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Low-frequency plasma waves in a radiative dusty magnetoplasma

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Abstract
The impact of electromagnetic (thermal) radiation on the dispersive low-frequency waves is examined in a radiative dusty magnetoplasma. For this purpose, a magneto-hydrodynamic model along with the Maxwell equations is employed to describe the three-component radiative dusty plasma that contains inertialess electrons, dynamical positive ions and negatively charged static dust particulates. The behavior of the electrostatic and electromagnetic waves significantly changes in a plasma medium when a radiation pressure is taken into consideration, as compared to the waves in vacuum. After seeking a plane wave solution, a general dispersion relation is obtained to investigate different limiting cases of the low-frequency modes propagating parallel, perpendicular and oblique to the external magnetic field direction both analytically and numerically. The calculations reveal that the usual thermal and acoustic speeds (c_T, c_a) do not remain constant even if the temperature is kept constant, because of the radiation pressure ($\tilde{\alpha}_e$) which strongly depends on the equilibrium number density as well. The present results may prove a useful understanding for the new features of the dispersive dust-ion-acoustic and compressional Alfven waves in astrophysical dusty magnetoplasmas.

Keywords: astrophysical plasma, dusty plasma, waves

(Some figures may appear in colour only in the online journal)

1. Introduction

The Alfvén wave models can be utilized as an important tools [1] to describe the upper solar atmosphere heating. A necessary element for such models is an effective and permanently acting source that not only excites these waves but also generates the waves throughout solar atmosphere. The existence of high and low amplitude drift and kinetic Alfven oscillations near the Earth plasma surroundings has been confirmed by many spacecraft observations [2–4]. Specifically, the waves recorded by Cluster [5] have shown the spatial scales of the order of ion Larmor radius with a wave impedance equal to local Alfven velocity. The Alfven and sound speeds can substantially vary in an inhomogenious magnetosphere. In a linear magneto-hydromagnetic limit, the hydrodynamic waves are used to model the sudden magnetospheric perturbations [6]. However,the compressional Alfvén waves(CAWs) that are the low-frequency waves in comparison with the ion-cyclotron (ω_c) and upper-hybrid waves (ω_{UH}) i.e. $\omega < \omega_c$, ω_{UH}, propagating in a magnetosphere across the magnetic shells to act as a source for generation of transverse waves [7] and ionospheric dissipation [8]. The dispersion properties of the low-frequency Alfvén and magnetosonic waves [9, 10] have successfully been discussed in a magnetized dusty plasma within the framework of magneto-hydrodynamic theory. Authora in [11] considered the oblique propagation of shear Alfven-like waves in self-gravitating, weakly ionized inhomogeneous dusty magnetoplasmas. and revealed the unstable waves with a wavelength (wave period) of the order 10AU (30 000 years) owing to the dust self-gravitational effects. Subsequently, Amin [12] carried out both analytical and numerical analyzes to examine the linear and nonlinear compressional Alfven waves in an electron-hole semiconductor plasmas and showed unique features involving the amplitude modulation and modulation instability. Influence of plasma parameters on the low frequency ion cyclotron and ion acoustic waves are investigated in [13]. More recently low frequency modes observed in lunar wake are focused in [14], kinetic Alfven waves excited by ion beam in the Earth's

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magnetosphere was studied in [15] to investigate the influence of plasma parameters on growth of low frequency waves, further it was shown in [16] that plasma parameters have significant signature on the helium cyclotron instability.

In the past, thermal radiation or black body radiation has received a considerable attention due to its crucial relevance in astronomical/astrophysical objects and stars. Specifically, stellar luminosity can be explained with the utilization of radiation theory. It has been found [17] that radiation energy and pressure not only play an important role in internal energy and pressure in very high-temperature tenuous plasmas but also affect significantly the collective waves and instabilities in plasmas. In this context, electromagneto-hydrodynamic equations have been developed for investigating relativistic plasmas and photon gas [18] to assume photons in thermal equilibrium with plasma. The theory of thermal radiation has mostly been discussed in the past [19–21] for one-component systems. However, originally, Planck has proposed the radiation to be a collection of photons (known as black body radiation) with a dispersion relation in vacuum as $\omega = ck$, where $\omega(k)$ represents the frequency (wave number) of the radiation and c is the speed of light in vacuum. In particular, thermodynamics [22] due to electromagnetic radiations gets more influence in plasma medium rather than in vacuum. Considering a charged plane surface of a radiative dusty plasma [17], the impact of radiation pressure was examined on the Jeans instability [23], hence was sown in two analysis [17, 23] that radiation energy and pressure significantly affect the collective modes and associated instabilities in plasmas, thus playing a crucial role in the internal energy and pressure in a very high-temperature tenuous plasma. More recently [24], the radiation pressure corrections were made to dense radiative dusty plasma to show that the nonlinear properties of the circularly polarized dust Alfven waves.

It well-known that an electron executes a quantum jump in the electric field of ion, may emit a photon or it may also absorb a photon, thus attaining additional kinetic energy. The expression for the pressure of high density and low temperature plasma i.e. $\hbar\omega_{ps}/T_s > >1$ [22], involving the electromagnetic radiation in a plasma medium, can be expressed in terms of radiation energy density (u_{Rs}), as

$$3p_{Rs} = u_{Rs} \equiv \frac{0.202 T_s^4 b_s^{3/2}}{(\hbar c)^3} \tag{1}$$

with $b_s = \hbar\omega_{ps}/T_s$. Here the subscript '$s$' essentially stands for the sth plasma species (s equals e for electrons and i for ions). The plasma oscillation frequency, thermal temperature (in energy units) and scaled Planck constant are denoted by $\omega_{ps}[=(4\pi n_{s0} q_s^2/m_s)^{1/2}]$, T_s and $\hbar$, respectively, Eliminating the parameter b_s from equation (1), we readily arrives at the following relation

$$u_{Rs} = \alpha_s T_s^{5/2} \text{ with } \alpha_s = \frac{0.202(\hbar\omega_{ps})^{3/2}}{(\hbar c)^3}. \tag{2}$$

It is pertinent to mention that radiation pressure due to plasma particles (via equation (1)) is to be taken into account in the momentum equation besides the usual thermal pressure [23] $p_{T_s}(=n_s T_s)$.

