

# EMERGENT SPIN AND DUALITY OF TIME

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**Abstract.** A new approach to the solution of the well-known proton spin puzzle is formulated on the ground of the innovative concepts of Emergent Spin and natural Time introduced earlier.

A. When Goudsmit and Uhlenbeck proposed the spin hypothesis (George E. Uhlenbeck and Samuel A. Goudsmit, *Nature*, **117**(2938), 264-265(1926)), they had in mind a mechanical picture of the Top (rotating rigid body with fixed point). This picture had earlier been considered by Kronig as well. However, it was soon recognized that such an approach could not be realized in a sphere of our immediate experience, connected with movement in space. What does it mean from the our days state of affairs?

From classical mechanics it follows that for solution of a certain range of problems it is possible not to consider the internal structure of the object in the question and accept that it is at the same internal state (concept of a "point particle"). With the discovery of quantum mechanics we know that this internal structure exists in the form of a wave field and at this fundamental level the internal symmetry plays the same defining role as the external symmetry (a transformation of internal symmetry affects functions of the field and does not touch upon the coordinates). Hence, the constructive idea of Kronig, Goudsmith and Uhlenberck can be considered from a new point of view as internal symmetry of Intrinsic Top, which is defined by the following dual laws:

$$[S_j, S_k] = i\varepsilon_{jkl}S_l, \quad S_1^2 + S_2^2 + S_3^2 = s(s+1) = \frac{3}{4}, \quad (1)$$

$$[\tilde{S}_j, \tilde{S}_k] = i\varepsilon_{jkl}\tilde{S}_l, \quad \tilde{S}_1^2 + \tilde{S}_2^2 + \tilde{S}_3^2 = \frac{3}{4}, \quad [S_j, \tilde{S}_k] = 0. \quad (2)$$

At the fundamental (field - theoretical) level the idea of Spin as intrinsic form of the quantum Spherical Top angular momentum was not realized during the period of the quantum mechanics foundation. And now, in quantum mechanics and particle physics, Spin is considered as an intrinsic form of orbital angular momentum with the laws analogous to those of the quantum orbital angular momentum:

$$[S_j, S_k] = i\varepsilon_{jkl}S_l, \quad S_1^2 + S_2^2 + S_3^2 = s(s+1) = \frac{3}{4}.$$



One intrinsic angular momentum instead of two, as it should be for the Intrinsic Top;  $|+>$ ,  $|-> \Rightarrow |++>, |+>|+>, |+->, |-+>, |-->$ . One can put in correspondence to these laws a visual picture of rotation, when the axis of rotation is constant during the motion. In general, it is not the case (instantaneous axis of rotation).

A deeply hidden structure that can be put in correspondence to equations (1) and (2) was recognized in the paper (Ivanhoe B. Pestov. Emergent Spin and New Physics. J.Phys: Conf.Series **938**, 012034 (2017)) and presented at the DSPIN17. It was shown that the idea of Spin as an intrinsic form of the symmetry group of quantum Spherical Top can be realized on a set of the fundamental and simplest geometrical quantities, which form the spin field and themselves do not exhibit this property. Hence, Spin as Intrinsic Top is emergent property and this is the reason to call this phenomenon Emergent Spin. "Emergent Spin:" means that Spin is emergent property and Intrinsic Top.

Our goal here is to represent some aspects of a new approach to the well known **proton spin puzzle**, which is defined by the concept of Emergent Spin and duality of natural Time. The concept of natural Time was at first introduced in the book: Ivanhoe B. Pestov, in Horizons in World Physics **248**, Chapter 1 (2005) and duality of natural Time was presented at DSPIN05 (see I.B.Pestov, Proc. of XI Advanced Research Workshop on High Energy Spin Physics. Dubna:JINR,2006, p.228). We demonstrate that in a certain sense the well-known idea of "rotating rigid body" (also mentioned as the Top) of classical mechanics is as fundamental as the idea of "mass point", i. e. the first concept can be reduced to the second one. It is possible since there is a deep connection between the symmetry group of the quantum Spherical Top and the dual Time. Hence our next step is to represent very shortly the essence of Time and its duality.

B.Since the field of real numbers  $R$  is continuous and irrelevant to all forms of physical matter, we can define on this ground continuous and irrelevant natural geometry  $R^n$ , in which a point is defined as an  $n$ -tuple of real numbers  $x = (x^1, x^2, \dots, x^n)$ , and the distance function is introduced as usual  $d(x, y) = \sqrt{(x^1 - y^1)^2 + \dots + (x^n - y^n)^2}$ .

