Journal of Physics: Conference Series **1435** (2020) 012023 doi:10.1088/1742-6596/1435/1/012023

Quantum Field Effects of Acceleration and Rotation: the Chiral Vortical Effect and the Unruh Effect

Georgy Y Prokhorov[1,*], **Oleg V Teryaev**[1,2,3,†] **and Valentin I Zakharov**[2,4,5,‡]

[1] Joint Institute for Nuclear Research, 141980 Dubna, Russia
[2] Institute of Theoretical and Experimental Physics, NRC Kurchatov Institute, 117218 Moscow, Russia
[3] Dubna International University, 141980 Dubna, Russia
[4] School of Biomedicine, Far Eastern Federal University, 690950 Vladivostok, Russia
[5] Moscow Institute of Physics and Technology, 141700 Dolgoprudny, Russia

E-mail: *`prokhorov@theor.jinr.ru`, †`teryaev@jinr.ru`, ‡`vzakharov@itep.ru`

Abstract. The effects of vorticity and acceleration can be studied in the framework of a general fundamental approach based on the Zubarev density operator. The calculation of axial current is related to Chiral Vortical Effect (CVE) and hyperon polarization, while the calculation of acceleration corrections to energy density leads to the Unruh effect. The nonperturbative formula for CVE of massive fermions and the possibility of stabilization and suppression of the axial current in the plasma of elementary particles due to the appearance of an effective mass, depending on temperature, is discussed.

1. Introduction

The study of the effects associated with the rotation and acceleration of the medium is of great interest both from the theoretical point of view, which shows the relationship of these effects with the most fundamental quantum field properties of matter [1, 2], and from the point of view of experiment , where the possible manifestation of these effects in the observables obtained at particle accelerators is discussed [3, 4, 5], in particular, the hyperon polarization.

Currently, the fundamental method has been developed, which allows one to study the quantum field effects of rotation and acceleration, on the basis of a general theoretical approach. This is the method of the Zubarev density operator [6]. An application of this method to the study of the discussed effects can be found in the literature [7, 8, 9].

This paper presents the results for contributions to observables up to the fourth order. In particular, corrections to the energy-momentum tensor make it possible to study the Unruh effect in a novel way [9]. The main attention in this paper is devoted to corrections related to the vorticity in the axial current. Based on these corrections, a general nonperturbative formula for the axial current in a rotating medium in a state of global thermodynamic equilibrium, in which the vorticity enters as a chemical potential, is substantiated [8]. It is shown that this formula reproduces both the standard formula for CVE and the known corrections to CVE.

An analysis of the predictions of the quantum-statistical approach for the thermodynamics of a medium with acceleration and vorticity allows us to conclude that vorticity plays the role of the real chemical potential, and acceleration plays the role of the imaginary chemical potential

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd

[10]. In the general case, when acceleration and vorticity are present simultaneously, "effective" vorticity and "effective" acceleration [10] contribute to the chemical potential. Replacing the acceleration in chemical potential with effective acceleration is in agreement with considerations related to the formation of an event horizon in a non-inertial system.

In conclusion, the interesting possibility of stabilization and suppression of the axial current as the result of the appearance of an effective thermal mass of particles due to collective phenomena in a plasma, proportional to temperature, is discussed. Based on this, the possibility of interpreting some data obtained on the lattice [11] is discussed.

2. Calculation of corrections associated with the rotation and acceleration of the medium, Unruh effect

The Zubarev density operator for a medium in a state of global thermal equilibrium has the form [6, 7]

$$\hat{\rho} = \frac{1}{Z}\exp\Big\{ -\beta_\mu(x)\hat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\hat{J}^{\mu\nu}_x + \zeta\hat{Q}\Big\}\,, \tag{1}$$

where the effects associated with acceleration and rotation are contained in a term with thermal vorticity ϖ.