In this work, we consider a three-component dusty magnetoplasma to account for electron-ion radiation pressures apart from their thermal pressures. Utilizing the linearized magnetohydrodynamical (MHD) model [25] along with Maxwell equations, we shall obtain a generalized dispersion relation for the electromagnetic waves and aim to analyze different limiting cases of the modes propagating parallel, perpendicular, and oblique to the external magnetic field direction both analytically and numerically.

The layout of manuscript is organized as follows: In section 2, we solve the MHD model equations along with the Maxwell equations and derive a general dispersion relation for electromagnetic waves with electron and ion radiation pressures. It is found that the angular frequencies and wave phase speeds associated with the parallel, perpendicular, and obliquely propagating modes are significantly modified by the radiation pressures. Section 3 illustrates parametric analyzes and numerical findings involving the electrostatic dust-ion-acoustic waves (DIAWs) and radiative compressional Alfven waves (RCAWs) in radiative dusty magnetoplasmas., while section 4 summarizes the main results.

2. Governing plasma model and formalism

We consider the compressional Alfvén waves that propagate in the y–z plane with wave vector $k = (0, k_y, k_z)$ in a magnetized dusty plasma, whose constituents are the inertialess electrons, inertial ions, and static dust grains. An external magnetic field having the strength B_0 is applied along the z-axis with magnetic field perturbations $B_1 = (B_x, B_y, 0)$ in x–y plane. The quasi-neutrality at equilibrium imposes the condition of the form $n_{i0} = n_{e0} + Z_{d0} n_{d0}$, where Z_{d0} being the dust charging state, e, i, and d stand for the electron, ion and dust grain species, respectively. For our investigation, we assume here that the dusty grains are fixed in the background and appear only through quasi-neutrality, the equation of motion for a radiative plasma is given by

$$m_s n_s \frac{\partial \mathbf{U}_s}{\partial t} = q_s n_s \left[\mathbf{E} + \frac{1}{c}(\mathbf{U}_s \times \mathbf{B})\right] - \boldsymbol{\nabla}(p_{T_s} + p_{Rs}) - \sum_s \mathbf{R}_s. \tag{3}$$

Here $\mathbf{R}_s$ is the dissipative or frictional force term representing the momentum gained by the electron fluid due to collisions with ions if $\mathbf{R}_s = \mathbf{P}_{ei}[\equiv m_e n_e \nu_{ei}(\mathbf{u}_i - \mathbf{u}_e)]$ and the momentum gained by the ion fluid due to collisions with electrons [23] if $\mathbf{R}_s = \mathbf{P}_{ie}[\equiv m_i n_i \nu_{ie}(\mathbf{u}_e - \mathbf{u}_i)]$, ν_{ei} (ν_{ie}) stands for the electron-ion (ion-electron) collisional frequency and $\mathbf{u}_e(\mathbf{u}_i)$ denotes the electron (ion) fluid velocity. It is assumed that collisions among the electron and ion species are so frequent that collisional frequencies become dominant ($\partial_t \ll \nu_{ei}, \nu_{ie}$). However, in a situation, in which collisions can only be neglected when the frictional forces are balanced by the other terms in equation (3) and the fluid velocities of plasma species are equal, i.e. $\mathbf{u}_i \approx \mathbf{u}_e$. Consequently, the conservation of momenta then vanish, i.e. $\sum_s R_s = 0$, as can be seen in [26]. Hence the plasma is characterized by a well-known density relation [27] as

$$\frac{n_e}{n_{e0}} = \frac{n_i}{n_{i0}}. \tag{4}$$

Also in equation (3), p_{Ts} (p_{Rs}) is the usual thermal pressure (the radiation pressure) for the electron and ion species. It is well-known [23] that for ideal gas having constant specific heat, the entropy remains conserved. As the model in consideration is comprised of three subsystems, i.e. the electrons, ions and dust grains, so the entropy of each subsystem must be conserved. As a consequence, the Poisson's adiabatic relation between the number density (n_s) and temperature (T_s) for sth species with an adiabatic expansion or compression, can be expressed, as

$$\frac{T_s}{n_s^{2/3}} = C_s \equiv \frac{T_{s0}}{n_{s0}^{2/3}}, \tag{5}$$

where $C_s\left(=\frac{T_{s0}}{n_{s0}^{2/3}}\right)$ is a constant with T_{s0} and $n_{s0}^{2/3}$ as the equilibrium temperature and number density of species s. Multiplying and dividing the left and right hand sides in equation (5) by n_s and n_{s0}, respectively, we then immediately obtain a relation for thermal pressure as given by

$$p_{Ts} = p_{s0}\left(\frac{n_s}{n_{s0}}\right)^{5/3}, \tag{6}$$

where $p_{s0}(= n_{s0}T_{s0})$ is the equilibrium pressure of the plasma species. Similarly, substituting the expression of T_s from equation (5) into equation (2), we obtain the radiation pressure of plasma species

$$p_{Rs} = \frac{\alpha_s}{3}T_{s0}^{5/2}\left(\frac{n_s}{n_{s0}}\right)^{5/3}. \tag{7}$$

