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Quantum coherence and correlation dynamics of two-qubit system in spin bath environment*

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The quantum entanglement, discord, and coherence dynamics of two spins in the model of a spin coupled to a spin bath through an intermediate spin are studied. The effects of the important physical parameters including the coupling strength of two spins, the interaction strength between the intermediate spin and the spin bath, the number of bath spins and the temperature of the system on quantum coherence and correlation dynamics are discussed in different cases. The frozen quantum discord can be observed whereas coherence does not when the initial state is the Bell-diagonal state. At finite temperature, we find that coherence is more robust than quantum discord, which is better than entanglement, in terms of resisting the influence of environment. Therefore, quantum coherence is more tenacious than quantum correlation as an important resource.

Keywords: quantum coherence, quantum correlation, spin bath

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1. Introduction

Quantum properties of a composite quantum system can be characterized by several concepts including entanglement, quantum discord, and coherence.[1–4] It has been shown that quantum discord is generally more robust than entanglement.[5,6] Different from quantum discord, quantum coherence, which originates from the superposition principle of quantum states, is even more fundamental than quantum discord and plays an important role in different fields, such as thermodynamical systems,[7,8] biological systems,[9–11] transport theory,[12,13] and nanoscale physics.[14,15]

Although quantum coherence is an important resource, its quantification has not been solved for a long time until Baumgratz *et al*.[4] proposed a strict framework to quantify coherence and the measures of quantifying coherence based on relative entropy and l_1 norm were given. Their work has greatly promoted the development of this field. After that, intrinsic randomness of coherence,[16] the quantum coherence measure based on skewness information,[17] the discordlike bipartite coherence,[18] and the coherence weight[19] were proposed one after another.

There is a very subtle relationship between quantum correlation and quantum coherence, for example, the creation of quantum discord is bounded by the amount of quantum coherence consumed.[20] Besides, the conceptual implications and connections of quantum coherence, mutual incompatibility, and quantum correlations are revealed in Ref. [21]. One should note that quantum coherence can exist in a unilateral system, while quantum correlation generally exists in a bilateral or multilateral system.[22] In reality, stable quantum resources are of great significance to the development of quantum technology. However, a quantum system inevitably interacts with its surrounding environment and produces decoherence. How to use the robust quantum resources to establish coherence and correlation under the influence of environment is a main task at present. Therefore, the dynamics of several quantifications of coherence and correlation in different environments have attracted great attention and showed that coherence indicates the behaviors of quantum discord and classical correlation under incoherent quantum channels,[22] and quantum correlation as well as coherence can characterize quantum phase transition for the spin chain at zero temperature.[23–25] In addition, the coherence and correlation dynamics of the two-qubit system and tripartite systems coupled with the different environments have also been discussed.[26–28]

Currently, the main studies of the quantum coherence and correlation dynamics are performed at zero temperature conditions. Different from previous works, we study the thermal quantum coherence and correlation (entanglement and quantum discord) dynamics of two spins in the model of a spin coupled to a spin bath through an intermediate spin. The frozen

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quantum discord can be found whereas l_1 norm of coherence does not during the evolution process in the initial state of Bell-diagonal state. In addition, quantum coherence is more robust than discord and entanglement when the system is subjected to the environment, therefore, coherence can be as an robust resource to perform quantum technology.

This paper is organized as follows. In Section 2, we introduce the model and give the solution to obtain the reduced density matrix of two spins. In Section 3, we review the measures of quantum coherence and quantum correlation. In Section 4, we discuss quantum coherence and correlation dynamics of two spins. The last section is a summary of our results.

2. Model

The system we consider here is two spins coupled to a spin bath, where a spin A indirectly coupled to a spin bath through an intermediate spin B, as shown in the Fig. 1. The interaction Hamiltonian is given by[29] $H = H_{\mathrm{SI}} + H_{\mathrm{IB}}$, where

$$\begin{aligned} H_{\mathrm{SI}} &= \frac{\gamma}{2}(\sigma_+\tau_- + \sigma_-\tau_+), \\ H_{\mathrm{IB}} &= \frac{\alpha}{2\sqrt{N}}(\tau_+ J_- + \tau_- J_+). \end{aligned} \tag{1}$$

H_{SI} represents the interaction between the spin A and the intermediate spin B, where γ is the coupling constant; $\sigma_\pm$ and $\tau_\pm$ correspond to the creation and annihilation operators of the spins A and B, respectively. H_{IB} denotes the interaction between the intermediate spin and the spin bath, where α is the interaction strength between the intermediate spin and the spin bath; $J_\pm = \sum_{i=1}^{N} \sigma_i^\pm$ are the total angular momentum operator of the spin bath; $\sigma_i^\pm$ are the corresponding creation and annihilation operators of the i-th spin in the bath; N is the number of spins of the spin bath. We assume that all the spins in the bath have the same coupling strength to the intermediate spin and the value of spin is 1/2 in this paper.

