

Problems with experimental observation of violation of Wigner inequalities in a system of neutral kaons

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Abstract. In the current work we add background to a system of neutral kaons using a Werner state. We show that the violation of Wigner inequalities is only observable in experiment in case when the background contribution is less than 1%.

1. Introduction

In the beginning of the 20 century after quantum mechanics (where probability plays the main role) was created, scientists wonder on the fundamental principles of this theory. For instance: what is the nature of probability? Is it a fundamental property of our life? There were two very different and very convincing points of view, suggested by two groups of scientists. First group contained N. Bohr, W. Heisenberg, and W. Pauli. They considered probability as a fundamental property and so the Copenhagen interpretation of quantum mechanics was founded. The second group included A. Einstein, Louis de Broglie and others. They thought that probability is some temporary simplification of quantum mechanics and in the future it will be replaced by a new one, more common, quantum theory. This approach resulted in the Statistical interpretation of quantum mechanics. It means that macrodevices can not measure any characteristics of a microsystem, so results of the measurements are averaging of all values [1].

In 1935 A. Einstein, B. Podolsky, and N. Rosen created a list of properties of theory which have to change quantum mechanics. It is called “local realism” [2].

The concept of local realism based on three main principles:

- **Classical realism**
An aggregate of all physical characteristics (in classical terms) of a system exists jointly and is independent of an observer, even if the observer cannot simultaneously measure these characteristics with any classical measurement device.
- **Locality**
If two measurements are performed in spatially-separated points of the spacetime, then the readings of one classical device do not affect the readings of a second one in any way.
- **Freedom of choice**
The observer can freely choose any experimental parameters from the available ones.

A. Einstein, B. Podolsky, and N. Rosen have shown that the local realism is in the contradiction with entangled state theory. This contradiction is known as “EPR paradox”.



In 1964 J. Bell has invented an inequality (named after him) which allowed to test in experiment the local realism in quantum theory [3] [4]. Bell used the hypothesis of hidden parameters. However in 1969 J. F. Clauser, M. A. Horne, A. Shimony, and A. R. Holt modified this inequality without using the hidden parameter hypothesis [5]. Both inequalities contain correlators which is very difficult to calculate, especially in quantum field theory.

Finally in 1970 E. Wigner suggested an inequality for probability which is more suitable for calculations [6]. We will demonstrate this.

Let us consider a pair of fermions: “1” and “2” with anticorrelated spin projections:

$$n_{\pm}^{(1)} = n_{\mp}^{(2)},$$

where n_{\pm} is the state of the fermion with the spin projection $\pm \frac{1}{2}$ and $i = \{1, 2\}$ is the number of the fermion.

Then we can select three different non-parallel directions \vec{a} , \vec{b} and \vec{c} and use the triangle inequality. After that we will have the Wigner inequality in form:

$$w(a_+^{(2)}, b_+^{(1)} | A^{(2)}, B^{(1)}) \leq w(c_+^{(2)}, b_+^{(1)} | C^{(2)}, B^{(1)}) + w(a_+^{(2)}, c_+^{(1)} | A^{(2)}, C^{(1)}), \quad (1)$$

where “A”, “B”, and “C” is the macrodevices which perform measurements.

In 1968 an idea arised to check the Wigner inequalities in the system of neutral pseudoscalar mesons [7], for instance in kaons that appear in Bell state $|\psi^-\rangle$ [8] after $\phi(1020)$ -meson decay [9] [10].

These experimental data can confirm or disprove the local realism concept. Thus getting these data is an important step to the understanding of the nature of quantum physics.

2. Basis in a systems of neural kaons

As we will see, one can introduce three bases in a system of neutral kaons which is an analogy of the bases in a spin-projection space. Let us show how it was built. The first component of the kaon's bases is

$$|K^0\rangle = |d\bar{s}\rangle$$

and

$$|\bar{K}^0\rangle = |\bar{d}s\rangle.$$

This is a flavour basis.

Neutral kaons can decay into a pair of pions. Pions can be neutral or have a different charge. Although in both cases they have specific CP-symmetry but $|K\rangle$ and $|\bar{K}\rangle$ does not have it:

$$\hat{C}\hat{P}|K^0\rangle = |\bar{K}^0\rangle,$$

$$\hat{C}\hat{P}|\bar{K}^0\rangle = |K^0\rangle,$$

where \hat{C} is a charge-symmetry operator and \hat{P} is a space-symmetry operator.

The second basis is created as CP-conservation basis:

$$|K_1^0\rangle = \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}},$$

$$|K_2^0\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}}.$$

These states already have a certain CP-symmetry:

$$\hat{C}\hat{P} |K_1^0\rangle = |K_1^0\rangle,$$

$$\hat{C}\hat{P} |K_2^0\rangle = -|K_2^0\rangle.$$

In 1964 Christensen, Cronin, Fitch found the CP-violation in the decay of CP-oddstate K_2^0 [11]. So the third basis is created as K_L^0, K_S^0 -eigenstate of the mass and proper time of the kaons.

$$|K_L^0\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} (|K_2^0\rangle + \varepsilon |K_1^0\rangle) = p |K^0\rangle - q |\bar{K}^0\rangle,$$

$$|K_S^0\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} (|K_1^0\rangle + \varepsilon |K_2^0\rangle) = p |K^0\rangle + q |\bar{K}^0\rangle,$$

where ε is a CP-violation parameter, which was obtained experimentally [12]:

$$|\varepsilon| = (2,228 \pm 0,011) \times 10^{-3};$$

$$|Re(\varepsilon)| = (1,596 \pm 0,013) \times 10^{-3}.$$

And p and q are some complex parameters.

