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Uncertainty Analysis for the Estimated Hydraulic Conductivity from the Reduced Duration Pumping Test

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Abstract. Use of a homogeneous aquifer method to represent a heterogeneous aquifer for parameter estimation will inevitably result in uncertainties in the estimated parameter values. The objective of this paper is to evaluate the impacts of the reduced duration of the pumping test on the estimated hydraulic conductivity using the observations from pumping tests. Firstly, a heterogeneous hydraulic conductivity *K* field was generated using the Monte Carlo method. A MODFLOW groundwater flow model was then constructed to perform numerical pumping tests. The *K* value was estimated by the inverse method based on the Theis solution for an unconfined aquifer. The *K* fields were generated based on the hydraulic conductivity data sets collected for a karst aquifer in southwest of China. The effective hydraulic conductivity (*Ke*) was calculated to represent the hydraulic conductivity of a heterogeneous aquifer. For a single test in a heterogeneous aquifer, while the duration of the test is less than a critical value, the reduced duration will lead to the big uncertainty of the estimated hydraulic conductivity. However, *Ke* can achieve a relative stable value while the number of reduced duration tests is big enough to eliminate the impacts of the uncertainty of the separate *K* values.

1. Introduction

The study on the heterogeneity of an aquifer is important to the analysis of groundwater flow and contaminant transport. The heterogeneity of an aquifer can include the spatial variation of hydraulic properties, such as hydraulic conductivity (*K*), specific yield (S_y), and coefficient of dispersity. Moench et al. (2001) noted that the differences (errors) between simulated and measured drawdowns in groundwater models are caused by local variations of hydraulic properties, primarily in *K*. Sudicky (1986) also reported that *K* is a key factor in controlling groundwater flow and solute transport, and it can vary significantly over short distances.

In the last century, the methods to determine *K* for a homogenous aquifer were developed from the point scale to regional scale, such as the slug test, permeability test, pumping test and inverse modeling. Therefore, numerous approaches were proposed to determine the *K* values of a homogenous aquifer. However, for a heterogeneous aquifer, the spatial heterogeneity results in difficulty in the determination the *K* values accurately under different scales. In the field studies on the estimation of *K* values of a heterogeneous aquifer, many researchers focused on the scale effects of *K* (Sudicky 1986; Illman 2006) and the methods of upscaling *K* values from a small scale (Felletti et al. 2006). In most of these studies, the actual heterogeneous aquifers were assumed to be homogenous aquifers, therefore, when the approaches derived from a homogenous aquifer are applied to a heterogeneous aquifer, they might lead to large uncertainties in the *K* values.

Devlin and McElwee (2007) used numerical modeling to study the effects of measurement error on the estimations of horizontal hydraulic gradient, which led to more attention to the uncertainty analysis of field tests and measurements. In practice, some pumping tests may be forced to stop due to the short depth of pumping well, the energy disruption, and other unpredictable factors, especially in a karst aquifer with strong heterogeneity, which might result in fewer observations with reduced duration of pumping test. The reduced duration tests (RDTs) induce a concern whether the fewer observations may significantly affect the estimations of hydraulic conductivity of a heterogeneous aquifer. The impacts of the reduced duration of test on the estimates of hydraulic properties were studied by Moench et al. (2001), and they reported that results for an 8-hour test were as valid as the results for a 72-hour test, which was conducted in Cape Cod, Massachusetts. In our study, the impact of the reduced duration of pumping tests on the estimated hydraulic conductivity will be further studied in several hypothetic aquifers with different heterogeneities.

In general, the understanding of heterogeneity is generally based on numerous field investigations, such as pumping test, slug test, ground tomography and other geophysical works; however, it is impossible to perform field investigations at each test site in a small scale to analyze the hydraulic properties of an aquifer. Due to the lack of data and sampling biases of field tests, numerical simulations and laboratory tests are implemented to achieve some insights of nature, which easily satisfy the specific statistical assumptions (Zhang et al. 2007). In comparison with the lab tests and numerical simulations, for the known physics the numerical simulation is more convenient to conduct than the lab test (Zhang et al. 2007; Beckwith et al. 2003). On the other hand, pumping test was always used to yield reasonable estimates of the average hydraulic conductivity of an aquifer (Butler and Healey 1998; Moench et al. 2001). This study focused on parameter estimation from pumping tests, and the drawdown datasets were primarily produced by the numerical pumping tests.

