding term in Eq. (2) may be modified due to interactions.

The contribution of the term in the axial current that is proportional to the third power of angular velocity at  $\Omega R < 1$  is smaller than errors of calculation, but it clearly arises if the causality condition is violated for  $\Omega R \geq 1$  (see Figure).



**Figure 1.** Axial current along rotation axis in case  $\Omega R > 1$ .

### 3. Chiral torsional effect.

As it was mentioned above, in some sense the rotation may be treated as an external gauge field. The same situation holds for torsion and one can calculate the response of the axial current  $j^{5\mu}(x) = \bar{\psi}(x)\gamma^\mu\gamma^5\psi(x)$  to torsion in terms of the Wigner transformation of the Green function. We consider the case of vanishing spin connection and the nontrivial torsion encoded in the vielbein  $e_\mu^a(x) = \delta_\mu^a + \delta e_\mu^a(x)$ . The inverse vielbein is denoted here by  $E_a^\mu = \delta_a^\mu + \delta E_a^\mu$ , it obeys equation  $e_\mu^a(x)E_b^\mu(x) = \delta_b^a$ , which gives  $\delta e_b^a(x) \approx -\delta E_b^a$ . Momentum operator is to be substituted by  $p_a \rightarrow (\delta_a^b + \delta E_a^b(x))p_b \approx (p_a - \delta e_a^b(x)p_b)$ . And finally one can obtain:

$$j^{5k} = \frac{i}{2} T_{ij}^a \int \frac{d\omega d^3p}{(2\pi)^4} p_a \text{Tr} G \partial_{p_i} G^{-1} \partial_{p_j} G \gamma^5 \partial_{p_k} G^{-1} \quad (6)$$

$$T_{ij}^a = \partial_i e_j^a - \partial_j e_i^a, T_{ij}^0 = \epsilon_{fij}^0 S^f \quad (7)$$

$$\vec{j}^5 = \frac{T^2}{12} \vec{S} \quad (8)$$

This is the chiral torsional effect (CTE), which, in principle, could be observed in Weyl and Dirac semimetals, and also in some other condensed matter systems. It is worth mentioning that from the point of view of pure mathematics CVE and CTE have the same origin. Actually, the CVE may be considered as a particular case of the CTE (for the details see [21]).

### 4. Conclusion

To conclude, we considered the chiral vortical effect (CVE) and the chiral torsional effect (CTE). In both cases the axial currents appear. In the case of the CVE the axial current is caused by rotation. In the case of CTE the axial current is caused by the space - time torsion. The CTE may be thought, to some extent, as the generalization of the CVE. For the case of the CVE we investigated the dependence of the axial current on the finite volume. It appears, that the

conventional expression for the CVE appears close to the rotation axis only while far from the axis the finite volume corrections are dramatically large.

### *Acknowledgments*

Z.V.K. and M.A.Z. are grateful to G.E.Volovik for the correspondence and useful comments. The work of Z.V.K. was supported by Russian Science Foundation Grant No 16-12-10059.

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