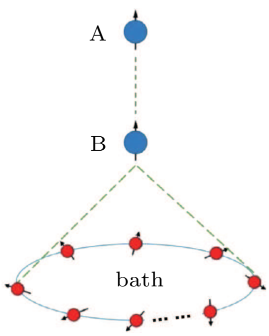

Fig. 1. Sketch of the model, the spin A interacts with an intermediate spin B, which interacts with the bath.

To describe the exact quantum coherence and correlation dynamics of two spins, we need to calculate the reduced density matrix of two spins and it can be calculated by tracing over the environmental degrees of freedom, namely,

$$\rho_{\mathrm{SI}}(t) = \mathrm{tr}_{\mathrm{B}}\{U(t)\rho_{\mathrm{tot}}(0)U(t)^\dagger\}, \tag{2}$$

or given in the form of environment states as

$$\rho_{\mathrm{SI}}(t) = \sum_{j,m} \nu(N,j)\langle j,m|\rho_{\mathrm{tot}}(t)|j,m\rangle, \tag{3}$$

where the initial state of the total system is setted as $\rho_{\mathrm{tot}}(\mathbf{0}) = \rho_{\mathrm{SI}}(\mathbf{0}) \otimes \rho_{\mathrm{B}}$, $U(t) = \exp(-\mathrm{i}Ht)$ is the time evolution operator, $\rho_{\mathrm{SI}}(0)$ is the initial state of the two-qubit system, the vectors $|j,m\rangle$ denote the eigenvectors of the bath operators J^2 and J_z, J_z denotes the z component of the total angular momentum J. The density matrix of the spin bath satisfies the Boltzmann distribution, *i.e.*, $\rho_{\mathrm{B}} = \mathrm{e}^{-\beta H_{\mathrm{B}}}/Z$, where β is inverse temperature, $H_{\mathrm{B}} = (g/N)[J^2 - J_z^2]$ is the energy of the bath with the neighbor interaction strength g of the bath spins,[30]

$$Z = \sum_{j,m} \nu(N,j) \exp\left\{-\frac{g\beta}{N}[j(j+1) - m^2]\right\}$$

is the partition function with the degeneracy

$$\nu(N,j) = \binom{N}{N/2+j} - \binom{N}{N/2+j+1}.$$

In addition, here the Boltzmann constant has been set to 1.

When the initial state of two spin qubits is prepared in the maximum entangled state as

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \tag{4}$$

the specific form of the reduced density matrix can be written as

$$\begin{aligned} \rho_{\mathrm{SI}}(t) = \frac{1}{2}\Big\{ & \frac{1}{Z}\mathrm{tr}_{\mathrm{B}}[\mathrm{e}^{-\mathrm{i}Ht}|00\rangle\,\mathrm{e}^{-\beta H_{\mathrm{B}}}\langle 00|\,\mathrm{e}^{\mathrm{i}Ht}] \\ & + \frac{1}{Z}\mathrm{tr}_{\mathrm{B}}[\mathrm{e}^{-\mathrm{i}Ht}|00\rangle\,\mathrm{e}^{-\beta H_{\mathrm{B}}}\langle 11|\,\mathrm{e}^{\mathrm{i}Ht}] \\ & + \frac{1}{Z}\mathrm{tr}_{\mathrm{B}}[\mathrm{e}^{-\mathrm{i}Ht}|11\rangle\,\mathrm{e}^{-\beta H_{\mathrm{B}}}\langle 00|\,\mathrm{e}^{\mathrm{i}Ht}] \\ & + \frac{1}{Z}\mathrm{tr}_{\mathrm{B}}[\mathrm{e}^{-\mathrm{i}Ht}|11\rangle\,\mathrm{e}^{-\beta H_{\mathrm{B}}}\langle 11|\,\mathrm{e}^{\mathrm{i}Ht}]\Big\}. \end{aligned} \tag{5}$$

