PAPER

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Bouncing a ball at rest on a surface

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Abstract

A ball at rest on a surface can be made to bounce up by pushing it down then releasing the downward force as fast as possible. Measurements and calculations are presented to show how it can best be done.

Keywords: tennis ball, collision, vertical bounce

Supplementary material for this article is available online

One of the tricks that a tennis player learns early on is to strike a ball at rest on the court with a tennis racquet. That way, the ball will bounce up without the player having to bend down to pick up the ball by hand. The player might need to strike the ball several times to achieve a satisfactory outcome, but the physics of the process deserves investigation. The basic physics is easy to understand. That is, when a ball is compressed, it stores elastic potential energy. That energy can be recovered, and the ball will bounce up, if the compression force is released faster than the ball can expand on its own. In the case of a tennis ball, the relevant expansion time is only about 0.002 seconds. A typical result, filmed at 300 fps, is shown in supplementary video Tennis.mov (available online at stacks.iop.org/PhysED/55/035021/mmedia).

Consider the problem shown in figure 1. A ball of mass m and stiffness k_2 is at rest on a horizontal surface. The stiffness of the ball is represented by two linear springs, one above and one below the ball, since both the top and the bottom of the stationary ball compress when m is struck by the falling ball. The falling ball has mass M and stiffness k_1 and is incident vertically at speed v_1. After the collision, the incident ball bounces upwards at speed v_2, and the ball that was at rest bounces up at speed V. The question is, what is the best combination of M and k_1 to maximise V, assuming v_1 remains constant?

To investigate the problem, I gathered up seven different balls and threw each one on each of the others, one at a time, and filmed each outcome. It was almost impossible to bounce a stationary billiard ball in that way. However, it was easy to bounce a soft rubber ball by impacting it with the billiard ball, as shown in supplementary video 9594.mov. The rubber ball was placed on top of a piezoelectric disk to measure the impact force on the disk, in order to compare with theoretical calculations, with good agreement. A related experiment, performed by many physics teachers, is to drop a basketball on the floor with a tennis ball at rest on top of the basketball [1]. But that is a different experiment and does not provide the answer I was looking for. I wanted the ball at the bottom to bounce as high as possible, not the ball at the top. Another version of that experiment, equally unhelpful, is to 'debounce' the bottom ball so it

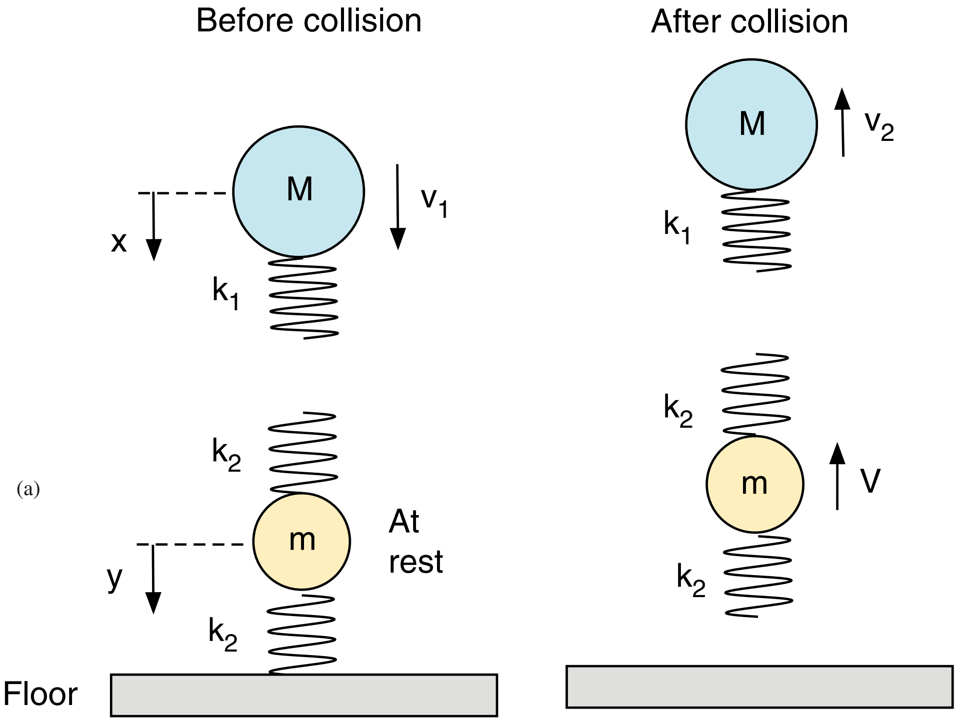

Figure 1. A ball initially at rest will bounce up at speed V if it is struck vertically by an incident ball.

does not bounce at all [2]. More relevant experiments have been done by colliding gliders on an air track [1, 3], but not with one of the gliders at rest against an end stop and not with gliders of variable mass and stiffness.

