Tensor meson contribution to the Lamb shift and hyperfine splitting in muonic hydrogen

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Abstract. We calculate the tensor meson exchange contribution to the interaction operator of muon and proton, which is determined by the tensor meson coupling with two photon state. For the construction of transition form factor $T \to \gamma\gamma$ we use the monopole parametrization over photon four-momenta and experimental data on the decay width $\Gamma_{T\gamma\gamma}$. It is shown that tensor meson $f_2(1270)$ exchange gives essential contribution to the Lamb shift in muonic hydrogen $\Delta E^{Ls}(2P - 2S)$ which should be taken into account for a comparison with precise experimental data.

1. Introduction

The problem of the proton charge radius, which arose after the CREMA experiments [1, 2], raised the question of a more accurate construction of the particle interaction operator in muon atoms and the inclusion of new contributions to this operator. Emerging new experimental data on the Lamb shift in electron hydrogen, as well as a new analysis of experiments already performed on the scattering of leptons by nucleons, show that the values of the proton charge radius obtained from electron and muon systems are approaching [3, 4]. The problem of the proton charge radius is gradually beginning to be solved. However, the analysis of new interactions between the proton and the muon is important for future more accurate experiments. Among the interactions of the proton and the muon there are those in which two virtual photons turn into a meson, which leads to an effective one-meson potential. The fusion of two photons in a meson with different quantum numbers can be described in terms of the quark model with light quarks. The calculation of the form factor of the transition of two photons into a meson can be performed within the framework of nonperturbative quantum chromodynamics. In the case of pseudoscalar and axial vector mesons, the results of a theoretical calculation of the transition form factor can be compared with the available experimental data. No experimental data are available for scalar and tensor mesons; therefore, model representations are used to calculate the amplitude of the interaction of the muon and proton. An important role is played by the inverse decay of the meson into two photons, the width of which is measured experimentally and is included in the estimation of the transition form factor at zero squared photon momenta.

In this work we continue our investigation [5, 6, 7] (see also [8, 9, 10, 11, 12]) of the role of one-meson exchange interactions in muonic hydrogen in the case of tensor mesons.

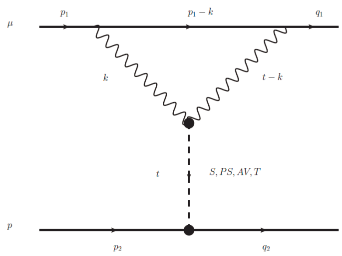

Figure 1. One-meson (pseudoscalar (PS), axial-vector (AV), scalar (S), tensor (T)) exchange interaction in muonic hydrogen.

2. General formalism

For tensor mesons consisting from light quarks the experimental analysis of decay angular distributions for $\gamma\gamma$ cross sections to $\pi^+\pi^-$, $\pi^0\pi^0$, K^+K^- have shown that the J=2 mesons are produced mainly in a state with helicity $\Lambda = 2$ [13]. We will assume further that hadronic light-by-light amplitude for tensor mesons is dominated be helicity $\Lambda = 2$ exchange. Then the amplitude of the process $\gamma^* + \gamma^* \to T$ (see Fig. 1) can be parametrised as follows [14]:

$$T^T_{\mu\nu\alpha\beta}(k_1,k_2) = e^2\frac{k_1k_2}{M}\mathcal{M}_{\mu\nu\alpha\beta}(k_1,k_2)\mathcal{F}_{T\gamma^*\gamma^*}(k_1^2,k_2^2), \tag{1}$$

where $\mathcal{F}_{T\gamma^*\gamma^*}(k_1^2,k_2^2)$ is a transition form factor, k_1, k_2 are four momenta of virtual photons,

$$\mathcal{M}_{\mu\nu\alpha\beta}(k_1,k_2) = \Big\{R_{\mu\alpha}(k_1,k_2)R_{\nu\beta}(k_1,k_2) + \frac{1}{8(k_1+k_2)^2\left[(k_1k_2)^2-k_1^2k_2^2\right]}R_{\mu\nu}(k_1,k_2)\times \tag{2}$$

$$\Big[(k_1+k_2)^2(k_1-k_2)_\alpha - (k_1^2-k_2^2)(k_1+k_2)_\alpha\Big]\times\Big[(k_1+k_2)^2(k_1-k_2)_\beta - (k_1^2-k_2^2)(k_1+k_2)_\beta\Big]\Big\},$$

$$R_{\mu\nu}(k_1,k_2) = -g_{\mu\nu} + \frac{1}{X}\left[(k_1k_2)(k_1^\mu k_2^\nu + k_2^\mu k_1^\nu) - k_1^2k_2^\mu k_2^\nu - k_2^2k_1^\mu k_1^\nu\right],\ X = (k_1k_2)^2 - k_1^2k_2^2.$$