The quantum corrections in acceleration a_μ and vorticity ω_μ can be calculated using (1), in the framework of the finite-temperature perturbation theory developed in [7]. In particular, in [7, 8] the axial current for Dirac fields was calculated

$$\langle j^5_\mu\rangle = \Big(\frac{1}{6}[T^2 + \frac{\omega^2}{4\pi^2}] + \frac{\mu^2}{2\pi^2} + \frac{a^2}{8\pi^2}\Big)\omega_\mu + \mathcal{O}(\varpi^5)\,, \tag{2}$$

where $a = \sqrt{-a_\mu a^\mu}$ and $\omega = \sqrt{-\omega_\mu\omega^\mu}$. In [9] the energy-momentum tensor for fermions was calculated in the limit $\omega_\mu = 0$, $\mu = 0$, $m = 0$

$$T^{\mu\nu} = \rho u_\mu u_\nu - \frac{\rho}{3}(g_{\mu\nu} - u_\mu u_\nu)\,, \quad \rho = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17a^4}{960\pi^2} + \mathcal{O}(a^6)\,. \tag{3}$$

As discussed in [9], the remarkable property of (3) is that the energy-momentum tensor vanishes at Unruh temperature

$$T^{\mu\nu}(T = \frac{a}{2\pi}) = 0\,, \tag{4}$$

which is, in fact, another formulation of the famous Unruh effect [2], obtained now from the point of view of quantum statistical mechanics.

Note that using the Wigner function [12], the following expression can be obtained for the energy density

$$\rho_{Wig} = 2\int\frac{d^3p}{(2\pi)^3}E_p\Big(\frac{1}{1+e^{\frac{E_p}{T}+\frac{ia}{2T}}} + \frac{1}{1+e^{\frac{E_p}{T}-\frac{ia}{2T}}}\Big)\,, \tag{5}$$

which, however, does not satisfy (4), which is due to the fact that the Wigner function [12] is approximate.

It is obvious, however, that the integral representation of the form [9]

$$\rho = 2\int\frac{d^3p}{(2\pi)^3}\Big(\frac{|\mathbf{p}|+ia}{1+e^{\frac{|\mathbf{p}|}{T}+\frac{ia}{2T}}} + \frac{|\mathbf{p}|-ia}{1+e^{\frac{|\mathbf{p}|}{T}-\frac{ia}{2T}}}\Big) + 4\int\frac{d^3p}{(2\pi)^3}\frac{|\mathbf{p}|}{e^{\frac{2\pi|\mathbf{p}|}{a}}-1}\,, \tag{6}$$

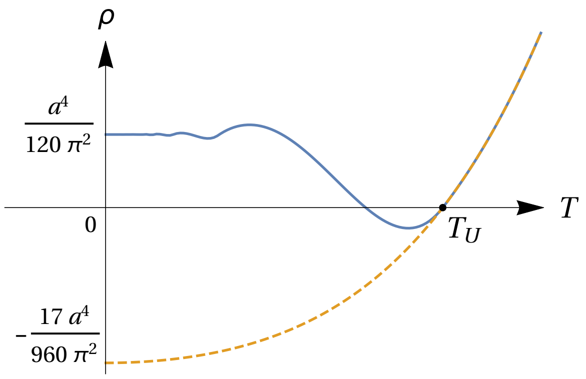

Figure 1. Energy density as a function of temperature. Integral representation (6) is shown by the solid blue line; the perturbative result (3) is shown by the dashed orange line.

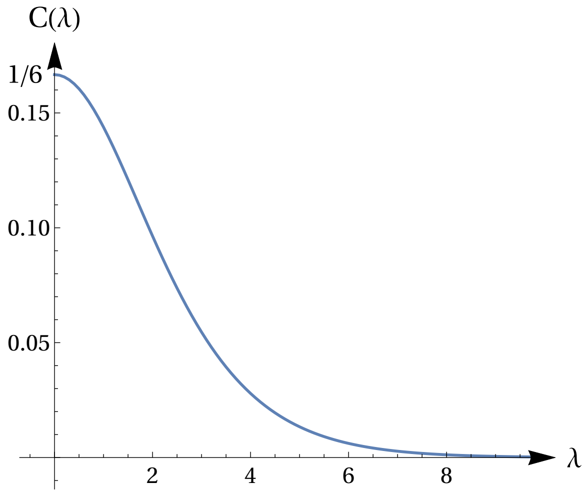

Figure 2. Stabilization and suppression of axial current due to thermal mass. The ration $C(\lambda) = \sigma/T^2$ as a function of λ.

automatically satisfies (4). Moreover, it turns out that for $T > T_U$ (6) exactly equals (3). Thus, the integrand in Eq. (6) looks like a good candidate for the distribution function of accelerated fermion gas. The plot of the function $\rho(T)$ in (6), compared with perturbative result (3) is shown in Fig. 1.