An approximate solution of the ball bounce problem could be found by colliding gliders on an air track where one glider is at rest near or against an end stop, as shown in figure 2. Furthermore, analytical solutions could be obtained by considering first the collision between M and m, then the collision of m with the end stop, then the collision of m with M. If m bounces back towards the end stop, further collisions would need to be considered to find the value of M that maximises the final rebound speed of m. A more realistic solution is given below to take into account the stiffness of the two masses.

Suppose that when the incident ball collides with the stationary ball in figure 1, the incident ball moves vertically down by a distance x and the centre of mass of the stationary ball moves down by a distance y. The bottom end of the

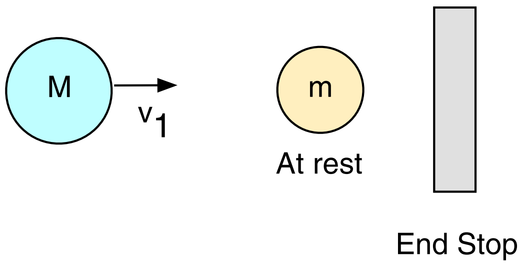

Figure 2. Equivalent air track experiment.

initially stationary ball compresses by a distance y so it exerts a downwards force $F_2 = k_2 y$ on the floor, and the floor exerts an equal and opposite upwards force on the stationary ball. The two springs in series above the ball can be regarded as a single spring of stiffness $k_s = k_1 k_2/(k_1 + k_2)$. If $k_1 \gg k_2$ then $k_s \approx k_2$. The incident ball compresses by a distance $x - y$ so springs k_1 and k_2 in series exert a downward force $F_1 = k_s(x - y)$ on the ball underneath it and an equal and opposite force on M above it. Since the gravitational

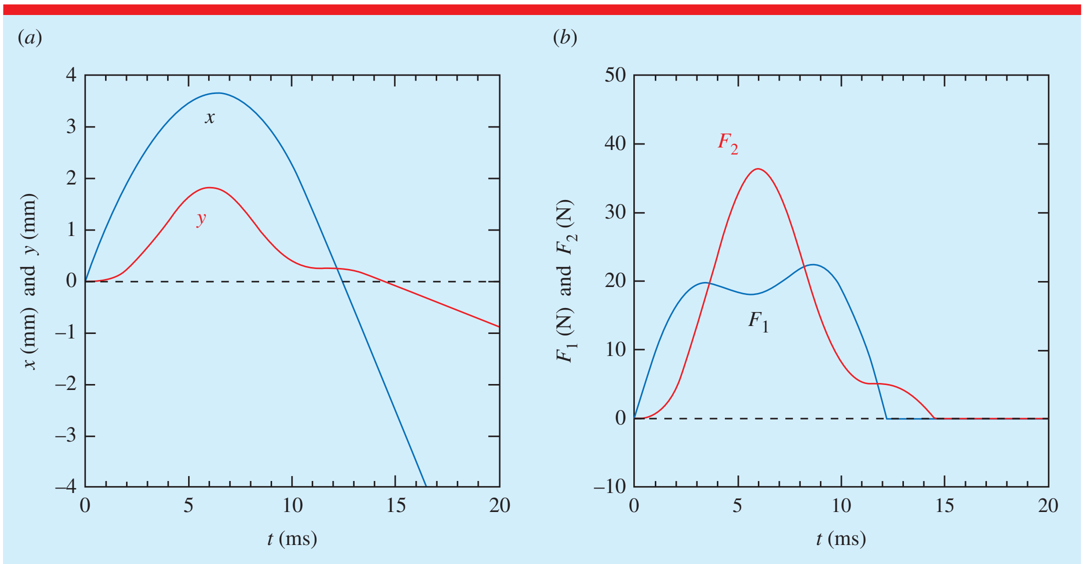

Figure 3. Numerical solutions showing (a) x and y versus t and (b) F_1 and F_2 versus t.

