

Use of two-parameter distributions for a joint assessment of the residual resource of building structures and engineering systems of buildings and structures

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Abstract. The article describes the use of two-parameter distributions for assessing the residual resource of engineering systems and building structures of buildings and structures. To estimate the residual resource, the authors propose using the Weibull distribution, beta distribution, gamma distribution, and student distribution. Scopes of each distribution are shown. In particular cases, these distributions turn into normal or exponential distributions, which are widely used to estimate the residual resource of building structures and engineering systems of buildings and structures. The authors proposed an algorithm for using two-parameter distributions for a joint assessment of the residual resource of building structures and engineering systems of buildings and structures. In this algorithm, the authors present the parameters by which it is necessary to choose one or another two-parametric distribution. Further, the authors determined how to estimate the element failure rate in the presence and absence of data on the number of failures. The authors also proposed a procedure for assigning the final value of the residual resource when using several distributions at once. The advantages and disadvantages of using two-parameter distributions for a joint assessment of the residual resource of building structures and engineering systems of buildings and structures are given. The authors also proposed ways to further improve it.

1. Introduction

The determination of the residual resource of buildings and structures is inevitably associated with a technical examination and calculation of a number of reliability indicators for its engineering systems and building structure elements (primarily, the probability of failure-free operation, gamma-percent or average operating time to failure, etc. [1÷ 14]) and, ultimately, with the establishment of the period of additional operation in accordance with the indicators specified in regulatory documents. Regulatory indicators for the required reliability are given in the current regulatory documents [15 ÷ 17].

The determination (calculation or forecast) of the residual resource or the establishment of the assigned resource is carried out in accordance with the established patterns of wear (changes in determining parameters) obtained by analyzing the mechanisms of damage development and conducting a comprehensive technical examination. Based on the assessments obtained and their



comparison with standard indicators, a decision is made on the possibility and timing of further safe operation of the facility.

For a homogeneous flow (for $\lambda = \text{const}$) for:

$$Q(t) = 1 - F(t) \quad (1)$$

where: λ – failure rate, year⁻¹; $P(t)$ – probability of failure-free operation; $F(t)$ – failure distribution function.

The determination of the residual resource of the elements of engineering systems and building structures is carried out according to the results of a technical survey of their actual condition on the date of the survey. Based on the calculation or the available statistical data on the failure rate of elements or systems, the main reliability indicators are obtained: the failure rate, the probability of failure-free operation, the gamma-percent operating time to failure, etc.

As indicators of reliability for elements and systems show, it is most advisable to use the following [18 ÷ 20]:

- probability of failure-free operation $P(t)$ of an object in the interval from 0 to time t inclusive;
- probability of failure in the interval from 0 to t ;
- distribution time between failures (for non-recoverable items) or failure (for recoverable items) $F(t)$;
- failure rate $\lambda(t)$ – the conditional probability density of failure;
- gamma-percent service life - the calendar duration of operation during which the object does not reach the limit state with probability γ , expressed as a percentage;
- gamma-percent resource - the total operating time during which the object does not reach the limit state with probability γ , expressed as a percentage;
- gamma-percent operating time to failure - the running time during which the failure of the object does not occur with probability, expressed as a percentage.

Based on the analysis of the value of T_γ the designated time of additional operation of the system is determined – T_n , during which (under certain additional conditions) the reliability indicator P – the probability of failure-free operation of the system will not be lower than the normalized value.

The positive contribution of the element is the contribution of the i -th element to the probability of failure of the real system in a year with its reliable failure:

$$\beta_i^+ = (P_F(t)_{p_i(t)=1}) - P_F(t) \quad (2)$$

The negative contribution of an element is the contribution of the i -th element to the probability of a real system failure in a year with its reliable failure:

$$\beta_i^- = -(P_F(t) - (P_F(t)_{p_i(t)=0})) \quad (3)$$

By the significance of element i is understood the difference between the value of the probabilistic characteristics of the system with the absolute reliability of element i and with its reliable failure, that is:

$$\beta_i = (P_F(t)_{p_i(t)=1}) - (P_F(t)_{p_i(t)=0}) \quad (4)$$

Here $(P_F(t)_{p_i(t)=1})$ – is the value of the probabilistic characteristic of the system for the absolute reliability of element i , and $(P_F(t)_{p_i(t)=0})$ – is for reliable failure of the element i on the considered interval t – the operating time.

