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Parameter estimation in cosmic string spacetime by using the inertial and accelerated detectors

Ying Yang[1], **Jieci Wang**[1], **Mengjie Wang**[1], **Jiliang Jing**[1,4] **and Zehua Tian**[2,3,4]

[1] Department of Physics, Key Laboratory of Low Dimensional Quantum Structures and Quantum Control of Ministry of Education, and Synergetic Innovation Center for Quantum Effects and Applications, Hunan Normal University, Changsha, Hunan 410081, People's Republic of China

[2] CAS Key Laboratory of Microscale Magnetic Resonance and Department of Modern Physics, University of Science and Technology of China, Hefei 230026, People's Republic of China

[3] Key Laboratory for Research in Galaxies and Cosmology, Chinese Academy of Science, Hefei 230026, People's Republic of China

E-mail: jljing@hunn.edu.cn and tianzh@ustc.edu.cn

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Abstract

Since cosmic string spacetime is locally flat but with nontrivial global topology characterized by a deficit angle, we devote to address the quantum bound to the estimation of the deficit angle parameter by using a two-level atom as a detector which is coupled to a massless scalar field. We show that the initial excited state of the detector is the optimal state, and quantum precision always obtains the maximum value when the detector evolves for a limited time. We find that the sensitivity in the predictions for the deficit angle parameter ν decreases with the increase of ν. We also note that a uniform, rectilinear accelerated motion does not improve the estimation accuracy, so we obtain the inertial detector is better than the uniform, rectilinear accelerated detector. Therefore, we provide a possibility for detecting the nontrivial global topology in the spacetime.

Keywords: metrology, quantum Fisher information, cosmic string spacetime

(Some figures may appear in colour only in the online journal)

[4] Author to whom any correspondence should be addressed.

1. Introduction

Among the most interesting consequences of phase transitions in gauge theories is the formation of a variety of topological defects [1–3]. Topology defects in the geometry of spacetime (such as cosmic strings) plays an important role in the evolution of the early Universe [3]. The evolution of a defect network perturbs the background spacetime, and those perturbations evolve and affect the contents of the Universe [3]. Inflationary perturbations were seeded primordially and then evolve passively, while defects induce perturbations actively during their whole existence [4].

Cosmic strings arose as topological defects during the symmetry breaking phase transitions in the early history of the Universe [1, 5, 6]. Some may still exist and may even be observable; others may have collapsed long ago. Cosmic strings models gained much attention in the last decades [7–12]. Deser *et al* 'T Hooft discussed the global properties of the (locally flat) geometries generated by moving point particles in $2 + 1$ dimensions, or equivalently by parallel moving cosmic strings in $3 + 1$ dimensions [13], and they also found that physical cosmic strings do not generate closed timelike curves [14]. There are also some typical works which are connected with quantization effects of the cosmic string spacetime [9, 10, 15, 16]. Tight constraints on the parameters of the cosmic strings by observations of gravitational wave signatures are given recently [17]. Current bounds on the string tension $G\mu$ from CMB experiments constrain its value to be below 10^{-7} [4, 17–19]. The observations from the COBE, WAMP, CMB, and PLANCK satellites also put constraints on the parameters of the model [20–28]. Cosmic strings have attracted considerable attention within the framework of string theory inspired cosmological models [29–32].

The simplest cosmic string spacetime is characterized by a flat metric with a deficit angle, which is described by an infinite, straight and static cylindrically symmetric cosmic string [33, 34]. The spacetime is locally flat outside the string but topologically nontrivial [34]. There are a lot of interesting and remarkable gravitational effects associated with cosmic string, such as gravitational lens [35], gamma ray bursts [36–39], the gravitational waves [40–42], and the high-energy cosmic rays [43].

In cosmic string spacetime, fields propagating are affected by its nontrivial topology [34]. In resent years, it is of great interest to focus on this perspective of the effect of the topological features on quantum systems. Here we mention some typical examples, such as the study of topological scattering in the context of quantum mechanics on a cone [44], the investigations on the interaction of a quantum system with conical singularities [45], quantum mechanics on topological defects of spacetimes [46], shifts in the energy levels due to the nontrivial topology [47], and the so-called gravitational Aharonov–Bohm effect [48]. Cosmic string spacetime is characterized by its nontrivial topological structure, and many quantum effects exhibit significant characteristics in such spacetime.

