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Influence of driving ways on measurement of relative phase in a two-atoms cavity system*

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We study the influence of driving ways on the interaction in a two-atoms cavity quantum electrodynamics system. The results show that driving ways can induce different excitation pathways. We show two kinds of significantly different excitation spectrums under two ways: driving cavity and driving atoms. We demonstrate that driving atoms can be considered as a method to obtain the position information of atoms. This research has very practical application values on obtaining the position information of atoms in a cavity.

Keywords: cavity quantum electrodynamics system, photon

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1. Introduction

Cavity quantum electrodynamics (QED) provides a platform to explore the quantum effects of light–matter interaction.[1–3] In strong-coupling regime, due to the atom–field interaction overwhelms dissipation of the system, cavity QED realizes the coherent exchange of a single photon energy between the atom and cavity. This quantum interaction progress has been observed in cavity QED experiment.[4–7] And many quantum effects also have been realized in experiments, including non-demolition measurement,[8–14] quantum gates,[15–19] quantum entanglement,[20–23] and manipulation of single atom.[24–27] In addition, non-classical light is also shown in a cavity QED system.[28–34] In their experiments, the non-classical effects, such as the anti-bunching phenomenon and sub-poissonian effect, are demonstrated in their results.

Recently, the two-atoms cavity QED system is widely studied in many articles.[35–45] Agarwal *et al.* showed a phenomenon in which the radiation can distinctly exceed the free-space superradiant behavior in a two-atoms cavity QED system by driving atoms. They call it hyperradiance.[39] They also found that the light field can be tuned from antibunched to (super-)bunched as well as nonclassical to classical behavior by merely modifying the atomic position.[44] Zhu *et al.* also proposed a scheme to realize the three-photon blockade. They found that out-of-phase coupling is more efficient to obtain three-photon blockade by driving the atoms.[42] The above works concentrate on the characters of radiation field and they considered only the case of driving atoms. This motivates us to exploring the basic properties in collective system under different driving ways.

In this paper, we consider a system in which two atoms are confined in a single mode cavity. We explore the transmission spectrum of the two-atoms cavity QED system at different radiation phases. Two driving ways, *i.e.*, driving cavity and driving atoms, are considered in our work. Our results demonstrate that the transition paths are significantly influenced by the driving way. And the phase information between two atoms is distinguished only by driving atoms. The results provide a method to measure the position information of atoms and also can guide the experiment to obtain non-classical light.

2. Theoretical method

In our scheme, the atoms–cavity system is described by the following quantum master equations

$$\frac{\partial}{\partial t}\rho = -\mathrm{i}[H,\rho] + \mathcal{L}_{\gamma}\rho + \mathcal{L}_{\kappa}\rho, \tag{1}$$

where ρ is the density operator of the atom–cavity system, H is the entire system Hamiltonian. The dynamics behaviors of the system are clearly demonstrated by this approach. As depicted in Fig. 1, under rotating-wave approximation, the system Hamiltonian can be expressed as $H = H_0 + H_{\mathrm{I}} + H_{\mathrm{d}}$, where $H_0 = \hbar\Delta_A(S_z^1 + S_z^2) + \hbar\Delta_{\mathrm{c}} a^{\dagger}a$ is the Hamiltonian of atoms and cavity field, $H_{\mathrm{I}} = \hbar\sum_{i=1,2} g_i(a^{\dagger}S_-^i + aS_+^i)$ is the interaction Hamiltonian between atoms and cavity, and $H_{\mathrm{d}} = \eta_{\mathrm{a}}\sum_{i=1,2}(S_-^i + S_+^i)$ or $H_{\mathrm{d}} = \eta_{\mathrm{c}}(a^{\dagger} + a)$ represents the interaction Hamiltonian with the coherent pumping field. Here, η_{a} and η_{c} are the strength of driving the atoms and the strength of driving the cavity, respectively.

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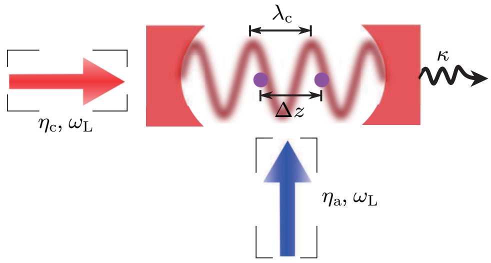

Fig. 1. (a) Sketch of atoms–cavity coupling system. Δ_z is the distance of two identical atoms [purple circles]. The frequency of pump field is $\omega_{\rm L}$. As depicted in sketch, the symbols $\eta_{\rm a}$ and $\eta_{\rm c}$ represent the Rabi frequency of atom driving and cavity driving, respectively. $\lambda_{\rm c}$ is the wavelength of cavity mode and κ is the dissipation of cavity.

