The oscillations of charged particle beam in crossed magnetic fields

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Abstract. In the present paper we are going to study the decrease of the velocity and density oscillations of the beam in the non-relativistic charged particle beams in the chosen configuration of the crossed field. This paper examines using a hydrodynamic approach and the Maxwell's processes inside the cold beam. Results show that in the framework of the present model one can obtain physical conditions when it is possible to observe the cooling effect for the charged particle beam.

1. Introduction

The interaction of different modes results in a selective transfer of energy/momentum from one degree of freedom to another, which can be caused by an anomalous transfer in the system. This effect can be achieved by proper choice of initial conditions and field configuration. In fact, the charged particle beam cooling is one type of this process.

The beam cooling is due to reduce the velocity of its particles in accompanying system, which moves with the average velocity of the particles [1]. This implies that the six-dimensional phase volume of the beam decreases in the phase space or the phase density of the beam increases. The beam cooling reduces the impact of diffusion processes**.** These include multiple scattering by residual gas in colliding beams experiments and the nonlinear effect of the coherent field of colliding bunches on the particles motion.

The difference between the existing methods of beam cooling is in the physics and its implementation. Each of the methods is better than the others for particles of a certain mass, a velocity and the range of twirling frequencies (for the processes in a circular accelerator). For example, the radiation damping [1-2] is based on the synchrotron radiation. This method is better suited for particles that can quickly reach the relativistic velocity range, i.e. for electrons and positrons. The right technical realization that is connected with the proper choice of magnetic structure provides a gradual decrease of deviations from the equilibrium point.

The electron cooling [3-6] is based on the effect of the energy transfer of total motion of charged particles to electrons that spreads with average speed of cooled particles.

In the process of collective motion, the Coulomb interaction of electrons and ions (protons) leads to a decrease of the energy of chaotic motion of the cooled particles. As a result, the phase volume that is occupied by the ion (proton) beam is compressed by an order of magnitude or more. The efficiency of energy transfer to electrons is much higher compared to the ionization cooling method [1,7] despite the fact that the electron beam density is significantly lower than the density of common targets. It is based on the interaction of particles and targets which is set in the path of its motion.

Stochastic cooling [1, 8] is more effective method for the ion (proton) beam with wide range of twirling frequencies. The realization is associated with obtaining information about the deviation of individual particles from the equilibrium position for the electromagnetic field. This field is created with these particles and the one is registered by the external pick-up electrodes. It would be of interest to try to combine these methods of particle beam cooling in merely one system and also reduce energy and particle losses. One can expect that some control parameter and the value of the external magnetic field affect the processes, which may be occurred inside the beam [9].

2. Formulation of the kinetic problem

Let us consider a non-relativistic, rotating, cylindrical, charged particle beam moving in crossed external magnetic fields depicted in Fig. 0, which was used before to accelerate plasma flows [10-12]. Here, the external magnetic field has a permanent radial component B_{0r} and the time-dependent axial component $B_{0z}(t)$ generating the nonstationary azimuthal electric field which shall increase the beam rotation with respect to the z-axis. The rotating particle beam interacts simultaneously with the permanent radial component B_{0r} that leads to momentum transfer from the azimuthal degree of the freedom to the axial direction.
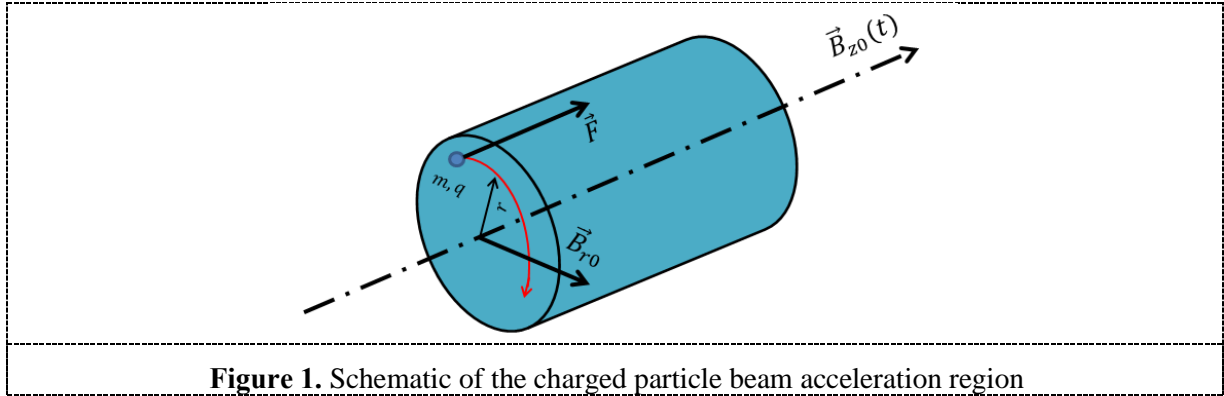
Figure 1. Schematic of the charged particle beam acceleration region