Our approach to the construction of the interaction operator is based on quasipotential method in quantum field theory [15, 16, 17]. Introducing the four-momentum t of the tensor meson we can present the muon-proton interaction amplitude due to two-photon fusion in the form:

$$i\mathcal{M} = \frac{4\pi Z\alpha}{16m_1^2m_2^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(p_1-k)^2-m_1^2}\mathcal{D}_{\mu\mu'}(t-k)\mathcal{D}_{\nu\nu'}(k)\mathcal{D}_T^{\alpha'\beta'\alpha\beta}(t)\mathcal{M}_{\mu'\nu'\alpha'\beta'}(k_1,k_2) \tag{3}$$

$$[\bar u(0)(\hat q_1+m_1)\gamma_\mu(\hat p_1-\hat k+m_1)\gamma_\nu(\hat p_1+m_1)u(0)][\bar v(0)(\hat p_2-m_2)\Gamma^{\alpha\beta}_{TNN}(\hat q_2-m_2)v(0)],$$

where the vertex function of tensor meson nucleon interaction is [18]

$$\Gamma^{\alpha\beta}_{TNN}(p_2,q_2) = \frac{G_{TNN}}{m_2}\left[(q_2+p_2)_\alpha\gamma_\beta + (q_2+p_2)_\beta\gamma_\alpha\right] + \frac{F_{TNN}}{m_2^2}(q_2+p_2)_\alpha(q_2+p_2)\beta,$$

and the tensor meson propogator is equal to

$$\mathcal{D}_T^{\mu\nu\alpha\beta}(t) = \frac{1}{t^2-M_T^2+i\varepsilon}\Big\{\frac{1}{2}(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\nu}g_{\alpha\beta})+ \tag{4}$$

$$\frac{1}{2}\left(g_{\mu\alpha}\frac{t^\nu t^\beta}{M_T^2}+g_{\nu\beta}\frac{t^\mu t^\alpha}{M_T^2}+g_{\mu\beta}\frac{t^\nu t^\alpha}{M_T^2}+g_{\nu\alpha}\frac{t^\mu t^\beta}{M_T^2}\right)+\frac{2}{3}\left(\frac{1}{2}g_{\mu\nu}+\frac{t^\mu t^\nu}{M_T^2}\right)\left(\frac{1}{2}g_{\alpha\beta}+\frac{t^\alpha t^\beta}{M_T^2}\right)\Big\}.$$

p_1, q_1; p_2, q_2 are four momenta of muon and proton correspondingly in initial and final states.

The introduction of projection operators onto the muon-proton states with certain values of the total momentum makes it possible to simplify the construction of the particle interaction operator in these states and use analytical systems such as Form [19]. Projecting the interaction amplitude on the S-state with $F = 1$ we obtain:

$$\mathcal{T}(^3S_1)=Tr\Big[\gamma_{\varepsilon_1}\frac{1+\gamma_0}{2\sqrt{2}}(\hat{q}_1+m_1)\gamma_\mu(\hat{p}_1-\hat{k}+m_1)\gamma_\nu(\hat{p}_1+m_1) \tag{5}$$

$$\frac{1+\gamma_0}{2\sqrt{2}}\gamma_{\varepsilon_2}(\hat{p}_2-m_2)\Gamma^{\alpha\beta}_{TNN}(\hat{q}_2-m_2)\Big]\frac{1}{3}(-g_{\varepsilon_1\varepsilon_2}+v_{\varepsilon_1}v_{\varepsilon_2})$$