In addition, we can obtain that[29] $U|00\rangle = U_{44}|00\rangle + U_{34}|01\rangle + U_{24}|10\rangle + U_{14}|11\rangle$,

$$U|11\rangle = U_{41}|00\rangle + U_{31}|01\rangle + U_{21}|10\rangle + U_{11}|11\rangle,$$

where the components of the time evolution operator are given as follows:

$$\begin{aligned} U_{11} &= 1 - \frac{\alpha^2}{4N}J_-\{(-1+\cosh(A_-))C_+F^{-1}G_-^{-2} \\ &\quad - (-1+\cosh(A_+))C_-F^{-1}G_+^{-2}\}J_+, \\ U_{21} &= \frac{\mathrm{i}\alpha}{2\sqrt{2N}}F^{-1}\{\sinh(A_+)C_-G_+^{-1} - \sinh(A_-)C_+G_-^{-1}\}J_+, \\ U_{31} &= \frac{\alpha\gamma}{4\sqrt{N}}F^{-1}\{-\cosh(A_+) + \cosh(A_-)\}J_+, \\ U_{41} &= \frac{\mathrm{i}\sqrt{2}\alpha^2\gamma}{8N}J_+F^{-1}\{\sinh(A_+)G_+^{-1} - \sinh(A_-)G_-^{-1}\}J_+, \end{aligned}$$

$$
\begin{aligned}
U_{14} &= \frac{\mathrm{i}\sqrt{2}\alpha^2\gamma}{8N} J_- F^{-1}\{\sinh(A_+)G_+^{-1} - \sinh(A_-)G_-^{-1}\}J_-,\\
U_{24} &= \frac{\alpha\gamma}{4\sqrt{N}} F^{-1}\{-\cosh(A_+) + \cosh(A_-)\}J_-,\\
U_{34} &= \frac{\mathrm{i}\alpha}{2\sqrt{2N}} F^{-1}\{\sinh(A_+)C_- G_+^{-1} - \sinh(A_-)C_+ G_-^{-1}\}J_-,\\
U_{44} &= 1 - \frac{\alpha^2}{4N} J_+\{(-1+\cosh(A_-))C_+ F^{-1} G_-^{-2}\\
&\quad -(-1+\cosh(A_+))C_- F^{-1} G_+^{-2}\}J_-,
\end{aligned} \tag{6}
$$

here

$$
\begin{aligned}
A_\pm &= \frac{G_\pm t}{\sqrt{2}}, \quad C_\pm = \frac{2\alpha^2 J_z \pm 4FN + \gamma^2 N}{4N},\\
G_\pm &= \frac{1}{2}\sqrt{\frac{-\alpha^2}{N}(J_+J_- + J_-J_+) - \gamma^2 \pm 4F},\\
F &= \frac{1}{4}\sqrt{4\frac{\alpha^4}{N^2}J_z^2 + 2\frac{\alpha^2\gamma^2}{N}(J_+J_- + J_-J_+) + \gamma^4}.
\end{aligned}
$$

Therefore we can obtain the reduced density matrix as

$$
\rho_{\mathrm{SI}}(t) = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}, \tag{7}
$$

where

$$
\begin{aligned}
\rho_{11} &= \frac{1}{2}\left(\frac{1}{Z}\mathrm{tr_B}\, U_{44}U_{44}^\dagger \mathrm{e}^{-\beta H_\mathrm{B}} + \frac{1}{Z}\mathrm{tr_B}\, U_{41}U_{41}^\dagger \mathrm{e}^{-\beta H_\mathrm{B}}\right),\\
\rho_{22} &= \frac{1}{2}\left(\frac{1}{Z}\mathrm{tr_B}\, U_{34}U_{34}^\dagger \mathrm{e}^{-\beta H_\mathrm{B}} + \frac{1}{Z}\mathrm{tr_B}\, U_{31}U_{31}^\dagger \mathrm{e}^{-\beta H_\mathrm{B}}\right),\\
\rho_{33} &= \frac{1}{2}\left(\frac{1}{Z}\mathrm{tr_B}\, U_{24}U_{24}^\dagger \mathrm{e}^{-\beta H_\mathrm{B}} + \frac{1}{Z}\mathrm{tr_B}\, U_{21}U_{21}^\dagger \mathrm{e}^{-\beta H_\mathrm{B}}\right),\\
\rho_{44} &= \frac{1}{2}\left(\frac{1}{Z}\mathrm{tr_B}\, U_{14}U_{14}^\dagger \mathrm{e}^{-\beta H_\mathrm{B}} + \frac{1}{Z}\mathrm{tr_B}\, U_{11}U_{11}^\dagger \mathrm{e}^{-\beta H_\mathrm{B}}\right),\\
\rho_{14} &= \frac{1}{2}\frac{1}{Z}\mathrm{tr_B}\, U_{44}U_{11}^\dagger \mathrm{e}^{-\beta H_\mathrm{B}},\\
\rho_{23} &= \frac{1}{2}\left(\frac{1}{Z}\mathrm{tr_B}\, U_{34}U_{24}^\dagger \mathrm{e}^{-\beta H_\mathrm{B}} + \frac{1}{Z}\mathrm{tr_B}\, U_{31}U_{21}^\dagger \mathrm{e}^{-\beta H_\mathrm{B}}\right),
\end{aligned}
$$