2. Materials and Methods

2.1. Aquifer Tests and the Solutions

Three pumping-recovery tests, nine slug tests and nine permeability tests (three constant-head tests and six falling-head tests) were conducted in an unconfined karst aquifer within the Houzhai basin from August 13th to 21st, 2008. The Houzhai basin is near the town of Puding, southwest of China. Drawdowns from the pumping and recovery periods were recorded in every ten minutes at the beginning of the tests. At the end of the pumping-recovery tests, the changes of drawdowns were minimal so that the groundwater flow in the aquifer was regarded as steady-state. The Dupuit solution for an unconfined aquifer was used to calculate the K value under the steady-state conditions. The drawdowns collected at each observation well in the pumping-recovery tests were used to estimate the hydraulic parameters using the Moench solution (Lu et al. 2011). Totally 13 K estimates were obtained using the Dupuit solution and the Moench solution based on the pumping-recovery tests. Both of the constant-head and falling-head tests were analyzed by Darcy's Law, and the slug tests were evaluated by the solutions of Hvorslev (1951) and Bouwer and Rice (1976). In all, 9 estimates and 18 estimates of K were obtained by the permeability tests and slug tests, respectively.

2.2. Numerical Simulations for the Pumping Test in a Heterogeneity Aquifer

2.2.1. Generation of the Random K Fields. Zhang et al. (2007) noted that the natural deposits often exhibit long range correlation. Webb and Anderson (1996) demonstrated that the pure random field with assumptions of spatial correlation and spatial trends might result in unrealistically smooth distribution of K. However, in a karst area, the spatial correlation exists slightly because of the strong heterogeneity of aquifer. Therefore, the Monte Carlo simulation is more feasible than the Gaussian sequential simulation in generating the random K fields in the study.

In order to study the impacts of the variability of an aquifer on the estimated K, the K field was given randomly according to the specific EK and Cv. The generation of random K value in this study followed the steps of the Monte Carlo method. The pseudorandom uniform numbers were produced by the algorithm described by Lehmer (1951), and the pseudorandom normal deviates were transformed from the uniform numbers using Box–Muller (1958) method. This random number generator has passed all the standard tests for random number generators.

2.2.2. Forward Modelling. The hypothetical pumping tests were conducted in heterogeneous aquifers generated based on the mean and deviation of *K* values from actual aquifer tests. MODFLOW was used to simulate the hydraulic heads in response to groundwater pumping. The simulated aquifer was unconfined, and the dimension of the model was 2500 m in length, 2500 m in width and 50 m in depth. The model area was discretized into 100 columns and 100 rows in the x and y directions, respectively. Since the determination of the vertical *K* was not the focus of this study, the model had only one layer of a depth of 50 m, thereby inducing a two-dimensional groundwater flow system. A pumping well was placed at the center of the model. The pumping test lasted for 24 hours with a constant pumping rate of 1728 m^3/d, which is the actual rate of a pumping test conducted near the town of Laoheitan. In all, the pumping duration was divided into 10 stress periods and 15 time steps in each stress period with a 1.1 multiplier used within each stress period. Therefore, 150 drawdowns were recorded at each of the two observation wells. In the study of the heterogeneous aquifer, two observation wells, labeled as Wmin and Wmax, were placed at different directions of the pumping well. Well Wmin was located in the short axis of the influence since pumping, and well Wmax was located in the long axis of the influence.

2.2.3. Estimation of the Hydraulic Conductivity. In this study, the Theis solution for the unconfined aquifer, labeled as Quasi-Theis, thereafter (Chen 2001), was used to calculate the hydraulic heads of the simulated aquifer, which is expressed as $h^2 = h_0^2 - \frac{Q}{2\pi K} W(u)$. Here, h_0 is the hydraulic head prior to pumping, *h* is the hydraulic head since pumping begins, and *W*(*u*) is the well function. Using of the Theis solution for this study is to document the difference between the derived *K* value from the simulated drawdown datasets and the actual *K* values built into the test heterogeneous aquifers. The gradient method reported by Chen and Ayers (1997) was regarded as the inverse method to estimate the *K* of the hypothetic aquifers, which was to minimize the square errors between the observed hydraulic heads from the forward modeling and the calculated hydraulic heads using the Quasi-Theis solution.