After the trace calculation we obtain that in the leading order in t the interaction amplitudes in $^3S_1$ and $^1S_0$ states are the following:

$$\mathcal{T}(^3S_1)=\mathcal{T}(^1S_0)=\frac{G_{TNN}}{M_T}4m_1k^4\left(1+\frac{k_0^4t^4}{[(kt)^2-k^2t^2]^2}+2\frac{k_0^2t^2}{[(kt)^2-k^2t^2]}\right) \tag{6}$$

The contribution to the ground state hyperfine splitting contains additional degrees t:

$$\mathcal{T}^{hfs}=\frac{4}{3}\frac{G_{TNN}}{M_Tm_2}t^2k^4\left(1-\frac{t^4k_0^4}{[(kt)^2-k^2t^2]^2}\right). \tag{7}$$

We use the dipole parametrization for the transition form factor

$$\mathcal{F}_{T\gamma^*\gamma^*}(k_1^2,k_2^2)=\mathcal{F}_{T\gamma^*\gamma^*}(0,0)\frac{\Lambda^4}{(k^2+\Lambda^2)^2},\ k_1=k,\ k_2=-k. \tag{8}$$

Making the transition to the Euclidean space we present the loop momentum integral in the form:

$$\mathcal{I}=\int_0^\infty\frac{d^4k}{k^4-4k_0^2m_1^2}\mathcal{F}_{T\gamma^*\gamma^*}(0,0)\frac{\Lambda^4}{(k^2+\Lambda^2)^2}\left(1+\frac{k_0^4t^4}{[(kt)^2-k^2t^2]^2}+2\frac{k_0^2t^2}{[(kt)^2-k^2t^2]}\right)= \tag{9}$$

$$=-\int_0^\pi\frac{\sin\psi d\psi}{4\left(a_1^2\cos\psi+a_1^2-2\right)^2}\left(a_1^2\cos 2\psi-2\ln\left(a_1^2\cos^2\psi\right)+a_1^2-2\right)\times$$

$$\left(\sin^3\psi-3\sin\psi\left(\cos^2\psi+3\right)+\cos^2\psi(\cos 2\psi-7)\ln\left(\frac{2}{\sin\psi+1}-1\right)\right)$$

where $a_1=\frac{2m_1}{\Lambda}$, $\mathcal{F}_{T\gamma^*\gamma^*}(0,0)=\frac{2\sqrt{5\Gamma_{\gamma\gamma}}}{\alpha\sqrt{\pi M_T}}$. Then the particle interaction operator for the Lamb shift and hyperfine splitting are

$$\Delta V_T^{LS}(r)=-\frac{16\alpha^2m_1G_{TNN}}{M_T}\frac{2\sqrt{5\Gamma_{\gamma\gamma}}}{\alpha\sqrt{\pi M_T}}\mathcal{I}\frac{1}{4\pi r}e^{-M_Tr}, \tag{10}$$

$$\Delta V_T^{HFS}(r)=\frac{8\alpha^2G_{TNN}}{3\pi m_2M_T}\frac{2\sqrt{5\Gamma_{\gamma\gamma}}}{\alpha\sqrt{\pi M_T}}\mathcal{J}\left(\delta(\mathbf{r})-\frac{M_T^2}{4\pi r}e^{-M_Tr}\right), \tag{11}$$

$$\mathcal{J}=\int_0^\pi\frac{\sin\psi d\psi}{(-2+a_1^2+a_1^2\cos 2\psi)^2}\left(a_1^2\cos 2\psi-2\ln\left(a_1^2\cos^2\psi\right)+a_1^2-2\right)\times$$

$$\left(\sin^3\psi + 7\sin\psi - 3\sin\psi\cos^2\psi + 2\cos^4\psi\ln\left(\frac{2}{\sin\psi+1} - 1\right)\right).$$

