Discharge flow of granular particles through an orifice on a horizontal hopper: Effect of the hopper angle*

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We experimentally investigate the effect of the hopper angle on the flow rate of grains discharged from a two-dimensional horizontal hopper on a conveyor belt. The flow rate grows with the hopper angle, and finally reaches a plateau. The curve feature appears to be similar for different orifice widths and conveyor belt-driven velocities. On the basis of an empirical law of flow rate for a flat-bottom hopper, we propose a modified equation to describe the relation between the flow rate and hopper angle, which is in a good agreement with the experimental results.

Keywords: granular flow, flow rate, hopper angle, Beverloo law

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1. Introduction

The improvement in flow rate of discrete particles passing through a bottleneck is of importance for plenty of fields, widely ranging from industrial transportation, traffic flow to evacuation of panicking crowds.[1–3] Usually, controlling the orifice size is a direct and convenient way to obtain the desired flow rate. However, when the orifice size cannot be changed in some particular situations, the optimization of flow rate under this constraint is a practical issue worthy of study.

In a vertical gravity-driven hopper flow, some methods have been adopted to improve the flow rate. Obviously, widening the orifice is the most effective way. It is found that the packing fraction and velocity of particles near the orifice simultaneously increase with the orifice size.[4] Changing the exit position can also increase the flow rate. Numerical results show that the flow rate remains constant when the exit is far from the wall and increases exponentially when the exit moves close to the wall.[5] Shaking the hopper is another method frequently used to enhance the flow rate.[6,7] By an external vibration, the arches near the orifice that block the flow become unstable and are readily broken. Also, an obstacle placed above an orifice with an optimal distance is used to suppress the formation of arches.[8–12] When the obstacle size and position are properly chosen, the flow rate can be increased up to 10%. The hopper angle has been proved to be another effective control parameter for the enhancement of the flow rate.[13–17] Based on an hourglass theory, a formula was derived for the flow rate of a narrow-angled hopper.[1] The formula gives an excellent qualitative prediction of the measured dependence of the mass flow rate on the density, orifice diameter, hopper angle, and angle of internal friction. However, it does not provide an accurate quantitative prediction, and the theoretical value from the formula is about twice the experimental measurement. Two of the reasons for the discrepancy are the assumptions that the hopper walls are smooth, and that the stresses fall to zero on a free-fall arch. The concept of free-fall arch is not proved by experiment and this could lead to unquantifiable uncertainties in the prediction. Later, the flow rate from an angled hopper with an obstacle above the orifice was further investigated on the basis of the researches above. The optimal flow rate was increased by up to 16% compared with that without an obstacle. Using the hourglass theory and a velocity-density relation, an empirical law that relates the flow rate, hopper angle and the obstacle position was proposed.[19]

For a horizontal hopper flow, the previous work mainly focused on a flat-bottom hopper. The orifice size, the driven velocity, and the initial packing fraction of particles have been considered as key factors for the improvement of the flow rate.[20–22] The effect of obstacles has been explored, showing that the obstacles on both sides of an orifice can effectively increase the flow rate by up to 15.6%.[24] In contrast to the intensive studies on the effect of hopper angles in the vertical gravity-driven system, the flow rate dependence on the hopper angle in a horizontal system still remains unknown, although this type of hopper flow, such as that driven by a conveyor belt, has wide applications in industry. In addition, traffic jam[25] or emergency evacuation management[3] could even draw inspiration from the optimal design of the orifice geometry.

In this work, our aim is to present a parametric study of a granular flow driven by a conveyor belt through a horizontal silo with different hopper angles. We experimentally report the effect of the hopper angle on the flow rate. The flow rate

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dependence is measured with different orifice sizes and conveyor belt-driven velocities. On the basis of an empirical law of flow rate from a flat-bottom vertical hopper, a modified law is proposed to directly relate the flow rate to the hopper angles. The potential application for the unjamming of particles near an orifice by means of the hopper angle is briefly discussed.

