

Scale-free distributions of waiting times for earthquakes

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Abstract

We investigate through detailed statistical analyses the dynamics of earthquakes with epicenters in Romania, Japan, California (USA), and Italy, and show that the distribution of waiting times, defined here akin to econophysics, as to implicitly include information about the magnitude of earthquakes, is very similar to the scale-free distributions observed in hydrodynamic turbulence and stock market dynamics. Our results show that the shape of the observed distributions depends on the size of the magnitude threshold δ , with a cut-off at small waiting times, and is sensitive with respect to the sign of the aforementioned threshold. In the case of earthquakes originating in Romania we also show that the distributions of waiting times for depths have the same power-law behaviour. Moreover, we show that the distributions of released daily energies calculated for Romania and California (USA) have a prominent scale-free nature, which reinforce the idea that seismic zones can be seen as critical systems.

Keywords: earthquakes, distribution of waiting times, inverse statistics, scale-free distributions

(Some figures may appear in colour only in the online journal)

1. Introduction

Since antiquity, earthquakes, like all destructive natural phenomena, played a pivotal role in reshaping the geographical landscape and our view of the world. Humanity's earliest records that mention earthquakes go back to 2000 B.C., but as one can imagine they have little value to the field of seismology. These accounts, however exaggerated, are nonetheless an excellent way to study the evolving understanding and knowledge of our ancestors [1].

The turning point on how we view seismic events was the Great Lisbon earthquake of November 1, 1755 [2]. This cataclysm and the following tsunami took the life of an estimated 70 000 people. Before the Lisbon earthquake most scholars still looked to Aristotle explanations on earthquakes. He was among the first to try to explain them based on natural causes, i.e. winds trapped beneath the Earth that shook the earths surface. The 1755 earthquake had a profound effect on

Voltaire who wrote the classical *Poème sur le désastre de Lisbonne* as a result. He was also among the first to adopt scientific ideas as he corresponded for two years with Newton and was able to understand the recent developments in Europe. For the next hundred years the study of seismic events saw a steady increase. With the advantage of trade routes and faster communication between communities, information about earthquakes could be gathered more easily and analyzed in a systematic way, thereby establish seismology as an independent field of study in the early 20th century.

It is very interesting to point out that on April 2019, the NASA InSight lander measured the first quake on Mars (a so-called *marsquake*), proving that Mars is still seismically active. Also, the Apollo missions placed seismometers on the surface of the Moon and it came as no surprise that *moonquakes*, quakes on the surface of the Moon, were registered.

Earthquakes are highly complex spatially and temporally correlated physical phenomena and we cannot study an individual event in the hopes of understanding the underlying dynamics. A strikingly simple relationship that is universal

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for earthquakes was first proposed by Charles Richter and Beno Gutenberg [3]. It states that the number of earthquakes with a magnitude greater than M is:

$$\log_{10} N = a - bM \quad (1)$$

The parameter b is commonly known as the b -value and it is close to 1 in most seismic regions. However, there are some variations in the b -value ranging from 0.5 to 2.5 depending on the geographical region where the earthquakes are measured. The extreme value of 2.5 was measured during earthquake swarms and the obvious interpretation is that we have a higher number of smaller earthquakes than larger ones. It is still debated why the b -value has spatial and temporal variations [4].

Aftershocks play a crucial role in seismology, as they are usually unpredictable and can cause more destruction than the main event. The decrease in time of the frequency of aftershocks is given by the Utsu–Omori law:

$$n(t) = k(c + t)^{-p} \quad (2)$$

where c , k , and p are constants [5], with $p \in [0.7, 1.5]$. The universality of the Gutenberg–Richter and Utsu–Omori laws can be considered as proof that earthquakes are critical phenomena as Christensen *et al* suggested in [6]. This helps to explain the scale-free nature of earthquake statistics and the complexity of temporal and spatial distributions.

All throughout nature we find phenomena akin to earthquakes, highly unpredictable and with a complex spatio-temporal evolution. From weather patterns and solar flares, to the World Wide Web and the citation network, a wide class of systems exhibit scale-free structure.

When an earthquake is measured, we usually retain information about the magnitude, the three spatial coordinates of the epicenter and the time of occurrence. Our goal is to investigate the inter-occurrence time without losing information on the spatial and magnitude distributions. To achieve this we follow the approach described in [7] and [8], where the concept of investment horizon was introduced for financial markets. The investment horizon represents the smallest time interval needed for an index to fluctuate by a given amount. With careful modifications we can use this concept to study the time interval between earthquakes without any loss of information. We rename the investment horizon as *waiting time* and define it in the following way: what is the time interval needed to find an earthquake of magnitude $M + \delta$, with δ a given constant, after an earthquake of magnitude M . Using this definition for the waiting time, we retain information about the time of occurrence and magnitude, but it can easily be modified to encompass the remaining three spatial coordinates. The aforementioned definition of waiting times is significantly different from similar ones used in the literature, see, e.g. [9–12], as we do not look at the time intervals elapsed between subsequent earthquakes, but focus on the time distance between a given trigger earthquake of magnitude M and the first subsequent earthquakes of magnitude larger than $M + \delta$. Naturally, for $\delta = 0$ our results are similar to those previously reported in the literature, and

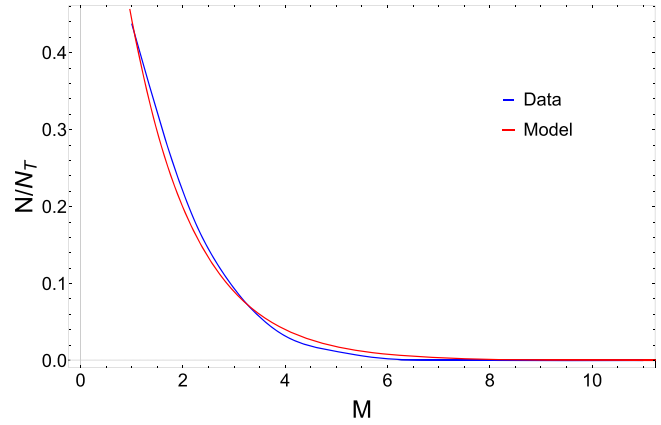


Figure 1. The Gutenberg–Richter law for Romanian earthquake data and for $b = 0.34$.