We suppose that these features will be able to lead to the decrease of the amplitude of such oscillations as a function of the density and velocity in the different macroscopic degrees of the freedom of the beam. There exists a certain relation between energy, frequency of natural oscillations of the beam and the value of the external magnetic field B_{0z} [13]. The beam shall explicitly depend on degrees of the freedom of the oscillation. Such behavior is directly opposite to the usual dynamics of natural oscillations of the plasma density in the beams.

We will study dynamics of an axis-symmetric ($\partial_\varphi \equiv 0$) homogenous charged particle beam consisting with the same particles of q charge and m mass. In this framework, a cold-fluid hydrodynamic approach is considered to decrease of an amplitude of such coupled nonlinear oscillations by Lorentz force only described in Fig. 1.

In view of linearity of Maxwell equations, we can present the total electric and magnetic fields of the system as the sum of the internal and external components :

$$B = B_0 + B^*, \tag{1}$$

$$E = E_0 + E^*. \tag{2}$$

It is assumed there is an external spatially homogeneous magnetic field which depends only on time:

$$B_0 = B_{0r}e_r + B_{0z}(t)e_z, \tag{3}$$

where $B_{0r} = B_{0r}(r)$ and $B_{0z} = B_{0z}(t)$ are some known, independent functions. Such configuration of

the external magnetic field can be obtained by using the permanent magnet, which creates a stationary radial magnetic field, and the solenoid, which creates predominantly an axial magnetic field that varies in time. In this paper, we put $B_{0r} =$ const for $0 \leq r \leq b$, here b is the characteristic size limiting the surface, because we study the simplest case only for an estimation of the possible effect as was said before. More detailed discussion of such configuration of the magnetic field is given in [11].

It should be noted that in the present paper an external magnetic field B_0 can create a vortex electric field E_0 while in the standard electrophysical devices electric E_0 and B_0 magnetic fields are independent and they can be created by different technical ways. From the induction equation for an external magnetic field B_0 written for in integral form:

$$\int_{\gamma} E_0 dl = -\frac{1}{c}\frac{\partial}{\partial t}\int_{S\gamma} B_0 dS, \tag{4}$$

To consider the axisymmetric of the flow presented in Fig. 1, we have:

$$E_{0\varphi} = -\frac{r}{2c}\frac{\partial B_{0z}}{\partial t}, \tag{5}$$

This means that the external electric field has only an azimuthal component. The intrinsic electric E^* and magnetic B^* fields are defined by the dynamic processes charged with a particle beam described by the standard cold-fluid model and Maxwell's equations.