Remarkable property of the integrals in (6), is their polynomiality (naturally appearing in hadronic physics due to properties of Radon transform [13]). Integrating in the complex plane, we obtain [14]

$$\begin{aligned}\rho &= \frac{a^4}{120\pi^2} + \frac{T^4}{\pi^2}\Big(\frac{\pi D}{\sin(\pi D)}\frac{(-iy)^4}{4} + 2iy\frac{\pi D}{\sin(\pi D)}\frac{(-iy)^3}{3}\Big)\Big|_{y=\frac{a}{2T}} \\ &= \frac{7\pi^2T^4}{60} + \frac{T^2a^2}{24} - \frac{17a^4}{960\pi^2} \qquad T > \frac{a}{2\pi}\,, \end{aligned} \tag{7}$$

where $D = \frac{\partial}{\partial(-iy)}$. From (7) it is easy to obtain (3) and immediately see the polynomiality, in particular, connected with the polynomiality of the weight functions in (6).

Analysis of the form of the Eq. (6) shows that acceleration a plays the role of an imaginary chemical potential $\mu \to \mu \pm ia/2$. This has far-reaching consequences related to the stability of states under Unruh temperature $T < T_U$ [14].

3. Nonperturbative formula for axial current

The formula (2) was derived for massless fields, however, the coefficients for $\omega^2\omega_\mu$, $a^2\omega_\mu$ were also obtained in the case of massive fermions $m \neq 0$. These coefficients allow us to establish the general nonperturbative formula [8] for the axial current of massive fermions in a rotating medium in the case $a_\mu = u_\nu\partial^\nu u_\mu = 0$

$$\begin{aligned}\langle j^5_\mu\rangle &= \int \frac{d^3p}{(2\pi)^3}\Big\{n_F(E_p - \mu - \frac{\omega}{2}) - n_F(E_p - \mu + \frac{\omega}{2}) + \\ &\qquad n_F(E_p + \mu - \frac{\omega}{2}) - n_F(E_p + \mu + \frac{\omega}{2})\Big\}\frac{\omega_\mu}{\omega}\,, \end{aligned} \tag{8}$$

where n_F is the Fermi distribution, and all mass effects are accumulated in the energy $E_p = \sqrt{\mathbf{p}^2 + m^2}$.

Although (8) has not yet been strictly derived, it seems very likely that this formula correctly describes the axial current in a rotating medium. We list the main reasons to trust the formula:

1. If we expand (8) in a series, obtaining the terms $\omega_\mu, \omega^2\omega_\mu$ for a finite mass, then there is an exact match with the results obtained from the density operator (1).

2. Eq. (8) was obtained analytically in the framework of another method based on the covariant Wigner function for particles with spin 1/2 [12].

3. For $m = 0$, the formula (2) is reproduced, where the linear term simply corresponds to the CVE [1], and the third-order term also was discussed previously [15].

4. For $m = 0$, the integrand itself also corresponds to the results of [15].

5. The first mass correction has the form

$$\langle j^5_\mu \rangle = \Big(\frac{T^2}{6} - \frac{m^2}{4\pi^2}\Big)\omega_\mu \,, \tag{9}$$

which exactly corresponds to the calculation result in the curved space [16]. This correspondence was already noted in [7].

6. In (8) the vorticity enters as a chemical potential, and the coefficient 1/2 corresponds to the spin of the particle. This role of vorticity is a confirmation of the results in the literature [17].

4. Vorticity and acceleration as new chemical potentials

In Section 2, we discussed the role of acceleration as an imaginary chemical potential $\mu \to \mu \pm ia/2$. At the same time, the previous section shows that vorticity plays the role of the real chemical potential $\mu \to \mu \pm \omega/2$.

