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Dynamical study of interacting Ricci dark energy model using Chevallier-Polarsky-Lindertype parametrization

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Abstract

The main focus of this paper is to investigate an interacting phenomena between dark matter-dark energy within a non-flat FLRW spacetime geometry bounded by a horizon with specific Ricci cut-off. We assume an interaction term $Q(z)$ between two dark components of the fluid and evaluate it analytically via physical quantities like energy density and pressure. We constraint the model parameters and found specific region of validity for these parameters. Considering 'Chevallier-Polarsky-Linder type parametrization' of the coincidence parameter $r(z)$, we observed that our model possess a future singularity of Type III. After finding the singular behavior, we examine the nature of our model via cosmological parameters like $q,\ r,\ s$. It is noted that our model is very close to Λ cold dark matter (ΛCDM) model.

Keywords: dark energy models, Statefinder parameters, future singularities

(Some figures may appear in colour only in the online journal)

1. Introduction

Latest observational data of Type 1a Supernovae and WMAP indicate that the current universe is transforming from decelerating stage to an accelerating one. Initially, it is supposed that an enigmatic kind of energy dubbed as dark energy (DE), having large negative pressure effecting the evolution of the universe. Later on, it is confirmed by CMBR. The most adopted and simplest DE component is the cosmological constant (Λ) with constant equation of state (EoS), i.e. '$\omega = -1$', well fitted with recent observational data. Yet, it suffers with two major issues: 'fine-tuning; cosmic coincidence'. The first puzzle is generated due to larger gaps between observed and anticipated value of Λ. The second problem can be explained as: 'In present time, our cosmos is experiencing an expansion when the energy densities of dark matter (DM) and DE are comparable. How this is possible?' To overcome such problems, a large number of DE models have been proposed in literature such as quintessence, k-essence, tachyon, phantom, holographic DE (HDE), Ricci DE (RDE) and Chaplygin gas (CG) etc [1]. However our knowledge about the past and future fate of the universe is quite limited till now. Still there is no certain unified scenario for such evolution of the universe. This motivates us to explore the DE problem by using some other procedure like, 'holographic principle (HP)'.

This principle is an attribute of quantum gravity, which states that 'the information contents of a space volume lie on the boundary surface that bound this volume'. This principle also suggests that the ultraviolet (UV) cutoff scale Λ is connected to its infrared (IR) cutoff scale L. Cohen *et al* [2] pointed out that for a system with size L and UV cutoff scale Λ without decaying into a black hole (BH), the quantum vacuum energy (ρ_Λ) of the system should not exceed the mass of BH with the same size. Mathematically, it is written as $L^3\rho_\Lambda \leqslant LM_p^2$, where $M_p^2 = 8\pi G$ is the reduced Planck mass. Applying this idea to cosmology, one can choose the largest L, which satisfies this inequality by assuming ρ_Λ as DE.

Holographic type DE model seems to be a reasonable choice as it solved some DE related issues but unfortunately this model attached with 'causality issue', i.e. future event horizon is supposed in this model. Gao *et al* [3] put forward an idea of proportionality relation between DE density (ρ_D) and Ricci scalar (R), i.e. RDE. This model is physically viable as it provides consistent results with latest astrophysical data. In addition, also eliminate the causality and cosmic coincidence issues. The scalar R for flat FRW universe is calculated as $6(\dot{H} + 2H^2)$, which leads to $\rho_{\mathrm{RDE}} = 3c^2(\dot{H} + 2H^2)$, where H is the 'Hubble parameter'.

Most of the investigation has been done by considering DE and DM as separate candidates, but there is no logic to neglect associations in the dark sector. It is found that cold DM (CDM) is decaying into DE, which favors the interaction between these two components. Many models with interacting DE have been investigated in literature. The possibility of DE-DM interaction is the usual phenomena and gives a richer dynamic. Arevalo *et al* [4] studied the interaction phenomena between DE-DM by considering a 'holographic Ricci-like DE model' in flat FRW spacetime. They noted that during cosmic evolution a change of sign involves in the interaction function. They obtained results that are well-fitted with the current observable universe. Aydiner [5] observed that the interaction between DE and DM can be modeled with the help of few types of 'non-linear Lotka–Volterra equations' suitable for cosmology. In previous years, a lot of work has been done on HDE/RDE and on their generalizations see [6–11]. del Campo *et al* [12] studied different HDE models from a unique point of view. They compared models for which the parameter H, the future event horizon or a quantity proportional to R are considered as an IR cutoff. Estimation of EoS parameter for all the three cutoffs are performed with the help of a 'Bayesian statistical analysis', using data from *supernovae type Ia* and the history of the H. The ΛCDM model is the significant triumph of the analysis. Som *et al* [13] studied DE models encouraged by the HP in homogeneous isotropic spacetime along with a DE density $\rho_{\mathrm{DE}} = 3(\alpha H^2 + \beta\dot{H})$ where α, β are constants. They worked by introducing different general types of interaction terms among three HDE models including HRDE and DM. In a spatially non-flat universe, Pasqua *et al* [14] analyzed the 'logarithmic entropy corrected' and 'power law' versions of the RDE model within Horava–Lifshitz gravity (HLG). For non-interacting and interacting RDE and DM, they get exact differential equations that admit the evolutionary form of the RDE density. Jawad [15] studied the behavior of 'pilgrim DE (PDE) conjecture', which can explain that phantom-like DE model holds the sufficient resistive force to prevent the formation of a BH. He considered the non-flat geometry, comprised of interacting CDM and 'ghost PDE (GPDE)'. In this scenario, he discussed the nature of DE model through famous cosmological parameters, statefinder pair, evolution parameter (ω_Λ), $\omega_\Lambda - \omega'_\Lambda$ planes and squared speed of sound.

Jimenez *et al* [16] explored interacting DE and DM models and provided new physics using Q-interaction. They determined their potential relation with cosmological future singularities and explored an interaction singularity named Q-singularity that can be mapped into every future singularity existed in the literature. Ali and Amir [17] reconstructed different scalar field models including K-essence, tachyon, quintessence and dilaton using modified HRDE model within Chern–Simon modified gravity. In this scenario, they observed the accelerating expanding cosmic behavior via declaration parameter (q). Zadeh *et al* [18] explored the cosmological effects of the sign-changeable interacting HDE model in the context of FRW universe by considering the Brans–Dicke theory of gravity. They choose three different cut-offs, i.e. the Granda–Oliveros, the future event horizon and the Ricci cut-offs and obtained the parameters such as ω, the density parameter (ω_D) and q. They observed that ω_D can cross the phantom divide line (PDL) only for future event horizon. Moreover, the stability of the sign-changeable interacting HDE model against perturbations has been checked.

Feng *et al* [19] considered interacting HDE model in the framework of a perturbed universe. It is the first work in literature in which generalized post-Friedmann approach is being used to avoid the large scale instability issue. They constrained the model by employing red-shift (RS) space distortions measurements. They found that for both of the cases: $Q = \beta H_0 \rho_c$ and $Q = \beta H \rho_c$, the interacting HDE model is more preferred over non-interacting model. Cruz and Lepe [20] considered a generalized function for c^2 term, which seems in the conventional expression for HDE model in curved FRW universe bounded by apparent horizon. They explored the slowly varying condition for c^2 term and obtained a range of validity for the HDE model. They found that the holographic cut-off is satisfactory to define late time cosmic evolution. The same authors [21] discussed the DE model by considering future event and particle horizons in flat FRW universe within a holographic background. They found that the model experienced genuine big rip singularity, when the DE density is drew by future horizon resulting parameter state cross PDL. Further, they analyzed that in the DE-DM interaction, the second law of thermodynamics (SLT) cannot be satisfied because of the production of negative entropy.