Based on the data on the assessment of the technical condition of the facility and its residual life, a reasonable decision is made about the possibility of further operation of the facility during the designated period (T_n).

2. Methods.

The use of two-parameter distributions will simultaneously determine the residual life of both building structures and engineering support systems, both together and separately.

Such two-parameter distributions that can be used in calculating the residual life of engineering systems and building structures of buildings and structures can be attributed:

- 1) Weibull distribution;
- 2) beta distribution;
- 3) Gamma distribution;
- 4) Student distribution (t-distribution);

Such a choice is explained by the universality of these distributions, which may be suitable for describing the operation of systems of different composition, layout, and purpose. Another advantage of such distributions can be considered that in particular cases they turn into simpler distributions, which are already used to estimate the residual life of both building structures and engineering systems of buildings and structures [21].

The **Weibull distribution** describes well the elements operating under dynamic loads (for example, the spread of the fatigue strength of steel or the limits of its elasticity). In addition, this distribution has found application in the description of complex technical systems.

The reliability function is described by the equation:

$$F(x) = \begin{cases} 0, & t < 0 \\ 1 - e^{-\lambda t^\alpha}, & t \geq 0 \end{cases} \quad (5)$$

The reliability function (probability of failure-free operation) is equal to:

$$P(x) = 1 - F(x) = e^{-\lambda t^\alpha} \quad (6)$$

α – curve shape parameter, set according to the survey results;

λ – curve scale parameter or in relation to construction, the probability of failure of an element of building structures;

At $\alpha = 1$ Weibull distribution turns into exponential (exponential), at $\alpha \approx 3,3$ – in a normal (Gaussian distribution).

Thus, the Weibull distribution is recommended for structural elements operating under dynamic loading conditions, as well as for machine parts and assemblies.

The remaining resource of engineering systems of capital construction facilities will be equal to:

$$t = \sqrt[\alpha]{\frac{-\ln P(x)}{\lambda}} \quad (7)$$

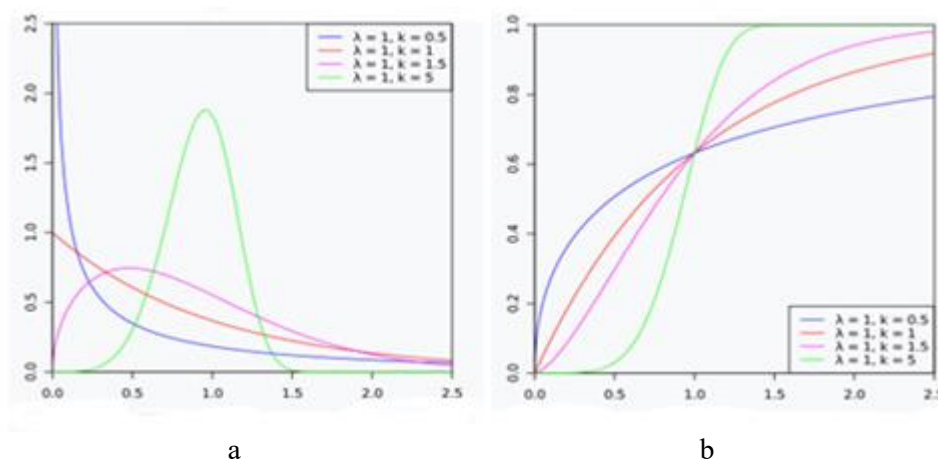


Figure 1. Probability density curves (a) and reliability function (b) of the Weibull distribution

The **beta-distribution** is used to describe random variables whose values are limited to a finite interval.

