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Thermodynamics in $f(R)$ theories of gravity with coupling between matter and geometry

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Abstract

The character of thermodynamics for $f(R)$ theories of gravity with coupling between matter and geometry in the FLRW universe has been studied in this paper. The result shows that on the apparent horizon, both the temporal and spatial component of the Friedmann equations can be written as the form of the first law of thermodynamics in the conservative context, but some additional terms are appeared due to the matter-geometry coupling and the non-equilibrium in the system. Moreover, the derived generalized second law of thermodynamics can not be always held in our considering case, which is different from the one given in general relativity. The condition to protect it is obtained, when we consider results from observational data and numerical solutions.

Keywords: coupling $f(R)$ gravity, thermodynamics, FLRW universe

1. Introduction

The result of the observational data from the type Ia supernovae (SNe Ia) [1, 2] indicates that our universe presently experiences an accelerating expansion, usually called the late-time cosmic acceleration. It makes the modern cosmology based on general relativity (GR) face troubles. Moreover, the high-precision observational data of cosmic microwave background (CMB) provided by the Wilkinson microwave anisotropy probe (WMAP) group [3–5] gives strong support to the existence of an inflationary period, which seems to be happened in the very early universe. This causes the situation of modern cosmology to get much more serious. In order to shed some lights on these observational issues, dark energy and modified theories of gravity are taken into consideration. However, up till now, the nature and the origin of dark energy are still not clear.

Alternative to dark energy, due to the simplicity and stability, $f(R)$ theories of gravity (see, for instance, [6–8] for reviews) are attractive candidates in modified theories of gravity, where $f(R)$ is an arbitrary function of the Ricci scalar R. To achieve the accelerating cosmic expansion, a great deal of study have been carried out in $f(R)$ theories of gravity (see, for instance, [9–14] and references therein). Moreover, under some conditions, the early-time and late-time cosmic acceleration can be unified (see, for instance, [10] and references therein). However, for a successful gravitational theory, besides attaining the stage of cosmic acceleration, other viability criteria, such as cosmological perturbations (see, for instance, [15–19] and references therein), energy conditions (see, for instance, [20–23] and references therein), instabilities (see, for instance, [22, 24, 25] and references therein) and the degeneracy problem of Lagrangian densities for a perfect fluid (see, for instance, [26–29] and references therein), should be also satisfied [7]. Among them, the character of thermodynamics is one of the most important aspects.

The gravitational field of black holes is the strongest in the nature and can be dealt with by GR. Considering the consequence of the quantum mechanics, black holes emit thermal radiations, just like the behavior of black bodies. Moreover, by introducing quantities of the area and the surface gravity of the black hole horizon, the laws of black holes mechanics are established in GR, which is very similar to the laws of the usual thermodynamics [30–32]. It is pointed out that the temperature, which is proportional to the surface gravity of the black hole horizon, the entropy, which is proportional to the area of the black hole horizon, and the mass of black holes satisfy the first law of thermodynamics [30].

The first law of thermodynamics of black holes indicates that there should be an association between thermodynamics of black holes and the Einstein equations since both the temperature and the entropy of black holes and the space-time structure are determined by purely geometric quantities. This relationship was found by Jacobson for the first time [33]. Then in the context of $f(R)$ theories of gravity [34], the general static spherically symmetric space-time [35] and scalar-tensor gravity [36–38], it was also discovered. While for the second law of thermodynamics, it has been widely studied in contexts of GR [39–49] and generalized theories of gravity [50, 51], where the second law of thermodynamics is usually called the generalized second law (GSL) of thermodynamics.

Since our universe can be seen as a thermodynamical system, the study on black holes can be extended to the framework of cosmology. It has been shown that the relationship between the first law of thermodynamics and the Friedmann equations with any spatial curvature was existed, but the horizon is the apparent horizon and the non-equilibrium thermodynamics should be taken into consideration [50, 52]. It is worth declaring that the apparent horizon and the event horizon are two kinds of horizons usually used in cosmology. They can not be distinguished clearly in some cases, which makes some difficulties in particular study [52]. However, since the event horizon does not protect the first and the second law of thermodynamics [47], we will only focus our concentration on the apparent horizon as the boundary of our universe in this work.

