

A dynamic aspiration-based interaction strategy blocks the spread of defections in social dilemma

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Abstract – Considering the time-varying selection criterions of neighbors, we propose a dynamic interaction strategy to determine interactive neighbors based on aspiration payoffs. Under this strategy, interactions decide the aspiration payoffs and interactive payoffs, after that, the group is divided into high-payoff and low-payoff subgroups. And individuals in the same subgroup constitute the interactions in turn, which indicates an internal feedback mechanism. Based on the spatial prisoner's dilemma game, simulations show that the introduction of this interaction strategy successfully delays the extinction of cooperators. And cooperation is enhanced at appropriate aspiration levels but suppressed with high temptations. Moreover, under an appropriate aspiration level, high-payoff cooperators are able to expand the scale of cooperator clusters and low-payoff cooperators maintain the stability of cooperative behaviors by both forming an isolation belt around cooperator clusters and promoting the appearance of high-payoff cooperators, which blocks the invasion of defectors and leads to a state of complete cooperation.

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Introduction. – Though the phenomenon of cooperation is widespread in real life, it is still a huge challenge to fully understand the internal mechanism [1,2]. Evolutionary game theory provides a powerful framework for this kind of problem [3–5]. Network reciprocity [6,7], one of the mechanisms to explain the emergence of cooperation, together with the classic games, describes the evolution of self-behaviors in social networks vividly. Network structure provides the game environment. A large number of studies indicate that the network structure plays an important role in the study of cooperative behaviors [6,8–13]. The incentive mechanism, another important factor to study cooperation by introducing other means such as punishment, also has significant effects on promoting cooperative behaviors [14–16].

Individuals usually interact with all neighbors to obtain accumulate payoffs. However, they tend to interact with some neighbors in reality, and these interactions change with time. In other words, the interactive network is a dynamic subset of the fixed social network. Based on these backgrounds, the influence of limited interaction on cooperation is gradually getting attention. Traulsen *et al.* [17]

used probability to expand deterministic interactions into random interactions in a finitely well-mixed group. Chen and Wang [18] utilized the reputation attribute of individuals with tolerance to select neighbors. Chen *et al.* [19] used the interaction probability p to select neighbors, and get an optimal strong interaction region. Further, Chen and Szolnoki [20] regarded personal wealth as the criterion for whether individuals participate in the game, and concluded that the cooperative behaviors can become the dominant strategy within a broader parameter interval. Julia Poncela *et al.* [21] used the concept of association ability to select interactive neighbors. Wu *et al.* [22] exploited degree-based and reputation-based mechanisms to select interactive neighbors.

Besides, aspiration is also useful for selecting partners. Aspiration [23] is firstly used in the deterministic win-stay-lose-shift (WSLS) strategy updating rule, which is called innovative strategy [24], and then mainly applied to fusion with other strategy updating rules [25–27] after changing into the form of owning fault tolerance [28]. Based on the aspiration, Perc and Wang [29,30] explored the synergistic effects of diverse strategy sources to be copied by individuals on cooperation. Shen *et al.* [31] focused on how the changing link weight based on aspiration

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affects cooperators. Li *et al.* [32] studied the influence of conditional participation on cooperative behaviors. Inspired by these researches, this paper studies the dynamic aspiration-based interaction strategy in the spatial prisoner's dilemma.

Model. – Different from other selections, interaction strategy here is a kind of iteration process. Individuals gain their current interactive payoffs and aspiration payoffs after playing games with their selected neighbors. These selected neighbors depend on both the aspiration payoffs and interactive payoffs in the last step. To be clear, we suppose that social network N describes connections among individuals, and the interactive network K depicts interactions among individuals. Initially, the interactive network K is the social network N , then becomes a dynamic sub-network of N during the selection process. We assume that neighbors of individual i in N and K are sets N_i and K_i^t in t step, respectively. The number of social neighbors and interactive neighbors of i are $|N_i| = n_i$ and $|K_i^t| = k_i^t$, respectively, here $k_i^t \in [0, n_i]$. The interactive payoff of i are $P_i^t = \sum_{j \in K_i^t} P_{ij}^{t-1}$ and the aspiration payoff of i is $P_{iA}^t = k_i^t \times A$ [33]. P_{ij}^{t-1} represents the interactive payoff of i after playing the game with the interactive neighbor j in $(t-1)$ step. A indicates the aspiration level of individual i . Thus, when $P_i^t \geq P_{iA}^t$, j is an interactive neighbor of i in $(t+1)$ step if and only if $P_j^t \geq P_{jA}^t$, and vice versa, namely individuals only play games with homogeneous neighbors. This selection rule of interactive neighbors is

