Volumetric Displacement in Large Deflection of Symmetrically Layered Piezoelectric Plate under Pretension

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Abstract. The volumetric displacement of a simply supported and symmetrically layered piezoelectric circular plate under lateral load and pretension in large deflection is studied. The approach extends von Karman's large deflection theory to a layered case and accounts for the associated piezoelectric force terms. The nonlinear governing equations based on the equilibrium were derived, but the arising nonlinear terms in governing equations were dropped, to have an analytical investigation. The solutions were found to be expressible with modified Bessel or standard Bessel functions, depending on whether it is a mechanical-load dominating or piezoelectric-load dominating condition. The previously derived expressions for lateral slopes and deflections are further manipulated to derive the analytical expressions for the volumetric displacements, following strictly the related recurrence relations. The presented approaches are implemented with typical silicon-based layer materials used in a sensing or actuating device. Various dimensions, especially the relative thickness of the piezoelectric layer, as well as different applied voltages and pretensions are considered to have an extensive parametric study. The results indicate that, piezoelectric effect is only apparent in a low pretension condition. Upon reaching a moderate pretension, the pretension effect may always be dominant over the effects of varying the applied voltage and relative thickness.

1. Introduction

Piezoelectric devices have long been utilized as micro transducers including sensors and actuators, especially micro pumps in practical applications. This type of devices has shown many advantages in characteristics such as rapid response, high-energy capacity, and excellent efficiency in power consumption [1]. In the recent years, similar application has been broadened and extended to the field of medical industry. In this area, a comparatively new application is to make use of the property of piezoelectricity of a piezoelectric material in pumping drug fluid as a micro pump for drug delivery [2]. The same pumping principle appears to already have a very industrialized application in automobile field for pumping fuel flow for fuel delivery [3]. Similar piezoelectric actuating mechanisms have also been employed in PZT energy transducers to harvest energy of sound wave [4]. For the consideration of exploiting sustainable energy and resource, energy harvesting floors based on similar PZT materials have been developed and commercialized as well [5]. These kinds of devices are commonly designed and fabricated in a layered configuration, or maybe bossed with a proof mass. Devices with such configuration have demonstrated better performances such as improved sensitivity and linearity over a conventional flat member without a center boss [6]. In practical applications, however, they may often undergo a large deflection condition. Thus, a more advanced approach beyond classical plate theory that bases on Kirchhoff hypothesis is required to evaluate their structural responses. In literature, there have been available quite a few studies dealing with this type of problems [7-8]. In practical fabrication, they are often made by simply depositing a piezoelectric sensing or actuating layer onto a substrate

[9-11]. Thus, a shallow-patched structure is quite common for this type of devices.

Another key issue in designing and fabricating these types of members appears to be the arising of initial tension, on the other hand. A severe pretension may cause a possible warpage in early stage and premature failure. The pressure sensitivity may thus be degraded for the case a pressure sensor [12]. A worth-to-note fact is, however, pretension induced buckling is also the most reliable mechanism in evaluating the residual stresses [13]. Apparently, problems of large deflection of a patched plate under pretension are of practical importance. In literature, however, related available studies often consider the cases of clamped-ended edges and the center boss is usually taken as a rigid part, thus neglecting the possible elastic deformation of the center boss. For a layered piezoelectric transducer such as a micro pump, a center boss may be of a thickness comparable to the annular region. The possible elastic deformation should be included in problem formulation in assessing their structural behaviors. On the other hand, a simply supported problem is deemed to represent the other limited case regarding end support for practical application, though it has been considered in less. Solutions for a simply supported case should be of definite significance as it may serve as an opposite bound in structural characterization. It can likely be of more practical merit in simulating a real application for piezoelectric miniaturized devices.