Motivated by these interesting research on topological defects in cosmic string spacetime, we would like to probe the topological defects in this spacetime. Since the spacetime is locally flat but with nontrivial global topology characterized by a deficit angle parameter, we now devote to address the quantum bound to the estimation of this parameter.

In classical metrology approaches, the statistical error is reduced by repeating the measurement and averaging the results, and the central limit theorem implies that the reduction is proportional to the square root of the number of repetitions [49–51]. Quantum metrology is the use of quantum techniques such as entanglement to yield higher statistical precision than purely classical approaches [51].

Due to experimental errors and uncertainties are unavoidable in reality, quantum estimation has been an active subject [52, 53]. With the application of quantum estimation theory(QET), one can estimate the parameter of interest with a higher precision which is beyond the standard classical limits [49]. Recently, quantum estimation has been applied widely in the entanglement detection [54], optimal quantum clock [55], measurement of gravity accelerations [56], clock synchronization [57], and so on. The estimation error is quantified by the quantum Cramér–Rao bound which is inversely proportional to quantum Fisher information (QFI), so enhancing the value of QFI for the estimated parameter has been an important issue in quantum estimation [52].

Here we are concerned with the parameter estimation in cosmic string spacetime. We intend to exploit local QET to find out the quantum measurement that maximizes the QFI which aims to evaluate the ultimate limits of precision in the estimation of the deficit angle parameter. We consider using a two-level atom as a detector to estimate the deficit angle parameter. By comparing the inertial detector with the accelerated detector, we devote to find some available conditions to improve the estimation of the deficit angle parameter.

This paper is organized as follows. In section 2, we review quantum scalar field in cosmic string spacetime. In section 3, we introduce the local QET and the model of a two-level detector. In section 4, we exploit the QET to cosmic string spacetime, and talk about parameter estimation for the deficit angle parameter with different detectors. Final remarks and conclusions are given in section 5.

2. Quantum scalar field in cosmic string spacetime

Let us briefly review the cosmic string scalar field. The simplest cosmic string spacetime is a static, straight cosmic string which can be described in the cylindrical coordinate system, and the metric is expressed as

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}r^2 - r^2\mathrm{d}\alpha^2 - \mathrm{d}z^2. \tag{1}$$

There are two types, local (gauged) and global cosmic strings, and the gravitational field of a gauge cosmic string is represented by a locally flat and cylindrically symmetric spacetime with a planar angle deficit [58]. We mainly interested in local cosmic strings and that the back-reaction is ignored. In equation (1), $0 \leqslant \alpha < \frac{2\pi}{\nu}$, $\nu = (1-4G\mu)^{-1}$ with G and μ being Newton constant and the mass per unit length of the string respectively and the value of ν is determined by the value of the mass density of the string which is in turn determined by the spontaneous symmetry breaking scale when the cosmic string was formed. The cosmic string spacetime we discussed which is locally flat but with nontrivial global topology characterized by a deficit angle. In this spacetime, the Klein–Gordon equation for a scalar field Φ is

$$(\partial_t{}^2 - \frac{1}{r}\partial_r(r\partial_r) - \frac{1}{r^2}\partial_\alpha{}^2 - \partial_z{}^2)\Phi(x) = 0. \tag{2}$$

Then one can obtain a complete set of field modes for the scalar field by solving equation (2)

$$u_j(t,\vec{x}) = \mathrm{e}^{-\mathrm{i}\omega t}u_j(\vec{x}), \tag{3}$$

with

$$u_j(\vec{x}) = \frac{1}{2\pi}\sqrt{\frac{\nu}{2\omega}}\mathrm{e}^{\mathrm{i}\kappa z}\mathrm{e}^{\mathrm{i}\nu m\alpha}J_{\nu|m|}(k_\perp r), \tag{4}$$

where $j = \{\kappa, m, k_\perp\}$, $\kappa \in (-\infty, \infty)$, $m \in Z$, $k_\perp \in (0, \infty)$ and $\omega^2 = \kappa^2 + k_\perp{}^2$. By defining the inner product of two mode functions

$$(u_j(t,\vec{x}), u_{j'}(t,\vec{x})) = -\mathrm{i}\int \mathrm{d}^3 x u_j(t,\vec{x})\overleftrightarrow{\partial_t} u^*_{j'}(t,\vec{x}), \tag{5}$$

one can obtain that

$$(u_j(t,\vec{x}), u_{j'}(t,\vec{x})) = \delta(\kappa-\kappa')\delta_{m,m'}\frac{\delta(k_\perp - k'_\perp)}{\sqrt{k_\perp k'_\perp}} \equiv \delta_{j,j'}. \tag{6}$$