$$j \in K_i^{t+1} \Leftrightarrow \begin{cases} P_j^t \geq P_{jA}^t, & \text{if } P_i^t \geq P_{iA}^t, \\ P_j^t < P_{jA}^t, & \text{if } P_i^t < P_{iA}^t. \end{cases}$$

The number k_i^{t+1} of interactive neighbors in $(t+1)$ step can be obtained by the relationship between interactive payoffs and aspiration payoffs of both individual i and his n_i neighbors in t step.

When $A > 0$, all individuals in the group are in a dynamic interaction game since individual i updates his strategy in each time step, and thus his interactive neighbors change with time. Consequently, though the aspiration level A is fixed, the interactive network K keeps changing. When $A = 0$, the interactive network K is always equivalent to the social network N during the selection process and this dynamic evolution selection process reduces to the classic spatial prisoner's dilemma game.

Individuals whose interactive payoffs are not lower than their aspiration payoffs are defined as high-payoff individuals, otherwise defined as low-payoff individuals. This interaction strategy indicates that a group is divided into high-payoff and low-payoff subgroups. The subgroups are depicted in fig. 1. As individuals in subgroups change with time, aspiration payoffs and interactive payoffs of individuals also alter with the number of interactive neighbors in this paper.

For simplicity, we simulate this interaction strategy in prisoner's dilemma game. The re-scaled payoff matrix

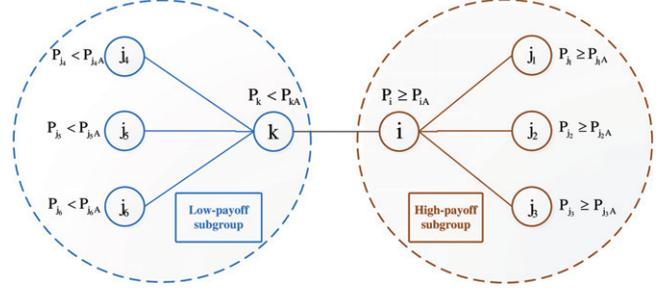


Fig. 1: A simple sketch of the interaction strategy in the evolutionary game. Small circles represent eight individuals in N . All solid lines represent the connections among individuals in N . All solid lines except for the black one constitute interactions among individuals in K . In addition, the solid line in blue (brown) means interactions in the low-payoff (high-payoff) subgroup, and the black one means there is no interaction between heterogeneous individuals. Therefore, both individuals i and k have four social neighbors, but only interact with three neighbors in the game.

depends on one single parameter b : $T = b$, $R = 1$, and $P = S = 0$. b represents the temptation to defect and is in the range $(1, 2]$. Individuals use the Fermi rule to update strategies. Thus, individual i in the above assumption randomly selects a learning object among n_i neighbors. Let the individual j be the learning neighbor selected by i in t step, then the individual i adopts the strategy of j with the probability of $W(S_j^t \rightarrow S_i^{t+1})$ in $(t+1)$ step:

$$W(S_j^t \rightarrow S_i^{t+1}) = \frac{1}{(1 + \exp((P_i^t - P_j^t)/\kappa))}, \quad (1)$$

where S_i^{t+1} and S_j^t means the strategy of individual i in $(t+1)$ step and j in t step, respectively. P_i^t and P_j^t is the interactive payoff of individual i and j in t step, respectively. The parameter κ represents the amplitude of noise and is set to 0.1 in the following simulation.

Simulation results. – We simulate this model on a square lattice of size 100×100 . Initially, strategies of individuals are randomly selected from cooperation (C) and defection (D) with the probability 0.5. Each individual updates his strategy synchronously after interactions. To overcome the randomness, the process is carried out for 100 Monte Carlo simulations and each contains 10000 time steps. Simulation results in the equilibrium state are averaged over the last 100 time steps.