We define γ as the spontaneous emisson rate of the atom in excited state, and $\omega_{\rm a}$ is the transition frequency of single two-level atom. In the above Hamiltonian, $\Delta_{\rm a} = \omega_{\rm L} - \omega_{\rm a}$ is the atom–laser detuning and $\Delta_{\rm c} = \omega_{\rm L} - \omega_{\rm c}$ is the cavity–laser detuning. Atom–cavity coupling strength is $g_i = g\cos(2\pi z_i/\lambda_{\rm c})$ associated with the positon of atoms where $\lambda_{\rm c}$ is the wavelength of cavity mode and z_i is the position of atom. In Eq. (1), the Liouvillian operators can be expressed as

$$\mathcal{L}_\kappa \rho = \frac{\kappa}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a), \tag{2}$$

$$\mathcal{L}_\gamma \rho = \frac{\gamma}{2}\sum_{i=1,2}(2S_-^i\rho S_+^i - S_+^i S_-^i\rho - \rho S_+^i S_-^i). \tag{3}$$

For a multiple atom system, the collective basis states may show the physical mechanism and dynamical behaviors clearly. Therefore, we use the collective states $|gg\rangle$, $|\pm\rangle = (|eg\rangle \pm |ge\rangle)/\sqrt{2}$, and $|ee\rangle$, to rewrite the Hamiltonian $H_{\rm I}$,[39,43]

$$\begin{aligned} H_{\rm I} = & \sum_{n=1}^{n}\sqrt{n}(g_1+g_2)|gg,n\rangle\langle +,n-1| \\ & + \sum_{n=1}^{n}\sqrt{n}(g_1-g_2)|gg,n\rangle\langle -,n-1| \\ & + \sum_{n=1}^{n}\sqrt{n-1}(g_1+g_2)|+,n-1\rangle\langle ee,n-2| \\ & - \sum_{n=1}^{n}\sqrt{n-1}(g_1-g_2)|-,n-1\rangle\langle ee,n-2| + \text{H.C.} \end{aligned} \tag{4}$$

3. Results and discussion

We assume the coupling strength $g_1 = G\cos(\phi_1)$ and $g_2 = G\cos(\phi_2)$ where the phases are defined as $\phi_1 = z_1/\lambda_{\rm c}$ and $\phi_2 = z_2/\lambda_{\rm c}$. Firstly, we consider the situations $\phi_2 = \pm\phi_1 + 2n\pi$, which means the coupling strength of two atoms is equal, $g_1 = g_2 = g$. So the relative position information of atoms are measured by the coupling interaction. We also assume $\omega_{\rm a} = \omega_{\rm c}$ which means $\Delta_{\rm a} = \Delta_{\rm c} = \Delta$. Under these conditions, the dressed states pictures of the systems can be easily obtained. The mean photon number $\langle a^\dagger a\rangle$ is obtained by numerically solving the master equation Eq. (1) under steady states.[46,47] As shown in Fig. 2, the physical mechanism of interactions between light and atoms can be clearly demonstrated. From the dressed states structures, the single-photon resonance and two-photon resonance can be found, $\Delta_{\rm 1ph} = \pm\sqrt{2}g$ and $\Delta_{\rm 2ph} = \pm\sqrt{6}/2g$ which agree with the results shown in Fig. 2(c). In Fig. 2(c), the parameter of driving strength $\eta_{\rm a}/\kappa = \eta_{\rm c}/\kappa = 1$ and the two groups of symmetry peaks correspond to single photon resonance and two-photon resonance under weak driving strength. Although the driving ways are hardly distinguished through the resonant peaks, the significant difference can be caused by driving ways at detuning $\Delta/g = 0$.