Now the field operator is expanded in terms of the complete set of field modes as

$$\Phi(t,\overrightarrow{x}) = \int \mathrm{d}\mu_j [a_j(t)u_j(\overrightarrow{x}) + a^+_j(t)u^*_j(\overrightarrow{x})], \tag{7}$$

with

$$\int \mathrm{d}\mu_j = \sum_{m=-\infty}^{\infty}\int_0^\infty \mathrm{d}k_\perp k_\perp \int_{-\infty}^{\infty}\mathrm{d}\kappa. \tag{8}$$

It is easy to verify that the creation and annihilation operators satisfy the following commutation relation

$$[a_j(t,\overrightarrow{x}), a^+_{j'}(t,\overrightarrow{x})] = \delta_{j,j'}. \tag{9}$$

Other commutators are equal to zero. Then the correlation function of scalar field in cosmic string spacetime can be expressed as

$$\begin{aligned}&\langle 0|\Phi(t,\overrightarrow{x})\Phi(t',\overrightarrow{x'})|0\rangle \\ &= \frac{\nu}{8\pi^2}\sum_{m=-\infty}^{\infty}\int_0^\infty \mathrm{d}k_\perp \int_{-\infty}^{\infty}\mathrm{d}\kappa \frac{k_\perp}{\omega}\mathrm{e}^{-\mathrm{i}\omega\Delta t}\mathrm{e}^{\mathrm{i}\kappa\Delta z}\mathrm{e}^{\mathrm{i}\nu m\Delta\alpha} \\ &\times J_{\nu|m|}(k_\perp r)J_{\nu|m|}(k_\perp r'),\end{aligned} \tag{10}$$

where $\omega = \sqrt{\kappa^2 + {k_\perp}^2}$, $\Delta t = t - t'$, $\Delta\alpha = \alpha - \alpha'$, and $\Delta z = z - z'$. According to the [59], equation (10) can be written as

$$\langle 0|\Phi(t,\overrightarrow{x})\Phi(t',\overrightarrow{x'})|0\rangle = \frac{q(1-\Lambda^{2\nu})}{1+\Lambda^{2\nu} - 2\Lambda^\nu\cos(\nu\Delta\alpha)}, \tag{11}$$

where

$$\Lambda = \frac{r_2 - r_1}{r_2 + r_1}, \tag{12}$$

with

$$r_1 = \sqrt{(r-r')^2 + (z-z')^2 - (t-t'-\mathrm{i}\varepsilon)^2}, \tag{13}$$

$$r_2 = \sqrt{(r+r')^2 + (z-z')^2 - (t-t'-\mathrm{i}\varepsilon)^2}, \tag{14}$$

and $q = \frac{\nu}{4\pi^2 r_1 r_2}$, ε is a positive infinite small real number.

3. Local quantum estimation theory and physical model

3.1. Local quantum estimation theory

Any inference strategy amounts to find an estimator, i.e. a mapping $\hat{\lambda} = \hat{\lambda}(x_1, x_2, ...x_n)$ from the set of measurement outcomes into the space of parameters. According to the Cramér–Rao theorem, the optimal estimators are bounded by the inequality [52, 60]

$$\mathrm{Var}(\lambda) \geqslant \frac{1}{NF_\lambda}, \tag{15}$$

where $\mathrm{Var}(\lambda) = E_\lambda[(\hat{\lambda} - \lambda)^2]$ is the variance of any estimator, N is the number of measurements and F_λ is the Fisher information(FI) of parameter λ. In equation (15), there is a lower bound on the mean square error of any estimator of the parameter λ. Upon maximizing the FI over all the possible quantum measurements, we have that the FI of any quantum measurement is upper bounded by the QFI, i.e. $F(\lambda) \leqslant H(\lambda) \equiv \mathrm{Tr}[\rho(\lambda)L(\lambda)^2]$, where ρ is the density matrix of the detector and $L(\lambda)$ represents the symmetric logarithmic derivative satisfying the partial differential equation $\partial\rho(\lambda) = \frac{1}{2}(L(\lambda)\rho(\lambda) + \rho(\lambda)L(\lambda))$, and $H(\lambda)$ is the QFI of parameter λ. Therefore, the quantum Cramér–Rao bound is expressed as

$$\mathrm{Var}(\lambda) \geqslant \frac{1}{NF_\lambda} \geqslant \frac{1}{NH_\lambda}. \tag{16}$$

The quantum Cramér–Rao bound provides the ultimate bound to precision in the estimation of parameter λ for a state of the family $\rho(\lambda)$.