This study is thus motivated to investigate the volumetric displacement in large deflection of a simply supported and piezoelectric layered plate under pretension. The approach extends von Karman's large deflection theory to consider a symmetrically layered piezoelectric plate. Thus, the associated piezoelectric force term will be included in formulating the problem. For a primary insight, however, the thus derived nonlinear governing equations were reduced to linear ones, by neglecting the arising nonlinear terms. This gives rise to a modified Bessel equation or standard Bessel equation for the lateral slope, depending on the relative magnitude of piezoelectric force term. The corresponding analytical solutions were developed by considering the boundary condition of the problem. The solutions for various geometrical responses, including lateral curvatures and deflections have been illustrated by the present authors in our previous work [14], all expressible in terms of either modified Bessel or standard Bessel functions. Following the solution of lateral deflections, one more tractable and feasible step is conducted in the present study to find the volumetric displacement, by integrating the solution of lateral defection over the entire plate. The significance of extending such analytical solutions is to provide a clue for detecting the quasi-static dosage for a single stroke in drug or fuel delivery applications [3], or maybe piezoelectric actuators operating alike. In real application, the importance and merit of finding the solution of volumetric displacement should worth further note, as it may also represent the key element for evaluating the acoustic compliance for an electro-acoustic device [15]. The developed approach will be implemented with typical silicon-based layer materials. A parametric study will be conducted to evaluate the effects of the ratio in thickness between the center piezoelectric layer and the substrate layer, as well as the magnitude of pretension and electric voltage applied across the piezoelectric layers. A recent work presented by Fox, et. al. [16] had ever evaluated the effect of dimension (radius and thickness) of an annular piezoelectric actuator film upon the behavior of lateral deflection of a circular plate, for different kinds of boundary conditions.   However, the influence due to initial tension was not considered.

2. Problem Description and Solution Method

In order to simulate a more practical situation compared to our previous investigation [14], a simply-supported isotropic circular plate symmetrically sandwiched between two piezoelectric outer layers polarized in the thickness direction is considered. It is subjected to an initial in-plane tension, N_0, a uniform lateral load, $p_z=p_0$, and an applied voltage V, across the piezoelectric layers polarized in the thickness direction, as shown in Figure 1. The governing equations based on force and moment equilibrium can be expressed by lateral slope, $w_{,r}$, in-plane force resultants, N_r and N_θ, and the moment resultants, M_r and M_θ, such that,

$$(rN_r)_{,r}-N_\theta = 0, \quad (rQ_r)_{,r}+(rN_r w_{,r})_{,r} = -p_0 r, \quad (rM_r)_{,r}-M_\theta - rQ_r = 0, \qquad (1)$$
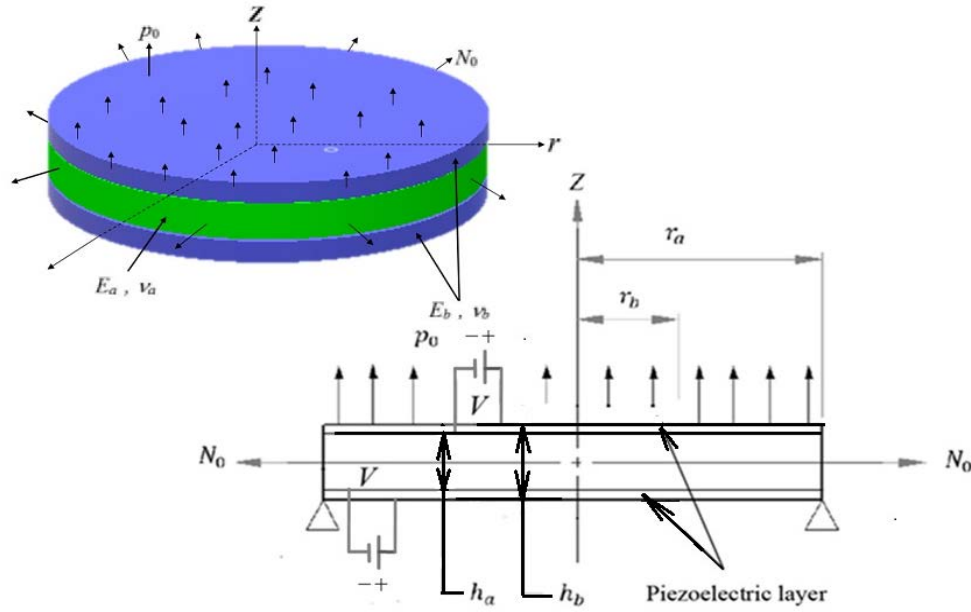
Figure 1. Simply Supported and Sandwiched Piezoelectric Layered Plate under Pretension in Lateral Load