In order to consider a set of magnetohydrodynamic equations in a radiative dusty plasma, we have to linearize the continuity equations for ions and electrons, respectively, as

$$\frac{\partial n_{i1}}{\partial t} + n_{i0}\nabla.\mathbf{u}_{i1} = 0, \tag{8}$$

and

$$\frac{\partial n_{e1}}{\partial t} + n_{e0}\nabla.\mathbf{u}_{e1} = 0. \tag{9}$$

In the present model, the dust particulates are assumed to be static, whereas we ignore the electron inertia in comparison with dynamical ions, and into account the conservation of momentum, i.e. $\sum_s R_s = 0$, equation (3) looks for inertialess electrons and inertial ions with thermal and radiation pressures becomes, respectively, as

$$0 = -e\left[\mathbf{E}_1 + \frac{\mathbf{u}_{e1}\times\mathbf{B}_0}{c}\right] - \frac{1}{n_{e0}}\boldsymbol{\nabla}\left[p_{e0}\frac{5}{3}\left(\frac{n_{e1}}{n_{e0}}\right) + \alpha_e T_{e0}^{5/2}\frac{5}{9}\left(\frac{n_{e1}}{n_{e0}}\right)\right], \tag{10}$$

and

$$\frac{\partial\mathbf{u}_{i1}}{\partial t} = \frac{e}{m_i}\left[\mathbf{E}_1 + \frac{\mathbf{u}_{i1}\times\mathbf{B}_0}{c}\right] - \frac{1}{m_i n_{i0}}\boldsymbol{\nabla}\left[p_{i0}\frac{5}{3}\left(\frac{n_{i1}}{n_{i0}}\right) + \alpha_i T_{i0}^{5/2}\frac{5}{9}\left(\frac{n_{i1}}{n_{i0}}\right)\right], \tag{11}$$

where the subscript 0 denotes the equilibrium values. We shall also make use of the following Maxwell's equations:

$$\frac{\partial\mathbf{B}_1}{\partial t} = -c\boldsymbol{\nabla}\times\mathbf{E}_1 \tag{12}$$

and

$$\frac{c}{4\pi e}\boldsymbol{\nabla}\times\mathbf{B}_1 = n_{i0}\mathbf{u}_{i1} - n_{e0}\mathbf{u}_{e1}. \tag{13}$$

Here c represents the speed of light, e is the charge of electrons, $E_1(B_1)$ is the perturbed electric (magnetic) field, and $u_{e1}(u_{i1})$ is the electron (ion) fluid velocity. Note that the displacement current term in equation (12) is ignored due to the fact that we have focused on the waves, whose phase speed is much smaller than the speed of light. Calculating $\mathbf{u}_{e1} = \frac{n_{i0}}{n_{e0}}\mathbf{u}_{i1} - \frac{c}{4\pi e n_{e0}}(\boldsymbol{\nabla}\times\mathbf{B}_1)$ from equation (13) and putting it into equation (10), then by using equation (4) we obtain

$$\mathbf{E}_1 = -\frac{1}{c}(\mathbf{u}_{i1}\times\mathbf{B}_0)\frac{n_{i0}}{n_{e0}} + \frac{1}{4\pi e n_{e0}}(\boldsymbol{\nabla}\times\mathbf{B}_1)\times\mathbf{B}_0 - \frac{1}{e n_{e0}}\left(\frac{5}{3}p_{e0} + \frac{5}{3}\frac{\alpha_e}{3}T_{e0}^{5/2}\right)\boldsymbol{\nabla}\frac{n_{e1}}{n_{e0}}. \tag{14}$$

Substituting equation (14) into equation (11) and utilizing the equilibrium charge neutrality condition $\frac{Z_{d0}n_{d0}}{n_{e0}} = \frac{n_{i0}}{n_{e0}} - 1$, we eventually arrive at

$$\frac{\partial\mathbf{u}_{i1}}{\partial t} = -\Omega_R(\mathbf{u}_{i1}\times\hat{z}) + \frac{1}{4\pi m_i n_{e0}}\{(\boldsymbol{\nabla}\times\mathbf{B}_1)\times\mathbf{B}_0\} - \left(c_a^2\boldsymbol{\nabla} + \frac{n_{e0}}{n_{i0}}c_T^2\boldsymbol{\nabla}\right)\frac{n_{e1}}{n_{e0}}. \tag{15}$$

with

$$c_a = \left(\frac{5}{3}\frac{p_{e0} + \tilde{\alpha}_e}{m_i n_{e0}}\right)^{1/2} \text{ and } c_T = \left(\frac{5}{3}\frac{p_{i0} + \tilde{\alpha}_i}{m_i n_{i0}}\right)^{1/2}, \tag{16}$$

where $\tilde{\alpha}_e = \frac{\alpha_e}{3}T_{e0}^{5/2}$ and $\tilde{\alpha}_i = \frac{\alpha_i}{3}T_{i0}^{5/2}$. In deriving equation (14) we have make use of equation (4) i.e. $\frac{n_{e1}}{n_{e0}} = \frac{n_{i1}}{n_{i0}}$. Here $\Omega_R(=\omega_{ci}Z_{d0}n_{d0}/n_{e0})$ is the Rao cut-off frequency with ion-cyclotron frequency $\omega_{ci}(=eB_0/m_i c)$ [28]. We have noticed that the effect of dust is associated to the finite cut-off frequency at the infinite wavelength ($k = 0$) limit and the ion drift velocity in the compressional magnetic field perturbation is inversely proportional to the dust charge density. It may be important to mention that Rao cut-off frequency was first time introduced by Rao in [10], representing the boundary in a system's frequency response at which the index of refraction goes to zero or $k = 0$. It may be noticed that both the ion-acoustic and thermal speeds, i.e. c_a and c_T are strongly modified by the radiation pressure effects. Now substituting equation (14) into equation (12) and using identity