Recently considering the coupling between matter and geometry, a more general type of $f(R)$ gravity has been proposed [53]. This proposal has been extensively explored in various aspects (see, for instance, [19, 26, 27, 53–56] and references therein). It is worth mentioning that due to the existence of the coupling between matter and geometry, the conservation equations can not be generally promised. In the present work, to extend previous works, it is interesting to investigate the effect of the non-minimal coupling on the property of thermodynamics on the apparent horizon of the FLRW universe in $f(R)$ theories of gravity.

The present paper is organized as follows. The framework of our discussions is briefly reviewed in the following section. In section 3, in order to study the relationship between the first law of thermodynamics and the Friedmann equations with any spatial curvature on the apparent horizon of the FLRW universe, two different approaches are taken into consideration in $f(R)$ theories of gravity with coupling between matter and geometry. In section 4, the GSL of thermodynamics for $f(R)$ theories of gravity with coupling is explored in the FLRW universe. Conclusions are given in the last section.

2. Framework

A more general action of $f(R)$ theories of gravity involving coupling between matter and geometry is given by [53]

$$S=\int\{\frac{1}{2}f_1(R)+[1+\lambda f_2(R)]L_m\}\sqrt{-g}\mathrm{d}^4x, \tag{1}$$

where $f_i(R)$ $(i=1,2)$ are arbitrary functions of the Ricci scalar R, L_m is the Lagrangian density of matter and λ is a constant to denote the coupling strength between matter and geometry. When the coupling strength between matter and geometry vanishes, action (1) reduces to the context of $f(R)$ gravity without coupling. Besides, the framework of Einstein gravity can be reproduced by taking $f_1(R)=R$. Note that the units are taken as $8\pi G=c=1$ and will be used throughout this work.

Varying the action (1) with respect to the metric tensor, the field equations are

$$\begin{aligned} &f_{1R}(R)R_{\mu\nu}-\tfrac{1}{2}f_1(R)g_{\mu\nu}+(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu)f_{1R}(R)=-2\lambda f_{2R}(R)L_mR_{\mu\nu}\\ &+2\lambda(\nabla_\mu\nabla_\nu-g_{\mu\nu}\Box)L_mf_{2R}(R)+[1+\lambda f_2(R)]T_{\mu\nu}, \end{aligned} \tag{2}$$

where $f_{iR}(R)\equiv \mathrm{d}f_i(R)/\mathrm{d}R$ $(i=1,2)$, $\Box\equiv\nabla_\alpha\nabla^\alpha$, ∇_μ is the usual covariant derivative associated with the Levi-Civita connection of the metric and $T_{\mu\nu}$ is the matter energy-momentum tensor which is defined as

$$T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}. \tag{3}$$

Taking the covariant divergence of the field equation (2), the generalized conservation equations are

$$\nabla^\mu T_{\mu\nu}=\frac{\lambda f_{2R}}{1+\lambda f_2}[g_{\mu\nu}L_m-T_{\mu\nu}]\nabla^\mu R, \tag{4}$$

where the Bianchi identities and the geometrical identities [57], $[\nabla_\mu,\nabla_\nu]f_{iR}=(\nabla_\mu\nabla_\nu-\nabla_\nu\nabla_\mu)f_{iR}=R_{\mu\nu}\nabla^\mu f_{iR}$, are used. It is obvious that since the matter-geometry coupling exists, the conservation law of matter no longer holds in our considering case. However, in principle, a conservative context can be constructed, if $f_2(R)$ is a constant or the Lagrangian density of matter L_m does not explicitly depend on the metric.

Considering the spatially homogenous and isotropic FLRW spacetime,

$$\mathrm{d}s^2=-\mathrm{d}t^2+a(t)^2(\frac{\mathrm{d}r^2}{1-kr^2}+r^2\mathrm{d}\Omega^2), \tag{5}$$

where $a(t)$ is the scale factor, t is the cosmic time, k is the spatial curvature constant ($k=1,0$ and -1 correspond to a closed, flat and open universe, respectively) and $\mathrm{d}\Omega^2\equiv\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2$

is the metric of 2-dimensional sphere with the unit radius, and taking the form of the matter energy-momentum tensor as a perfect fluid, i.e.