Figure 2 shows the density of cooperators as a function of iterations for distinct aspiration levels. Compared with the results in classic spatial prisoner's dilemma game represented by $A = 0$ [34,35], the introduction of this aspiration-based interaction strategy has successfully slowed the spread of defections. When A is 0.3 and 0.5, respectively, cooperators decrease first, then increase, and finally keep a certain proportion, and the group reaches a state of complete cooperation when $A = 0.7$. Thus, there

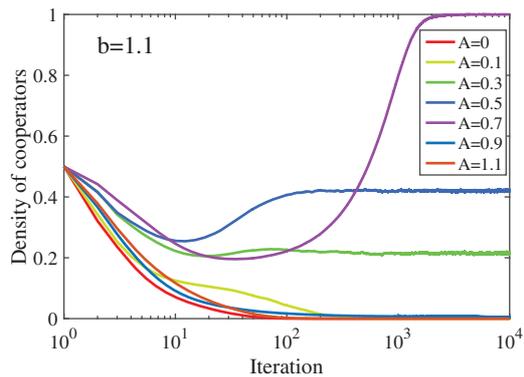


Fig. 2: Time evolution of the density of cooperators for different aspirations. We use the square lattice of size 100×100 as the social network N . The temptation b is 1.1. The aspiration level A changes from 0.1 to 1.1 at intervals of 0.2. When $A = 0$, the game reduces to the classic one [34]. Group reaches complete cooperation when $A = 0.7$.

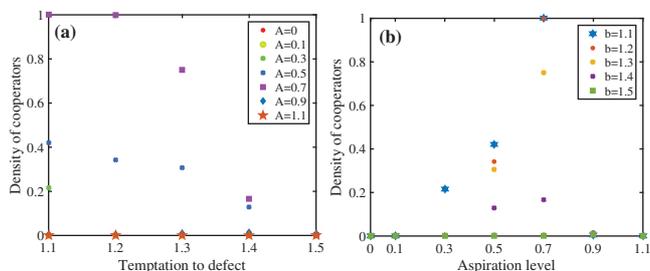


Fig. 3: Equilibrium density of cooperators as a function of temptation b under varied aspiration levels in (a) and aspiration level A under different temptations in (b). A lies in the range $[0.1, 1.1]$ at intervals of 0.2, and $A = 0$ is given as a contrast. b lies in the range $[1.1, 1.5]$ at intervals of 0.1. Cooperators are enhanced at appropriate aspiration levels and are suppressed with high temptations.

exists an appropriate aspiration level that makes cooperation become the dominant strategy in the group.

Figure 3(a) shows the density of cooperators as a function of temptation b for different aspiration levels. On the whole, the density of cooperators shifts when A is 0.3, 0.5 and 0.7, respectively, and cooperators decrease with the temptation b . The result means that at an appropriate aspiration level, the density of cooperators is greatly increased. Figure 3(b) shows the density of cooperators as a function of aspiration level A for different temptations b . When b is 1.5, the density of cooperators is always 0 regardless of A , which means high temptation restrains the positive effects of this interaction strategy on group cooperation. In other temptations, the density of cooperators increases first and then decreases with A , and reaches the highest cooperation level at 0.7. Thus, there exists an appropriate aspiration level that is able to minimize the influence of temptation b on cooperation to some extent.

In order to better understand cooperative behaviors, the snapshots of group cooperators in simulations under

three different aspiration levels are given. The group is divided into four kinds of individuals by introducing the aspiration-based interaction strategy. Taking individual i as an example, if $P_i > P_{iA}$, then i is defined as the strict high-payoff individual. If $P_i = P_{iA} > 0$, i is called the aspiration-payoff individual. If $P_i = P_{iA} = 0$, i is the isolated individual. And when $P_i < P_{iA}$, i is the low-payoff individual. That is, individuals either interact with the same level of neighbors or become isolated individuals who do not participate in the game. Here, the temptation b is set to 1.1. Each snapshot contains eight colors representing four kinds of individuals in two strategies (C or D). Dark colors mean cooperators, bright colors represent defectors. The corresponding types and colors of individuals are listed in table 1. And for high-payoff individual i , the demanding minimum number ω_i of cooperators in K_i is $\lceil k_i \times A \rceil$ and $\lceil (k_i \times A)/b \rceil$ when $S_i = C$ and $S_i = D$, respectively. $\lceil x \rceil$ is the ceiling function.

