Discrete-analytical nonlinear analysis with improved computation accuracy for steel frame lateral response evaluation

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Abstract. The structural performance evaluation under the monotonic increase of a lateral load due the whole loading process in modern earthquake engineering practice can be performed with the help of different methods and software and can be dramatically represented by «Base reaction – Roof displacement» relationship, which is also known as capacity curve. This kind of relationship is used by nonlinear static Pushover-analysis for seismic performance evaluation of buildings considering any possible damage, which is acceptable by serviceability requirements of a building. However, there are many problems arising due to physically nonlinear analysis of structure, primarily associated with features of the employing finite elements analysis software. Main problems are rather long analysis time among the necessity of high performance computing, or shortcomings of nonlinear analysis methods inherent to the program used. One of the most important shortcomings of structural analysis software is the necessity of single-handed assignment of a failure mechanism for the structure by nonlinear hinges formation zones emplacement to elements of analytical model. This circumstance may lead to results substantially different from reality. These problems can be resolved with the aid of discrete-analytical methods.

1. Introduction

The objective of this investigation is capacity evaluation method for structure under the monotonic increasing of a lateral load with the help of discrete-analytical nonlinear analysis (DANA). In the paper [1] the capacity curve developing method with the aid of ultimate analysis approach realized in Ing+ software is proposed. However, the curve developed by this method has a bilinear character obtained from 2 points. If more detailed representation of «Base reaction – Roof displacement[1]» relationship is necessary, then another software shall be used, for example ANSYS, that, unfortunately, needs time-consuming and high performance computations (HPC). Therefore, for building response evaluation subjected to lateral load action and its capacity curve creation discrete-analytical approaches can be used, for example, hybrid finite elements method DANA [2, 3]. This method allows reducing computation time and improving the accuracy of stresses determination.

As it is shown in the work [1], the capacity curve can be used in earthquake engineering for optimization by economic criteria [4] rather for preliminary dynamic analysis of a SDOF equivalent

[1] Roof displacement – the displacement of the top point of building selected above its mass center

system [5] or nonlinear static Pushover-analysis (NSPA). Thus, algorithms for capacity curve developing realized in different structural analysis software are generally based on direct subsequent approach (DSA) of nonlinear procedure which implies that the program traces due the iterational process the rate of criticality of any factor (performance level) which is responsible for the plastic hinge formation in the element of the analytical model. One of the main problem of this approach is that the solution generally becomes approximate though it brings accurate results in many simple cases, e.g. *n*-story 2D frames or soft ground story buildings. It happens because the engineer have to assign for elements of the analytical model plastic hinges formation zones basing on his own engineering judgement that may lead to the worse case results. Another problem is a big amount of different discrepancies appeared due the nonlinear hinges modeling process and setting up «Force – Deformation» diagram properties [6] which have a big variablity of parameters. Finally, the failure criterion is not clear and stable that is evidenced by a wide variety of possible results that can be obtained from physically the same initial conditions. Thus, the problem of software verificationm denoted in [7] as well by an analytical procedure is obvious here. These shortcomings could be eliminated with the aid of DANA as well.

2. Analysis methods

The essence of the proposed method among with the accurate (by ANSYS) and approximate (by SAP2000) analysis consists in the finite elements exclusion and its substitution for integration domains. As the result, the analytical model is obtained which consists from rod macroelements with nonlinear stiffness contribution over their length. Thus, the amount of finite elements method (FEM) equations can be reduced by several digits. One of the key feature of this method is nonlinear stiffness of a rod section specifying. This can be done by usage of tangent stiffness matrix or integral function of section state law. For the case of 2D-problem, the nonlinear stiffness of a rectangular section is defined by formula