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + p g_{\mu\nu}, \tag{6}$$

where ρ is the energy density, p is the pressure and the four-velocity U_μ satisfies the conditions $U_\mu U^\mu = -1$ and $U^\mu U_{\mu;\nu} = 0$, the Friedmann equations with any spatial curvature in $f(R)$ theories of gravity with coupling between matter and geometry can be obtained as follows:

$$H^2 + \frac{k}{a^2} = \frac{1}{3F}[\frac{1}{2}RF - \frac{1}{2}f_1(R) - 3H\dot{F} + (1 + \lambda f_2(R))\rho], \tag{7}$$

$$\dot{H} - \frac{k}{a^2} = -\frac{1}{2F}[\ddot{F} - H\dot{F} + (1 + \lambda f_2(R))(\rho + p)], \tag{8}$$

where the dot implies the derivative with respect to the cosmic time, $F \equiv f_{1R}(R) + 2\lambda f_{2R}(R)L_m$ and $H \equiv \dot{a}(t)/a(t)$ denotes the Hubble rate. Moreover, noting the Einstein convention for summation, equation (4) yields

$$\dot{\rho} + 3H(\rho + p) = -\frac{\lambda f_{2R}}{1 + \lambda f_2}(L_m + \rho)\dot{R}. \tag{9}$$

It is obvious that the condition for constructing the conservative context is to take the Lagrangian density of matter to be opposite to the energy density of the perfect fluid, i.e. $L_m = -\rho$, otherwise the feature and the generality of our considering case will be lost. We will take it in the following discussions.

3. First law of thermodynamics

In this section, we will study the relationship between the first law of thermodynamics and the Friedmann equations with any spatial curvature on the apparent horizon of the FLRW universe in $f(R)$ theories of gravity with coupling between matter and geometry.

Taking the spherical symmetry into consideration, the metric (5) can be rewritten as

$$\mathrm{d}s^2 = h_{ab}\mathrm{d}x^a\mathrm{d}x^b + \widetilde{r}^2\mathrm{d}\Omega^2, \tag{10}$$

where $\widetilde{r} = ra(t), x^0 = t, x^1 = r$ and the two-dimensional metric $h_{ab} = \mathrm{diag}(-1, a(t)^2/1 - kr^2)$. The dynamical apparent horizon is determined by the relation $h^{ab}\partial_a\widetilde{r}\partial_b\widetilde{r} = 0$, which indicates $\nabla\widetilde{r} = 0$ on the apparent horizon. Then the radius of the apparent horizon can be obtained as

$$\widetilde{r}_A = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}}. \tag{11}$$

When $k = 0$, the radius of the apparent horizon has the same value as the one of the Hubble horizon.

Since the FLRW universe can be seen as a thermodynamical system with the apparent horizon as its boundary, we assume that definitions of the temperature and the entropy on the horizon of black holes can be generalized to the apparent horizon. As the Hawking temperature is proportional to the surface gravity of the black hole horizon, the related temperature on the apparent horizon can be defined as

$$T = \frac{|\ \kappa\ |}{2\pi}, \tag{12}$$

where the surface gravity κ is given by

$$\begin{aligned} \kappa &= \tfrac{1}{2\sqrt{-h}}\partial_a(\sqrt{-h}h^{ab}\partial_b\widetilde{r}_A) \\ &= -\tfrac{1}{\widetilde{r}_A}(1-\tfrac{\dot{\widetilde{r}}_A}{2H\widetilde{r}_A}). \end{aligned} \tag{13}$$

In order to avoid a negative temperature, the absolute value sign has been introduced.

In GR, according to the Bekenstein–Hawking relation [30–32], the entropy of the black hole is given by $S = \mathscr{A}/4G$, where $\mathscr{A}$ is the area of the black hole horizon. In the context of $f(R)$ theories of gravity without coupling, the entropy of the black hole is assumed as $S = \mathscr{A}f_R(R)/4G$ [58, 59]. Later, in the framework of modified theories of gravity, Wald proposed that the entropy of the black hole with bifurcate Killing horizons is a Noether charge entropy [58]. It is then proved to be equal to a quarter of the horizon area in units of the effective gravitational coupling [60], i.e. $S = \mathscr{A}/4G_{\mathrm{eff}}$, where G_{eff} is the effective gravitational coupling.