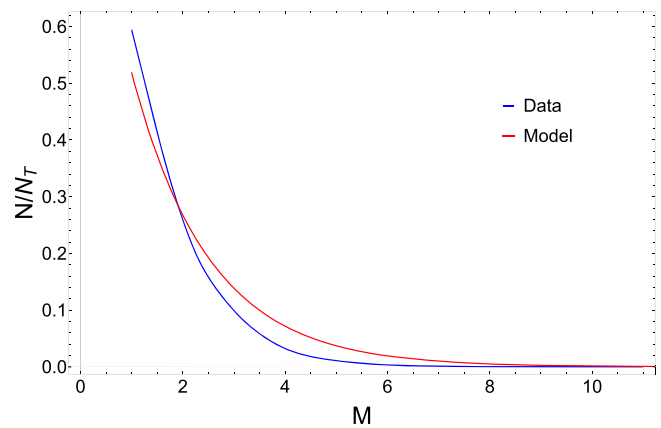


Figure 2. The Gutenberg–Richter law for Californian earthquake data and for $b = 0.29$.

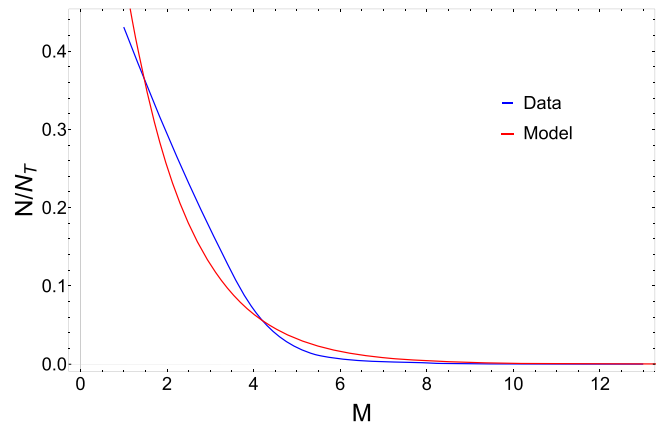


Figure 3. The Gutenberg–Richter law for Japanese earthquake data and for $b = 0.29$.

exhibit very prominent scale-free distributions, but from the viewpoint of seismic risk assessment the interesting results are those for $\delta \neq 0$.

In this paper we analyze four earthquake databases which cover Romania, Japan, Italy, and California (USA), see [13–16]. This choice was made in order to have a large enough sample size and to check that we find the same structure of distributions even if the fault structures and the

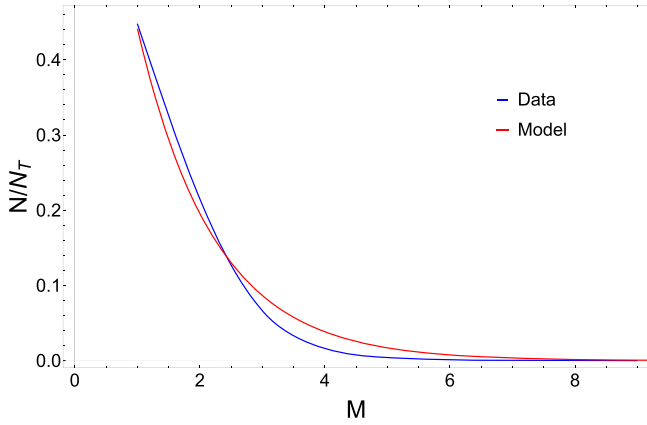


Figure 4. The Gutenberg–Richter law for Italian earthquake data and for $b = 0.31$.

seismic activity for the analyzed regions are very different from each other. We show that the magnitude and the depth waiting time distributions present the same scaling laws. These preliminary results enforce the idea that earthquakes are critical phenomena and also that the spatial and temporal distributions of epicenters have a dynamical origin.

2. Inverse statistics and databases analysis

As mentioned before, the processing speed of computers and the capacity of storing vast amounts of data in real time saw an exponential increase in the last few decades. From social networks to climate change, the modern world is driven by *big data*. It is in our immediate interest to develop tools that analyze these databases and to extract useful and applicable results. Most databases are generated by the nonlinear interaction of independent variables of complex dynamical systems. It is from these nonlinear correlations that the unpredictability of complex phenomena rises and our difficulty in having a deeper understanding of the underlying mechanisms.

In most cases, phenomena with a large number of independent variables have Gaussian distributions, however there are many other systems that deviate from this type of distribution. Among them we count the fluctuations of the financial market, the variations of wave heights, the earthquake magnitudes, etc. One way to look at these systems is through the lens of inverse statistics. By this method we switch the roles of the variables. The fluctuating variable becomes constant and the constant variable becomes the fluctuating one.

Our approach follows that used for the study of financial markets and draws from the [7] and [8]. The traditional way of interpreting financial data is to measure the profit gained after a fixed time interval Δt . With the use of inverse statistics they turn the problem around and asked what is the time interval Δt (now the fluctuating variable) needed such that a variation δ of the fixed variable to be reached. For any given process $\{I_t\}_{t \geq 0}$ the waiting time τ_δ is defined in the following

way [8]:

$$\tau_\delta = \begin{cases} \inf \left\{ s > 0, \ln \frac{I_{t+s}}{I_t} \geq \delta \right\}, & \delta > 0 \\ \inf \left\{ s > 0, \ln \frac{I_{t+s}}{I_t} \leq -\delta \right\}, & \delta < 0 \end{cases} \quad (3)$$

,where we have implicitly assumed that τ_δ is independent of t . In this way we obtain the probability distribution of τ_δ .

Using this definition we take the earthquake magnitude as the fixed variable and we want to find the smallest time interval until an earthquake with a given magnitude is measured. In other words, we compute the distributions of waiting times τ_δ such that an event with a magnitude at a given moment, $M(t)$ to increase or decrease by a fixed value δ . For this case, we rewrite the waiting time as:

$$\tau_\delta = \begin{cases} \inf \{ \Delta t, M(t + \Delta t) - M(t) \geq \delta \}, & \delta > 0 \\ \inf \{ \Delta t, M(t + \Delta t) - M(t) \leq -\delta \}, & \delta < 0 \end{cases} \quad (4)$$

.From this definition of the waiting time it is obvious that we can replace the magnitude with any of the other variable used to characterize earthquakes. For now we only calculate the distribution of waiting times for magnitudes and depths, but the analysis is generic.

We note that a similar analysis of waiting times was performed by Baran *et al* in [17] to understand the chaotic behaviour of the logistic map, seen as a prototypical example of deterministic dissipative systems.

Our analysis is performed on four distinct earthquake databases: Romania, Japan, Italy, and California (USA). These choices were made in order to have a large enough sample size of events and to check that the distributions are very similar, even if the geophysical structure of each seismic zone is different. Because the databases vary between each other in the number of events per time interval, we are also able to verify that we recover the same scaling laws of the waiting times for different orders of magnitude.