Let us set intrinsic electric field of the system as:

$$E^* = r\varepsilon_r(t)e_r + r\varepsilon_\varphi(t)e_\varphi, \tag{6}$$

where $\varepsilon_r(t)$ and $\varepsilon_\varphi(t)$ are some unknown functions, which describes changes of radial and azimuthal components of the intrinsic electric field of the beam in time, and density as:

$$n = n(t), \tag{7}$$

We set the upper limit for $v_r(t)$, $v_\varphi(t)$ and $v_z(t)$ velocities on the order $: 0.1c$ because of we have to restrict on the non-relativistic case. In particular, let the velocity components of the beam represent as:

$$v = rA(t)e_r + rC(t)e_\varphi + rD(t)e_z, \tag{8}$$

where $A(t)$, $C(t)$ и $D(t)$ are associated with the radial, axial, and azimuthal velocity components. Also, it should be noted that the model worked out is the Brillouin model of the beam[14], which is usually used in the dynamics of such a beam [15]-[17]. However, with nonstationary azimuthal electric field with a constant radial magnetic field, this provides a redistribution of momentum from azimuthal to axial component.

By inserting (6–8) into standard cold-fluid model and Maxwell's equations we have nonlinear system of ordinary differential equations, written for in dimensionless form [18]:

$$\frac{dA}{dlt} + \left(A^2 - C^2\right) = sgn(q)\left[\varepsilon_r + \chi C\left(B_{0z} + B_z\right)\right], \tag{9}$$

$$\frac{dC}{dt} + 2AC = sgn(q)\left[\varepsilon_\varphi + \chi\left(B_{0r}D - \left(B_{0z} + B_z\right)A\right) - \frac{\chi}{2}\frac{dB_{0z}}{dt}\right], \tag{10}$$

$$\frac{dD}{dt} + AD = -sgn(q)\chi B_{0r}C, \tag{11}$$

$$\frac{dn}{dt} + 2nA = 0,$$
(12)

$$\frac{dB_z}{dt} = -\frac{2}{\chi}\varepsilon_\varphi,$$
(13)

$$\frac{d\varepsilon_r}{dt} = -\text{sgn}(q)nA,$$
(14)

$$\frac{d\varepsilon_\varphi}{dt} = -\text{sgn}(q)nC,$$
(15)

where χ is the dimensionless variable:

$$\chi = \omega_c/\omega_b.$$
(16)

here $\omega_c = |q|B_0/mc$ и $\omega_L = c/R_0$ are the frequency of the cyclotron resonance and characteristic frequency, respectively. Dimensionless value χ determines the different physical scripta for the system.

We used the initial density n_0, the initial radius of charged particle beam R_0, the inverse Langmuir is given by frequency $\omega_b^{-1} = (4\pi n_0 q^2/m)^{-1/2}$, where q and m are a charge and a mass of the particle, as the natural scale of density, coordinates, and time. All velocities are normalized by the $\tilde{V} = R_0\omega_b$, the electric field is normalized to the $\tilde{E} = (4\pi n_0 m\tilde{V}^2)^{1/2}$, and the magnetic field is normalized by the amplitude of external magnetic field B_0.

This system of nonlinear ODEs describes the nonlinear processes in a charged particle beam occurring in crossed magnetic fields. It should be noted again that $B_{0z}(t)$ is some known function, but its form will be defined afterward.

An approach of such idea in real accelerators requires spatial localization into space. This means that the transverse size of the beam doesn't need to be more than the transverse size of the beam pipe of the facility. So we should establish the moving boundary $R(t)$ for the beam, which separates the beam particles from the vacuum [18]:

$$R(t) = \frac{1}{\sqrt{n(t)}}.$$
(17)

3. Dynamics of the coupled nonlinear oscillations

As previously mentioned, the coupled azimuthal, radial and axial nonlinear oscillations can increase with the external fields in the system shown in Fig. 1. Let us consider the model (6–16) where an axial component of the external magnetic field is changed by a harmonic law:

$$B_{0z} = B_{0z}\sin(ht),$$
(18)