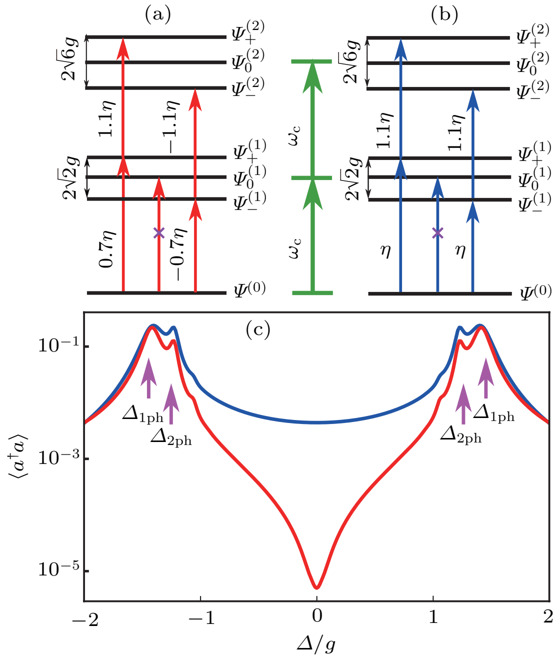

Fig. 2. Energy levels and transitions of the system for $\Delta\phi = \phi_2 \mp \phi_1 = 2n\pi$. Atom–cavity system can either be excited via driving cavity [red, see panel (a)] or driving atoms [blue, see panel (b)]. The transition strength between states is described by the thick colored lines (red and blue). The symbols $\omega_{\rm c}$, η, Δ, and g correspond to cavity frequency, driving strength, detuning, and coupling strength, respectively. The mean photon number $\langle a^\dagger a\rangle$ for driving the cavity (red $\eta_{\rm c}/\kappa = 1.0$) or driving the atoms (blue, $\eta_{\rm a}/\kappa = 1.0$) is shown in panel (c). The normalized parameters, $\gamma/\kappa = 1.0$ and $g_1/\kappa = g_2/\kappa = g/\kappa = 15$ are taken in panel (c).

This phenomenon can be explained easily from the quantum interference and perturbation theory. Under weak driving strength, the photons mainly occupy the single photon state, *i.e.*, $\langle a^\dagger a\rangle \approx P_1$ where P_1 is the population of state $|gg,1\rangle$. The first excited eigen states of $H_{\rm I}$ are expressed as $\Psi_\pm^{(1)} = (|+,0\rangle \pm |gg,1\rangle)/\sqrt{2}$. So the state $|gg,1\rangle$ can be expressed as: $|gg,1\rangle = (\Psi_+^{(1)} - \Psi_-^{(1)})/\sqrt{2}$. The transition probability amplitudes, $|gg,0\rangle \longrightarrow \Psi_+^{(1)}$ and $|gg,0\rangle \longrightarrow \Psi_-^{(1)}$, are defined as C_+ and C_-, respectively. Under the first-order approximation, the transition probability amplitudes of the two driving ways can be given as

$$C_{\rm atom} \propto (C_+ - C_-) = \frac{\eta}{-\Delta_{\rm 1ph}} - \frac{\eta}{\Delta_{\rm 1ph}}, \tag{5}$$

$$C_{\rm cavity} \propto (C_+ - C_-) = \frac{0.7\eta}{-\Delta_{\rm 1ph}} - \frac{-0.7\eta}{\Delta_{\rm 1ph}}. \tag{6}$$

Therefore, the occupation probability of the single photon state by driving the atoms is $P_1 \propto |C_{\text{atom}}|^2 \neq 0$, and the occupation probability by driving the cavity is $P_1 \propto |C_{\text{cavity}}|^2 \approx 0$ which is shown as a dip at $\Delta/g = 0$ in Fig. 2(c) [red line].