The effective hydraulic conductivity, labeled as *Ke*, was generally regarded as the hydraulic conductivity of the heterogeneous aquifer (Renard and de Marsily 1997), and *Ke* can be calculated from the discrete *K* values obtained from the previous parameter estimations. The equations proposed by Gutjahr et al. (1978) were adopted in this study because they are simple to conduct without considering the boundary conditions. *Ke* was calculated as follow, respectively,

$$K_e = K_g[1 + (\frac{1}{2} - \frac{1}{n})\sigma_Y^2],\ K_g = \left(\prod_{i=1}^{N} K_i\right)^{1/N} \tag{1}$$

Where σ_Y^2 the variance of LnK, Kg is is the geometric mean of *K* values, *n* is the dimensionality of media, *N* is the number of the *K* values. Since the numerical modeling was two-dimensional in the study, *Ke* is equal to *Kg*.

Ke represent the estimated hydraulic conductivity of the heterogeneous aquifer. *EK* and *Cv* for the *K* of model grids represent the average *K* and the variability of the hypothetical aquifer, respectively. In the numerical simulations, the *EK* values were set to be 10, 50 and 100 m/d, respectively, and the *Cv* values of given *K* at each mean value were set to be 0.1 (small variance), 0.5 (medium variance) and 1 (large variance). The *K* values of the girds in the model were assigned from the ten thousand *K* values. Under the condition of a set of the assigned *EK* and *Cv* values, one hundred realizations were run to obtain an estimate of *Ke* for an observation well. The impacts of the aquifer heterogeneity were assessed by generation of the heterogeneous *K* fields, running the numerical pumping tests in these heterogeneous aquifers and performing inverse modeling for *K* estimation.

3. Results and Discussion

3.1. The Estimated K Values in the Karst Aquifer

Figure 3 shows the wide range of the estimated *K* values from 10^{-7} to 10^{3} m/d in the karst aquifer located in Houzhai basin. The estimated *K* values range from 26.74 to 469 m/d, 6.39×10^{-5} to 20.7 m/d, and 6.67×10^{-7} to 32.5 m/d in terms of the pumping tests, permeability tests, and slug tests, respectively. The results show that the estimated *K* values from the pumping tests are greater than those from the permeability tests and slug tests, which may be attributed to the fact that the scales of pumping tests are larger than those of other aquifer tests (Rovey 1998). In the meanwhile, the *Cv* of the estimated *K* values from the pumping tests are lower than those from other tests, which indicates that pumping tests may yield a higher *K* value with less variability.

Scale effect of *K* means that *K* values vary with the data support (Zhang et al. 2007). The estimated *K* values from the pumping-recovery tests, permeability tests and slug tests were plotted versus the support volume (Schulze-Makuch et al. 1999) and the nominal length (Illman 2006) to evaluate the scale effect and characterize the variability of the *K* values. For the pumping tests, the pumpage in the pumping test and the radius of influence were regarded as the support volume and the nominal length, respectively. For the permeability tests and slug tests, the volume of the injected water and the depth of the test hole were considered as the support volume and the nominal length, respectively. Butler and Healey (1998) suggested that the fact of different *K* values with different measurement scales cannot be attributed to the scale effect without the examination of hydraulic tests. Here, when the support volume is regarded as the scale of measurement, the observed *K* values have an increasing trend with the support volume (Figure 1a). On the other hand, due to the lack of field investigations in the karst aquifer, when the nominal length is regarded as the scale of measurement, there is no increasing trend of *K* values with the nominal length (Figure 1b).