For a two-level atom system, the atomic state can be expressed in the Bloch sphere as

$$\rho = \frac{1}{2}(I + \boldsymbol{\omega} \cdot \boldsymbol{\sigma}), \tag{17}$$

where $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)$ denotes the Bloch vector, and $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is the Pauli matrices. Then the QFI can be written as [61]

$$H_X = \begin{cases} |\partial_X \boldsymbol{\omega}|^2 + \frac{(\boldsymbol{\omega}\cdot\partial_X\boldsymbol{\omega})^2}{1-|\boldsymbol{\omega}|^2}, & |\boldsymbol{\omega}| < 1, \\ |\partial_X \boldsymbol{\omega}|^2, & |\boldsymbol{\omega}| = 1. \end{cases} \tag{18}$$

3.2. Dynamical evolution of a two-level detector

In this section, we would like to discuss the evolution of the two-level atom. The total Hamiltonian of the detector-field system can be described as

$$H = H_S + H_F + H_I, \tag{19}$$

where $H_S = \frac{1}{2}\hbar\omega_0\sigma_3$ is the Hamiltonian of the two-level atom, H_F denotes the Hamiltonian of the free scalar field and H_I is the interaction Hamiltonian between the atom and the scalar field. Here $\hbar\omega_0$ represents the atomic energy spacing, and σ_3 is the Pauli matrix. The interaction Hamiltonian can be described as

$$H_I = \mu m(\tau)\Phi(x(\tau)) = \mu[\sigma_+(\tau) + \sigma_-(\tau)]\Phi(x(\tau)), \tag{20}$$

where μ is the coupling constant, $m(\tau)$ is the monopole matrix of the detector whose spacetime coordinates are given by $x(\tau)$. $\sigma_+(\tau)$ and $\sigma_-(\tau)$ are the atomic rising and lowering operator, respectively. Here $\Phi(x(\tau))$ corresponds to the scalar field operator.

Supposing the initial total density matrix of the system is $\hat{\rho}_{\text{tot}}(0) = \rho(0) \otimes |0\rangle\langle 0|$, where $\hat{\rho}(0)$ denotes the initial density matrix of the atom, and $|0\rangle\langle 0|$ is the vacuum state. In the limit of weak coupling, the master function of the atom is given by the Kossakowski–Lindblad form [62]

$$\frac{\partial \rho(\tau)}{\partial \tau} = -\mathrm{i}[H_{\text{eff}}, \rho(\tau)] + \mathcal{L}[\rho(\tau)], \tag{21}$$

with

$$\begin{aligned} H_{\text{eff}} &= \frac{1}{2}\Omega\sigma_z = \frac{1}{2}\{\omega_0 + \mu^2 \mathrm{Im}(\Gamma_+ + \Gamma_-)\}\sigma_z, \\ \mathcal{L}[\rho(\tau)] &= \sum_{j=1}^{3}[2L_j\rho L_j^\dagger - L_j^\dagger L_j \rho - \rho L_j^\dagger L_j], \end{aligned} \tag{22}$$

where $\Gamma_\pm = \int_0^\infty \mathrm{e}^{\mathrm{i}\omega_0\Delta\tau} G^+(\Delta\tau \pm \mathrm{i}\epsilon)\mathrm{d}\Delta\tau$, $L_1 = \sqrt{\frac{\gamma_-}{2}}\sigma_-$, $L_2 = \sqrt{\frac{\gamma_+}{2}}\sigma_+$, $L_3 = \sqrt{\frac{\gamma_z}{2}}\sigma_z$, $\gamma_\pm = \mu^2 \int_{-\infty}^{\infty} \mathrm{e}^{\mp\mathrm{i}\omega_0\Delta\tau} G^+(\Delta\tau - \mathrm{i}\epsilon)\mathrm{d}s$, $\gamma_z = 0$, where $G^+(x - x') = \langle 0|\Phi(x)\Phi(x')|0\rangle$ is the field correlation function, and $\Delta\tau = \tau - \tau'$. Equation (21) characterizes the evolution of the detector. If a two-level atom is initially prepared in an arbitrary state $|\psi(0)\rangle = \cos\frac{\theta}{2}|+\rangle + \mathrm{e}^{\mathrm{i}\phi}\sin\frac{\theta}{2}|-\rangle$, where θ, ϕ denote the weight parameter and phase parameter, and $|-\rangle, |+\rangle$ represent the ground state and excited state of the atom respectively. According to the equations (21) and (17), we obtain the evolution of the Bloch vector as follows