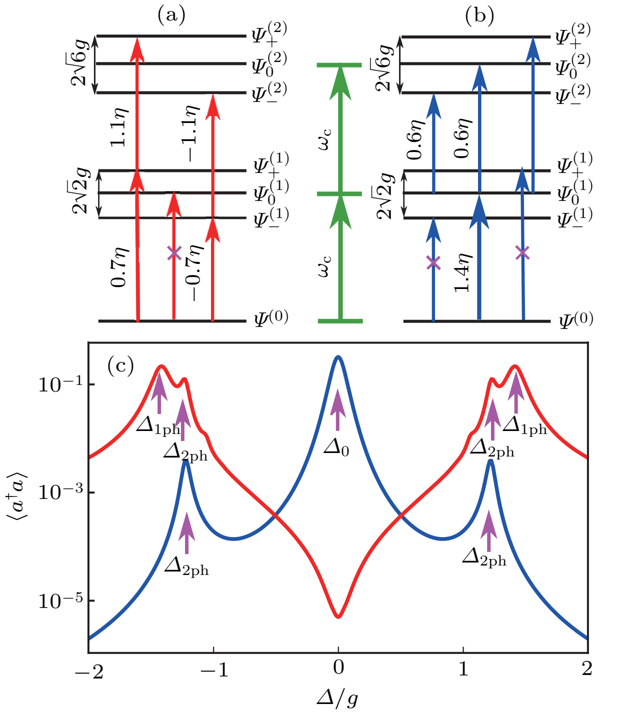

Fig. 3. Energy levels and transitions of the system for $\Delta\phi = \phi_2 \mp \phi_1 = (2n+1)\pi$. Atom–cavity system can either be excited via driving cavity [red, see panel (a)] or driving atoms [blue, see panel (b)]. The transition strength between states is described by the thick colored lines (red and blue). The symbols ω_c, η, Δ, and g correspond to cavtiy frequency, driving strength, detuning, and coupling strength, respectively. The mean photon number $\langle a^\dagger a\rangle$ for driving the cavity (red $\eta_c/\kappa = 1.0$) or driving the atoms (blue, $\eta_a/\kappa = 1.0$) is shown in panel (c). The normalized parameters, $\gamma/\kappa = 1.0$ and $g_1/\kappa = g_2/\kappa = g/\kappa = 15$ are taken in panel (c).

To be different from the above cases, we also pay attention to the case at $\phi_2 = \pm\phi_1 + (2n+1)\pi$. The dressed states structures and some of transition paths are displayed in Figs. 3(a) and 3(b). The mean photon number is also plotted in Fig. 3(c) at $\eta_c/\kappa = \eta_a/\kappa = 1.0$. The single photon resonance peaks at $\Delta_{1\text{ph}} = \pm\sqrt{2}g$ and two photon resonance peaks at $\Delta_{2\text{ph}} = \pm\sqrt{6}/2g$ are shown by red curves. The dip at $\Delta/g = 0$, which is similar to that in Fig. 2(c), can also be interpreted by quantum interference. However, there are three resonance peaks at $\Delta/g = 0$ and $\Delta_{2\text{ph}} = \pm\sqrt{6}/2g$ under atoms driven. There is no single photon resonance peaks. The differences caused by two driving ways are easily demonstrated by the transition between dressed states. The dressed states are shown as $\Psi_\pm^{(1)} = (|-,0\rangle \pm |gg,1\rangle)/\sqrt{2}$ and $\Psi_0^{(1)} = |+,0\rangle$. Therefore, the transition paths are $|gg,0\rangle \xrightarrow{\text{driving the cavity}} |gg,1\rangle$ and $|gg,0\rangle \xrightarrow{\text{driving the atoms}} |+,0\rangle$. The forbidden transition $\Psi^{(0)} \longrightarrow \Psi_\pm^{(1)}$ causes that the peaks are disappeared at $\Delta_{1\text{ph}} = \pm\sqrt{2}g$ and the resonance transition $\Psi^{(0)} \longrightarrow \Psi_0^{(1)} \longrightarrow \Psi_0^{(2)} \longrightarrow \cdots$ and two photon excitation $\Psi_0 \longrightarrow \Psi_0^{(1)} \longrightarrow \Psi_\pm^{(2)}$ result in the peaks at $\Delta/g = 0$ and $\Delta_{2\text{ph}} = \pm\sqrt{6}/2g$, respectively. The above results show that quite different photon blockade phenomena are expected through different driving methods.[43]

By comparing Fig. 2(a) and Fig. 3(a), the excitation spectra are expected to be exactly the same. And the significantly different spectra are also expected under driving the atoms. We plot the mean photon number at different phases under different driving ways in Fig. 4. The phase of the first atom is set as $\phi_1 = 2n\pi$ and the coupling strength of the first atom is defined as $g_1 = G\cos(\phi_1)/\kappa = 15$. As shown in Fig. 4(a), there is no difference between $\phi_2 = \phi_1 + 2n\pi$ and $\phi_2 = \phi_1 + (2n+1)\pi$ under driving the cavity. This means that two phases are degenerate for driving the cavity. We can not distinguish these phases by driving the cavity. However, it shows two different curves due to different phases by driving the atoms in Fig. 4(b). Therefore, it may be a method to measure the phases. We also display the results of $\phi_2 = \phi_1 + (1/3+2n)\pi$ and $\phi_2 = \phi_1 + (2/3+2n)\pi$ under driving the cavity [Fig. 4(c)] and driving the atoms [Fig. 4(d)]. The results indicate that driving the atoms is a scheme to obtain the information of atoms position which can not be realize by driving the cavity.

