Dalitz decays of vector mesons in relativistic quark model

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Abstract. In the course of work, basing on the previously developed method for calculating integral representations of decay constants for pseudoscalar and vector mesons in the frame of point form of Poincaré-covariant quark model, the research of ω– and ϕ–mesons decays form-factors for Dalitz $V(q\bar{Q}) \to P(q\bar{Q})\ell^+\ell^-$ decays has been carried out. It is shown that the usage of the pseudoscalar density constant with integral representations of decay constants $P(q\bar{Q}) \to \ell\nu_\ell$, $V(q\bar{Q}) \to \ell^+\ell^-$ and $\ell \to P(q\bar{Q})\nu_\ell$ leads to parameters which can be used for the study of $V(q\bar{Q}) \to P(q\bar{Q})\ell^+\ell^-$ decay form-factors. As a result of the work, the behaviour of $F_{\omega\pi\gamma^*}(t)$ and $F_{\phi\eta\gamma^*}(t)$ form-factors compared with the experimental data in the interval $q < 0.5$ GeV is researched.

Introduction

Lately high energy physics has been marked by high-accuracy results both in hadronic and leptonic decay of light pseudoscalar and vector bound quark-antiquark systems (mesons). Since such systems are purely relativistic the phenomenological description of various characteristics requires the involvement of corresponding models.

Poincaré-invariant quantum mechanics (further PiQM) is one of a number of approaches which could be applied for the study of bounded systems. This model is a natural relativistic generalization of Schröedinger quantum mechanics [1, 2] or quantum mechanics, based on Poincaré group. Indicated fact makes approaches, based on PiQM, most common, which gives the opportunity to use them for the description of the characteristics for relativistic u–, d– and s–quark systems.

Theoretical research in the framework of point form of PiQM for light mesons are represented to a lesser extent than other forms of dynamics. Despite the developed mathematical technique for calculation of electromagnetic form-factors of mesons [3, 4] and successful applying this form of PiQM for the estimation of nucleon characteristics [5], a considerable difference between theoretical predictions and experimental data led to the appearance of various modifications of this form of dynamics. However, a comparative analysis carried out in the work [6] showed that the calculations in the framework of both the point form and its various modifications lead to a discrepancy with experimental data on $\pi^\pm$ – meson. It follows that the development of a point form PiQM is an important and relevant task of hadron physics.

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1. Basic parameters of the model

The procedure for obtaining integral representations of constant of leptonic decay of pseudoscalar ($I = P$) and vector ($I = V$) mesons is particularized by the authors in [7, 8, 9]. The finite expressions of constants decay in point form of PiQM can be written as:

$$f_I(m_q, m_{\bar{Q}}, \beta^I_{q\bar{Q}}) = \sqrt{\frac{3}{2}}\frac{1}{\pi}\int_0^\infty \mathrm{dk}\ \mathrm{k}^2\ \Phi(\mathrm{k}, \beta^I_{q\bar{Q}})\sqrt{\frac{W^+_{m_q}(\mathrm{k})\ W^+_{m_{\bar{Q}}}(\mathrm{k})}{M_0(\mathrm{k})\ \omega_{m_q}(\mathrm{k})\,\omega_{m_{\bar{Q}}}(\mathrm{k})}}\times \tag{1}$$

$$\left(1 + a_I\frac{\mathrm{k}^2}{W^+_{m_q}(\mathrm{k})\,W^+_{m_{\bar{Q}}}(\mathrm{k})}\right),\ a_P = -1,\ a_V = 1/3,$$

$$W^\pm_m(\mathrm{k}) = \omega_m(\mathrm{k}) \pm m,\ \omega_m(\mathrm{k}) = \sqrt{\mathrm{k}^2 + m^2},\ \mathrm{k} = |\mathbf{k}|\,,\ M_0(\mathrm{k}) = \omega_{m_q}(\mathrm{k}) + \omega_{m_{\bar{Q}}}(\mathrm{k}).$$

In expression (1) the wave function is resticted by normalization condition

$$\sum_{\ell,s}\int_0^\infty \mathrm{dk}\ \mathrm{k}^2\left|\Phi^J_{\ell s}(\mathrm{k})\right|^2 = 1. \tag{2}$$