2. Experimental details

The experimental establishment consists of a conveyor belt above which a two-dimensional, hourglass-shaped frame is fixed, with 1750 Plexiglas disk particles confined in this frame, as shown in Fig. 1. A motor drives the belt at a constant velocity ranging from 3 cm/s to 15 cm/s. A rectangular orifice with width D is located at the center of the bottom wall of the frame. The hopper angle between the horizontal and the hopper bottom can be varied from $\theta = 0^\circ$ (flat-bottom hopper) to $\theta = 90^\circ$ (pipeline hopper). The initial packing fraction of the dense-packing disks is set to be $\phi_0 = 0.86 \pm 0.02$ by using a previous protocol.[24] A steady flow status is always maintained since the orifice is larger than a critical size at which clogging occurs ($D \geqslant 6d$, d is the diameter of the disk particle).[23]

A high-resolution video camera hung above the frame records the temporal evolution of the disk number inside the frame, and thus an average flow rate during the steady flow is extracted as before.[20,24] The measurement is repeated 6 times for each data point, and the flow rate is computed as an arithmetic average of these trials.

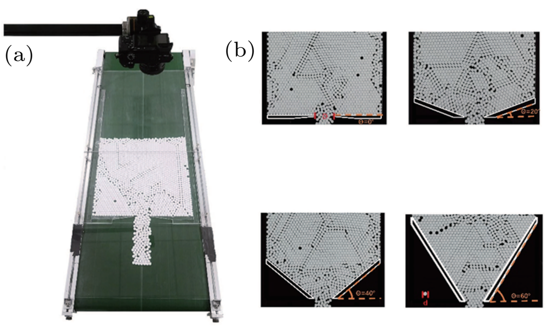

Fig. 1. (a) Schematic diagram of the experimental establishment. Disk particles on a conveyor belt are driven to flow through an orifice. A high-resolution camera is used to record the number of the particles in the frame. (b) Illustration of the hopper angle. By adjusting the position of the bottom wall (highlighted by white lines), different hopper angles from 0° to 80° can be obtained. $d = 11$ mm is the diameter of the disk particle used in the experiment.

3. Results and discussion

We report that the dependence of the flow rate on the hopper angle with a given orifice width $D = 10d$, as shown in Fig. 2(a). Here we examine the dependence curve for a wide range of belt-driven velocities; these flow rate–hopper angle curves look similar and the curve feature is robust: the flow rate monotonically increases with the hopper angle, until it reaches a plateau. In order to show the dependent feature more clearly, the data in Fig. 2(a) are normalized by the belt-driven velocity (Fig. 2(b)). Compared with the flow rate from a flat-bottom hopper ($\theta = 0^\circ$), the flow rate increases by about 13%–15% when $\theta > 50^\circ$. It is worth pointing out that the improvement in the flow rate solely due to the geometrical modification of the orifice is not a marginal effect. This is because the theoretical maximal value of the flow rate is $Q = \frac{4\phi V D}{\pi d^2}$,[20,23] where ϕ is the initial packing fraction of the dense-packing particles (note ϕ is not the packing fraction of flowing particles near the orifice), V the belt velocity, D the orifice width, and d the particle diameter. If the orifice size and the belt velocity are fixed, the theoretically ideal flow rate is just 23% larger than that for a flat-bottom hopper, where the flow rate of a flat-bottom hopper $Q = \frac{4\phi V}{\pi d}(\frac{D}{d} - k)$,[20,23] $k = 1.86$ for our measurement. Generally, the value of k was in a range of 1–3.5 in previous literature. The value of k has been found to be dependent on the particle and hopper properties in a range of $1 < k < 2$.[26] If the flowing particles consist of sands, the value of k can reach 2.9.[27] For a horizontal flat-bottom hopper flow system, $k = 2.1 \pm 0.2$ is similar to that in our system.[20] If the conveyer belt velocity is very large, the flow rate is related to the friction between the conveyer belt and flowing particles, and $k = 3.23$.[28]