In a curved space-time the trouble with singularities is very subtle and a broad literature deals with this problem from different perspectives. Apart of previous bibliography, recently, a new technique of DE-DM interaction under a holographic approach within curved FLRW spacetime has been proposed in [22] in which the interaction term is not the usual one. According to the observational results of Planck, BAO, SNIa and H_0 [23, 24], they considered a positive interaction function with a 'Chevallier-Polarsky-Linder (CPL)' type parametrization of the coincidence parameter and realized that the considered model admits a future singularity of Type III. Also, obtained the crossing of PD with the aid of some cosmological parameters constrained with astrophysical data of DE survey year I results mentioned in [25] and Planck Power Spectra, Planck Lensing, and BAO published in [26]. So, motivated by [22], our aim is to check the role of RDE model to discuss the singular universe when the effects of spatial curvature of the spacetime are included and to alleviate the coincidence problem by introducing an interaction function between RDE and DM under non-flat FRW model. To discuss future singularity, we consider CPL-type Parametrization of the coincidence parameter $r(z)$. In this scenario, the interaction term $Q(z)$ will be calculated analytically through physical quantities, i.e. we did not consider a specific parametrization for the Q-terms as is often done in literature.

The arrangement of this paper is as under: in section 2, we will develop basic dynamical equations for RDE-DM interacting scheme in the framework of non-flat FRW model. We will calculate some quantities of interest at cosmological level such as the coincidence and deceleration parameters using Ricci cut-off in section 3. Using some recent observational data given in published papers [23–26], we will determine the specific range of values for each cosmological parameter at present time, we mainly focus on the EoS parameter of DE. In section 4, we will discuss about future singularity coming from CPL-type of parametrization and after admitting singularity, we will perform statefinder diagnostic for our model. The results are summarized in the last section.

2. Interacting DM-DE scheme

In this section, we are going to describe briefly the dynamics of interacting scheme for the DM-DE components. We have a tendency to show that beneath the selection of a cut-off, which is given by *Hubble* scale in the form of ρ_{DE}, we are able to construct a particular interaction term between these dark components, which is function of some model parameters as well as the cosmological RS. In the case of non-flat FLRW spacetime, the Friedmann constraint can be defined as follows

$$E^2(z) = \frac{1}{3H_0^2}(\rho_{\mathrm{DE}}(z) + \rho_{\mathrm{DM}}(z)) + \Omega_k(z), \tag{1}$$

where $E(z) = \frac{H(z)}{H_0}$ be a normalized *Hubble* parameter, $1 + z = \frac{a_0}{a}$, ρ_{DM}, the energy density of DM and ρ_{DE} be the DE density. Also, Ω_k is the curvature parameter described as $\Omega_k(z) = \Omega_k(0)(1+z)^2$, where $\Omega_k(0) = -\frac{k}{a_0^2 H_0^2}$. The continuity equations in the form of energy densities are defined as

$$\rho'_{\mathrm{DE}} - 3\left(\frac{1+\omega_{\mathrm{DE}}}{1+z}\right)\rho_{\mathrm{DE}} = \frac{Q}{H_0 E(z)(1+z)}, \tag{2}$$

$$\rho'_{\mathrm{DM}} - \left(\frac{3}{1+z}\right)\rho_{\mathrm{DM}} = -\frac{Q}{H_0 E(z)(1+z)}, \tag{3}$$

where ω_{DM} is supposed to be zero in the above equations and prime represents derivative with respect to z. Also, the Q-term concludes the behavior of the interaction between DM and DE.

Differentiating equation (1) with respect to z and simplifying, we get

$$1 + \frac{\omega_{\mathrm{DE}}(z)}{1+r(z)} = \frac{2E^2(z)}{3\rho_{\mathrm{DE}}(1+r(z))}\left[\frac{3H_0^2}{2}\frac{(1+z)\mathrm{d}\ln E^2(z)}{\mathrm{d}z} - \Omega_k(0)\left(\frac{1+z}{E(z)}\right)^2\right]. \tag{4}$$

We can evaluate an expression used in the above equation using (1), given as follows

$$\frac{E^2(z)}{\rho_{\mathrm{DE}}(1+r(z))} = \left[3H_0^2 - \Omega_k(0)\left(\frac{1+z}{E(z)}\right)^2\right]^{-1},$$

putting this expression in the above equation (4), we get

$$1 + \frac{\omega_{\mathrm{DE}}(z)}{1+r(z)} = \frac{2}{3}\left(\frac{1}{2}(1+z)\frac{\mathrm{d}\ln E^2(z)}{\mathrm{d}z} - \Omega_k(0)\left(\frac{1+z}{E(z)}\right)^2\right)\left[1 - \Omega_k(0)\left(\frac{1+z}{E(z)}\right)^2\right]^{-1}, \tag{5}$$

where $r(z)$ is said to be the coincidence parameter defined as $r = \frac{\rho_{\mathrm{DM}}}{\rho_{\mathrm{DE}}}$. Also, equation (5) can be written in the form of deceleration parameter, using following definition of q (by converting formula of q in terms of z) as

$$1 + q(z) = \frac{1}{2}(1+z)\left(\frac{\mathrm{d}\ln E^2(z)}{\mathrm{d}z}\right), \tag{6}$$

we get modified equation (5) as under

$$1+\frac{\omega_{\rm DE}(z)}{1+r(z)}=\frac{2}{3}\left(1+q(z)-\Omega_k(0)\left(\frac{1+z}{E(z)}\right)^2\right)\left[1-\Omega_k(0)\left(\frac{1+z}{E(z)}\right)^2\right]^{-1}. \tag{7}$$

Performing the calculations for equation (7) at present time i.e. at $z=0$, we can have an approximation for the present value of q as follows

$$q_0=\frac{1}{2}\left(1+\frac{3\omega_{\rm DE,0}}{1+r_0}\right)(1-\Omega_k(0)). \tag{8}$$

If we assume the expression given in (7) along with values of $\Omega_k(0)$ and $r_0=\frac{\rho_{\rm DM,0}}{\rho_{\rm DE,0}}$ that are given in [23, 26] at $z=0$, we get an interval for $\omega_{\rm DE,0}\in[-1.4746,-0.008\,746]$, which represents the crossing from phantom region ($\omega<-1$) to quintessence region ($\omega>-1$).

3. Ricci cut-off for DE

In this section, we will evaluate some physical quantities like $\rho_{\rm DE}$, $\rho_{\rm DM}$ assuming that universe is bounded by a horizon with Ricci cut-off $L=\frac{1}{R}$. The RDE density is given as follows

$$\rho_{\rm DE}=\frac{3c^2}{8\pi}R, \tag{9}$$

where R for flat FRW universe is given as

$$R=-6(\dot{H}+2H^2),$$

where, c is a positive constant, which is given in the interval $0<c^2<1$ in order to define an expanding universe. To explain the behavior of RDE, this parameter c has an important role. Moreover, according to recent observations, this choice of $\rho_{\rm DE}$ has a good fit results as shown in [27]. Now, we substituting value of R in equation (9), we get $\rho_{\rm DE}$ in terms of z (using expression of $E(z)$) as

$$\rho_{\rm DE}(z)=-\frac{9c^2}{4\pi H_0}\left(2H_0E^2(z)+E'(z)\right). \tag{10}$$

Now, we are able to calculate the quantity $\rho_{\rm DM}$ via equations (1) and (10) in terms of z as

$$\rho_{\rm DM}(z)=3H_0^2E^2(z)\left[1-\Omega_k(0)\left(\frac{1+z}{E(z)}\right)^2+\frac{3c^2}{2\pi H_0^2}\right]+\frac{9c^2}{4\pi H_0}E'(z). \tag{11}$$

Dividing equations (11) to (10) and after some simple calculations the parameter $r(z)$ may be written as

$$r(z)=-1-\frac{1}{3c^2(2H_0E^2(z)+E'(z))}\left[4\pi H_0^3E^2(z)\left(1-\Omega_k(0)\left(\frac{1+z}{E(z)}\right)^2\right)\right]. \tag{12}$$

