

# Interferences driven by the few-cycle laser field in the photodetachment of $\text{Cl}^-$ ion

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## Abstract

The time-dependent semiclassical theory is extended to study the photodetachment of negative ions with outer  $p$ -state electrons. The photodetachment cross section of  $\text{Cl}^-$  ion is specially studied exposed to a few-cycle laser field. Considering the influence of laser field effects, the photodetachment cross section is found to oscillate complicatedly with increasing laser intensity and frequency. We demonstrate that different types of closed orbit have different impacts on the interference structures in the photodetachment cross section. Detailed analysis of the  $t$ - $t_i$  map of the detached electron reveals a remarkably correlation with the initial phase in the laser field. Additionally, the profiles of photodetachment cross section show strong dependence on the optical cycle number  $N$  in the laser field, indicating that it is an important parameter for controlling the photodetachment of negative ion. The method adopted in this work is universal, and can be applied to study the photodetachment of other halide negative ions in the laser field, such as  $\text{F}^-$  ion,  $\text{Br}^-$  ion, etc This work provides a new method for the experimentalists to coherent control of the photodetachment process of negative ion using a few-cycle laser field.

Keywords: Photodetachment,  $\text{Cl}^-$  ion, few-cycle laser field

(Some figures may appear in colour only in the online journal)

## 1. Introduction

Photodetachment of halide negative ions in strong laser field has attracted a lot of attention in the field of atomic physics. In the past several years, many researchers have used the multi-cycle laser pulses to study the photodetachment of halide negative ions [1–8]. Recently, with the advancement of the laser technology, theoreticians begin to probe the photodetachment process of halide negative ions by intense phase-controlled few-cycle infrared laser pulses [9–16]. These laser pulses are defined in terms of amplitude, frequency, the phase and the cycle number. The photodetachment dynamics of negative ion can be regulated by variation of these parameters in the laser pulse.

For negative ions, the outer electron is bound by short-range potential and the long-range Coulomb effect is absent

[17, 18]. At present, a successful analytical method developed for the description of the photodetachment of negative ions in the strong field is the closed orbit theory (COT) [19–21]. The original COT is developed for the photoabsorption spectra of Rydberg atom in the magnetic field and consists of considering the action of the ionic Coulombic field on the photoionized electron [20, 21]. Later, this theory has been extended to study the photodetachment of  $\text{H}^-$  in the strong electric field [22], in perpendicular and parallel electric and magnetic fields [23, 24], in crossed electric and magnetic fields [25], or in a gradient electric field [26], etc It has been shown that COT gives very reliable results for the photodetachment cross section of  $\text{H}^-$  ion in the electric field [22] and is consistent with that of the quantum mechanical result [27] or the experimental result [28]. Following the general picture depicted by COT, the external field can control the photodetachment process of negative ion by returning back an outgoing electron wave to the source region. This leads to a

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quantum interference signature. Analytically the interference signature arises from the superposition of the returning electron wave with the initial outgoing wave near the source center, which is intrinsically included in the final expression describing the photodetachment cross section. Each closed orbit of the detached electron corresponds to one sinusoidal term in the oscillating photodetachment cross section. Such quantum interference effects in the photodetachment cross section have been observed and analyzed in experimental studies on the photodetachment of negative ions in the external electric fields, such as  $\text{H}^-$ ,  $\text{S}^-$ ,  $\text{Cl}^-$ , etc [28–30].

In these previous experimental and theoretical studies, the external fields are static. With the development of the strong field physics, people began to investigate the photodetachment of the negative ion in the time-dependent field. Many types of specific field profiles, like a microwave field or a low-frequency laser pulse, can be used to study the time-dependent electric field effect on the photodetachment of the negative ion. Recently, with the realization of the single-cycle terahertz pulse in the experiment, researchers have attempted to explore the photodetachment of negative ion using this kind of laser pulse [31, 32]. For this kind of laser pulse, there is only one cycle. The adjustable parameters are the laser pulse intensity and the duration. In the last few years, increasing interests have been put on the coherent control of photodetachment process of the negative ion by using the few-cycle laser pulses. For instance, in 2013, Shearer and Monteith have studied the photodetachment of  $\text{F}^-$  using the few-cycle femtosecond laser pulse [33]. In that work, they studied the photoelectron energy spectra and differential detachment probability at fixed optical cycle  $N = 4$ . As to the variation of the photoelectron energy spectra and differential detachment probability with the optical cycle, they did not give a detailed analysis.

Inspired by this work, we study the coherent control for the photodetachment dynamics of negative ion using the wave-form-controlled few-cycle laser field based on the time-dependent COT. An analytical formula for calculating the time-dependent photodetachment cross section of  $\text{Cl}^-$  ion in the few-cycle laser field has been put forward. The behavior of the oscillatory structure in the photodetachment cross section is found to be highly correlated with the laser intensity, the frequency, initial phase, and the optical cycle of the laser field. Analysis of the photodetachment cross section indicates that quantum interference effects get weakened as the number of the optical cycle is increased. The semiclassical method used in this work is universal. It would also be very interesting to extend this method to study the photodetachment of other halide negative ions, such as  $\text{F}^-$  ion,  $\text{Br}^-$  ion, etc. In general, the motivation of this work can be concluded as follows: First, in contrast to the traditional study of photodetachment of negative ion in a static electric field, the current application of a few-cycle laser field provides an intuitive insight in understanding the electron interferences from a time-dependent viewpoint, and may guide the experimentalists to the coherent control of the photodetachment dynamics of negative ion in combing with a wave-form-controlled few-cycle laser field. Second, our work may also provide further insight in strong field and ultrafast physics where the electron dynamics

manipulated by a few-cycle laser field is often studied. An immediate application of our theoretical method would be the photoionization dynamics of neutral atoms in the driving laser field by including the long-range Coulomb potential. Finally, our work is not only extended the semiclassical closed orbit theory but also has many applications in the fields of the atomic and molecular physics, such as in exploring the ionization dynamics of Rydberg atom, controlling the alignment and orientation of polar molecules, etc.

Atomic units (which are abbreviated as a.u.) are used throughout this work unless indicated otherwise.

## 2. The photodetachment cross section of $\text{Cl}^-$ ion in the driving few-cycle laser field

We consider  $\text{Cl}^-$  ion is interacting simultaneously with a weak laser field plus an wave-form-controlled few-cycle laser field. We assume both the laser fields are linearly polarized along the  $z$ -axis. The weak laser field has the following specific form [31, 32]:

$$f_L(t) = \frac{1}{2} \left[ \tanh\left(\frac{t-t_u}{t_L}\right) - \tanh\left(\frac{t-t_d}{t_L}\right) \right], \quad (1)$$

where  $f_L(t)$  represents the slowly varying envelope of the weak laser field, and  $t_u, t_d, t_L$  are the parameters. The parameters  $t_u$  and  $t_d$  indicate the time when the laser field is turned on and off, respectively. In this work, we choose  $t_d = -t_u = 3.0$  ps,  $t_L = 80T_L$ , in which  $T_L$  denotes the oscillating period of the laser used for the photodetachment. The photon energy  $E_p = \hbar\omega_L = 3.67$  eV, then  $T_L = 1.12$  fs. The harmonic frequency of this laser field is  $\omega_L = 0.1349$  a.u. The pulse shape of the weak laser light is shown in figure 1(a).