Using the expression for the constant of pseudoscalar density [7]

$$g_P\left(m_q, m_{\bar{Q}}, \beta^P_{q\bar{Q}}\right) = \sqrt{\frac{3}{2}}\frac{1}{\pi}\int_0^\infty \mathrm{dk}\ \mathrm{k}^2\ \Phi\left(\mathrm{k}, \beta^P_{q\bar{Q}}\right)\sqrt{\frac{M_0(\mathrm{k})}{\omega_{m_q}(\mathrm{k})\,\omega_{m_{\bar{Q}}}(\mathrm{k})}}\times \tag{3}$$

$$\left(\sqrt{W^+_{m_q}(\mathrm{k})\,W^+_{m_{\bar{Q}}}(\mathrm{k})} + \sqrt{W^-_{m_q}(\mathrm{k})\,W^-_{m_{\bar{Q}}}(\mathrm{k})}\right)$$

and oscillator wave function $\Phi^{\mathrm{os}}(\mathrm{k},\beta) = \frac{2}{\pi^{1/4}\ \beta^{3/2}}\ \exp\left[-\frac{\mathrm{k}^2}{2\beta^2}\right]$ from the experimental data on decays of pseudoscalar $\pi^\pm$, $K^\pm$ – mesons and heavy $\tau^\pm$–lepton in vector $\rho^\pm$, $K^{*\pm}$ – mesons one can obtain following basic parameters:

$$m_u = (219.4 \pm 9.6)\ \mathrm{MeV},\ m_d = (221.9 \pm 9.6)\ \mathrm{MeV},\ m_s = (416.9 \pm 61.2)\ \mathrm{MeV}, \tag{4}$$

$$\beta^P_{u\bar{d}} = (367.93 \pm 25.10)\ \mathrm{MeV}, \beta^V_{u\bar{d}} = (311.95 \pm 2.14)\ \mathrm{MeV},\ \beta^P_{u\bar{s}} = (375.53 \pm 19.66)\ \mathrm{MeV},$$

$$\beta^V_{u\bar{s}} = (313.62 \pm 24.22)\ \mathrm{MeV}.$$

For the further calculations with the use of a weak isotopic symmetry violation, we assume that

$$\begin{aligned}\beta^V_{u\bar{u}} &= \beta^V_{u\bar{d}} + \triangle\beta_{u\bar{d}},\ \beta^V_{d\bar{d}} = \beta^V_{u\bar{d}} - \triangle\beta_{u\bar{d}},\\ \beta^V_{d\bar{s}} &= \beta^V_{u\bar{s}} - \triangle\beta_{u\bar{d}},\ \beta^P_{d\bar{s}} = \beta^P_{u\bar{s}} + \triangle\beta_{u\bar{d}},\end{aligned} \tag{5}$$

where $\triangle\beta_{u\bar{d}} \simeq m_d - m_u = (2.5 \pm 0.2)$ MeV.

2. Integral representation of $V \to P\gamma^*$ decay constant

The parametrization of the matrix element for the transition of a vector (pseudoscalar) meson $V(P)$ into a pseudoscalar (vector) meson $P(V)$ with 4-momenta $Q = \{Q_0, \mathbf{Q}\}$ ($Q^2 = M^2$, $V = Q/M$) and $Q' = \{Q'_0, \mathbf{Q}'\}$ ($Q'^2 = M'^2$, $V' = Q'/M'$) with the emission of a virtual γ^* is given by [10]

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$$\langle \mathbf{Q}', M_P|\hat{J}^\alpha(0)|\mathbf{Q}, 1\lambda_V, M_V\rangle = \frac{\mathrm{i}}{(2\pi)^3} g_{VP\gamma^*} \frac{\epsilon^{\alpha\nu\rho\sigma}\varepsilon_\nu(\lambda_V)V_\rho V'_\sigma}{\sqrt{4V_0V'_0}}\sqrt{M_V M_P}. \tag{6}$$

