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Efficient scheme for remote preparation of arbitrary n-qubit equatorial states

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Recently, a scheme for deterministic remote preparation of arbitrary multi-qubit equatorial states was proposed by Wei *et al.* [*Quantum Inf. Process.* **17** 70 (2018)]. It is worth mentioning that the construction of mutual orthogonal measurement basis plays a key role in quantum remote state preparation. In this paper, a simple and feasible remote preparation of arbitrary n-qubit equatorial states scheme is proposed. In our scheme, the success probability will reach unit. Moreover, there are no coefficient constraint and auxiliary qubits in this scheme. It means that the success probabilities are independent of the coefficients of the entangled channel. The advantage of our scheme is that the mutual orthogonal measurement basis is devised. To accomplish the quantum remote state preparation (RSP) schemes, some new sets of mutually orthogonal measurement basis are introduced.

Keywords: mutual orthogonal measurement basis, remote state preparation, equatorial states, success probability

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1. Introduction

In quantum communication and quantum computation, quantum entanglement is a unique resource of the implementation. So far, some schemes have been proposed experimentally and theoretically to solve the problem of multi-mode entanglement works, such as multi-mode squeezing of parametrically amplified multi-wave mixing in atomic ensemble or atomic-like medium.[1–4] Quantum communication technology is considered as an important technology in the field of quantum information, such as quantum teleportation (QT),[5–12] remote state preparation (RSP),[13–26] quantum key distribution (QKD),[27–29] and so on. In RSP, the sender can transmit a known state to the receiver and it has less classical communication cost than QT. In order to improve the security in RSP, a variety of schemes of controlled RSP (CRSP),[30–37] joint RSP (JRSP),[38–48] controlled bidirectional RSP (CBRSP),[49–53] and controlled JRSP (CJRSP)[54–56] are presented.

In recent years, many protocols have been proposed for preparing equatorial states. In 2011, Chen *et al.*[57] proposed a probabilistic joint remote preparation of a two-particle high-dimensional equatorial state. In 2015, Choudhury *et al.*[58] presented a joint RSP scheme for preparing two-qubit equatorial states. At the same year, a scheme for joint remote preparation of multi-qubit equatorial states was proposed by Li *et al.*[59] A year later, Wei *et al.*[60] proposed a deterministic RSP scheme to remotely prepare arbitrary multi-qubit equatorial states. Recently, Sun *et al.*[53] realized an asymmetric bidirectional transmission of two- and four-qubit equatorial states synchronously.

Inspired by some RSP schemes in Refs. [53,58–60], we present a scheme for remotely preparing arbitrary multi-qubit equatorial states from the sender Alice to the receiver Bob, in which n two-qubit maximally entangled states are used as the quantum channel. The characteristics of our scheme is that we find a simple form of measurement basis to realize the RSP.

The rest of this paper is organized as follows. In Section 2, an efficient scheme for remotely preparing n-qubit equatorial states is presented via n two-qubit maximally entangled states. Moreover, we demonstrate some concrete examples of preparing equatorial states to make our scheme clear and feasible. Finally, a brief discussion and a concluding summary are drawn in Section 3.

2. Remote preparation of n-qubit equatorial states

In order to interpret our protocol better, now we begin to realize the remote preparation of arbitrary n-qubit equatorial states. Suppose there are two participants named Alice and Bob, the former is the sender and the latter is the receiver, who are situated in separated sites. Presume that Alice wants to help Bob prepare arbitrary n-qubit equatorial states. Generally, it has the form as given below

$$|\chi\rangle = \frac{1}{(\sqrt{2})^n}\sum_{v=0}^{2^n-1}(\mathrm{e}^{\mathrm{i}\theta_v}|g_n\cdots g_2 g_1\rangle),$$
$$g_l \in \{0,1\}, \quad v=\sum_{l=1}^{n} g_l\cdot 2^{l-1}. \tag{1}$$

In the above formula, the decimal form of the binary state $g_n\cdots g_2 g_1$ is v and the range of θ_v ($v=0,1,\ldots,2^n-1$) is

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$0 \le \theta_v \le 2\Pi$.[60] Moreover, θ_0 is set to be zero. The maximum entangled channel which composes of two-qubit states can be expressed as

$$|\psi\rangle_{A_nB_n} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_nB_n}, \quad n = 1,2,3,\ldots,n, \tag{2}$$

where Alice possesses the particle A, the particle B belongs to Bob. Alice needs to perform n-qubit projective measurements on her particles $(1,2,3,\ldots,n)$ which, under the mutual orthogonal measurement bases $\{|\Gamma_d\rangle|d = 1,2,\ldots,2^n\}$, can be expressed as