The external few-cycle driving laser field is constructed by a time-dependent vector potential [33]:

$$A(t) = A_0 \left[ \sin^2\left(\frac{\omega t}{2N}\right) \sin(\omega t + \alpha) \right]. \quad (2)$$

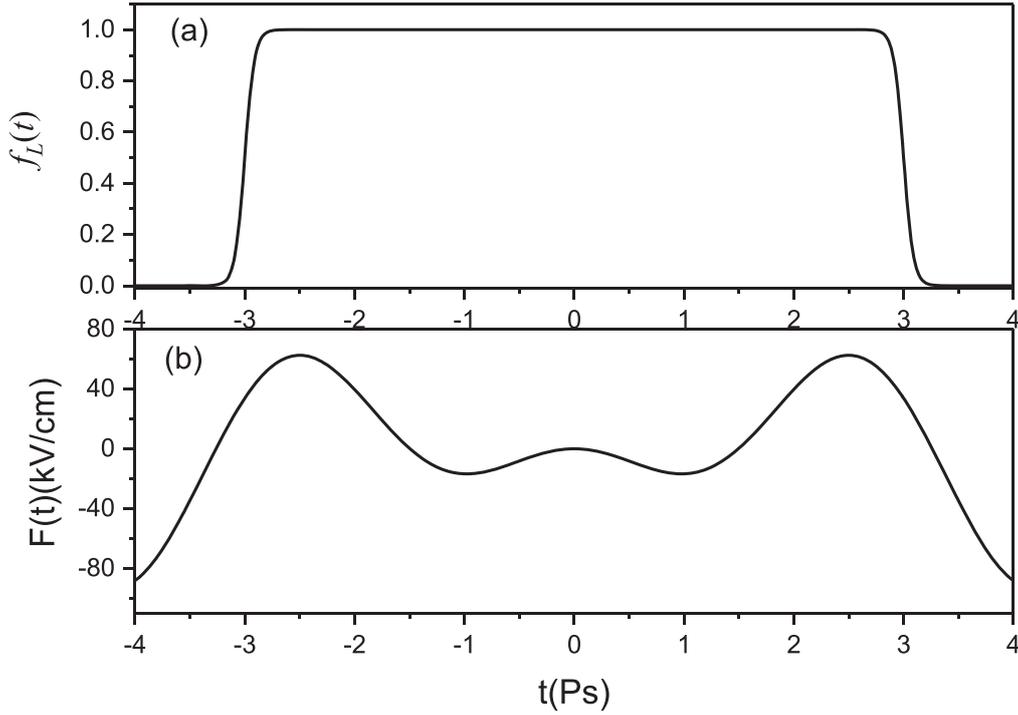
Here  $A_0$  is the peak value of the vector potential:  $A_0 = F_0/\omega$ , where  $F_0$  is the peak value of the laser field intensity. The vector potential  $A(t)$  is turned on and off at  $t = -3.0$  ps and  $t = 3.0$  ps, respectively. The frequency, optical cycle and the initial phase of the few-cycle driving laser field are denoted by  $\omega, N$  and  $\alpha$ , respectively.

The few-cycle laser field corresponds to the vector potential is:

$$F(t) = -\frac{dA(t)}{dt} = A_0 \left[ -\frac{\omega}{2} \cos(\omega t + \alpha) + \frac{\omega_1}{4} \cos(\omega_1 t + \alpha) + \frac{\omega_2}{4} \cos(\omega_2 t + \alpha) \right], \quad (3)$$

where  $\omega_1 = \left(1 + \frac{1}{N}\right)\omega$ ,  $\omega_2 = \left(1 - \frac{1}{N}\right)\omega$ . The few-cycle laser field profile is shown in figure 1(b).

The physical picture description of the photodetachment process of  $\text{Cl}^-$  ion can be divided into two steps: first, the binding electron in the  $\text{Cl}^-$  ion will be photodetached by absorbing one photon from the weak laser field, and then the



**Figure 1.** The laser light field configurations for the present system. (a) The weak laser field  $f_L(t)$ . (b) The external few-cycle driving laser field  $F(t)$ , the laser field parameters  $A_0 = 0.5$ ,  $\omega = 3.63 \times 10^{-5}$  a.u.,  $N = 2$  and  $\alpha = 0$ .

motion of the photodetached electron will be dominated by the driving laser field. After the electron is photo-detached from  $\text{Cl}^-$  ion, a steady stream of outgoing electron waves with a fixed energy is produced. These waves are propagating along the classical trajectories in all directions. If the laser field is strong enough, it can control the photodetachment process of negative ion by reflecting back an outgoing electron wave to the source region where the initial bound state is localized. This kind of electron trajectory is called the closed orbit. Since the driving laser field is pointing along the  $z$ -axis, it is obvious that only the detached electron emitted along the  $z$  axis can be bounced back by the external field to the origin to form a closed orbit. All other electron's trajectory traveling off the  $z$  axis goes away and never returns to the origin. The interference effect between the returning wave traveling along each closed orbit with the initial outgoing wave causes the oscillation in the photodetachment cross section.

It has been shown in [34] that the photodetachment cross section of negative ion in the time-dependent external field can be written as:

$$\sigma(E, t) = -\frac{4\pi E_p}{c} \text{Im}\langle I(t) | \psi(t) \rangle, \quad (4)$$

where  $c$  is the light speed,  $E_p$  is the photon energy:  $E_p = E_0 + E_b$ ,  $E_0$  is the initial kinetic energy of the detached electron,  $E_0 = k_0^2/2$ , and  $E_b = 29138.3(5) \text{ cm}^{-1} = 3.612 \text{ eV}$  is the binding energy of  $\text{Cl}^-$  ion [29]. In equation (4),  $I(t)$  denotes the interaction of  $\text{Cl}^-$  ion with the weak laser field  $f_L(t)$ :

$$I(t) = f_L(t) e^{-iE_0 t} D \varphi_i. \quad (5)$$

Where  $D$  is the dipole operator, which is dependent on the laser polarization. For  $z$ -polarized laser light,  $D = z$ . In equation (5),

$\varphi_i$  is the initial wave function of  $\text{Cl}^-$  ion. For the photodetachment of  $\text{Cl}^-$  in the external field, the detached electron wave source is an  $s$ -wave. For  $s$ -wave photodetachment,  $D\varphi_i$  can be written as a  $\delta$  source function:  $D\varphi_i = \lambda \delta(\vec{r})$  [35]. Here  $\lambda$  is a constant, and is related to the bound state wave function of  $\text{Cl}^-$  ion. In this work, we choose  $\lambda = 43.71 \text{ a.u.} = 23.12 \text{ nm}$ .

In equation (4),  $\psi(t)$  is the detached electron wave function at arbitrary time  $t$ , which can be divided into two parts:

$$\psi(t) = \psi_{dir}(r, t) + \psi_{ret}(r, t). \quad (6)$$

The first part  $\psi_{dir}(r, t)$  is the directly outgoing wave initially going out from the ion source at each time instant  $t$ :

$$\psi_{dir}(r, t) = f_L(t) \psi_{out}(r) e^{-iE_0 t}, \quad (7)$$

where  $\psi_{out}(r)$  satisfies the following equation:

$$\left( E - \frac{p^2}{2} \right) \psi_{out}(r) = \lambda \delta(\vec{r}), \quad (8)$$

Using the Green's function method [36], we obtain:

$$\psi_{out}(r) = -\frac{\lambda}{2\pi} \frac{e^{ik_0 r}}{r}. \quad (9)$$

The second term  $\psi_{ret}(r, t)$  in equation (6) is the electron wave returned back by the driving laser field, which is a sum including all the returning waves travelling along all the possible closed-orbits:

$$\psi_{ret}(r, t) = \sum_j \psi_{ret}^j(r, t), \quad (10)$$

Here,  $\psi_{ret}^j(r, t)$  is the returning wave along the  $j$ -th closed orbit.