When the aspiration level A is 0.1, snapshots in fig. 4 display a clear change process of strategies. Firstly, dark blue and green disappear significantly. Then, green is the color of most nodes around the remaining dark blue clusters. Orange and yellow keep the status of increase during this process. Finally, the result in the snapshot is represented as the coexistence of yellow and orange. Figure 5 is given to further verify these changes in fig. 4. In left subfigure, all types of cooperators are in the process of fading out. The trend of strict high-payoff cooperators is consistent with the density of group cooperators. In the right subfigure, strict high-payoff defectors eventually disappear. The trend of low-payoff defectors is in agreement with the density of defectors in the group, and there also exists a few proportion of isolated defectors in equilibrium state. Thus, it can be inferred that the density of cooperators and defectors depends mainly on strict high-payoff individuals and low-payoff individuals, respectively.

Based on the model and on fig. 5, changes in fig. 4 are analyzed. As is shown in fig. 4(a), the group owns vast strict high-payoff individuals and a few low-payoff ones, which represents a high chance to meet strict high-payoff ones. Strict high-payoff cooperators (dark blue) easily turn into strict high-payoff defectors (green) or low-payoff defectors (yellow) if contacting with strict high-payoff defectors. And low-payoff cooperators (cyan) are easy to transform into isolated defectors (orange) when surrounded by strict high-payoff defectors (green). These transformations result in the significantly drop of strict high-payoff cooperators (dark blue) in fig. 4(b), which increases the probability to meet strict high-payoff defectors. For aspiration-payoff defectors (light orange), they will turn into low-payoff defectors (yellow) if surrounded by strict high-payoff defectors (green). For strict high-payoff defectors (green), they either become low-payoff defectors (yellow) when the interactive cooperators are less than ω_i , or change into isolated defectors (orange) when surrounded by low-payoff defectors, or become strict high-payoff cooperators (dark blue) when the interactive

Table 1: The corresponding types and colors of individuals.

Payoffs		$P_i = P_{iA}$			
		$P_i > P_{iA}$	$k_i > 0$	$k_i = 0$	$P_i < P_{iA}$
Cooperators	Types	Strict high-payoff ^(a)	Aspiration-payoff	Isolated	Low-payoff
	Colors	Dark blue ^(b)	Blue	Light blue	Cyan
	Value of colors	0 ^(c)	1	2	3
Defectors	Types	Strict high-payoff	Aspiration-payoff	Isolated	Low-payoff
	Colors	Green	Light orange	Orange	Yellow
	Value of colors	4	5	6	7

(a) When $P_i > P_{iA}$, i is a strict high-payoff individual.

(b) The color of strict high-payoff cooperators is dark blue.

(c) The value 0 stands for the color of dark blue.

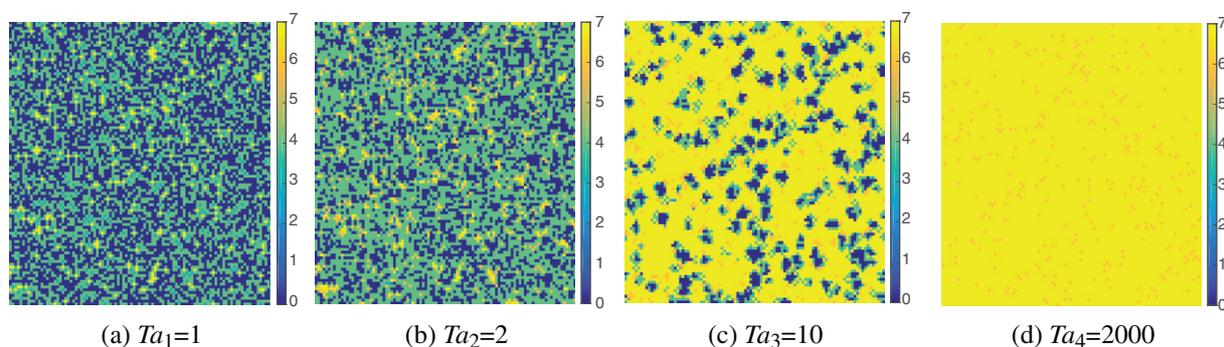


Fig. 4: Snapshots during the evolutionary game when A is 0.1. The color bar provides the corresponding number of colors, which is shown in table 1. Ta_i means the i -th time step. The time step is 1, 2, 10 and 2000 for Ta_1 – Ta_4 , respectively. At last, orange and yellow coexist in equilibrium.