where B_{0z} is the amplitude of the external magnetic field and h is a dimensionless characteristic frequency. Here we get the process with a constant radial component of magnetic field directed outside of the beam. It means we will use the relation $B_{0r} = 1$ for all further numerical solutions.
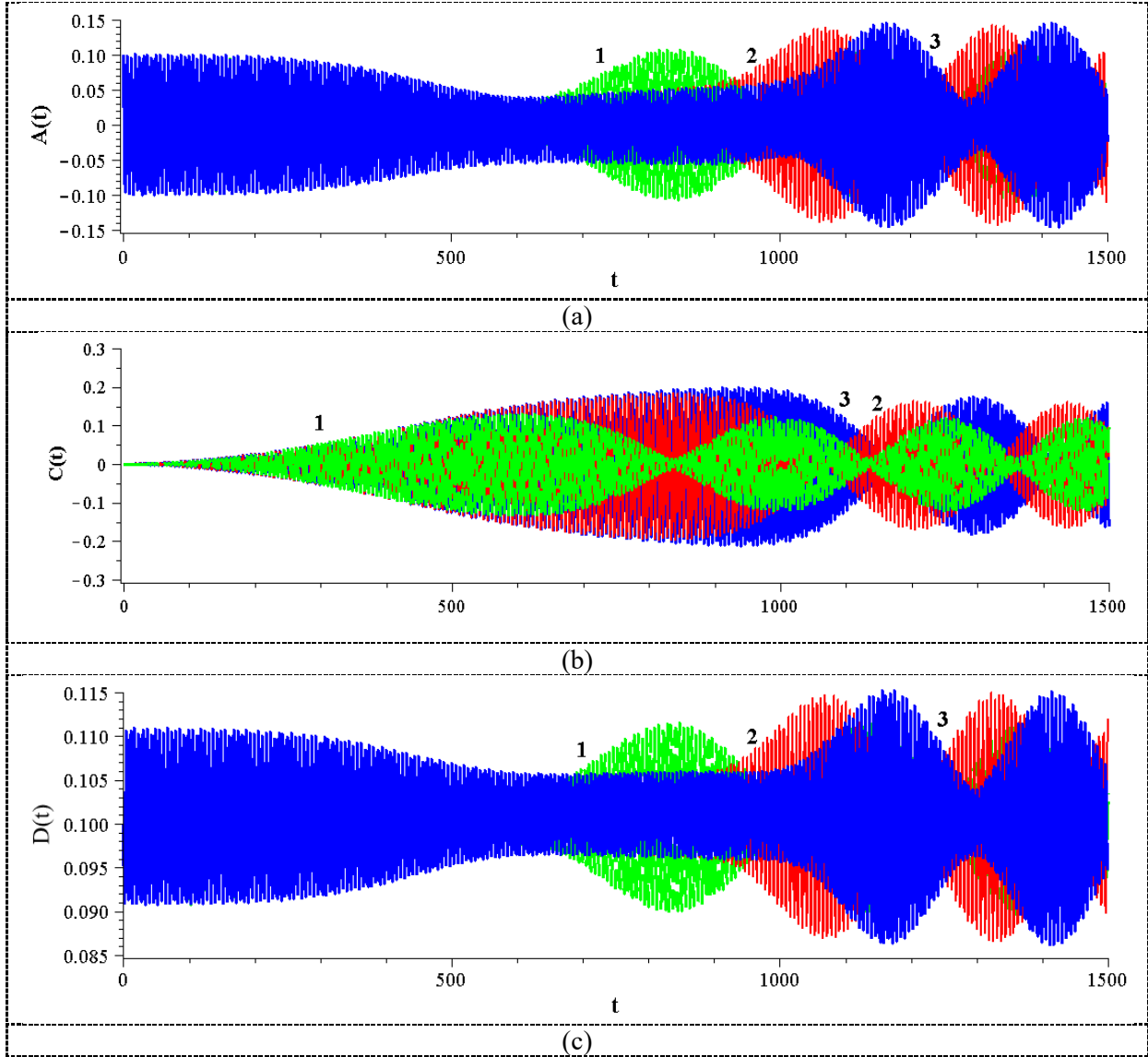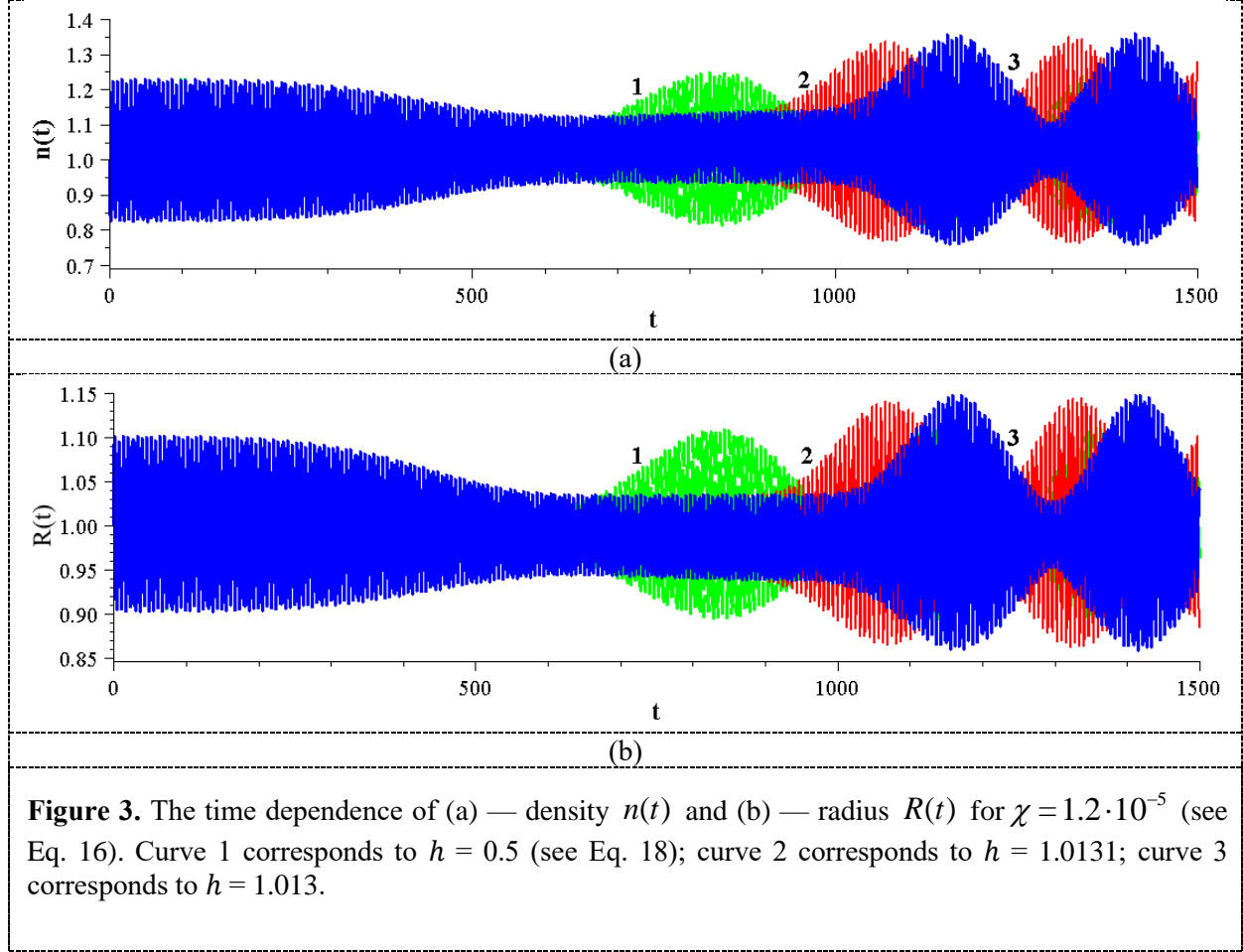
Figure 2. The time dependence of (a) — $A(t)$, (b) — $C(t)$ and (c) — $D(t)$ (see Eq. 8) for $A(0) = D(0) = 0.1, C(0) = 0$ and for $\chi = 1.2 \cdot 10^{-5}$ (see Eq. 16). Curve 1 corresponds to $h = 0.5$ (see Eq. 18); curve 2 corresponds to $h = 1.0131$; curve 3 corresponds to $h = 1.013$.