Therefore, the total photodetachment cross section can be decomposed into two terms:

$$\sigma(E, t) = \sigma_0(E, t) + \sigma_{osc}(E, t), \quad (11)$$

where  $\sigma_0(E, t)$  is the background term without the external field [34]:

$$\begin{aligned} \sigma_0(E, t) &= -\frac{4\pi E_p}{c} \text{Im} \langle I(t) | \psi_{dir}(r, t) \rangle \\ &= \frac{2\lambda^2 k_0 E_p}{c} f_L^2(t). \end{aligned} \quad (12)$$

The second term  $\sigma_{osc}(E, t)$  is the oscillating part:

$$\sigma_{osc}(E, t) = -\frac{4\pi E_p}{c} \sum_j \text{Im} \langle I(t) | \psi_{ret}^j(r, t) \rangle. \quad (13)$$

To obtain  $\psi_{ret}^j(r, t)$  driven back by the laser field, we draw a small spherical surface around the negative ion. The outgoing wave on this spherical surface is not obviously affected by the external laser field [35]:

$$\psi_{out}(R, \theta_i, \varphi_i) = -\frac{\lambda}{2\pi} \frac{e^{ik_0 R}}{R}. \quad (14)$$

Here,  $R$  is the radius of the spherical surface. When this wave travels into the external laser field, we can use the semiclassical method to construct the wave function. The semiclassical wave travelling along the  $j$ -th closed orbit can be written as [31]:

$$\begin{aligned} \psi_{sc}^j(r, t) &= f_L(t_i) \psi_{out}(R, \theta_i, \phi_i) \\ &\times A_j \exp[i(S_j - E_0 t_i - \lambda_j \pi/2)], \end{aligned} \quad (15)$$

where  $A_j$  is the semiclassical amplitude of the wave function,  $S_j$  denotes the classical action, which are described in the following section, and  $\lambda_j$  is the Maslov index.

When the outgoing wave returns back to the source region by the driving laser field, it behaves like a plane wave and approximately written as:

$$\psi_{ret}^j(r, t) = f_L(t_i) e^{-iE_0 t} \tilde{\psi}_{ret}^j(r, t), \quad (16)$$

where  $\tilde{\psi}_{ret}^j(r, t)$  is related to the plane wave:

$$\tilde{\psi}_{ret}^j(r, t) = -\frac{\lambda}{2\pi} N_{co}^j e^{\pm ik_{ret} z}. \quad (17)$$

In the above equation,  $N_{co}^j$  is a matching factor and  $k_{ret}$  is the returning electron momentum. According to the semiclassical approximation, we get:

$$N_{co}^j = \frac{A_j}{R} \exp \left[ i \left( S_j + E_0(t - t_i) - \lambda_j \frac{\pi}{2} \right) \right]. \quad (18)$$

Substituting the returning wave equation (16) into equation (13), after carrying out the overlap integral  $\langle I(t) | \psi_{ret}^j(r, t) \rangle$ , we obtain:

$$\begin{aligned} \sigma_{osc}(E, t) &= \frac{6\lambda^2 E_p}{c} g_j f_L(t) f_L(t_i) \\ &\times \frac{A_j}{R} \exp \left[ i \left( S_j + E_0(t - t_i) - \lambda_j \frac{\pi}{2} \right) \right] \end{aligned} \quad (19)$$

Where  $g_j$  is a factor:

$$g_j = \begin{cases} +1 & \theta_{ret} = \theta_i \\ -1 & \theta_{ret} \neq \theta_i \end{cases}. \quad (20)$$

Combing equations (12) and (19), we finally have the total time-dependent photodetachment cross section of  $\text{Cl}^-$  ion in a few-cycle laser field:

$$\begin{aligned} \sigma(E, t) &= \sigma_0(E, t) + \sigma_{osc}(E, t) = \sigma_0 \\ &\times \left( f_L^2(t) + \sum_j 3g_j f_L(t) f_L(t_i) \right. \\ &\left. \times \frac{A_j}{k_0 R} \sin \left[ S_j + E_0(t - t_i) - \lambda_j \frac{\pi}{2} \right] \right) \end{aligned} \quad (21)$$

From this formula, we find in order to calculate the photodetachment cross section, we should find out all the closed orbits of the detached electron in the driving laser field.

### 3. Classical motion of the detached electron in the driving laser field

#### 3.1. The Hamiltonian and closed orbit of the detached electron

Let the driving laser field point along the  $z$ -axis, the Hamiltonian governing the motion of the detached electron in the few-cycle driving laser field can be written as (using cylindrical coordinates  $(\rho, z, \varphi)$ ) [31]:

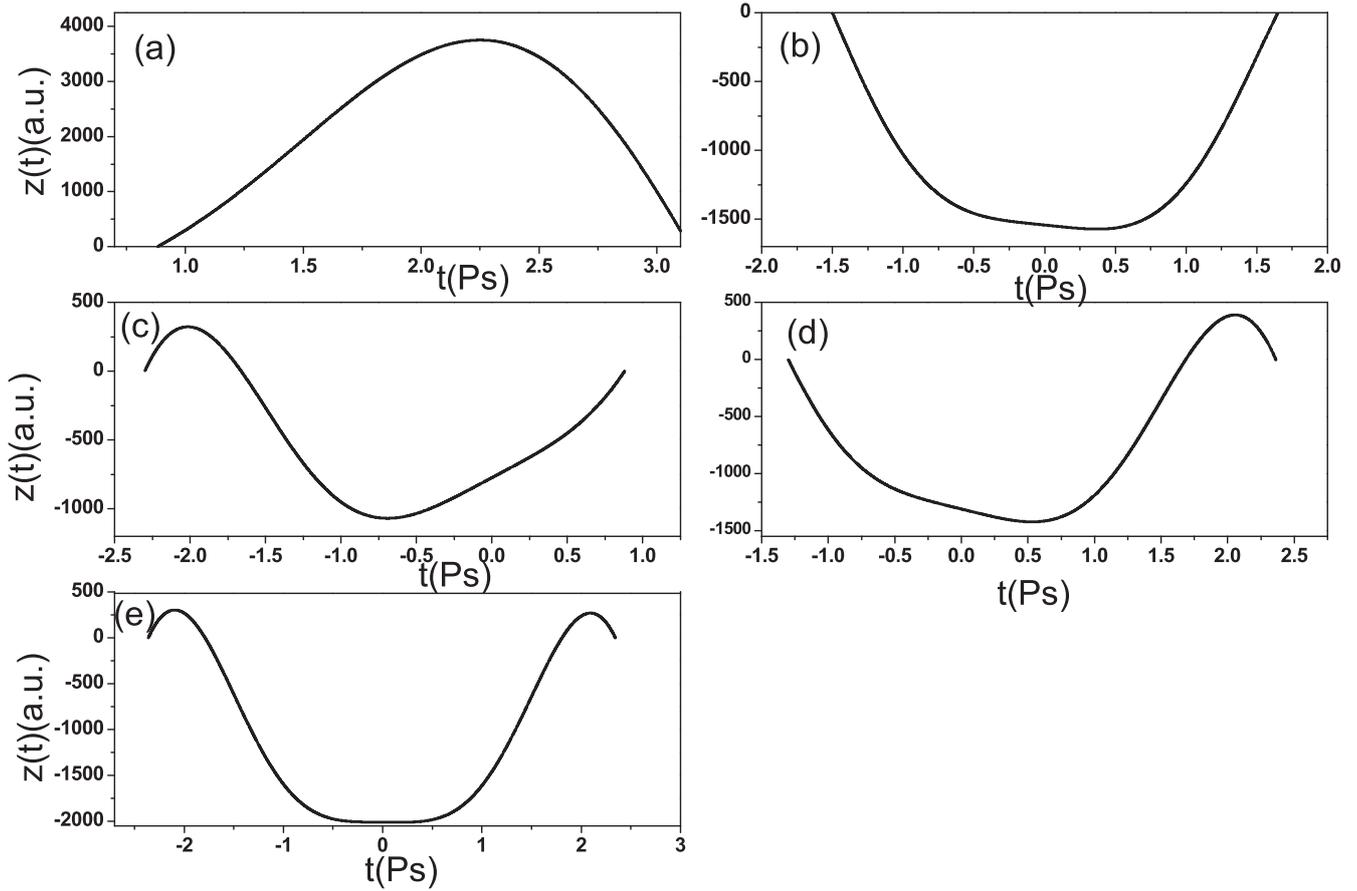
$$\begin{aligned} H(\rho, z, t, p_\rho, p_z, p_t) &= \frac{1}{2} p_\rho^2 \\ &+ \frac{1}{2} p_z^2 + F(t)z + p_t, \end{aligned} \quad (22)$$