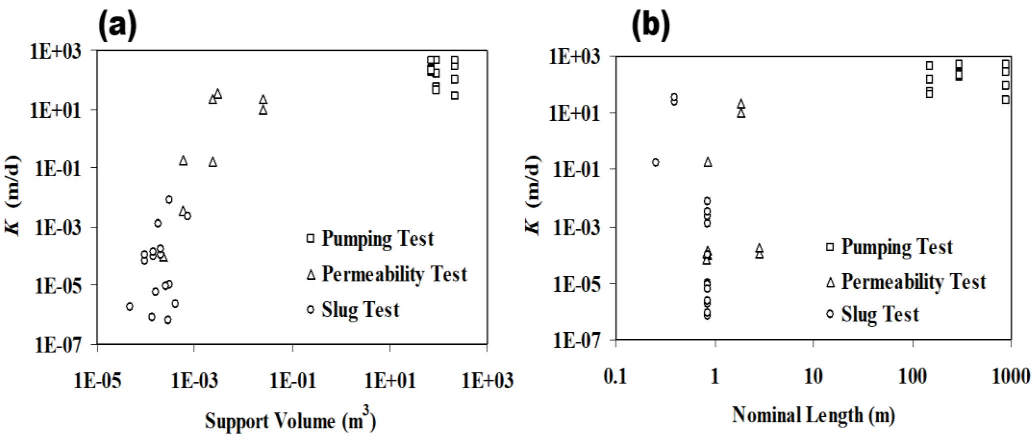

Figure 1. The Hydraulic Conductivities through the Field Tests with the Support Volume (a) and the Nominal Length (b). The Support Volume is calculated by the Involved Water Volume in Each Test. The nominal length represents the depth of test hole in permeability test and slug test, and represents the radius of influence in pumping test, respectively.

All the estimated *K* values show a log-normal distribution, which can be tested through the Kolmogorov-Smirnov (K-S) test (Figure 2a). The mean of log-K is -1.20 and the deviation of log-K is 3.22. The mean and *Cv* of the estimated *K* are 80.81 m/d and 1.73, respectively. In terms of the pumping tests, 13 different *K* values were estimated, and they are in a normal distribution from the K-S test (Figure 2b). The mean and C*v* of the 13 estimated *K* values are 223.35 m/d and 0.71, respectively. Sanchez-Vila et al. (1996) suggested a log-normal distribution for the *K* values in the aquifers. However, due to the non-stationarity of the log-normal distribution, extreme variations of the *K* field may lead to the nonconvergence of the computation in numerical models. In addition, since the

K value of a model grid represents the hydraulic conductivity of the entire grid, including the matrix and fissure within the area, the normal distribution for the 13 estimated *K* values from pumping tests may be more suitable for numerical simulation in this study. Therefore, the normal distribution was selected as the probability distribution function in the following numerical simulations.

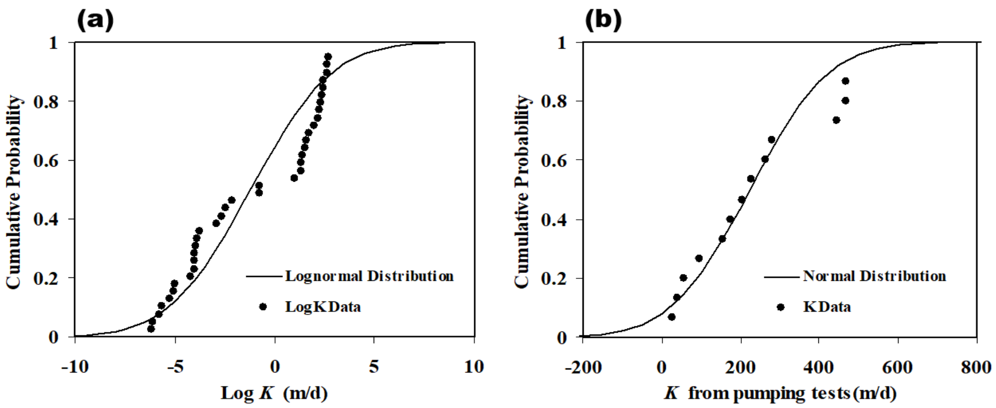

Figure 2. The hydraulic conductivities through the field tests with the support volume (a) and the nominal length (b). The support volume is calculated by the involved water volume in each test. The nominal length represents the depth of test hole in permeability test and slug test, and represents the radius of influence in pumping test, respectively.

3.2. Occurrences of Reduced Duration Test by Aquifer Heterogeneity

Given the *EK* and *Cv* values from the previous pumping-recovery tests in the karst aquifer, thousands of different *K* values representing the heterogeneous *K* field were generated randomly based on the Monte-Carlo method. As *Cv* increases, the variability of the given *K* distribution becomes stronger, and the probability of that the pumping well is located in a grid with extremely small *K* become greater. Because the pumping rate was constant in the numerical models, the grid cell with a small *K* value placed the pumping well will occasionally change into a dry cell in the process of the simulation. Since the durations of these tests were not equal to one day, here, these reduced duration tests (RDTs) were not taken into account to estimate *K*. However, in the next section, numerous artificial reduced duration tests were conducted to study the impacts of the reduced duration tests on the estimated hydraulic conductivity. Table 1 gives the occurrences of RDTs at different *EK* and *Cv* values. The occurrences increase as it has a small *EK* and a large *Cv*.