Where Q_r is the transverse shear force resultant, and Ns and Ms are the force resultants and moment resultants, respectively, with the following common definitions based on laminate constitutive laws,

$$\begin{Bmatrix} N_r \\ N_\theta \end{Bmatrix} = \begin{bmatrix} A_l & A_t \\ A_t & A_l \end{bmatrix} \begin{Bmatrix} \varepsilon_r^\circ \\ \varepsilon_\theta^\circ \end{Bmatrix}; \quad \begin{Bmatrix} M_r \\ M_\theta \end{Bmatrix} = \begin{bmatrix} D_l & D_t \\ D_t & D_l \end{bmatrix} \begin{Bmatrix} \kappa_r \\ \kappa_\theta \end{Bmatrix} \tag{2}$$

In the above, As and Ds are the elements of the extensional and bending stiffness matrix, respectively, and both are expressible in terms of moduli and thickness of the composted layers of the plate. The subscript r (θ) represents a radial (circumferential) component, and the subscript l (t) for stiffness components represents a diagonal (off-diagonal) component. In addition, ε_α° and κ_α ($\alpha = r, \theta$) are the mid-plane strain and curvature components defined by the radial displacement, u, and the lateral deflection, w, such that,

$$\varepsilon_r^\circ = u,_r + (w,_r)^2 / 2, \quad \varepsilon_\theta^\circ = u/r; \quad \kappa_r = -w,_{rr}, \quad \kappa_\theta = -w,_r / r. \tag{3}$$

2.1. Nonlinear Governing Equations

For the problem of lateral loading after in-plane pretension, an incremental form for the equilibrium equations can be derived, following the approach presented previously [14]. For completeness, the previously illustrated equations are cited as follow,

$$\begin{cases} \hat{N}_r,_r + \dfrac{(\hat{N}_r - \hat{N}_\theta)}{r} = 0; \\ w,_{rrr} + \dfrac{w,_{rr}}{r} - \dfrac{w,_r}{r^2} - \dfrac{1}{D_l}\left(\hat{N}_r - N_r^P + N_0\right)w,_r = \dfrac{p_0 r}{2 D_l} \\ r\hat{N}_\theta,_r + (\hat{N}_\theta - \hat{N}_r) + A_q\left(N_\theta^P - N_r^P\right) + \dfrac{1}{2\bar{A}_l}(w,_r)^2 = 0 \end{cases} \tag{4}$$

Where a caped superscript and the superscript "*P*" represent incremental force and piezoelectric

force terms, respectively. In additions, $A_q = 1 - \overline{A_t}/\overline{A_l}$ and $\overline{A_t} = -A_t\overline{A_l}/A_l$ are all expressible in terms of extensional stiffness of the layered plate. Employing a non-dimensional scheme similar to that defined by Sheplak and Dugundji [8], these equations can further be simplified and merged to read.

$$\begin{cases} \xi^2\theta'' + \xi\theta' - [\xi^2(k^2 + 12S_r - N_r^p) + 1]\theta = 6P\xi^3 \\ \xi^2 S_r'' + 3\xi S_r' = -\dfrac{a^2 A_q}{D_l}(N_\theta^P - N_r^P) - \dfrac{h^2\theta^2}{2\overline{A_l}D_l'\xi} \end{cases} . \tag{5}$$