Although the seismic activity does not follow a regular and deterministic pattern, on large enough time scales the average number of earthquakes is of the same order of magnitude. Looking at each individual database, high variations in activity was found and that they did not respect the Gutenberg–Richter law. The reason for these variations is that for long periods of time many events with magnitude lower than two were not properly registered in the databases. A clear example of this is the Romanian database, where before the 7.2 magnitude earthquake on March 4th 1977 and for several years after, earthquakes with magnitude small than two are scarcely included. Thus, we impose a cutoff $m_W \geq 2$ for all earthquake magnitudes. Some authors suggested that there are no differences between the mechanisms that generate earthquakes and aftershocks and for our study we consider this to be true. This means that when we calculate the waiting times, no constraints are imposed on the time intervals between earthquakes. We performed our analysis on the entire seismic zone, without dividing it in a grid of variable cell size as our interest lies in the general behavior of the

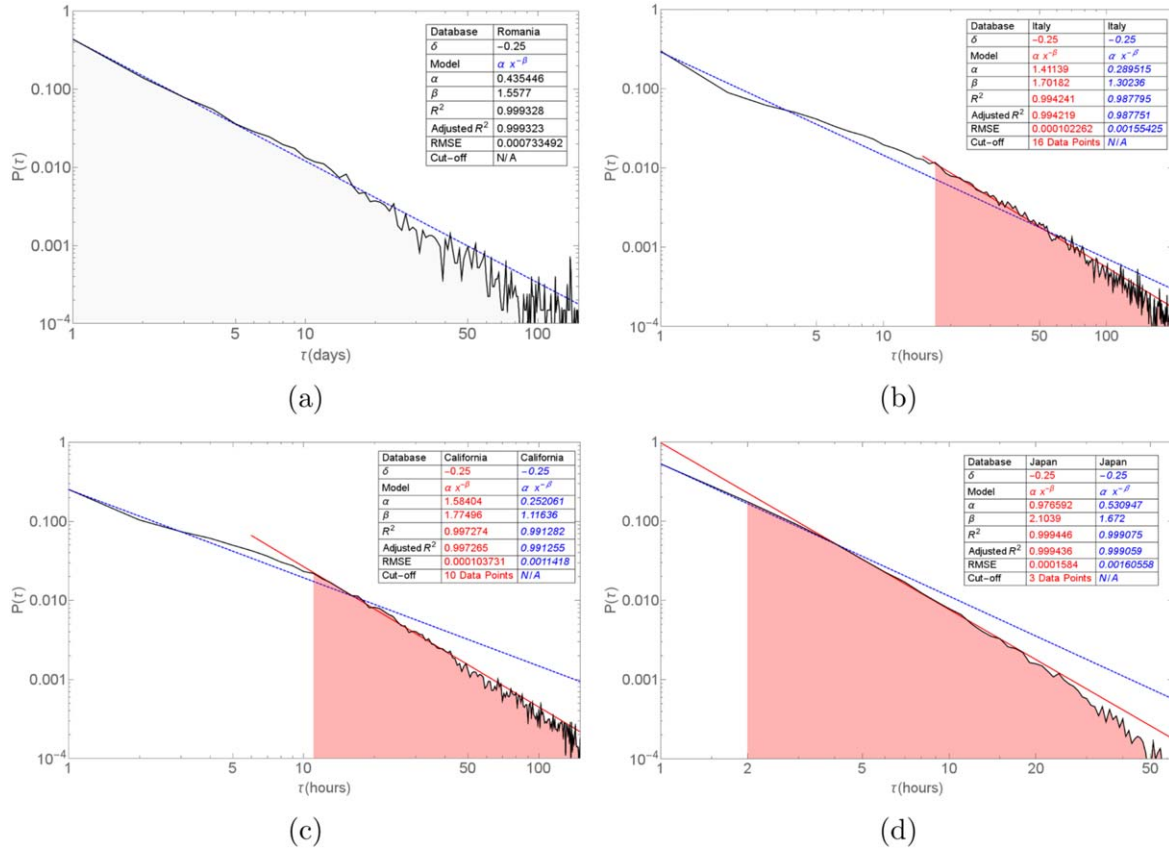


Figure 5. Waiting time distributions for a variation of magnitude of $\delta = -0.25$. The databases are: (a) Romania, (b) Italy, (c) California (USA), and (d) Japan. For each distribution of waiting time we have shown two scale-free fits: (i) a global one, in blue (dashed line and italic fonts), which accounts for all points in the distribution, and (ii) an optimal fit (technically, one which maximizes the adjusted R^2), in red (full line, normal fonts), which includes only the points in the hatched region. For panel (a) the two fits coincide.

distributions independent of spatial correlations. The data sets used for the earthquakes in Romania, California, Japan, and Italy, used in our investigations respect the Gutenberg–Richter law, as evidenced in figures 1–4.

2.1. Romanian seismic zone and database

Across Romania we can identify several active seismic zones and they are grouped such that their activity and stress field are uniform. In reducing the seismic hazard it is of vital importance in identifying and characterizing these active seismic regions. The most active and dangerous zone in Romania, Vrancea region, is located at the intersection between three tectonic units: the East European plate, Intra-Alpine and Moesian subplates [18]. The majority of earthquakes are concentrated at intermediate depths, 60–200 km, with an almost vertical distribution. This is due to the process of subduction, the slip of the lithospheric plate in the asthenosphere plate. There are only two other zones in the world, Bucaramanga in Columbia and Hindu Kush in Afghanistan, that have the same geophysical structure. If we refine our observation we can identify two different zones of activity, roughly situated between 80–100 km and 120–160 km. The largest earthquakes measured in the last century occurred in these two zones. However, in the interval of 100–120 km there is a relatively calm zone where very few earthquakes are

measured. Two main explanations have been put forward: either the rock is very soft and no earthquakes can be produced, or the zone is very dense and still accumulating energy. It is the uniqueness of the Vrancea region that validates our assumption that the structure of the medium where earthquakes occur plays no role in the behavior of the distribution of waiting times. The Romanian database spans a time interval of 41 years and contains a number of 19 277 events. Given this, the waiting time will be expressed in days. For this curated database we find the b -value to be equal to $b = 0.34$.