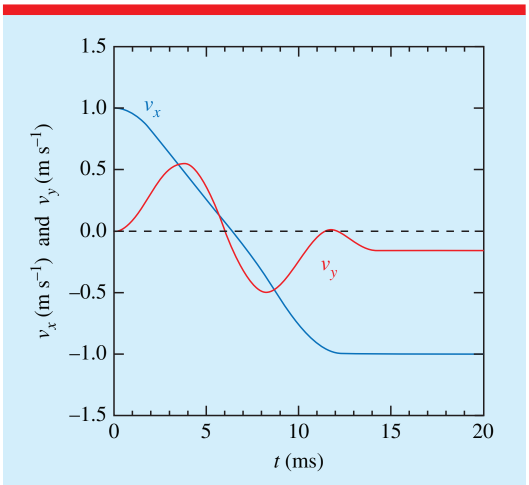

Figure 4. Solutions showing v_x and v_y vs t.

force on the masses is much smaller than F_1 and F_2 during the impact, the equations of motion are given by

$$M\frac{d^2x}{dt^2} = -k_s(x-y) \tag{1}$$

and

$$m\frac{d^2y}{dt^2} = k_s(x-y) - k_2 y \tag{2}$$

ignoring energy losses in each ball. In practice, all balls lose energy when they compress and expand, but that effect is ignored for simplicity in order to get approximate solutions of the problem. Equations (1) and (2) cannot be solved simply by analytical means, but are easy to solve numerically.

Typical solutions are shown in figures 3 and 4 for a case where $M = 100$ g, $m = 50$ g, $k_1 = k_2 = 20$ kN m^{-1} and $v_1 = 1$ m s^{-1}. Results are presented to show x and y versus t, F_1 and F_2 versus t and also $v_x = \mathrm{d}x/\mathrm{d}t$ and $v_y = \mathrm{d}y/\mathrm{d}t$ versus time. The end result is that m bounces up at 0.161 m s^{-1}, in a slightly complicated way. The incident ball starts slowing down as soon as it hits the stationary ball, and comes to a stop after 6.5 ms. After that time, the incident ball moves upwards, so its velocity changes sign. Meanwhile, the initially stationary ball compresses by a maximum distance about 1.8 mm, and the force F_2 reaches a maximum after 6 ms. At $t = 12$ ms, $x = y$ so F_1 decreases to zero and remains zero since the two balls lose contact after 12 ms. However, the lower ball is still compressed, with a small value of $y > 0$, so it accelerates upwards from 12 to 14.5 ms until it loses contact with the floor.

The rebound speed of the lower ball, V, is shown in figure 5 as a function of k_1 for several values of M when $v_1 = 1$ m s^{-1}, $m = 50$ g

