Finite-time Mittag–Leffler synchronization of fractional-order delayed memristive neural networks with parameters uncertainty and discontinuous activation functions*

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The finite-time Mittag–Leffler synchronization is investigated for fractional-order delayed memristive neural networks (FDMNN) with parameters uncertainty and discontinuous activation functions. The relevant results are obtained under the framework of Filippov for such systems. Firstly, the novel feedback controller, which includes the discontinuous functions and time delays, is proposed to investigate such systems. Secondly, the conditions on finite-time Mittag–Leffler synchronization of FDMNN are established according to the properties of fractional-order calculus and inequality analysis technique. At the same time, the upper bound of the settling time for Mittag–Leffler synchronization is accurately estimated. In addition, by selecting the appropriate parameters of the designed controller and utilizing the comparison theorem for fractional-order systems, the global asymptotic synchronization is achieved as a corollary. Finally, a numerical example is given to indicate the correctness of the obtained conclusions.

Keywords: fractional-order delayed memristive neural networks (FDMNN), parameters uncertainty, discontinuous activation functions, finite-time Mittag–Leffler synchronization

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1. Introduction

Over the past decade, the integer-order memristive neural networks (IMNN) have been developed in an unprecedented way and widely used in various fields, such as signal and image processing,[1,2] algorithm optimization,[3] classification and automatic control,[4] and so on. At the same time, the relevant dynamic behaviors have also attracted the attention of many scholars.[5–8] As a generalization of integer-order calculus (IC), fractional-order calculus (FC) can be dated back to the 17th century. Compared with IC operators, FC operators not only have hereditary and memory characteristics, but also can increase the degree of freedom to improve the performance of the system. So far, FC has been generally applied in neural networks,[9,10] recognition systems,[11,12] communication systems,[13] viscoelasticity of the material,[14] and so on. What is more important, it is necessary to introduce FC operators into memrisitve neural networks to construct a novel fractional-order memristive neural networks (FMNN), which more accurately describe the dynamic performance of the networks. Some interesting results about FMNN have been investigated, such as Refs. [15–17].

Undeniably, time delays are unavoidable in electronic and electric circuits due to finite switching speed of the amplifiers in electronic components. Moreover, time delays are one of the important reasons producing instability or oscillation of the systems.[18] Taking such facts into account, time delays should be considered in the FMNN. In the published papers,[17,19,20] time delays have been studied as the main considering object. In fact, neurons may have different communication delays, therefore the study on multiple time delays FMNN has more profound theoretical meanings and applications.[21] It is well known that the system parameters may fluctuate within a certain range due to inaccuracy of the model, environmental noise, external disturbances, and other factors. Meanwhile, parameters uncertainty can produce poor dynamic performance for the systems, such as instability, oscillation, chaos, large steady-state error, and so on.

In addition, to the best of our knowledge, the activation functions of many literature[17,22–24] were assumed to be Lipschitz, continuous or continuously differentiable. However, the activation functions of FMNN are usually discontinuous. The main reason is that signal output of neuron and information transmission are discontinuous in actual models.[25,26] In Ref. [25], Forti and Nistri pointed out that the model with discontinuous activation functions can highlight some crucial dynamical behaviors, such as the phenomenon of convergence in finite-time toward the equilibrium point, the presence of sliding modes along discontinuity surfaces, and so on. Considering the fact that various influencing factors could appear when FMNN are applied to the engineering fields such as classification and pattern recognition, it is desirable to explore the dynamical performance about fractional-order delayed mem-

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ristive neural networks (FDMNN) with parameters uncertainty and discontinuous activation functions.

Synchronization can be regarded as a typical collective behavior, which refers to the coordination of events in a system, and the phenomenon of consistency and unification in time. At noted, synchronization not only can be found in many physical systems, such as power converters and biological systems,[27] but also has been applied to a wide variety of engineering applications like image encryption and information processing.[28] In order to achieve the synchronization, most of the results are obtained to ensure the asymptotic stability of error systems.[29–31] However, asymptotic synchronization denotes that it takes infinite time from the trajectories of the response to the trajectory of the drive system.[22,23,32,33] In fact, it is more desirable for networks to reach synchronization in a finite-time and achieve optimization in convergence time in physical and engineering.[34–36] Hence, it is necessary to investigate the finite-time synchronization of FMNN.

Up to now, the finite-time synchronization of FMNN has been studied in previous literature.[37–41] In Ref. [37], by designing a simple linear feedback controller, the finite-time synchronization of FMNN was derived according to Gronwall–Bellmaan inequality. The finite-time synchronization of FDMNN was achieved by utilizing Lyapunov theory, norm properties, and linear feedback controller in Ref. [38]. In Ref. [39], the authors dealt with the finite-time synchronization for a class of FMNN by considering discontinuous activation functions, employing the Young inequality, and applying the fractional-order Lyapunov stability theory. Some sufficient criteria were obtained to ensure the finite-time projective synchronization of FDMNN by utilizing the linear feedback controller and employing Gronwall–Bellman integral inequality and Volterra-integral equation in Ref. [40]. By using Laplace transform, the generalized Gronwalls inequality and linear feedback control technique, the finite-time Mittag–Leffler synchronization of FMNN was achieved in Ref. [41]. However, it is noteworthy that there are few results on the finite-time Mittag–Leffler synchronizations for a class of FDMNN with parameters uncertainty and discontinuous activation functions.

Inspired by the aforementioned discussions, the finite-time Mittag–Leffler synchronization is investigated for fractional-order delayed memristive neural networks with parameters uncertainty and discontinuous activation functions in this paper. Based on non-smooth analysis theory and the properties of fractional-order Lyapunov functions, the synchronization conditions are put forward under the framework of Filippov. The crucial contributions of this paper are at the following aspects:

(i) Compared with previous results, our model considers the parameters uncertainty and discontinuous activation functions. So, our system is more general.

(ii) By designing a new type of discontinuous feedback controller, some sufficient criteria for the synchronization in finite-time are obtained. Meanwhile, the upper bound of the setting time is explicitly evaluated.

(iii) By simplifying the designed controller, the asymptotic synchronization of FDMNN with parameters uncertainties and discontinuous activation functions is realized as a corollary.

(iv) In this paper, the assumptions about activation functions are more general. Moreover, our results extend the existing results in Refs. [19,32,39].

The organization of this paper is summarized as follows. In Section 2, the system models and some preliminaries are introduced. In Section 3, some sufficient criteria of the finite-time Mittag–Leffler synchronization are established by using the theory of fractional-order differential equations with discontinuous right-hand sides. Subsequently, numerical simulations are given to describe the effectiveness of the obtained conclusions in Section 4. Finally, conclusions are drawn in Section 5.

2. Preliminaries and system description

2.1. Caputo fractional-order derivative

Definition 1[42] The fractional-order integral of order α for an integrable function $g(t):[0,+\infty)\to\mathbb{R}$ is denoted by

$$ {}_0I_t^{\alpha}g(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(t-\zeta)^{\alpha-1}g(\zeta)\,\mathrm{d}\zeta, $$

where $\alpha>0$, and $\Gamma(\cdot)$ is the Gamma function which is defined by

$$ \Gamma(x)=\int_0^{\infty}\mathrm{e}^{-t}t^{x-1}\,\mathrm{d}t,\ \big(\mathrm{Re}(x)>0\big), $$

where $\mathrm{Re}(x)$ denotes the real part of x.

Definition 2[42] The Caputo fractional-order derivative of order α for a function $g(t)\in C^n\big([0,+\infty),\mathbb{R}\big)$ is denoted by

$$ {}_0^CD_t^{\alpha}g(t)=\frac{1}{\Gamma(n-\alpha)}\int_0^t\frac{g^{(n)}(\zeta)}{(t-\zeta)^{\alpha-n+1}}\,\mathrm{d}\zeta, $$

where $t\geq 0$ and $n-1<\alpha<n$, $n\in\mathbb{N}_+$. Specifically, when $0<\alpha<1$,

$$ {}_0^CD_t^{\alpha}g(t)=\frac{1}{\Gamma(1-\alpha)}\int_0^t\frac{g'(\zeta)}{(t-\zeta)^{\alpha}}\,\mathrm{d}\zeta. $$

The relevant properties of Caputo fractional-order derivative are as follows.