The  $\varphi$  motion has been separated because of the cylindrical symmetry of the system.

The  $z$  component of the angular momentum is a constant of motion due to the symmetry, and has been set to zero. In equation (22),  $p_t$  is a conjugate momentum corresponding to the classical dynamical variable  $t$  [31, 32]. Since the Hamiltonian is time-dependent, in order to solve the Hamiltonian canonical equations, we introduce an 'evolution time'  $\tau$  to describe the electron motion in the driving laser field,  $\tau = t - t_i$ . Here,  $t_i$  is the initial time of the electron trajectory and  $t$  denotes the real, laboratory time. The initial conditions are as follows:  $t(\tau = 0) = t_i$ ,  $p_t(\tau = 0) = -E$ . In addition, we add two motion equations related to the classical dynamical variable  $t$  and its conjugate momentum  $p_t$ :  $dt/d\tau = 1$ ,  $dp_t/d\tau = -\partial H/\partial t$ .

Supposing the  $\text{Cl}^-$  ion is localized at the origin, the classical motion equations of the detached electron in the few-cycle driving laser field has the following form:

$$\begin{cases} \rho(t) = k_0 \sin \theta_i (t - t_i) \\ z(t) = \left\{ k_0 \cos \theta_i - A_0 \left[ \sin^2 \left( \frac{\omega t_i}{2N} \right) \right. \right. \\ \left. \left. \times \sin(\omega t_i + \alpha) \right] \right\} \cdot (t - t_i) + \int_{t_i}^t A(t') dt'. \end{cases} \quad (23)$$



**Figure 2.** Some closed orbits for the detached electron in the few-cycle laser field. The photon energy  $E_p = 3.67$  eV. The parameters in the laser field are as follows:  $A_0 = 0.5$ ,  $\omega = 3.63 \times 10^{-5}$  a.u.,  $N = 2$  and  $\alpha = 0$ . (a) the up closed orbit; (b) the down closed orbit; (c) the up-down closed orbit; (d) the down-up closed orbit; (e) the up-down-up closed orbit.

Where  $k_0$  is the initial momentum of the detached electron, and  $\theta_i$  denotes the outgoing angle of the detached electron relative to the  $+z$  axis.

For the closed orbit, at the final time  $t$ ,  $\rho(t) = 0$  and  $z(t) = 0$ . Let  $\rho(t) = 0$ , we get  $\theta_i = 0$  or  $\theta_i = \pi$ . Let  $z(t) = 0$ , we get:

$$\left\{ k_0 \cos \theta_i - A_0 \left[ \sin^2 \left( \frac{\omega t_i}{2N} \right) \sin(\omega t_i + \alpha) \right] \right\} \cdot (t - t_i) + \int_{t_i}^t A(t') dt' = 0. \quad (24)$$

Through numerical calculation of equation (24), we obtain the initial time  $t_i$  and the returning time  $t$  for each closed orbit. The number of the closed orbit depends sensitively on the driving laser field. For example, in figure 2, we choose the laser field parameters  $A_0 = 0.5$ ,  $\omega = 3.63 \times 10^{-5}$  a.u.,  $N = 2$  and  $\alpha = 0$ . We found five different types of closed orbits. To be clear, we name each type of closed orbit according to the direction it propagates. Figure 2(a) shows the closed orbit emits along the  $+z$  axis with the initial outgoing angle  $\theta_i = 0$ , after a small period of time, it returns back to the origin with the returning angle  $\theta_{ret} = \pi$ . we call it the up orbit. Figure 2(b) is the down orbit, which travels along the  $-z$  axis at first, with the initial outgoing angle  $\theta_i = \pi$ , and returns back with  $\theta_{ret} = 0$ . Figure 2(c) shows the up-down closed orbit, which

travels along the up orbit first, then the down orbit. The returning angle equals the outgoing angle,  $\theta_{ret} = \theta_i = 0$ . Figure 2(d) is called the down-up orbit, which travels along the down orbit first, then the up orbit,  $\theta_i = \theta_{ret} = \pi$ . Figure 2(e) is called the up-down-up closed orbit. The outgoing angle  $\theta_i = 0$ , and the returning angle  $\theta_{ret} = \pi$ .

### 3.2. Semiclassical amplitude and classical action

The semiclassical amplitude  $A_j$  is defined as the ratio of Jacobian at the initial time and the final time:  $A_j = \left| \frac{J_f(t=t)}{J_i(t)} \right|^{1/2}$ , where  $J(t)$  can be written as:

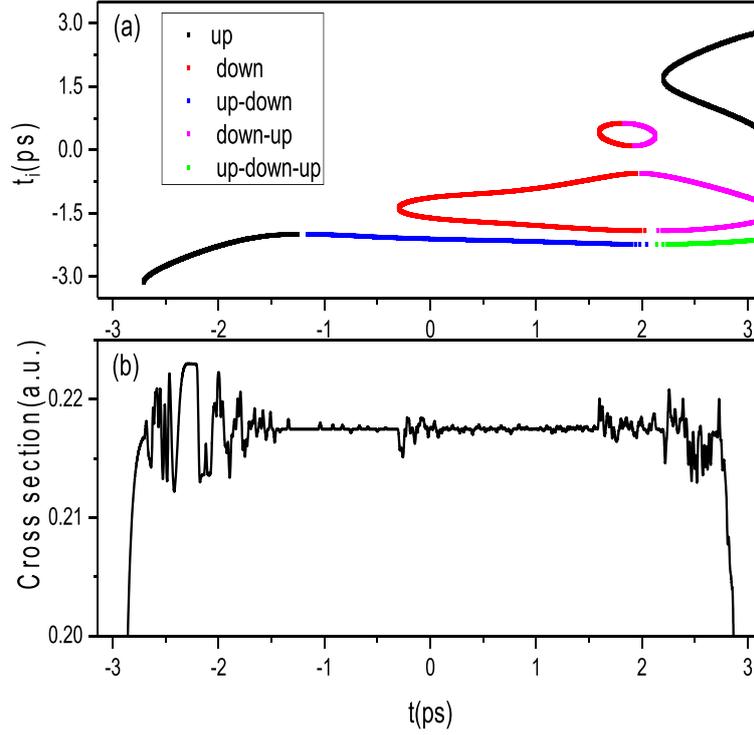
$$J(t) = \rho(t) \det \left( \frac{\partial(\rho, z, t)}{\partial(t_i, \theta_i, \tau)} \right). \quad (25)$$

