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Identifying anomalous Floquet edge modes via bulk–edge correspondence*

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Floquet engineering appears as a new protocol for designing topological states of matter, and features anomalous edge modes pinned at quasi-energy π/T with vanished topological index. We propose how to predict the anomalous edge modes via the bulk Hamiltonian in frequency space, and use Zak phase to quantitatively index the topological properties. The above methods are clarified by the example of time periodic Kitaev chain with chemical potential of harmonic driving and pulse driving, and topological phase transitions are manifested at different driving frequencies.

Keywords: Floquet energineering, bulk-edge correspondence, Zak phase

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1. Introduction

Topological state of matter is an intriguing topic for years, and many fruitful methods on endowing quantum systems with nontrivial topological properties have been brought up, such as spin orbit coupling,[1–12] shaking optical lattice,[13–19] and exercising external magnetic field.[20–25] Among all these methods, Floquet engineering appears as a most profound one, for the generation of anomalous edge modes. The main idea of Floquet engineering is to drive the physical parameters periodically in time. Under periodical boundary conditions, the system not only loops around in real space, but also in time. Hence, it is quite natural to think up that traditional one-dimensional topological index must be altered to characterize the driven system. Meanwhile, we should also notice that the driven system can transit quantized energy with external driving field, and thus features non-equilibrium properties.[26–29] With all these in mind, we can imply from Bloch theorem (Floquet theorem in time space) that periodically driving will enforce the energy to winds around, and there is no well defined lowest energy as well as ground states.

One typical way to identify topological properties is via edge modes. In the driven system, there are two kinds of edge modes, located respectively at quasi energy 0, π/T. In the following, we will show that the driving term will generate the π gap due to the bands avoid crossing. According to the Floquet theorem, the quasi energy are folded into the energy Brillouin zone $[-\frac{\pi}{T},\frac{\pi}{T}]$. With above in consideration, edge modes can be quite hard to recognize when the driving frequency get extremely low, where all energy shall be maintained within a tiny window. In this case, one usually utilizes a long chain of big size to distinguish the edge modes from bulk states in energy, which is very costly in calculation. To overcome this problem is the central part of this paper.

We utilize the Floquet–Schrödinger equation, and manifest the bulk spectrum in frequency space. Such a spectrum is not folded, and carries the full information of topological properties. The close and reopen of the 0, π/T gap in frequency space can predict the emergence of two kinds of edge modes. Meanwhile, we also propose how to quantitatively denote topological phase transitions via Zak phase. To illustrate our method, we take a time-periodic Kitaev chain with chemical potential of different driving forms as an example.

2. Harmonic driving Kitaev chain

We consider a time periodic Kitaev chain with harmonic driving chemical potential in thermodynamic limit

$$\begin{aligned} H = & \sum_j \Big[-w(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) - \mu(t)\Big(c_j^\dagger c_j - \frac{1}{2}\Big) \\ & + c_j c_{j+1}\Delta + c_{j+1}^\dagger c_j^\dagger \Delta \Big], \end{aligned} \tag{1}$$

where $c_j^\dagger$ represents creating a fermion at site j; w is the hopping amplitude, Δ denotes the pairing between the nearest sites, $\mu(t)$ is the driving chemical potential, and here we take $\mu(t) = \mu_0 + \mu_1 \cos(\omega t + \phi)$. Since there is no ground state for the driven system, all information of topological properties are contained within the evolving operator

$$U(T) = \frac{1}{T}\mathcal{T} \mathrm{e}^{-\mathrm{i}\int_0^T H(t)\,\mathrm{d}t}, \tag{2}$$

where $\mathcal{T}$ represents the time ordering. In Fig. 1(a), we have set hopping amplitude $w = 1$, it is manifested that in con-