To demonstrate the possibility of the behavior, which was described in the previous chapter, we analytically consider an electron beam ($\mathrm{s}gn(q) = -1$) expanding in the external magnetic field with frequency h, which changes in the big range. To understand the dependent results of the numerical solution on the χ value, it is convenient to rewrite relation (16) as:

$$n_0 = \frac{B_{0\mathrm{z}}^2}{4\pi mc^2 \chi^2}, \tag{19}$$

which allows us to estimate ranges for n_0 and $B_{0\mathrm{z}}$ respectively. For example, if we let $\chi = 1.2 \cdot 10^{-5}$ and $B_{0\mathrm{z}} = 10\,\mathrm{T}$, we will have the density $n_0 \approx 10^4\,\mathrm{cm}^{-3}$ and the value of the characteristic time is $\omega_\mathrm{b}^{-1} = 2 \times 10^{-7}\,\mathrm{s}$. The present paper is based on characteristics of real facilities and due to this we will

consider $\chi \leq 10^{-4}$ (see. Eq. 16) [18]. The external magnetic field plays a weak role in the dynamics of the process, but there is a generation of the intrinsic magnetic field (see Eq. 13).

Figure 3. The time dependence of (a) — density $n(t)$ and (b) — radius $R(t)$ for $\chi = 1.2 \cdot 10^{-5}$ (see Eq. 16). Curve 1 corresponds to $h = 0.5$ (see Eq. 18); curve 2 corresponds to $h = 1.0131$; curve 3 corresponds to $h = 1.013$.

Initial values of radial and axial velocities were taken $v_r(0) = v_z(0) = 2 \times 10^{-3} c$ that approximately corresponds to $1\ eV$. We neglect the azimuthal velocity component in the electron beam at the initial moment of time for simplicity, i.e. we put $v_\varphi(t = 0) = 0$. Let us set value $\hat{V} = 0.02c$ for a more convenient graphic representation. In this case initial values of velocity components will equal $A(0) = D(0) = 0.1$ and $C(0) = 0$.

Initial values of perturbed electric and magnetic fields were examined with $\varepsilon_r(0) = \varepsilon_\varphi(0) = B_z^*(0) = 0$. These conditions at the initial state in the quasineutral plasma shift, the density of electrons approximate for the density of ions $n_e \approx n_i$ but only one type of particles will leave the shift and will be accelerated.
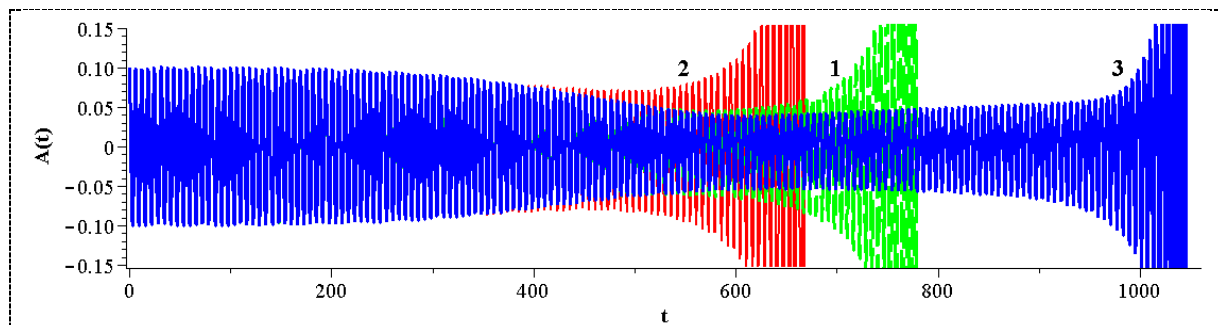
Figure 4. The time dependence of $A(t)$ for $A(0) = 0.1$ and for $\chi = 1.2 \cdot 10^{-5}$ (see Eq. 16). Curve 1 corresponds to h = 1.0015; curve 2 corresponds to h = 1.007; curve 3 corresponds to h = 1.013.