with

$$
\rho_{14} = \rho_{41}^*, \quad \rho_{23} = \rho_{32}^*.
$$

3. Quantum coherence and correlation

The coherence properties of a quantum state are generally attributed to the off-diagonal elements of its density matrix relative to the selected reference basis. Baumgratz *et al*. proposed a strict framework to quantify coherence and the measure of l_1 norm of coherence was given, which is related to the non-diagonal elements of the density matrix. The l_1 norm of coherence is given by[4]

$$
C_{l_1}(\rho) = \sum_{i\neq j} |\, \rho_{i,j}\, |, \tag{8}
$$

where $\rho_{i,j}$ is the off-diagonal elements of ρ.

The concurrence can be used to measure the entanglement of two spins and given by[1]

$$
C = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{9}
$$

where the quantities λ_i $(i = 1,2,3,4)$ is the square roots of the eigenvalues of $\rho_{\mathrm{AB}}(\sigma_y \otimes \sigma_y)\rho_{\mathrm{AB}}^*(\sigma_y \otimes \sigma_y)$ in descending order.

Quantum discord was proposed[2,3] to capture the non-classical correlation in a system and is defined as the difference between total correlation and classical correlation

$$
D(\rho_{\mathrm{AB}}) = I(\rho_{\mathrm{AB}}) - C(\rho_{\mathrm{AB}}), \tag{10}
$$

where $I(\rho_{\mathrm{AB}})$ can be used as a measure of total correlation that includes quantum correlation and classical correlation and given by

$$
I(\rho_{\mathrm{AB}}) = S(\rho_\mathrm{A}) + S(\rho_\mathrm{B}) - S(\rho_{\mathrm{AB}}), \tag{11}
$$

where $S(\rho) = -\mathrm{tr}(\rho\log_2\rho)$ is the von Neumann entropy and ρ_A or ρ_B is the reduced density matrix of ρ_{AB}. The classic correlation is given by

$$
C(\rho_{\mathrm{AB}}) = \sup_{\{B_k\}} I(\rho_{\mathrm{AB}} \mid \{B_k\}), \tag{12}
$$

based on the given measurement basis B_k on subsystem B, where a variant of quantum mutual information is given as

$$
I(\rho_{\mathrm{AB}} \mid \{B_k\}) = S(\rho_\mathrm{A}) - \sum_k p_k S(\rho_k). \tag{13}
$$

After measurement the state of subsystem A change to $\rho_k = \frac{1}{p_k}(I\otimes B_k)\rho_{\mathrm{AB}}(I\otimes B_k)$, obtaining the outcome k on B with the probability $p_k = \mathrm{tr}(I\otimes B_k)\rho_{\mathrm{AB}}(I\otimes B_k)$. By making use of the method with simplifying complex optimization process,[31] we can obtain

$$
D(\rho_{\mathrm{AB}}) = S(\rho_\mathrm{B}) - S(\rho_{\mathrm{AB}}) + \min(N_1, N_2), \tag{14}
$$

where

$$
\begin{aligned}
S(\rho_\mathrm{B}) &= -(\rho_{11}+\rho_{33})\log_2(\rho_{11}+\rho_{33})\\
&\quad -(\rho_{22}+\rho_{44})\log_2(\rho_{22}+\rho_{44}), \qquad (15)\\
S(\rho_{\mathrm{AB}}) &= -\sum_{i=1}^{4}\lambda_i\log_2\lambda_i, \qquad (16)
\end{aligned}
$$