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trast to the nondriven Kitaev chain, there still exist Majorana edge modes for $\mu_0 > 2w$. Intuitively, Majorana edge modes in real space can be derived from the bulk Hamiltonian. To manifest this, we turn to use the time-independent effective Hamiltonian $U(T) = \frac{1}{T}\,\mathrm{e}^{-\mathrm{i}TH_{\rm eff}(k)}$. Figure 1(b) denotes the time-independent effective Hamiltonian at $\mu_0 = 3$. The total number of Majorana $0, \frac{\pi}{T}$ edge modes is supposed to be equal to the Zak phase of the filled bands,

$$\varphi_{\rm Zak} = \frac{1}{2\pi\mathrm{i}}\int_{-\pi}^{\pi} \mathrm{d}k \langle u_k|\partial_k|u_k\rangle, \tag{3}$$

where $|u_k\rangle$ is the eigenstate of the filled bands. In Fig. 1(a), we denote the Zak phase by w, and for $\mu_0 \in [0,1]$, we have $w = -1$ with the existence of Majorana zero modes. However, for $\mu_0 \in [1,2]$, Majorana 0 modes and Majorana π modes coexist with vanished Zak phase. Such edge modes are anomalous edge modes, and can not be obtained via traditional bulk-edge correspondence. Indeed, inspired by Ref. [29], we shall notice that the time-independent effective Hamiltonian is not available when there are degenerate points in phase band $\phi(k,t)$, where $U_n(k,t) = \sum_n \mathrm{e}^{-\mathrm{i}\phi_n(k,t)}$. The degeneracy of phase bands suggests that it can not be smoothly deformed to a flat band, which corresponds to the time-independent effective Hamiltonian. Generally, such a degeneracy is common with low frequency driven systems, see Figs. 2(a) and 2(b).

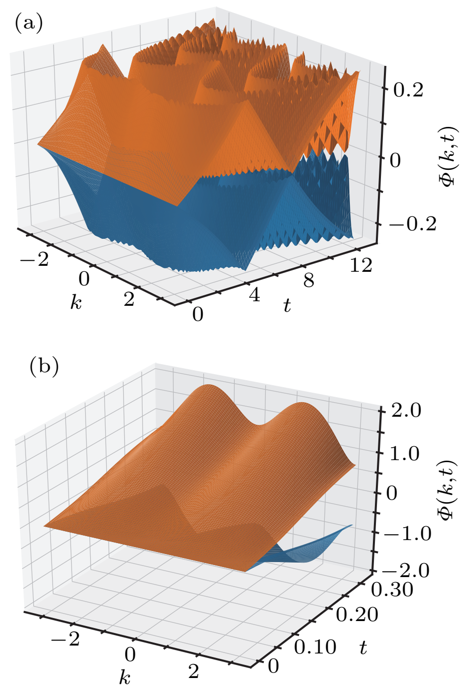

Fig. 2. The phase bands with $w = 1, \mu_0 = 0.5, \mu_1 = 0.5, \Delta = 2$. (a) Degenerate points in phase band with low driving frequency $\omega = 0.5$. (b) Non-degenerate points in phase bands with high driving frequency $\omega = 20$.