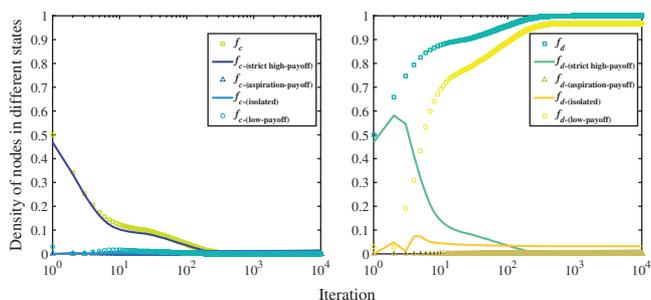


Fig. 5: Time evolution of the density of individuals in different states when $A = 0.1$. The subfigure on the left depicts the density of cooperators, and the right one is the density of defectors. For example, in the left subfigure, f_c means the density of cooperators in group, $f_{c-(strict\ high-payoff)}$ is the density of strict high-payoff cooperators in the group, and curves in the right subfigure are similar with the left one, f_d means the density of defectors in group, $f_{d-(strict\ high-payoff)}$ is the density of strict high-payoff defectors in group. Only low-payoff defectors and isolated defectors remain finally.

payoffs of strict high-payoff cooperators are higher than those of defectors. These changes lead to the high drop of strict high-payoff defectors (green) and low increase of strict high-payoff cooperators (dark blue) in fig. 4(c), and

the latter slows down the extinction of strict high-payoff cooperators (dark blue). Low-payoff defectors also get improved as is shown in fig. 4(b) and (c), where yellow is always increasing. On the one hand, the increased low-payoff defectors accelerate the process of both low-payoff cooperators (cyan) turning into low-payoff defectors (yellow) and strict high-payoff defectors (green) changing into isolated ones (orange). On the other hand, low-payoff defectors (yellow) either keep unchanged after interacting with homogeneous defectors, or become high-payoff defectors (green) after interacting with low-payoff cooperators, or change into isolated defectors (orange) when there is no interaction. Finally, only isolated defectors (orange) and low-payoff defectors (yellow) remain in fig. 4(d). It should be noted that during the process, though a few strict high-payoff cooperators are able to spread cooperations by forming turbulent cooperator clusters, these cooperator clusters finally disappear as their surrounding strict high-payoff cooperators are gradually invaded by defectors.

When A equals 0.5, fig. 6 shows that there are five kinds of colors in the group at first. Then green and dark blue decrease while yellow and orange increase. Different from fig. 4, dark blue clusters are not only surrounded by green but also cyan, and then start to expand with time. Finally,

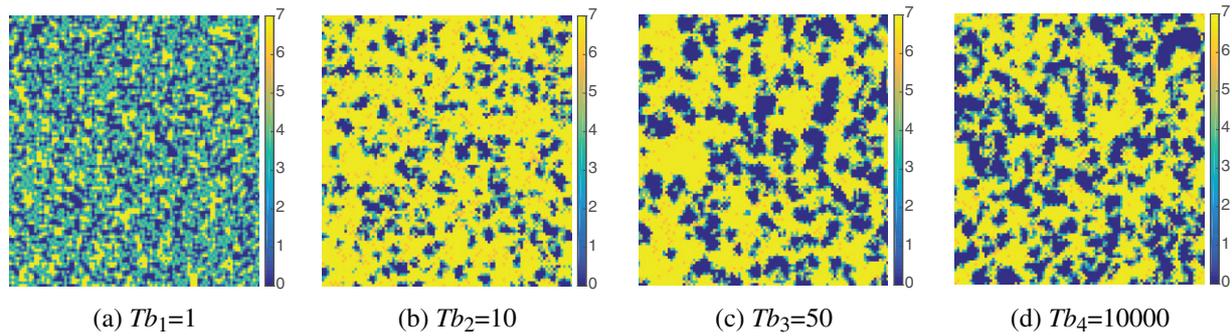


Fig. 6: Snapshots of individuals when $A = 0.5$. Here, the time step is 1, 10, 50 and 10000 for Tb_1 – Tb_4 , respectively. None of these eight colors disappears in equilibrium.