and

$$
\begin{aligned}
N_1 &= -x\log_2 x - (1-x)\log_2(1-x), \qquad (17)\\
N_2 &= -\rho_{11}\log_2\left(\frac{\rho_{11}}{\rho_{11}+\rho_{33}}\right) - \rho_{22}\log_2\left(\frac{\rho_{22}}{\rho_{22}+\rho_{44}}\right)\\
&\quad -\rho_{33}\log_2\left(\frac{\rho_{33}}{\rho_{33}+\rho_{11}}\right) - \rho_{44}\log_2\left(\frac{\rho_{44}}{\rho_{44}+\rho_{22}}\right), \qquad (18)
\end{aligned}
$$

here λ_i is the eigenvalues of ρ_{AB},

$$
x = \frac{1+\sqrt{(\rho_{11}-\rho_{44}+\rho_{22}-\rho_{33})^2 + 4(|\,\rho_{14}\,| + |\,\rho_{23}\,|)^2}}{2}.
$$

4. Results and discussion

We study the evolutive behaviors of concurrence, quantum discord, and quantum coherence by numerical simulations. The dynamic evolutions of them with different ratios of γ and α are plotted in Figs. 2 and 3. When the coupling strength of two spins exceeds the interaction strength between the two qubits system and the spin bath, *i.e.*, $\gamma/\alpha > 1$, we find that entanglement shows sudden death phenomenon,[32] while quantum discord dies slowly after a period of oscillation and its survival time increases as the γ/α ratio becomes large, as shown in Fig. 2. Coherence keeps oscillating at high levels in the same time. From Fig. 2, we can clearly find that when both entanglement and quantum discord are zero, coherence still exists and remains quantum properties. When the coupling strength of two spins is less than or equal to the interaction strength between the two qubits system and the spin bath, *i.e.*, $\alpha/\gamma \geq 1$, entanglement still exhibits sudden death phenomenon as shown in Fig. 3. With the increase of the α/γ ratio, the coherence and correlation of the system will be destroyed. The strong interaction between the system and the bath can further accelerate the death of quantum entanglement and discord. The survival time of l_1 norm of coherence is longer than that of quantum discord. Therefore, l_1 norm of coherence is the most robust in the three ways to measure quantum properties.[26]

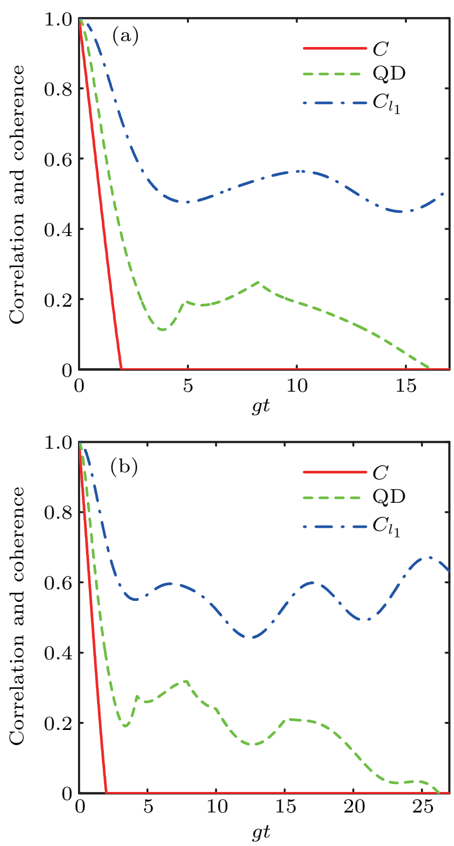

Fig. 2. The quantum entanglement C (red solid line), quantum discord QD (green dashed line), and the l_1 norm of coherence C_{l_1} (blue dot–dashed line) as a function of scaled time gt with the different coupling ratio γ/α, (a) $\gamma/\alpha = 1.25$, (b) $\gamma/\alpha = 1.5$. Other parameters: $\alpha = g$, $N = 50$, $\beta = 1$.