2.2. Californian seismic zone and database

California is one of the most complex seismic zone in the world and it contains many tectonic features. Among them we count the San Andreas Fault, the Cascadia subduction zone or the Mt. Lassen and Mt. Shasta active volcanoes. The most studied is the San Andreas Fault that represents the boundary between the North American plate and the Pacific plate. It is considered a transform fault, which means that the two plates slide past each other and the motion is predominantly horizontal. These type of faults are also known as conservative plate boundaries because they do not create or destroy the lithosphere. The seismic events in California occur at depths from 1 to 40 km with the bulk of epicenters focusing between

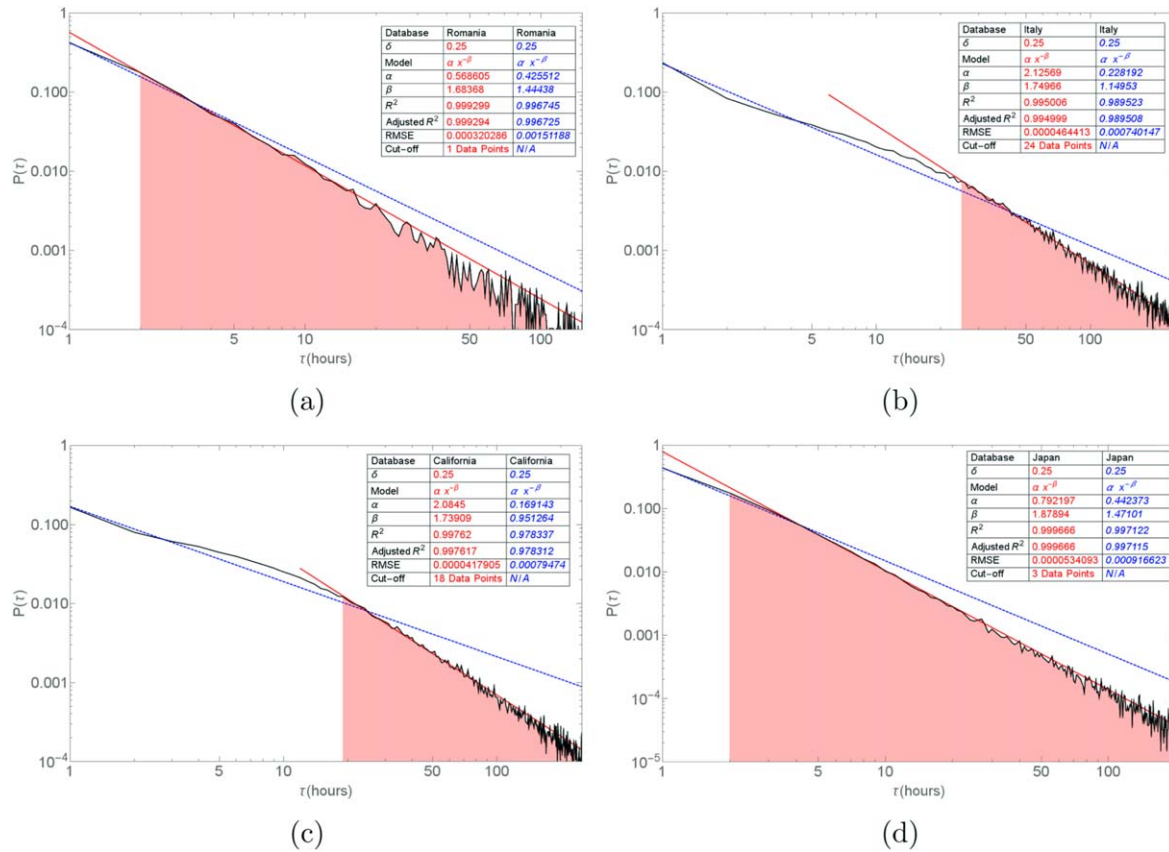


Figure 6. Waiting time distributions for a variation of magnitude of $\delta = +0.25$. The databases are: (a) Romania, (b) Italy, (c) California (USA), and (d) Japan. The details are as in figure 5.

2 and 12 km. The database we work with contains 134 171 events measured over a time period of 49 years and the waiting time is measured in hours.

2.3. Japanese seismic zone and database

The Japanese island is the best studied and monitored seismic zone in the world. With an average of 1500 earthquakes per year it is easily understandable why people pay such close attention to this region and why Japan is at the forefront in seismic hazard prevention. This high and complex seismic activity is due to the fact that Japan is situated at the intersection of four major tectonic plates. The two oceanic plates, Pacific and Philippine, are subducting under the two continental plates, Eurasian, and Okhotsk. The plate boundary is responsible for the high seismic activity and also for some of the most destructive earthquakes. As an example, the earthquake of magnitude 9 which took place on March 2011 occurred at the plate interface in North-East Japan.

The complex area of plate convergence gives rise to several types earthquakes: interplate earthquakes (along the tectonic plate interfaces), intraslab earthquakes (in the subduction plates) and inland crustal earthquakes. We also mention that earthquakes are frequently measured around volcanic areas. The depths at which earthquakes are measured spans a very wide interval, from surface earthquakes to very deep ones between 600 and 700 km. We used a database with events measured between 1985 and 1998. Even if we have a

shorter time interval than in the previous cases, the number of earthquakes registered (namely, 186 125) is enough for our analysis.

2.4. Italian seismic zone and database

Italy has a long tradition in measuring and investigating earthquakes. The detailed and refined record keeping was possible because the entire Italian territory has a medium to high seismic hazard. The high seismic activity is a result of the convergence of the African and Eurasian plate that runs through the center of Italy, forming an arc in the Ionian sea and continuing towards Sicily. For most of its length, this plate boundary is a collision margin that is responsible only for earthquakes and the creation of the two mountain chains (Alps in the north and the Apennines throughout the peninsula). There is also a subduction zone, the Calabrian Arc, located in southern Italy. It was the movement of this subduction zone, in a south-eastern direction, that formed the Mt. Etna and Mt. Vesuvius volcanoes. In analyzing the Italian region we work with a database of 94 744 registered earthquakes in the time interval between 1985 and 2019.

3. Results

In this section we present our results for the distribution of waiting times for the magnitude of earthquakes for all four