Property 1[42] For $n-1<\alpha<n$, $n\in\mathbb{N}_+$, if $g(t)\in C^n([0,+\infty),\mathbb{R})$, then

$$ {}_0I_t^{\alpha}{}_0^CD_t^{\alpha}g(t)=g(t)-\sum_{k=0}^{n-1}\frac{g^{(k)}(0)}{k!}t^k, $$

particularly, $0<\alpha<1$, one have

$$ {}_0I_t^{\alpha}{}_0^CD_t^{\alpha}g(t)=g(t)-g(0). $$

Property 2[42] For any constants w_1 and w_2, if $g(t)$, $f(t) \in C^n([0,+\infty),\mathbb{R})$, and $\alpha > 0$, the linearity of Caputo fractional-order derivative can be written as

$$ {}_0^C D_t^\alpha (w_1 g(t) + w_2 f(t)) = w_1 {}_0^C D_t^\alpha g(t) + w_2 {}_0^C D_t^\alpha f(t). $$

Property 3[42] For $g(t) \in C^n([0,+\infty),\mathbb{R})$, the Laplace transform of the Caputo fractional-order derivative is

$$ \mathscr{L}\{{}_0^C D_t^\alpha g(t); s\} = s^\alpha G(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} g^{(k)}(0), $$

where $n-1 < \alpha < n$, $n \in \mathbb{N}_+$, $G(s) = \mathscr{L}\{g(t)\}$, and s is the variable in Laplace domain. In particular, when $0 < \alpha < 1$, then

$$ \mathscr{L}\{{}_0^C D_t^\alpha g(t); s\} = s^\alpha G(s) - s^{\alpha-1} g(0). $$

Lemma 1[43] If $g(t) \in C^1([0,+\infty),\mathbb{R})$, the following inequality holds almost everywhere

$$ {}_0^C D_t^\alpha |g(t)| \le \operatorname{sign}(g(t)) {}_0^C D_t^\alpha g(t), \quad 0 < \alpha < 1. $$

Lemma 2[44] (comparison principle) Considering the following linear fractional-order differential inequality with time delays

$$ \begin{cases} {}_0^C D_t^\alpha f(t) \le -\eta f(t) + \sum_{j=1}^{n} l_j f(t-\tau_j), & 0 < \alpha \le 1, \\ f(t) = \vartheta(t), & t \in [-\tau, 0], \end{cases} \tag{1} $$

where $f(t)$ is continuous and nonnegative in $(0,+\infty)$, and $\vartheta(t) > 0$, $t \in [-\tau, 0]$, $\tau = \sup_{1\le j\le n}\{\tau_j\}$, $\tau_j \ge 0$ is the transmission delay. And the linear fractional-order systems with time delays

$$ \begin{cases} {}_0^C D_t^\alpha g(t) = -\eta g(t) + \sum_{j=1}^{n} l_j g(t-\tau_j), & 0 < \alpha \le 1, \\ g(t) = \vartheta(t), & t \in [-\tau, 0], \end{cases} \tag{2} $$

where $g(t)$ is continuous and nonegative in $(0,+\infty)$. If $\eta > 0$ and $l_j > 0$, then

$$ f(t) \le g(t), \quad \forall t \in (0,+\infty). $$

2.2. System description

In this subsection, a class of FDMNN with the parameters uncertainty and discontinuous activation functions as the drive system is defined by

$$ \begin{aligned} {}_0^C D_t^\alpha x_i(t) \ =\ & -(d_i + \Delta d_i(t)) x_i(t) + \sum_{j=1}^{n} (a_{ij}(x_j(t)) \\ & + \Delta a_{ij}(t)) f_j(x_j(t)) + \sum_{j=1}^{n} (b_{ij}(x_j(t-\tau_j)) \\ & + \Delta b_{ij}(t)) g_j(x_j(t-\tau_j)) + I_i, \end{aligned} \tag{3} $$

where $\alpha \in (0,1)$, $t \ge 0$ and $i, j = 1,2,\ldots,n (i,j \in \mathbb{N}_+)$; $x(t) = (x_1(t), x_2(t), \ldots,\ x_n(t))^{\mathrm{T}} \in \mathbb{R}^n$ denotes the vector of neuron states; $d_i > 0$ is the self-regulation parameters neuron; $f_j(\cdot)$ and $g_j(\cdot)$ denote the discontinuous activation functions; $\tau_j \ge 0$ is the transmission delay; I_i denotes the external input; $x_i(s) = \phi_i(s) \in C([-\tau,0],\mathbb{R})$ is the initial condition of system (3), where $\tau = \sup_{1\le j\le n}\{\tau_j\}$; $\Delta d_i(t)$, $\Delta a_{ij}(t)$, and $\Delta b_{ij}(t)$ are the parameter uncertainties and bounded, defined by

$$ \begin{aligned} &|\Delta d_i(t)| \le \omega_i, \\ &|\Delta a_{ij}(t)| \le \varpi_{ij}, \\ &|\Delta b_{ij}(t)| \le \rho_{ij}, \end{aligned} $$

where $\omega_i \ge 0$, $\varpi_{ij} \ge 0$, and $\rho_{ij} \ge 0$. Meanwhile, $d_i > \omega_i$. $a_{ij}(x_j(t))$ and $b_{ij}(x_j(t-\tau_j))$ are the connection memristive weights, defined as

$$ a_{ij}(x_j(t)) = \begin{cases} \hat{a}_{ij}, & |x_j(t)| > T_j, \\ \check{a}_{ij}, & |x_j(t)| < T_j, \end{cases} $$

$$ b_{ij}(x_j(t-\tau_j)) = \begin{cases} \hat{b}_{ij}, & |x_j(t-\tau_j)| > T_j, \\ \check{b}_{ij}, & |x_j(t-\tau_j)| < T_j, \end{cases} $$

$a_{ij}(\pm T_j) = \hat{a}_{ij}$ or $\check{a}_{ij}$ and $b_{ij}(\pm T_j) = \hat{b}_{ij}$ or $\check{b}_{ij}$ for $i,j = 1,2,\ldots,n$, where $T_j > 0$ is switching jump of memristion; $\hat{a}_{ij}$, $\check{a}_{ij}$, $\hat{b}_{ij}$, and $\check{b}_{ij}$ are any constants. The activation functions $f_i(\cdot)$ and $g_i(\cdot)$ satisfy the following assumption.

Assumption H1 For $i \in \mathbb{N}_+$, the activation functions $f_i(t)$ (and $g_i(t)$) have at most a finite number of jump discontinuities ρ_k (and ς_k). Moreover, there are finite right and left limits, $f_i(\rho_k^+)$ (and $g_i(\varsigma_k^+)$) and $f_i(\rho_k^-)$ (and $g_i(\varsigma_k^-)$) respectively.

Similarly, the response system is defined as

$$ \begin{aligned} {}_0^C D_t^\alpha y_i(t) \ =\ & -(d_i + \Delta d_i(t)) y_i(t) + \sum_{j=1}^{n} (a_{ij}(y_j(t)) \\ & + \Delta a_{ij}(t)) f_j(y_j(t)) + \sum_{j=1}^{n} (b_{ij}(y_j(t-\tau_j)) \\ & + \Delta b_{ij}(t)) g_j(y_j(t-\tau_j)) + I_i + u_i(t), \end{aligned} \tag{4} $$

where $i \in \mathbb{N}_+$ and $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^{\mathrm{T}}$; $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^{\mathrm{T}}$ denotes the control input; $y_i(s) = \varphi_i(s) \in C([-\tau,0],\mathbb{R})$ is the initial condition of system (4); $a_{ij}(y_j(t))$ and $b_{ij}(y_j(t-\tau_j))$ are denoted by

$$ a_{ij}(y_j(t)) = \begin{cases} \hat{a}_{ij}, & |y_j(t)| > T_j, \\ \check{a}_{ij}, & |y_j(t)| < T_j, \end{cases} $$

$$ b_{ij}(y_j(t-\tau_j)) = \begin{cases} \hat{b}_{ij} & |y_j(t-\tau_j)| > T_j, \\ \check{b}_{ij} & |y_j(t-\tau_j)| < T_j. \end{cases} $$

$a_{ij}(\pm T_j) = \hat{a}_{ij}$ or $\check{a}_{ij}$ and $b_{ij}(\pm T_j) = \hat{b}_{ij}$ or $\check{b}_{ij}$ for $i,j = 1,2,\cdots,n$, where switching jumps $T_j > 0$, weights $\hat{a}_{ij}$, $\check{a}_{ij}$, $\hat{b}_{ij}$, and $\check{b}_{ij}$ are any constants.

The drive system (3) and response system (4) are discontinuous due to the existence of discontinuous activation functions and memristors. Hence, we introduce the concept of Filippov solution to analyze the above systems.

Definition 3[45] If there exists a neighborhood N of x_0 such that $F(N) \subseteq M$ for any open set M containing $F(x_0)$, a set-valued map F with nonempty values is said to be upper-semi-continuous at $x_0 \in E \subseteq \mathbb{R}^n$. $F(x)$ is said to have a closed (convex, compact) image if for each $x \in E$, $F(x)$ is closed (convex, compact).

Let the set-valued maps be as follows:

$$K[a_{ij}(x_j(t))] = \begin{cases} \hat{a}_{ij}, & |x_j(t)| > T_j, \\ co\{\hat{a}_{ij},\ \check{a}_{ij}\}, & |x_j(t)| = T_j, \\ \check{a}_{ij}, & |x_j(t)| < T_j, \end{cases}$$

$$K[b_{ij}(x_j(t-\tau_j))] = \begin{cases} \hat{b}_{ij}, & |x_j(t-\tau_j)| > T_j, \\ co\{\hat{b}_{ij}, \check{b}_{ij}\}, & |x_j(t-\tau_j)| = T_j, \\ \check{b}_{ij}, & |x_j(t-\tau_j)| < T_j, \end{cases}$$

where $i, j \in \mathbb{N}_+$, and co denotes the convex closure of a set. Clearly, $K[a_{ij}(x_j(t))]$ and $K[b_{ij}(x_j(t-\tau_j))]$ are all closed, convex, and compact in $x_j(t)$ and $x_j(t-\tau_j)$ respectively.

From Assumption H1, $f_i(\cdot)$ and $g_i(\cdot)$ possess only isolated jump discontinuities. Hence, for all $x \in \mathbb{R}^n$,

$$K[f(x)] = (K[f_1(x_1)], K[f_2(x_2)], \ldots, K[f_n(x_n)])^{\mathrm{T}},$$
$$K[g(x)] = (K[g_1(x_1)], K[g_2(x_2)], \ldots, K[g_n(x_n)])^{\mathrm{T}},$$

where $K[f_i(x_i)] = co\{\min\{f_i(x_i^+), f_i(x_i^-)\}, \max\{f_i(x_i^+), f_i(x_i^-)\}\}$ and $K[g_i(x_i)] = co\{\min\{g_i(x_i^+), g_i(x_i^-)\}, \max\{g_i(x_i^+), g_i(x_i^-)\}\}$.