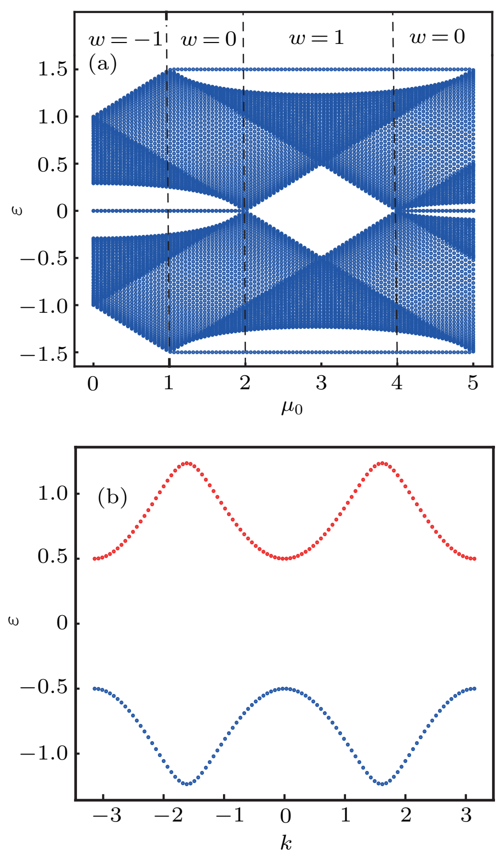

Fig. 1. (a) Real space Floquet spectrum as a function of μ_0 with harmonic driving in chemical potential where $w = 1$, $\Delta = 0.5$, $\mu_1 = 4$, $\omega = 3$. The Zak phase of the effective Hamiltonian varies with different regions of μ_0, and anomalous Majorana $0, \frac{\pi}{T}$ edge modes can exists with vanished Zak phase. (b) The bulk spectrum of Floquet effective Hamiltonian, where $w = 1$, $\Delta = 0.5$, $\mu_0 = 3$, $\mu_1 = 4$, $\omega = 3$.

Meanwhile, we shall keep in mind that $|u_k\rangle$ captures no periodic properties of time, and thus can not characterize the topological features. Notice that the driven system distinguishes itself by the windings in energy. If we construct a parameter space corresponding to energy which contains the winding features, the bulk-edge correspondence may work again. According to the Floquet theorem $\psi(t) = \mathrm{e}^{-\mathrm{i}\varepsilon_n t}\chi_n(t)$, we have $\chi_n(t) = \chi_n(t+T)$. Substituting them into the Schrödinger equation, we arrive at $[H - \mathrm{i}\partial_t]\chi_n(t) = \varepsilon_n\chi_n(t)$. It seems quite tricky that quasi energy ε_n here does not need to be folded into quasi energy Brillouin zone. Indeed, such a folding can be recovered and there is gauge freedom in the Floquet theorem,

$$\chi'_n(t) = \mathrm{e}^{-\mathrm{i}p\omega t}\chi_n(t). \tag{4}$$

We shall see $\chi'_n(t+T) = \chi_n(t)$, and the corresponding Floquet–Schrödinger equation now becomes $[H - i\partial_t]\chi'_n(t) = (\varepsilon_n + p\omega)\chi'_n(t)$. Notice that $\chi_n(t)$ has periodicity T, we can apply the Fourier transformation $\chi_n(t) = \sum_m \mathrm{e}^{\mathrm{i}m\omega t}\chi_n(m)$. Put the above Fourier expansion into the Floquet–Schrödinger equation, and with a little integration, we can construct the Floquet–Schrödinger equation in frequency space

$$\sum_m H_{m,m'}(k)\chi_n(m) = \varepsilon_n\chi_n(m'). \tag{5}$$

It is exhibited that all the static part contributes to the diagonal block in $H_{m,m'}(k)$

$$H_{m=m'} = H_0(k) + m\omega I, \tag{6}$$

where I is the identity matrix, and $H_0(k) = -(w\cos k + \frac{\mu_0}{2})\sigma_k^z + \Delta\sin k\sigma_k^y$ originates from the static part of Hamiltonian, and driving part of the Hamiltonian contributes to

$$H_{m=m'+1} = \mathrm{e}^{\mathrm{i}\phi}\frac{-\mu_1}{4}\sigma_k^z, \quad H_{m=m'-1} = \mathrm{e}^{-\mathrm{i}\phi}\frac{-\mu_1}{4}\sigma_k^z. \tag{7}$$