$$T(\tau)=\begin{cases}\dfrac{\sigma_S bh^3}{4\varepsilon_S}\cdot\dfrac{1}{3}, \tau\le 1\\[2ex] \dfrac{\sigma_S bh^3}{4\varepsilon_S}\cdot\dfrac{1}{3\tau(x)^3}\cdot\left[1+a\left(\tau(x)^3-1\right)\right], \tau>1\end{cases}, a=\frac{E_{pl}}{E}, \tag{1}$$

$$\tau=\frac{\chi h}{2\varepsilon_s}, \tag{2}$$

where ε_s – yield strain, σ_s – yield stress, χ – curvature of rod at the section under consideration, b, h – rectangular section characteristics; E_{pl} – tangent (plasticity) modulus. The stiffness matrix of a rod is defined by the shape function (Figure 1)

$$N_1=1-\frac{x}{l}; N_2=(1-\frac{3x^2}{l^2}+\frac{2x^3}{l^3}); N_3=(x-\frac{2x^2}{l}+\frac{x^3}{l^2}), \tag{3}$$

$$N_4=\frac{x}{l};\ N_5=(\frac{3x^2}{l^2}-\frac{2x^3}{l^3});\ N_6=(-\frac{x^2}{l}+\frac{x^3}{l^2}). \tag{4}$$

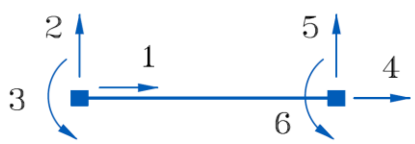

Figure 1. Degrees of freedom

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The formula for the stiffness matrix of a bending rod can be obtained after differentiating the potential strain energy with respect to displacements

$$\left[K^e\right]=EI\int_0^l \frac{d^2[N]^{\mathrm{T}}}{dx^2}\frac{d^2[N]}{dx^2}dx=EI\int_0^l \begin{bmatrix} N_2'' \\ N_3'' \\ N_5'' \\ N_6'' \end{bmatrix}\begin{bmatrix} N_2'' & N_3'' & N_5'' & N_6'' \end{bmatrix}dx=EI\int_0^l \begin{bmatrix} N_2''N_2'' & N_2''N_3'' & N_2''N_5'' & N_2''N_6'' \\ N_3''N_2'' & N_3''N_3'' & N_3''N_5'' & N_3''N_6'' \\ N_5''N_2'' & N_5''N_3'' & N_5''N_5'' & N_5''N_6'' \\ N_6''N_2'' & N_6''N_3'' & N_6''N_5'' & N_6''N_6'' \end{bmatrix}dx \tag{5}$$

Substitute *EI* for *T*(τ) and carry it under the integral sign. With due regard to axial stiffness ratios the formula for determination of stiffness matrix by the shape function associated with an iteration step will be

$$\left[K^e\right]=\int_0^l \begin{bmatrix} \frac{F}{I}\cdot N_1'N_1' & 0 & 0 & \frac{F}{I}\cdot N_1'N_4' & 0 & 0 \\ 0 & N_2''N_2'' & N_2''N_3'' & 0 & N_2''N_5'' & N_2''N_6'' \\ 0 & N_3''N_2'' & N_3''N_3'' & 0 & N_3''N_5'' & N_3''N_6'' \\ \frac{F}{I}\cdot N_4'N_1' & 0 & 0 & \frac{F}{I}\cdot N_4'N_4' & 0 & 0 \\ 0 & N_5''N_2'' & N_5''N_3'' & 0 & N_5''N_5'' & N_5''N_6'' \\ 0 & N_5''N_2'' & N_6''N_3'' & 0 & N_6''N_5'' & N_6''N_6'' \end{bmatrix}\cdot T(\tau)\,dx, \tag{6}$$

where *F* – section area; *I* – second area moment.