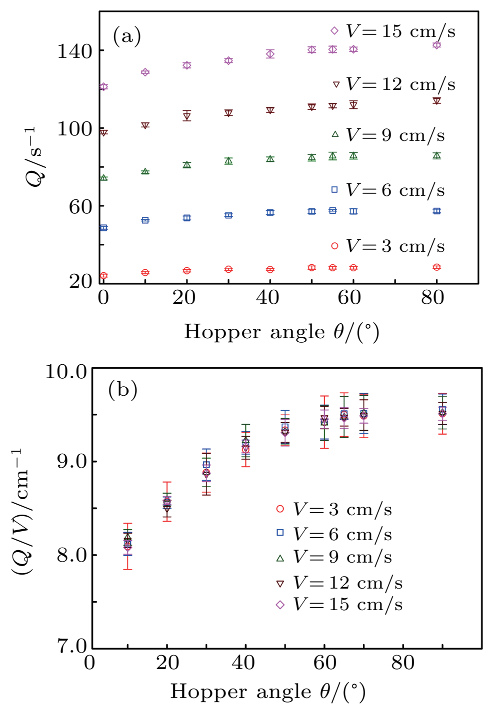

Fig. 2. (a) The flow rate Q versus hopper angle θ for different conveyor belt velocities. The orifice width $D = 10d$. The size of the error bars is almost the same as that of the symbols. (b) Dependence of normalized flow rate on the hopper angle shows the curve feature in (a) more clearly.

In the previous literatures, an empirical law equivalent to the 2D Beverloo's law,[23] $Q = \frac{4\phi V}{\pi d}(\frac{D}{d} - k)$, was proposed to describe the flow rate of particles on a conveyor belt passing through an orifice.[20,23,24] However, the law is only applicable to a flat-bottom horizontal hopper on a conveyor belt. To interpret the results in Fig. 2, we adopt a modified model for a two-dimensional flat-bottom vertical hopper proposed by Mancok and Janda *et al.*[4,29] The flow rate Q in this model is derived based on the self-similar velocity and density profiles at the orifice, $Q = C'\sqrt{gR}\phi_0(1-\alpha e^{-\beta R})R$, where C' is a constant depending on the particle diameter and on the curvature of the density profile, R is the half-width of the orifice, ϕ_0 is the asymptotic value of the packing fraction for big orifices, α and β are fitting parameters. Due to the similar self-similarity in the velocity and density profiles at the orifice for the horizontal hopper (see the supplementary materials), we assume that the flow rate in the two-dimensional flat-bottom horizontal hopper shares the same mathematical form as that in the vertical hopper,

$$Q = C' v_b \phi_0 (1-\alpha e^{-\beta D})D, \tag{1}$$

where C', ϕ_0, α, and β have the same physical meanings as those in the vertical hopper model, $D = 2R$ is the orifice width, and v_b is the belt velocity. To examine the speculation, the reduced flow rate $Q^* = \frac{Q}{v_b \phi_0 D}$ is plotted as a function of the orifice width in units of the particle diameter d, ($d = 1.1$ cm is a constant in our experiment), as shown in Fig. 3. The solid line in this figure is the fitting curve with an exponential form: $C'(1-\alpha e^{-\beta D/d})$, where $C' = 0.95$, $\alpha = 0.74$, and $\beta = 0.19$. An excellent agreement between the experimental flow rate and the prediction in Eq. (1) confirms the assumption that the flow rate can be well described by Eq. (1).

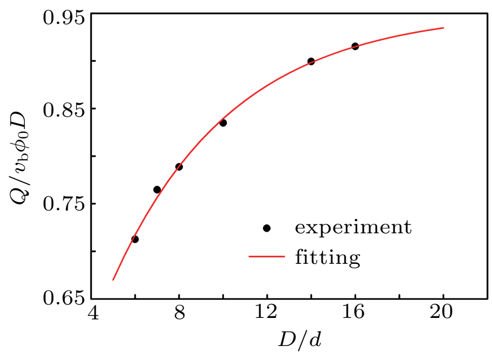

Fig. 3. Reduced flow rate Q^* as a function of the orifice width D in units of the particle diameter d. The solid line is a fitting curve which follows the form of an exponential function $C'(1-\alpha e^{-\beta D/d})$, where $C' = 0.95$, $\alpha = 0.74$, and $\beta = 0.19$.