$$\begin{aligned}
&|\Gamma_1\rangle_{A_1A_2\cdots A_n} = I_{A_1} \otimes I_{A_2} \otimes I_{A_3} \cdots \otimes I_{A_n}|\Gamma_1\rangle_{A_1A_2\cdots A_n},\\
&|\Gamma_2\rangle_{A_1A_2\cdots A_n} = \sigma_{A_{1z}} \otimes I_{A_2} \otimes I_{A_3} \cdots \otimes I_{A_n}|\Gamma_1\rangle_{A_1A_2\cdots A_n},\\
&|\Gamma_3\rangle_{A_1A_2\cdots A_n} = I_{A_1} \otimes \sigma_{A_{2z}} \otimes I_{A_3} \cdots \otimes I_{A_n}|\Gamma_1\rangle_{A_1A_2\cdots A_n},\\
&|\Gamma_4\rangle_{A_1A_2\cdots A_n} = I_{A_1} \otimes I_{A_2} \otimes \sigma_{A_{3z}} \cdots \otimes I_{A_n}|\Gamma_1\rangle_{A_1A_2\cdots A_n},\\
&\cdots\\
&|\Gamma_{2^n}\rangle_{A_1A_2\cdots A_n} = \sigma_{A_{1z}} \otimes \sigma_{A_{2z}} \otimes \sigma_{A_{3z}} \cdots \otimes \sigma_{A_{nz}}|\Gamma_1\rangle_{A_1A_2\cdots A_n}.
\end{aligned} \tag{3}$$

The whole system which consists of the entanglement state can be written as

$$\begin{aligned}
&|\psi\rangle_{A_1B_1} \otimes |\psi\rangle_{A_2B_2} \otimes \cdots \otimes |\psi\rangle_{A_nB_n}\\
&= \frac{1}{n}[|\Gamma_1\rangle_{A_1\cdots A_n}|\varphi_1\rangle_{B_1\cdots B_n} + \cdots + |\Gamma_{2^n}\rangle_{A_1\cdots A_n}|\varphi_{2^n}\rangle_{B_1\cdots B_n}].
\end{aligned} \tag{4}$$

Then Alice announces her measurement outcome through the classical channel, if Alice's measurement outcome is $|\Gamma_{2^n}\rangle_{A_1\cdots A_n}$, the state of particles $B_1\cdots B_n$ will collapse into $|\varphi_{2^n}\rangle_{B_1\cdots B_n}$. To fulfill the scheme, Bob would select the appropriate unitary operations $\sigma_{B_{1z}} \otimes \sigma_{B_{2z}} \otimes \cdots \otimes \sigma_{B_{nz}}$ to reconstruct the initial state. Analogously, for other cases, Bob can also employ appropriate unitary operations to recover the prepared state.

It can be obtained from the permutation and combination calculation formula that when Alice wants to help Bob prepare arbitrary n-qubit equatorial states, the possibility of σ_z appearing in the measurement bases can be presented as

$$C_n^0 + C_n^1 + C_n^2 + \cdots + C_n^n = 2^n. \tag{5}$$

For example, when $n = 3$, the possibility of σ_z appearing in the measurement bases is 8. So in this formula, it can be found that there are 2^n ways to use the σ_z to make the different measurement bases.

In order to show the clarity of our protocol, meanwhile, we discuss the remote preparation of three-qubit and four-qubit equatorial states respectively.

2.1. Examples of remote preparation of three-qubit equatorial state

Suppose the sender Alice would like to help the receiver Bob prepare the following three-qubit equatorial state

$$|\chi\rangle = \frac{1}{2\sqrt{2}}(e^{i\theta_0}|000\rangle + e^{i\theta_1}|001\rangle + e^{i\theta_2}|010\rangle + e^{i\theta_3}|011\rangle + e^{i\theta_4}|100\rangle + e^{i\theta_5}|101\rangle + e^{i\theta_6}|110\rangle + e^{i\theta_7}|111\rangle). \tag{6}$$

The quantum channel which between Alice and Bob is three two-qubit entangled states

$$|\psi\rangle_{A_1A_2A_3B_1B_2B_3} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_1B_1}\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_2B_2}\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_3B_3}. \tag{7}$$

Now, Alice measures the particles A_1, A_2, and A_3 under the basis

$$\{|\Gamma_1\rangle_{A_1A_2A_3}, |\Gamma_2\rangle_{A_1A_2A_3}, |\Gamma_3\rangle_{A_1A_2A_3}, |\Gamma_4\rangle_{A_1A_2A_3}, |\Gamma_5\rangle_{A_1A_2A_3}, |\Gamma_6\rangle_{A_1A_2A_3}, |\Gamma_7\rangle_{A_1A_2A_3}, |\Gamma_8\rangle_{A_1A_2A_3}\},$$