$$\begin{aligned} \omega_1(\tau) &= \sin\theta\cos(\Omega\tau + \phi)\mathrm{e}^{-\frac{1}{2}A\tau}, \\ \omega_2(\tau) &= \sin\theta\sin(\Omega\tau + \phi)\mathrm{e}^{-\frac{1}{2}A\tau}, \\ \omega_3(\tau) &= \cos\theta\mathrm{e}^{-A\tau} - \frac{B}{A}(1 - \mathrm{e}^{-A\tau}), \end{aligned} \tag{23}$$

with $A = \frac{1}{4}(\gamma_+ + \gamma_-)$ and $B = \frac{1}{4}(\gamma_+ - \gamma_-)$. It is worth noting that A and B are related to the field correlation function that depends on the property of spacetime. As a consequence of the interaction between the detector and field, the information of cosmic string spacetime could be encoded into the quantum state of the detector. By performing the measurements on the detector, we can infer the related parameter through the relevant outcomes.

4. Parameter estimation in cosmic string spacetime

Now, we intend to exploit local QET to find out the quantum measurement that maximizes the QFI which aims to evaluate the ultimate limits of precision in the estimation of the deficit angle parameter. We devote to explore how the precision in the estimation of the parameter changes when a two-level detector interacts with cosmic string scalar field. In the next, we will discuss the case that the detector which is very close to the string to obtain the analytical results. Here, we use the wire-approximation.

4.1. Parameter estimation for an inertial detector

We now probe the cosmic string spacetime by using the inertial detector. Since the spacetime is locally (but not globally) flat, the trajectory of the detector is written in Minkowski coordinates, which is described as

$$\begin{aligned} t = \gamma\tau, \quad z = z_0 + v\gamma\tau, \\ r = \text{constant}, \quad \alpha = \text{constant}, \end{aligned} \tag{24}$$

where $\gamma = (1-v^2)^{-\frac{1}{2}}$ is the Lorentz factor, v is the velocity of the detector, and τ is the proper time of the detector. According to the definition of the Wightman function, we obtain

$$G(\tau - \tau') = \frac{q(1+\Lambda^\nu)}{1-\Lambda^\nu}, \tag{25}$$

with

$$\Lambda = \frac{r_2 - r_1}{r_2 + r_1}, \tag{26}$$

and

$$r_1 = \sqrt{(r-r')^2 + (z-z')^2 - (t-t'-\mathrm{i}\varepsilon)^2}, \tag{27}$$

$$r_2 = \sqrt{(r+r')^2 + (z-z')^2 - (t-t'-\mathrm{i}\varepsilon)^2}, \tag{28}$$

where $q = \frac{\nu}{4\pi^2 r_1 r_2}$, and ε is a positive infinite small real number. By taking the trajectory (24) into equation (25), we obtain the Wightman function at $r \to 0$ as

$$G(\Delta\tau) = -\frac{\nu}{4\pi^2}\frac{1}{(\Delta\tau - \mathrm{i}\varepsilon)^2}. \tag{29}$$

Accordingly, we obtain

$$\gamma_- = \frac{\nu\mu^2\omega_0}{2\pi}, \tag{30}$$

and

$$\gamma_+ = 0. \tag{31}$$

Then, we obtain

$$A = B = \frac{\nu\gamma_0}{4}, \tag{32}$$

where $\gamma_0 = \frac{\mu^2\omega_0}{2\pi}$ is the spontaneous emission rate of the atom. In order to investigate the precision of quantum estimation for the deficit angle parameter ν, we will calculate the QFI of it, and devote to find the optimal estimation conditions. Based on the above result, we now substitute equation (23) into (18), and use the result from equation (32). Then we obtain the QFI of ν as follows

$$H_\nu = \frac{\mathrm{e}^{-\nu\tau}\tau^2\cos^2\frac{\theta}{2}(2\mathrm{e}^{\nu\tau} - 1 + \cos\theta)}{2(\mathrm{e}^{\nu\tau} - 1)}. \tag{33}$$