Definition 4[45,46] If the following conditions are valid (i) $x_i(t)$ is continuous on $[-\tau, T)$ and absolutely continuous on $[0, T)$. (ii) $x_i(t)$ satisfies

$$\begin{aligned} {}_0^C D_t^{\alpha} x_i(t) \in & -(d_i + \Delta d_i(t)) x_i(t) + \sum_{j=1}^{n} (K[a_{ij}(x_j(t))] \\ & + \Delta a_{ij}(t)) K[f_j(x_j(t))] + \sum_{j=1}^{n} (K[b_{ij}(x_j(t-\tau_j))] \\ & + \Delta b_{ij}(t)) K[g_j(x_j(t-\tau_j))] + I_i, \end{aligned} \tag{5}$$

then, a function $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^{\mathrm{T}}$: $[-\tau, T] \hookrightarrow \mathbb{R}^n$, $T \in (0, +\infty)$ is a solution of the drive system (3) on $[-\tau, T]$, $\tau = \sup\limits_{1 \le j \le n} \{\tau_j\}$, $\tau_j \ge 0$ is the transmission delay.

Based on the measurable selection theorem,[46] if $x_i(t)$ and $y_i(t)$ are a solution of the systems (3) and (4) respectively, then there exist the measurable functions $\varepsilon_j(t) \in K[f_j(x_j(t))]$, $\varepsilon_j(t) \in K[f_j(y_j(t))]$, $\varepsilon'_j(t) \in K[g_j(x_j(t-\tau_j))]$, $\varepsilon'_j(t) \in K[g_j(y_j(t-\tau_j))]$, $\mathring{a}'_{ij}(t) \in K[a_{ij}(x_j(t))]$, $\mathring{a}_{ij}(t) \in K[a_{ij}(y_j(t))]$, $\mathring{b}'_{ij}(t) \in K[b_{ij}(x_j(t-\tau_j))]$, and $\mathring{b}_{ij}(t) \in K[b_{ij}(y_j(t-\tau_j))]$, such that

$$\begin{aligned} {}_0^C D_t^{\alpha} x_i(t) = & -(d_i + \Delta d_i(t)) x_i(t) + \sum_{j=1}^{n} (\mathring{a}'_{ij}(t) + \Delta a_{ij}(t)) \varepsilon_j(t) \\ & + \sum_{j=1}^{n} (\mathring{b}'_{ij}(t) + \Delta b_{ij}(t)) \varepsilon'_j(t) + I_i, \end{aligned} \tag{6}$$

and

$$\begin{aligned} {}_0^C D_t^{\alpha} y_i(t) = & -(d_i + \Delta d_i(t)) y_i(t) + \sum_{j=1}^{n} (\mathring{a}_{ij}(t) + \Delta a_{ij}(t)) \varepsilon_j(t) \\ & + \sum_{j=1}^{n} (\mathring{b}_{ij}(t) + \Delta b_{ij}(t)) \varepsilon'_j(t) + I_i + u_i(t). \end{aligned} \tag{7}$$

In order to obtain our results, it is necessary to give the following assumption for the discontinuous activation functions.

Assumption H2 For any $i = 1, 2, \ldots, n$, suppose that there exist constants $F_i \ge 0$, $G_i \ge 0$, $L_i \ge 0$, and $M_i \ge 0$, such that

$$|\varepsilon_i(t) - \varepsilon_i(t)| \le F_i |y_i(t) - x_i(t)| + L_i,$$
$$|\varepsilon'_i(t) - \varepsilon'_i(t)| \le G_i |y_i(t-\tau_i) - x_i(t-\tau_i)| + M_i,$$

where $\varepsilon_i(t) \in K[f_i(x_i(t))]$, $\varepsilon_i(t) \in K[f_i(y_i(t))]$, $\varepsilon'_i(t) \in K[g_i(x_i(t-\tau_i))]$, and $\varepsilon'_i(t) \in K[g_i(y_i(t-\tau_i))]$.

Remark 1 The relevant dynamic behaviors of fractional-order memristive neural networks have been investigated in Refs. [22–24]. However, the activation functions are Lipschitz-continuous and satisfy $f_j(\pm T_j) = g_j(\pm T_j) = 0$. In our paper, the activation functions are discontinuous and $f_j(\pm T_j) = g_j(\pm T_j) = 0$ are removed. Hence, our assumptions about activation functions are more general and reasonable in practical application.

Lemma 3 Under the conditions that Assumptions H1 and H2 hold, then

$$\begin{aligned} & |\mathring{a}_{ij}(t)\varepsilon_j(t) - \mathring{a}'_{ij}(t)\varepsilon_j(t)| \\ \le & \bar{a}_{ij} F_j |y_j(t) - x_j(t)| + \bar{a}_{ij} L_j \\ & + |\hat{a}_{ij} - \check{a}_{ij}| (F_j T_j + L_j + \varepsilon_j^*), \end{aligned} \tag{8}$$

$$\begin{aligned} & |\mathring{b}_{ij}(t)\varepsilon'_j(t) - \mathring{b}'_{ij}(t)\varepsilon'_j(t)| \\ \le & \bar{b}_{ij} G_j |y_j(t-\tau_j) - x_j(t-\tau_j)| + \bar{b}_{ij} M_j \\ & + |\hat{b}_{ij} - \check{b}_{ij}| (G_j T_j + M_j + \varepsilon_j^*), \end{aligned} \tag{9}$$

where $i, j = 1, 2, \ldots, n$, $\bar{a}_{ij} = \max\{|\hat{a}_{ij}|, |\check{a}_{ij}|\}$, $\bar{b}_{ij} = \max\{|\hat{b}_{ij}|, |\check{b}_{ij}|\}$, $\varepsilon_j^* = \max\{|f_j(0^-)|, |f_j(0^+)|\}$, and $\varepsilon_j^* = \max\{|g_j(0^-)|, |g_j(0^+)|\}$.

Proof In order to proof Lemma 3, there are four cases to be considered.

Case 1 $|x_j(t)| < T_j$ and $|y_j(t)| < T_j$, one has

$$\begin{aligned} & |\mathring{a}_{ij}(t)\varepsilon_j(t) - \mathring{a}'_{ij}(t)\varepsilon_j(t)| \\ = & |\check{a}_{ij}\varepsilon_j(t) - \check{a}_{ij}\varepsilon_j(t)| \\ \le & |\check{a}_{ij}| F_j |y_j(t) - x_j(t)| + |\check{a}_{ij}| L_j \\ \le & \bar{a}_{ij} F_j |y_j(t) - x_j(t)| + \bar{a}_{ij} L_j \\ & + |\hat{a}_{ij} - \check{a}_{ij}| (F_j T_j + L_j + \varepsilon_j^*); \end{aligned} \tag{10}$$

Case 2 $|x_j(t)| > T_j$ and $|y_j(t)| > T_j$, then

$$\begin{aligned} & |\mathring{a}_{ij}(t)\varepsilon_j(t) - \mathring{a}'_{ij}(t)\varepsilon_j(t)| \\ = & |\hat{a}_{ij}\varepsilon_j(t) - \hat{a}_{ij}\varepsilon_j(t)| \end{aligned}$$

$$\begin{aligned}&\le |\hat{a}_{ij}|F_j|y_j(t)-x_j(t)|+|\hat{a}_{ij}|L_j\\&\le \bar{a}_{ij}F_j|y_j(t)-x_j(t)|+\bar{a}_{ij}L_j\\&\quad+|\hat{a}_{ij}-\check{a}_{ij}|(F_jT_j+L_j+\varepsilon_j^*);\end{aligned}\tag{11}$$

Case 3 $|x_j(t)|<T_j$ and $|y_j(t)|>T_j$, one has

$$\begin{aligned}&|\mathring{a}_{ij}(t)\varepsilon_j(t)-\mathring{a}'_{ij}(t)\varepsilon_j(t)|\\=&|\hat{a}_{ij}\varepsilon_j(t)-\check{a}_{ij}\varepsilon_j(t)|\\\le&|\hat{a}_{ij}|F_j|y_j(t)-x_j(t)|+|\hat{a}_{ij}|L_j\\&+|\hat{a}_{ij}-\check{a}_{ij}|F_j|x_j(t)|+|\hat{a}_{ij}-\check{a}_{ij}|(L_j+\varepsilon_j^*)\\\le&\bar{a}_{ij}F_j|y_j(t)-x_j(t)|+\bar{a}_{ij}L_j\\&+|\hat{a}_{ij}-\check{a}_{ij}|(F_jT_j+L_j+\varepsilon_j^*);\end{aligned}\tag{12}$$

Case 4 $|x_j(t)|>T_j$ and $|y_j(t)|<T_j$, then

$$\begin{aligned}&|\mathring{a}_{ij}(t)\varepsilon_j(t)-\mathring{a}'_{ij}(t)\varepsilon_j(t)|\\=&|\check{a}_{ij}\varepsilon_j(t)-\hat{a}_{ij}\varepsilon_j(t)|\\\le&|\check{a}_{ij}|F_j|y_j(t)-x_j(t)|+|\hat{a}_{ij}|L_j\\&+|\hat{a}_{ij}-\check{a}_{ij}|F_j|y_j(t)|+|\hat{a}_{ij}-\check{a}_{ij}|(L_j+\varepsilon_j^*)\\\le&\bar{a}_{ij}F_j|y_j(t)-x_j(t)|+\bar{a}_{ij}L_j\\&+|\hat{a}_{ij}-\check{a}_{ij}|(F_jT_j+L_j+\varepsilon_j^*);\end{aligned}\tag{13}$$

from expressions (10)–(13), inequality (8) holds. Similarly, we can prove that inequality (9) holds. This completes the proof of Lemma 3.