$\nabla \times (\nabla f) = 0$, we are left with an equation

$$\frac{\partial \mathbf{B}_1}{\partial t} = \frac{n_{i0}}{n_{e0}}[-\mathbf{B}_0(\boldsymbol{\nabla}.\mathbf{u}_{i1}) + (\mathbf{B}_0.\boldsymbol{\nabla})\mathbf{u}_{i1}]. \tag{17}$$

The perpendicular component of equation (15) gives

$$\frac{\partial \mathbf{u}_{i\perp}}{\partial t} = \frac{-(\mathbf{B}_0.\boldsymbol{\nabla})\mathbf{B}_1}{4\pi m_i n_{e0}} - \Omega_R(\mathbf{u}_{i\perp} \times \hat{z}) - V_{\text{eff}}^2 \boldsymbol{\nabla}_\perp \frac{n_{e1}}{n_{e0}}, \tag{18}$$

and the z-component of equation (15) becomes as

$$\frac{\partial u_{iz}}{\partial t} = -V_{\text{eff}}^2 \frac{\partial}{\partial z}\frac{n_{e1}}{n_{e0}}. \tag{19}$$

From equation (17)

$$\frac{\partial \mathbf{B}_1}{\partial t} = -\frac{n_{i0}}{n_{e0}}\mathbf{B}_0 \boldsymbol{\nabla}_\perp.\mathbf{u}_{i\perp}, \tag{20}$$

where $V_{\text{eff}} = \left(c_a^2 + \frac{n_{e0}}{n_{i0}}c_T^2\right)^{1/2}$ is the effective speed due to thermal and radiation pressures. Now taking the time derivative of equation (18) on both sides and also eliminating the term $\frac{\partial \mathbf{u}_{i\perp}}{\partial t}$ by using (i.e. equation (18)) the back substitution as well as taking divergence of $u_{i\perp}$, we finally arrive at the following

$$\begin{aligned}
&\left(\frac{\partial^2}{\partial t^2} + \Omega_R^2\right)\boldsymbol{\nabla}_\perp.\mathbf{u}_{i\perp} \\
&= -\frac{B_0}{4\pi m_i n_{e0}}\left[\left(\Omega_R \boldsymbol{\nabla}_\perp.(\hat{z} \times \boldsymbol{\nabla}_\perp) + \nabla_\perp^2 \frac{\partial}{\partial t}\right)B_1\right] \\
&\quad + \Omega_R V_{\text{eff}}^2 \boldsymbol{\nabla}_\perp.(\boldsymbol{\nabla}_\perp \times \hat{z})\frac{n_{e1}}{n_{e0}} - \frac{\partial}{\partial t}V_{\text{eff}}^2 \nabla_\perp^2 \frac{n_{e1}}{n_{e0}}.
\end{aligned} \tag{21}$$

Equation (21) can be simplified by utilizing equation (20) and identity $\nabla_\perp.(\hat{z} \times \nabla_\perp \phi) = 0$, here ϕ represents the scalar function

$$\left(\frac{\partial^2}{\partial t^2} + \Omega_R^2 - \nabla_\perp^2 V_A^2\right)B_1 = B_0 \frac{n_{i0}}{n_{e0}} V_{\text{eff}}^2 \nabla_\perp^2 \frac{n_{e1}}{n_{e0}}. \tag{22}$$

Here the Alfvén speed is given by

$$V_A = \left(\frac{n_{i0}}{n_{e0}}\right)^{1/2} \frac{B_0}{(4\pi m_i n_{e0})^{1/2}}.$$

Upon using the condition $n_{i1} = \frac{n_{i0}}{n_{e0}} n_{e1}$, we can express the ion continuity equation (8) as

$$\frac{n_{i0}}{n_{e0}}\frac{\partial n_{e1}}{\partial t} + n_{i0}(\nabla_\perp.\mathbf{u}_{i\perp}) + n_{i0}\frac{\partial}{\partial z}u_{iz} = 0. \tag{23}$$

Making use of (19) and (21) into (23) and multiplying the resultant equation by $\left(\frac{\partial^2}{\partial t^2} + \Omega_R^2\right)$, we finally arrive at

$$\begin{aligned}
&\left\{\left(\frac{\partial^2}{\partial t^2} + \Omega_R^2\right)\left(\frac{n_{i0}}{n_{e0}}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}V_{\text{eff}}^2 \frac{n_{i0}}{n_{e0}}\right)\right. \\
&\left. - \frac{\partial^2}{\partial t^2}V_{\text{eff}}^2 \nabla_\perp^2 \frac{n_{i0}}{n_{e0}}\right\}n_{e1} = \frac{n_{i0}}{n_{e0}}\frac{B_0 \nabla_\perp^2}{4\pi m_i}\frac{\partial^2 B_1}{\partial t^2}.
\end{aligned} \tag{24}$$

After seeking the plane wave solution in equations (22) and (24), and solving them together, a general dispersion relation for low-frequency radiative Alfven waves is obtained in a magnetized radiative dusty plasma, as

$$\begin{aligned}
&[(\omega^2 - \Omega_R^2)(\omega^2 - k_z^2 V_{\text{eff}}^2) - k_\perp^2 V_{\text{eff}}^2 \omega^2] \\
&\times (\omega^2 - \Omega_R^2 - k_\perp^2 V_A^2) = k_\perp^4 V_A^2 V_{\text{eff}}^2 \omega^2.
\end{aligned} \tag{25}$$