It is clear that  $R^1, R^2, R^3$  provide a new irrelevant representation of such things as the Euclidian straight line, plane and space, respectively. However,  $R^1, R^2, R^3$  admit simple and clear generalization and, hence,  $R^n$  is a very important geometry which can be considered as the underlying structure of any investigation in the field of geometry and physics. On the basis of natural geometry more complicated geometries may be constructed in which a point is defined as a point of some  $n$ -dimensional surface in the space  $R^N$ ,  $n < N$ .

It is evident that the variables (Cartesian coordinates)  $x^1, x^2, \dots, x^n$  in the definition of point  $R^n$  should be considered on an absolutely equal footing. Hence, it is unclear how to introduce the so-called space coordinates and time coordinate (a space-time structure of  $R^n$ ) in the framework of natural geometry alone. To do this, we need to give a definition of natural Time as an entity which is tightly connected with all natural dynamical processes and is as simple as possible from a geometrical point of view. To make it easier to perceive the definition given below, let us appeal to physical intuition. We know very well the physical phenomena connected with the temperature and pressure difference. We speak about the gradient of temperature and pressure and presuppose that values of these physical quantities are known for any point of some region of the Euclidian space. From a geometrical point of view we deal with a scalar field that is invariant with respect to all admissible transformations of coordinates. Now it is natural to suppose that there is a field of moments of Time and an area of phenomena defined by the gradient of Time.

*Definition:* a moment of natural Time is a number that we put in correspondence to any point of the reference space  $R^n$ . Hence, a moment of Time is defined by the equation  $t = f(x^1, x^2, \dots, x^n) = f(x)$ .

All points of the reference space that correspond to the same moment of Time  $t$  constitute

physical space  $S(t)$ . A point of  $S(t)$  is defined by the equation  $f(x^1, x^2, \dots, x^n) = f(x) = t = \text{constant}$ . This one-parameter family of physical spaces defines causality or determinism of physical reality itself.

The gradient of Time is the covector field  $\mathbf{t}$  with the components  $t_i = (\nabla f)_i = \partial_i f$ . In what follows we will consider case  $n = 4$ .

In the simplest case a scalar potential of natural Time  $f(x)$  is a solution for the equations:

$$\left(\frac{\partial f}{\partial x^1}\right)^2 + \left(\frac{\partial f}{\partial x^2}\right)^2 + \left(\frac{\partial f}{\partial x^3}\right)^2 + \left(\frac{\partial f}{\partial x^4}\right)^2 = 1, \quad f(\lambda x^1, \lambda x^2, \lambda x^3, \lambda x^4) = \lambda f(x^1, x^2, x^3, x^4).$$

The first equation can be considered as the definition of uniformity of natural Time and the second one take into account the possibility changing the scale. These equations have two solutions:

$$f(x) = a_i x^i = a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4,$$

where  $\mathbf{a} = (a^1, a^2, a^3, a^4)$  is a unit constant vector ( $\mathbf{a}|\mathbf{a} = 1$ ), and

$$f(x) = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2}.$$

From the equations

$$f(x) = a_i x^i = a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 = t = \text{constant},$$

and

$$f(x) = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2} = \tau = \text{constant}$$

we see that in one case the physical space is familiar three-dimensional Euclidian space  $E^3$  and in another case the new physical space is three-dimensional Riemannian space of constant positive curvature, i.e. the 3d-sphere  $S^3$ . The physical (mass) points are to be identified with the points belonging to the three-dimensional Euclidian space  $E^3$ , but the points belonging to the 3d-sphere  $S^3$  should be put in correspondence to the Spherical Tops. Indeed, the symmetries of Euclidian space can be composed of translations and rotations and the symmetries of the 3d-sphere  $S^3$  coincide with symmetries of the Spherical Top (1) and (2). In other words, geometrical points in Euclidian and Riemann spaces have different physical meaning. Thus, from the duality of Time it follows that any known particle can be put in correspondence to a dual particle moving in the dual Time. We see that there are two space-time structures (two different times) on the same reference space. The first space-time structure provides a more deep understanding of the so called Special Theory of Relativity. The dual space-time structure describes the general rotational motion as motion in dual Time. Hence, it is natural to put forward the idea of dual approach to the world of elementary particles which can explain the existence of leptons and quarks, lepton-quark symmetry and confinement (if we identify dual particles with quarks).