Definition 5[33] If the equilibrium point of the system (3) is global Mittag–Leffler stable, the system (3) is said to be global Mittag–Leffler stable.

Lemma 4[47,48] If the system (3) has an equilibrium point x^* and there exists a Lyapunov function $V(t,x(t)):[0,+\infty)\times\mathbb{D}\longrightarrow\mathbb{R}^n$ and class-k functions v_j $(j=1,2,3)$ satisfying

$$v_1(||x(t)||)\le V(t,x(t))\le v_2(||x(t)||),\tag{14}$$

$${}_0^CD_t^\alpha V(t,x(t))\le -v_3(||x(t)||),\tag{15}$$

where $\alpha\in(0,1)$ and the origin is included in a domain of $\mathbb{D}\subseteq\mathbb{R}^n$. So, the equilibrium point x^* of the system (3) is asymptotically stable.

Lemma 5[34] Let $V(t)$ be a continuous function on $[0,+\infty)$ and the following inequality holds

$${}_0^CD_t^\alpha V(t)\le\gamma V(t),$$

where $0<\alpha<1$ and γ is a constant. Then

$$V(t)\le V(0)E_\alpha(\gamma t^\alpha),$$

where $E_\alpha(\cdot)$ denotes the one-parameter Mittag–Leffler function.

3. Main results

The synchronization error is defined as $e(t)=y(t)-x(t)$ from the drive system (6) and response system (7), the error system can be written as

$$\begin{aligned}&{}_0^CD_t^\alpha e_i(t)\\=&-(d_i+\Delta d_i(t))e_i(t)+\sum_{j=1}^n(\mathring{a}_{ij}(t)\varepsilon_j(t)-\mathring{a}'_{ij}(t)\varepsilon_j(t))\\&+\sum_{j=1}^n\Delta a_{ij}(t)(\varepsilon_j(t)-\varepsilon_j(t))+\sum_{j=1}^n\Delta b_{ij}(t)(\varepsilon'_j(t)-\varepsilon'_j(t))\\&+\sum_{j=1}^n(\mathring{b}_{ij}(t)\varepsilon'_j(t)-\mathring{b}'_{ij}(t)\varepsilon'_j(t))+u_i(t).\end{aligned}\tag{16}$$

Next, the Mittag–Leffler synchronization of FDMNN with the parameters uncertainty and discontinuous activation functions are obtained by designing a new type of discontinuous feedback controller. In addition, the upper bound of the setting time of the global Mittag–Leffler synchronization in finite-time is explicitly evaluated.

The discontinuous feedback controller $u_i(t)$ is defined as

$$u_i(t)=-k_i^*e_i(t)-q_i^*\operatorname{sign}(e_i(t))-\iota_i\operatorname{sign}(e_i(t))|e_i(t-\tau_i)|,\tag{17}$$

where $i=1,2,\ldots,n$, $k_i^*>0$, $q_i^*>0$, and $\iota_i>0$.

Remark 2 Since our system is discontinuous system with parameters uncertainty and discontinuous activation functions, a new discontinuous delayed feedback controller is proposed to deal with such system. On the one hand, it can be applied to solve the parameters uncertainty because of the significant changes of the environment, on the other hand, by analyzing such controller, the reliability and safety of system can be obtained.

Theorem 1 Suppose that the Assumptions H1 and H2 hold, if the following inequalities hold

$$\begin{cases}\lambda^*>0,\\\beta^*>0,\\Q_i^*>0,\end{cases}$$

where $i=1,2,\ldots,n$, $0<\alpha<1$, $r_i>0$, $Q_i^*=r_i\{q_i^*-\sum_{j=1}^n(\bar{a}_{ij}L_j+|\hat{a}_{ij}-\check{a}_{ij}|(F_jL_j+L_j+\varepsilon_j^*))-\sum_{j=1}^n\varpi_{ij}L_j-\sum_{j=1}^n\rho_{ij}M_j-\sum_{j=1}^n(\bar{b}_{ij}M_j+|\hat{b}_{ij}-\check{b}_{ij}|(G_jM_j+M_j+\varepsilon_j^*))\}$, $\lambda^*=\min_{1\le i\le n}\{r_i(d_i+k_i^*-\omega_i)-\sum_{j=1}^n r_j\bar{a}_{ji}F_i-\sum_{j=1}^n r_j\varpi_{ji}F_i\}$, and $\beta^*=\min_{1\le i\le n}\{r_i\iota_i-\sum_{j=1}^n r_j\bar{b}_{ji}G_i-\sum_{j=1}^n r_j\rho_{ji}G_i\}$. Then the drive system (3) and response system (4) are global Mittag–Leffler synchronization in finite-time based on the controller (17), and the time upper bound T^* is evaluated by

$$T^*\le\left(\frac{\sum_{i=1}^n r_i|e_i(0)|\Gamma(\alpha+1)}{\sum_{i=1}^n Q_i^*}\right)^{1/\alpha}.$$

Proof Constructing the following Lyapunov function

$$V(t)=\sum_{i=1}^{n} r_i|e_i(t)|, \tag{18}$$

where $r_i>0$.

Based on Lemma 1, one has

$${}_0^C D_t^{\alpha} V(t)\leq \sum_{i=1}^{n} r_i \operatorname{sign}(e_i(t)) {}_0^C D_t^{\alpha} e_i(t). \tag{19}$$

From expressions (16) and (17), inequality (19) can be written as

$$\begin{aligned}
&{}_0^C D_t^{\alpha} V(t)\\
&\leq \sum_{i=1}^{n} r_i \operatorname{sign}(e_i(t))\{-(d_i+\Delta d_i(t))e_i(t)\\
&\quad+\sum_{j=1}^{n}(\mathring{a}_{ij}(t)\varepsilon_j(t)-\mathring{a}'_{ij}(t)\varepsilon_j(t))+\sum_{j=1}^{n}\Delta a_{ij}(t)(\varepsilon_j(t)-\varepsilon_j(t))\\
&\quad+\sum_{j=1}^{n}\Delta b_{ij}(t)(\varepsilon'_j(t)-\varepsilon'_j(t))+\sum_{j=1}^{n}(\mathring{b}_{ij}(t)\varepsilon'_j(t)-\mathring{b}'_{ij}(t)\varepsilon'_j(t))\\
&\quad-k_i^* e_i(t)-q_i^*\operatorname{sign}(e_i(t))-\iota_i \operatorname{sign}(e_i(t))|e_i(t-\tau_i)|\}\\
&\leq \sum_{i=1}^{n} r_i(-d_i-k_i^*+\omega_i)|e_i(t)|\\
&\quad+\sum_{i=1}^{n}\sum_{j=1}^{n} r_i|\mathring{a}_{ij}(t)\varepsilon_j(t)-\mathring{a}'_{ij}(t)\varepsilon_j(t)|\\
&\quad+\sum_{i=1}^{n}\sum_{j=1}^{n} r_i|\Delta a_{ij}(t)||(\varepsilon_j(t)-\varepsilon_j(t))|\\
&\quad+\sum_{i=1}^{n}\sum_{j=1}^{n} r_i|\Delta b_{ij}(t)||\varepsilon'_j(t)-\varepsilon'_j(t)|\\
&\quad+\sum_{i=1}^{n}\sum_{j=1}^{n} r_i|\mathring{b}_{ij}(t)\varepsilon'_j(t)-\mathring{b}'_{ij}(t)\varepsilon'_j(t)|\\
&\quad-\sum_{i=1}^{n} r_i q_i^*-\sum_{i=1}^{n} r_i\iota_i|e_i(t-\tau_i)|.
\end{aligned} \tag{20}$$

According to Lemma 3, one has

$$\begin{aligned}
&{}_0^C D_t^{\alpha} V(t)\\
&\leq \sum_{i=1}^{n} r_i(-d_i-k_i^*+\omega_i)|e_i(t)|-\sum_{i=1}^{n} r_i\iota_i|e_i(t-\tau_i)|\\
&\quad+\sum_{i=1}^{n}\sum_{j=1}^{n} r_j(\bar{a}_{ji}+\varpi_{ji})F_i|e_i(t)|\\
&\quad+\sum_{i=1}^{n}\sum_{j=1}^{n} r_j(\bar{b}_{ji}+\rho_{ji})G_i|e_i(t-\tau_i)|\\
&\quad+\sum_{i=1}^{n} r_i[-q_i^*+\sum_{j=1}^{n}\varpi_{ij}L_j+\sum_{j=1}^{n}\rho_{ij}M_j\\
&\quad+\sum_{j=1}^{n}(\bar{a}_{ij}L_j+|\hat{a}_{ij}-\check{a}_{ij}|(F_jL_j+L_j+\varepsilon_j^*))\\
&\quad+\sum_{j=1}^{n}(\bar{b}_{ij}M_j+|\hat{b}_{ij}-\check{b}_{ij}|(G_jM_j+M_j+\varepsilon_j^*))].
\end{aligned} \tag{21}$$

According to Theorem 1, one has

$$\begin{cases}
\lambda^*=\min\limits_{1\leq i\leq n}\{r_i(d_i+k_i^*-\omega_i)-\sum_{j=1}^{n} r_j\bar{a}_{ji}F_i\\
\qquad-\sum_{j=1}^{n} r_j\varpi_{ji}F_i\}>0,\\
\beta^*=\min\limits_{1\leq i\leq n}\{r_i\iota_i-\sum_{j=1}^{n} r_j\bar{b}_{ji}G_i-\sum_{j=1}^{n} r_j\rho_{ji}G_i\}>0,\\
Q_i^*>0,
\end{cases} \tag{22}$$

where $Q_i^*=r_i\{q_i^*-\sum_{j=1}^{n}(\bar{a}_{ij}L_j+|\hat{a}_{ij}-\check{a}_{ij}|(F_jL_j+L_j+\varepsilon_j^*))-\sum_{j=1}^{n}\varpi_{ij}L_j-\sum_{j=1}^{n}\rho_{ij}M_j-\sum_{j=1}^{n}(\bar{b}_{ij}M_j+|\hat{b}_{ij}-\check{b}_{ij}|(G_jM_j+M_j+\varepsilon_j^*))\}$.