It is important to mention here that equation (25) is significantly modified by the thermal as well as radiation pressures and static dust particulates. For a simple electron-ion plasma, we assume $\Omega_R \to 0$, $n_{i0} \simeq n_{e0}$ in equation (25), as a consequence we are left with an equation

$$\omega^4 - \omega^2(k^2 \bar{V}_{\text{eff}}^2 + k_\perp^2 \bar{V}_A^2) + k_z^2 k_\perp^2 \bar{V}_A^2 \bar{V}_{\text{eff}}^2 = 0, \tag{26}$$

where $\bar{V}_A = \frac{B_0}{(4\pi m_i n_{e0})^{1/2}}$ is the Alfven speed, which physically shows that both wave and particles are moving together however, at high field or low density, the velocity of the Alfvén wave approaches the speed of light, and the Alfvén wave may be an ordinary electromagnetic wave, $\bar{V}_{\text{eff}} = (c_a^2 + c_T^2)^{1/2}$ is the effective speeds in an electron-ion plasma. Equation (26) represents a quadratic equation having an analytic solution of the form

$$\begin{aligned}
\omega^2 &= \frac{1}{2}(k^2 \bar{V}_{\text{eff}}^2 + k_\perp^2 \bar{V}_A^2) \\
&\pm \frac{1}{2}\{(k^2 \bar{V}_{\text{eff}}^2 + k_\perp^2 \bar{V}_A^2)^2 - 4k_z^2 k_\perp^2 \bar{V}_A^2 \bar{V}_{\text{eff}}^2\}^{1/2}.
\end{aligned} \tag{27}$$

Next, we now investigate the dispersive properties of radiative compressional Alfven waves for three particular cases, the parallel, perpendicular, and oblique propagations. For this purpose, let us substitute $k_z = k\cos\theta$ and $k_\perp = k\sin\theta$ to re-write equation (25) in following form

$$\begin{aligned}
&\omega^6 - \omega^4(2\Omega_R^2 + k^2 V_A^2 \sin^2\theta + k^2 V_{\text{eff}}^2) \\
&\quad + \omega^2(\Omega_R^4 + k^2 V_A^2 \Omega_R^2 \sin^2\theta + 2k^2 V_{\text{eff}}^2 \Omega_R^2 \cos^2\theta \\
&\quad + k^2 V_{\text{eff}}^2 \Omega_R^2 \sin^2\theta + k^4 V_A^2 V_{\text{eff}}^2 \sin^2\theta\cos^2\theta) \\
&= k^2 V_{\text{eff}}^2 \Omega_R^4 \cos^2\theta + k^4 V_A^2 V_{\text{eff}}^2 \Omega_R^2 \sin^2\theta\cos^2\theta.
\end{aligned} \tag{28}$$

This is the 6th degree polynomial equation, representing a modified general dispersion for RCAWs in radiative dusty plasma.

2.1. Parallel Propagation ($\theta = 0$)

To analyze equation (28) analytically, we first consider the parallel propagation by assuming $\theta = 0$ and obtain from equation (28)

$$(\omega^2 - \Omega_R^2)^2(\omega^2 - k^2 V_{\text{eff}}^2) = 0 \tag{29}$$

with possible solutions as

$$\omega = \pm\Omega_R \quad \text{and} \quad \omega = \pm k\left(c_a^2 + \frac{n_{e0}}{n_{i0}}c_T^2\right)^{1/2}. \tag{30}$$

Since the positive roots are the physical roots in contrast to negative ones, therefore only the positive real frequencies in equation (30) represent the cut-off frequency and a modified dust-ion acoustic waves, respectively. For null dust number density, we have $\Omega_R = 0$, $n_{i0} \simeq n_{e0}$ and as a result, one can easily obtain an ion-acoustic wave that is significantly modified with electron and ion radiation pressures. Moreover, for numerical analysis, we normalize equation (30) in the presence of dust component using the scaled parameters, $\tilde{k} = \frac{kV_{Ti}}{\omega_{pi}}$ and $\tilde{\omega} = \frac{\omega}{\omega_{pi}}$, as

$$\tilde{\omega} = \frac{\tilde{k}}{V_{Ti}}\left(c_a^2 + \frac{n_{e0}}{n_{i0}}c_T^2\right)^{1/2}, \tag{31}$$

where $V_{Ti}\left[=\left(\frac{5}{3}\frac{T_{i0}}{m_i}\right)^{1/2}\right]$ is the usual ion-thermal speed.

2.2. Perpendicular propagation ($\theta = \frac{\pi}{2}$)

In case of perpendicular propagation, we choose an angle $\theta = \pi/2$ in equation (28) to obtain the following equation

$$\omega^4 - \omega^2(2\Omega_R^2 + k^2V_A^2 + k^2V_{eff}^2) + \Omega_R^2(\Omega_R^2 + k^2V_A^2 + k^2V_{eff}^2) = 0 \tag{32}$$

with solutions

$$\omega^2 = \frac{1}{2}(2\Omega_R^2 + k^2V_A^2 + k^2V_{\rm eff}^2) \pm \frac{1}{2}\left\{\begin{matrix}(2\Omega_R^2 + k^2V_A^2 + k^2V_{\rm eff}^2)^2\\ -4\Omega_R^2(\Omega_R^2 + k^2V_A^2 + k^2V_{\rm eff}^2)\end{matrix}\right\}^{1/2}. \tag{33}$$