The above formula can be further simplified as [31]:

$$J(t) = \rho(t) \left( \frac{\partial \rho}{\partial t_i} \right)_{\theta_i, t} \left( \frac{\partial z}{\partial \theta_i} \right)_{\rho, t}. \quad (26)$$

For the initially outgoing spherical wave, we can obtain:  $J(t = t_i) = k_0 R^2 \sin \theta_i$ .

With the help of equation (23), the partial derivative of  $\rho$  with respect to  $t_i$  is:  $\left( \frac{\partial \rho}{\partial t_i} \right)_{\theta_i, t} = k_0 \sin \theta_i$ . The partial derivative



**Figure 3.** (a) The  $t-t_i$  map for the detached electron's closed orbit in the few-cycle laser field. The photon energy  $E_p = 3.67$  eV. The parameters in the laser field are as follows:  $A_0 = 0.5$ ,  $\omega = 3.63 \times 10^{-5}$  a.u.,  $N = 2$  and  $\alpha = 0$ . Different types of closed orbit are denoted by different lines. (b) The time-dependent photodetachment cross section of  $\text{Cl}^-$  ion in the few-cycle laser field.

of  $z$  with respect to  $\theta_i$  can be written as:

$$\begin{aligned} \left(\frac{\partial z}{\partial \theta_i}\right)_{\rho,t} &= \left(\frac{\partial z}{\partial \theta_i}\right)_{t_i,t} + \left(\frac{\partial z}{\partial t_i}\right)_{\theta_i,t} \left(\frac{\partial t_i}{\partial \theta_i}\right)_{\rho,t} \\ &= -k_0 \sin \theta_i (t - t_i) \\ &\quad - [k_0 \cos \theta_i - F(t_i)(t - t_i)] \text{ctg} \theta_i (t - t_i). \end{aligned} \quad (27)$$

Therefore,

$$\begin{aligned} J(t) &= \left(\frac{\partial z}{\partial \theta_i}\right)_{t_i,t} + \left(\frac{\partial z}{\partial t_i}\right)_{\theta_i,t} \left(\frac{\partial t_i}{\partial \theta_i}\right)_{\rho,t} \\ &= k_0 \sin \theta_i (t - t_i)^2 (-k_0 \sin \theta_i) \{-k_0 \sin \theta_i \\ &\quad - [k_0 \cos \theta_i - F(t_i)(t - t_i)] \text{ctg} \theta_i\} \end{aligned} \quad (28)$$

Finally, we obtain the semiclassical amplitude corresponding to the  $j$ -th closed orbit:

$$A_j = \frac{R}{t - t_i} \left| \frac{1}{k_0 [k_0 - F(t_i) \cos \theta_i (t - t_i)]} \right|^{1/2} \quad (29)$$

Here,  $F(t_i)$  is the few-cycle laser field at the initial time  $t_i$ :

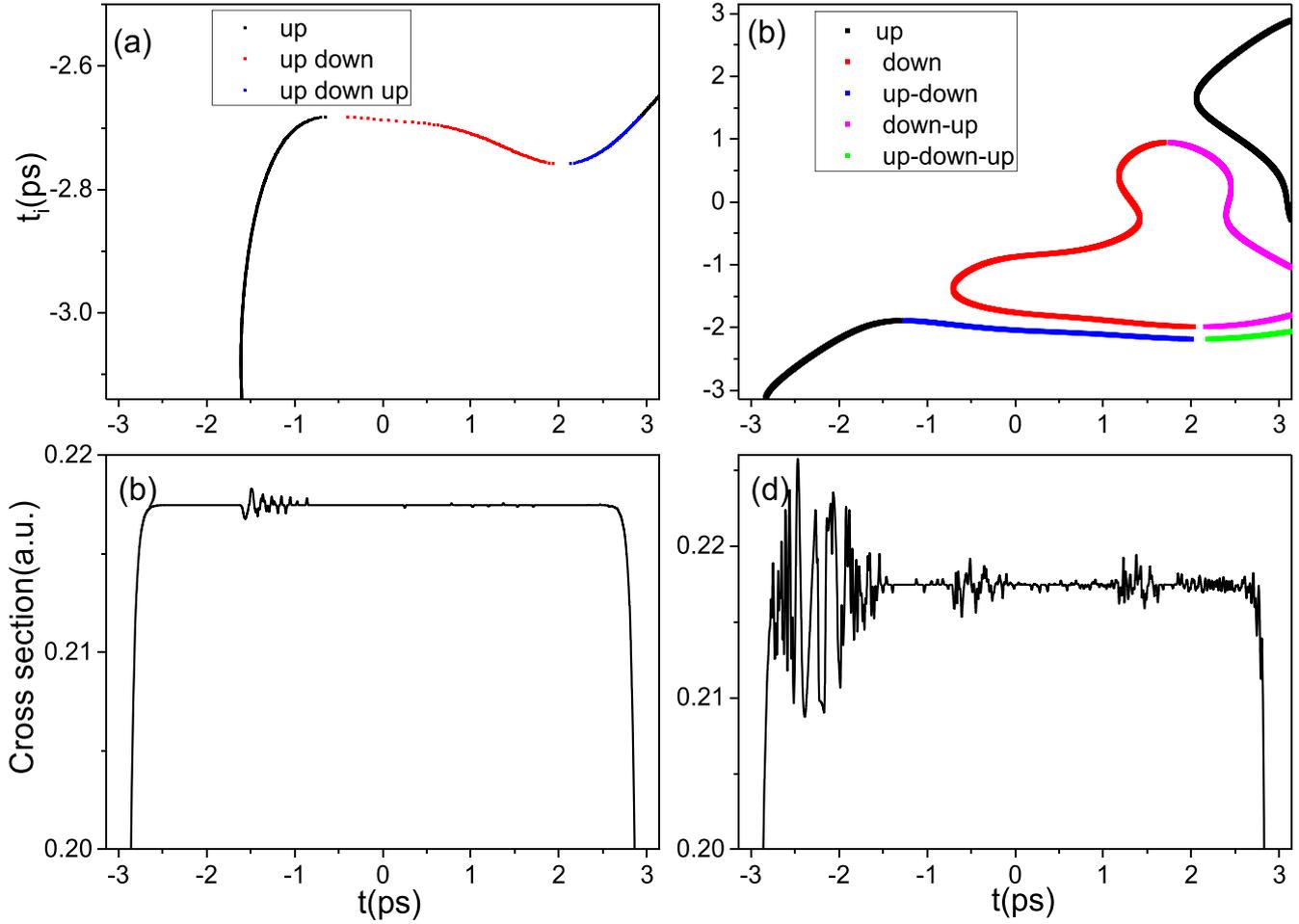
$$\begin{aligned} F(t_i) &= A_0 \left[ -\frac{\omega}{2} \cos(\omega t_i + \alpha) + \frac{\omega_1}{4} \cos(\omega_1 t_i + \alpha) \right. \\ &\quad \left. + \frac{\omega_2}{4} \cos(\omega_2 t_i + \alpha) \right] \end{aligned} \quad (30)$$