The distributed load *p* is translated into the equivalent one on the step

$$\left\{P^e\right\}=\int_0^l [N]^{\mathrm{T}}\, p\cdot\frac{T(\tau)}{EI}\,dx=p\int_0^l \begin{bmatrix} N_2 \\ N_3 \\ N_5 \\ N_6 \end{bmatrix}\cdot\frac{T(\tau)}{EI}\,dx\,. \tag{7}$$

After stiffness coefficient k_{ij} determination the stiffness matrix in global coordinate system is defined for a macro element at each step. Stiffness coefficients of focused rods in a node are summarized. Next, in incremental form for a load increment FEM equations are solved for the entire system at the step, and bending moments increments *ΔM(x)* at sections are calculated. The increment of curvature lengthwise of rods is defined as:

$$\Delta\chi(x)=\frac{\Delta M(x)}{T(\tau)}, \tag{8}$$

Deformations increment in outermost fibers:

$$\Delta\varepsilon(x)=\Delta\chi(x)\frac{h}{2}. \tag{9}$$

Stress and strain at a step is given as

$$\varepsilon(x)=\varepsilon(x)+\Delta\varepsilon(x) \tag{10}$$

$$\sigma(x)=\begin{cases}\sigma(x)+E_{pl}\Delta\varepsilon(x), \tau>1 \\ \sigma(x)+E\Delta\varepsilon(x), \tau\le 1\end{cases}. \tag{11}$$

Thereafter, a new local stiffness matrix of a rod macro element is performed by integration (6) considering section stiffness mutation. After this, the computation cycle is repeated.

At the end of computations the capacity curve based on obtained results can be developed. For this purpose, it is necessary to get out from the program the ensemble of results, which are lateral force (base reaction) values and related roof displacements. This population of related values is nothing but characteristic points that are necessary for structure capacity curve developing.

If the analysis is run by SAP2000, then the acceptance criteria and nonlinear parameters for hinges in all structural elements need to be determined. For plastic materials the behavior type is ductile, i.e. the ultimate element deformation shall be attributed to nonlinear «Moment – Rotation» or interaction relationship (Figure 2). The ultimate rotation value (ordinate of point C) can be derived by FEMA 356 guidelines or from experiment or analysis data [8]. The ultimate moment at this point is assumed to be 1.25 of its yielding value.

Point	Moment/SF	Rotation/SF
D-	-0,2	-6
C-	-1,25	-6
B-	-1	0
A	0	0
B	1,	0,
C	1,25	6,
D	0,2	6,
E	0,2	8,

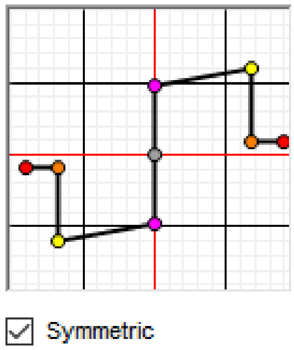

Figure 2. Nonlinear «Moment – Rotation» relationship for ductile type of element in SAP2000

The analogic kind of relationships shall be established for all elements of the analytical model, which are subjected to plastic yielding due to the proportional increase of a lateral load. Due the computational process, the program designates plastic hinges formation zones from those assigned by the user, the criticality of associated element performance level and registers the lateral load value related to a certain step of analysis. The analysis stops when the displacement of the associated node at the top of structure reaches the predefined value. SAP2000 depicts the capacity curve as the result of completed NSPA.

3. Comparison of computation results

3.1. Characteristics of structure

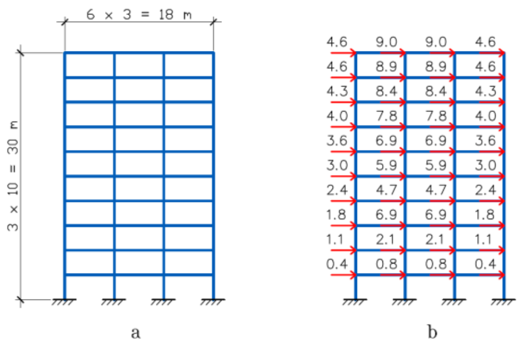

Figure 3. a – General frame configuration; b – Lateral load distribution

The numerical simulation is performed for 10-story 2D 3-span steel frame (Figure 3a). The height of all columns is 3 m, all beams length is 6 m. Total height of the frame is 30 m, total length – 18 m. Beams and columns has the rectangular section form 200x100 mm. Material of frame is steel with yield strength 240 MPa and 10% bilinear isotropic hardening.