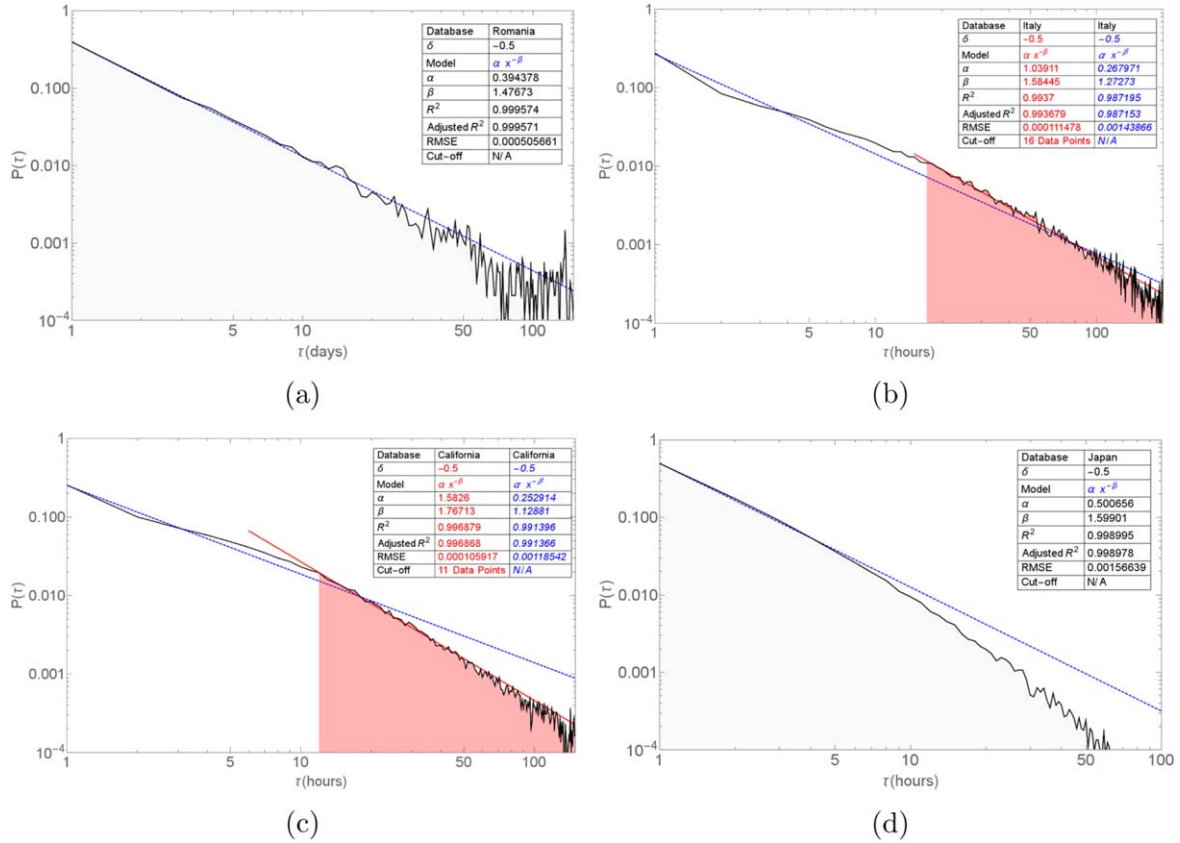


Figure 7. Waiting time distributions for a variation of magnitude of $\delta = -0.5$. The databases are: (a) Romania, (b) Italy, (c) California (USA), and (d) Japan. The details are as in figure 5. For panels (a) and (d) the two fits coincide.

earthquake databases. We also show the results for the distribution of waiting times considering the depth of the earthquakes, but here the results are limited to the Romanian database. The current results complement our preliminary investigations of the distributions of earthquakes from the Vrancea region reported in [17].

In figures 5–11 we show our results for the distribution of waiting times, while in figure 12 we show the distributions of released daily energies. For each distribution of waiting times we have shown two scale-free fits: (i) a global one, in blue (dashed line and italic fonts), which accounts for all points in the distribution, and (ii) an optimal fit (technically, one which maximizes the adjusted R^2), in red (full line, normal fonts), which includes only the points in the hatched region. In some cases, explicitly mentioned in the captions of the figures, the two fits are identical.

The scale-free nature of distributions of waiting times reported in figures 5–10 can be taken as evidence that earthquakes are self-organized systems [19]. The minimal description for such a system includes ‘(1) a medium which has (2) a disturbances propagating through it, causing (3) a modification of the medium, such that eventually (4) the medium is in a critical state, and (5) the medium is modified no more’ [19]. Please notice that we do not touch upon the actual formation of earthquakes in microscopic models, and rely on the scale-free distribution of waiting times as evidence for the emergence of the self-organized criticality. In fact,

there are many models which describe the emergence of the self-organized critical state, see [20, 21] and most importantly [22] which introduces a now classical Olami–Feder–Christensen self-organized critical model for earthquakes.

This spontaneous order of a seismic zone tells us that after an earthquake of magnitude M the probability to observe an event of magnitude $M \pm \delta$ decreases with the waiting time as a power-law. It is important to notice that we observe an asymmetry between negative and positive variations, i.e. δ , which has also been reported for other classes of complex systems. In the case of very small variations, see figures 5 and 6, we see little difference between the distribution for positive values of δ and that for negative values of δ . Increasing, however, the value of the variation magnitude δ , and thus accounting for the entire seismic sequence (main shock and aftershocks), it is easily observed that the asymmetry between distributions becomes larger for larger values of δ (see figures 7–10). Moreover, let us mention that the the observed distributions of waiting times are more similar to a pure scale-free distribution for negative values of δ than for positive values, an aspect which can also be derived from the fact that for some negative values of δ the aforementioned two fits are identical.

The prominent scale-free nature of the distributions in figures 5–11 reinforce the idea that seismic zones can be seen as critical systems (see [9–12]) using two observables which