in which the bases are given by

$$\begin{aligned}
|\Gamma_1\rangle_{A_1A_2A_3} &= \frac{1}{2\sqrt{2}}(e^{-i\theta_0}|000\rangle + e^{-i\theta_1}|001\rangle + e^{-i\theta_2}|010\rangle + e^{-i\theta_3}|011\rangle\\
&\quad + e^{-i\theta_4}|100\rangle + e^{-i\theta_5}|101\rangle + e^{-i\theta_6}|110\rangle + e^{-i\theta_7}|111\rangle),\\
|\Gamma_2\rangle_{A_1A_2A_3} &= \sigma_{A_{1z}} \otimes I_{A_2} \otimes I_{A_3}|\Gamma_1\rangle_{A_1A_2A_3}, \qquad |\Gamma_3\rangle_{A_1A_2A_3} = I_{A_1} \otimes \sigma_{A_{2z}} \otimes I_{A_3}|\Gamma_1\rangle_{A_1A_2A_3},\\
|\Gamma_4\rangle_{A_1A_2A_3} &= I_{A_1} \otimes I_{A_2} \otimes \sigma_{A_{3z}}|\Gamma_1\rangle_{A_1A_2A_3}, \qquad |\Gamma_5\rangle_{A_1A_2A_3} = \sigma_{A_{1z}} \otimes \sigma_{A_{2z}} \otimes I_{A_3}|\Gamma_1\rangle_{A_1A_2A_3},\\
|\Gamma_6\rangle_{A_1A_2A_3} &= \sigma_{A_{1z}} \otimes I_{A_2} \otimes \sigma_{A_{3z}}|\Gamma_1\rangle_{A_1A_2A_3}, \qquad |\Gamma_7\rangle_{A_1A_2A_3} = I_{A_1} \otimes \sigma_{A_{2z}} \otimes \sigma_{A_{3z}}|\Gamma_1\rangle_{A_1A_2A_3},\\
|\Gamma_8\rangle_{A_1A_2A_3} &= \sigma_{A_{1z}} \otimes \sigma_{A_{2z}} \otimes \sigma_{A_{3z}}|\Gamma_1\rangle_{A_1A_2A_3}.
\end{aligned} \tag{8}$$

With the mutual orthogonal measurement bases Eq. (8), we can rewrite the entangled state as

$$\begin{aligned}
|\psi\rangle_{A_1A_2A_3B_1B_2B_3} = \frac{1}{2\sqrt{2}}[&|\Gamma_1\rangle_{A_1A_2A_3}|\varphi_1\rangle_{B_1B_2B_3} + |\Gamma_2\rangle_{A_1A_2A_3}|\varphi_2\rangle_{B_1B_2B_3} + |\Gamma_3\rangle_{A_1A_2A_3}|\varphi_3\rangle_{B_1B_2B_3} + |\Gamma_4\rangle_{A_1A_2A_3}|\varphi_4\rangle_{B_1B_2B_3}\\
&+ |\Gamma_5\rangle_{A_1A_2A_3}|\varphi_5\rangle_{B_1B_2B_3} + |\Gamma_6\rangle_{A_1A_2A_3}|\varphi_6\rangle_{B_1B_2B_3} + |\Gamma_7\rangle_{A_1A_2A_3}|\varphi_7\rangle_{B_1B_2B_3} + |\Gamma_8\rangle_{A_1A_2A_3}|\varphi_8\rangle_{B_1B_2B_3}].
\end{aligned} \tag{9}$$

After Alice makes the three-particle projective measurement on her particles A_1, A_2, and A_3 under the basis

$$\{|\Gamma_1\rangle_{A_1A_2A_3},\ |\Gamma_2\rangle_{A_1A_2A_3},\ |\Gamma_3\rangle_{A_1A_2A_3},\ |\Gamma_4\rangle_{A_1A_2A_3},\ |\Gamma_5\rangle_{A_1A_2A_3},\ |\Gamma_6\rangle_{A_1A_2A_3},\ |\Gamma_7\rangle_{A_1A_2A_3},\ |\Gamma_8\rangle_{A_1A_2A_3}\}.$$

Without loss of generality, suppose Alice's measurement result is $|\Gamma_3\rangle_{A_1A_2A_3}$, the state of particles B_1, B_2, B_3 will collapse into

$$|\varphi_3\rangle_{B_1B_2B_3} = \frac{1}{2\sqrt{2}}(\mathrm{e}^{\mathrm{i}\theta_0}|000\rangle + \mathrm{e}^{\mathrm{i}\theta_1}|001\rangle - \mathrm{e}^{\mathrm{i}\theta_2}|010\rangle - \mathrm{e}^{\mathrm{i}\theta_3}|011\rangle + \mathrm{e}^{\mathrm{i}\theta_4}|100\rangle + \mathrm{e}^{\mathrm{i}\theta_5}|101\rangle - \mathrm{e}^{\mathrm{i}\theta_6}|110\rangle - \mathrm{e}^{\mathrm{i}\theta_7}|111\rangle). \quad (10)$$

From Eq. (10), Bob executes appropriate unitary operations $I_{B_1} \otimes \sigma_{B_{2z}} \otimes I_{B_3}$ on his qubits B_1, B_2, and B_3 after receiving the result from Alice. The state which Alice wants to prepare can always be obtained. Here, Alice's measurement outcomes, Bob's collapse states, and the corresponding unitary operations are listed in Table 1.