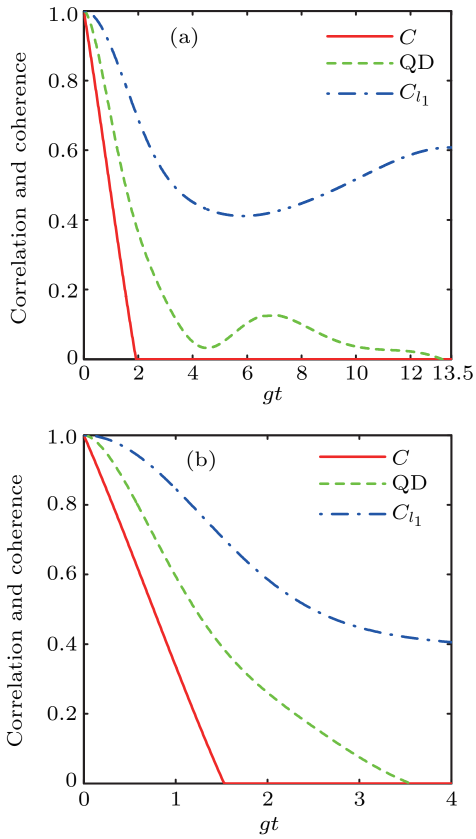

Fig. 3. The quantum entanglement C (red solid line), quantum discord QD (green dashed line), and the l_1 norm of coherence C_{l_1} (blue dot–dashed line) as a function of scaled time gt with the different coupling ratio α/γ, (a) $\alpha/\gamma = 1$, (b) $\alpha/\gamma = 1.25$. Other parameters: $\gamma = g$, $N = 50$, $\beta = 1$.

Figure 4 shows the effects of the number of spins in the bath on concurrence, quantum discord, and l_1 norm of coherence. When N increases from 20 to 50 even 100, surprisingly, entanglement is revival for a while after death due to the increase of the γ/α ratio compared with Fig. 2 and its dynamics are not sensitive for different N. At the same time, we can also know that the survival time of quantum discord is extended when N increases from 20 to 50. However, it is not sensitive to the change of N when N increases from 50 to 100. On the other hand, coherence remains oscillating and does not show too much change for different N.

To show the effects of temperatures on concurrence, quantum discord and l_1 norm of coherence, we plotted the dynamics of them in different β cases in Fig. 5. We can find that quantum entanglement, quantum discord and l_1 norm of coherence all show the decaying behavior of the oscillations. As the temperature decreases, the decay becomes slow. At the same temperature, the decay of quantum discord is slower than entanglement. Compared with entanglement and quantum discord, the decay of coherence is the slowest. It is clear that low temperature is beneficial to the maintenance of quantum correlation and coherence. Furthermore, figure 5 shows that l_1 norm of coherence is better than entanglement and quantum discord in terms of resistance to temperature effects.

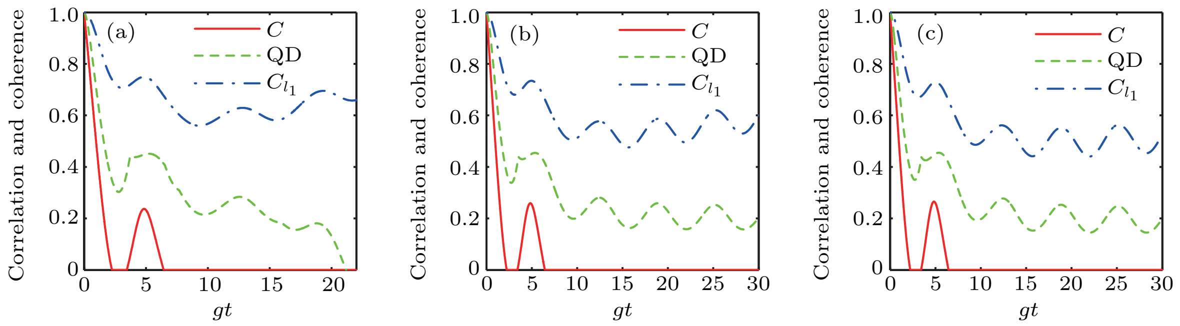

Fig. 4. The quantum entanglement C (red solid line), quantum discord QD (green dashed line), and the l_1 norm of coherence C_{l_1} (blue dot–dashed line) as a function of scaled time gt with different N, (a) $N=20$, (b) $N=50$, (c) $N=100$. Other parameters: $\alpha=g$, $\gamma/\alpha=2$, $\beta=1$.