The integral reliability is described by the equation:

$$F(x) = \int_0^1 \frac{G(a+b)*x^{a-1}*(1-x)^{b-1}}{G(a)*\Gamma(b)} dx \quad (8)$$

The reliability function (probability of failure-free operation) is equal to:

$$P(x) = 1 - F(x) = 1 - \int_0^1 \frac{G(a+b)*t^{a-1}*(1-t)^{b-1}}{G(a)*\Gamma(b)} dt \quad (9)$$

$G(a)$ and $G(b)$ – gamma functions.

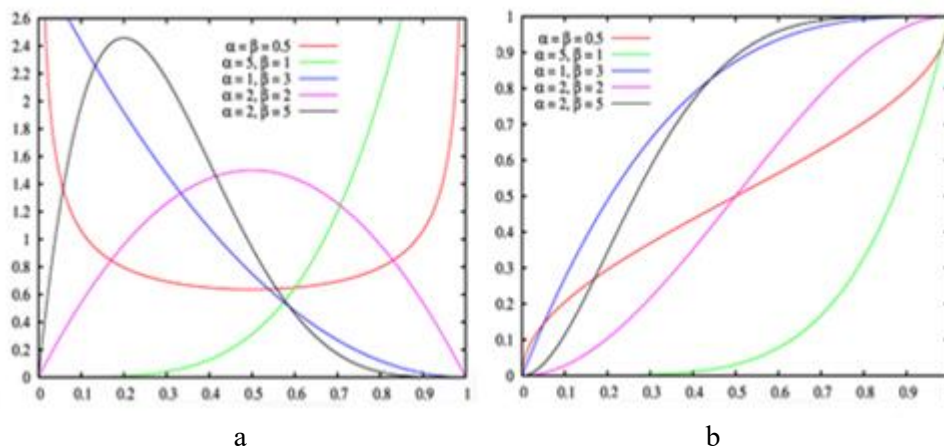


Figure 2. Probability density curves (a) and reliability function (b) beta distribution

Beta-distribution is good to use when considering the limited-time service life of building structures, for example, the time between overhauls and engineering systems.

Gamma-distribution is used to describe the occurrence of failures during normal operation of the system with redundancy and aging of elements.

The integral function of reliability is:

$$F(x, \alpha, \lambda) = \frac{\lambda^\alpha}{G(a)} \int_0^t t^{\alpha-1} * e^{-\lambda*t} * dt \quad (10)$$

The reliability function (probability of failure-free operation) is equal to:

$$P(x) = 1 - F(x) = 1 - \frac{\lambda^\alpha}{G(a)} \int_0^x x^{\alpha-1} * e^{-\lambda*x} * dx \quad (11)$$

$\alpha > 0$ – curve shape parameter, set according to the survey results;

If the parameter α takes an integer value, then such a gamma distribution is also called the Erlang distribution.

$\lambda > 0$ – curve scale parameter;

Gamma function:

$$G(a) = \int_0^\infty t^{\alpha-1} * e^{-t} * dt \quad (12)$$

At $\alpha = 1$ the gamma distribution turns into exponential (exponential), at $\alpha \rightarrow \infty$ – in a normal (Gaussian) distribution.