From expressions (21) and (22), one has

$$\begin{aligned}
{}_0^C D_t^{\alpha} V(t)&\leq -\lambda^* V(t)-\beta^* V(t-\tau)-\sum_{i=1}^{n} Q_i^*\\
&\leq -\lambda^* V(t).
\end{aligned} \tag{23}$$

Based on Lemma 5, we have

$$V(t)\leq V(0)E_{\alpha}(-\lambda^* t^{\alpha}). \tag{24}$$

Hence, inequality (24) can be concluded as

$$\sum_{i=1}^{n} r_i|e_i(t)|\leq \sum_{i=1}^{n} r_i|\varphi_i(0)-\phi_i(0)|E_{\alpha}(-\lambda^* t^{\alpha}). \tag{25}$$

From inequality (25), we can obtain

$$||e(t)||_1=||\varphi(0)-\phi(0)||_1\frac{\max_{1\leq i\leq n}\{r_i\}}{\min_{1\leq i\leq n}\{r_i\}}E_{\alpha}(-\lambda^* t^{\alpha}),$$

which implies that the equilibrium point $e^*=0$ is Mittag–Leffler stable. Based on the Definition 5, the system (16) is Mittag–Leffler stable, which means that it completes the proof of global Mittag–Leffler synchronization. In the following, the upper bound of the setting time of the global Mittag–Leffler synchronization in finite-time will be given.

From expressions (21) and (22), one also has

$${}_0^C D_t^{\alpha} V(t)\leq -\sum_{i=1}^{n} Q_i^*. \tag{26}$$

There exists a function $\Lambda(t)\geq 0$ such that

$${}_0^C D_t^{\alpha} V(t)+\Lambda(t)=-\sum_{i=1}^{n} Q_i^*. \tag{27}$$

Using Property 2, expression (27) can be written as

$$V(t)-V(0)+{}_0 I_t^{\alpha}\Lambda(t)=-{}_0 I_t^{\alpha}\sum_{i=1}^{n} Q_i^*. \tag{28}$$

From Definition 1, one has

$${}_0 I_t^{\alpha}\Lambda(t)=\frac{1}{\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}\Lambda(\zeta)\,\mathrm{d}\zeta. \tag{29}$$

Since $\Lambda(t) \geq 0$ for $\zeta \in [0,t]$, $(t-\zeta)^{\alpha-1} \geq 0$ and $\Gamma(\alpha) > 0$. Then

$$ {}_0I_t^{\alpha}\Lambda(t) \geq 0. \tag{30}$$

Combining expression (30) and $V(t) \geq 0$, we can obtain

$$ -V(0) \leq -{}_0I_t^{\alpha}\sum_{i=1}^{n} Q_i^*. \tag{31}$$

Similarly, based on Definition 1, one has

$$\begin{aligned} -{}_0I_t^{\alpha}\sum_{i=1}^{n} Q_i^* &= -\frac{1}{\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}\Big(\sum_{i=1}^{n} Q_i^*\Big)\,\mathrm{d}\zeta \\ &= \frac{-\sum\limits_{i=1}^{n} Q_i^*}{\Gamma(\alpha+1)}t^{\alpha}. \end{aligned} \tag{32}$$

Combining expressions (31) and (32), one also has

$$ -V(0) \leq \frac{-\sum\limits_{i=1}^{n} Q_i^*}{\Gamma(\alpha+1)}t^{\alpha}. \tag{33}$$

After simplification, one has

$$ t \leq \left(\frac{\sum\limits_{i=1}^{n} r_i|e_i(0)|\Gamma(\alpha+1)}{\sum\limits_{i=1}^{n} Q_i^*}\right)^{1/\alpha}, \tag{34}$$

where $Q_i^* = r_i\{q_i^* - \sum\limits_{j=1}^{n}(\bar{a}_{ij}L_j + |\hat{a}_{ij} - \check{a}_{ij}|(F_jL_j + L_j + \varepsilon_j^*)) - \sum\limits_{j=1}^{n}\varpi_{ij}L_j - \sum\limits_{j=1}^{n}\rho_{ij}M_j - \sum\limits_{j=1}^{n}(\bar{b}_{ij}M_j + |\hat{b}_{ij} - \check{b}_{ij}|(G_jM_j + M_j + \varepsilon_j^*))\}$. And it implies that the upper bound of the setting time of the global Mittag–Leffler synchronization in finite-time is denoted as

$$ T^* \leq \left(\frac{\sum\limits_{i=1}^{n} r_i|e_i(0)|\Gamma(\alpha+1)}{\sum\limits_{i=1}^{n} Q_i^*}\right)^{1/\alpha}. \tag{35}$$

Therefore, the state trajectories of error system (16) will converge to the origin in finite-time. This completes proof.

When the parametric uncertainties $\Delta d_i(t) = 0$, $\Delta a_{ij}(t) = 0$, and $\Delta b_{ij}(t) = 0$. As a special case of Theorem 1, the following result can be obtained.

Corollary 1 Under the Assumptions H1 and H2, if the following inequalities hold

$$\begin{cases} \lambda^* > 0, \\ \beta^* > 0, \\ Q_i^* > 0, \end{cases}$$

where $i = 1,2,\ldots,n$, $0 < \alpha < 1$, $r_i > 0$, $Q_i^* = r_i\{q_i^* - \sum\limits_{j=1}^{n}(\bar{a}_{ij}L_j + |\hat{a}_{ij} - \check{a}_{ij}|(F_jL_j + L_j + \varepsilon_j^*)) - \sum\limits_{j=1}^{n}(\bar{b}_{ij}M_j + |\hat{b}_{ij} - \check{b}_{ij}|(G_jM_j + M_j + \varepsilon_j^*))\}$, $\lambda^* = \min_{1\leq i\leq n}\{r_i \ (d_i + k_i^*) - \sum\limits_{j=1}^{n} r_j\bar{a}_{ji}F_i\}$ and $\beta^* = \min_{1\leq i\leq n}\{r_i\iota_i - \sum\limits_{j=1}^{n} r_j\bar{b}_{ji}G_i\}$. Then the drive system (3) and response system (4) are global Mittag–Leffler synchronization in finite-time based on the controller (17), and the time upper bound T^* is evaluated as

$$ T^* \leq \left(\frac{\sum\limits_{i=1}^{n} r_i|e_i(0)|\Gamma(\alpha+1)}{\sum\limits_{i=1}^{n} Q_i^*}\right)^{1/\alpha}.$$

Remark 3 Compared with Theorem 1 of FDMNN in Ref. [19], Corollary 1 not only does not require boundedness for discontinuous activation functions, but also gives the time upper bound of synchronization. Hence, the results (see Theorem 1) of FDMNN[19] can be directly obtained from Corollary 1.

When the parametric uncertainties $\Delta d_i(t) = 0$, $\Delta a_{ij}(t) = 0$, and $\Delta b_{ij}(t) = 0$, and the transmission delays $\tau_i = 0$. Based on the controller (17), the following result can be obtained.

Corollary 2 If the Assumptions H1, H2, and the following inequalities hold

$$\begin{cases} \lambda^* > 0, \\ Q_i^* > 0, \end{cases}$$

where $i = 1,2,\ldots,n$, $0 < \alpha < 1$, $r_i > 0$, $Q_i^* = r_i\{q_i^* - \sum\limits_{j=1}^{n}(\bar{a}_{ij}L_j + |\hat{a}_{ij} - \check{a}_{ij}|(F_jL_j + L_j + \varepsilon_j^*))\}$, and $\lambda^* = \min_{1\leq i\leq n}\{r_i(d_i + k_i^*) - \sum\limits_{j=1}^{n} r_j\bar{a}_{ji}F_i\}$. Then the drive system (3) and response (4) can be achieved to global Mittag–Leffler synchronization in finite-time. Meanwhile, the time upper bound T^* is evaluated as

$$ T^* \leq \left(\frac{\sum\limits_{i=1}^{n} r_i|e_i(0)|\Gamma(\alpha+1)}{\sum\limits_{i=1}^{n} Q_i^*}\right)^{1/\alpha}.$$

Remark 4 When neglecting the effects of time delays and parameters uncertainty, the finite-time synchronization results (see Theorem 1) of FMNN[39] are the special case of Corollary 2.