This is the dispersion relation of the radiative compressional Alfven waves in cold dusty magnetoplasmas. Here + and − signs indicate the fast and slow modes. Equation (33) can be expressed in normalized form as

$$\tilde{\omega} = \left[\frac{1}{2}\left(\frac{2\Omega_R^2}{\omega_{pi}^2} + \frac{\tilde{k}^2\bar{V}_A^2}{V_{Ti}^2} + \frac{\tilde{k}^2\bar{V}_{\rm eff}^2}{V_{Ti}^2}\right) + \frac{1}{2}\left\{\begin{matrix}\left(\frac{2\Omega_R^2}{\omega_{pi}^2} + \frac{\tilde{k}^2\bar{V}_A^2}{V_{Ti}^2} + \frac{\tilde{k}^2\bar{V}_{\rm eff}^2}{V_{Ti}^2}\right)^2 - \\ \frac{4\Omega_R^2}{\omega_{pi}^2}\left(\frac{2\Omega_R^2}{\omega_{pi}^2} + \frac{\tilde{k}^2\bar{V}_A^2}{V_{Ti}^2} + \frac{\tilde{k}^2\bar{V}_{\rm eff}^2}{V_{Ti}^2}\right)\end{matrix}\right\}^{1/2}\right]^{1/2}. \tag{34}$$

2.3. Oblique propagation

In this case, if we ignore the dust component, i.e. $\Omega_R \to 0$ and $n_{e0} \simeq n_{i0}$. Then equation (28) may be simplified, as

$$\omega^4 - \omega^2(k^2\bar{V}_A^2\sin^2\theta + k^2\bar{V}_{\rm eff}^2) + k^4\bar{V}_A^2\bar{V}_{\rm eff}^2\sin^2\theta\cos^2\theta = 0. \tag{35}$$

This is the dispersion equation of the radiative compressional Alfven waves in a dust-free magnetoplasma, which accounts for thermal and radiation pressures. The solutions of (35) can be expressed, as

$$\omega^2 = \frac{1}{2}(k^2\bar{V}_A^2\sin^2\theta + k^2\bar{V}_{\rm eff}^2) \pm \frac{1}{2}\{(k^2\bar{V}_A^2\sin^2\theta + k^2\bar{V}_{\rm eff}^2)^2 - 4k^4\bar{V}_A^2\bar{V}_{\rm eff}^2\sin^2\theta\cos^2\theta\}^{1/2}. \tag{36}$$

In a normalized form, the above equation (36) leads to

$$\tilde{\omega} = \left[\frac{1}{2}\left(\frac{\tilde{k}^2\bar{V}_A^2}{V_{Ti}^2}\sin^2\theta + \frac{\tilde{k}^2\bar{V}_{\rm eff}^2}{V_{Ti}^2}\right) + \frac{1}{2}\left\{\left(\frac{\tilde{k}^2\bar{V}_A^2}{V_{Ti}^2}\sin^2\theta + \frac{\tilde{k}^2\bar{V}_{\rm eff}^2}{V_{Ti}^2}\right)^2 - 4\frac{\tilde{k}^4\bar{V}_A^2\bar{V}_{\rm eff}^2}{V_{Ti}^4}\sin^2\theta\cos^2\theta\right\}^{1/2}\right]^{1/2}. \tag{37}$$

3. Parametric analysis and discussion

Numerical illustration helps us in explaining the electromagnetic/electrostatic waves with more reliable findings, consistent to an application area. Thus, the present section solves numerically equations (31), (34) and (37) to examine the impact of thermal and radiation pressures on the wave speed and associated frequencies of radiative compressional Alfven waves (RCAWs). Here RCAWs waves are propagating parallel, perpendicular and obliquely to the external magnetic field. For this purpose, as an example, we have chosen typical parameters from dense astrophysical environments [29, 30] with typical density and temperature as $n_{i0} = 1.001 \times 10^{31}$ cm^{-3}, $n_{e0} = 1\times 10^{31}$ cm^{-3}, $n_{d0} = 10^{-4}\ n_{i0}$, $Z_{d0} = \frac{n_{i0} - n_{e0}}{n_{d0}}$, $T_e \geqslant T_i = (9 \times 10^8 - 2 \times 10^9)$K.

The substitution of the above values along with $T_{e0} = 9 \times 10^8$ K, $T_{i0} = 1 \times 10^7$ K, $B_0 = 1 \times 10^6 G$ and the wave number $k = 10^9$ cm^{-1} in equation (31); one can easily compute other parameters like the usual ion-thermal speed $V_{Ti} \sim 2.3 \times 10^7$ cm s^{-1}, the electron plasma frequency $\omega_{pe} = 1.7 \times 10^{20}$ s^{-1}, the ion plasma frequency $\omega_{pi} = 4.1 \times 10^{18}$ s^{-1}, the ion-thermal length $\lambda_{Ti}(=\frac{V_{Ti}}{\omega_{pi}}) \sim 5.63 \times 10^{-12}$ cm, the equilibrium electron-thermal pressure $p_{e0} = 1.24 \times 10^{24}$ dyne cm^{-2}, the equilibrium ion-thermal pressure $p_{i0} = 1.38 \times 10^{22}$ dyne cm^{-2} with electron-radiation pressure $\tilde{\alpha}_e = \frac{\alpha_e}{3}T_{e0}^{5/2} = 9.4 \times 10^{20}$ dyne cm^{-2}, ion-radiation pressure $\tilde{\alpha}_i = \frac{\alpha_i}{3}T_{i0}^{5/2} = 4.4 \times 10^{13}$ dyne cm^{-2} and thermal speed $c_T = 3.711\ 13 \times 10^7$ cm s^{-1}. It is pertinent here to note that radiation pressure due to electrons is quite large as compared to the radiation pressure of ions ($\tilde{\alpha}_e >> \tilde{\alpha}_i$). Therefore, the ion-radiation pressure can be comparatively ignored in the present analysis. For taking into account the parallel propagation with $\tilde{k}(=\frac{kV_{Ti}}{\omega_{pi}}) = 0.02$, the modifiedr444 corresponding frequency and acoustic speed slightly get reduced in magnitude to $\omega = 0.300\ 15\omega_{pi}$ and $c_a = \left(\frac{5}{3}\frac{p_{e0}}{m_i n_{e0}}\right)^{1/2} = 3.520\ 68 \times 10^8$ cm s^{-1}. Hence, the radiation pressure not only modifies the phase speed but also enhances the angular frequency of the longitudinal DIAWs, as displayed in figure 1. It may be observed that modifications appear more prominent at shorter wavelengths, accounting for radiation pressure effect.