3.2. Numerical simulation

The frame is subjected to the action of lateral load applied to its analytical model according to the inertia forces distribution associated with the frame fundamental mode (Figure 3b). Then, the analyses by methods prescribed above are performed.

3.3. Results and discussion

Based on obtained results 3 capacity curves are plotted (Figure 4) and comparative results are given (Table 1).

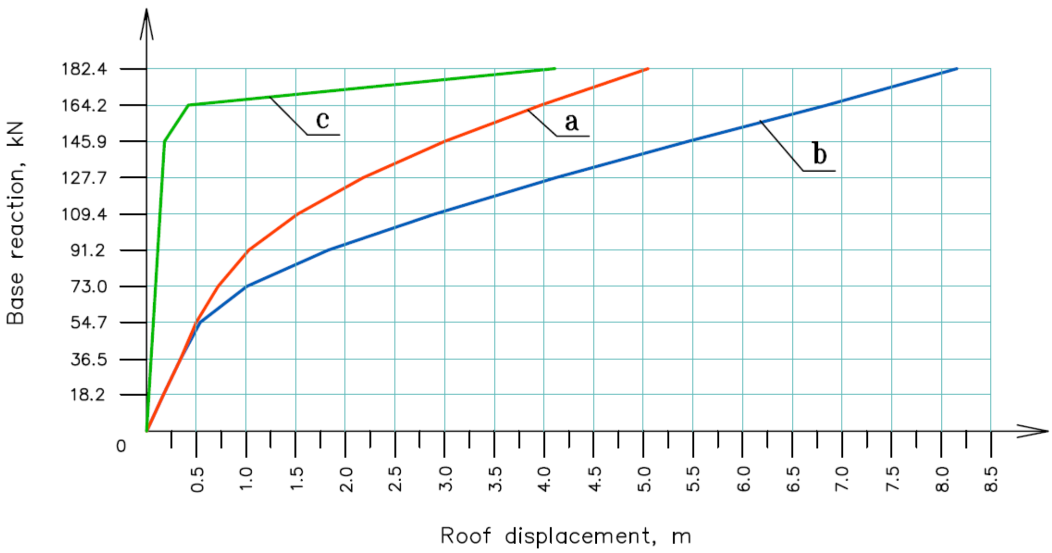

Figure 4. Capacity curves: a – DANA; b – ANSYS; c – SAP2000

Table 1. Comparative results of nonlinear analyses

Program	Amount of finite elements	Computation time, s	Max of Base Reaction, kN	Max of Roof Displacement, m
ANSYS	1000	120	182.4	8.3
SAP2000	1000	66	182.4	4.1
DANA	1000	1	182.4	5.1

Figure 5 demonstrates that analyses by ANSYS and DANA provide significantly accurate and similar solutions in the linear and near-linear region since nonlinear analyses give as different results as the computation goes on. The accuracy here depends upon many factors, e.g. from distribution of stresses by section area or from finite elements fragmentation of the rod. In the model with perfect elastic-plastic characteristic (SAP2000) the solution can be obtained faster, but with less accuracy. One of the reasons is that the program confines the «Force – Deformation» diagram in yield and ultimate displacement boundaries that are specified by building codes and guidelines. Furthermore, all programs demonstrate sufficiently different results in the region of ultimate load action.

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Conclusion

The proposed method improves nonlinear analysis solution accuracy and significantly reduces computation time through decreasing the amount of equations in the system. It makes it possible to perform the problem with infinitely remote boundaries as well. The increasing of a problem order is achieved by applying analytical expressions for a rod stiffness determination.

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