The related non-dimensional quantities are defined as follow:

$$\xi = \frac{r}{a} , \quad W = \frac{w}{h} , \quad U = \frac{u}{h}, \quad P = \frac{p_0 a^4}{hD_l'} , \qquad \theta = \frac{dW}{d\xi} = \frac{aw_{,r}}{h} , \quad \Psi = \frac{d\theta}{d\xi} = \frac{a^2 w_{,rr}}{h} ,$$

$$\theta' = \frac{d\theta}{d\xi}, \quad \dots \text{etc}, \qquad k = \sqrt{\frac{N_0 a^2}{D_l}} , \qquad (S_r, S_\theta) = (\hat{N}_r, \hat{N}_\theta)\frac{a^2}{D_l'} , \qquad (D_l', D_t') = 12(D_l, D_t).$$

2.2. Linear Problem and Analytical Geometrical Responses

Due to the presence of nonlinear terms including θ^2 and $S_r * \theta$, apparently, trying to solve the above nonlinear equations is formidable. For a preliminary insight, a simplified linear study, i. e., the case of small deflection is considered. The non-dimensional force resultant S_r may be small as well, so that the nonlinear term of product, $S_r * \theta$, can be negligible. In this manner, the first of previous equations for lateral slope can be reduced to the following linear one, i. e.

$$\xi^2\theta'' + \xi\theta' - [\xi^2(k^2 - N_r^p) + 1]\theta = 6P\xi^3, \tag{6}$$

Where ξ is the normalized radial coordinate. Depending upon the sign of the term, $k^2 - N_r^P$, this equation can be reformed as either one of the following two cases :

(i) For $(k^2 - N_r^p = k_m^2 > 0)$, i. e., **mechanical-load dominating** case, the previous equation may clearly be cast in a form as a modified Bessel equation, i. e.,

$$\xi^2\theta'' + \xi\theta' - [k_m^2\xi^2 + 1]\theta = 6P\xi^3. \tag{7}$$

Boundary conditions of the problem to be satisfied include: (i) Free lateral slope at the center of the plate due to axis-symmetry, i.e, $\xi = 0: \theta = 0$; and (ii) Free radial moment resultant at support ends, i. e, $\xi = 1 : M_r = 0$. Considering the asymptotic behaviors of the modified Bessel functions at infinity, the solution for θ can be derived in terms of modified Bessel functions, i. e.,

$$\theta(\xi) = \frac{6P}{k_m^2}[D_I I_1(k_e\xi) - \xi], \quad D_I = \frac{(D_t + D_l)}{\left[(D_t - D_l)I_1(k_m) + D_l k_e I_0(k_m)\right]} \tag{8}$$

Where $I_1(k\xi)$ is the modified Bessel function of the 1st kind. Subsequently, taking another step of integration and differentiation, respectively, the lateral deflection and curvature can be obtained following the recurrence relationships of the modified Bessel functions, i. e.,

$$W(\xi) = \int_1^\xi \theta(\xi)d\xi = \frac{6P}{k_m^2}\left\{\frac{D_I}{k_m}\left[I_0(k_m\xi) - I_0(k_m)\right] - \frac{1}{2}(\xi^2 - 1)\right\}, \tag{9}$$

$$\Psi(\xi) = \frac{6P}{k_m^2}\left\{D_I\left[kI_0(k_m\xi) - \frac{1}{\xi}I_1(k_m\xi)\right] - 1\right\}. \qquad (10)$$

Likewise, the solution of volumetric displacement (V_D) can further be derived, by integrating the lateral deflection over the entire plate based on similar recurrence relations of I_1 and J_1 [18], as well as the definition of (V_D). Thus, for mechanical-load dominating case,

$$V_{D(M)} = \int_0^1 (2\pi\xi)*W(\xi)d\xi = \int_0^1 (2\pi\xi)*\frac{6P}{k_m^2}\left\{\frac{D_I}{k_m}\left[I_0(k_m\xi) - I_0(k_m)\right] - \frac{1}{2}(\xi^2 - 1)\right\}$$

$$= \frac{12\pi P}{k_m^2}\left\{\frac{D_I}{k_m}\left[\int_0^1 \xi\bullet I_0(k_m\xi)d\xi - I_0(k_m)\int_0^1 \xi d\xi\right] - \int_0^1 \frac{1}{2}(\xi^3 - \xi)d\xi\right\}$$