Based on above mentioned facts, in our considering case, we assume that the associated entropy on the apparent horizon can be expressed as

$$S = \frac{\mathscr{A}}{4G}F, \tag{14}$$

where $\mathscr{A} = 4\pi\widetilde{r}_A^2$ is the area of the apparent horizon. When the matter-geometry coupling disappears, the expression (14) reduces to the case of $f(R)$ gravity without coupling, which is just the same as the one in [58, 59]. Furthermore, the entropy of black holes in Einstein gravity can be obtained by taking $f_1(R) = R$.

Taking the time derivative of the radius of the apparent horizon (11) and using the Friedmann equation (8), we can obtain

$$Fd\widetilde{r}_A = \frac{1}{2}\widetilde{r}_A^3H[\ddot{F} - H\dot{F} + (1+\lambda f_2(R))(\rho+p)]\mathrm{d}t, \tag{15}$$

where $\mathrm{d}\widetilde{r}_A$ is the infinitesimal change in the radius of the apparent horizon during the time interval $\mathrm{d}t$. Then taking the differential of the entropy (14) and combining the relationship (15), it reaches

$$\frac{1}{2\pi\widetilde{r}_A}\mathrm{d}S = (4\pi\widetilde{r}_A^3)H[\ddot{F} - H\dot{F} + (1+\lambda f_2(R))(\rho+p)]\mathrm{d}t + \frac{2\widetilde{r}_A}{4G}\mathrm{d}F. \tag{16}$$

Multiply the factor $(1-\frac{\dot{\widetilde{r}}_A}{2H\widetilde{r}_A})$ on both sides of the relationship (16), then one can get

$$\begin{aligned} T\mathrm{d}S = {}& T\tfrac{\mathscr{A}}{4G}\widetilde{r}_A^2H(\ddot{F} - H\dot{F})\mathrm{d}t + (1+\lambda f_2(R))(4\pi\widetilde{r}_A^3)H[(\rho+p)\mathrm{d}t \\ & -\tfrac{1}{2H\widetilde{r}_A}(\rho+p)\mathrm{d}\widetilde{r}_A] + T\tfrac{\mathscr{A}}{4G}\mathrm{d}F. \end{aligned} \tag{17}$$

The total energy of matter inside the apparent horizon with a sphere radius $\widetilde{r}_A$ is given by

$$E = \rho V, \tag{18}$$

where $V = \frac{4}{3}\pi\widetilde{r}_A^3$ is the volume of the sphere. Then the differential of the total energy of matter is

$$\mathrm{d}E = 4\pi\widetilde{r}_A^2\rho\mathrm{d}\widetilde{r}_A - 4\pi\widetilde{r}_A^3H(\rho+p)\mathrm{d}t, \tag{19}$$

where the conventional relationship between the density ρ and the pressure p is used. Substituting the relationship (19) into the expression (17), we find

$$TdS = (1 + \lambda f_2(R))(-dE + WdV) + T\frac{\mathscr{A}}{4G}[\widetilde{r}_A^2 H(\ddot{F} - H\dot{F}) + \dot{F}]dt, \tag{20}$$

where $W = \frac{1}{2}(\rho - p)$ is the work density [61, 62].

It is clear that the universal form, $dE = TdS + WdV$, contained in GR on the apparent horizon of FLRW universe does not hold for our considering case, where some additional terms are appeared. On the right side of the relationship (20), additional ones in the first term arise from the coupling between matter and geometry inside the apparent horizon as the existence of the constant λ. While besides the matter-geometry coupling, additional ones in the second term come from modified geometry terms, which may be interpreted as a entropy production of the non-equilibrium developed internally in the system according to [34, 63].