Corollary 3 Under the Assumptions H1 and H2, the drive system (3) is synchronized in finite-time with the response system (4) under the delayed feedback controller (17), if the following inequalities hold

$$\begin{cases} \lambda^* > 0, \\ \beta^* > 0, \\ Q_i^* > 0, \end{cases}$$

where $i = 1,2,\ldots,n$, $0 < \alpha < 1$, $Q_i^* = q_i^* - \sum\limits_{j=1}^{n}(\bar{a}_{ij}L_j + |\hat{a}_{ij} - \check{a}_{ij}|(F_jL_j + L_j + \varepsilon_j^*)) - \sum\limits_{j=1}^{n}\varpi_{ij}L_j - \sum\limits_{j=1}^{n}\rho_{ij}M_j - \sum\limits_{j=1}^{n}(\bar{b}_{ij}M_j + |\hat{b}_{ij} - \check{b}_{ij}|(G_jM_j + M_j + \varepsilon_j^*))$, $\lambda^* = \min_{1\leq i\leq n}\{d_i + k_i^* - \omega_i - \sum\limits_{j=1}^{n}\bar{a}_{ji}F_i - \sum\limits_{j=1}^{n}\varpi_{ji}F_i\}$, and $\beta^* = \min_{1\leq i\leq n}\{\iota_i - \sum\limits_{j=1}^{n}\bar{b}_{ji}G_i -$

$\sum_{j=1}^{n}\rho_{ji}G_i\}$. Moreover, the settling time for synchronization of FDMNN with parameters uncertainty and discontinuous activation functions is obtained as

$$T^*\leq\left(\frac{\sum_{i=1}^{n}|e_i(0)|\Gamma(\alpha+1)}{\sum_{i=1}^{n}Q_i^*}\right)^{1/\alpha}.$$

Proof Let $r_i=1$ in $V(t)=\sum_{i=1}^{n}r_i|e_i(t)|$, one has

$$V(t)=\sum_{i=1}^{n}|e_i(t)|.$$

Hence, we can obtain Corollary 3 by utilizing a similar approach in the proof of Theorem 1.

Remark 5 Note that the existence of the term $\mathrm{sign}(\cdot)$ in controller (17) can produce chartering phenomenon, especially when the errors of system (16) vary around zero. In order to deal with the problem of the harmful chattering, the symbolic function $\mathrm{sign}(\cdot)$ can be replaced by the following saturation function[49] in numerical simulation or practical application:

$$\mathrm{sat}(e_i(t),\nu)=\begin{cases}1, & \frac{e_i(t)}{\nu}\geq 1,\\ \frac{e_i(t)}{\nu}, & -1<\frac{e_i(t)}{\nu}<1,\\ -1, & \frac{e_i(t)}{\nu}\leq -1,\end{cases}$$

where $i=1,2,\ldots,n$ and $\nu>0$ is a small constant.

When neglecting the effect of $e_i(t-\tau_i)$ in expression (17), the following controller is designed to achieve the global asymptotic synchronization with the corresponding FDMNN:

$$u_i(t)=-k_ie_i(t)-q_i\mathrm{sign}(e_i(t)), \tag{36}$$

where $i=1,2,\ldots,n$, $k_i>0$, $q_i>0$, and $\mathrm{sign}(\cdot)$ denotes symbolic function.

Corollary 4 Under the Assumptions H1 and H2, choosing proper parameters k_i and q_i, the following relationships hold

$$\begin{cases}Q_i>0,\\ 0<\sum_{i=1}^{n}\beta_i<\lambda\sin(\alpha\pi/2),\end{cases}$$

where $i=1,2,\ldots,n$, $0<\alpha<1$, $Q_i=q_i-\sum_{j=1}^{n}(\bar{a}_{ij}L_j+|\hat{a}_{ij}-\check{a}_{ij}|(F_jL_j+L_j+\varepsilon_j^*))-\sum_{j=1}^{n}\varpi_{ij}L_j-\sum_{j=1}^{n}\rho_{ij}M_j-\sum_{j=1}^{n}(\bar{b}_{ij}M_j+|\hat{b}_{ij}-\check{b}_{ij}|(G_jM_j+M_j+\varepsilon_j^*))$, $\lambda=\min_{1\leq i\leq n}\{d_i+k_i-\omega_i-\sum_{j=1}^{n}\bar{a}_{ji}F_i-\sum_{j=1}^{n}\varpi_{ji}F_i\}$, and $\beta_i=\sum_{j=1}^{n}\bar{b}_{ji}G_i+\sum_{j=1}^{n}\rho_{ji}G_i$. Then, the drive system (3) and the response system (4) will achieve the global asymptotic synchronization.

Proof Constructing the following Lyapunov function

$$V(t)=\sum_{i=1}^{n}|e_i(t)|. \tag{37}$$

Similar to the above method, one has

$$\begin{aligned}{}_0^CD_t^\alpha V(t)\leq&\sum_{i=1}^{n}\mathrm{sign}(e_i(t)){}_0^CD_t^\alpha e_i(t)\\
\leq&\sum_{i=1}^{n}(-d_i-k_i+\omega_i)|e_i(t)|\\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}|\mathring{a}_{ij}(t)\varepsilon_j(t)-\mathring{a}'_{ij}(t)\varepsilon_j(t)|\\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}|\Delta a_{ij}(t)||(\varepsilon_j(t)-\varepsilon_j(t))|\\
&-\sum_{i=1}^{n}q_i+\sum_{i=1}^{n}\sum_{j=1}^{n}|\Delta b_{ij}(t)||\varepsilon'_j(t)-\varepsilon'_j(t)|\\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}|\mathring{b}_{ij}(t)\varepsilon'_j(t)-\mathring{b}'_{ij}(t)\varepsilon'_j(t)|\\
\leq&\sum_{i=1}^{n}(-d_i-k_i+\omega_i)|e_i(t)|+\sum_{i=1}^{n}\sum_{j=1}^{n}(\bar{a}_{ji}+\varpi_{ji})F_i|e_i(t)|\\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}(\bar{b}_{ji}+\rho_{ji})G_i|e_i(t-\tau_i)|\\
&+\sum_{i=1}^{n}\Bigg[-q_i+\sum_{j=1}^{n}\varpi_{ij}L_j+\sum_{j=1}^{n}\rho_{ij}M_j\\
&+\sum_{j=1}^{n}(\bar{a}_{ij}L_j+|\hat{a}_{ij}-\check{a}_{ij}|(F_jL_j+L_j+\varepsilon_j^*))\\
&+\sum_{j=1}^{n}(\bar{b}_{ij}M_j+|\hat{b}_{ij}-\check{b}_{ij}|(G_jM_j+M_j+\varepsilon_j^*))\Bigg].\end{aligned} \tag{38}$$

According to Corollary 4, we obtain that

$$\begin{aligned}Q_i=&q_i-\sum_{j=1}^{n}\varpi_{ij}L_j-\sum_{j=1}^{n}\rho_{ij}M_j\\
&-\sum_{j=1}^{n}(\bar{a}_{ij}L_j+|\hat{a}_{ij}-\check{a}_{ij}|(F_jL_j+L_j+\varepsilon_j^*))\\
&-\sum_{j=1}^{n}(\bar{b}_{ij}M_j+|\hat{b}_{ij}-\check{b}_{ij}|(G_jM_j+M_j+\varepsilon_j^*))\\
>&0.\end{aligned} \tag{39}$$

From expressions (38) and (39), one has

$${}_0^CD_t^\alpha V(t)\leq-\lambda V(t)+\sum_{j=1}^{n}\beta_jV(t-\tau_j), \tag{40}$$

where $\lambda=\min_{1\leq i\leq n}\{d_i+k_i-\omega_i-\sum_{j=1}^{n}\bar{a}_{ji}F_i-\sum_{j=1}^{n}\varpi_{ji}F_i\}$ and $\beta_i=\sum_{j=1}^{n}\bar{b}_{ji}G_i+\sum_{j=1}^{n}\rho_{ji}G_i$.

Considering the following system

$${}_0^CD_t^\alpha W(t)=-\lambda W(t)+\sum_{j=1}^{n}\beta_jW(t-\tau_j), \tag{41}$$

where $W(t)\geq 0$ and the initial value condition of $W(t)$ is consistent with $V(t)$.

According to Lemma 2, one has

$$0 < V(t) \le W(t), \quad \forall t \in [0, +\infty).$$

Obviously, $W^* = 0$ is an equilibrium point of the system (41). Next, we will prove that the equilibrium point of system (41) is the global asymptotic stable, *i.e.*, $\lim_{t\to+\infty} W(t) = 0$.