Where since $\int_0^1 \xi\bullet I_0(k_m\xi)d\xi = \frac{1}{k_m^2}\int_0^1 (\eta_m)\bullet I_0(\eta_m)d(\eta_m) = \frac{1}{k_m^2}\left[(\eta_m)\bullet I_1(\eta_m)\right]_0^1 = \frac{I_1(k_m)}{k_m}$, we may

have

$$V_{D(M)} = \frac{12\pi P}{k_m^2}\left\{\frac{D_I}{k_m}\left[\frac{I_1(k_m)}{k_m} - \frac{I_0(k_m)}{2}\right] + \frac{1}{8}\right\} \qquad (11)$$

(ii) For $N_r^p - k^2 = k_c^2 > 0$, or **piezoelectric force dominating** situation, on the other hand, the problem reduces to a standard Bessel equation, i. e.,

$$\eta^2\theta'' + \eta\theta' + [\eta^2 - 1]\theta = 6P\eta^3/k_c^3, \qquad (12)$$

Where $k_c^2 = N_r^p - k^2$; and $\eta = k_c\xi$. The solution for lateral slope can be similarly formulated to read,

$$\theta(\xi) = \frac{6P}{k_c^2}\left[\xi - D_J J_1(k_c\xi)\right], \quad D_J = \frac{(D_t + D_l)}{\left[(D_t - D_l)J_1(k_c) + D_l k_c J_0(k_c)\right]} \qquad (13)$$

The corresponding solutions for lateral deflection and curvature can subsequently be derived, i. e.,

$$W(\xi) = \int_1^\xi \theta(\xi)d\xi = \frac{6P}{k_c^2}\left\{\frac{1}{2}(\xi^2 - 1) + \frac{D_J}{k_c}\left[J_0(k_c\xi) - J_0(k_c)\right]\right\}, \quad \Psi(\xi) = \frac{6P}{k_c^2}\left\{1 - D_J\left[k_c J_0(k_c\xi) - \frac{1}{\xi}J_1(k_c\xi)\right]\right\} \quad (14)$$

In the above, $J_0()$ and $J_1()$ are the Bessel functions of the 1st kind with order 0 and 1, respectively. Thus, the volumetric displacement can be similarly derived to take the form as,

$$V_{D(E)} = \int_0^1 (2\pi\xi)*W_{bE}(\xi)d\xi = \int_0^1 (2\pi\xi)*\frac{6P}{k_c^2}\left\{\frac{1}{2}(\xi^2 - 1) + \frac{D_J}{k_c}\left[J_0(k_c\xi) - J_0(k_c)\right]\right\}$$

$$= \frac{12\pi P}{k_c^2}\left\{\int_0^1 \frac{1}{2}(\xi^3 - \xi)d\xi + \frac{D_J}{k_c}\left[\int_0^1 \xi\bullet J_0(k_c\xi)d\xi - J_0(k_c)\int_0^1 \xi d\xi\right]\right\}$$

In the last equation, since $\int_0^1 \xi\bullet J_0(k_c\xi)d\xi = \frac{1}{k_c^2}\int_0^1 (\eta_c)\bullet J_0(\eta_c)d(\eta_c) = \frac{1}{k_c^2}\left[(\eta_c)\bullet J_1(\eta_c)\right]_0^1 = \frac{J_1(k_c)}{k_c}$, a

few more steps of manipulations can give rise to the expression of volumetric displacement for this case that,

$$V_{D(E)} = \frac{12\pi P}{k_c^2} \left\{ \frac{D_J}{k_c} \left[\frac{J_1(k_c)}{k_c} - \frac{J_0(k_c)}{2} \right] - \frac{1}{8} \right\} \tag{15}$$