Before proceeding, we shall notice that particle-hole symmetry can be inherited during the driving process $\Pi H(k)\Pi^{-1} = -H(-k)$, $\Pi = \sigma_x\kappa$. κ is the complex conjugator. Hence, the quasi energy 0, $\frac{\pi}{T}$ gaps are supposed to be closed at high symmetric point $k = 0$, π. Meanwhile, we need to point out that the relative phase ϕ in the time periodic chemical potential can be gauged out by properly choosing the initial point of evolving $t' = t + \frac{\phi}{\omega}$. Since $U(T,0) = U(T + \frac{\phi}{\omega}, \frac{\phi}{\omega})$, different initial phase ϕ results in same topological properties. Similar to the way we obtain the Bloch state with periodical potential in real space, here we truncate $m = -3,-2,-1,0,1,2,3$. Figure 3(a) depicts the quasi energy $\frac{\pi}{T}$, 0 gap with $k = 0$. Figure 3(b) depicts the quasi energy $\frac{\pi}{T}$, 0 gap with $k = \pi$. It is exhibited that $\frac{\pi}{T}$ gaps close and reopen with $k = 0$ at $\mu_0 = 1$. Correspondingly, we see that in Fig. 1(a), Majorana $\frac{\pi}{T}$ edge modes start to appear from $\mu_0 = 1$. With this in mind, we observe from Fig. 3, $\frac{\pi}{T}$, 0 gaps close and reopen at $\mu_0 = 1,2,4,5$. Correspondingly, in Fig. 1(a), topological phase transitions happen at the same place.

To denote the topological phase transition quantitatively, we define a new topological invariant based on the Zak phase: $\nu_{0,\pm\frac{\pi}{T}} = \mathrm{mod}(\sum_{i=1}^{i=n}\varphi_{\mathrm{Zak}}^n, 2)$, where n denotes n-th bands right below quasi energy $0, \pm\frac{\pi}{T}$. We observe that if $\nu_{0,\pm\frac{\pi}{T}} = 1$, it is supposed to be edge modes pinned at energy $0, \pm\frac{\pi}{T}$. If $\nu_{0,\pm\frac{\pi}{T}} = 0$, no edge mode exist, and the system appears to be topologically trivial. In our cases, with truncation at $m = -3$. The first six bands are located below quasi energy $-\frac{\pi}{T}$, first seven band located below quasi energy 0, and first eight band located below quasi energy $\frac{\pi}{T}$. To illustrate this, we take $\mu_0 = 0.5$ (other parameters are fixed the same as those in Fig. 1(a)). The corresponding Zak phase of first eight bands is $[-1,-1,-1,-1,-1,-1,-1,-1]$. Hence, $\nu_{\pm\frac{\pi}{T}} = 0, \nu_0 = 1$, there are only Majorana 0 modes, and no Majorana $\frac{\pi}{T}$ modes. For $\mu_0 = 1.5$, the Zak phase are $[-1,0,0,0,0,0,0,0]$. Therefore, $\nu_0 = 1$, $\nu_{\pm\frac{\pi}{T}} = 1$, there are both Majorana 0 modes, and Majorana $\frac{\pi}{T}$ modes. The above results coincide with that of Fig. 1(a).

With all the above methods, we are capable of studying topological phase transitions as a function of driving frequency ω readily, which is very costly in numerics with the conventional method. In Fig. 4(a), we fix the $\mu_0 = 3$, $\mu_1 = 4$ (other parameters are the same as those in Fig. 1(a)), and varies the driving frequency. Here δ represents the magnitude of the gaps at high symmetric point. Every time $\delta = 0$ implies that there are topological phase transitions, which can be implied by the emergence of Majorana edge modes. It is exhibited from Fig. 4(a) that at low driving frequency, topological phase transitions occur quite frequently. We shall figure out that when $\omega \geq 1$, the magnitude of quasi energy 0 gap become constant $\delta = 1$. Such results originate from the following: when the driving frequency is large enough, the intrinsic energy scale of the Kitaev chain is no larger than size of the quasi energy Brillouin zone, which means that energy does not need to be folded.