Now differentiating equation (11) with respect to z, we get

$$\rho'_{\rm DM}(z)=\frac{1}{4\pi H_0}\Big[-24\pi H_0^3(1+z)\Omega_k(0)+24\pi H_0^3E(z)E'(z)+36H_0c^2E(z)E'(z)+9c^2E''(z)\Big], \tag{13}$$

inserting above calculated expression of $\rho'_{\rm DM}(z)$ in equation (3) and after some calculations, we arrived at

$$(1+z)\frac{\mathrm{d}\ln E^2(z)}{\mathrm{d}z} = 3 + \frac{2(1+z)^2\Omega_k(0)}{E^2(z)} + \frac{9c^2}{2\pi H_0^2} + \frac{9c^2E'(z)}{4\pi H_0^3E^2(z)} - \frac{Q}{3H_0^3E^3(z)} - \frac{3\Omega_k(0)(1+z)^2}{E^2(z)} - \frac{3c^2E'(z)(1+z)}{\pi H_0^2E(z)} - \frac{3c^2E''(z)(1+z)}{4\pi H_0^3E^2(z)}. \tag{14}$$

Putting above result in equation (4) and after a straightforward calculation, we get interaction term as a function of z as follows

$$\frac{Q(z)}{9H_0^3E^3(z)} = 1 + \frac{3c^2}{2\pi H_0^2}\left[1 + \frac{E'(z)}{2H_0E^2(z)} - \frac{E''(z)(1+z)}{6H_0E^2(z)} - \frac{2\pi H_0^2\Omega_k(0)(1+z)^2}{3c^2E^2(z)} - \frac{2E'(z)(1+z)}{3E(z)}\right] - \left(1 + \frac{\omega_{\rm DE}(z)}{1+r(z)}\right)\left(1 - \Omega_k(0)\left(\frac{1+z}{E(z)}\right)^2\right). \tag{15}$$

At $z = 0$, the term $Q(z)$ is reduced to

$$Q_0 = 9(1-\Omega_k(0)) + \frac{9c^2}{\pi} - 9(1-\Omega_k(0))\left(1 + \frac{\omega_{\rm DE,0}}{1+r_0}\right). \tag{16}$$

If $Q_0 > 0$, then energy flows from DE to DM and vice versa for $Q_0 < 0$. As perceived, the Q-term written in equation (15), which is constructed through physical quantities, i.e. we did not consider a specific parametrization for the Q-term as is often done. In [4] by using the best fit value of the cosmological parameters, included the previous construction may have the benefit of constraining the interaction term. It is significant to note that the value of Q-term determines the rate at which the universe expanding as $r(z)$ decreases.

Now, from equation (1), we can write the following expression

$$\rho_{\rm DE} + \rho_{\rm DM} = 3H^2(z) - \Omega_k(z). \tag{17}$$

Equation (10) can be modified as follows

$$\rho_{\rm DE}(z) = -\frac{9c^2(2H^2(z) + H'(z))}{4\pi H_0^2}. \tag{18}$$

Differentiating $r(z)$ with respect to z and using equations (17) and (18), we get

$$\frac{r'}{r} = -\frac{3\omega_{\rm DE}}{1+z} - \frac{Q}{H_0E(z)(1+z)}\left(\frac{4\pi H_0^2(3H^2(z) - \Omega_k(z))}{-9c^2\rho_{\rm DM}(2H^2(z) + H'(z))}\right), \tag{19}$$

using chain rule, $r' = \frac{\dot{r}}{\dot{z}}$, in the above equation, we have

$$\frac{\dot{r}}{r} = 3H(z)\omega_{\rm DE}(z) - Q(z)\left(\frac{4\pi H_0^2(3H^2(z) - \Omega_k(z))}{9c^2\rho_{\rm DM}(z)(2H^2(z) + H'(z))}\right). \tag{20}$$

The rate of change for the coincidence parameter can be varied as $\Omega_k(z)$ decreases or increases. In the above equation, assuming $Q = 0$ along with $\omega_{\rm DE} = -1$, the Λ-CDM model can be recovered where $\dot{r} = -3Hr$ [22]. Furthermore, at present time taking $\dot{r} = 0$ in the above equation, we can solve to obtain a specific value r_0 given as under

$$r_0 = \left[6H_0^2\left(c^2\rho_{\mathrm{DM},0}\omega_{\mathrm{DE},0} - 2H_0c^2 + 2\pi H_0\omega_{\mathrm{DE},0}(1-\Omega_{k,0})\right) + c^2(6H_0'\rho_{\mathrm{DM},0}\omega_{\mathrm{DE},0} + 4H_0\Omega_{k,0}) + 4\pi H_0\omega_{\mathrm{DE},0}\Omega_{k,0}(\Omega_{k,0}-1)\right] \times \left[c^2\left(4H_0(3H_0^2-\Omega_{k,0}) - 3\rho_{\mathrm{DM},0}\omega_{\mathrm{DE},0}(2H_0^2+H_0')\right)\right]^{-1}. \tag{21}$$

Using equation (21), we can constraint r_0 at specific values of other model parameters at present time.

From equation (16) and the positivity condition, $Q_0 > 0$, we get

$$\omega_{\mathrm{DE},0} < (1+r_o)\left(\frac{c^2}{\pi(1-\Omega_{k,0})}\right), \tag{22}$$

where $\omega_{\mathrm{DE},0} < -0.666\,485$ or $\omega_{\mathrm{DE},0} < -0.666\,007$ and $\Omega_k(0) = 0.000^{+0.005(k=-1)}_{-0.005(k=1)}$ [23, 26]. It is important to point out that the parameter $\omega_{\mathrm{DE},0}$ can take positive values, if $k=1$, which could define a decelerated expansion. Using already mentioned expression given in equation (6), we can find the value of $q(z)$ from equation (14) as

$$q(z) = \frac{1}{2}\left[1 + \frac{2(1+z)^2\Omega_k(0)}{E^2(z)} + \frac{9c^2}{2\pi H_0^2} + \frac{9c^2E'(z)}{4\pi H_0^3E^2(z)} - \frac{3c^2E''(z)(1+z)}{4\pi H_0^3E^2(z)} - \frac{3\Omega_k(0)(1+z)^2}{E^2(z)} - \frac{3c^2E'(z)(1+z)}{\pi H_0^2E(z)} - \frac{Q}{3H_0^3E^3(z)}\right]. \tag{23}$$

While at present time, it is calculated that $q_0 < \frac{1}{2}$. By using equation (23), we get interaction term in the following form

$$Q(z) < \frac{27c^2H_0E^3(z)}{2\pi} + \frac{27c^2E(z)E'(z)}{4\pi} - \frac{9c^2E(z)E''(z)(1+z)}{4\pi} - \frac{9c^2H_0E^2(z)E'(z)(1+z)}{\pi} - 3H_0^3\Omega_k(0)E(z)(1+z)^2. \tag{24}$$

It is significant to point out that the above changes in the sign of Q-interaction term strongly depends on the sign of $\Omega_k(z)$. Moreover, a modification in the sign of Q interaction term used to determine the phase transitions (sign changes in heat capacities) along the cosmic evolution as well as provide data to confirm the validity of SLT [28, 29].