It is interesting that the QFI is independent of quantum phase ϕ. Thus, the QFI in fact should be written as $H_\nu(\nu, \tau, \theta, \omega_0)$, while we adopt the notation H_ν for simplify here. We will work with dimensionless quantities by rescaling time

$$\tau \longmapsto \widetilde{\tau} \equiv \gamma_0\tau. \tag{34}$$

For convenience, we continue to term $\widetilde{\tau}$ as τ.

4.2. Parameter estimation for a uniformly accelerated detector

The trajectory of the uniformly accelerated two-level atom is described as [63]

$$\begin{aligned} t &= \frac{1}{a}\sinh a\tau, \quad z = \frac{1}{a}\cosh a\tau, \\ r &= \text{constant}, \quad \alpha = \text{constant}. \end{aligned} \tag{35}$$

By taking the trajectory (35) into equation (25), we obtain the Wightman function of massless scalar fields in cosmic string spacetime at $r \to 0$ as

$$G(\Delta\tau) = -\frac{a^2\nu}{16\pi^2}\frac{1}{\sinh^2(\frac{a\Delta\tau}{2} - \mathrm{i}\varepsilon)}. \tag{36}$$

Accordingly, we obtain

$$\gamma_- = \frac{\nu\mu^2\omega_0}{2\pi}(1 + \frac{1}{\mathrm{e}^{\frac{2\pi\omega_0}{a}} - 1}), \tag{37}$$

and

$$\gamma_+ = -\frac{\nu\mu^2\omega_0}{2\pi}(1 + \frac{1}{\mathrm{e}^{\frac{-2\pi\omega_0}{a}} - 1}). \tag{38}$$

Then, we obtain

$$A = \frac{\nu\gamma_0}{4}\coth\frac{\pi\omega_0}{a}, \qquad B = \frac{\nu\gamma_0}{4}, \tag{39}$$

where $\gamma_0 = \frac{\mu^2\omega_0}{2\pi}$ is the spontaneous emission rate of the atom. Similarly, we will calculate the QFI of it, and devote to find the optimal estimation conditions. We now substitute equation (23) into (18), and use the result from equation (39). Then we calculate the QFI of ν as follows

$$H_\nu = \frac{w^{-1}\tau^2\gamma_0^2[w^2g^2\tan^2\theta + \cosh^2(\frac{\pi\omega_0}{a})(1+g)(w(3+\cos(2\theta)+4g) - (1+g)\sin^2\theta)]}{(w-1)[4w - 1 + 2\cos(2\theta)\cosh^2(\frac{\pi\omega_0}{a}) + 3\cosh(\frac{2\pi\omega_0}{a}) + 4\cos\theta\sinh(\frac{2\pi\omega_0}{a})]}, \tag{40}$$

where $w = \mathrm{e}^{\nu\tau\gamma_0\coth(\frac{\pi\omega_0}{a})}$, $g = \cos\theta\coth(\frac{\pi\omega_0}{a})$. Here the QFI is also independent of quantum phase ϕ. It only depends on the parameters ν, τ, a, θ and ω_0. Similarly, the QFI in fact should be written as $H_\nu(\nu, \tau, \theta, \omega_0, a)$, while we adopt the notation H_ν for simplify here. We will work with dimensionless quantities by rescaling time and acceleration

$$\tau \longmapsto \widetilde{\tau} \equiv \gamma_0\tau, \quad a \longmapsto \widetilde{a} \equiv \frac{a}{\omega_0}. \tag{41}$$

For convenience, we continue to term $\widetilde{a}$ and $\widetilde{\tau}$, respectively, as a and τ.

4.3. A comparison of the inertial detector and the accelerated detector

In order to find the conditions to improve the estimation, we would like to analyze and compare the inertial and accelerated cases. In the following, several figures will be presented to better illustrate our conclusions.