After some calculation from the relation (6) one can obtain the integral representation of the radiative decay constant for neutral ($m_{\bar{Q}} = m_q$) mesons [11, 12]

$$g_{VP\gamma^*}(t, m_q, \beta^V_{q\bar{q}}, \beta^P_{q\bar{q}}) = \frac{1}{4\pi}\sum_{\nu_1,\nu'_1}\int \mathrm{d}\mathbf{k}\,\sqrt{\frac{3+4\nu_1(\lambda_V-\nu_1)}{4}}\frac{\nu'_1}{\sqrt{M_0(\mathrm{k})}}\Phi(\mathrm{k},\beta^V_{q\bar{q}})\times \tag{7}$$
$$\frac{1}{\omega_{m_q}(\mathrm{k})}\Bigg(\bar{u}_{\nu'_1}(\mathbf{k}_2, m_q)B(\boldsymbol{v}_Q)\left(K^*(\lambda_V)\cdot\Gamma_q\right)u_{\nu_1}(\mathbf{k}, m_q)\frac{1}{\sqrt{\varpi^2_{12}(\mathrm{k},t)-1}}\frac{\Phi^*(\mathrm{k}_2,\beta^P_{q\bar{q}})}{\sqrt{M_0(\mathrm{k}_2)}}\times$$
$$D_{-\nu'_1,\lambda_V-\nu_1}(\mathbf{n}_{W_2}(\mathbf{k},\boldsymbol{v}_Q)) + \bar{v}_{\lambda_V-\nu_1}(\mathbf{k}, m_{\bar{q}})B(-\boldsymbol{v}_Q)\left(K^*(\lambda_V)\cdot\Gamma_{\bar{q}}\right)v_{-\nu'_1}(\mathbf{k}_1, m_{\bar{q}})\times$$
$$\frac{1}{\sqrt{\varpi^2_{12}(\mathrm{k},t)-1}}\frac{\Phi^*(\mathrm{k}_1,\beta^P_{q\bar{q}})}{\sqrt{M_0(\mathrm{k}_1)}}D_{\nu'_1,\nu_1}(\mathbf{n}_{W_1}(\mathbf{k},\boldsymbol{v}_Q))\Bigg),\;\; K(\lambda_V) = \sqrt{\frac{\varpi^2(\mathrm{k},t)-1}{2}}\times\{0,\lambda_V,i,0\}.$$

Additional relations for (7) are given by

$$\mathbf{k}_{1,2} = \mathbf{k}\pm\boldsymbol{v}_Q\left(\left(\varpi(\mathrm{k},t)+1\right)\omega_{m_{q,\bar{q}}}(\mathrm{k}) - \mathrm{k}\sqrt{\varpi^2(\mathrm{k},t)-1}\cos\theta_\mathrm{k}\right),\;\; \boldsymbol{v}_Q = \frac{\mathbf{V}_Q}{V_0} \tag{8}$$

and

$$\varpi(\mathrm{k},t)^2 = \left(V\cdot V'\right)^2 = 1 - \frac{t}{4\left(\omega^2_{m_q}(\mathrm{k}) - \mathrm{k}^2\cos^2\theta_\mathrm{k}\right)},\;\; t = (Q-Q')^2. \tag{9}$$

3. Numerical research and discussion

Fixing the vertex in (7) with expression

$$\Gamma^\mu_q = F_1(q^2)\gamma^\mu + \frac{1}{2m_q}F_2(q^2)\sigma^{\mu\nu}q_\nu, \tag{10}$$

where form-factors $F_1(q^2)$ and $F_2(q^2)$ in the limit $t\to 0$ are defined as

$$F_1(0)+F_2(0) = \mu_q,\;\; \mu_q = \frac{e_q}{2m_q}(1+\kappa_q), \tag{11}$$

with the values of anomalous quark magnetic moments [9]

$$\kappa_u = -0.123,\;\; \kappa_d = -0.088,\;\; \kappa_s = -0.198 \tag{12}$$

the form-factors of neutral mesons research becomes possible with the parameters, obtained in section **1**.