The classical action  $S_j$  in equation (15) is defined as an integral along each closed orbit:

$$\begin{aligned} S_j &= \int p dq = \int_{t_i}^t p_\rho d\rho + \int_{t_i}^t p_z dz + \int_{t_i}^t p_t dt = \left\{ \frac{1}{2} k_0^2 \right. \\ &\quad \left. + \frac{1}{2} \left[ A_0 \sin^2 \left( \frac{\omega t_i}{2N} \right) \sin(\omega t_i + \alpha) \right]^2 \right. \\ &\quad \left. - \left[ A_0 \sin^2 \left( \frac{\omega t_i}{2N} \right) \sin(\omega t_i + \alpha) \right] k_0 \cos \theta_i \right\} \\ &\quad \times (t - t_i) - \frac{1}{2} \int_{t_i}^t A^2(t') dt' \end{aligned} \quad (31)$$

#### 4. Results and discussions

Firstly, we choose the laser field parameter  $N = 2$ ,  $\omega = 3.63 \times 10^{-5}$  a.u.,  $A_0 = 0.5$  a.u.,  $\alpha = 0.0$ . figure 3(a) shows the  $t-t_i$  map for different type of closed orbit in the driving laser field. The photodetachment cross section is shown in figure 3(b). It is found due to the influence of the laser field, the electron wave can be driven back to the source region. The returning electron wave interferes with the outgoing wave near the source center, which causes the oscillatory structures in the photodetachment cross section. Through the comparison with the  $t-t_i$  map shown in figure 3(a), we can see that the up orbit plays the crucial role in the oscillatory structures of the photodetachment cross section.

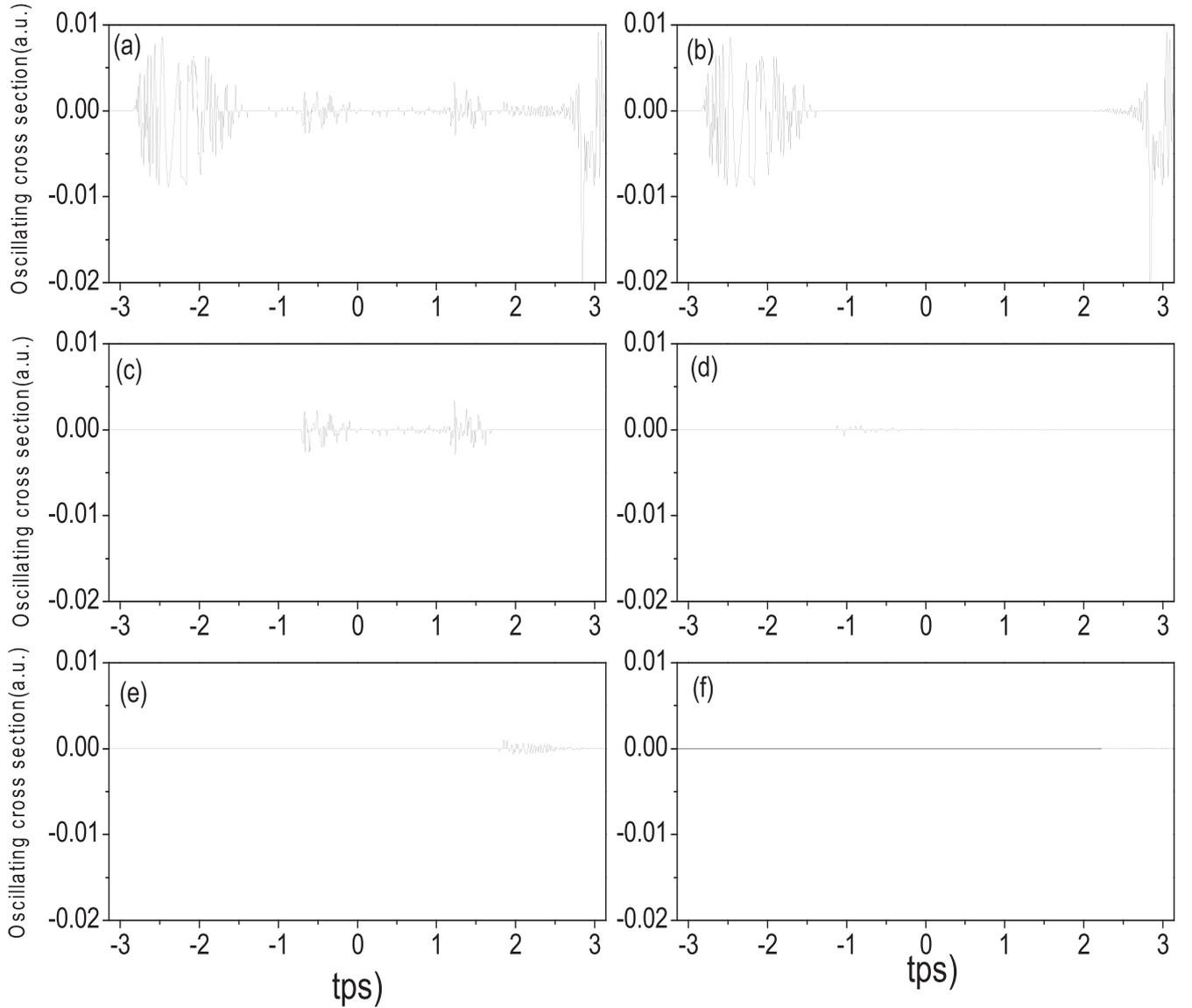


**Figure 4.** (a), (b) The  $t-t_f$  map for the detached electron's closed orbit in the few-cycle laser field with the peak value of the vector potential  $A_0 = 0.1$ , and  $A_0 = 0.8$  respectively. (c), (d) The time-dependent photodetachment cross section of  $\text{Cl}^-$  ion in the laser field corresponding to (a) and (b). Different types of closed orbit are denoted by different lines. The other parameters in the laser field are as follows:  $\omega = 3.63 \times 10^{-5}$  a.u.,  $N = 2$  and  $\alpha = 0$ .

If we change the peak value of the vector potential  $A_0$ , the oscillatory structures in the photodetachment cross section vary accordingly. In figure 4, we calculate the photodetachment cross section with the peak value of the vector potential  $A_0$  is less or larger than 0.5 a.u. The other parameters in the laser field are the same as given in figure 3. Figure 4(a) shows the  $t-t_f$  map for the detached electron's closed orbit in the few-cycle laser field with the peak value of the vector potential  $A_0$  is relatively small,  $A_0 = 0.1$  a.u. Under this condition, the intensity of the few-cycle laser field is relatively small, the laser field makes very little influence on the movement of the detached electron. Only three types of closed orbit exist. The corresponding photodetachment cross section is shown in figure 4(c). It is found that the photodetachment cross section exhibits a smooth background term plus a series of small oscillations, and the oscillatory structure is only limited in a small region. As we increase the peak value of the vector potential to  $A_0 = 0.8$  a.u., the laser field intensity becomes strengthened. As we show in figure 4(b), the number of the closed orbit for the detached electron becomes increased. Five different types of closed orbit for the detached electron appear. The oscillatory structure in the photodetachment cross

section becomes much more complicated, as we can see from figure 4(d). The reason can be analyzed using the COT: As the peak value of the vector potential increases at fixed frequency, the intensity of the laser field gets increased. The oscillating electric field force acting on the detached electron becomes significant, after the electron is emitted from the origin, it will return back to the origin after a short period of time, then the interference effect of the returning electron wave with the initial outgoing wave gets strengthened, which causes the complicated oscillatory structure in the photodetachment cross section.

