

Effect of Inhomogeneity of FGM Coating on Contact with Adhesion under the Axisymmetric Indenter

Fan Yang and Tiejun Liu*

Department of Mechanics, Inner Mongolia University of Technology, Hohhot 010051, China.

Email: liutiejun@imut.edu.cn; 285381416@qq.com

Abstract. The contact with adhesion between the FGM coating and axisymmetric indenter is probed into in the present paper. Coating is made of FGM with linear varying modulus. Poisson's ratios of coating and substrate are chosen to 1/3. The Maugis-type adhesive model is used to describe the adhesive interaction in the adhesion zone. According to the basic equations for axisymmetric problem, the controlling singular integral equations of the contact problem with adhesion are gained by making use of Hankel integral transformation. These integral equations are solved by applying a numerical collocation technique. The results show that the contact damage and deformation on the coating surface can be reduced by adjusting the inhomogeneity of the coating.

1. Presentation

Functionally graded material (FGM) is often used as coating or interface layers to resist contact damage or delete the stress concentrating [1, 2]. With the development of information technology, FGM is considered to be applied in the micro-nano field. The influences of adhesion are more obvious with the contact size of materials decreases [3]. During these years, many scholars have done lots of research on contact problem for functionally graded materials. Ke and Wang [4, 5] solved the contact problem of FGM coating with any function by using the multi-layered model which requires the elastic modulus of FGM in each layer vary according to the linear function. Liu and wang [6, 7] developed the model to explore the contact problems of functionally graded coating under axisymmetric indenter. Recently, Balci and Dag [8] consider the dynamic frictional contact mechanics of a FGM coating loaded by a rigid moving cylindrical punch based on the analytical method. By considering the adhesion effect which presented in JKR adhesion model, Jin et al [9] solved the axisymmetric contact problem for power-law gradient materials by considering the adhesion effect. Li and Liu [10, 11] applied the Maugis model to investigate the impact of adhesion on the contact problem between a functionally graded coating in which material parameter is characterized by exponentially varying modulus and a cylindrical indenter. Meanwhile, Liu et al [12] solved full adhesive contact problem for FGM coating by assuming the interfacial friction sufficient to prevent any slip taking place.

In this paper, the contact with adhesion for FGM coating with linear varying modulus under a cylindrical indenter is considered. The singular integral equations which describe the adhesive contact problem are gained by using the Hankel integral transformation. The Maugis' adhesion model is adopted to depict the adhesive interaction in the adhesion zone. An iteration method is applied to solve the singular integral equations. The effect of inhomogeneity of FGM coating on the contact behavior is analyzed.



2. Formulation

The contact geometry is shown in Figure 1a, in which a rigid spherical indenter is loaded on the surface of functionally graded coating. The homogenous materials substrate coated by FGM film with thickness h . The ratio of Poisson is chosen to 0.33. The shear modulus of the substrate is an invariant constant. The shear modulus of the graded coating is assumed to vary as linear function:

$$\mu(z) = \left(\frac{\mu_0 - \mu^*}{h} \right) z + \mu^* \quad (0 \leq z \leq h) \tag{1}$$

Where μ_0 refer to the elastic parameter of the coating surface and μ^* represent the elastic parameter of the substrate.

According to reference [6, 12], the non-dimension normal displacement can be expressed as:

$$\begin{aligned} \frac{3}{4} A\bar{r} = & \frac{1}{\pi} \int_{-1}^1 f(\bar{t}) \frac{1}{\bar{t} - \bar{r}} d\bar{t} + \frac{1}{\pi} \int_{-1}^1 f(\bar{t}) \frac{\log|\bar{t} - \bar{r}|}{2\bar{r}} d\bar{t} + \frac{1}{\alpha_1} \int_{-1}^1 f(\bar{t}) L(\bar{r}, \bar{t}) d\bar{t} \\ & + \frac{\lambda}{2\alpha_1} \left(\frac{A}{H} \right) m K_3(\bar{r}) + \frac{2}{\pi} \lambda m \left(E\left(\frac{\bar{r}}{m}\right) - K\left(\frac{\bar{r}}{m}\right) \right) \frac{1}{\bar{r}} \end{aligned} \tag{2}$$