Table 1. Occurrences of Reduced Duration Test (RDT) Owing to the Variability of the Random K Field. One hundred realizations were performed with each set of EK and Cv.

Cv	0.1			0.5			1		
EK (m/d)	10	50	100	10	50	100	10	50	100
Occurrences	0	0	0	6	3	1	19	16	14

3.3. Impacts of RDTs on the Estimated K

As the description of the numerical simulation in the previous section, due to the strong spatial variability of *K*, the model cell will become dry during pumping, resulting in the RDTs. In practice, various reasons can cause RDTs. With respect to the estimate of *K*, the direct impact of RDTs is expressed by the shorter length of the observations from the reduced duration of tests. In this section, the impacts of reduced duration tests on the estimated *K* of a single pumping test and on the *Ke* value through one hundred realizations are discussed.

3.3.1. *Impact on the Estimated K Using the Datasets from a RDT.* According to the results of the numerical tests, there are two identified types of pumping tests (see Figure 3). The calculated heads and observed ones from the first type of pumping test easily agree well; however, the calculated head curve is difficult to fit the observed heads in the other type of pumping test. In terms of the two types of pumping tests, a series of artificially reduced observations were used to estimate *K*, such as 10% off, 20% off, 30% off and up to 90% off, respectively. The comparison of the observed time-head with the calculated time-head and the goodness of fit for the two curves are shown in Figure 3.