Thus, one would expect that in a more general case of a medium with acceleration and vorticity, the replacement $\mu \to \mu \pm \omega/2 \pm ia/2$ works. However, as shown in [10], the replacement $\mu \to \mu \pm \omega/2 \pm ia/2$ will be only in the case of parallel acceleration and vorticity $(\omega_\mu a^\mu)^2 = \omega^2 a^2$, while in the general case we should talk about "effective" acceleration g_a and "effective" vorticity g_ω

$$g_\omega = \frac{1}{\sqrt{2}}\Big(\sqrt{(a^2-\omega^2)^2 + 4(\omega_\mu a^\mu)^2} - a^2 + \omega^2\Big)^{1/2} \,,$$
$$g_a = \frac{1}{\sqrt{2}}\Big(\sqrt{(a^2-\omega^2)^2 + 4(\omega_\mu a^\mu)^2} + a^2 - \omega^2\Big)^{1/2} \,, \tag{10}$$

and the chemical potential is modified as follows $\mu \to \mu \pm g_\omega/2 \pm ig_a/2$.

In particular, g_a and g_ω equal a and ω, while in the case of perpendicular acceleration and vorticity $\omega_\mu a^\mu = 0$ effective acceleration is equal to zero for $a < \omega$. The absence of the imaginary part of the chemical potential removes instability below the temperature $\widetilde{T}_U = g_a/(2\pi)$ [10, 14]. The physical reason of this situation becomes quite clear if we consider the features of motion corresponding to $\omega_\mu a^\mu = 0$ and $a < \omega$ solving kinematical equations of motion and constructing the world lines. It can be shown that in this case it turns out that the movement becomes periodic and occurs along a flat spiral, the velocity varies in finite limits below speed of light [18]. Obviously, there is no event horizon in the system associated with such a movement, which explains the absence of thermodynamic instability. At the same time, when $a > \omega$, an event horizon appears, and, as a result, instability occurs at $\widetilde{T}_U$.

Journal of Physics: Conference Series **1435** (2020) 012023 doi:10.1088/1742-6596/1435/1/012023

5. Stabilization and suppression of axial current in a plasma of elementary particles

An interesting consequence of (8) is the ability to stabilize the axial current as a function of temperature. We expand (8) in a series of ω and consider the linear term at $\mu = 0$

$$\langle \hat{j}_5^\lambda(x)\rangle^{(1)} = \sigma(m,T)\omega^\lambda\,, \quad \sigma(m,T) = T^2 C(\frac{m}{T}) = \frac{T^2}{\pi^2}\int_0^\infty dx\, x^2 \frac{e^{\sqrt{x^2+(m/T)^2}}}{\left(1+e^{\sqrt{x^2+(m/T)^2}}\right)^2}\,, \tag{11}$$

where it turned out to be convenient to introduce the function $C(\frac{m}{T}) = \sigma(m,T)/T^2$, corresponding to the ratio of chiral vortical conductivity to the square of temperature, which depends only on the ratio m/T. In fact, (11) was obtained earlier in [7]. Obviously, $C(\frac{m}{T})$ does not depend on temperature only in two special cases

$$C(\frac{m}{T}) = \text{const}\,, \quad \text{if} \quad m=0\,, \quad \text{or} \quad m = \lambda T\,, \lambda = \text{const}\,, \tag{12}$$

in particular $C(0) = 1/6$. The second relation in (12) is typical for plasma physics - due to collective phenomena in the plasma, particles can acquire an effective mass proportional to the temperature [19]

$$m_{eff} = \lambda T\,, \tag{13}$$

where the coefficient λ depends on the details of the theory under consideration and, in particular, on the interaction constants.

In the case (13), the suppression of the coefficient C in (12), (13) is reached: it becomes less than its value at zero $C(0) = 1/6$

$$C(\lambda) \leq 1/6\,, \tag{14}$$

for any $\lambda \geq 0$, as follows from the form of the function $C(\lambda)$ in (11) and shown in the Fig. 2. Thus, the condition (13) is a necessary and sufficient condition for the stabilization of the ratio of chiral vortical conductivity to the square of the temperature σ/T^2, and this ratio is always suppressed.