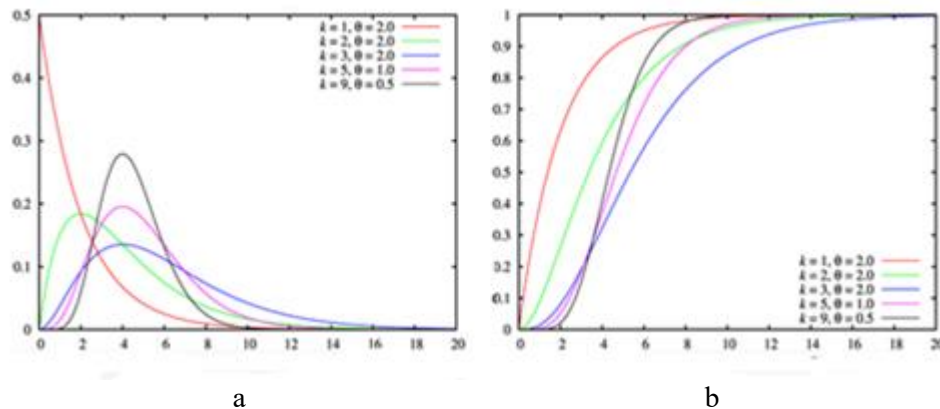


Figure 3. Probability density curves (a) and reliability function (b) of the gamma-distribution.

This distribution is recommended to be used when the life of building structures and engineering systems has exceeded the regulatory period established in regulatory documents or a project, as well as for engineering systems with redundancy.

Student's distribution (t-distribution) is recommended to be used when the life of the building structures and engineering systems has exceeded the regulatory period established in regulatory documents or the project.

The integral reliability function is:

$$F(x) = \frac{1}{2^{\frac{k-1}{2}} * G(a) * \sqrt{k * \pi}} * \int_{-\infty}^x \left(1 + \frac{x^2}{k}\right)^{-\frac{k-1}{2}} dx \quad (13)$$

The reliability function (probability of failure-free operation) is equal to:

$$P(x) = 1 - F(x) = 1 - \frac{1}{2^{\frac{k-1}{2}} * G(a) * \sqrt{k * \pi}} * \int_{-\infty}^x \left(1 + \frac{x^2}{k}\right)^{-\frac{k-1}{2}} dx \quad (14)$$

At $k \rightarrow \infty$ the Student distribution turns into a normal (Gaussian distribution).

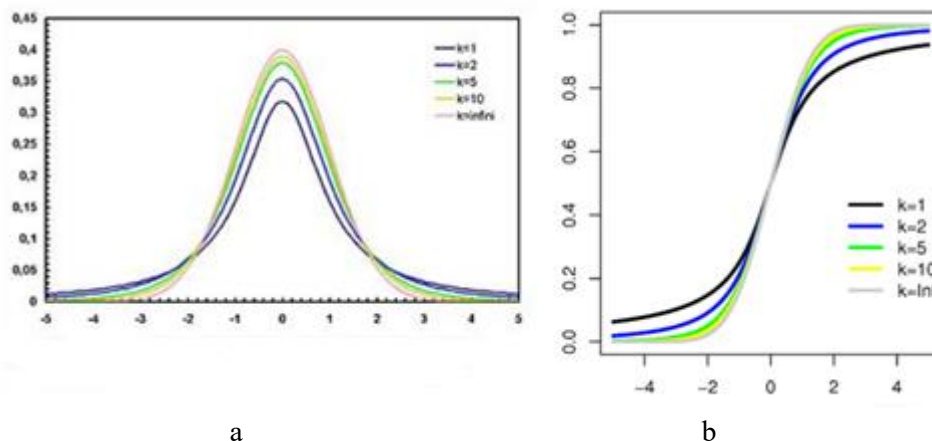


Figure 4. Probability density curves (a) and reliability function (b) of Student distribution

3. Results and Discussion.

Having briefly reviewed the distribution data, the authors propose the following algorithm for calculating the residual life of building structures and engineering systems using two-parameter distributions.

The procedure for calculating the residual life of engineering systems and building structures using two-parameter distributions.

1. *The definition of elements (systems) of engineering and technical support for buildings and structures and building structures for which the residual resource will be considered.*

2. *The choice of two-parameter distributions for estimating the residual resource.*

When choosing one or another distribution, it is necessary to be guided by the following parameters: the type of system (simple or complex, with or without redundancy, serial or parallel connection of elements, etc.), the dominant influence factor (aging, wear, dynamic impact, etc.) and other parameters and factors that were identified during the survey.

