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ERRATUM

Erratum: On quantumness in multi-parameter quantum estimation (2019 *J. Stat. Mech.* 094010)

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In equation (12), there is a small error in the definition of the Holevo Cramér Rao Bound, which should be written (for the notation see the original article)

$$C_H(W) := \min_{\{X_\mu\}}\{\mathrm{tr}(W\mathbf{Re}Z) + ||\sqrt{W}\mathbf{Im}Z\sqrt{W}||_1\}, \tag{1}$$

where the second term was written as $||W\mathbf{Im}Z||_1$, in the original article. All the remaining results are unchanged. The bounds in equation (15) are still correct, but need further clarifications. Indeed equation (15) is justified by the following chain of inequalities

$$\begin{aligned} \mathcal{D}(W) &\leqslant 2||\sqrt{W}\,J^{-1}\mathcal{U}J^{-1}\sqrt{W}||_1 \leqslant 2||J^{-1/2}WJ^{-1}\mathcal{U}J^{-1/2}||_1 \\ &\leqslant 2\,||J^{-1/2}WJ^{-1/2}||_1\;\;||J^{-1/2}\mathcal{U}J^{-1/2}||_\infty = \mathrm{tr}(WJ^{-1})R. \end{aligned} \tag{2}$$

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If one takes $A = \sqrt{W}\, J^{-1}\mathcal{U}J^{-1/2}$ and $B = J^{-1/2}\sqrt{W}$, the second inequality follows from proposition IX.I.1 of [1], which states that for any matrix A and B with AB normal,

$$||AB||_1 \leqslant ||BA||_1, \tag{3}$$

and indeed, $AB = \sqrt{W}\, J^{-1}\mathcal{U}J^{-1}\sqrt{W}$ is skew-symmetric. The third inequality in equation (2) is an application of Hölder's inequality for Shatten-p norms. The last equality of equation (2) follows from the positive semi-definiteness of $J^{-1/2}WJ^{-1/2} = (W^{-1/2}J^{-1/2})^{\dagger}(W^{-1/2}J^{-1/2})$ and the cyclic property of the trace, from which $||J^{-1/2}WJ^{-1/2}||_1 = \mathrm{tr}(J^{-1/2}WJ^{-1/2}) = \mathrm{tr}(WJ^{-1})$. Finally, in the proof of equation (17) in the original article, it is shown that $R := ||i2J^{-1}\mathcal{U}||_\infty = ||i2J^{-1/2}\mathcal{U}J^{-1/2}||_\infty$.

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References

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[2] Albarelli F and Datta A 2019 (arXiv:1911.11036)