4. Future singularity

Here, we will discuss about the presence of future singularity in this dynamical model. The most highlighted point is that the mentioned singularity is only assists in a curved universe. We assume a CPL-type parametrization for $r(z)$ that helps us to recognize the occurrence of singularity in the RS. We are working with a Type III future singularity but not with a genuine big rip, moreover, we can observe that cosmic evolution induced by the existence of previous singularity differs from that singularity, which is attained by Λ. We prove that the Q-term will remain positive through the cosmic evolution using the previous results. Using equation (12), we can form $E(z)$ in terms of $r(z)$ as given below:

$$E(z) = \frac{2\pi H_0^3 \Omega_k(0)}{3c^2 r_0^3} \left[r_0^2(1+z)^2 + 2r_0(r_0+r_c)(1+z) + 2(r_0+r_c)^2 \ln(r(z)-r_c) \right], \tag{25}$$

here a constant quantity, $r_c = \frac{1-c^2}{c^2}$. The expression $E(z)$ became singular at given point $r(z) = r_c$. Evaluating equation (25) at $z = 0$, we can find an expression for r_c, which is given in terms of r_0 and $\Omega_k(0)$ as

$$r_c = \frac{1-c^2}{c^2} = \frac{9(1+r_0)}{4\pi(\Omega_k(0)-1)}. \tag{26}$$

Now for $r(z)$, we will assume a CPL-type parametrization given as follows [30]

$$r(z) = r_0 + \epsilon_0 \frac{z}{1+z}, \tag{27}$$

where $\epsilon_0 = r_0'$ can be obtained from equation (27) after taking its derivative with respect to z. One can see that at $z = -1$, the preceding parametrization will become singular for high values of z for which $r(z)$ has a bounded nature and declares a linear behavior for low values of z, also shows the sensitive behavior to observational data [31]. Comparing the equations (26) and (27), we can find the following value of z_s at which $E(z)$ will become singular

$$z_s = -\left[\frac{4\pi r_0^2(1-\Omega_k(0)) + 9(1+r_0)(2r_0-r_c)}{4\pi r_0(r_0+\epsilon_0)(1-\Omega_k(0)) + 9\epsilon_0(1+r_0)(1+(2r_0-r_c)/\epsilon_0)} \right]. \tag{28}$$

In the future, we must have $-1 < z_s < 0$ for a singular behavior. From the previous condition and by using equations (26) and (27), we get

$$r(z) - r_c = \epsilon_0 \left[\frac{z - z_s}{(1+z_s)(1+z)} \right] \geqslant 0 \Longrightarrow z \geqslant z_s. \tag{29}$$

It is noted that $r \to r_c$ as the limit $z \to z_s$ in the above expression. Evaluating previous equation at present time, we get an inequality which must be satisfied

$$-\epsilon_0 \frac{z_s}{(1+z_s)} \geqslant 0, \tag{30}$$

and is consistent with the interval $-1 < z_s < 0$. The equation (25) can be re-written using previous results

$$E(z) = \frac{4\pi H_0^3 \Omega_k(0)(r_0+r_c)}{3c^2 r_0^2}(1+z) + \frac{4\pi H_0^3 \Omega_k(0)(r_0+r_c)^2}{3c^2 r_0^3} \ln\left(\frac{z-z_s}{\eta c^2(1+z)} \right) + \frac{2\pi H_0^3 \Omega_k(0)}{3c^2 r_0}(1+z)^2, \tag{31}$$

where $\eta = \frac{(1+z_s)}{c^2 \epsilon_0} > 0$ if $\epsilon_0 > 0$. The behavior of equation (31) can be examined from figure 1. Left plot of figure 1 shows behavior of $E^2(z)$ as z progresses for specific values of the model parameters taken from literature [23, 26]. It is clear from the trajectory plotted in the left graph, $E^2(z) - z$ that during closed universe ($\Omega_k(0) = -0.005$ for $k = 1$), $E^2(z)$ has decreasing behavior as z transits from past to future era. Right plot of figure 1 is plotted for $\Omega_k(0) = 0.005$, which generates negative values of $E^2(z)$. The solid line in figure 1 shows the point of singularity z_s. Note that a change in the sign of the $\Omega_k(0)$ can cause two types of cosmic evolution but in both cases the value z_s is the same.

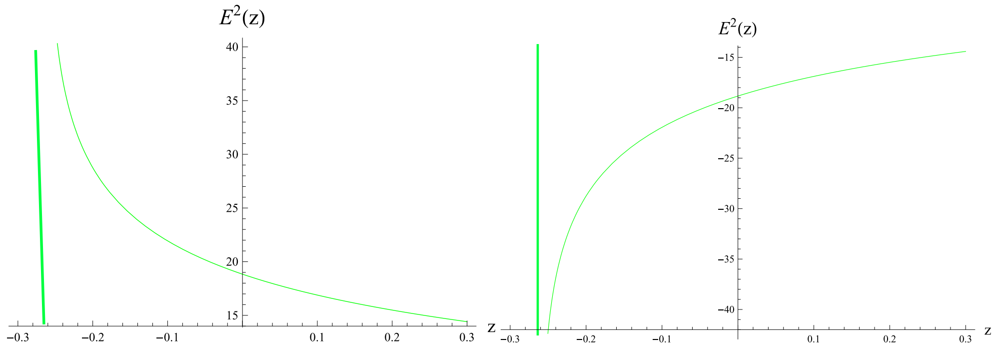

Figure 1. Left plot of $E^2(z)$ versus z for $\Omega_k(0) = -0.005$, $c = 0.681\,476$, $\epsilon_0 = 0.2$, $z_s = -0.26$, $H_0 = 1$, $r_0 = 0.11$, $r_c = 0.4674$; right plot of $E^2(z)$ versus z for $\Omega_k(0) = 0.005$.

Inserting $E(z) = \frac{H}{H_0}$, $H = \frac{\dot{a}}{a}$ along with $1 + z = \frac{a_o}{a}$ in equation (31) and perform some simplifications, we get an analytical solution for $a(t)$ as follows

$$a(t) = \exp\left[\left(\frac{4\pi H_0^4 \Omega_k(0)(r_0 + r_c)^2}{3c^2 r_0^3 - 4\pi H_0^4 \Omega_k(0)(r_0 + r_c)(r_0 a_0)}\right) \ln\left(\frac{a_0}{1 + z_s}\right) t\right]. \tag{32}$$

It is clear from equation (32), as $\Omega_k(0) \to 0$ then $a(t)$ converges to a constant value, which shows a static universe. This type of universe model is in conflict with the SLT, which is one of the famous universal laws of nature [32], so a singular universe along with a negative value of $\Omega_k(z)$ is more preferred explanation of cosmic expansion. Furthermore, since both of the energy densities $\rho_{\rm DM}$ and $\rho_{\rm DE}$ are directly associated to the quantity $E(z)$ and its first derivative over the Friedmann constraint as $z \to z_s$, the $\rho_{\rm DM}$, $\rho_{\rm DE} \to \infty$, therefore the quantity $P_{\rm DE}$ also diverges. From this behavior, we have concluded that our model admits a future singularity of Type III similar to HDE [33, 34].

Now, we are able to evaluate equation (4) in terms of z only using obtained solution of $E(z)$ given in equation (31) as

$$\begin{aligned}
1 + \frac{\omega_{\rm DE}(z)}{1 + r(z)} = &\Big[12(r_0 + r_c)r_0^3\theta(z)(1+z)^2 + 4(r_0 + r_c)^2 r_0^2\theta(z)(1+z)\{(1 + \theta(z)) \\
&+ 2\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\} + 8(r_0 + r_c)^3 r_0\{(1+z)(\theta(z) - 1)^2 \\
&+ \theta(z)\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\} + 4(r_0 + r_c)^4(\theta(z) - 1)^2 + 4r_0^4\theta(z)(1+z)^3 \\
&+ \frac{9c^4 r_0^6\theta(z)(1+z)^2}{2\pi^2 H_0^6\Omega_k(0)}\Big]\Big[24(r_0 + r_c)^3 r_0\theta(z)\ln(\eta^{-1}c^{-2}\theta^{-1}(z)) \\
&+ 12(r_0 + r_c)r_0^3\theta(z)(1+z)^2 + 12(r_0 + r_c)^2 r_0^2\theta(z)(1+z)\{1 \\
&+ \ln(\eta^{-1}c^{-2}\theta^{-1}(z))\} + 12(r_0 + r_c)^4\frac{\theta(z)}{1+z}\ln(\eta^{-1}c^{-2}\theta^{-1}(z)) \\
&- \frac{27c^4 r_0^6\theta(z)(1+z)}{4\pi^2 H_0^6\Omega_k(0)} + 3r_0^4\theta(z)(1+z)^3\Big]^{-1},
\end{aligned} \tag{33}$$