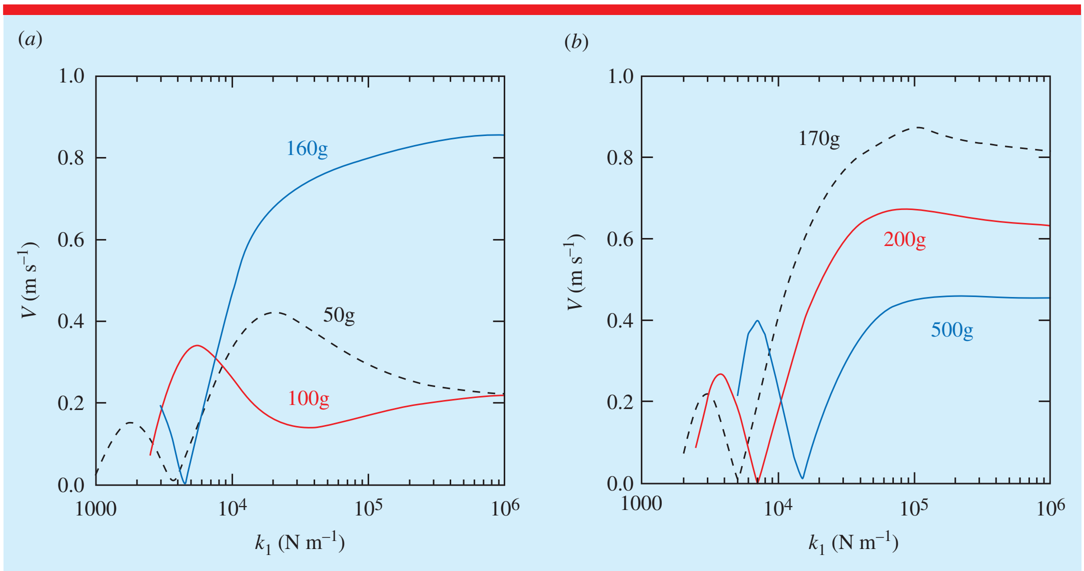

Figure 5. Solutions showing V versus k_1 when $m = 50$ g and $k_2 = 20$ kN m^{-1} and when (a) $M = 50$g, 100 g or 160 g and (b) $M = 170$ g, 200 g or 500 g.

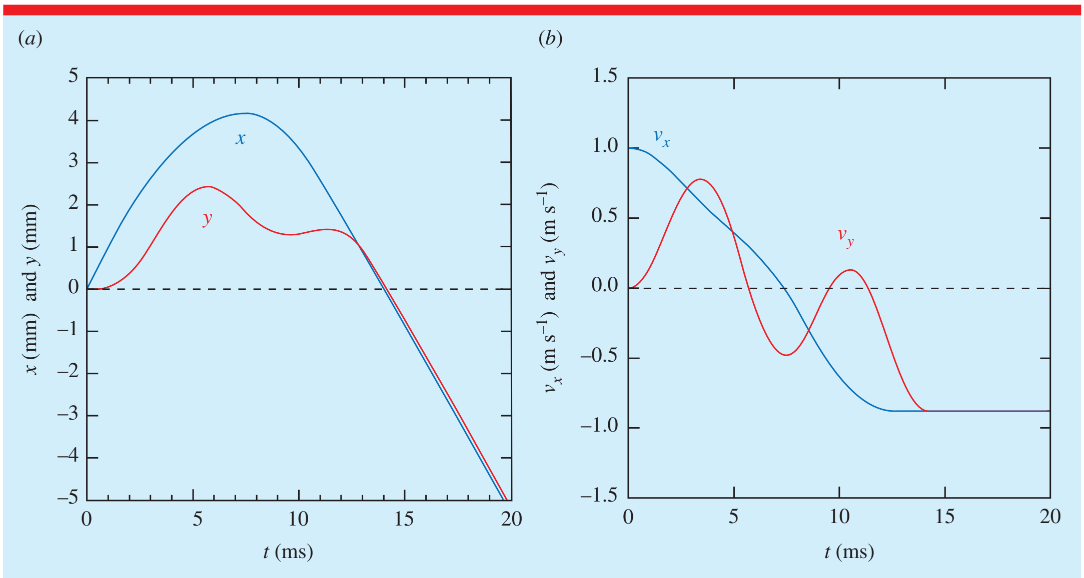

Figure 6. Solutions with $M = 170$ g and $k_1 = 1 \times 10^5$ N m^{-1} showing (a) x and y versus t and (b) v_x and v_y versus t.

and $k_2 = 20$ kN m^{-1}. These parameters are approximately those for a tennis ball. The rebound speed depends on both the mass and stiffness of the incident ball, the highest rebound speeds occurring when the mass of the incident ball is about 160 g or 170 g.

A surprising result is that V can be almost as large as the speed of the incident ball (1 m s^{-1}) but it cannot be larger. The maximum rebound speed of the incident ball is 1.0 m s^{-1} if all the initial energy is retained by the incident ball and the stationary ball remains at rest after the collision.

If the stationary ball bounces up at a higher speed than the incident ball (i.e. with $V > v_2$) then the faster ball will catch up with the slower ball and a second collision will occur, resulting in a final speed $V < v_2$. A result where $V = 0.88$ m s^{-1} is shown in figure 6. In that case, $M = 170$ g and $k_1 = 1 \times 10^5$ N m^{-1}. Both balls bounce up at almost the same speed, losing contact when $x = y$ at $t = 13$ ms. At that time, the lower ball is still compressed so it accelerates upwards and reaches nearly the same speed as the incident ball.

Even though the numerical solutions may be beyond high school physics students, the problem could still be investigated experimentally using a variety of different balls. The main problem then would be to explain the results in terms of compression and expansion of the two balls. High speed video film helps to explain the results.

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Rod Cross is an honorary Associate Professor in physics with research interests in the physics of sport and experiments for physics teaching.