3. Numerical Remarks

To implement the developed approach, a simply-supported and symmetrically layered plate composed of silicon (Si) substrate layer sandwiched between two piezoelectric outer layers of either poly-silicon (Poly-Si) or silicon-dioxide (SiO_2) were considered during the course of this study. The material properties (Young's moduli and Poisson's ratios) for the layers are listed in Table 1 [18]. The case with poly-silicon (Poly-Si) outer layers simulates a nearly monolithic plate (NMP), while that with the SiO_2 outer layers represents a typically layered plate. An outer radius of 500 (μm) for the plate is considered. The thickness of the substrate layer (h_a) is taken to be 10 (μm) and that of the piezoelectric layer (h_p) is set to be 0.25, 0.5, 1.0 or 2.5 (μm), respectively; to have a total thickness ratio of $h_b/h_a =$ 1.05, 1.1, 1.2, and 1.5 between the entire sandwiched plate and that of the middle substrate layer. In this way, hence, only comparatively thin piezoelectric layers were considered, to avoid the risk that the polarization orientation may be affected as the electric voltage is applied. The pretension parameter with k_a=1, 10, and 50 was implemented, similar to Sheplak and Dugundji [8]. Meanwhile, piezoelectric constant of the symmetric patches is taken to be d_{31}=50 pm/V [16] and the applied voltage is considered to be V=1, 5 and 10 (Volt) [11].

Table 1. Material Properties for the Layers of Piezoelectric Plates

Layer Material	Young's Modulus (Gpa)	Poisson's Ratio
PolySi	170	0.22
Si	165	0.27
SiO_2	75	0.17

For demonstration, however, the afore-mentioned nearly monolithic plate (NMP) was implemented first. In the case of a relatively low applied voltage, V=1 (volt), simulating a nearly pure mechanical loading condition, the solutions of lateral slope and deflection for various initial tensions versus central deflection (W_0) are presented in Figure 2 and 3, respectively. The results following CPT (Classical Plate Theory) obtained by taking the layered plate as a single layer with Young's modulus, E=170 (GPa) and the same Poisson's ratio as the silicon layer without initial tension is included for comparison. Apparently, the present solutions for the case of very low initial tension (k=1) correlate well with the CPT results and thus the present approach is validated.
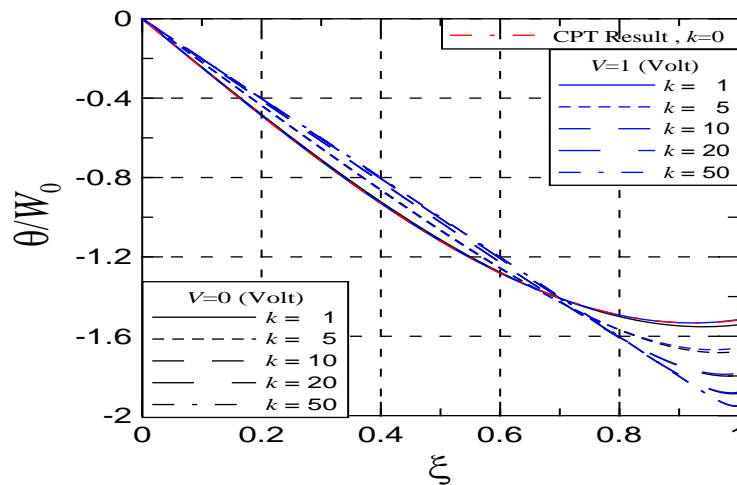