In order to examine the impact of electron density on radiation pressure ($\tilde{\alpha}_e$), we may fix the electron temperature $T_{e0} = 9 \times 10^8$ K and $n_{e0} = 1 \times 10^{31}$ cm^{-3} to find

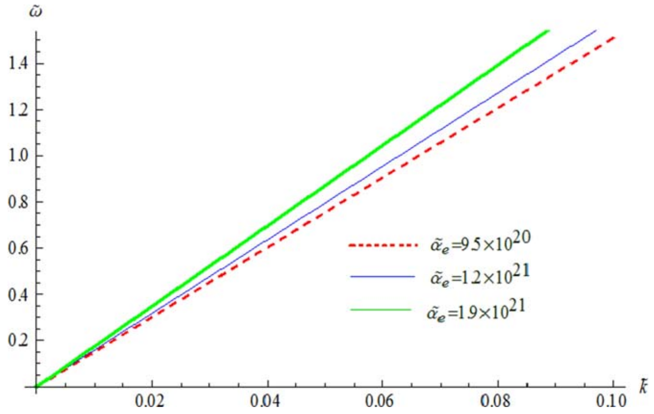

Figure 1. Normalized angular frequency $\tilde{\omega}\left(=\frac{\omega}{\omega_{pi}}\right)$ of DIAWs (as given by equation (31)) is plotted against the normalized wave number $\tilde{k}(=\frac{kV_{Ti}}{\omega_{pi}})$ for different electron radiation pressure values, $\tilde{\alpha}_e = 9.5 \times 10^{20}$ dyne cm^{-2} (red dashed curve), $\tilde{\alpha}_e = 1.2 \times 10^{21}$ dyne cm^{-2} (blue thin curve), and $\tilde{\alpha}_e = 1.9 \times 10^{21}$ dyne cm^{-2} (green thick curve). These values correspond to the electron temperatures $T_{e0} = 9 \times 10^8$K, 1×10^9 K and 1.2×10^9 K, respectively, with other fixed parameters, as $n_{e0} = 10^{31}$ cm^{-3}, $n_{i0} = 1.001 \times 10^{31}$ cm^{-3}, $B_0 = 1 \times 10^6$G, and $T_{i0} = 10^7$ K.

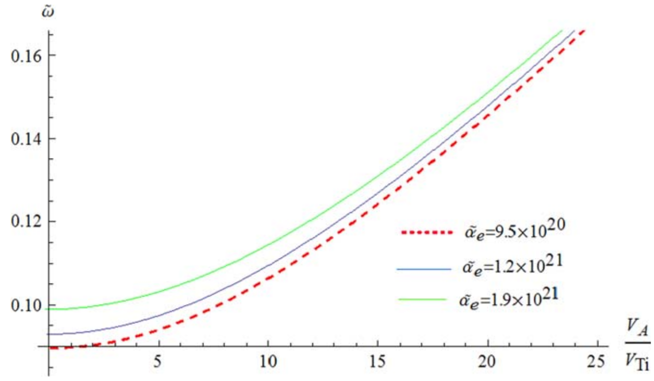

Figure 2. Normalized angular frequency $\tilde{\omega}$ of the compressional mode (as given by equation (34)) is plotted against the normalized Alfvén speed $(\frac{V_A}{V_{Ti}})$ for varying the electron radiation pressure through $\tilde{\alpha}_e$, i.e. $\tilde{\alpha}_e = 9.5 \times 10^{20}$ dyne cm^{-2} (red dashed curve), $\tilde{\alpha}_e = 1.2 \times 10^{21}$ dyne cm^{-2} (blue thin curve), and $\tilde{\alpha}_e = 1.9 \times 10^{21}$ dyne cm^{-2} (green thick curve) with $k_\perp = 10^9$ cm^{-1}.

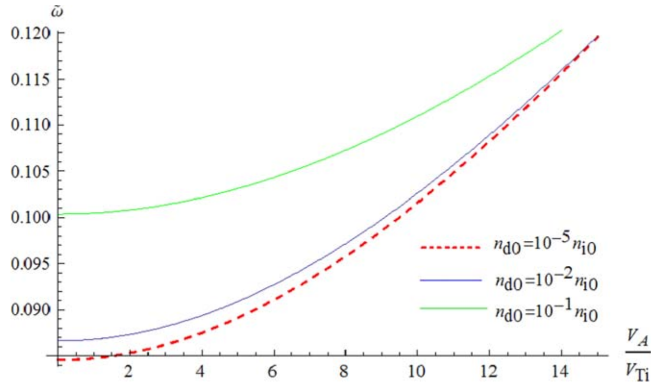

Figure 3. Normalized frequency $\tilde{\omega}$ of the RCAWs (as given by equation (34)) is shown as function of the normalized Alfvén speed $(\frac{V_A}{V_{Ti}})$ for different dust number densities, $n_{d0} = 10^{-5}$ n_{i0} (red dashed curve), $n_{d0} = 10^{-2}$ n_{i0} (blue thin curve), $n_{d0} = 10^{-1}$ n_{i0} (green thick curve).