where $\theta(z) = \frac{(1+z)}{(z - z_s)}$. Consequently, by using aforementioned equation, we get a divergent behavior for $\omega_{\rm DE}$, which can be written as $\omega_{\rm DE}(z \to z_s) \to -\infty$, negative sign is due to range of z_s. For early universe, the RS parameter $z \to \infty$. By using this assumption in CPL parametrization, we obtained $r(z \to \infty) \to r_0 + \epsilon_0$ and the right hand side of equation (33) becomes $\frac{4}{3}$ while the bounded value of $\omega_{\mathrm{DE}}(z \to \infty) \to \frac{1+r_0+\epsilon_0}{3}$. We observed that in early universe, ω_{DE} takes greater values than those attained at present time.

To evaluate the interaction term Q, put equations (33) in (15) and after solving it, we get

$$
\begin{aligned}
\frac{Q(z)}{3H_0^3} = 3\Bigg[&\frac{4\pi^2 H_0^6\Omega_k^2(0)}{9c^4 r_0^2}\Bigg(\frac{4(r_0+r_c)^2(1+z)^2}{r_0^2} + \frac{4(r_0+r_c)^4}{r_0^4}(\ln(\eta^{-1}c^{-2}\theta^{-1}(z)))^2\\
&+(1+z)^4 + \frac{4(r_0+r_c)}{r_0}(1+z)^3 + \frac{8(r_0+r_c)^3(1+z)}{r_0^3}\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\\
&+\frac{4(r_0+r_c)^2(1+z)^2}{r_0^2}\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\Bigg)\Bigg]^{3/2}\Bigg[\Bigg(12(r_0+r_c)r_0^3\theta(z)(1+z)^2\\
&\times(2\pi H_0^2+3c^2-1)+4(r_0+r_c)^2 r_0^2\theta(z)(1+z)\{6\pi H_0^2\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\\
&+6\pi H_0^2+9c^2+9c^2\ln(\eta^{-1}c^{-2}\theta^{-1}(z))-1-\theta(z)+2\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\}\\
&+8(r_0+r_c)^3 r_0\theta(z)\{6\pi H_0^2\ln(\eta^{-1}c^{-2}\theta^{-1}(z))+9c^2\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\\
&-(1+z)(\theta(z)-1)^2-\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\}-4(r_0+r_c)^4(\theta(z)-1)^2\\
&+12(r_0+r_c)^4\frac{\theta(z)}{1+z}\ln(\eta^{-1}c^{-2}\theta^{-1}(z))(2\pi H_0^2+3c^2)+r_0^4\theta(z)(1+z)^3(6\pi H_0^2\\
&+9c^2-4)-\frac{9c^4 r_0^6\theta(z)(1+z)}{2\pi H_0^4\Omega_k(0)}\left(3+\frac{9c^2}{2\pi H_0^2}+\frac{1+z}{\pi H_0^2}\right)\Bigg)(2\pi H_0^2\triangle)^{-1}\Bigg]\\
&+\frac{2\pi H_0^3\Omega_k^2(0)(1+z)^2}{c^2 r_0}\Bigg[\frac{2(r_0+r_c)(1+z)}{r_0}+\frac{2(r_0+r_c)^2}{r_0}\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\\
&+(1+z)^2\Bigg]\Bigg[\Bigg(4(r_0+r_c)^2 r_0^2\theta(z)(1+z)\{-2+\theta(z)-\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\}\\
&+8(r_0+r_c)^3 r_0\{(1+z)(\theta(z)-1)^2-2\theta(z)\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\}\\
&+4(r_0+r_c)^4\{(\theta(z)-1)^2-\frac{3\theta(z)}{1+z}\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\}\\
&+r_0^4\theta(z)(1+z)^3+\frac{9c^4 r_0^6\theta(z)(1+z)}{2\pi^2 H_0^6\Omega_k(0)}(z+\frac{5}{2})\Bigg)\triangle^{-1}\Bigg]\\
&+\frac{8\pi H_0^4\Omega_k^2(0)}{3c^2 r_0^2}\Bigg[\Bigg(\Bigg\{\frac{2(r_0+r_c)(1+z)}{r_0}+\frac{2(r_0+r_c)^2}{r_0^2}\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\\
&+(1+z)^2\Bigg\}\Bigg\{(1+z)+\frac{(r_0+r_c)}{r_0}+\frac{(r_0+r_c)^2(\theta(z)-1)}{r_0^2(1+z)}\Bigg\}\Bigg)\Bigg(\frac{3}{4H_0}\\
&-\frac{2\pi H_0^3\Omega_k(0)(1+z)}{3c^2 r_0}\Bigg\{\frac{2(r_0+r_c)(1+z)}{r_0}+\frac{2(r_0+r_c)^2}{r_0^2}\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\\
&+(1+z)^2\Bigg\}\Bigg)\Bigg],
\end{aligned}
\tag{34}
$$

where

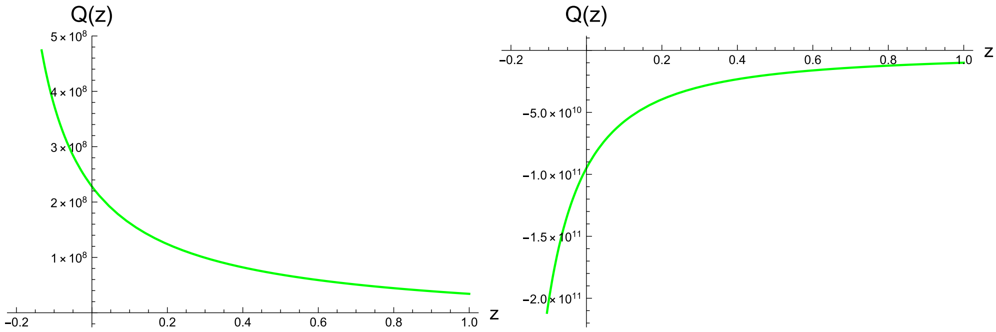

Figure 2. Left plot $Q(z)$ versus z for $\Omega_k(0) = -0.005$, $c = 0.681\,476$, $\epsilon_0 = 0.2$, $z_s = -0.26$, $H_0 = 1$, $r_0 = 0.11$, $r_c = 0.4674$; right plot for $\Omega_k(0) = 0.005$.

$$\begin{aligned}\triangle &= 12(r_0+r_c)r_0^3\theta(z)(1+z)^2 + 12(r_0+r_c)^2r_0^2\theta(z)(1+z)\{1+\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\}\\ &+24(r_0+r_c)^3r_0\theta(z)\ln(\eta^{-1}c^{-2}\theta^{-1}(z)) + 12(r_0+r_c)^4\frac{\theta(z)}{1+z}\ln(\eta^{-1}c^{-2}\theta^{-1}(z))\\ &-\frac{27c^4r_0^6\theta(z)(1+z)}{4\pi^2H_0^6\Omega_k(0)} + 3r_0^4\theta(z)(1+z)^3.\end{aligned}$$

It is obvious that Q can be positive and negative defining two different regions of cosmos. The calculated interaction term Q is plotted versus z in left and right plot of figure 2. It is important to note that the value of $\Omega_k(z)$ shows a crucial role in order to have real Q-function. The left plot of figure 2 verifies the natural phenomena that interaction term remains positive throughout the domain of z for $\Omega_k(0) = -0.005$ during closed universe. As $Q > 0$ represents the conversion of DE dominated era to DM dominated era. Although the observations suggests that Q must be positive, but right plot of figure 2 for $\Omega_k(0) = 0.005$ is not in good agreement with recent observations [25]. So, in a closed universe, DM is converted in to DE. We have to assume the possible implications at thermodynamic level for late-time universe, this is to study the fulfillment of SLT and the occurring possibility of phase transitions.