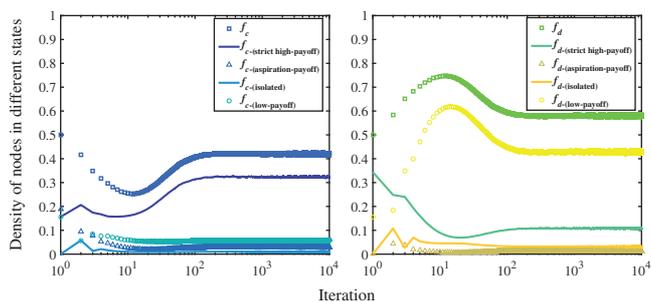


Fig. 7: Time evolution of the density of individuals in different states when A is 0.5. Definitions of symbols are the same as in fig. 5. All kinds of cooperators and defectors coexist in the group.

all colors coexist in the square lattice. Figure 7 depicts the density of individuals in eight colors. No type of individuals disappears in the equilibrium state. Cooperators and defectors coexist in the group, but there are mainly defectors.

Combining fig. 7, changes of snapshots are analyzed when A is 0.5. Figure 6(a) and fig. 7 show that the increased aspiration level brings aspiration-payoff individuals and more low-payoff individuals. Both the aspiration-payoff cooperators and low-payoff individuals reduce the interactions between strict high-payoff defectors and cooperators. But the difference is that the former delay the process of strict high-payoff cooperators turning into defectors while the latter prevent it. When updating strategies, the probability of strict high-payoff cooperators turning low-payoff defectors into cooperators is increased. Interactions among low-payoff cooperators boost the emergence of strict high-payoff cooperators. Therefore, strict high-payoff cooperators are surrounded by a few aspiration-payoff cooperators and low-payoff cooperators gradually occupy a certain proportion in fig. 6(b). And there are more individuals of strict high-payoff cooperators compared with fig. 4(c). As time goes by, based on the support of aspiration-payoff and low-payoff individuals, scattered strict high-payoff cooperator clusters try to expand, and those strict high-payoff defectors gathering around strict high-payoff cooperators are able to keep

high-payoff state. Therefore, cooperators and defectors are in a dynamic death-birth process, and finally these changes enable cooperators to coexist with defectors in a relative stable mode, as is shown in fig. 6(c) and (d). Thus, low-payoff individuals not only maintain cooperations, but also consolidate defections.

When A is 0.7, this process is embodied in fig. 8. Firstly, the dark blue is surrounded by abundant cyan and a little green, and yellow is distributed throughout the square lattice. As time goes by, the scattered dark blue clusters gradually expand into compact clusters and finally cover yellow. And, in fig. 9, all types of individuals except for the strict high-payoff cooperators gradually disappear in the group.

Based on fig. 9, the changes in fig. 8 are analyzed. The distribution of colors in fig. 8(a) shows that the increased aspiration level makes low-payoff individuals increase again. Strict high-payoff defectors turn homogeneous cooperators into strict high-payoff defectors or low-payoff defectors. Only those strict high-payoff cooperators remain who gain higher interactive payoffs than strict high-payoff defectors, and form cooperator clusters. Though low-payoff defectors can convert into strict high-payoff ones when interactive cooperators are not less than ω_i , the strict high-payoff defectors either become low-payoff defectors when the interactive cooperators are less than ω_i , or change into isolated defectors when surrounded by low-payoff defectors, or turn into strict high-payoff cooperators when updating strategies. Low-payoff cooperators easily change into defectors if connecting with strict high-payoff defectors when updating strategies. Only those low-payoff cooperators remain who are around cooperator clusters, and form an isolation belt among high-payoff individuals. Thus, the increased low-payoff individuals enable strict high-payoff cooperators to form compact clusters mostly surrounded by low-payoff individuals, as is shown in fig. 8(b). As time passes, these stable cooperator clusters motivate the surrounding low-payoff defectors and isolated defectors to turn into low-payoff cooperators and isolated cooperators, respectively. And the improved low-payoff cooperators promote the appearance of high-payoff cooperators, which helps