Figure 2. Comparison of Lateral Slope for Nearly Monolithic Plate with CPT Solutions
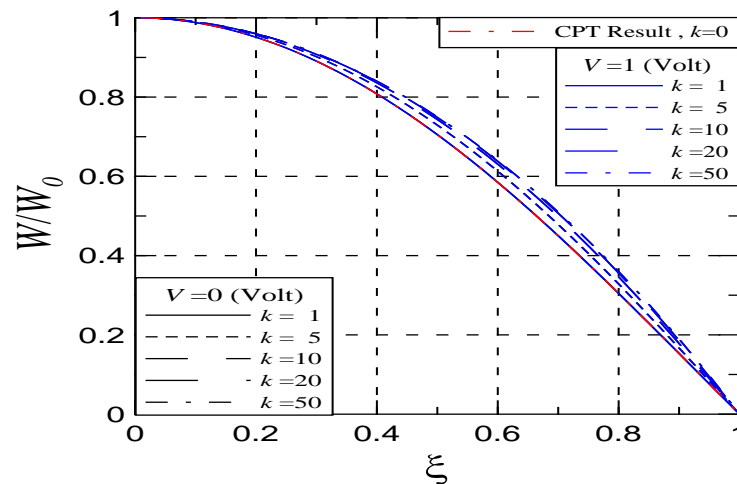
Figure 3. Comparison of Lateral Deflection for Nearly Monolithic Plate with CPT Solutions

Volumetric displacement (V_D) for the same plate under various applied voltages are evaluated for thickness ratios of h_b/h_a = 1.05, 1.1, 1.2 and 1.5 between the total sandwiched plate and the substrate layer. The results are plotted against the magnitude of pretension ($k=k_a$), as shown in Figure 4 to 7, respectively. Due to the inherent normalization scheme, it should be noted that, the present solutions for (V_D) are also normalized quantities relative to exactly the dimension of structural volume of the considered sandwiched plates. The obtained solutions show that V_D always have the highest magnitude when no pretension ($k=k_a=0$) is involved. The thinner the piezoelectric layers, the greater the piezoelectric effect is observed. This implies that not only a larger V_D will be obtained at the left end for the shown curves; it also shows a very sensitive change for V_D as the applied voltage (V) is varied. Under a low pretension and applied voltage, on the other hand, varying the thickness of the piezoelectric layers seems to have an apparent influence on the results of (V_D). In a low initial tension, conceivably, raising the applied voltage may rapidly increase the volumetric displacement, especially for a thin piezoelectric layer cases. Yet, as the pretension grows, volumetric displacement (V_D) may decrease apparently. The decreasing of (V_D) relative to pretension further shows that pretension will dominate over both the piezoelectric effect and the geometrical size (layer thickness) effect of the sandwiched plate. The volumetric displacement will eventually vanish, as the pretension keeps increasing to become relatively large.
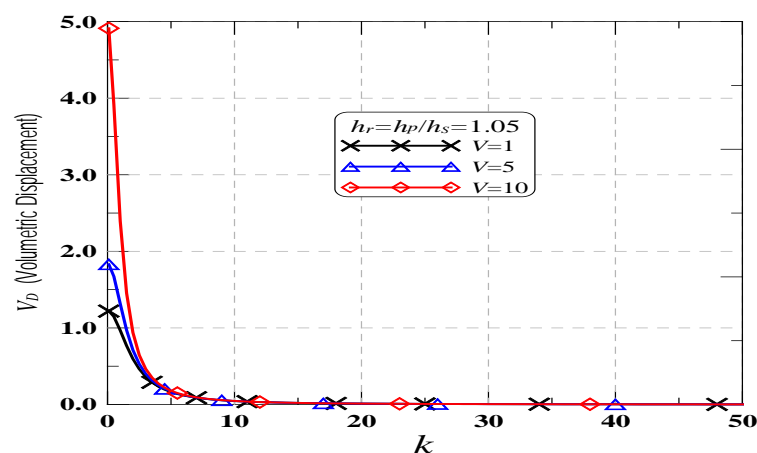