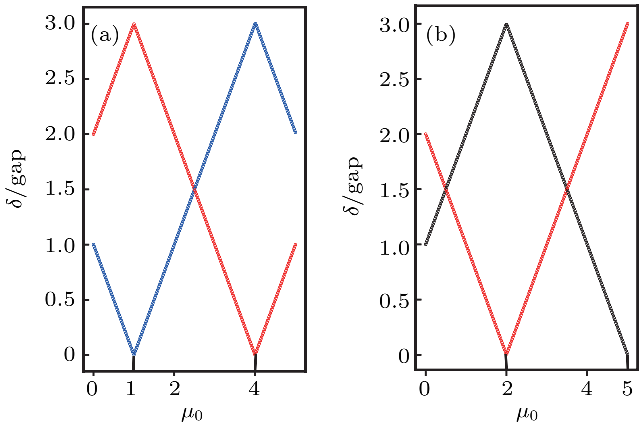

Fig. 3. (a) The magnitude of quasi energy $\frac{\pi}{T}$ gap (blue line), 0 gap (red line) as a function of chemical potential μ_0 with $k = 0$, $w = 1$, $\Delta = 0.5$, $\mu_1 = 4$, $\omega = 3$. (b) The magnitude of quasi energy $\frac{\pi}{T}$ gap (black line), 0 gap (red line) as a function of chemical potential μ_0 with $k = \pi$, $w = 1$, $\Delta = 0.5$, $\mu_1 = 4$, $\omega = 3$.

In this scenario, the driven system is much alike a non-driven one. To prove this, we consider a non-driven Kitaev chain (just put $\mu_1 = 0$) at $k = \pi$, the 0 gap is supposed to be $\delta = 2|w\cos k + \frac{\mu_0}{2}| = 1$, which is consistent with Fig. 4(a). In Fig. 4(b), we have enlarged $\mu_0 = 5$, it can be seen that the topological phase transition becomes even more complicated, and driving frequency has to be a larger value for the system to get approach to a non-driven Kitaev chain. When $\omega \geq 7$, energy 0 gap at $k = 0$ becomes constant $\delta = 2|w\cos k + \frac{\mu_0}{2}| = 7$. Comparing Figs. 4(a) and 4(b), we can find that when the intrinsic energy scale is much larger than the driving frequency, Majorana edge modes emerge and disappear quite frequently, and are not stabilized. In Fig. 4(c), we fix $\mu_0 = 3$ and enlarge $\mu_1 = 10$. We shall observe that topological phase transitions are suppressed, and Majorana edge modes only merge (or disappear) at $\omega = 2.38$. It is necessary to mention that the energy gap also indicates the tunneling between different edges. When $\delta \to 0$, edge modes become close to the bulk.