Where $K(\cdot)$ and $E(\cdot)$ indicate respectively the first and second kinds of the complete elliptic integral [13], and

$$L(\bar{r}, \bar{t}) = \frac{1}{8} \left(\frac{A}{H} \right)^2 |\bar{t}| K_1(\bar{r}, \bar{t}) + \frac{\alpha_1}{\pi} \left(K_2(\bar{r}, \bar{t}) - \frac{\log|\bar{t} - \bar{r}|}{2\bar{r}} \right) \tag{3}$$

$$K_1(\bar{r}, \bar{t}) = \int_0^{\infty} \left[\frac{\bar{s}}{2h} M_{11} \left(\frac{\bar{s}}{2h}, h \right) - \alpha_1 \right] (-\bar{s}) J_0 \left(\frac{A \bar{s} \bar{t}}{H} \right) J_1 \left(\frac{A \bar{s} \bar{r}}{H} \right) d\bar{s} \tag{4}$$

$$K_2(\bar{r}, \bar{t}) = \frac{h(\bar{r}, \bar{t}) - 1}{\bar{t} - \bar{r}}, \quad h(\bar{r}, \bar{t}) = \begin{cases} \left| \frac{\bar{t}}{\bar{r}} \right| E\left(\frac{\bar{t}}{\bar{r}}\right), & |\bar{t}| < |\bar{r}| \\ \frac{\bar{t}^2}{\bar{r}^2} E\left(\frac{\bar{r}}{\bar{t}}\right) - \frac{\bar{t}^2 - \bar{r}^2}{\bar{r}^2} K\left(\frac{\bar{r}}{\bar{t}}\right), & |\bar{t}| > |\bar{r}| \end{cases} \tag{5}$$

$$K_3(\bar{r}) = (-1) \int_0^{\infty} \left[\frac{\bar{s}}{2h} M_{11} \left(\frac{\bar{s}}{2h}, h \right) - \alpha_1 \right] J_1 \left(\frac{A \bar{s}}{H} \right) J_1 \left(\frac{A \bar{s} \bar{r}}{H} \right) d\bar{s} \tag{6}$$

With J_p is the first kind of the p th order Bessel functions [14].

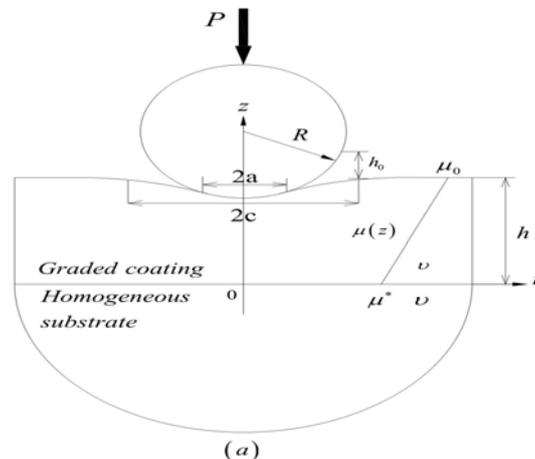


Figure 1. Functionally Graded Coated Substrate under the Rigid Spherical Indenter

In order to obtain the solution of the singular integral equation (2), the geometric relationship for the gap of the contacting bodies at $r=c$ and a is needed [15]. Using the normalizations, the additional equation is given by

$$-\left[\frac{2}{\lambda A} + \frac{\pi}{2} A(1-m^2)\right] = \frac{\pi}{3\alpha_1} \left(\frac{A}{H}\right) \int_{-1}^1 f(\bar{t}) \bar{t} K_4(\bar{r}, \bar{t}) d\bar{t} \Big|_{\bar{r}=1}^{\bar{r}=m} + \frac{4}{3} \int_{-1}^1 \frac{|\bar{t}|}{\bar{r}} f(\bar{t}) K\left(\frac{\bar{t}}{\bar{r}}\right) d\bar{t} \Big|_{\bar{r}=1}^{\bar{r}=m} + \lambda \frac{4\pi}{3\alpha_1} m K_5(\bar{r}) \Big|_{\bar{r}=1}^{\bar{r}=m} + \frac{8}{3} \lambda m \left(E(1) - E\left(\frac{1}{m}\right) \right) \quad (7)$$

Where

$$K_4(\bar{r}, \bar{t}) = \int_0^{\infty} \left[\frac{\bar{s}}{2h} M_{11}\left(\frac{\bar{s}}{2h}, h\right) - \alpha_1 \right] J_0\left(\frac{A}{H} \frac{\bar{s}\bar{t}}{2}\right) J_0\left(\frac{A}{H} \frac{\bar{s}\bar{r}}{2}\right) d\bar{s} \quad (8)$$

$$K_5(\bar{r}) = \int_0^{\infty} \left[\frac{\bar{s}}{2h} M_{11}\left(\frac{\bar{s}}{2h}, h\right) - \alpha_1 \right] \frac{1}{\bar{s}} J_1\left(\frac{A}{H} m \frac{\bar{s}}{2}\right) J_0\left(\frac{A}{H} \frac{\bar{s}\bar{r}}{2}\right) d\bar{s} \quad (9)$$