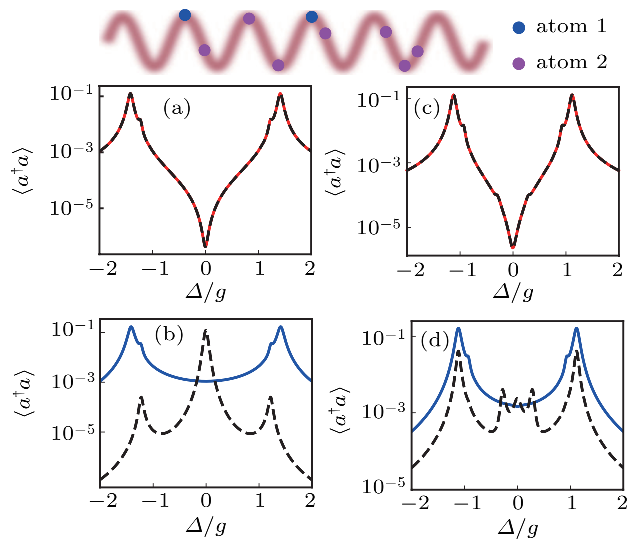

Fig. 4. The mean photon number $\langle a^\dagger a\rangle$ under different driving ways. Panels (a) and (c) correspond to driving the cavity, $\langle a^\dagger a\rangle$ of driving the atoms is shown in panels (b) and (d). The solid lines and dashed lines in both panels (a) and (b) are $\Delta\phi = 2n\pi$ and $\Delta\phi = (2n+1)\pi$, respectively. The solid lines and the dashed lines in panels (b) and (d) correspond to $\Delta\phi = (1/3+2n)\pi$ and $\Delta\phi = (2/3+2n)\pi$, respectively. The normalized parameters, $\eta_c/\kappa = 0.5$, $\eta_a/\kappa = 0.5$, $\gamma/\kappa = 1.0$, and $G/\kappa = g_1/\kappa = 15$ are taken. The diagrams of relative atom's position in cavity are shown in the top of the figure.

In Fig. 5, the mean photon number $\langle a^\dagger a\rangle$ is shown as a function of ϕ_2 and the detuning. There always are two phases which have the same spectrum under driving the cavity in Figs. 5(a) and 5(b). But we can distinguish ϕ_2 by driving the atoms. Therefore driving the atoms is a simple method to measure the phase of the atoms. Firstly, we can have a preliminary analysis by the resonant peaks at single photon detuning and two-photon detuning under weak driving strength. Secondly, the relative position information can further be concluded by the peaks at detuning $\Delta/g = 0$.

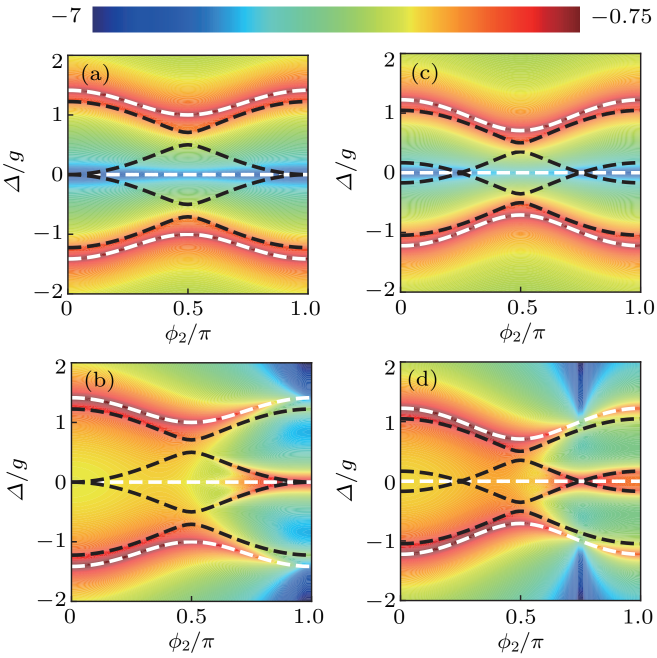

Fig. 5. The logarithm of $\langle a^{\dagger}a\rangle$ to the base 10 as a function of $\phi_2 + 2n\pi$ and the detuning. Panels (a) and (b) correspond to driving the cavity ($\eta_a/\kappa = 0.5$) and driving the atoms ($\eta_c/\kappa = 0.5$) correspond to panels (c) and (d). Panels (a) and (c) correspond to phase $\phi_1 = 2n\pi$. Panels (b) and (d) correspond to phase $\phi_1 = (2n+1/4)\pi$. The white dashed line and black dashed lines correspond to eigen energy values in single photon space and in two-photon space, respectively. The other parameters are the same as those in Fig. 2.

4. Conclusion

In summary, we have shown the interaction of a two-atoms collective system by different driving ways. We analyzed the physical transition paths in dressed states presentation. We investigated the differences of the excitation spectrum by driving the cavity and driving the atoms. The results demonstrate that driving the atoms can be used as a measurement method to obtain the relative position information of the two atoms. This measurement only needs to analyze transmission spectrum. This study has very practical application values on achieving the measurement of atom position and provides a train of thought to study the interaction progress in a cavity QED system.

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