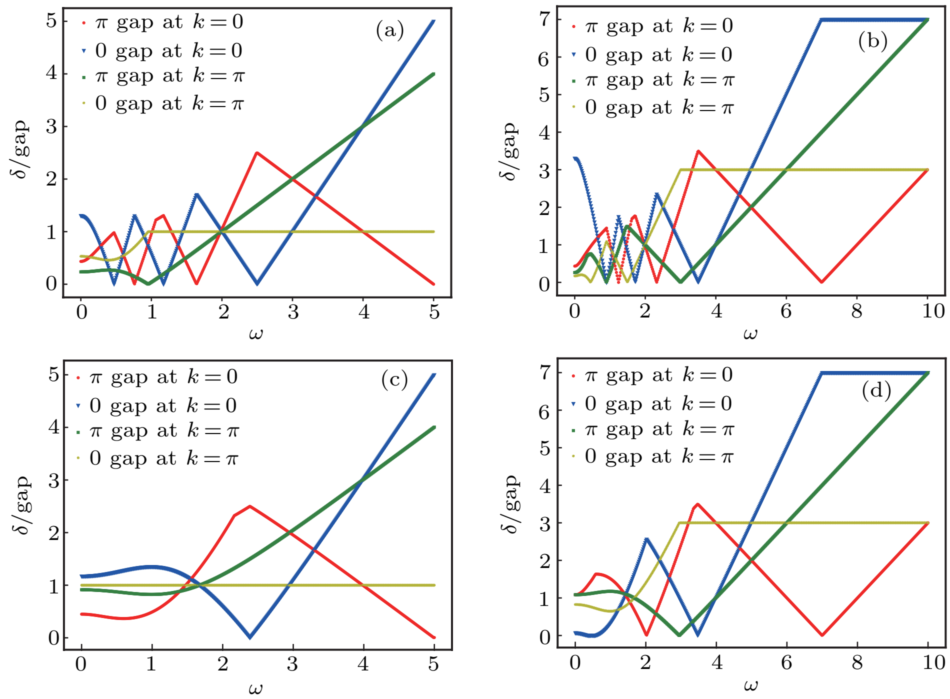

Fig. 4. The magnitude of quasi energy $\frac{\pi}{T}$, 0 gap, at high symmetric point $k=0$, $k=\pi$ as a function of driving frequency. Parameters are fixed the same as those in Fig. 1(a) except for μ_0, μ_1: (a) $\mu_0=3$, $\mu_1=4$. (b) $\mu_0=5$, $\mu_1=4$. (c) $\mu_0=3$, $\mu_1=10$. (d) $\mu_0=5$, $\mu_1=10$.

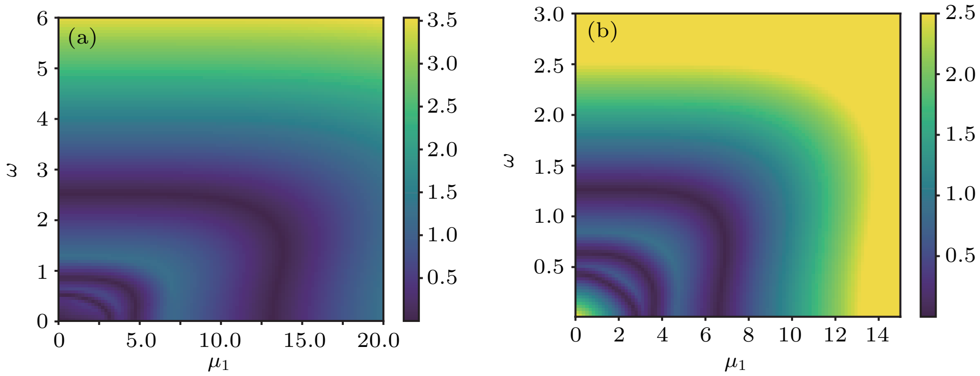

Fig. 5. The magnitude of quasi energy gaps (color) as a function of driving frequency ω, and the strength of driving μ_1 with $k=0$, $\mu_0=0.5$, $t=1$, $\Delta=0.5$ (a) for the $\pm\frac{\pi}{T}$ gap amplitude and (b) for the 0 gap amplitude.

We proceed to explicitly demonstrate that how the strength of time periodic driving will affect the topological properties. In Figs. 5(a) and 5(b), we exhibit how the $\frac{\pi}{T}$, 0 gaps reacts with tilting driving strength and driving frequency. The dark blue line circles out different topological phases, in which it can be seen that with larger driving strength, it needs much higher driving frequency to induce topological phase transitions. Combined with Figs. 4(c) and 4(d), Figs. 5(a) and 5(b) can manifest that enlarging the driving amplitude will stabilize the system, make topological phases easier to recognize.