Table 1. The measurement outcomes of Alice, Bob's collapse states and the unitary operators of Bob.

Alice's measurements	Bob's collapse states	Bob's unitary operations
$\lvert\Gamma_1\rangle_{A_1A_2A_3}$	$(1/2\sqrt{2})(\mathrm{e}^{\mathrm{i}\theta_0}\lvert000\rangle + \mathrm{e}^{\mathrm{i}\theta_1}\lvert001\rangle + \mathrm{e}^{\mathrm{i}\theta_2}\lvert010\rangle + \mathrm{e}^{\mathrm{i}\theta_3}\lvert011\rangle$ $+\mathrm{e}^{\mathrm{i}\theta_4}\lvert100\rangle + \mathrm{e}^{\mathrm{i}\theta_5}\lvert101\rangle + \mathrm{e}^{\mathrm{i}\theta_6}\lvert110\rangle + \mathrm{e}^{\mathrm{i}\theta_7}\lvert111\rangle)$	$I_{B_1} \otimes I_{B_2} \otimes I_{B_3}$
$\lvert\Gamma_2\rangle_{A_1A_2A_3}$	$(1/2\sqrt{2})(\mathrm{e}^{\mathrm{i}\theta_0}\lvert000\rangle + \mathrm{e}^{\mathrm{i}\theta_1}\lvert001\rangle + \mathrm{e}^{\mathrm{i}\theta_2}\lvert010\rangle + \mathrm{e}^{\mathrm{i}\theta_3}\lvert011\rangle$ $-\mathrm{e}^{\mathrm{i}\theta_4}\lvert100\rangle - \mathrm{e}^{\mathrm{i}\theta_5}\lvert101\rangle - \mathrm{e}^{\mathrm{i}\theta_6}\lvert110\rangle - \mathrm{e}^{\mathrm{i}\theta_7}\lvert111\rangle)$	$\sigma_{B_{1z}} \otimes I_{B_2} \otimes I_{B_3}$
$\lvert\Gamma_3\rangle_{A_1A_2A_3}$	$(1/2\sqrt{2})(\mathrm{e}^{\mathrm{i}\theta_0}\lvert000\rangle + \mathrm{e}^{\mathrm{i}\theta_1}\lvert001\rangle - \mathrm{e}^{\mathrm{i}\theta_2}\lvert010\rangle - \mathrm{e}^{\mathrm{i}\theta_3}\lvert011\rangle$ $+\mathrm{e}^{\mathrm{i}\theta_4}\lvert100\rangle + \mathrm{e}^{\mathrm{i}\theta_5}\lvert101\rangle - \mathrm{e}^{\mathrm{i}\theta_6}\lvert110\rangle - \mathrm{e}^{\mathrm{i}\theta_7}\lvert111\rangle)$	$I_{B_1} \otimes \sigma_{B_{2z}} \otimes I_{B_3}$
$\lvert\Gamma_4\rangle_{A_1A_2A_3}$	$(1/2\sqrt{2})(\mathrm{e}^{\mathrm{i}\theta_0}\lvert000\rangle - \mathrm{e}^{\mathrm{i}\theta_1}\lvert001\rangle + \mathrm{e}^{\mathrm{i}\theta_2}\lvert010\rangle - \mathrm{e}^{\mathrm{i}\theta_3}\lvert011\rangle$ $+\mathrm{e}^{\mathrm{i}\theta_4}\lvert100\rangle - \mathrm{e}^{\mathrm{i}\theta_5}\lvert101\rangle + \mathrm{e}^{\mathrm{i}\theta_6}\lvert110\rangle - \mathrm{e}^{\mathrm{i}\theta_7}\lvert111\rangle)$	$I_{B_1} \otimes I_{B_2} \otimes \sigma_{B_{3z}}$
$\lvert\Gamma_5\rangle_{A_1A_2A_3}$	$(1/2\sqrt{2})(\mathrm{e}^{\mathrm{i}\theta_0}\lvert000\rangle + \mathrm{e}^{\mathrm{i}\theta_1}\lvert001\rangle - \mathrm{e}^{\mathrm{i}\theta_2}\lvert010\rangle - \mathrm{e}^{\mathrm{i}\theta_3}\lvert011\rangle$ $-\mathrm{e}^{\mathrm{i}\theta_4}\lvert100\rangle - \mathrm{e}^{\mathrm{i}\theta_5}\lvert101\rangle + \mathrm{e}^{\mathrm{i}\theta_6}\lvert110\rangle + \mathrm{e}^{\mathrm{i}\theta_7}\lvert111\rangle)$	$\sigma_{B_{1z}} \otimes \sigma_{B_{2z}} \otimes I_{B_3}$
$\lvert\Gamma_6\rangle_{A_1A_2A_3}$	$(1/2\sqrt{2})(\mathrm{e}^{\mathrm{i}\theta_0}\lvert000\rangle - \mathrm{e}^{\mathrm{i}\theta_1}\lvert001\rangle + \mathrm{e}^{\mathrm{i}\theta_2}\lvert010\rangle - \mathrm{e}^{\mathrm{i}\theta_3}\lvert011\rangle$ $-\mathrm{e}^{\mathrm{i}\theta_4}\lvert100\rangle + \mathrm{e}^{\mathrm{i}\theta_5}\lvert101\rangle - \mathrm{e}^{\mathrm{i}\theta_6}\lvert110\rangle + \mathrm{e}^{\mathrm{i}\theta_7}\lvert111\rangle)$	$\sigma_{B_{1z}} \otimes I_{B_2} \otimes \sigma_{B_{3z}}$
$\lvert\Gamma_7\rangle_{A_1A_2A_3}$	$(1/2\sqrt{2})(\mathrm{e}^{\mathrm{i}\theta_0}\lvert000\rangle - \mathrm{e}^{\mathrm{i}\theta_1}\lvert001\rangle - \mathrm{e}^{\mathrm{i}\theta_2}\lvert010\rangle + \mathrm{e}^{\mathrm{i}\theta_3}\lvert011\rangle$ $+\mathrm{e}^{\mathrm{i}\theta_4}\lvert100\rangle - \mathrm{e}^{\mathrm{i}\theta_5}\lvert101\rangle - \mathrm{e}^{\mathrm{i}\theta_6}\lvert110\rangle + \mathrm{e}^{\mathrm{i}\theta_7}\lvert111\rangle)$	$I_{B_1} \otimes \sigma_{B_{2z}} \otimes \sigma_{B_{3z}}$
$\lvert\Gamma_8\rangle_{A_1A_2A_3}$	$(1/2\sqrt{2})(\mathrm{e}^{\mathrm{i}\theta_0}\lvert000\rangle - \mathrm{e}^{\mathrm{i}\theta_1}\lvert001\rangle - \mathrm{e}^{\mathrm{i}\theta_2}\lvert010\rangle + \mathrm{e}^{\mathrm{i}\theta_3}\lvert011\rangle$ $-\mathrm{e}^{\mathrm{i}\theta_4}\lvert100\rangle + \mathrm{e}^{\mathrm{i}\theta_5}\lvert101\rangle + \mathrm{e}^{\mathrm{i}\theta_6}\lvert110\rangle - \mathrm{e}^{\mathrm{i}\theta_7}\lvert111\rangle)$	$\sigma_{B_{1z}} \otimes \sigma_{B_{2z}} \otimes \sigma_{B_{3z}}$