The typical evolution of coupled oscillations will be considered for the case $\chi = 1.2 \cdot 10^{-5}$. Fig.2 and Fig.3 represents examples of time evolution of the nonlinear oscillations for h = 0.5; 1.0131; 1.013 cases. The decrease of oscillations occurs initially (t < 900) in both radial and azimuthal components, but later (t > 900) there are formed wave trains (packets) as shown in Fig. 2 (curve 3). It should be stressed that the increase of the amplitude of the azimuthal component is occurred when there is decrease in other degrees of the freedom of the beam and vice versa. This process demonstrates the possibility of the energy/momentum exchange from one to another mode of the beam.

The decrease of the amplitude of the velocity components provides the decroese of the density oscillations $n(t)$ of the beam (see Fig. 3(a)) and also the amplitude of oscillations of the radius R(t) is positive limited (see Fig. 3(b)) for the whole time range. It is worth to mention, that the decrease of the amplitude of the density, velocity components with time does not depend on the h value for the beam thus it is non-resonant. As will be shown below, this behavior differs from generation processes.

One can see a behavior of the radial component in the resonant frequency range of the field in Fig. 4 (curve 3), where initially (t < 900) occur, the behaviour is similar to processes in Fig. 2 but later (t > 900), instead of formation of wave trains (packets) there is the growth of the amplitude of the velocity components (packets).

As noted in the paper [18], this process are observed for the axial velocity component and the density until density collapse when it appears in the beam. It should be stressed that the formation of the wave trains (packets) an the decrease of the amplitude of coupled oscillations might be nothing but the process of energy accumulation by the beam despite the possible realization of the method as one of the beam cooling methods.

4. Conclusion

Based on the hydrodynamic description, the decrease of the amplitude of coupled nonlinear oscillations of the velocity/density in non-relativistic ($\beta < 0.1$), cold, rotating, charged particle beams in crossed magnetic fields for was studied.

The anomalus energy/momentum exchange between different macroscopic degrees of the beam freedom occurs in external crossed magnetic fields with the configuration shown in Fig. 1. This effect leads to the increase the kinetic energy of the beam and to azimuthal acceleration of the charged particles. Meanwhile we considered the Brillouin model of the beam. This beam spreads in the system where there are same conditions for internal fields as at the initial state in the quasineutral plasma shift.

One can see that the the amplitude of the axial and radial velocity components oscillations decreases simultaneously (see Fig. 2 and Fig. 3) followed by wave trains (packets) formation in the system.The increase of the amplitude of the azimuthal component is occurred when there is decrease

of the radial and axial components. This process demonstrates the energy/momentum exchange from one macroscopic degree of the freedom to another one.

In addition, the present results demonstrate that the density of the beam has the similar behaviour as the amplitude of radial and axial velocities, but at the same time the radius of the beam is positive limited for the whole time range. This demonstrates the opportunity to use the process in real accelerators, where transverse size is limited by the size of the vacuum chamber.

One can see that the amplitude of the density, velocity components oscillations increases (see curves in Fig. 4) instead of wave trains (packets) formation when the frequency of the external field is close to the frequency of natural oscillations of the beam (the difference between them is less than 2%.). This process can be interesting in the future only from the point of view of electromagnetic radiation generation and it is described in more detail in [18].

Results demonstrate the possibility of the amplitude decrease of the coupled density, velocity components oscillations for non-relativistic homogenuos axis-symmetric electron beam already at initial stage. This method can be considered as one of the charged particle beam cooling methods. It should be noted that there was an effort to define the necessary conditions for beam cooling, but the process itself was not the subject of the study.

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