Substituting equations (25) in (6), we obtain $q(z)$ as given below

$$q(z) = \frac{1}{2}\left[\frac{r_0^2(1+z)^2 + 2(r_0+r_c)^2\left(\frac{1+z}{r(z)-r_c}r'(z) - \ln(r(z)-r_c)\right)}{\frac{r_0^2}{2}(1+z)^2 + r_0(r_0+r_c)(1+z) + 2(r_0+r_c)^2\ln(r(z)-r_c)}\right]. \quad (35)$$

To check the singular behavior of $q(z)$, we have to convert the above expression in terms of z and z_s by putting equations (31) into (6) as under

$$q(z) = \frac{(1+z)^2 + \frac{2(r_0+r_c)^2}{r_0^2}\left[\left(\frac{1+z_s}{z-z_s}\right) - \ln\left(\frac{\epsilon_0(z-z_s)}{(1+z_s)(1+z)}\right)\right]}{(1+z)^2 + \frac{2(r_0+r_c)}{r_0}(1+z) + \frac{2(r_0+r_c)^2}{r_0^2}\ln\left(\frac{\epsilon_0(z-z_s)}{(1+z_s)(1+z)}\right)}. \quad (36)$$

Note that $q(z\to z_s)\to-\infty$ and $q(z\to\infty)\to 0$, i.e. the universe expanding more rapidly. If we evaluate $\theta(z)$ at $z=0$ then we get $\theta_0 = -\frac{1}{z_s}$, which is always positive. At present time, the expression of $q(z)$ can be modified as follows

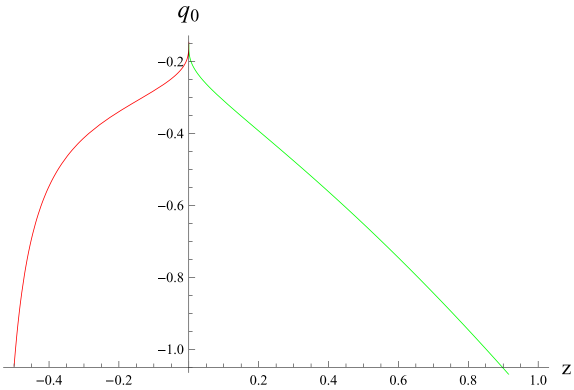

Figure 3. Plot of q_0 verses $z = z_s$ for $\Omega_k(0) = -0.005$; $\epsilon = 0.2$ (red) and $\Omega_k(0) = 0.005$; $\epsilon = -0.7$ (green).

$$q_0 = -\left[\frac{-1 + 2\left(\frac{(r_0+r_c)^2}{r_0}\right)\left(-\theta_0(1+z_s) + \ln\left(\frac{-\epsilon_0 z_s}{1+z_s}\right)\right)}{1 + \frac{2(r_0+r_c)}{r_0} + \frac{2(r_0+r_c)^2}{r_0^2}\ln\left(\frac{-\epsilon_0 z_s}{1+z_s}\right)}\right] < 0. \tag{37}$$

The above inequality is proved in figure 3 for specified values of the model parameters. We have checked that this expression is independent of $\Omega_k(0)$. The term q_0 remains negative in the range $-1 < z < 1$ for any positive or negative value of $\Omega_k(0)$.

Equations (33) and (34) at present time has the following expression

$$\begin{aligned}\omega_{\mathrm{DE},0} = \Bigg[-1 + \Bigg(& 12(r_0+r_c)r_0^3\theta_0 + 4(r_0+r_c)^2 r_0^2\theta_0\{(1+\theta_0) + 2\ln(\eta^{-1}c^{-2}\theta_0^{-1})\} \\ & + 8(r_0+r_c)^3 r_0\{(\theta_0-1)^2 + \theta_0\ln(\eta^{-1}c^{-2}\theta_0^{-1})\} + 4(r_0+r_c)^4(\theta_0-1)^2 + 4r_0^4\theta_0 \\ & - \frac{9c^4 r_0^6\theta_0}{2\pi^2 H_0^6\Omega_k(0)}\Bigg)\Bigg(12(r_0+r_c)r_0^3\theta_0 + 12(r_0+r_c)^2 r_0^2\theta_0\{1 + \ln(\eta^{-1}c^{-2}\theta_0^{-1})\} \\ & + 24(r_0+r_c)^3 r_0\theta_0\ln(\eta^{-1}c^{-2}\theta_0^{-1}) + 12(r_0+r_c)^4\theta_0\ln(\eta^{-1}c^{-2}\theta_0^{-1}) \\ & - \frac{27c^4 r_0^6\theta_0}{4\pi^2 H_0^6\Omega_k(0)} + 3r_0^4\theta_0\Bigg)^{-1}\Bigg](1+r_0),\end{aligned} \tag{38}$$

$$
\begin{aligned}
\frac{Q_0}{H_0^3} &= 9\Bigg[\frac{4\pi^2 H_0^6\Omega_k^2(0)}{9c^4 r_0^2}\Bigg(1+\frac{4(r_0+r_c)^2}{r_0^2}+\frac{4(r_0+r_c)^4}{r_0^4}(\ln(\eta^{-1}c^{-2}\theta_0^{-1}))^2 \\
&+\frac{4(r_0+r_c)}{r_0}+\frac{8(r_0+r_c)^3}{r_0^3}\ln(\eta^{-1}c^{-2}\theta_0^{-1})+\frac{4(r_0+r_c)^2}{r_0^2}\ln(\eta^{-1}c^{-2}\theta_0^{-1})\Bigg)\Bigg]^{3/2} \\
&\times\Bigg[\Bigg(12(r_0+r_c)r_0^3\theta_0(2\pi H_0^2+3c^2-1)+4(r_0+r_c)^2 r_0^2\theta_0\{6\pi H_0^2+9c^2 \\
&+\ln(\eta^{-1}c^{-2}\theta_0^{-1})(9c^2+6\pi H_0^2+2)-1-\theta_0\}-4(r_0+r_c)^4(\theta_0-1)^2 \\
&+8(r_0+r_c)^3 r_0\theta_0\{\ln(\eta^{-1}c^{-2}\theta_0^{-1})(6\pi H_0^2+9c^2-1)-(\theta_0-1)^2\} \\
&+12(r_0+r_c)^4\theta_0\ln(\eta^{-1}c^{-2}\theta_0^{-1})(2\pi H_0^2+3c^2)+r_0^4\theta_0(6\pi H_0^2+9c^2-4) \\
&-\frac{9c^4 r_0^6\theta_0}{2\pi H_0^4\Omega_k(0)}\left(3+\frac{9c^2}{2\pi H_0^2}+\frac{1}{\pi H_0^2}\right)\Bigg)\Big(\{2\pi H_0^2\}\{\nabla\}\Big)^{-1}\Bigg] \\
&+\frac{6\pi H_0^3\Omega_k^2(0)}{c^2 r_0}\left[\frac{2(r_0+r_c)}{r_0}+\frac{2(r_0+r_c)^2}{r_0}\ln(\eta^{-1}c^{-2}\theta_0^{-1})+1\right]\Bigg[\Bigg(r_0^4\theta_0 \\
&+4(r_0+r_c)^2 r_0^2\theta_0\{-2+\theta_0-\ln(\eta^{-1}c^{-2}\theta_0^{-1})\}+8(r_0+r_c)^3 r_0\{(\theta_0-1)^2 \\
&-2\theta_0\ln(\eta^{-1}c^{-2}\theta_0^{-1})\}+4(r_0+r_c)^4\{(\theta_0-1)^2-3\theta_0\ln(\eta^{-1}c^{-2}\theta_0^{-1})\} \\
&+\frac{45c^4 r_0^6\theta_0}{4\pi^2 H_0^6\Omega_k(0)}\Bigg)\Big(\nabla\Big)^{-1}\Bigg]+\frac{8\pi H_0^4\Omega_k^2(0)}{c^2 r_0^2}\Bigg[\Bigg(\Bigg\{\frac{2(r_0+r_c)^2}{r_0^2}\ln(\eta^{-1}c^{-2}\theta_0^{-1}) \\
&+\frac{2(r_0+r_c)}{r_0}+1\Bigg\}\Bigg\{1+\frac{(r_0+r_c)}{r_0}+\frac{(r_0+r_c)^2(\theta_0-1)}{r_0^2}\Bigg\}\Bigg)\Bigg(\frac{3}{4H_0} \\
&-\frac{2\pi H_0^3\Omega_k(0)}{3c^2 r_0}\Bigg\{\frac{2(r_0+r_c)}{r_0}+\frac{2(r_0+r_c)^2}{r_0^2}\ln(\eta^{-1}c^{-2}\theta_0^{-1})+1\Bigg\}\Bigg)\Bigg],
\end{aligned} \tag{39}
$$