3. Pulsed driving Kitaev chain

In the previous part, we have considered the Kitaev chain with harmonic driving in chemical potential. Now we turn to the case of pulse driving: $\mu(t)=\mu_0+\mu_1\delta(t-NT)$. A direct consequence of this will be manifested as follows:

$$H_{m'\neq m}=\frac{-\mu_1}{2}\sigma_k^z. \tag{8}$$

Here $H_{m'\neq m}$ does not need to be a tridiagonal matrix. Considering this, it seems that the above truncations at $m=3$ are not accurate enough to give the topological phase transitions. In Fig. 6(a), we show the Majorana edge modes by diagonalizing the Floquet operator $U(T)$. Figures 6(b), 6(c), and 6(d) show the magnitude of the 0, $\frac{\pi}{T}$ gap at high symmetric point $k=0$, π, with truncation $m=3,5,8$. It appears although enlarging the truncation will slightly alter the gaps, where we shall admit that accurate critical points of topological phase transitions can not be gained with finite m, a general figure of topological properties can still be obtained via our methods.

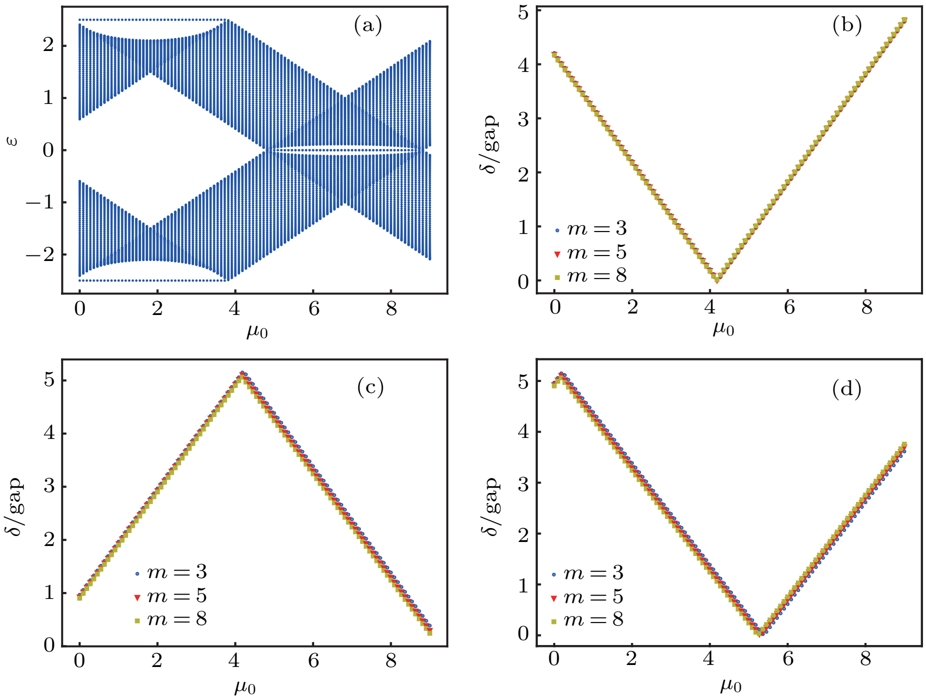

Fig. 6. (a) Real space Floquet spectrum as a function of μ_0 with pulsed driving in chemical potential, where $w=1$, $\Delta=0.5$, $\mu_1=4$, $\omega=5$. (b) The magnitude of $\frac{\pi}{T}$ gap at $k=\pi$ with truncation $m=3,5,8$, $\delta=0$, $\mu_0=4.2$, where the other parameters are fixed the same as those in (a). (c) The magnitude of 0 gap at $k=\pi$ with truncation $m=3,5,8$, $\delta=0.24$, $\mu_0=9$. (d) The magnitude of 0 gap at $k=0$ with truncation $m=3,5,8$ for $\delta=0$, $\mu_0=5.3$.

4. Conclusion

We have shown how to construct the bulk-edge correspondence for identifying the anomalous edge modes in a driven system. Our methods are capable of dealing with Floquet phase transitions at low driving frequency, and save the numerical calculations from avoiding using large size of lattice. We use time periodic Kitaev chain with harmonic, pulsed driving in chemical potential to illustrate our method, and prove its generality. Our works enrich the study of topological phase transitions in the driven system.

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