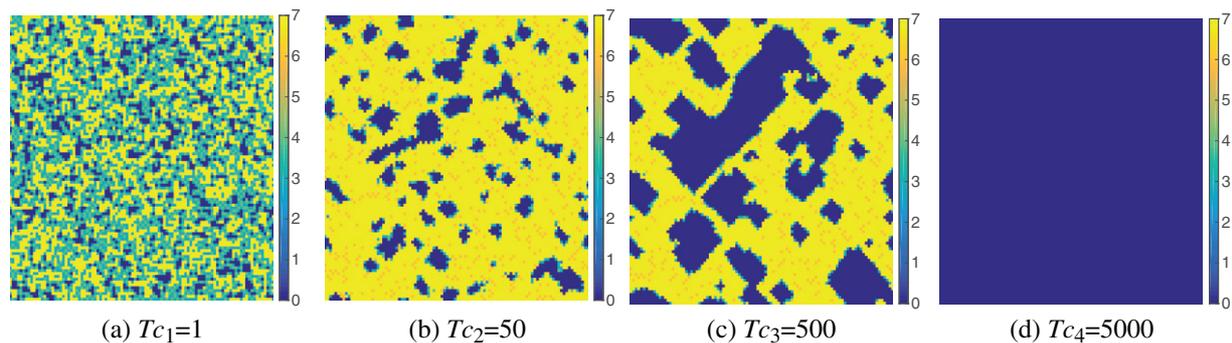


Fig. 8: Snapshots of individuals when A is 0.7. Here, the time step is 1, 50, 500 and 5000 for Tc_1 – Tc_4 , respectively. Only dark blue exists in equilibrium.

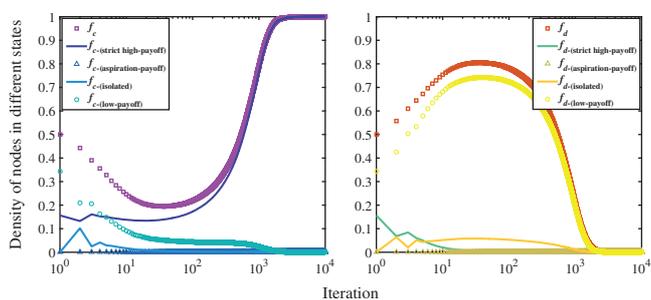


Fig. 9: Time evolution of the density of individuals in different states when A is 0.7. Definitions of symbols are the same as in fig. 5. Only strict high-payoff cooperators remain at last.

expand the cooperator clusters in turn. As a result, transformations between cooperator clusters and low-payoff individuals form a positive feedback to enhance the emergence of strict high-payoff cooperators. Hence, the dark blue clusters in fig. 8(b) gradually expand to those in fig. 8(c). Finally the group reaches a state of full cooperation in fig. 8(d).

Therefore, according to the analysis and results, different aspiration levels bring distinct probabilities of the above events, which constitutes diverse interactive processes.

Conclusion. – Considering the limited time and energy in reality, individuals usually choose suitable neighbors to interact. Therefore, this paper proposes a dynamic aspiration-based interaction strategy. During this process, the group is divided into high-payoff individuals and low-payoff individuals based on the interactive payoffs and aspiration payoffs. Interactions only happen among homogeneous individuals. We apply this strategy of selecting neighbors to the prisoner’s dilemma on square lattice. Individuals get their aspiration payoffs according to the number of interactive neighbors, and use the Fermi rule to update strategies. Simulations show that under the same temptation, there exists an appropriate level which makes the group reach a complete cooperation state. The increase of temptation suppresses the positive effect of interaction strategy on cooperation. Under

the appropriate aspiration level, the isolation belt between homogeneous individuals blocks the invasion of defectors. Interactions among low-payoff cooperators promote the appearance of high-payoff cooperators. The blocked interactions with strict high-payoff cooperators lead to the decline of interactive payoffs, which makes high-payoff defectors turn into low-payoff defectors. On the one hand, high-payoff cooperators transformed from low-payoff cooperators are absorbed by high-payoff cooperator clusters, which expands the scale of clusters. On the other hand, low-payoff defectors gradually transform into low-payoff cooperators, which maintains the stability of cooperation and greatly increases the density of strict high-payoff cooperators by promoting the interactions among low-payoff cooperators. In addition, according to the simulation results, too many low-payoff individuals cannot make cooperation become the dominant strategy. Therefore, there may exist an appropriate varying range of aspiration level that brings the highest group cooperation.

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