2.2. Examples of remote preparation of four-qubit equatorial state

Suppose that Alice wishes to prepare a four-qubit equatorial state for Bob, which can be written as

$$\begin{aligned}|\chi\rangle = {} & \frac{1}{4}(\mathrm{e}^{\mathrm{i}\theta_0}|0000\rangle + \mathrm{e}^{\mathrm{i}\theta_1}|0001\rangle + \mathrm{e}^{\mathrm{i}\theta_2}|0010\rangle + \mathrm{e}^{\mathrm{i}\theta_3}|0011\rangle \\ & + \mathrm{e}^{\mathrm{i}\theta_4}|0100\rangle + \mathrm{e}^{\mathrm{i}\theta_5}|0101\rangle + \mathrm{e}^{\mathrm{i}\theta_6}|0110\rangle + \mathrm{e}^{\mathrm{i}\theta_7}|0111\rangle + \mathrm{e}^{\mathrm{i}\theta_8}|1000\rangle + \mathrm{e}^{\mathrm{i}\theta_9}|1001\rangle + \mathrm{e}^{\mathrm{i}\theta_{10}}|1010\rangle + \mathrm{e}^{\mathrm{i}\theta_{11}}|1011\rangle \\ & + \mathrm{e}^{\mathrm{i}\theta_{12}}|1100\rangle + \mathrm{e}^{\mathrm{i}\theta_{13}}|1101\rangle + \mathrm{e}^{\mathrm{i}\theta_{14}}|1110\rangle + \mathrm{e}^{\mathrm{i}\theta_{15}}|1111\rangle). \end{aligned} \quad (11)$$

The quantum channel can be made up of four two-qubit entangled states, which are presented as

$$|\psi\rangle_{A_1A_2A_3A_4B_1B_2B_3B_4} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_1B_1}\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_2B_2}\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_3B_3}\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_4B_4}. \quad (12)$$