$\tilde{\alpha}_e = 9.4 \times 10^{20}$ dyne cm^{-2}. And a reduction occurs in radiation pressure $\tilde{\alpha}_e = 1.6 \times 10^{20}$ dyne cm^{-2}, when the density $n_{e0} = 1 \times 10^{30}$ cm^{-3} is reduced. So, the parametric analysis clearly show that the variation of radiation pressure $\tilde{\alpha}_e$ via either number density or temperature modify the angular frequency $\tilde{\omega}$ of RCAWs effectively.

In particular, figure 2 displays the variation of the radiation pressure ($\tilde{\alpha}_e$) on the normalized angular frequency of the RCAWs as a function of the normalized radiative Alfvén speed. Note that the acoustic speed c_a as well as the effective speed ($V_{\rm eff}^2$) go on increasing as long as the radiation pressure increases, thus leading to the enhancement of the frequency and phase speed of the RCAWs. Similarly, the normalized angular frequency of the compressional mode (as given by equation (33)) is represented against the normalized radiative Alfvén speed for different dust concentration in figure 3. It shows that an increase in the dust number density causes to increase the normalized angular frequency of the RCAWs through the cut-off dust frequency Ω_R.

To examine how the thermal radiation speed alters the frequency of obliquely propagating compressional Alfven waves in a radiative dusty plasma, we parametrically analyze equation (37) with different angles of propagation. For $\theta = \frac{\pi}{5}$ and taking $T_{e0} = 9 \times 10^8$ K, $\bar{V}_A = 10V_{Ti}$ and $\tilde{k}_\perp$=0.005 fixed, the effective speed becomes $\bar{V}_{\rm eff} = \left(c_a^2 + \frac{5}{3}\frac{p_{i0}}{m_i n_{i0}}\right)^{1/2} = 3.541\,53 \times 10^8$ cm s^{-1} and angular frequency of the RCAWs $\omega = 0.087\,490\,4\omega_{pi}$ with modified acoustic speed $3.522\,03 \times 10^8$ cm s^{-1}. Whereas the effective speed in the absence of radiation yields $V_{\rm eff} = 3.540\,17 \times 10^8$ cm s^{-1} with a corresponding angular frequency $\omega = 0.301\,811\omega_{pi}$. However, for changing $\theta = \frac{\pi}{3}$, we trace the angular frequency $\omega = 0.095\,633\omega_{pi}$. The parametric analysis further elaborates that as we increase the angles of propagation, the corresponding angular frequency also increases. This result is also evident from the numerical plot in figure 4. For small values of the normalized Alfven speed, the angle of propagation has not very prominent impact on the frequency of the RCAWs, but it affects quite significantly at large normalized Alfven speeds. So we have shown via numerical analysis that for in the crust of white dwarf, having $T_s > 1.5 \times 10^7$ K and $n_s > 10^{32}$m^{-3}, the radiation pressure become much more important inspite of the fact that the radiation pressure is only a few thousandths of the gas pressure, since the net effect is small.

4. Summary

To sum up, we have studied the dispersive properties of the electromagnetic/electrostatic waves in a radiative dusty magnetoplasma that essentially accounts for thermal and radiation pressures of the plasma species. After solving the one-fluid adiabatic MHD model along with the Maxwell equations and seeking a plane wave solution to all perturbed quantities, we have obtained a modified general dispersion relation for the

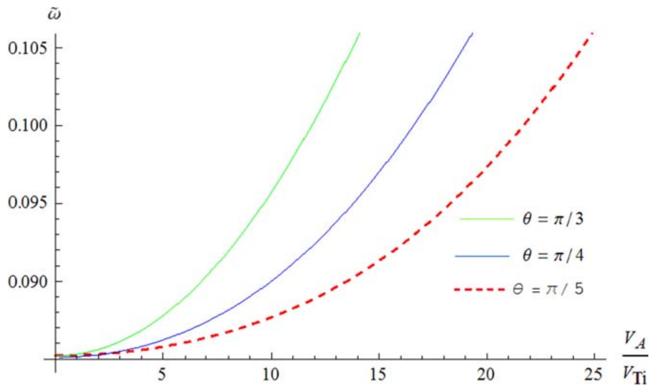

Figure 4. Normalized frequency $\tilde{\omega}$ of the RCAWs (as given by equation (37)) is displayed as function of the normalized Alfvén speed ($\frac{\tilde{V}_A}{V_{Ti}}$) for changing the angles of propagation at $\theta = \frac{\pi}{5}$ (red dashed curve), $\theta = \frac{\pi}{4}$ (blue thin curve), and $\theta = \frac{\pi}{3}$ (green thick curve).

electromagnetic/electrostatic waves and analyzed different limiting cases for the modes propagating parallel, perpendicular and oblique to the external magnetic field direction both analytically and numerically. It is revealed that the usual thermal and acoustic speeds (c_T, c_a) have not only strong dependence on temperature but also on the equilibrium number density because of the radiation pressure ($\tilde{\alpha}_e$). The main findings stem from the general dispersion relation, showing the influence of the radiation pressure on the dispersive low-frequency dust-ion-acoustic and compressional Alfven waves in dusty magnetoplasmas. Even for a dust-free plasma, we have assumed $\Omega_R = 0$,that implies that $n_{i0} \simeq n_{e0}$ and as a result, one can easily obtain an ion-acoustic wave modified significantly with the electron and ion radiation pressures. Similarly, the dispersion relation for the fast and slow compressional Alfven waves are examined for thermal and radiation pressure effects in dusty magnetoplasmas. Thus, the wave frequencies and associated phase speeds are found to be affected by thermal radiation pressure. However, the variation in the angle of propagation mitigates these frequencies and phase speeds of the plasma waves. The present results are important for understanding the new features of the dispersive electrostatic/electromagnetic waves in dusty astrophysical dense plasmas.

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