On the basis of the modified flow rate expression for the two-dimensional flat-bottom vertical hopper $Q = C'\sqrt{gR}\phi_0(1-\alpha e^{-\beta R})R$, a physical model to describe the effect of the hopper angle on the flow rate has been proposed.[16] In this model, the hopper angle is considered to be influential in the radius of curvature of an arch $\tilde{R} = R/\cos\theta$ at the orifice. The new flow rate expression is $Q = C'\sqrt{gR}\phi_0(1-\alpha e^{-\frac{\beta R}{d\cos\theta}})R$, where d is the particle diameter and θ is the hopper angle. Due to the similar geometrical feature at the orifice between the horizontal and vertical systems, we assume that the mathematical form of this law still holds for the horizontal hopper,

$$Q = C' v_b \phi_0 \left(1-\alpha e^{-\frac{\beta D}{d\cos\theta}}\right) D. \tag{2}$$

In Fig. 4, the experimental flow rates for different hopper angles and orifice sizes are scaled to compare with the prediction of Eq. (2). The data points collapse fairly well and reveal an excellent agreement with the fitting curve from Eq. (2), which suggests the expression in Eq. (2) can describe the flow rate well for all hopper angles.

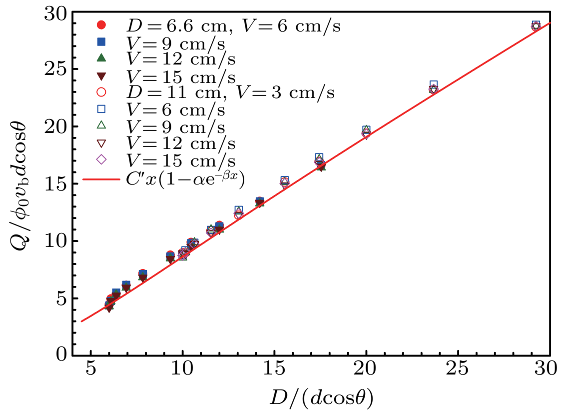

Fig. 4. Scaled flow rate $Q^* = \frac{Q}{\phi_0 v_b d\cos\theta}$ as a function of $D/(d\cos\theta)$ for different belt velocities and orifice sizes. The solid symbols are for an orifice width $D = 6d = 6.6$ cm, and the hollow symbols for $D = 10d = 11$ cm. All data points collapse together to form a master curve. The solid curve is a fitting line which follows an exponential function $C'x(1-\alpha e^{-\beta x})$, where $x = \frac{D}{d\cos\theta}$, $C' = 0.95$, $\alpha = 0.74$, and $\beta = 0.19$.

Recent work pointed out that the hopper angle may significantly affect the clogging probability of a particle through an orifice. By increasing the hopper angle, the clogging probability can be reduced by three orders of magnitude.[26] Therefore, the increase of the hopper angle leads to a resultant decrease of temporary and permanent clogging arches which consist of a few clogging particles developing at the orifice. An appropriate increase of the hopper angle will probably maintain the flowing status of particles without a complete arrest, even if the orifice size is smaller than the clogging size in a flat-bottom hopper.

4. Conclusion

We present experimental results of the effect of the hopper angle on the flow rate of grains discharged from a two-dimensional horizontal silo on a conveyor belt. The flow rate grows with the hopper angle, and finally reaches a plateau. Based on an original empirical law, we propose a modified equation which can well describe the dependence of the flow rate on the hopper angle within a wide parameter range. The results reported in this work show a practical method for the improvement of the flow rate through a bottleneck, especially for the case where the orifice size and driven velocity have to be restricted. In addition, an increase in the hopper angle will probably be beneficial to maintain flowing of discrete objects through a narrow exit even when the bottleneck is smaller than a critical size of clogging in a flat-bottom hopper. The feature is probably not only applicable to the conveyor belt system, but to other systems of discrete particles flowing through a constriction such as pedestrian, active agents, and mirco- or nano-particles.

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