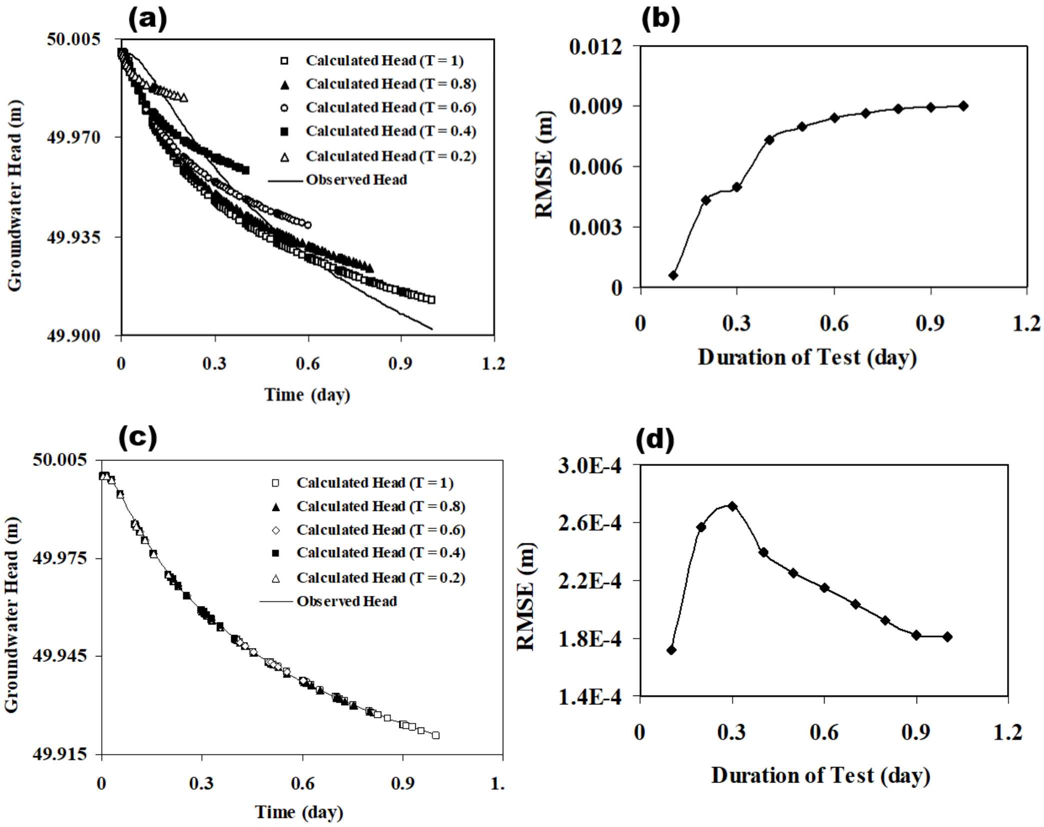

Figure 3. Comparison of the Observed Head with the Calculated Heads in Two Types of Pumping Test for a Heterogeneous Aquifer. (a) shows the observed time-head and calculated time-head using the estimated K with different observed duration, and the groundwater flow is significantly affected by the heterogeneity of aquifer in this type of pumping test. (b) shows the root mean square error (RMSE) between the calculated values and observed ones in the case of (a). (c) shows the observed time-head and calculated time-head, and the groundwater flow is not affected by the heterogeneity of aquifer obviously in this type of pumping test. (d) shows the RMSE in the case of (c).

Figure 3a shows the observed time-head and the calculated time-head of the first type of pumping test. The groundwater flow is significantly affected by the heterogeneity of aquifer (see Figure 3a) so that the curve of time-head is different with the curve in theory. The dissimilarity between the two curves results in the difficulty in fitting the curve of observed time-head using analytical solution. Though the root mean square error (RMSE) between the calculated heads and observed heads drops significantly (see Figure 3b) when the duration of pumping test decreases, it cannot be concluded that the more accurate estimation is achieved with the fewer observations. Because the reduced duration with the same pumping rate will lead to a smaller cone of depression, the differences between observed heads and calculated heads are less than the cases with longer pumping duration. Therefore, the smaller RMSE value resulted from the smaller drawdowns. With respect to the long-duration prediction, the estimates of *K* from the reduced duration test in terms of this type of pumping test will produce greater errors of the drawdowns.

The drawdown curves of the second type of pumping test are shown in Figure 3c. Since the groundwater flow is not affected obviously by the heterogeneity, the drawdown curves are generally fitted well by the analytical solutions. The absolute values of RMSE (Figure 3d) are one order of magnitude lower than the RMSE values of the first type of pumping test (see Figure 3b). In Figure 3d, when the duration is less than 0.3 day, the RMSE has lower value, which may result from the smaller drawdowns. While the duration is larger than 0.3 day, the RMSE value decreases with the increasing duration. The results indicate the accuracy of the estimated *K* increases with the increasing duration of test while the duration is greater than a critical value.

In particular, the different length of observations were used to estimate the *K* value while *EK* was equal to 100 m/d, and *Cv* was equal to 0.5 and 1, respectively. Figure 4 shows the estimated *K* varied by the duration of test at well Wmin and well Wmax. The estimates of *K* at each well can reach a relative stable value when the duration is greater than 0.6 day. Meanwhile, the influences of estimates are shown in Figure 4 while the duration is less than 0.4 day. The results indicate that a long duration test will reduce the uncertainty of the estimate of *K* in a heterogeneous aquifer. Moreover, the accuracy of the estimate cannot be improved significantly while the duration of test is larger than a critical value (specifically 0.6 day in this case). Real-time estimation of the K value can be regarded as an efficient correction method, and it increases the efficient of pumping tests and reduces the cost of field work.

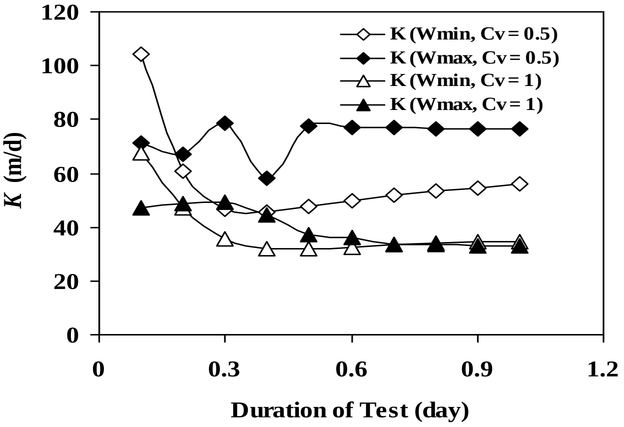

Figure 4. The Estimated K Using the Datasets of Different Durations at Observation Well Wmin and Wmax while the EK value is equal to 100 m/d, and the Cv values are equal to 0.5 and 1, respectively.