In order to see the contribution of each type of the electron's closed orbit to the photodetachment cross section, we calculate the oscillating cross section  $\sigma_{osc}(E, t)$ . The parameters in the laser field are the same as in figure 4(d). Figure 5(a) is the total oscillating cross section  $\sigma_{osc}(E, t)$  caused by all five types closed orbit of the detached electron shown in figure 4(b), however, figures 5(b)–(f) shows the contribution of the up, down, up-down, down-up and the up-down-up closed orbit to  $\sigma_{osc}(E, t)$ , respectively. From this figure, we can see that the oscillation caused by the up closed orbit is the most obvious, followed by the down and the

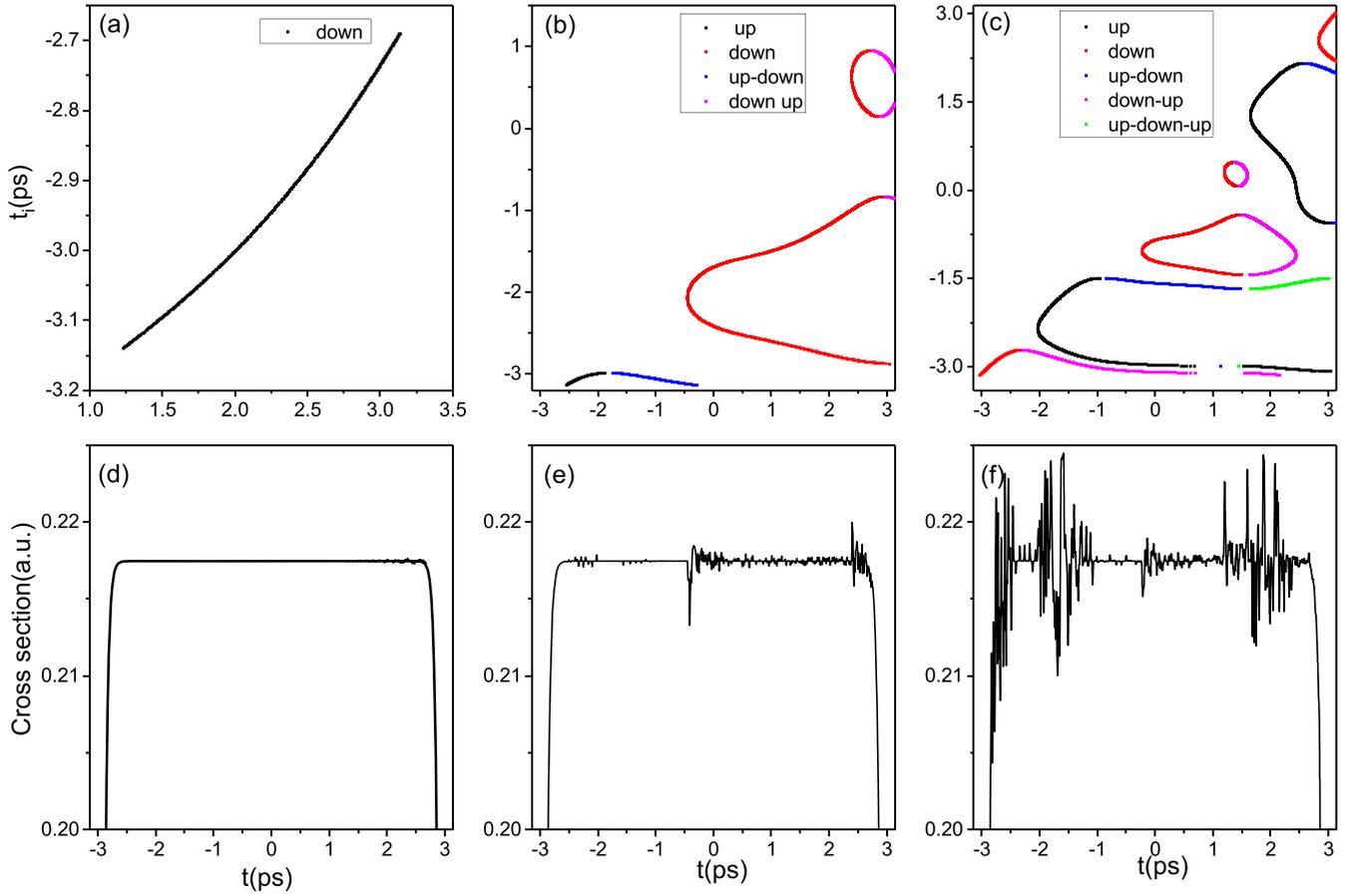


**Figure 5.** The oscillating photodetachment cross section of  $\text{Cl}^-$  ion in the few-cycle laser field. The parameters in the laser field are as follows:  $A_0 = 0.8$ ,  $\omega = 3.63 \times 10^{-5}$  a.u.,  $N = 2$  and  $\alpha = 0$ . (a) The total oscillating cross section  $\sigma_{osc}(E, t)$  caused by all the closed orbits. (c-f) The oscillating cross section induced by the up, down, up-down, down-up, and up-down-up closed orbit respectively.

down-up closed orbit. The up-down and up-down-up closed orbits only have small contribution to the cross section, and their influences can be neglected. The reason is as follows: from the  $t-t_i$  map for each type of closed orbit shown in figure 4(b), we find that the up, down and down-up closed orbits dominate a large part in the whole  $t-t_i$  map, thus their contributions to the oscillating cross section are significant. However, the up-down and up-down-up closed orbits are limited in small regions in the whole  $t-t_i$  map, they only produce a little modulation on the oscillating cross section.

Next, we show the influence of the laser field frequency on the photodetachment cross section. Suppose the other parameters in the laser field are as follows:  $N = 2$ ,  $A_0 = 0.5$  a.u.,  $\alpha = 0.0$ . The results are shown in figure 6. In figure 6(a), we plot the  $t-t_i$  map for the detached electron's closed orbit in the laser field with the frequency

$\omega = 1.21 \times 10^{-5}$  a.u. Under this condition, the laser field  $F(t)$  described by equation (3) is less than 0, i.e., the laser field always points along the  $-z$  axis in the given evolution time. Thus only the down closed orbit for the detached electron appears. We find that the down orbit is limited in a small region and its evolution time is relatively long. The corresponding photodetachment cross section is shown in figure 6(d). It is shown that the laser field has a minor influence on the photodetachment cross section. Both the oscillating amplitude and the oscillating region in the photodetachment cross section are very small. The oscillatory structure in the photodetachment cross section is nearly invisible, and the total photodetachment cross section approximates to the case without the driving laser field. With the increase of the laser field frequency, the value of the laser field  $F(t)$  could be positive or negative, which makes the