In order to help the receiver Bob prepare the equatorial state remotely, the sender Alice needs to perform the four-qubit projective measurements on her particles A_1, A_2, A_3, and A_4 under the bases $|\Gamma_j\rangle$ ($j = 1, 2, 3, \ldots, 16$) as follows:

$$\begin{aligned}|\Gamma_1\rangle_{A_1A_2A_3A_4} = {} & \frac{1}{4}(\mathrm{e}^{-\mathrm{i}\theta_0}|0000\rangle + \mathrm{e}^{-\mathrm{i}\theta_1}|0001\rangle + \mathrm{e}^{-\mathrm{i}\theta_2}|0010\rangle + \mathrm{e}^{-\mathrm{i}\theta_3}|0011\rangle \\ & + \mathrm{e}^{-\mathrm{i}\theta_4}|0100\rangle + \mathrm{e}^{-\mathrm{i}\theta_5}|0101\rangle + \mathrm{e}^{-\mathrm{i}\theta_6}|0110\rangle + \mathrm{e}^{-\mathrm{i}\theta_7}|0111\rangle \\ & + \mathrm{e}^{-\mathrm{i}\theta_8}|1000\rangle + \mathrm{e}^{-\mathrm{i}\theta_9}|1001\rangle + \mathrm{e}^{-\mathrm{i}\theta_{10}}|1010\rangle + \mathrm{e}^{-\mathrm{i}\theta_{11}}|1011\rangle \\ & + \mathrm{e}^{-\mathrm{i}\theta_{12}}|1100\rangle + \mathrm{e}^{-\mathrm{i}\theta_{13}}|1101\rangle + \mathrm{e}^{-\mathrm{i}\theta_{14}}|1110\rangle + \mathrm{e}^{-\mathrm{i}\theta_{15}}|1111\rangle), \end{aligned}$$

$$|\Gamma_2\rangle_{A_1A_2A_3A_4} = \sigma_{A_{1z}} \otimes I_{A_2} \otimes I_{A_3} \otimes I_{A_4}|\Gamma_1\rangle_{A_1A_2A_3A_4}, \qquad |\Gamma_3\rangle_{A_1A_2A_3A_4} = I_{A_1} \otimes \sigma_{A_{2z}} \otimes I_{A_3} \otimes I_{A_4}|\Gamma_1\rangle_{A_1A_2A_3A_4},$$

$$
\begin{aligned}
&|\Gamma_4\rangle_{A_1A_2A_3A_4} = I_{A_1}\otimes I_{A_2}\otimes\sigma_{A_{3z}}\otimes I_{A_4}|\Gamma_1\rangle_{A_1A_2A_3A_4}, && |\Gamma_5\rangle_{A_1A_2A_3A_4} = I_{A_1}\otimes I_{A_2}\otimes I_{A_3}\otimes\sigma_{A_{4z}}|\Gamma_1\rangle_{A_1A_2A_3A_4},\\
&|\Gamma_6\rangle_{A_1A_2A_3A_4} = \sigma_{A_{1z}}\otimes\sigma_{A_{2z}}\otimes I_{A_3}\otimes I_{A_3}|\Gamma_1\rangle_{A_1A_2A_3A_4}, && |\Gamma_7\rangle_{A_1A_2A_3A_4} = \sigma_{A_{1z}}\otimes I_{A_2}\otimes\sigma_{A_{3z}}\otimes I_{A_4}|\Gamma_1\rangle_{A_1A_2A_3A_4},\\
&|\Gamma_8\rangle_{A_1A_2A_3A_4} = \sigma_{A_{1z}}\otimes I_{A_2}\otimes I_{A_3}\otimes\sigma_{A_{4z}}|\Gamma_1\rangle_{A_1A_2A_3A_4}, && |\Gamma_9\rangle_{A_1A_2A_3A_4} = I_{A_1}\otimes\sigma_{A_{2z}}\otimes\sigma_{A_{3z}}\otimes I_{A_4}|\Gamma_1\rangle_{A_1A_2A_3A_4}\\
&|\Gamma_{10}\rangle_{A_1A_2A_3A_4} = I_{A_1}\otimes\sigma_{A_{2z}}\otimes I_{A_3}\otimes\sigma_{A_{4z}}|\Gamma_1\rangle_{A_1A_2A_3A_4}, && |\Gamma_{11}\rangle_{A_1A_2A_3A_4} = I_{A_1}\otimes I_{A_2}\otimes\sigma_{A_{3z}}\otimes\sigma_{A_{4z}}|\Gamma_1\rangle_{A_1A_2A_3A_4},\\
&|\Gamma_{12}\rangle_{A_1A_2A_3A_4} = \sigma_{A_{1z}}\otimes\sigma_{A_{2z}}\otimes\sigma_{A_{3z}}\otimes I_{A_4}|\Gamma_1\rangle_{A_1A_2A_3A_4}, && |\Gamma_{13}\rangle_{A_1A_2A_3A_4} = \sigma_{A_{1z}}\otimes\sigma_{A_{2z}}\otimes I_{A_3}\otimes\sigma_{A_{4z}}|\Gamma_1\rangle_{A_1A_2A_3A_4},\\
&|\Gamma_{14}\rangle_{A_1A_2A_3A_4} = \sigma_{A_{1z}}\otimes I_{A_2}\otimes\sigma_{A_{3z}}\otimes\sigma_{A_{4z}}|\Gamma_1\rangle_{A_1A_2A_3A_4}, && |\Gamma_{15}\rangle_{A_1A_2A_3A_4} = I_{A_1}\otimes\sigma_{A_{2z}}\otimes\sigma_{A_{3z}}\otimes\sigma_{A_{4z}}|\Gamma_1\rangle_{A_1A_2A_3A_4},\\
&|\Gamma_{16}\rangle_{A_1A_2A_3A_4} = \sigma_{A_{1z}}\otimes\sigma_{A_{2z}}\otimes\sigma_{A_{3z}}\otimes\sigma_{A_{4z}}|\Gamma_1\rangle_{A_1A_2A_3A_4}. &&
\end{aligned}
\tag{13}
$$