where

$$
\begin{aligned}
\nabla &= 12(r_0+r_c)r_0^3\theta_0+12(r_0+r_c)^2 r_0^2\theta_0\{1+\ln(\eta^{-1}c^{-2}\theta_0^{-1})\}+24(r_0+r_c)^3 r_0\theta_0 \\
&\times\ln(\eta^{-1}c^{-2}\theta_0^{-1})+12(r_0+r_c)^4\theta_0\ln(\eta^{-1}c^{-2}\theta_0^{-1})-\frac{27c^4 r_0^6\theta_0}{4\pi^2 H_0^6\Omega_k(0)}+3r_0^4\theta_0.
\end{aligned}
$$

From equations (37)–(39), we can observe that the both conditions: cosmic evolution determined by the quintessence fluid can be always sure by fulfilling condition $\eta\theta_0^2 > 0$ and the positive interaction term Q at present time.

Finally, using equation (27) for $r(z)$ and its derivative with respect to z at present time along with differentiating both sides of equation (25) with respect to z, we have

$$
r'(z)=\frac{r(z)-r_c}{(r_0+r_c)^2}\left[\frac{3c^2 r_0^3}{4\pi H_0^3\Omega_k(0)}E'(z)-r_0^2(1+z)-r_0(r_0+r_c)\right]. \tag{40}
$$

Now equation (40) can be modified using the condition $r_0'=\epsilon_0$ as

$$
\epsilon_0=\frac{1}{(r_0+r_c)^2}\left[\frac{3c^2 r_0^3(r_0-r_c)}{4\pi\Omega_k(0)}E_0'-2r_0^3+r_0 r_c(r_0+r_c)\right]. \tag{41}
$$

Using equations (10) and (11) at $z=0$, then we have a an important result for $E'(z)$ as given below

$$
E_0'=\frac{3}{2}\left(1+\frac{\omega_{\mathrm{DE},0}}{1+r_0}\right)(1-\Omega_k(0))+\Omega_k(0). \tag{42}
$$

Using this value in the above equation, we determined the value of ϵ_0 as follows

$$\epsilon_0 = \frac{1}{(r_0+r_c)^2}\left[\frac{3c^2r_0^3(r_0-r_c)}{4\pi} + \frac{9c^2r_0^3(r_0-r_c)}{8\pi\Omega_k(0)}\left(1+\frac{\omega_{\text{DE},0}}{1+r_0}\right)(1-\Omega_k(0)) - 2r_0^3 + r_0r_c(r_0+r_c)\right]. \tag{43}$$

Note that, we can obtained in a similar way the whole analysis developed for the considered model, to find a suitable value for parameter ϵ_0 in which $\Omega_k(z)$ has a crucial role. It is important to point out that for quintessence model $\epsilon_0 > 0$ for flat and closed universe. While in phantom region, $\Omega_k(0) > 0$ for which $\epsilon_0 < 0$.

A relationship between ϵ_0 (calculated in the above equation) and $\Omega_k(0)$ can be seen from both three dimensional graphs of figure 4. The model parameters are constrained in the caption of the figure whereas the value of $\omega_{\text{DE},0}$ has been taken from [25]. The left graph is plotted for quintessence region taking value of $\omega_{\text{DE},0} > -1$ while right plot admits the phantom region ($\omega_{\text{DE},0} < -1$). We can see from the left plot that ϵ_0 starts from zero and goes on decreasing with an increment in $\Omega_k(0)$ in the interval $-0.005 < \Omega_k(0) < 0$ (representing the closed universe). In the open universe where $0 < \Omega_k(0) < 0.005$, the graph has monotonic behavior as $\epsilon_0 > 0$ increases firstly with the increase of $\Omega_k(0)$ for $0 < r_0 < 0.2$ after that ϵ_0 has decreasing behavior for some region then again increasing and process continues. In phantom region (right plot of figure 4), the curves show the same behavior as of quintessence. In region $-0.005 < \Omega_k(0) < 0$, an inversely proportional relation holds between ϵ_0 and $\Omega_k(0)$ while for $0 < \Omega_k(0) < 0.005$, the curve shows monotonic behavior firstly increases to a specific value then decreases while space of validity r_0 increases as compared to previous description for quintessence in the left plot.

4.1. Statefinder diagnosis

In this section, we will discuss about the Satefinder diagnosis for our model. The Statefinder diagnostic pair is an effective geometrical tool that is used to compare the properties of a DE model with other DE models and then check how far this model from the Λ-CDM. As the universe has an accelerating expansion, so the rates of $a(t)$ are important to investigate the cosmic behavior. In more general sense, these can be used to examine the fluid that filled the universe with the effects of other parameters involved in its expression. These parameters r and s are defined as under, respectively

$$r = \frac{\dddot{a}}{aH^3} = \frac{\ddot{H}}{H^3} + 3\frac{\dot{H}}{H^2} + 1, \quad s = \frac{r-1}{3(q-\frac{1}{2})}. \tag{44}$$

For ΛCDM model, the above parameters corresponds to a fixed point $(r,s) = (1,0)$ in the $s-r$ plane.

Taking the derivatives of $a(t)$ (calculated in equation (32)) up to third order, we are able to evaluate the expressions of r and s. These expressions directly reduced to ΛCDM limit. The declaration parameter q is equal to -1, which represents an accelerating expansion of the universe.

The values found from ΛCDM model ($q = -1$) are always greater than current values. It is observed that we will get a static universe in our RDE model by assuming a null curvature parameter.

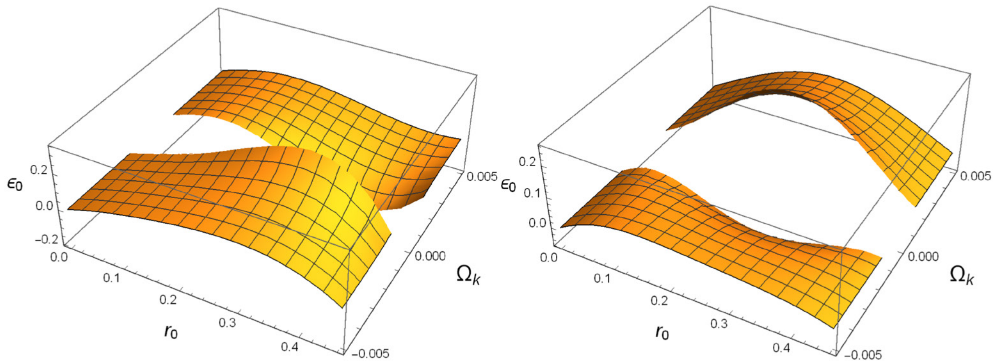

Figure 4. Left plot ϵ_0 in terms of r_0 and $\Omega_k(0)$ for $c = 0.681\,476$, $\omega_{\mathrm{DE},0} = -0.95$, $r_c = 0.4674$; Right plot for $c = 0.681\,476$; $\omega_{\mathrm{DE},0} = -1.5$, $r_c = 0.4674$.