C.Below we exhibit the the system of equations which should be considered as a basis for the systematical development of quantum mechanics and quantum electrodynamics of the dual particles :

$$\begin{aligned} \frac{1}{\tau}(D + \frac{3}{2})\kappa &= \text{div } \mathbf{K} - m \mu \\ \frac{1}{\tau}(D + \frac{3}{2})\lambda &= \text{div } \mathbf{L} - m \nu \\ \frac{1}{\tau}(D + \frac{3}{2})\mu &= \text{div } \mathbf{M} + m \kappa \\ \frac{1}{\tau}(D + \frac{3}{2})\nu &= \text{div } \mathbf{N} + m \lambda \end{aligned} \tag{3}$$

$$\begin{aligned} \frac{1}{\tau}(D + \frac{3}{2})\mathbf{K} &= -\text{rot } \mathbf{L} + \text{grad } \kappa + m \mathbf{M} \\ \frac{1}{\tau}(D + \frac{3}{2})\mathbf{L} &= \text{rot } \mathbf{K} + \text{grad } \lambda + m \mathbf{N} \\ \frac{1}{\tau}(D + \frac{3}{2})\mathbf{M} &= \text{rot } \mathbf{N} + \text{grad } \mu - m \mathbf{K} \\ \frac{1}{\tau}(D + \frac{3}{2})\mathbf{N} &= -\text{rot } \mathbf{M} + \text{grad } \nu - m \mathbf{L}, \end{aligned} \tag{4}$$

where

$$\tau = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2}, \quad D = x^i \partial_i.$$

The operators of vector analysis in the case in question are defined as follows:

$$(\text{rot } \mathbf{M})^i = \frac{1}{\tau} e^{ijkl} x_j \partial_k M_l, \quad (i, j, k, l = 1, 2, 3, 4)$$

where  $e^{ijkl}$ , is the Levi-Civita tensor normalized as follows  $e_{1234} = \sqrt{g} = 1$ ;

$$(\text{grad } \varphi)_i = \Delta_i \varphi, \quad \Delta_i = \partial_i - \frac{x_i}{\tau^2} D, \quad \text{rot grad} = 0, \quad \text{div rot} = 0$$

and  $(\mathbf{x}|\mathbf{K}) = (\mathbf{x}|\mathbf{L}) = (\mathbf{x}|\mathbf{M}) = (\mathbf{x}|\mathbf{N}) = 0$ .

Two scalars  $\kappa$  and  $\mu$ , two pseudo-scalars  $\lambda$  and  $\nu$ , two vectors  $\mathbf{K}$  and  $\mathbf{M}$ , two pseudo-vectors  $\mathbf{L}$  and  $\mathbf{N}$  are defined by the complex spin field

$$\psi = (\psi, \psi_i, \psi_{ij}, \psi_{ijk}, \psi_{ijkl})$$

as follows :

$$\kappa = t^i \psi_i, \quad \mu = \psi, \quad \lambda = \frac{1}{3!} t_l e^{ijkl} \psi_{ijk}, \quad \nu = \frac{1}{4!} e^{ijkl} \psi_{ijkl},$$

$$\mathbf{K}^i = h_m^i \psi^m, \quad \mathbf{M}^i = t_k \psi^{ki}, \quad \mathbf{L}^i = \frac{1}{3!} h_m^i e^{mjkl} \psi_{jkl}, \quad \mathbf{N}^i = t_k \tilde{\psi}^{ki},$$

where  $h_j^i = \delta_j^i - t^i t_j$ ,  $\tilde{\psi}^{ki} = \frac{1}{2} e^{kijl} \psi_{jl}$ .

The inverse mapping has the form

$$\psi = \mu, \quad \psi_i = \mathbf{K}_i + \kappa t_i, \quad \psi_{ij} = t_i \mathbf{M}_j - t_j \mathbf{M}_i + e_{ijkl} t^k \mathbf{N}^l,$$

$$\psi_{ijk} = e_{mijk} \mathbf{L}^m + t^m e_{ijkm} \lambda, \quad \psi_{ijkl} = e_{ijkl} \nu.$$

The Maxwell equations for dual photons read

$$\frac{1}{\tau} (D + 2) \mathbf{H} = -\text{rot } \mathbf{E}, \quad \frac{1}{\tau} (D + 2) \mathbf{E} = \text{rot } \mathbf{H},$$

$$(\mathbf{x}|\mathbf{E}) = (\mathbf{x}|\mathbf{H}) = 0, \quad \text{div } \mathbf{E} = \text{div } \mathbf{H} = 0,$$

where

$$E_i = \frac{x^k}{\tau} F_{ik}, \quad H_i = \frac{x^k}{\tau} \bar{F}_{ik}, \quad F_{ij} = \partial_i A_j - \partial_j A_i, \quad \bar{F}_{ij} = \frac{1}{2} e_{ijkl} F^{kl}.$$

D. We suggest to consider leptons on the ground of the one space-time structure and connect quarks (dual particles) with the dual space-time structure on the same four-dimension Euclidian space. We put forward a conjecture that this beautiful duality is adequate to the nature of things since it gives a simple and evident explanation of lepton-quark symmetry, quark confinement and baryon number conservation. The confinement and the baryon number conservation simply mean that the quarks (dual particles) cannot change their space-time structure because they are doomed for the eternal natural rotation.

We propose equations (3) and (4) to unravel the spin puzzle.