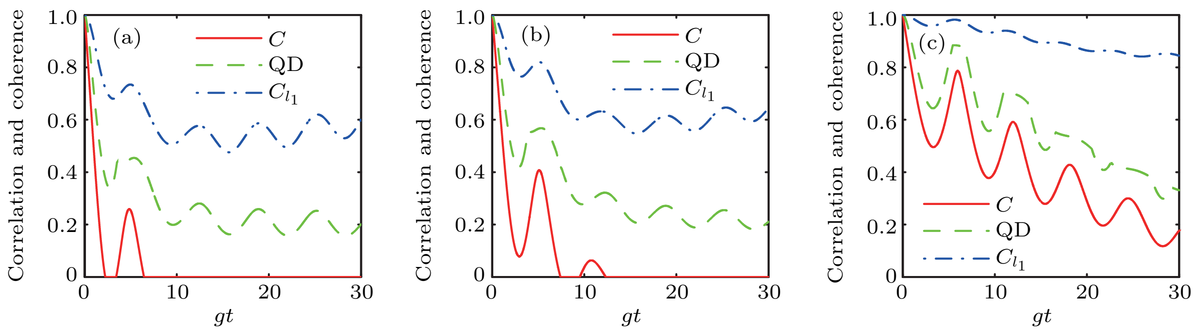

Fig. 5. The quantum entanglement C (red solid line), quantum discord QD (green dashed line), and the l_1 norm of coherence C_{l_1} (blue dot–dashed line) as a function of scaled time gt with different β, (a) $\beta=1$, (b) $\beta=10$, (c) $\beta=100$. Other parameters: $\alpha=g$, $\gamma/\alpha=2$, $N=50$.

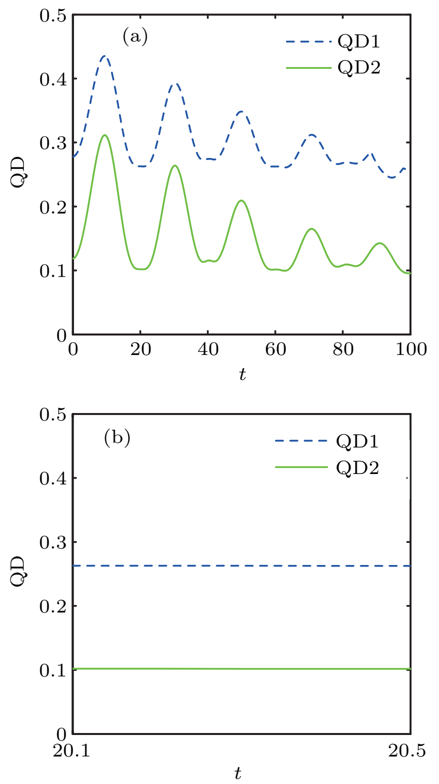

Fig. 6. The quantum discord QD as a function of time t, (a) QD1 (blue dashed line): $c_1=1$, $c_2=-0.6$, $c_3=0.6$, and QD2 (green solid line): $c_1=1$, $c_2=-0.4$, $c_3=0.4$. (b) The freezing phenomenon in panel (a) is magnified. Other parameters: $\gamma/\alpha=3$, $N=50$, $\beta=1$, $g=0.1$.

Without loss of generality, the initial state can also be prepared in Bell diagonal state, which can be written as $\rho_{AB}(0)=(1/4)(\boldsymbol{I}+\sum_{i=1}^{3}c_i\boldsymbol{\sigma}_A^i\otimes\boldsymbol{\sigma}_B^i)$. The components of the time evolution operator U acting on the basis $|01\rangle$ and $|10\rangle$ are given in Appendix A. Quantum discord shows a damped oscillations as a function of time and the freezing phenomenon between two adjacent peaks as shown in Fig. 6. The frozen quantum discord has been reported in Ref. [33]. In addition, we can find that QD1 is always larger than QD2 so it is important to choose an appropriate parameter to get a larger value of frozen quantum discord. On the other hand, the γ/α ratio can significantly affect the evolution behavior of coherence. When $\gamma/\alpha=1$, the oscillations of coherence are very slow. When $\gamma/\alpha=3$, the oscillation period of coherence becomes much shorter than before.

The relationship between quantum discord and entanglement has been extensively studied in various systems in the past.[34–36] The feature that quantum discord is more robust than entanglement is also widely found. Because the entanglement measure only reflects a part of the nature of quantum correlation, and the discord more reflects the global characteristics of quantum correlation. In quantum information resource theory, the relationship among quantum coherence, discord and entanglement is extremely close. Quantum coherence originates from the superposition principle of quantum states, the existence of quantum coherence is the precondition for the survival of quantum discord and entanglement, which explains why quantum coherence still exists when both the discord and entanglement are zero. As we know, a quantum system interacting with the environment can cause decoherence of quantum states. From the expression of l_1 norm of coherence, we can know that the decoherence of the quantum states is reflected in the disappearance of the off-diagonal elements of the density matrix. When all the off-diagonal elements of the density matrix of a quantum system disappear, the quantum system is no longer coherent at this time due to the influence of environment.