Thus these three kaon-state bases correspond to some spin-projection bases on these three different directions. Kaon's flavour, CP-symmetry and mass and proper time of kaons is the dichotomic values in the system of neutral kaons. Similar bases could be built in a system of any neutral pseudoscalar mesons.

3. Twelve Wigner inequalities

$K^0 \bar{K}^0$ -pairs can be created in the decay of $\phi(1020)$ -meson. They are produced in Bell state:

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}} \left(|K^{(2)}\rangle \otimes |\bar{K}^{(1)}\rangle - |\bar{K}^{(2)}\rangle \otimes |K^{(1)}\rangle \right).$$

It is an analog of anticorrelation in a spin-projection's bases.

Twelve different inequalities are possible:

$$w(K^{(2)}, K_L^{(1)}) \leq w(K_2^{(2)} K_L^{(1)}) + w(K^{(2)}, K_2^{(1)}); \quad (2)$$

$$w(K^{(2)}, K_H^{(1)}) \leq w(K_2^{(2)} K_H^{(1)}) + w(K^{(2)}, K_2^{(1)}); \quad (3)$$

$$w(\bar{K}^{(2)}, K_L^{(1)}) \leq w(K_2^{(2)} K_L^{(1)}) + w(\bar{K}^{(2)}, K_2^{(1)}); \quad (4)$$

$$w(\bar{K}^{(2)}, K_H^{(1)}) \leq w(K_2^{(2)} K_H^{(1)}) + w(\bar{K}^{(2)}, K_2^{(1)}); \quad (5)$$

$$w(K^{(2)}, K_2^{(1)}) \leq w(K_L^{(2)} K_2^{(1)}) + w(K^{(2)}, K_L^{(1)}); \quad (6)$$

$$w(K^{(2)}, K_1^{(1)}) \leq w(K_L^{(2)} K_1^{(1)}) + w(K^{(2)}, K_L^{(1)}); \quad (7)$$

$$w(\bar{K}^{(2)}, K_2^{(1)}) \leq w(K_L^{(2)} K_2^{(1)}) + w(\bar{K}^{(2)}, K_L^{(1)}); \quad (8)$$

$$w(\bar{K}^{(2)}, K_1^{(1)}) \leq w(K_L^{(2)} K_1^{(1)}) + w(\bar{K}^{(2)}, K_L^{(1)}); \quad (9)$$

$$w(K_2^{(2)}, K_L^{(1)}) \leq w(K^{(2)} K_L^{(1)}) + w(K_2^{(2)}, K^{(1)}); \quad (10)$$

$$w(K_2^{(2)}, K_H^{(1)}) \leq w(K^{(2)} K_H^{(1)}) + w(K_2^{(2)}, K^{(1)}); \quad (11)$$

$$w(K_1^{(2)}, K_L^{(1)}) \leq w(K^{(2)} K_L^{(1)}) + w(K_1^{(2)}, K^{(1)}); \quad (12)$$

$$w(K_1^{(2)}, K_H^{(1)}) \leq w(K^{(2)} K_H^{(1)}) + w(K_1^{(2)}, K^{(1)}). \quad (13)$$

Every probability entering these inequalities can be calculated using the standard quantum mechanics formula:

$$w(\alpha^{(2)}, \beta^{(1)}) = \left| \left\langle \alpha^{(2)} \right| \otimes \left\langle \beta^{(1)} \right| \Psi^- \right|^2.$$

After calculations we will have only four inequalities instead of twelve (2) – (13). They can be represented using the CP-violation parameter ε :

Thus inequalities (2) and (8) turn into

$$|Re(\varepsilon)| \geq -1, \quad (14)$$

inequalities (3) and (9) into

$$|Re(\varepsilon)| \geq -|\varepsilon|^2, \quad (15)$$

inequalities (4), (6), (11) and (12) into

$$|Re(\varepsilon)| \leq 1 \quad (16)$$

and inequalities (5), (7), (10) and (13) into

$$|Re(\varepsilon)| \leq |\varepsilon|^2, \quad (17)$$

It is obvious that inequalities (14) – (16) can not be violated because of ε values. But the last one (17) may be strongly violated; as it was shown in [9]. It was suggested to check Wigner inequalities in system of neutral kaons.

4. Background in a system of neutral kaons

Previous calculations were made for pure state kaons. But there is always a background in a real experiment.

A simplest model of background is a Werner state [13]. It is described by density matrix:

$$\hat{\rho}^{(W)} = x |\Psi^- \rangle \langle \Psi^-| + \frac{1}{4} (1-x) \hat{1}, \quad (18)$$

where $\hat{1}$ is the identity matrix 4×4 and $|\Psi^- \rangle$ is the Bell's state.

We will call parameter x a “purity parameter”. Parameter x is proportional to the number of $K^0 \bar{K}^0$ -pairs after $\phi(1020)$ -meson decay. It is in the range $[0,1]$. If x equals 1 it means that the state of kaons is completely pure and the density matrix is just a projector to the $|\Psi^- \rangle$ state.

Inequalities (5), (7), (10), and (13) for Werner state are represented by following inequality:

$$x \left(1 + 2|Re(\varepsilon)| - |\varepsilon|^2 \right) \leq 1 + |\varepsilon|^2, \quad (19)$$

After that we can calculate the values of the parameter x which allow experimental detecting of violation of Wigner inequalities:

$$x \leq \frac{1 + |\varepsilon|^2}{1 + 2|Re(\varepsilon)| - |\varepsilon|^2}; \quad (20)$$

$$x \leq (0,996828 \pm 0,000026). \quad (21)$$

Thus for detecting violation of Wigner inequalities (5), (7), (10), and (13) the purity of kaon system have to be greater than 99%.

5. Conclusion

In this work twelve Wigner inequalities were considered and it was shown that only four of them can be violated. But in a real experiment these violations can be observed only if the contribution of the background is less than 1%.

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