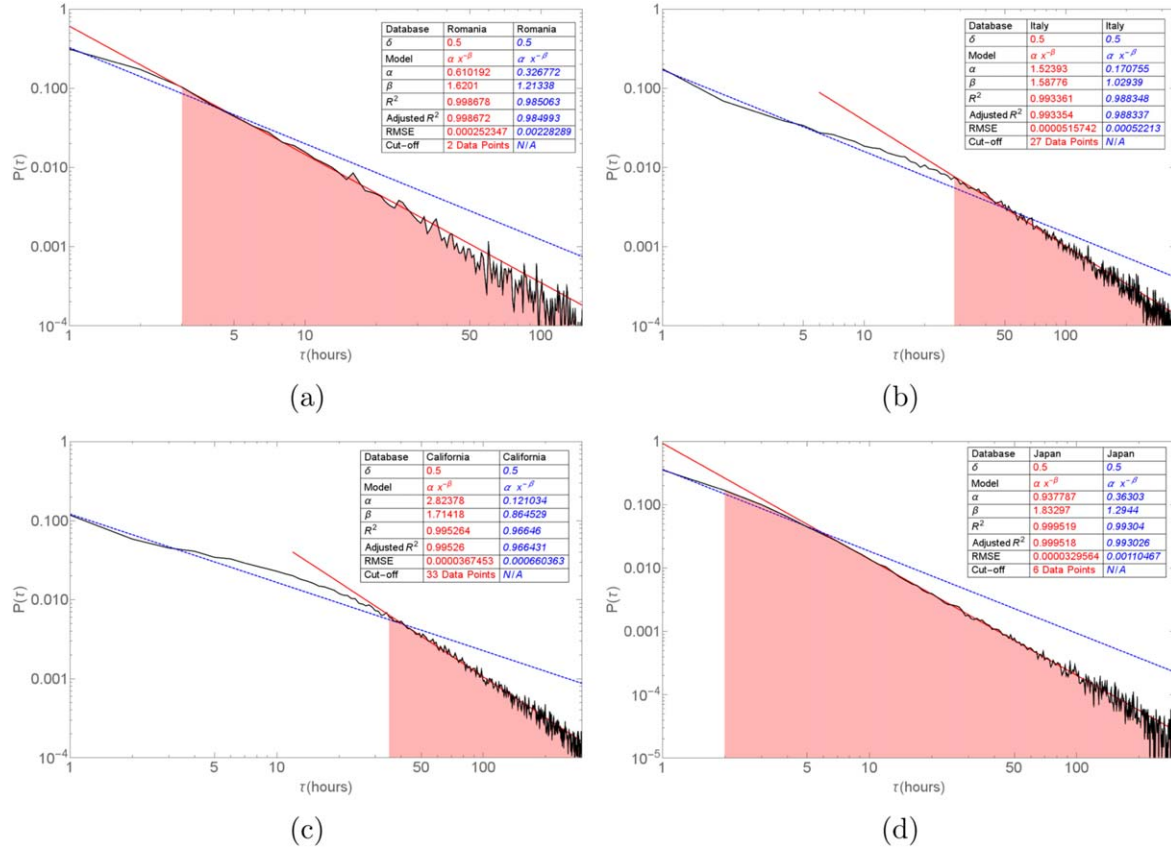


Figure 8. Waiting time distributions for a variation of magnitude of $\delta = +0.5$. The databases are: (a) Romania, (b) Italy, (c) California (USA), and (d) Japan. The details are as in figure 5.

have not been previously studied, namely the aforementioned waiting times for magnitudes and depths.

Our numerical investigations have shown a power-law distribution of the waiting times, with

$$P(\tau) = \alpha \tau^{-\beta}, \quad (5)$$

where τ is the waiting time and α and β two numerical coefficients. In determining the slope, α , of the power-law distribution we used the previous results for the waiting times with different values of the magnitude variation.

We remark that an exponent of similar value $\beta \approx 1.5$ was reported in the behaviour of the optimal horizon distribution of financial markets [23].

We have also calculated the distributions of waiting times for depths (as opposed to the above magnitudes) for the Romanian earthquake database using variations of $\delta = \pm 5$ km and $\delta = \pm 60$ km. The results of this analysis and the ensuing power-law distributions are presented in figure 8.

Lastly, to strengthen the image of seismic zones as critical systems, we show in figure 12 the distribution of released daily energies⁴ for earthquakes originating in Romania, see panel (a) of figure 12, and California, USA, see panel (b) of figure 12, which have a prominent scale-free nature with a scaling exponent around 2. For both seismic regions we have

discarded from our analysis small earthquakes of magnitude $M < 2$, as most catalogues treat inconsistently earthquakes of this magnitude. We will report elsewhere a detailed study on the distribution of the daily released energy, focusing on distinct sub-regions which take into account the clustering of earthquakes epicenters with respect to depth, latitude and longitude.

4. Conclusions

We have reported here a series of statistical analysis for earthquakes with epicenters in Romania, Japan, California (USA), and Italy, and have shown that the distribution of waiting times is scale-free in nature, having a small dependence on the size of the magnitude threshold Δ , with a cut-off at small waiting times, and being more sensitive with respect to the sign of the aforementioned threshold. Our definition of waiting times draws from econophysics and is different from similar ones used in the literature, as we do not look at the time intervals elapsed between subsequent earthquakes, but focus on the time distance between a given trigger earthquake of magnitude M and the first subsequent earthquakes of magnitude larger than $M + \delta$.

Also on the side of scale-free distributions, we have shown that for earthquakes originating from Romania the same power-law distribution describes the distribution of

⁴ The energy E (calculated in kilograms of explosives) released by an earthquake of magnitude M is computed through the formula $\ln(E) = 5.24 + 1.44 \cdot M$.

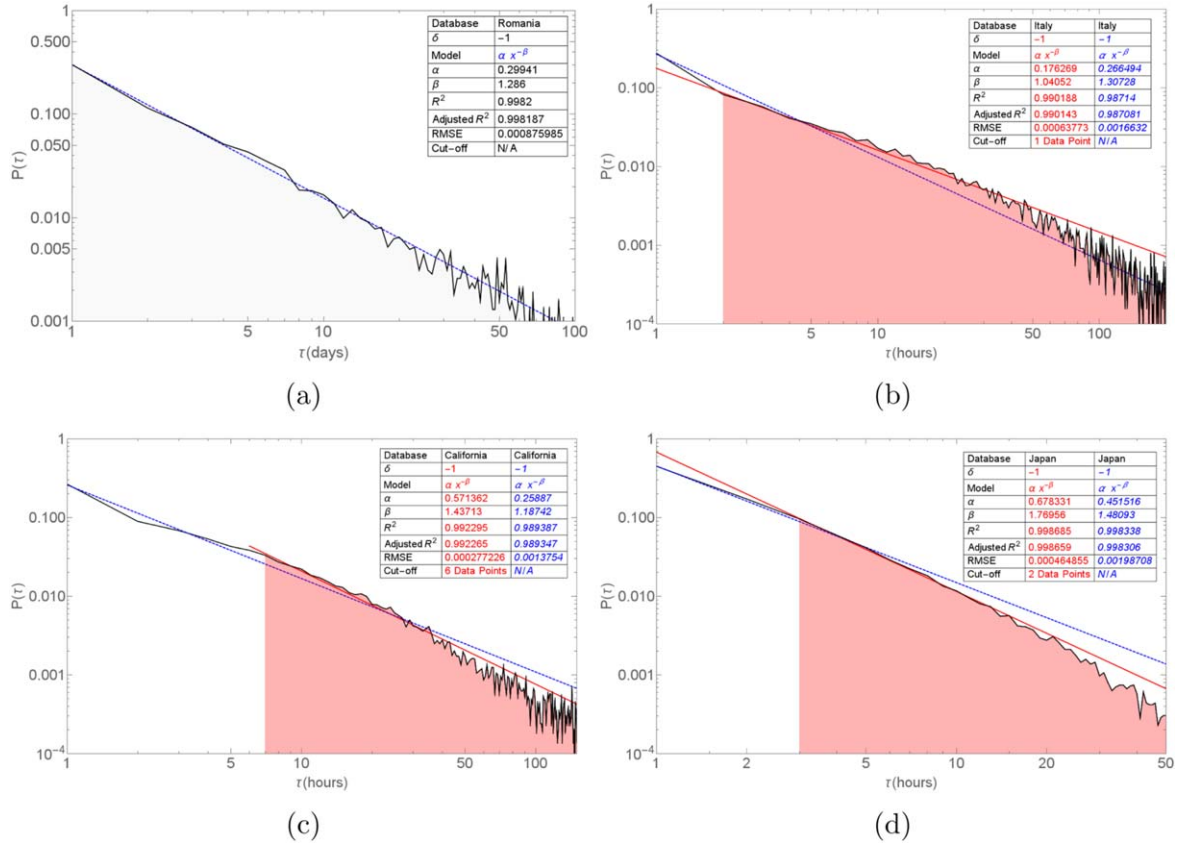


Figure 9. Waiting time distributions for a variation in magnitude of $\delta = -1$. The databases are: (a) Romania, (b) Italy, (c) California (USA), and (d) Japan. The details are as in figure 5. For panel (a) the two fits coincide.

waiting times for depths. For both magnitudes and depths the observed distributions of waiting times are sensitive with respect to the sign of the threshold. These results reinforce the idea that seismic zones can be seen as critical systems, an idea which is further strengthened by the distributions of released daily energies calculated for Romania and California, USA, which have a prominent scale-free nature.