Firstly, to clarify which initial state is the best probe state for the estimation of ν, we plot the QFI as a function of the initial state parameter θ with different effective time τ in figure 1. The left panel is for the inertial detector, and the right panel is for the accelerated detector

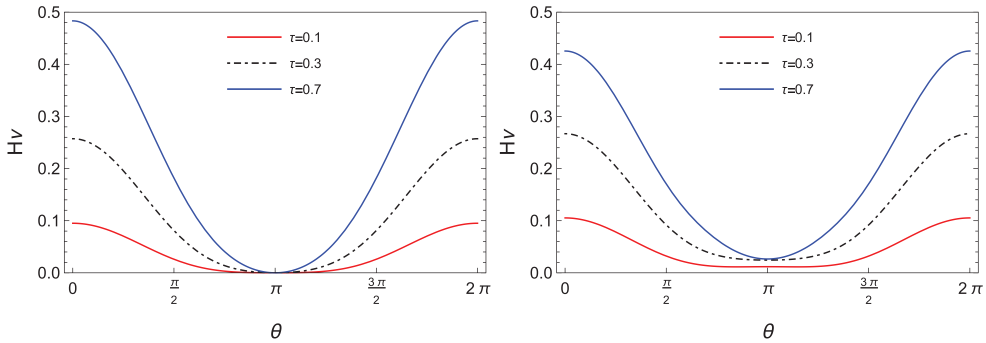

Figure 1. QFI of the deficit angle parameter ν as a function of the initial state parameter θ with fixed values of the effective time τ. The left panel is for the inertial detector, and the right panel is for the accelerated detector with the effective acceleration $a = 3$. We take the string tension $G\mu = 10^{-7}$, and the effective time $\tau = 0.1, 0.3, 0.7$, respectively.

with the effective acceleration $a = 3$. Obviously, for the two cases, both the QFI approaches its maximum value with an initial excited state of the detector. Therefore, we arrive a conclusion that the maximum sensitivity in the predictions for the deficit angle parameter ν can be obtained by initially preparing the detector in its excited state, which means that the excited state is the best probe state. In addition, we find that the maximum value of QFI for the inertial detector is higher than that for the accelerated detector.

We plot QFI of the deficit angle parameter ν as a function of the effective time τ with the optimal values of initial parameter θ for different detectors in figure 2. The left panel is for the inertial detector, and the right panel is for the accelerated detector with the effective acceleration $a = 3, 5, 8$. As is shown in figure 2, for different detectors, the QFI always achieves the maximum value when the detector evolves for a limited time, so we obtain that we can improve the precision for parameter ν by choosing appropriate detecting time. In addition, we find that the maximum value of QFI is higher for the inertial detector by comparing it with the uniform accelerated detector, and we obtain that accelerated motion does not improve this precision. We choose the string tension $G\mu = 10^{-7}$ according to the current bounds on the string tension from CMB experiments data [4]. We approximate the distance to the string to zero to get an analytic solution, while in the specific experimental operation, the magnitude of distance to the core can be taken around the order of $r \sim 10^{-6}$ m. From figure 2, we find out that an inertial detector is better. Here, we will give the optimal observation time and the QFI of the parameter. According to the [64], γ_0/ω_0 is of the order of 10^{-6}, and a typical transition frequency of the hydrogen atom is around $\omega_0 \sim 10^{15}\,\mathrm{s}^{-1}$. For the initial state parameter $\theta = 0$, we calculate the optimal evolution time is around 1.58×10^{-9} s, and H_ν is around 0.65 for the inertial detector.

Though the parameter ν for a real cosmic string spacetime is only slightly larger than unity, the investigations on cosmic string spacetimes with other ν also caught much attention [9, 65, 66]. In order to illustrate how the estimation is affected by different deficit angle, we plot the QFI as a function of the effective time τ with different ν in figure 3. The left panel is for the inertial detector, and the right panel is for the accelerated detector with the effective acceleration $a = 3$. With the increase of the parameter ν, the maximum value of QFI is decreased. As we have discussed above, the maximum value of QFI is higher for the inertial detector by comparing it with the accelerated detector.