In order to denote additional terms, three notations are introduced as follows:

$$E_c \equiv \lambda f_2(R)E, \tag{21a}$$

$$W_c \equiv \lambda f_2(R)W, \tag{21b}$$

$$d\overline{S} \equiv -\frac{\mathscr{A}}{4G}[\widetilde{r}_A^2 H(\ddot{F} - H\dot{F}) + \dot{F}]dt. \tag{21c}$$

Then the expression (20) can be rewritten as

$$TdS = -dE - dE_c + WdV + W_c dV - Td\overline{S}. \tag{22}$$

If we can take $\widetilde{W} = W + W_c$ as the effective work density and $d\widetilde{E} = d(E + E_c)$ and $d\widetilde{S} = d(S + \overline{S})$ as the infinitesimal change of the effective energy and the effective entropy on the apparent horizon of the FLRW universe during the time interval dt, respectively, the expression (22) can be written as

$$Td\widetilde{S} = -d\widetilde{E} + \widetilde{W}dV. \tag{23}$$

It is worth stressing that when the matter-geometry coupling disappears, the expression (20) will be reduced to the context of $f(R)$ theories of gravity without coupling, which is consistent with the one given in [37]. Moreover, the traditional first law of thermodynamics in GR can be achieved, when $f_1(R) = R$.

On the other hand, the first law of thermodynamics for $f(R)$ theories of gravity with coupling between matter and geometry on the apparent horizon of the FLRW universe can also be obtained in the following way.

From the entropy of Einstein gravity, i.e. $S_E = \mathscr{A}/4G$, the thermodynamical fluid δQ can be given as

$$\delta Q = TdS_E = -3V\frac{d(H^2 + \frac{k}{a^2})}{dt}dt - 3VH\frac{(\dot{H} - \frac{k}{a^2})^2}{H^2 + \frac{k}{a^2}}dt. \tag{24}$$

In general, the Friedmann equations in any theories of gravity can be expressed as that in GR, i.e.

$$H^2 + \frac{k}{a^2} = \frac{1}{3}\rho_{\rm eff}, \tag{25}$$

$$\dot{H} - \frac{k}{a^2} = -\frac{1}{2}(\rho_{\text{eff}} + p_{\text{eff}}), \tag{26}$$

where $\rho_{\text{eff}} = \rho + \rho_{\text{fe}}$ and $p_{\text{eff}} = p + p_{\text{fe}}$ are the effective energy density and the effective pressure respectively, where ρ and p are the energy density and the pressure of ordinary matter, and ρ_{fe} and p_{fe} are the energy density and the pressure of other matter fields and energy components, respectively. From the functional point of view, the Friedmann equation (25) can be written as

$$H^2 + \frac{k}{a^2} = \mathscr{H}(\rho, \rho_{\text{fe}}). \tag{27}$$

Then the expression (24) reads

$$T\mathrm{d}S_E = -3V(\frac{\partial \mathscr{H}}{\partial \rho}\dot{\rho} + \frac{\partial \mathscr{H}}{\partial \rho_{\text{fe}}}\dot{\rho}_{\text{fe}})\mathrm{d}t - 3VH\frac{(\dot{H} - \frac{k}{a^2})^2}{H^2 + \frac{k}{a^2}}\mathrm{d}t. \tag{28}$$

Multiplying the factor $(3\partial\mathscr{H}/\partial\rho)^{-1}$ on both sides of the expression (28), it follows that

$$T\frac{1}{3}\frac{1}{\frac{\partial \mathscr{H}}{\partial \rho}}\mathrm{d}S_E = -V\dot{\rho}\mathrm{d}t - V\frac{1}{\frac{\partial \mathscr{H}}{\partial \rho}}\frac{\partial \mathscr{H}}{\partial \rho_{\text{fe}}}\dot{\rho}_{\text{fe}}\mathrm{d}t - VH\frac{(\dot{H} - \frac{k}{a^2})^2}{H^2 + \frac{k}{a^2}}\frac{1}{\frac{\partial \mathscr{H}}{\partial \rho}}\mathrm{d}t. \tag{29}$$

Taking the conservative equation into consideration, one can prove that

$$-VH\frac{(\dot{H} - \frac{k}{a^2})^2}{H^2 + \frac{k}{a^2}}\frac{1}{\frac{\partial \mathscr{H}}{\partial \rho}} = -\frac{1}{2}(\rho + p)\dot{V}. \tag{30}$$

Then the expression (29) can be written as

$$T\frac{1}{3}\frac{1}{\frac{\partial \mathscr{H}}{\partial \rho}}\mathrm{d}S_E = -\mathrm{d}E + W\mathrm{d}V - V\frac{1}{\frac{\partial \mathscr{H}}{\partial \rho}}\frac{\partial \mathscr{H}}{\partial \rho_{\text{fe}}}\dot{\rho}_{\text{fe}}\mathrm{d}t. \tag{31}$$