5. Some possible modifications

In literature, a large number of DE models has been proposed. Some of them are the generalized and the modified versions of the HDE and the RDE models. Different authors worked on different interacting and non-interacting DE models in various scenarios. Wei [35] reconsidered the idea of HDE and proposed a new model of DE, known as PDE ($\rho_{\mathrm{PDE}} = 3n^2 m_p^{4-s} L^{-s}$; $s \leqslant 2$ for $s = 2$, it reduced to HDE model). One of its key points is the formation of the BH and based on the speculation that the repulsive force contributed by the phantom-like DE ($\omega < -1$) is strong enough to prevent the formation of the BH. They also considered the cosmological constraints on PDE by using the latest observational data of Type Ia supernovae (SNIa) alone and then the Supernova Cosmology Project (SCP) collaboration released the updated Union2.1 compilation, which consists of 580 SNIa. Recently, using relation of non-additive entropy for the non-extensive systems introduced by Tsallis and holographic hypothesis, which led to the suggestion of a DE density in the form $\rho = BL^{2\delta-4}$, where B is an unknown parameter [36]. Similarly, a lot of work have been done on DE-DM interaction in modified theories of gravity such as Kaluza–Klein, Brane-world Models, Horava–Lifshitz cosmology, $f(R)$, $f(T)$, $f(R,T)$, $f(G,T)$, gravity etc.

We can extend our work to these DE models interacting with DM under the framework of non-flat FRW model. In this way, we may be able to find out corresponding physical quantities through which the future behavior can be predicted that either these models would admit a future singularity or not. Further, we can check that how far these model are from ΛCDM model? In case of modified theories of gravity, we will get generalized results that can be compared with general relativity results. We will come back with this scenario in near future.

6. Conclusion

This paper is devoted to explore the nature of the universe via interacting RDE-DM model within curved FLRW background. The future singularity is being discussed using a special type of parametrization, i.e. CPL parametrization of the parameter $r(z)$. The evaluated expressions of energy densities of both components and pressure of RDE lead us to prove that our model admits a future singularity of type III at a specific point $z = z_s$. From the dynamics of the model and observations of DE survey year I results [25] and Planck Power Spectra, Planck

Lensing, and BAO results [26] can be established that the DE EoS parameter can take values within the quintessence-phantom region at present time, i.e. the cosmic evolution has an accelerated expansion. The value for the ω-parameter can change if we consider different values for the curvature parameter ($\Omega_k(0)$).

We obtained an analytical solution of $E^2(z)$, which is strongly dependent upon $\Omega_k(z)$. It is graphically represented in figure 1 for closed ($k = 1$) and open universe ($k = -1$). The main results of this figure are listed as:

- It is clear from the left $E^2(z) - z$ trajectory that in a closed universe ($\Omega_k(0) = -0.005$ for $k = 1$), $E^2(z)$ has decreasing behavior for constraint model parameters $\{r_0, r_c\}$ as z transits from past to future era.
- Right plot of figure 1 is plotted for $\Omega_k(0) = 0.005$ (for $k = -1$) for which the function attains negative values.
- It is found that the range $-1 < z_s < 0$ is not valid for all values of the pair of parameters $\{r_0, r_c\}$. Considering appropriate values for the aforementioned pair and keeping them fixed, we find that ϵ_0 plays an important role in the manifestation of the singular behavior. When this parameter decreases the singularity can take place close to the far future ($z = -1$) otherwise, the singularity is closer to the present time ($z = 0$) as ϵ_0 increases.
- It is noted that a change in the sign of the $\Omega_k(0)$ can cause two types of cosmic evolution but in both cases, the value z_s (the singularity point) remains the same.

We have calculated the expression of $Q(z)$ analytically and explored its nature graphically. In our case, the sign of function $Q(z)$ is uniquely dependent upon the value of $\Omega_k(0)$, and it plays a crucial role to have real Q-function. The function Q is plotted versus z in figure 2 for $\Omega_k(0) = -0.005$ and $\Omega_k(0) = 0.005$ keeping the other parameters fixed as mentioned in figure 1.

- As observed from left plot of figure 2, we have a monotonically increasing Q function from the recent past to early times with a singular behavior at some future value of the RS (z_s). The positivity of Q implies that DE is transforming into DM all the time, compatible with the Planck, BAO, SNIa and H_0 data [23, 24] and positive values work in the direction of solving the cosmological coincidence problem. It is checked that the behavior remains the same if we consider other appropriate values for $\{r_0, r_c\}$ and ϵ_0.
- According to the observations, $Q < 0$ would violate the SLT. In our case, right plot of figure 2 for $\Omega_k(0) = 0.005$ shows $Q(z) < 0$, which is not in good agreement with Planck, BAO, SNIa and H_0 data [23, 24].

It is important to mention that in a similar way to the obtained throughout the analysis developed for RDE model, the parameter $\Omega_k(0)$ plays an important role in determining an acceptable value for the parameter ϵ_0. A relationship between ϵ_0 and $\Omega_k(0)$ can be seen from figure 4. The left graph is plotted for quintessence region ($\omega_{\mathrm{DE},0} > -1$) while right plot admits the phantom region ($\omega_{\mathrm{DE},0} < -1$), where the value of $\omega_{\mathrm{DE},0} = -0.95$ has been taken from DE survey year I results mentioned in [25] and $\omega_{\mathrm{DE},0} = -1.56$ from Planck Power Spectra, Planck Lensing, and BAO [26].

- In quintessence region, for $\Omega_k(0) < 0$, the parameter $\epsilon_0 < 0$ coming from CPL parametrization.
- $\epsilon_0 > 0$ is attained for open and flat universe as shown in right plot. While for HDE model, $\epsilon_0 > 0$ for closed and flat universe obtained in [22].

The behavior of deceleration parameter q for closed and open universe is shown in figure 3.

- The RDE model always have negative values for the parameter q, which implies accelerated expansion in the whole region $-1 < z < 1$.
- The condition for ΛCDM model, i.e. $q = -1$ is also attained for both early and late universe as compared to HDE model [22].
- With the appropriate selection of the model parameters, we performed the Statefinder diagnosis, which showed that RDE model with this interacting scheme exactly generates the ΛCDM model. Using evaluated scale factor, one can easily checked that the expressions of r and s parameters are directly valued $\{1, 0\}$. For HDE model, the ΛCDM model can not be achieved as the authors claimed in [22] that their model represents an over acceleration.

In a general perspective, we observed that:

- The closed universe model provides more physical results as compared to an open one.
- With a Ricci cutoff, this work produce much complicated and lengthy results than a holographic cutoff considered in [22] but these results are more physical than HDE model.
- A comparison with the flat universe can not be performed since the quantities obtained are trivialized in the corresponding limit, therefore the future singularity can be obtained only in the non-flat universe within RDE model description.
- The accelerating expansion is the big picture of the universe evolution has been attained in this model and RDE is proved to be a promising candidate for its explanation. Based on previous results, we can conclude that to induce a phantom behavior in the model, the value of curvature parameter must be negative in spite of the previous value for curvature parameter describes a closed universe, this is due to the existence of the future singularity for which universe will not collapse as we have attained in the standard cosmology.

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We would like to dedicate this paper to Prof. Muhammad Sharif on his silver jubliee of supervising PhD Scholars. May he live healthy and blessed long life.

Conflict of interest

We have no conflict of interest.

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