3. *Determining the failure rate.*

In the general case, the failure rate for one element for which several distributions are used at once will be different. This must be taken into account in the calculation.

In cases where failure cases of an element (system) were recorded, the calculation is carried out according to the formula:

$$\lambda = \frac{n}{N_{ave} \cdot t} = \frac{n}{(N-n) \cdot t} = \frac{f(t)}{P(t)} \quad (15)$$

where N – total number of products in question;

$f(t)$ – failure rate — number of elements that failed at time t per unit time;

$P(t)$ – number of elements that did not fail at time t ;

n – number of failed elements during time t ;

t – time interval;

N_{ave} - average number of properly working elements over time t ;

$$N_{ave} = \frac{N_i + N_{i+1}}{2} \quad (16)$$

N_i - number of properly working elements at the beginning of operation;

N_{i+1} - number of properly working elements at the time of the survey.

In cases where failures were not recorded during the service life conservatively, one failure for a certain period of their operation is taken to calculate the reliability indicator. This period can be assigned according to regulatory documents or documents for this system.

In this case, first find the average failure rate by the formula:

$$\lambda_{ave} = \frac{1}{T_{norm}} \quad (17)$$

For the calculated then take the gamma-percent failure rate calculated by the formula:

$$\lambda_\gamma = \lambda_{ave} \cdot k \quad (18)$$

where k is the quantile of the exponential distribution.

It is also necessary when calculating failures for a common reason.

Then the total value of the failure rate can be calculated using the β -factor method [22]:

$$\lambda_\Sigma = \lambda_{cal} + \lambda_{op} \quad (19)$$

where λ_{cal} – calculated failure rate calculated according to paragraph 3 of the procedure for calculating the residual resource of engineering systems [22];

λ_{op} – intensity of common cause failures [22].

4. *Determination of the curve shape parameter.*

This parameter is determined on the basis of the survey data and the study of the available design, executive and operational documentation.

5. *We set the probability of failure-free operation for the selected elements.*

This value is set based on the data that were obtained during the technical examination of engineering systems and building structures of buildings and structures.

6. *Find the residual resource of the element (system).*

7. *We analyze the obtained values and assign the final value of the residual resource of the element*

(system).

It is necessary to evaluate the scatter of the obtained values. The assessment is carried out relatively medium. The choice of the average value, relative to which the spread of residual life is estimated, is explained by the fact that in the theory of mathematical statistics it is considered as the closest to the true value.

The average value is:

$$T_{ult.ave.} = \frac{T_{ult\ 1} + \dots + T_{ult\ i}}{i} \quad (20)$$

$T_{ult\ i}$ – value of the residual resource, determined by the i -th distribution;

i – number of distributions used.

The estimation of the spread of values relative to the average according to the formula:

$$\Delta = \frac{T_{ult\ i} - T_{ult.ave.}}{T_{ult.ave.}} * 100\% \quad (21)$$

If the spread does not exceed 20%, then the final value is taken as the average value. If it exceeds, then it is necessary to recalculate.

4. Conclusions

The algorithm for calculating the residual life of building structures and engineering systems using two-parameter distributions was developed.

This algorithm for calculating the residual life of engineering systems and building structures of buildings and structures has its advantages and disadvantages.

Advantages:

- Versatility. This method can be applied to all elements without exception.
- Abstract. The use of abstract source data (the probability of uptime and failure rate) allows taking into account many different factors of a technical and operational nature.

Disadvantages:

- Subjectivity. The input data (probability of failure-free operation and failure rate) and the choice of calculation method are made by the inspector himself. Therefore, the accuracy of this method greatly depends on the qualifications and experience of the one who conducts the survey and calculation.

In conclusion, I would like to note ways to improve this algorithm

First way. The application of methods of logical and probabilistic modeling for calculating the residual life of engineering systems and building structures. This will improve the results of calculating the residual resource when taking into account the interaction of different systems and elements.

Second way. The application together with other methods of calculating the residual resource to compare the results obtained in order to increase their accuracy and reliability.

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