The left side of the above expression can be expressed as

$$T\frac{1}{3}\frac{1}{\frac{\partial \mathscr{H}}{\partial \rho}}\mathrm{d}S_E = Td(\frac{1}{3}\frac{1}{\frac{\partial \mathscr{H}}{\partial \rho}}S_E) - T\frac{1}{3}S_E d(\frac{1}{\frac{\partial \mathscr{H}}{\partial \rho}}). \tag{32}$$

If the entropy can be redefined as

$$S' \equiv \frac{1}{3}\frac{1}{\frac{\partial \mathscr{H}}{\partial \rho}}S_E, \tag{33}$$

we can obtain

$$T\mathrm{d}S' = -\mathrm{d}E + W\mathrm{d}V - T\mathrm{d}\overline{S}', \tag{34}$$

where $\mathrm{d}\overline{S}'$ is defined as

$$\mathrm{d}\overline{S}' \equiv -\frac{1}{T}[-V\frac{1}{\frac{\partial \mathscr{H}}{\partial \rho}}\frac{\partial \mathscr{H}}{\partial \rho_{\text{fe}}}\dot{\rho}_{\text{fe}}\mathrm{d}t + T\frac{1}{3}S_E d(\frac{1}{\frac{\partial \mathscr{H}}{\partial \rho}})]. \tag{35}$$

Consequently, the form of the first law of thermodynamics in generalized theories of gravity can be expressed as

$$T\mathrm{d}S' + T\mathrm{d}\overline{S}' = -\mathrm{d}E + W\mathrm{d}V. \tag{36}$$

Formulations of the entropy S' and the entropy production $\mathrm{d}\overline{S}'$ depend on particular theories of gravity. The reason of the presence of the entropy production $\mathrm{d}\overline{S}'$ has been mentioned above. In our considering context, expressions of S' and $\mathrm{d}\overline{S}'$ are just the same as expressions (14) and (21*c*).

4. GSL of thermodynamics

The second law of thermodynamics for $f(R)$ theories of gravity with coupling between matter and geometry in the FLRW universe is explored as follows. In contexts of modified theories of gravity, the second law of thermodynamics is usually called the GSL of thermodynamics. In order to study the GSL, all kinds of the entropy should be taken into consideration. From the expression of the first law of thermodynamics (23), it is clear that the entropy on the apparent horizon and the entropy production of the non-equilibrium should be included. Besides them, the entropy for all fluids of matter, field and energy inside the apparent horizon should be also contained, which can be given by the Gibb's equation as

$$T_m \mathrm{d}S_m = d(\rho V) + p\mathrm{d}V = V\mathrm{d}\rho + (\rho + p)\mathrm{d}V, \tag{37}$$

where T_m denotes the temperature of the total energy inside the apparent horizon, which usually is not equal to the temperature on the apparent horizon T due to the energy flow between them. One assumes that $T_m = bT$ [50, 64], where b is the temperature parameter and satisfies $0 < b < 1$ to ensure that the value of temperature is positive and less than the temperature on the apparent horizon. When $b = 1$, the energy flow between the apparent horizon and inside it vanishes, which indicates that the energy on the apparent horizon and inside it are in the thermal equilibrium. In our discussions, the continuity equation is protected so that $b = 1$.

According to the statement of the second law of thermodynamics, the GSL in our considering case can be proposed as

$$\dot{S} + \mathrm{d}\dot{\overline{S}} + \dot{S}_m \geqslant 0. \tag{38}$$

Then using expressions (23) and (37), the inequality (38) reads

$$\begin{aligned} &-\tfrac{\lambda f_{2R}}{1+\lambda f_2} 8\pi (H^2 + \tfrac{k}{a^2})^{-\frac{3}{2}} [\ddot{H} + 2H\dot{H} + 2H(\dot{H} - \tfrac{k}{a^2})][-3F(\dot{H} + H^2) \\ &+ \tfrac{1}{2} f_1 + 3H\dot{F}] + \tfrac{1}{1+\lambda f_2} 2\pi (H^2 + \tfrac{k}{a^2})^{-\frac{3}{2}} H[2\lambda f_2 - (1 - \lambda f_2) \tfrac{\dot{H} - \frac{k}{a^2}}{H^2 + \frac{k}{a^2}}][\\ &-2F(\dot{H} - \tfrac{k}{a^2}) - \ddot{F} + H\dot{F}] \geqslant 0. \end{aligned} \tag{39}$$