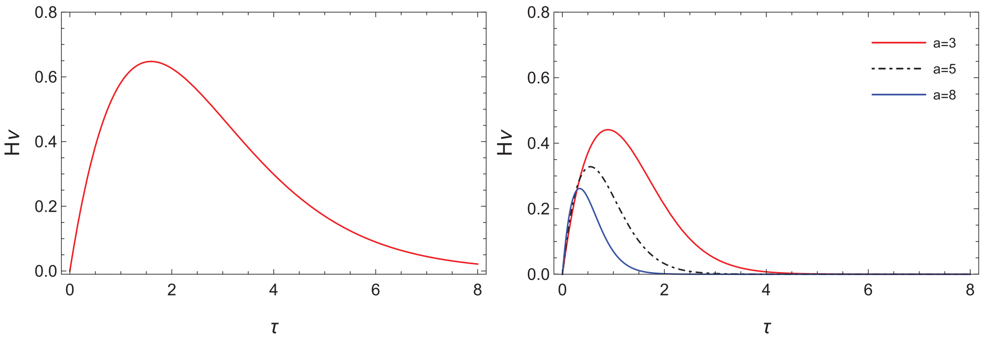

Figure 2. QFI of the deficit angle parameter ν as a function of the effective time τ. The left panel is for the inertial detector, and the right panel is for the accelerated detector with the effective acceleration $a = 3, 5, 8$. We take the string tension $G\mu = 10^{-7}$, and the initial state parameter $\theta = 0$.

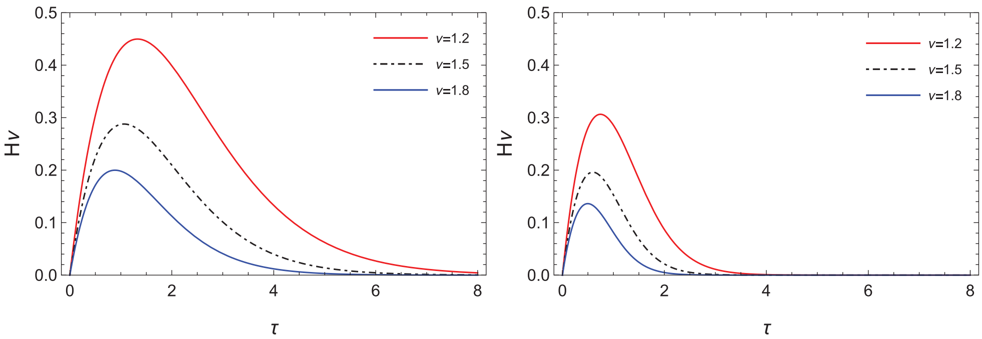

Figure 3. QFI of the deficit angle parameter ν as a function of the effective time τ with fixed values of ν. The left panel is for the inertial detector, and the right panel is for the accelerated detector with the effective acceleration $a = 3$. We take the initial state parameter $\theta = 0$, the acceleration $a = 3$, and the different deficit angle parameter $\nu = 1.2, 1.5, 1.8$, respectively.

5. Conclusion

We are interested in detecting the topological defects in cosmic string spacetime. Based on this motivation, we use a two-level atom which is coupled to the scalar field as a probe to estimate the deficit angle parameter ν, since the nontrivial global topology in this spacetime is characterized by the parameter ν. To address the quantum bound to the estimation of this parameter, we investigate the QFI of it, and we perform measurements on the two-level detector and maximize the value of QFI over all possible detector preparations and evolution times. Since it is very difficult to give a general analytic expression in such spacetime, we discuss the case that the atom is very close to the string to obtain the analytical results. For the estimation of parameter ν, we find some consistent conclusions for different detectors. We obtain that the QFI approaches its maximum value with an initial excited state of the detector, which tells us that the excited state is the best probe state. We show that the QFI always achieves the maximum value when the detector evolves for a limited time. We note that the sensitivity in the predictions for the deficit angle parameter ν decreases with the increase of ν. We also prove that the optimal QFI decreases with the increase of the acceleration, which shows that inertial detector is better than uniform, rectilinear accelerated detector for the estimation, but maybe circular detector is an even better choice, we would like to discuss it in the following work. Therefore, we provide a possibility to detect the nontrivial global topology in the cosmic string spacetime. We should acknowledge, for the atom which is very close to the string, there are still some limitations, for example, scaling problems will arise. There is a strategy which is proposed by 't Hooft, by considering conformal invariant gravity. Then the metric close to the string can we written as $g_{\mu\nu} = \Omega^2 \bar{g}_{\mu\nu}$ with $\bar{g}_{\mu\nu}$ Minkowski metric, where Ω^2 contains all the scale dependence. More detail please see the [58].

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ORCID iDs

Jiliang Jing https://orcid.org/0000-0002-2803-7900

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