The non-dimension normal load is expressed as

$$\frac{P}{\pi w R} = -\left(\frac{1}{2} \pi A^2 \left(\int_{-1}^1 f(\bar{t}) |\bar{t}| d\bar{t} + m^2 \lambda \right) \right) \quad (10)$$

The non-dimension penetration depth can be determined

$$\frac{\delta}{(\pi^2 w^2 R / K^2)^{1/3}} = \frac{A^2}{2} - \left(\frac{A^2}{3\alpha_1 H} \int_{-1}^1 f(\bar{t}) |\bar{t}| K_4(1, \bar{t}) d\bar{t} + \frac{4A}{3\pi} \int_{-1}^1 f(\bar{t}) |\bar{t}| K(\bar{t}) d\bar{t} + \frac{4A}{3\alpha_1} \lambda m K_5(1) + \frac{8A}{3\pi} \lambda m E\left(\frac{1}{m}\right) \right) \quad (11)$$

3. Numerical Analysis and Results

Base on the above analysis, the approximate expression of integral equations (2), (7), (10) and (11) can be solved by using the Gauss-Chebyshev formula [14]. The effect of the shear modulus ratio on load vs. contact radius, load vs. maximum indentation depth, surface stress distribution of coating and the ratio of adhesion zone to contact zone are analyzed in this section.

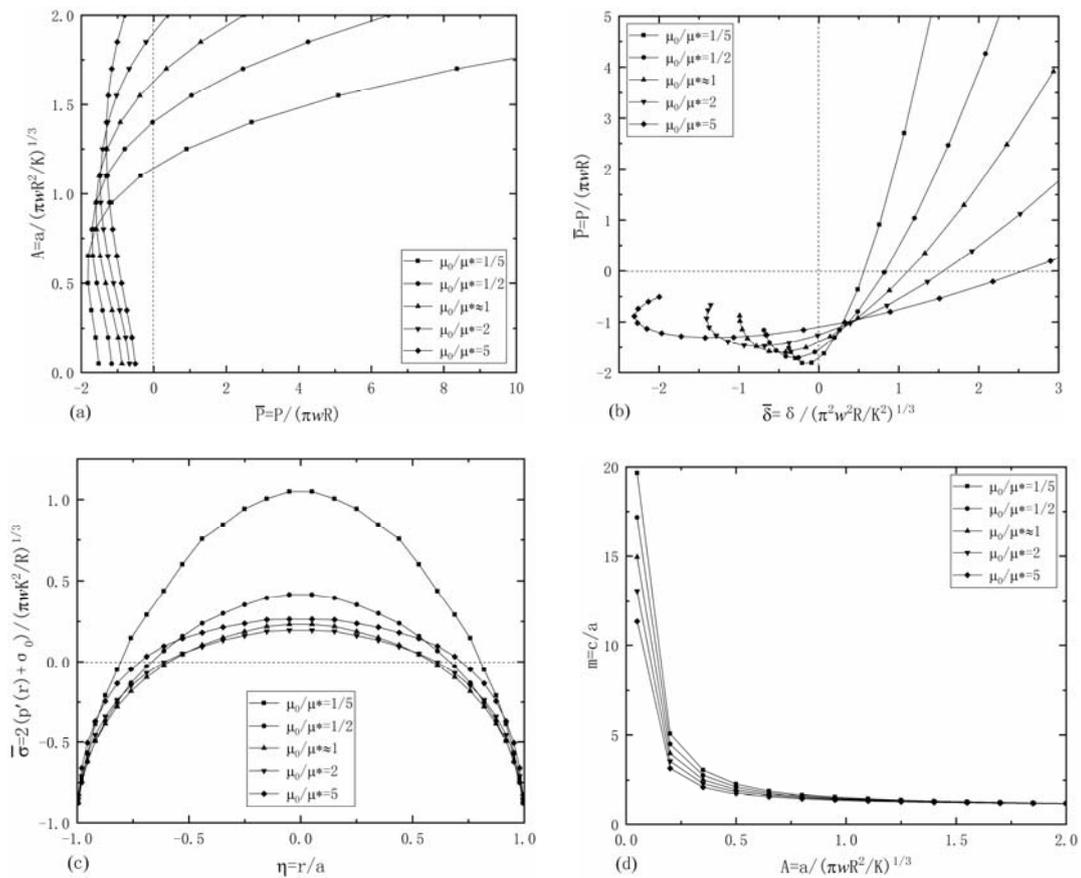


Figure 2. Effect of the shear modulus ratio μ_0/μ^* on \bar{P} vs. A (a) and \bar{P} vs. $\bar{\delta}$ (b), the non-dimension surface stress distribution of coating $\bar{\sigma}$ ($A=0.8$) (c) and the ratio of adhesion zone to contact zone m (d) when $\lambda=1$, $H=1$.