It is important to note that stabilization and suppression of the ratio σ/T^2, were observed on the lattice. In particular, in [11], it was shown that this ratio is stabilized at high temperatures and is much less than $1/6$, resulting in the appropriate reduction of hyperon polarization [20]. As we see, precisely the same behaviour can be obtained if (8) is used instead of standard massless CVE formula and if it is assumed that the particles (quarks in [11]) have an effective thermal mass. It would be interesting to have a more detailed test of the applicability of the proposed mechanism of stabilization and suppression of σ/T^2 to explain the data [11].

6. Conclusion

The results of calculating corrections related to the vorticity and acceleration in the medium for Dirac fields, relevant for hyperon polarization, are presented, based on a fundamental approach with the Zubarev density operator. Corrections related to acceleration in energy-momentum tensor allow us to show the Unruh effect for fermions.

The corrections related to the vorticity in the axial current make it possible to substantiate the general nonperturbative formula for CVE with massive fermions for a medium in a state of global thermodynamic equilibrium and with zero acceleration. This formula reproduces the formula for CVE and the known corrections to it obtained in the literature as limiting cases. It receives motivation from the point of view of other methods, in particular, it can be precisely deduced from the Wigner function.

The results obtained using quantum statistical methods allow us to conclude that the vorticity plays the role of the real chemical potential, and the acceleration plays the role of the imaginary chemical potential. In the more general case, when there is both acceleration and vorticity, more complex combinations constructed from vorticity and acceleration contribute to the chemical potential: effective acceleration and vorticity can be distinguished.

The possibility of stabilization and suppression of the ratio σ/T^2 of the chiral vortical conductivity to the square of the temperature as the result of the formation of the effective thermal mass due to collective effects in the plasma is discussed. This mechanism allows us to explain qualitatively the behaviour of chiral conductivity obtained on the lattice, while a detailed quantitative analysis should be performed in the future.

Acknowledgments
The work was supported by Russian Science Foundation Grant No 16-12-10059.

[1] Kharzeev D E, Landsteiner K, Schmitt A and Yee H U 2013 *Lect. Notes Phys.* **871**
[2] Crispino L C B, Higuchi A and Matsas G E A 2008 *Rev. Mod. Phys.* **80** 787
[3] Sorin A and Teryaev O 2017 *Phys. Rev.* C **95** 1 011902
[4] Becattini F, Cao G and Speranza E 2019 *Eur. Phys. J.* C **79** 9 741
[5] Castorina P, Kharzeev D and Satz H 2007 *Eur. Phys. J.* C **52** 187
[6] Zubarev D N, Prozorkevich A V, Smolyanskii S A 1979 *TMF* 40:3 394-407; *Theoret. and Math. Phys.* 40:3 821-831
[7] Buzzegoli M, Grossi E and Becattini F 2017 *JHEP* **1710** 091
[8] Prokhorov G Y, Teryaev O V and Zakharov V I 2019 *JHEP* **1902** 146
[9] Prokhorov G Y, Teryaev O V and Zakharov V I 2019 *Phys. Rev.* D **99** 7 071901
[10] Prokhorov G, Teryaev O and Zakharov V 2018 *Phys. Rev.* D **98** 7 071901
[11] Braguta V, Chernodub M N, Goy V A, Landsteiner K, Molochkov A V and Polikarpov M I 2014 *Phys. Rev.* D **89** 7 074510
[12] Becattini F, Chandra V, Del Zanna L and Grossi E 2013 *Annals Phys.* **338** 32
[13] Teryaev O V 2001 *Phys. Lett.* B **510** 125
[14] Prokhorov G Y, Teryaev O V and Zakharov V I 2019 Instability at Unruh temperature as manifestation of singularity in complex momentum plane *Preprint arXiv*:1906.03529 [hep-th]
[15] Vilenkin A 1980 *Phys. Rev.* D **21** 2260
[16] Flachi A and Fukushima K 2018 *Phys. Rev.* D **98** 9 096011
[17] Jiang Y and Liao J 2016 *Phys. Rev. Lett.* **117** 19 192302
[18] Korsbakken J I and Leinaas J M 2004 *Phys. Rev.* D **70** 084016
[19] Laine M and Vuorinen A 2016 *Lect. Notes Phys.* **925** 1
[20] Baznat M, Gudima K, Sorin A and Teryaev O 2018 *Phys. Rev.* C **97** 4 041902