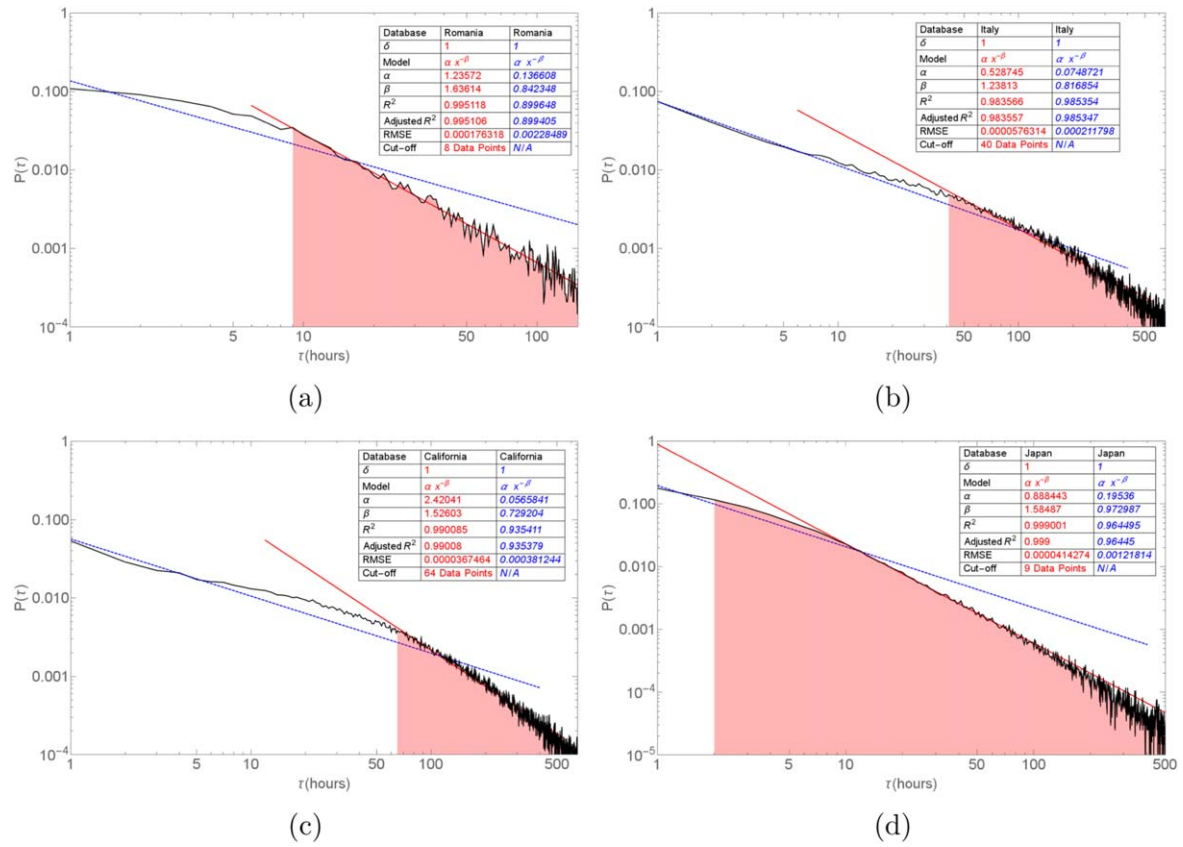


Figure 10. Waiting time distributions for a variation in magnitude of $\delta = +1$. The details are as in figure 5.

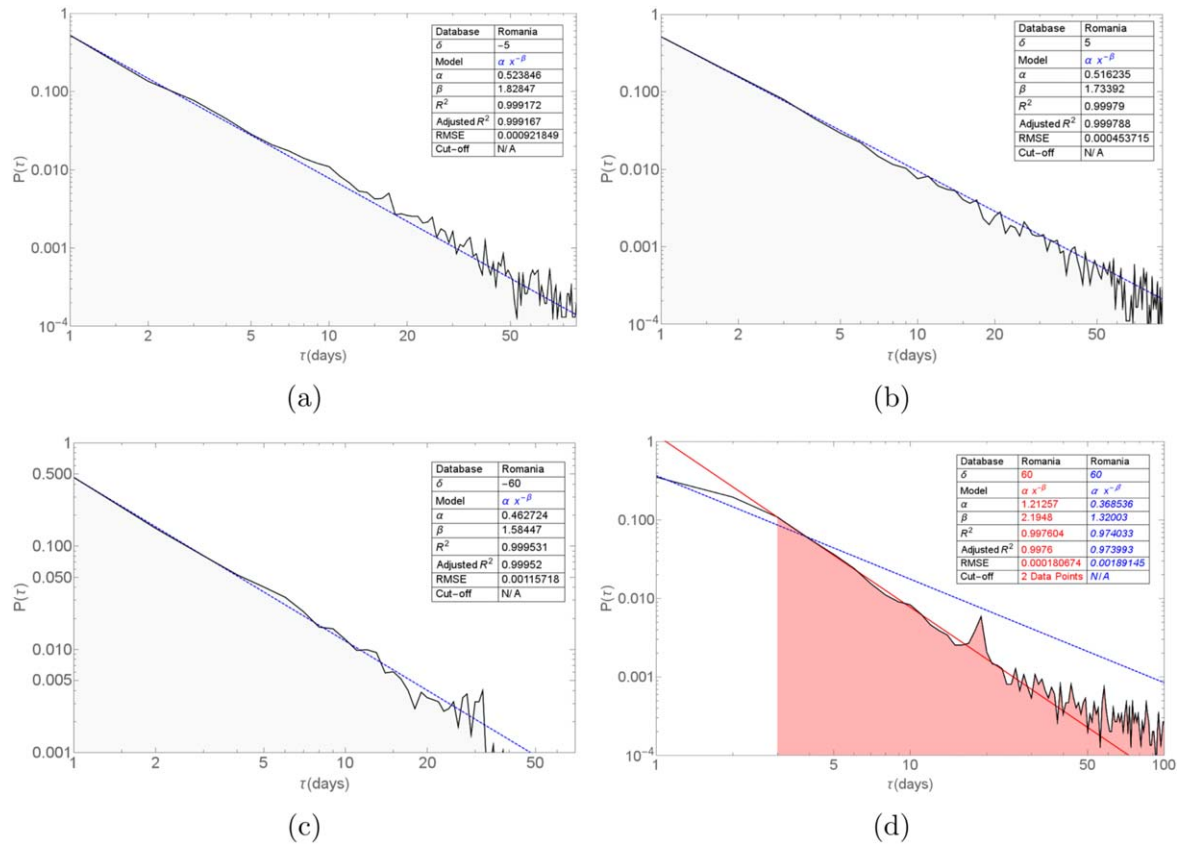


Figure 11. Distributions of waiting times for depths using variations of $\delta = \pm 5$ km (see panels (a) and (b)) and $\delta = \pm 60$ km (see panels (c) and (d)) for the Romanian database. For each distribution of waiting time we have shown two scale-free fits: (i) a global one which accounts for all points in the distribution, and (ii) an optimal fit (technically, one which maximizes the adjusted R^2) which includes only the points in the hatched region. For panel (a), (b), and (c) the two fits coincide.