Figure 2 show that the effect of the shear modulus ratio μ_0/μ^* on \bar{P} vs. A (a) and \bar{P} vs. $\bar{\delta}$ (b), the non-dimension surface stress distribution of coating $\bar{\sigma}$ ($A=0.8$) (c) and the ratio of adhesion zone to contact zone m (d) when $\lambda=1$, $H=1$. It can be found from figure 2a that the pull-off force determined by the vertical tangents increase with the decrease of the shear modulus ratio. The similar results can be gained from figure 2b in which the horizon tangents determine the pull-off force. In figure 2b, the indentation depth increases with the shear modulus ratio increases for the fixed pressure. Figure 2c demonstrates that the surface stress $\bar{\sigma}$ gradually transitions from compressive stress to tensile stress along the radial direction because of the effect of adhesion. It can also be seen from figure 2c that the shear modulus ratio has a great influence on the coating surface stress distribution. This behavior indicates that the contact behavior can be improved by adjusting the stiffness of the coating. In figure 2d, the ratio of adhesion zone to contact zone m increase with the decrease of the shear modulus ratio.

4. Conclusions

The impacts of shear modulus ratio on the contact behavior of between a graded coating and a spherical indenter are analyzed. The numerical results demonstrated that the gradient index (denoted by μ_0/μ^*) of graded coating can alter the pull-off force and distribution of surface stress. The results in this paper provide a method to suppress contact deformation and damage by using the FGM coating.

5. Acknowledgment

The work described in the present paper was supported by National Natural Science Foundation of China under Grant Nos. 11662011 and 11811530067.

6. References

- [1] Suresh S 2001 Graded materials for resistance to contact deformation and damage *Sci.* **292** 2447-51
- [2] Zhang J, Zhang Y L, Fu Y Q, Li T and Meng J C 2019 Effect of HfC-SiC transition layer on the ablation resistance of SiC/HfC-SiC/HfC multi-layer coating for C/C composites *Vacuum* **169** 108886
- [3] Elloumi R, Kallel-Kamoun I and El-Borgi S 2010 A fully coupled partial slip contact problem in a graded half-plane *Mech. Mater.* **42** 417-428
- [4] Ke L L and Wang Y S 2006 Two-dimensional contact mechanics of functionally graded materials with arbitrary spatial variations of material properties *Int. J. Solids Struct.* **43** 5779-98
- [5] Ke L L and Wang Y S 2007 Two-dimensional sliding frictional contact of functionally graded materials *Eur. J. Mech. A-solid* **26** 171-188
- [6] Liu T J, Wang Y S and Zhang C Z 2008 Axisymmetric frictionless contact of functionally graded materials *Arch. Appl. Mech.* **78** 267-282
- [7] Liu T J, Wang Y S and Xing Y M 2012 Fretting contact of two elastic solids with graded coatings under torsion *Int. J. Solids Struct.* **49** 1283-93
- [8] Balci M N and Dag S 2019 Solution of the dynamic frictional contact problem between a functionally graded coating and a moving cylindrical punch *Int. J. Solids Struct.* **161** 267-281
- [9] Jin F and Guo X 2010 Non-slipping adhesive contact of a rigid cylinder on an elastic power-law graded half-space *Int. J. Solids Struct.* **47** 1508-21
- [10] Li P X and Liu T J 2018 The adhesive contact problem between a graded coated half-space and a cylindrical indenter by using a Maugis mode *J. Adhes. Sci. Technol.* **32** 2494-2508
- [11] Liu T J and Li P X 2019 Two-dimensional adhesion mechanics of a graded coated substrate under a rigid cylindrical punch based on a PWEMML model *Appl. Math. Model.* **69** 1-14
- [12] Liu T J, Yang F, Yu H F and Aizikovitch S M 2019 Axisymmetric adhesive contact problem for functionally graded materials coating based on the linear multi-layered model *Mech. Based Des. Struc.* 1-18
- [13] Hancock H 1958 *Elliptic Integrals* (New York: Dover Publications)
- [14] Andrews G E, Askey R and Roy R 1999 *Special Functions* (London: Cambridge University Press)
- [15] Sergici A O, Adams G G and Sinan Müftü 2006 Adhesion in the contact of a spherical indenter with a layered elastic half-space *J. Mech. Phys. Solids* **54** 1843-61