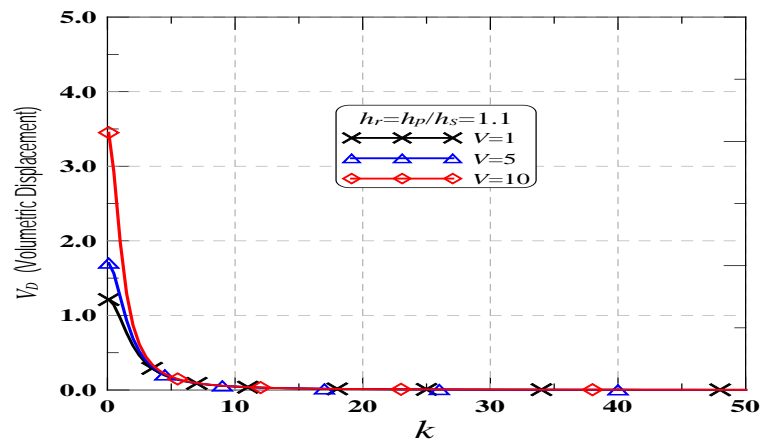**Figure 4.** Volumetric Displacement of [Poly-Si/Si/Poly-Si] Plate, h_r=1.05

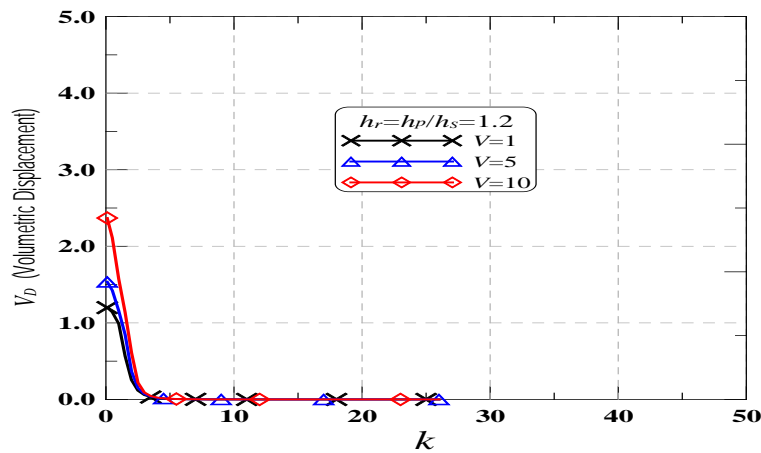**Figure 5.** Volumetric Displacement of [Poly-Si/Si/Poly-Si] Plate, h_r=1.1

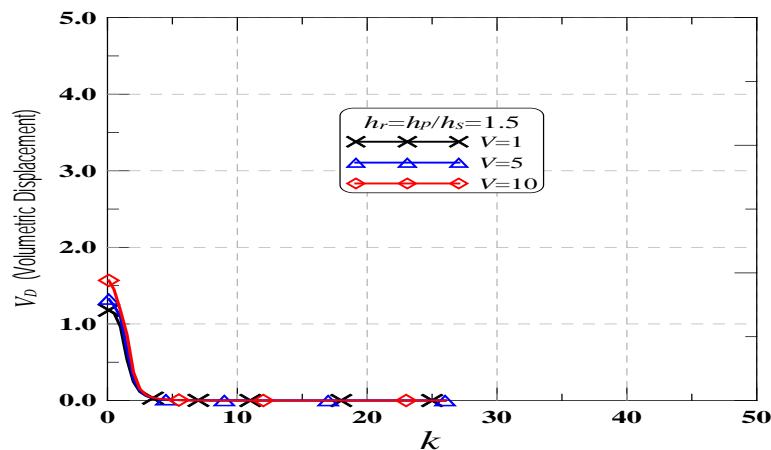**Figure 6.** Volumetric Displacement of [Poly-Si/Si/Poly-Si] Plate, h_r=1.2

Figure 7. Volumetric Displacement of [Poly-Si/Si/Poly-Si] Plate, h_r=1.5

4. Conclusion

The problem volumetric displacement in large deflection of a pre-stressed piezoelectric and sandwiched plate due to lateral load is studied. For a nearly monolithic plate under a very low applied voltage, the obtained solutions agree with available solutions for the case of pure mechanical loading and thus the presented approach is checked. The solutions for typically layered plate show that,

piezoelectric effect and geometric size effect of the layered plate may come into play when it is in a low pretension.   Upon reaching a moderate magnitude for the pretension, the pretension effect will dominate over the effects of the applied voltage and relative thickness ratios of the layers.

5. References

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