Based on Property 3, the Laplace transformation of the system (41) can be written as

$$\begin{aligned} & s^{\alpha} W(s) - s^{\alpha-1} W(0) \\ = & -\lambda W(s) + \int_0^{+\infty} \sum_{j=1}^{n} \beta_j \mathrm{e}^{-st} W(t-\tau_j)\,\mathrm{d}t. \end{aligned} \tag{42}$$

The characteristic equation of the system (42) can be written as

$$s^{\alpha} + \lambda - \sum_{j=1}^{n} \beta_j \mathrm{e}^{-s\tau_j} = 0. \tag{43}$$

Assuming that equation (43) has pure imaginary root $s = \sigma i = |\sigma|(\cos(\pi/2) + \mathrm{i}\sin(\pm\pi/2))$, where σ is a real number. Substituting it into Eq. (43), one has

$$\begin{aligned} & |\sigma| \left[\cos\frac{\pi}{2} + \mathrm{i}\sin\left(\pm\frac{\pi}{2}\right) \right] + \lambda \\ = & \sum_{j=1}^{n} \beta_j [\cos(\sigma\tau_j) - \mathrm{i}\sin(\sigma\tau_j)]. \end{aligned} \tag{44}$$

Discussing the real part and imaginary part of Eq. (44) respectively, we have

$$\begin{cases} |\sigma| \cos\dfrac{\alpha\pi}{2} + \lambda = \sum\limits_{j=1}^{n} \beta_j \cos(\sigma\tau_j), \\ |\sigma| \sin\left(\pm\dfrac{\alpha\pi}{2}\right) = -\sum\limits_{j=1}^{n} \beta_j \sin(\sigma\tau_j), \end{cases} \tag{45}$$

By utilizing the properties of trigonometric functions, we have

$$\begin{aligned} & \left(\sum_{j=1}^{n} \beta_j \cos(\sigma\tau_j)\right)^2 + \left(\sum_{j=1}^{n} \beta_j \sin(\sigma\tau_j)\right)^2 \\ = & \sum_{i=1}^{n}\sum_{j=1}^{n} \beta_i\beta_j \cos(\sigma(\tau_i - \tau_j)). \end{aligned} \tag{46}$$

Then, combining the real part and imaginary part, one has

$$\begin{aligned} & \left(|\sigma|\cos\frac{\alpha\pi}{2} + \lambda\right)^2 + \left(|\sigma|\sin\left(\pm\frac{\alpha\pi}{2}\right)\right)^2 \\ = & \sum_{i=1}^{n}\sum_{j=1}^{n} \beta_i\beta_j \cos(\sigma(\tau_i - \tau_j)). \end{aligned} \tag{47}$$

Considering $|\sigma|^{\alpha}$ as variable of Eq. (47), the discriminant of Eq. (47) can be written as

$$\begin{aligned} \Delta = & \left(2\lambda\cos\frac{\alpha\pi}{2}\right)^2 - 4\left(\lambda^2 - \sum_{i=1}^{n}\sum_{j=1}^{n} \beta_i\beta_j \cos(\sigma(\tau_i - \tau_j))\right) \\ = & 4\left(\sum_{i=1}^{n}\sum_{j=1}^{n} \beta_i\beta_j \cos(\sigma(\tau_i - \tau_j)) - \lambda^2 \sin^2\left(\pm\frac{\alpha\pi}{2}\right)\right) \\ \le & 4\left(\left(\sum_{i=1}^{n}\beta_i\right)^2 - \lambda^2\sin^2\left(\pm\frac{\alpha\pi}{2}\right)\right). \end{aligned} \tag{48}$$

Since $0 < \alpha < 1$ and $0 < \sum_{i=1}^{n} \beta_i < \lambda \sin(\alpha\pi/2)$, $\Delta < 0$ which implies that equation (47) has no solution, *i.e.*, the characteristic equation of Eq. (41) has no pure imaginary roots for any $\tau_j > 0$.

Based on Lemma 4, the zero solution of system (41) is Lyapunov-global asymptotic stable. According to Lemma 2, the zero solution of error system (16) is Lyapunov-global asymptotic stable. Hence, the global asymptotic stable of error system (16) can be achieved. This completes the proof.

Remark 6 In Ref. [32], while the activation functions are Lipshits-continuous and $f_j(\pm T_j) = g_j(\pm T_j) = 0$, the asymptotic synchronization results (see Theorem 3) of FDMNN can be obtained. Hence, Corollary 4 can be regarded as a generalization of the synchronization results (see Theorem 3) of FDMNN.[32]

Remark 7 It is well known that time delays are unavoidable in practice engineering due to finite switching speeds of the amplifiers. However, in most papers,[33,50] the dynamic performances of FMNN have been studied without considering time delays. In our paper, the synchronization of FMNN with multiple time delays is investigated. At the same time, we also give a scheme to deal with the problems about FMNN with multiple time delays.

Remark 8 Obviously, the controller (36) is a special case of the controller (17). Based on controller (17), both finite-time Mittag–Leffler synchronization criterion (*i.e.*, Theorem 1) and global asymptotic synchronization criteria (*i.e.*, Corollary 4) are established. Apparently, three groups of adjustable parameters (k_i^*, q_i^*, and ι_i) are involved in controller (17), which implies that the desired performance (finite-time Mittag–Leffler synchronization and asymptotic synchronization) can be achieved by selecting the appropriate parameters to satisfy the proposed conditions in Theorem 1 or Corollary 4. However, considering the ease of implementation of the designed controller in actual engineering, both the controllers (17) and (36) are necessary to study according to different engineering requirements.

4. Numerical examples

In this section, an examples is provided to illustrate the validity of results obtained in this paper.

Example 1 Consider the three-dimensional fractional-order delayed memristive neural networks with the parameters uncertainty and discontinuous activation functions as the drive system, which is defined as

$${}_0^C D_t^{\alpha} x_i(t) = -(d_i + \Delta d_i(t))x_i(t) + \sum_{j=1}^{3} (a_{ij}(x_j(t))$$

$$+\Delta a_{ij}(t))f_j(x_j(t))+\sum_{j=1}^{3}(b_{ij}(x_j(t-\tau_j))$$
$$+\Delta b_{ij}(t))g_j(x_j(t-\tau_j))+I_i, \tag{49}$$

where $\alpha = 0.98$, $i = 1,2,3$; The activation functions $f_i(x_i(t)) = \sin(x_i(t)) + 0.05\text{sign}(x_i(t))$, $g_i(x_i(t-\tau_i)) = \sin(x_i(t-\tau_i)) + 0.05\text{sign}(x_i(t-\tau_i))$; The parameters of the system (49) are selected as $d_1 = d_2 = d_3 = 2.5$;

$$a_{11}(x_1(t)) = \begin{cases} 1.2, & |x_1(t)| > 0.1, \\ 1, & |x_1(t)| \le 0.1, \end{cases}$$
$$a_{12}(x_2(t)) = \begin{cases} -2.2, & |x_2(t)| > 0.1, \\ -2, & |x_2(t)| \le 0.1, \end{cases}$$
$$a_{13}(x_3(t)) = \begin{cases} 3.8, & |x_3(t)| > 0.1, \\ 3, & |x_3(t)| \le 0.1, \end{cases}$$
$$a_{21}(x_1(t)) = \begin{cases} 1.5, & |x_1(t)| > 0.1, \\ -2, & |x_1(t)| \le 0.1, \end{cases}$$
$$a_{22}(x_2(t)) = \begin{cases} 1.8, & |x_2(t)| > 0.1, \\ -1.5, & |x_2(t)| \le 0.1, \end{cases}$$
$$a_{23}(x_3(t)) = \begin{cases} -2, & |x_3(t)| > 0.1, \\ 2.5, & |x_3(t)| \le 0.1, \end{cases}$$
$$a_{31}(x_1(t)) = \begin{cases} 0.8, & |x_1(t)| > 0.1, \\ -1, & |x_1(t)| \le 0.1, \end{cases}$$
$$a_{32}(x_2(t)) = \begin{cases} 5, & |x_2(t)| > 0.1, \\ -4.7, & |x_2(t)| \le 0.1, \end{cases}$$
$$a_{33}(x_3(t)) = \begin{cases} 3.5, & |x_3(t)| > 0.1, \\ -2, & |x_3(t)| \le 0.1, \end{cases}$$
$$b_{11}(x_1(t-\tau_1)) = \begin{cases} -2 & |x_1(t-\tau_1)| > 0.1, \\ 1 & |x_1(t-\tau_1)| \le 0.1, \end{cases}$$
$$b_{12}(x_2(t-\tau_2)) = \begin{cases} 0.5, & |x_2(t-\tau_2)| > 0.1, \\ -1, & |x_2(t-\tau_2)| \le 0.1, \end{cases}$$
$$b_{13}(x_3(t-\tau_3)) = \begin{cases} -1, & |x_3(t-\tau_3)| > 0.1, \\ -4, & |x_3(t-\tau_3)| \le 0.1, \end{cases}$$
$$b_{21}(x_1(t-\tau_1)) = \begin{cases} -5.5, & |x_1(t-\tau_1)| > 0.1, \\ 3, & |x_1(t-\tau_1)| \le 0.1, \end{cases}$$
$$b_{22}(x_2(t-\tau_2)) = \begin{cases} -2, & |x_2(t-\tau_2)| > 0.1, \\ 1.5, & |x_2(t-\tau_2)| \le 0.1, \end{cases}$$
$$b_{23}(x_3(t-\tau_3)) = \begin{cases} -2.5, & |x_3(t-\tau_3)| > 0.1, \\ 3, & |x_3(t-\tau_3)| \le 0.1, \end{cases}$$
$$b_{31}(x_1(t-\tau_1)) = \begin{cases} -2.5, & |x_1(t-\tau_1)| > 0.1, \\ 3, & |x_1(t-\tau_1)| \le 0.1, \end{cases}$$
$$b_{32}(x_2(t-\tau_2)) = \begin{cases} -2, & |x_2(t-\tau_2)| > 0.1, \\ 1.8, & |x_2(t-\tau_2)| \le 0.1, \end{cases}$$
$$b_{33}(x_3(t-\tau_3)) = \begin{cases} 3, & |x_3(t-\tau_3)| > 0.1, \\ -2, & |x_3(t-\tau_3)| \le 0.1. \end{cases}$$

The parametric uncertainties $\Delta d_i(t) = \Delta a_{ij}(t) = \Delta b_{ij}(t) = 0.1\sin(t)$; The external input $I_1 = I_2 = I_3 = 0$, and the transmission delays $\tau_1 = 0.1$, $\tau_2 = 0.08$, and $\tau_3 = 0.11$; The initial condition of the system (49) is $x(0) = (-0.2, 0.15, -0.1)^{\mathrm{T}}$. Under the above parameters, the dynamical behavior of the drive system (49) is depicted in Fig. 1.