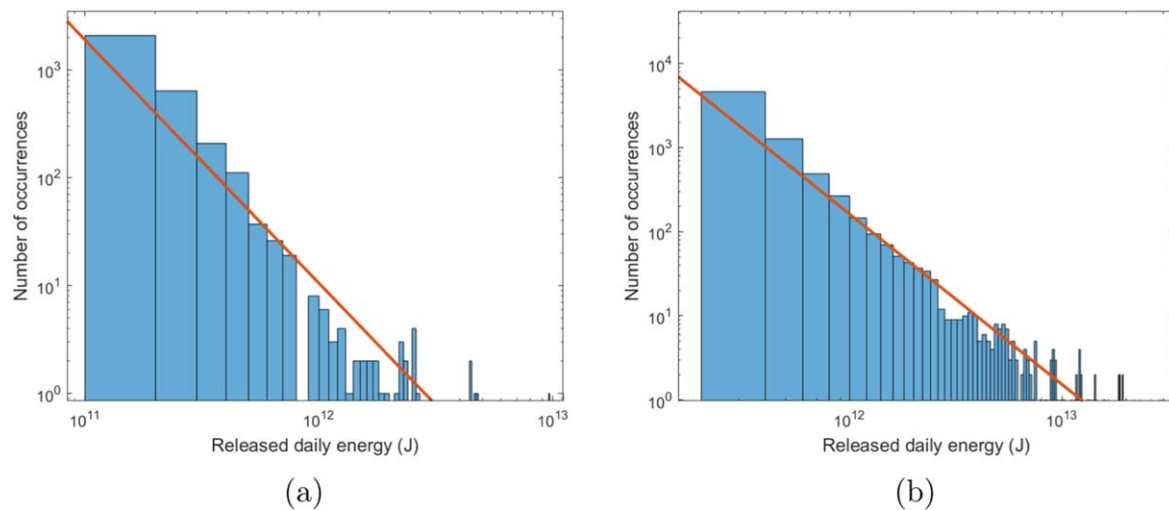


Figure 12. Distributions released daily energies for earthquakes originating in Romania, see panel (a), and California, USA, see panel (b). The scale-free fit for earthquakes originating Romania yields a scaling exponent β of 2.25, while for California, USA, the scaling exponent is 2.02.

References

- [1] Severn R T 2012 Understanding earthquakes: from myth to science *Bull. Earthq. Eng.* **10** 351–66
- [2] Reinhardt O and Oldroyd D R 1983 Kant's theory of earthquakes and volcanic action *Ann. Sci.* **40** 247–72
- [3] Gutenberg B and Richter C F 1944 *Bull. Seismol. Soc. Am.* **34** 185–8
- [4] Schorlemmer D, Wiemer S and Wyss M 2005 Variations in earthquake-size distribution across different stress regimes *Nature* **437** 539–42
- [5] Omori F and College J 1985 *Sci. Imper. Univ. Tokyo* **7** 111–200

- [6] Christensen K, Danon L, Scanlon T and Bak P 2002 Unified scaling law for earthquakes *Proc. Natl Acad. Sci. USA* **99** 2509–13
- [7] Simonsen I, Jensen M H and Johansen A 2002 Optimal investment horizons *Eur. Phys. J. B* **27** 583–6
- [8] Siven J V and Lins J T 2009 Temporal structure and gain-loss asymmetry for real and artificial stock indices *Phys. Rev. E* **80** 057102
- [9] Corral A 2004 Long-term clustering, scaling, and universality in the temporal occurrence of earthquakes *Phys. Rev. Lett.* **92** 108501
- [10] Davidsen J and Goltz C 2004 Are seismic waiting time distributions universal? *Geophys. Res. Lett.* **31** L21612
- [11] Kawamura H, Hatano T, Kato N, Biswas S and Chakrabarti B K 2012 Statistical physics of fracture, friction, and earthquakes *Rev. Mod. Phys.* **84** 839
- [12] Zhou Y, Chechkin A, Sokolov I M and Kantz H 2016 A model of return intervals between earthquake events *Europhys. Lett.* **114** 60003
- [13] ROMPLUS database of earthquakes in Romania is publicly available at <http://infp.ro/romplus>. This database is curated by The National Institute for Earth Physics, Măgurele, Romania
- [14] The database of earthquakes in Japan is freely available at <https://eic.eri.utokyo.ac.jp/db/junec/index.html> and is curated by The Earthquake Research Institute, University of Tokyo, through cooperation with regional centers and observatories
- [15] The database of earthquakes in Italy is freely available at <https://ingv.it>
- [16] The database of earthquakes in California is freely available at <https://scedc.caltech.edu> and is curated by The California Institute of Technology through cooperation with regional centers and observatories
- [17] Baran V, Zus M, Bonasera A and Paturca A 2015 Quantifying the folding mechanism in chaotic dynamics *Rom. J. Phys.* **60** 1263–77
- [18] Mutihac V and Ionesi L 1974 *Geology of Romania* (Bucharest: Technical Press) in Romanian
- [19] Flyvbjerg H 1996 Simplest possible self-organized critical system *Phys. Rev. Lett.* **76** 940–3
- [20] Bak P and Tang C 1989 Earthquakes as a self-organized critical phenomenon *J. Geophys. Res.* **94** 635–7
- [21] Kanamori H and Heaton T H 2000 Microscopic and macroscopic physics of earthquakes *Geocomplexity and the Physics of Earthquakes. Geophysical Monograph. No. 120* (Washington, DC: American Geophysical Union) pp. 147–63
- [22] Olami Z, Feder H J S and Christensen K 1992 Self-organized criticality in a continuous, nonconservative cellular automaton modeling earthquakes *Phys. Rev. Lett.* **68** 1244
- [23] Balogh E, Simonsen I, Nagy B Sz and Neda Z 2010 Persistent collective trend in stock markets *Phys. Rev. E* **82** 066113