3.3.2. Impact of RDTs on the Estimated Ke. Numerous numerical tests were conducted to observe the effect of reduced durations on the estimated *Ke*. The *EK* value was set to be 100 m/d, and the *Cv* values were set to be 1 and 0.5, respectively. One hundred realizations were conducted to calculate a *Ke* value. In order to evaluate the relation between *Ke* and duration of test more accurately, anther one hundred realizations were run to obtain more estimates for each observation well with the same *EK* and *Cv* value.

Figure 5 gives the *Ke* values varied by the different durations of the tests. The different durations represent the length of observations 0% off, 10% off, 30% off, 60% off and 90% off, respectively. When *Cv* is equal to 1, the estimated *Ke* values are smaller than the corresponding estimates when *Cv* is 0.5. Because two *Ke* values were estimated in terms of an observation well in this section and there are two observation wells in the model, each point in Figure 5 represents the average value of the four *Ke* values from well Wmin and well Wmax. We cannot derive any significant trend from the results; therefore, the *Ke* values calculated from large numbers of tests are slightly influenced by the duration. That may result from that the averages of many tests (greater than or equal to one hundred realizations in this study) eliminate the random fluctuations of single estimated *K* of RDT.

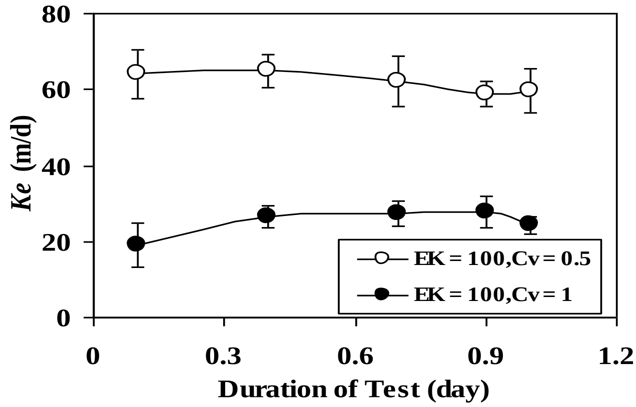

Figure 5. The estimated Ke varied by duration of test while the EK value is equal to 100 m/d, and the Cv values are equal to 0.5 and 1, respectively. The average value and error bar of each point are calculated from four separate values.

4. Summary and Conclusions

The estimated hydraulic conductivity is strongly affected by the heterogeneity of aquifers and the choice of determination methods. Especially for a pumping test, the duration of the test leads to the significant variation to the estimated hydraulic conductivity. In this study, the heterogeneity of the natural aquifer was observed using the field data collected by the field tests in Puding, southwest of China. The numerical model was used to study the impacts of the heterogeneity of aquifer on the estimated hydraulic conductivity.

The results of the field tests, including pumping tests, permeability tests and slug tests, indicate that the estimated hydraulic conductivities range widely from 10^{-7} to 10^{3} m/d. The estimates of *K* exhibit the strong heterogeneity of the karst aquifer. The results of pumping tests are generally larger than the results of permeability tests and slug tests. With respect to the support volume as the measurement scale, the scale effect is shown in our datasets. Due to the paucity of the nominal length as the measurement scale, the structure trend of hydraulic conductivity versus the nominal length does not display explicitly. All the estimated *K* values were log-normally distributed. However, the K estimations from pumping tests show the normal distribution. It should be noted that the estimated *K* from pumping tests are much smoother compared with all the estimates of *K*.

For a heterogeneous aquifer, while the drawdown curve is difficult to fit using a theory curve in the whole pumping duration, the reduced duration will bring high uncertainty of estimated hydraulic conductivity even the goodness of fit between the observed heads and simulated ones becomes better with the reduction of the duration. In terms of a pumping test conducted in a heterogeneous aquifer, the estimated hydraulic conductivity is influenced by the length of observation. A critical duration of test can be obtained to achieve a stable estimate of *K* and to reduce the uncertainty of single estimation. At the same time, the reduced duration of test can make a great economic benefit through the reducing cost of field work. However, the duration of test is not of importance to evaluating an effective hydraulic conductivity under the condition of large numbers of tests, because the random influences may be eliminated between each other. In other words, with respect to the effective hydraulic conductivity of a heterogeneous aquifer, reduced duration tests cannot change the estimates significantly.

5. Acknowledgement

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