This is the GSL of $f(R)$ theories of gravity with coupling between matter and geometry in the FLRW universe. It is obviously that the GSL can not be always held. To obtain this result, the Friedmann equations (7) and (8) have been used. When $\lambda = 0$, the inequality (39) reduces to the case of $f(R)$ theories of gravity without coupling, which is different from the one in [50], where the GSL can be always held. Moreover, if we further take $f_1(R) = R$, the second law of thermodynamics for GR in the FLRW universe can be attained.

According to the observational data from the Large Scale Structure [65, 66] and the result of numerical solutions in [19], the spatial part of our universe is flat and the coupling between matter and geometry should be weak. Hence, when $k = 0$ and $\lambda \sim 0$, the inequality (39) reads

$$2F\dot{H}^2 + \ddot{F}\dot{H} - H\dot{F}\dot{H} \geqslant 0. \tag{40}$$

It is clear that the condition to protect the inequality (40) mainly depends on the function F and its time derivatives.

5. Conclusions

In this paper, the character of thermodynamics for $f(R)$ theories of gravity with coupling between matter and geometry in the FLRW universe has been investigated. By extending the definition of the temperature and the entropy on the horizon of black holes to the apparent horizon of the FLRW universe, and taking the condition for the conservation law, the spatial component of the Friedmann equations can be written as the form of the first law of thermodynamics, but some additional terms are appeared. Due to the matter-geometry coupling inside the apparent horizon, additional terms related to the energy and the work density arise as expected. While the rest additional terms can be seen as the entropy production of the non-equilibrium developed internally in the system. On the other hand, by introducing the Clausius relation, $\delta Q = T\mathrm{d}S$, to the apparent horizon of the FLRW universe, the temporal component of the Friedmann equations can also be written as the form of the first law of thermodynamics, where additional terms are also appeared. It is clear that, if the FLRW universe can be seen as a thermodynamical system with the apparent horizon as its boundary, the Friedmann equations can be written as the form of the first law of thermodynamics in $f(R)$ theories of gravity with coupling between matter and geometry. When the matter-geometry coupling disappears, the form of the first law of thermodynamics in $f(R)$ theories of gravity without coupling can be obtained. Moreover, when $f_1(R) = R$, the traditional first law of thermodynamics in GR can be achieved.

In order to study the second law of thermodynamics in our considering context, which is usually called GSL, all kinds of the entropy should be taken into consideration. Besides the entropy provided by the apparent horizon and the non-equilibrium, the entropy for all fluids of matter, field and energy inside the apparent horizon should be also contained. According to the statement of the second law, the total entropy of an isolated system can never decrease over time, we have derived the GSL for $f(R)$ theories of gravity with coupling between matter and geometry in the FLRW universe. The result shows that the GSL can not be always held, which is different from the one in GR. When we consider the spatial part of our universe is flat and the coupling between matter and geometry should be weak, the condition to protect the reduced GSL can be obtained.

In this work, results are obtained in the conservative context. However, when the conservation of the matter energy-momentum tensor is not preserved, the conventional relationship between the density and the pressure no longer holds. Actually, this situation can occur in any theory of gravity, where metric couples to another field or the extra degree of freedom appears (see [67] as an example alternative to $f(R)$ theories of gravity). In our considering case, it leads to the result given in equation (18), which right side is not equal to zero. It is clear that the extra term depends on the form of the arbitrary function $f_2(R)$ and the Lagrangian density of matter and the coupling strength between matter and geometry. Then this term will appear in equation (29) and eventually affect the result given in equation (20). Since the additional term essentially arises from the coupling between matter and geometry, it will be in the first term on the right side of equation (20). This is the effect of the deviation from the conservation on the first law of thermodynamics in our considering case. Moreover, the coupling between matter and geometry could cause a source or sink to the fluid, which can be a classical form or in the form of the particle production. This should be noted as the physical consequence of the violation from the conservation of the matter energy-momentum tensor.

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