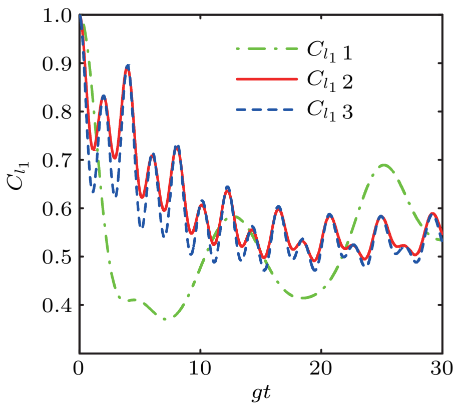

Fig. 7. The l_1 norm of coherence C_{l_1} as a function of scaled time gt. C_{l_1} 1 (green dot–dashed line): $\gamma/\alpha = 1$, $c_1 = 1$, $c_2 = -0.6$, $c_3 = 0.6$; C_{l_1} 2 (red solid line): $\gamma/\alpha = 3$, $c_1 = 1$, $c_2 = -0.6$, $c_3 = 0.6$; C_{l_1} 3 (blue dashed line): $\gamma/\alpha = 3$, $c_1 = 1$, $c_2 = -0.4$, $c_3 = 0.4$. Other parameters: $N = 50$, $\beta = 1$, $g = 0.1$.

5. Conclusions

In this paper, the quantum entanglement, discord, and coherence dynamics of two qubits system in the model of a spin coupled to a spin bath through an intermediate spin are studied. Our results show that, when the coupling strength of two spins exceeds the interaction strength between the system and the bath, the larger the γ/α ratio, the better the coherence and correlation of the system. When the coupling strength of two spins is less than the interaction strength between the system and the bath, the larger the α/γ ratio, the faster the coherence and correlation dynamics of the system disappear. When N reaches a certain number, the coherence and correlation dynamics of the system are not sensitive to the number of spins in the bath. The lower the temperature, the more favorable it is to maintain the quantum coherence and correlation of the system. The freezing phenomenon of quantum discord can be observed whereas l_1 norm of coherence does not when the initial state is Bell-diagonal state. In addition, quantum discord is more robust than entanglement, and l_1 norm of coherence is the most robust of the three due to the existence of quantum coherence is the precondition for the survival of quantum discord and entanglement. Our results are also instructive for a comprehensive understanding of the relationship between the quantum coherence and correlation for open quantum system.

Appendix A

The components of the time evolution operator are given as follows:[29]

$$U|01\rangle = U_{43}|00\rangle + U_{33}|01\rangle + U_{23}|10\rangle + U_{13}|11\rangle,$$
$$U|10\rangle = U_{42}|00\rangle + U_{32}|01\rangle + U_{22}|10\rangle + U_{12}|11\rangle,$$

$$U_{43} = \frac{\mathrm{i}\alpha}{2\sqrt{2N}} J_+ F^{-1}\{\sinh(A_+)C_- G_+^{-1} - \sinh(A_-)C_+ G_-^{-1}\},$$
$$U_{33} = U_{22} = \frac{1}{2} F^{-1}\{-\cosh(A_+)C_- + \cosh(A_-)C_+\},$$
$$U_{23} = U_{32} = \frac{\mathrm{i}\gamma}{2\sqrt{2}} F^{-1}\{-\sinh(A_+)G_+ + \sinh(A_-)G_-\},$$
$$U_{13} = \frac{\alpha\gamma}{4\sqrt{N}} J_- F^{-1}\{-\cosh(A_+) + \cosh(A_-)\},$$
$$U_{42} = \frac{\alpha\gamma}{4\sqrt{N}} J_+ F^{-1}\{-\cosh(A_+) + \cosh(A_-)\},$$
$$U_{12} = \frac{\mathrm{i}\alpha}{2\sqrt{2N}} J_- F^{-1}\{\sinh(A_+)C_- G_+^{-1} - \sinh(A_-)C_+ G_-^{-1}\}.$$

The other variables have given before.

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