Thus, the quantum channel can be rewritten as

$$
|\psi\rangle_{A_1A_2A_3A_4B_1B_2B_3B_4} = \frac{1}{4}[|\Gamma_1\rangle_{A_1A_2A_3A_4}|\varphi_1\rangle_{B_1B_2B_3B_4} + |\Gamma_2\rangle_{A_1A_2A_3A_4}|\varphi_2\rangle_{B_1B_2B_3B_4} + \cdots + |\Gamma_{16}\rangle_{A_1A_2A_3A_4}|\varphi_{16}\rangle_{B_1B_2B_3B_4}]. \tag{14}
$$

After Alice's measurements, she sends the results to the intend receiver Bob via classical channel. If Alice's measurement result is $|\Gamma_{16}\rangle_{A_1A_2A_3A_4}$, the state of Bob will collapse into

$$
\begin{aligned}
|\varphi_{16}\rangle_{B_1B_2B_3B_4} = \frac{1}{4}(&\mathrm{e}^{\mathrm{i}\theta_0}|0000\rangle - \mathrm{e}^{\mathrm{i}\theta_1}|0001\rangle - \mathrm{e}^{\mathrm{i}\theta_2}|0010\rangle + \mathrm{e}^{\mathrm{i}\theta_3}|0011\rangle\\
&- \mathrm{e}^{\mathrm{i}\theta_4}|0100\rangle + \mathrm{e}^{\mathrm{i}\theta_5}|0101\rangle + \mathrm{e}^{\mathrm{i}\theta_6}|0110\rangle - \mathrm{e}^{\mathrm{i}\theta_7}|0111\rangle - \mathrm{e}^{\mathrm{i}\theta_8}|1000\rangle + \mathrm{e}^{\mathrm{i}\theta_9}|1001\rangle + \mathrm{e}^{\mathrm{i}\theta_{10}}|1010\rangle - \mathrm{e}^{\mathrm{i}\theta_{11}}|1011\rangle\\
&+ \mathrm{e}^{\mathrm{i}\theta_{12}}|1100\rangle - \mathrm{e}^{\mathrm{i}\theta_{13}}|1101\rangle - \mathrm{e}^{\mathrm{i}\theta_{14}}|1110\rangle + \mathrm{e}^{\mathrm{i}\theta_{15}}|1111\rangle).
\end{aligned}
\tag{15}
$$

Based on receiving the classical information from Alice, Bob performs appropriate unitary operations $\sigma_{B_{1z}}\otimes\sigma_{B_{2z}}\otimes\sigma_{B_{3z}}\otimes\sigma_{B_{4z}}$ to convert the collapsed state into the target state. Other cases are elucidated in Table 2.

Table 2. The measurement outcomes of Alice, Bob's collapse states and the unitary operators of Bob.