For the pseudoscalar π^0, η – mesons and ϕ and ω – vector mesons we use the following mixing schemes [13]

$$\left\{\begin{array}{l}|\pi^0\rangle = |\psi_1\rangle,\\ |\eta\rangle = X_\eta|\psi_q\rangle + Y_\eta|\psi_s\rangle + Z_\eta|\psi_G\rangle,\end{array}\right.\quad \left\{\begin{array}{l}|\phi\rangle = \cos\phi_V|\psi_q\rangle - \sin\phi_V|\psi_s\rangle,\\ |\omega\rangle = \sin\phi_V|\psi_q\rangle + \cos\phi_V|\psi_s\rangle,\end{array}\right. \tag{13}$$

where $|\psi_1\rangle = (1/\sqrt{2})|u\bar{u}-d\bar{d}\rangle$, $|\psi_q\rangle = (1/\sqrt{2})|u\bar{u}+d\bar{d}\rangle$, $\psi_s = |s\bar{s}\rangle$ and X_η, Y_η, Z_η – elements of mixing matrix for pseudoscalar mesons with gluonium content $|\psi_G\rangle$ [14].

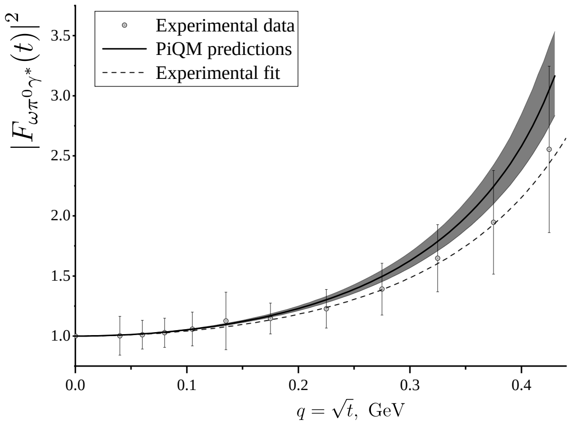

Figure 1. Behavior of the form-factor of the decay $\omega \to \pi^0\ell^+\ell^-$ for different q values. The experimental data with the fitting parameter $b_{\omega\pi^0} = (1.9 \pm 0.2)$ GeV^{-2} are taken from [15].

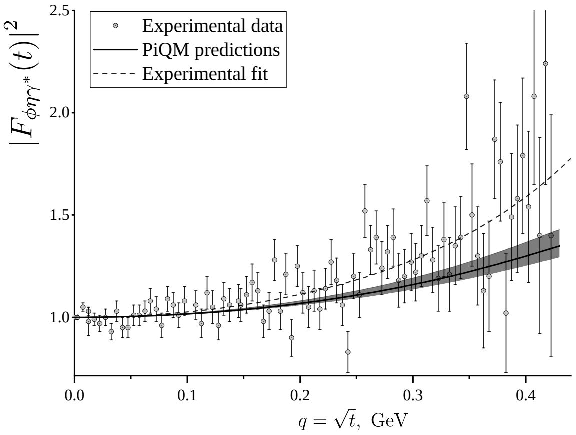

Figure 2. Behavior of the form-factor of the decay $\phi \to \eta\ell^+\ell^-$ for different q values. The experimental data with the fitting parameter $b_{\phi\eta} = (1.3 \pm 0.1)$ GeV^{-2} are taken from [16].

Choosing the form-factors $F_1(q^2)$, $F_2(q^2)$ in (10) as [17]

$$F_1(t) = \frac{e_q}{1 - \frac{\langle r_q \rangle^2}{6}\, t}, \; F_2(t) = \frac{e_q\ \kappa_q}{\left(1 - \frac{\langle r_q \rangle^2}{12}\, t\right)^2}, \; \langle r_q \rangle^2 = \frac{a}{m_q^2}, \; a = 0.3 \tag{14}$$

with mixing angle [18]

$$\theta_V = (31.92 \pm 0.2)^\circ, \; \theta_V = \phi_V - \arctan\sqrt{2} \tag{15}$$

and basic parameters of the model, calculated in section **1** from (7-9) one can obtain the behaviour of $\omega \to \pi\gamma^*$ and $\phi \to \eta\gamma^*$ form-factors (see pictures 1 and 2).

Conclusion

In the course of this work the authors have carried out the research of the behavior of form-factors of light neutral vector mesons under different transmitted to the leptonic pair momentums with account of anomalous magnetic quark moments, basing on the parameters that were obtained from leptonic decays of pseudoscalar and vector mesons. The analysis of the obtained results demonstrates that the suggested model describes the behaviour of radiative transition form-factors for ω – and ϕ – meson.

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