Considering the following system as the response system, which is defined by

$$\begin{aligned} {}_0^C D_t^{\alpha} y_i(t) = & -(d_i+\Delta d_i(t))y_i(t)+\sum_{j=1}^{3}(a_{ij}(y_j(t)) \\ & +\Delta a_{ij}(t))f_j(y_j(t))+\sum_{j=1}^{3}(b_{ij}(y_j(t-\tau_j)) \\ & +\Delta b_{ij}(t))g_j(y_j(t-\tau_j))+I_i+u_i(t), \end{aligned} \tag{50}$$

where the initial condition of the system (50) is $y(0) = (0.5, -0.5, 0.5)^{\mathrm{T}}$. Under these parameters, the phase trajectories of response system (50) without controller is shown in Fig. 2.

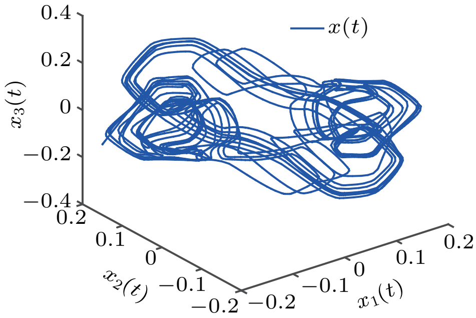

Fig. 1. Phase plot of drive system (49) with initial condition $x(0) = (-0.2, 0.15, -0.1)^{\mathrm{T}}$.

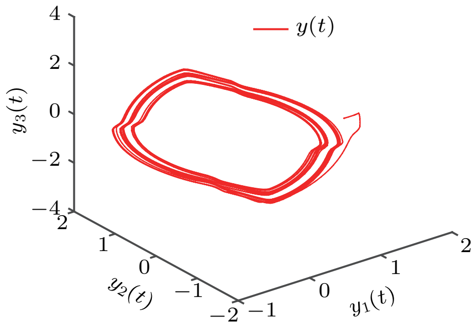

Fig. 2. Phase plot of response system (50) with initial condition $y(0) = (0.5, -0.5, 0.5)^{\mathrm{T}}$ and without the controller.

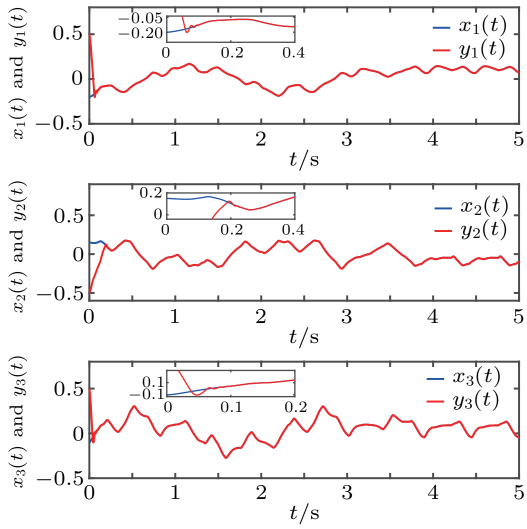

Fig. 3. State trajectories of drive–response system with the initial conditions $x(0) = (-0.2, 0.15, -0.1)^{\mathrm{T}}$ and $y(0) = (0.5, -0.5, 0.5)^{\mathrm{T}}$ under the controller (17).

In controller (17), the parameters can be choose as $k_1^* = 5$, $k_2^* = 4.1$, $k_3^* = 7.3$, $q_1^* = q_2^* = q_3^* = 8$, $\iota_1 = 7.3$, $\iota_2 = 10.8$, and $\iota_3 = 8.3$. It is easy to verify that these values satisfy the conditions of Theorem 1. The drive–response systems (49) and (50) can achieve global finite-time Mittag–Leffler synchronization under the controller (17), which is shown in Figs. 3 and 4. Meanwhile, the time bound $T^* = 0.2433$ is evaluated based on Theorem 1.

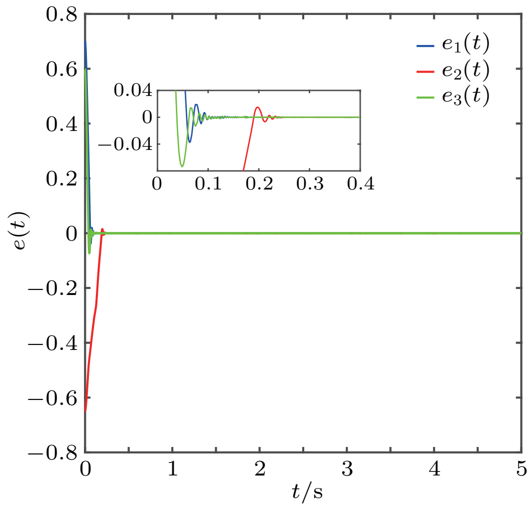

Fig. 4. State trajectories of synchronization errors $e_1(t)$, $e_2(t)$, and $e_3(t)$ under the controller (17).

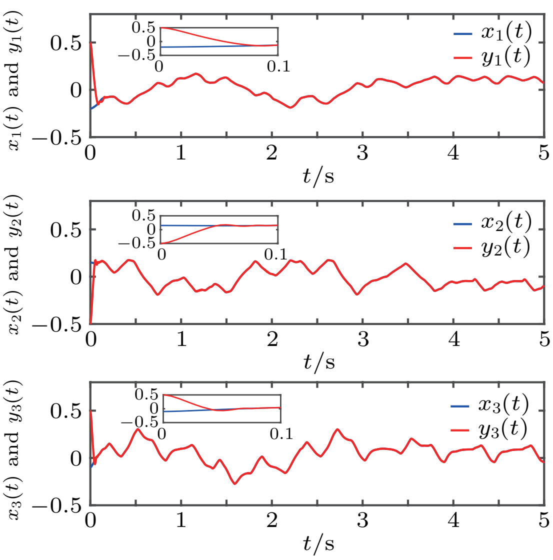

Fig. 5. State trajectories of drive–response system with the initial conditions $x(0) = (-0.2, 0.15, -0.1)^{\mathrm{T}}$ and $y(0) = (0.5, -0.5, 0.5)^{\mathrm{T}}$ under the controller (36).

Remark 9 To the best of our knowledge, there exist some interesting results about finite-time synchronization for fractional-order neural networks with discontinuous activation functions.[35,36,39,51,52] In Ref. [52], the authors had investigated the global Mittag–Leffler synchronization and finite-time synchronization for such systems by utilizing the discontinuous delayed feedback controller. It is worth noting that the main results in Ref. [52] can also deal with our system. Under the same initial values and control gains k^*, q^*, and ι, the settling time $T_1^* = 0.3059$ can be evaluated by using the obtained criteria in Ref. [52]. But the settling time $T^* = 0.2433$ by using our criteria. It is obvious that the upper bound of the settling time $T^* = 0.2433 < 0.3059$, the results of this paper are less conservative.

In controller (36), the parameters can be designed as $k_1 = k_2 = k_3 = 20$, $q_1 = 2.48$, $q_2 = 6.63$, and $q_3 = 7.17$. Obviously, these values satisfy the conditions of Corollary 4. The drive system (49) and response system (50) can achieve global asymptotic synchronization, which is demonstrated in Figs. 5 and 6. In Fig. 6, the synchronization errors converge to zero, which denote that the drive–response systems (49) and (50) are global asymptotic synchronization based on the controller (36).

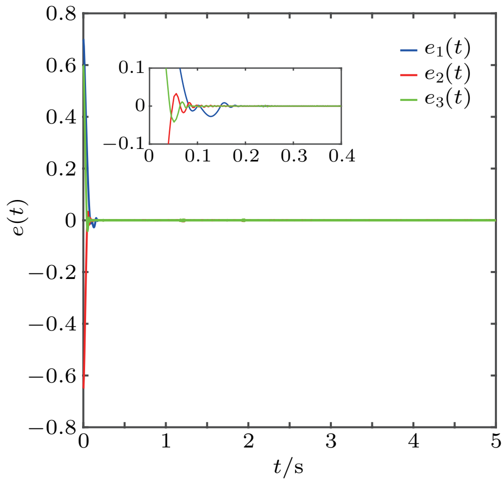

Fig. 6. State trajectories of synchronization errors $e_1(t)$, $e_2(t)$, and $e_3(t)$ under the controller (36).

5. Conclusion

The finite-time Mittag–Leffler synchronization for a class of FDMNN with parameters uncertainty and discontinuous activation functions has been considered. A series of sufficient conditions ensuring finite-time Mittag–Leffler synchronization of such systems are shown by designing a discontinuous feedback controller. In addition, the asymptotic synchronization has been achieved by using comparison theorem and selecting the appropriate parameters of designed controller. Compared with existing results, the obtained results of this paper are less conservative. It would be interesting to focus on the application of finite-time Mittag–Leffler synchronization of such discontinuous systems in image encryption. This topic goes beyond the scope of this paper and will be a challenging issue for future research.

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