Alice's measurements	Bob's collapse states	Bob's unitary operations
$\|\Gamma_1\rangle_{A_1A_2A_3A_4}$	$\|\varphi_1\rangle_{B_1B_2B_3B_4}$	$I_{B_1}\otimes I_{B_2}\otimes I_{B_3}\otimes I_{B_4}$
$\|\Gamma_2\rangle_{A_1A_2A_3A_4}$	$\|\varphi_2\rangle_{B_1B_2B_3B_4}$	$\sigma_{B_{1z}}\otimes I_{B_2}\otimes I_{B_3}\otimes I_{B_4}$
$\|\Gamma_3\rangle_{A_1A_2A_3A_4}$	$\|\varphi_3\rangle_{B_1B_2B_3B_4}$	$I_{B_1}\otimes\sigma_{B_{2z}}\otimes I_{B_3}\otimes I_{B_4}$
$\|\Gamma_4\rangle_{A_1A_2A_3A_4}$	$\|\varphi_4\rangle_{B_1B_2B_3B_4}$	$I_{B_1}\otimes I_{B_2}\otimes\sigma_{B_{3z}}\otimes I_{B_4}$
$\|\Gamma_5\rangle_{A_1A_2A_3A_4}$	$\|\varphi_5\rangle_{B_1B_2B_3B_4}$	$I_{B_1}\otimes I_{B_2}\otimes I_{B_3}\otimes\sigma_{B_{4z}}$
$\|\Gamma_6\rangle_{A_1A_2A_3A_4}$	$\|\varphi_6\rangle_{B_1B_2B_3B_4}$	$\sigma_{B_{1z}}\otimes\sigma_{B_{2z}}\otimes I_{B_3}\otimes I_{B_4}$
$\|\Gamma_7\rangle_{A_1A_2A_3A_4}$	$\|\varphi_7\rangle_{B_1B_2B_3B_4}$	$\sigma_{B_{1z}}\otimes I_{B_2}\otimes\sigma_{B_{3z}}\otimes I_{B_4}$
$\|\Gamma_8\rangle_{A_1A_2A_3A_4}$	$\|\varphi_8\rangle_{B_1B_2B_3B_4}$	$\sigma_{B_{1z}}\otimes I_{B_2}\otimes I_{B_3}\otimes\sigma_{B_{4z}}$
$\|\Gamma_9\rangle_{A_1A_2A_3A_4}$	$\|\varphi_9\rangle_{B_1B_2B_3B_4}$	$I_{B_1}\otimes\sigma_{B_{2z}}\otimes\sigma_{B_{3z}}\otimes I_{B_4}$
$\|\Gamma_{10}\rangle_{A_1A_2A_3A_4}$	$\|\varphi_{10}\rangle_{B_1B_2B_3B_4}$	$I_{B_1}\otimes\sigma_{B_{2z}}\otimes I_{B_3}\otimes\sigma_{B_{4z}}$
$\|\Gamma_{11}\rangle_{A_1A_2A_3A_4}$	$\|\varphi_{11}\rangle_{B_1B_2B_3B_4}$	$I_{B_1}\otimes I_{B_2}\otimes\sigma_{B_{3z}}\otimes\sigma_{B_{4z}}$
$\|\Gamma_{12}\rangle_{A_1A_2A_3A_4}$	$\|\varphi_{12}\rangle_{B_1B_2B_3B_4}$	$\sigma_{B_{1z}}\otimes\sigma_{B_{2z}}\otimes\sigma_{B_{3z}}\otimes I_{B_4}$
$\|\Gamma_{13}\rangle_{A_1A_2A_3A_4}$	$\|\varphi_{13}\rangle_{B_1B_2B_3B_4}$	$\sigma_{B_{1z}}\otimes\sigma_{B_{2z}}\otimes I_{B_3}\otimes\sigma_{B_{4z}}$
$\|\Gamma_{14}\rangle_{A_1A_2A_3A_4}$	$\|\varphi_{14}\rangle_{B_1B_2B_3B_4}$	$\sigma_{B_{1z}}\otimes I_{B_2}\otimes\sigma_{B_{3z}}\otimes\sigma_{B_{4z}}$
$\|\Gamma_{15}\rangle_{A_1A_2A_3A_4}$	$\|\varphi_{15}\rangle_{B_1B_2B_3B_4}$	$I_{B_1}\otimes\sigma_{B_{2z}}\otimes\sigma_{B_{3z}}\otimes\sigma_{B_{4z}}$
$\|\Gamma_{16}\rangle_{A_1A_2A_3A_4}$	$\|\varphi_{16}\rangle_{B_1B_2B_3B_4}$	$\sigma_{B_{1z}}\otimes\sigma_{B_{2z}}\otimes\sigma_{B_{3z}}\otimes\sigma_{B_{4z}}$

3. Conclusion

In this scheme, we give a new way by utilizing n two-qubit maximally entangled states as the entangled quantum channel to prepare arbitrary n-qubit equatorial states, the probability of success can be reached 100%. Compared with the other schemes,[53,57–60] our scheme makes remote preparation of arbitrary equatorial states more simple and convenient for simplifying the measurement basis and unitary transformations in our protocol. Additionally, our scheme has no coefficient constraint and auxiliary qubits. So the success probabilities in our scheme are independent of the coefficients of the entangled channel. Furthermore, we analysis the possibility of σ_z appearing in the measurement bases. It will be helpful for the following study of equatorial states. In addition, our scheme is demonstrated by theoretical knowledge and concrete instances. The protocol also has greatly improved the efficiency, because of needing fewer appropriate unitary transformations to get the desired state. Thus, we wish our scheme will be realized by experiment in future.

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