There are several tensor mesons which can contribute to LS and HFS [20], but in our calculation we take into account only the contribution of $f_2(1270)$ meson, since we know for it more or less reliable values of various parameters, including the coupling constant with the nucleon [18]. Using obtained potentials we can calculate contributions to the energy levels of muonic hydrogen:

$$\Delta E^{Ls}_{1S} = -\frac{16\alpha^5\mu^3 m_1 G_{TNN}}{\pi M_T}\frac{2\sqrt{5\Gamma_{\gamma\gamma}}}{\sqrt{\pi M_T}}\frac{1}{(M_T+2W)^2}\mathcal{I} = -0.0528\ meV, \tag{12}$$

$$\Delta E^{Ls}_{2S} = -\frac{2\alpha^5\mu^3 m_1 G_{TNN}}{\pi M_T}\frac{2\sqrt{5\Gamma_{\gamma\gamma}}}{\sqrt{\pi M_T}}\frac{(2M_T^2+W^2)}{2(M_T+2W)^4}\mathcal{I} = -0.0066\ meV, \tag{13}$$

$$\Delta E^{hfs}_{1S} = -\frac{8\alpha^5\mu^3 G_{TNN}}{3\pi^2 M_T m_2}\frac{2\sqrt{5\Gamma_{\gamma\gamma}}}{\sqrt{\pi M_T}}\left(1 - \frac{M_T^2}{(M_T+2W)^2}\right)\mathcal{J} = -0.0551\ \mu eV, \tag{14}$$

$$\Delta E^{hfs}_{2S} = -\frac{\alpha^5\mu^3 G_{TNN}}{3\pi^2 M_T m_2}\frac{2\sqrt{5\Gamma_{\gamma\gamma}}}{\sqrt{\pi M_T}}\left(1 - \frac{M_T^2(2M_T^2+W^2)}{2(M_T+W)^4}\right)\mathcal{J} = -0.0069\ \mu eV, \tag{15}$$

where $W = \mu\alpha$.

3. Conclusion

The obtained numerical values of the contributions to the Lamb shift and the hyperfine structure show that they are significant and must be taken into account in a more accurate comparison with experimental data. The contribution of the tensor meson is comparable to the contribution of the scalar σ-meson. Other tensor mesons apparently make a significantly smaller contribution, since their constant of interaction with the nucleon is much smaller. Experimental data [20] show that all tensor mesons have a significant decay width into a pair of pions, which interact well with the nucleon, therefore, such processes need to be investigated additionally. Work in this direction is in progress.

Acknowledgments

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References

[1] Antognini A et al. 2013 *Ann. Phys. (NY)* **331** 127
[2] Diepold M et al. 2018 *Ann. Phys. (NY)* **396** 220
[3] Bezginov N et al. 2019 *Science* **365** 1007
[4] Alarcon J M et al. 2019 *Phys. Rev.* C **99** 044303
[5] Dorokhov A E, Kochelev N I, Martynenko A P et al. 2017 *Phys. Part. Nucl. Lett.* **14** 857
[6] Dorokhov A E, Kochelev N I, Martynenko A P et al. 2018 *Phys. Lett.* B **776** 105
[7] Dorokhov A E, Martynenko A P, Martynenko F A et al. 2019 *Phys. Part. Nucl. Lett.* **16** 520
[8] Hagelstein F and Pascalutsa V 2016 *PoS* **CD15** 077
[9] Zhou H Q and Pang H R 2015 *Phys. Rev.* A **92** 032512
[10] Huong N T, Kou E, Moussallam B 2016 *Phys. Rev.* D **9** 114005
[11] Zhou H Q 2017 *Phys. Rev.* C **95** 025203
[12] Borisyuk D 2017 *Phys. Rev.* C **96** 055201
[13] Pennington M R et al. 2008 *Eur. Phys. J.* C **56** 1
[14] Pauk V and Vanderhaeghen M 2014 *EPJ* C **74** 3008
[15] Martynenko A P, Martynenko F A, and Faustov R N 2017 *Jour. Exp. Theor. Phys.* **124** 895
[16] Faustov R N, Martynenko A P, Martynenko G A, and Sorokin V V 2015 *Phys. Rev.* A **92** 052512
[17] Dorokhov A E, Martynenko A P, Martynenko F A, and Radzhabov A E 2018 *Eur. Phys. Jour.* A **54** 131
[18] Oh Y, and Lee T S H 2004 *Phys. Rev.* C **69** 025201.
[19] Vermaseren J A M 2000 e-preprint arXiv:math-ph/0010025
[20] Tanabashi M et al. 2018 *Phys. Rev.* D **98** 030001