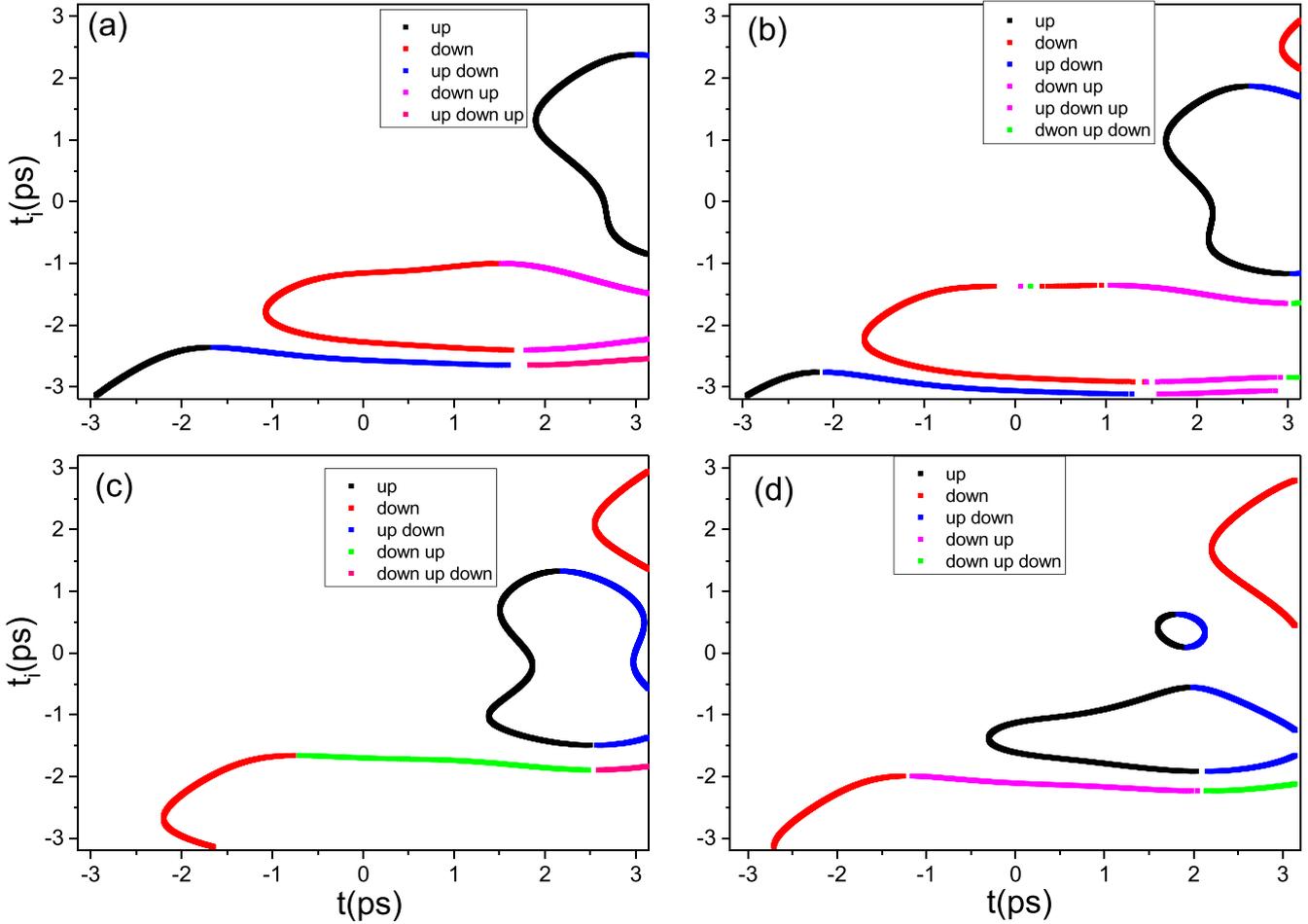


**Figure 6.** (a)–(c) The  $t$ - $t_i$  map for the detached electron's closed orbit in the few-cycle laser field with the frequency  $\omega = 1.21 \times 10^{-5}$  a.u.,  $\omega = 2.42 \times 10^{-5}$  a.u. and  $\omega = 3.63 \times 10^{-5}$  a.u. respectively. (c–e) The time-dependent photodetachment cross section of  $\text{Cl}^-$  ion in the laser field corresponding to (a)–(c). Different types of closed orbit are denoted by different lines. The other parameters in the laser field are as follows:  $A_0 = 0.5$ ,  $N = 2$  and  $\alpha = 0$ .

number of the detached electron's closed orbit become increased, and their contribution to the total photodetachment cross section gets significant. Figure 6(b) shows the  $t$ - $t_i$  map with the laser field frequency  $\omega = 2.42 \times 10^{-5}$  a.u. Four different types of detached electron's closed orbit exist. It can be further noted that the down closed orbit occupies a large part in the whole  $t$ - $t_i$  map, and it makes a great effect on the total photodetachment cross section, as we can see from figure 6(e) clearly. As we further increase the laser field frequency,  $\omega = 4.84 \times 10^{-5}$  a.u., the influence of the laser field on the photodetachment dynamics of  $\text{Cl}^-$  ion gets strengthened. There are five different types of detached electron's closed orbits, as we show in figure 6(c). Additionally, the laser field  $F(t)$  mostly points along the  $+z$  axis, so the up closed orbit dominates a large part in the  $t$ - $t_i$  map. The total photodetachment cross section is shown in figure 6(f). In addition to the down closed orbit, the up closed orbit also has a large contribution to the total photodetachment cross section, and the photodetachment cross section becomes much more complicated.

In the following, we discuss how the initial phase in the laser field affects the photodetachment process of  $\text{Cl}^-$  ion. Suppose the other parameters in the laser field are as follows:  $A_0 = 0.5$  a.u.,  $N = 2$ ,  $\omega = 3.63 \times 10^{-5}$  a.u. The results are

shown in figure 7 and figure 8. The  $t$ - $t_i$  map for the closed orbit of the detached electron is shown in figure 7. It is found as the initial phase in the laser field changes, the movement of the detached electron is varied correspondingly. For example, in figure 7(a), the initial phase  $\alpha = \pi/4$ , five different types of closed orbit appear. However, in figure 7(b),  $\alpha = \pi/2$ , the time-dependent vector potential becomes:  $A(t) = A_0 \sin^2\left(\frac{\omega t}{2N}\right) \cos \omega t$ . Under this condition, the  $t$ - $t_i$  map has changed a little. In addition to the five types of closed orbit appeared in figure 7(a), a new type of closed orbit, namely, the down-up-down orbit takes place. As the phase  $\alpha = 3\pi/4$ (figure 7(c)), the up-down-up orbit shown in figures 7(a) and (b) disappears. If  $\alpha = \pi$ , the time-dependent vector potential  $A(t)$  is anti-phase with the case of  $\alpha = 0$ . The  $t$ - $t_i$  map is shown in figure 7(d). Through a detailed comparison with figure 3(a), which is the case with the initial phase  $\alpha = 0$ , we find the  $t$ - $t_i$  map is nearly the same, except the closed orbits are in reverse orders from figure 3(a). That is to say, the  $t$ - $t_i$  curve for the up closed orbit shown in figure 3(a) becomes the  $t$ - $t_i$  curve for the down closed orbit in this figure. The  $t$ - $t_i$  curve for the other closed orbits can be analyzed in the same manner.

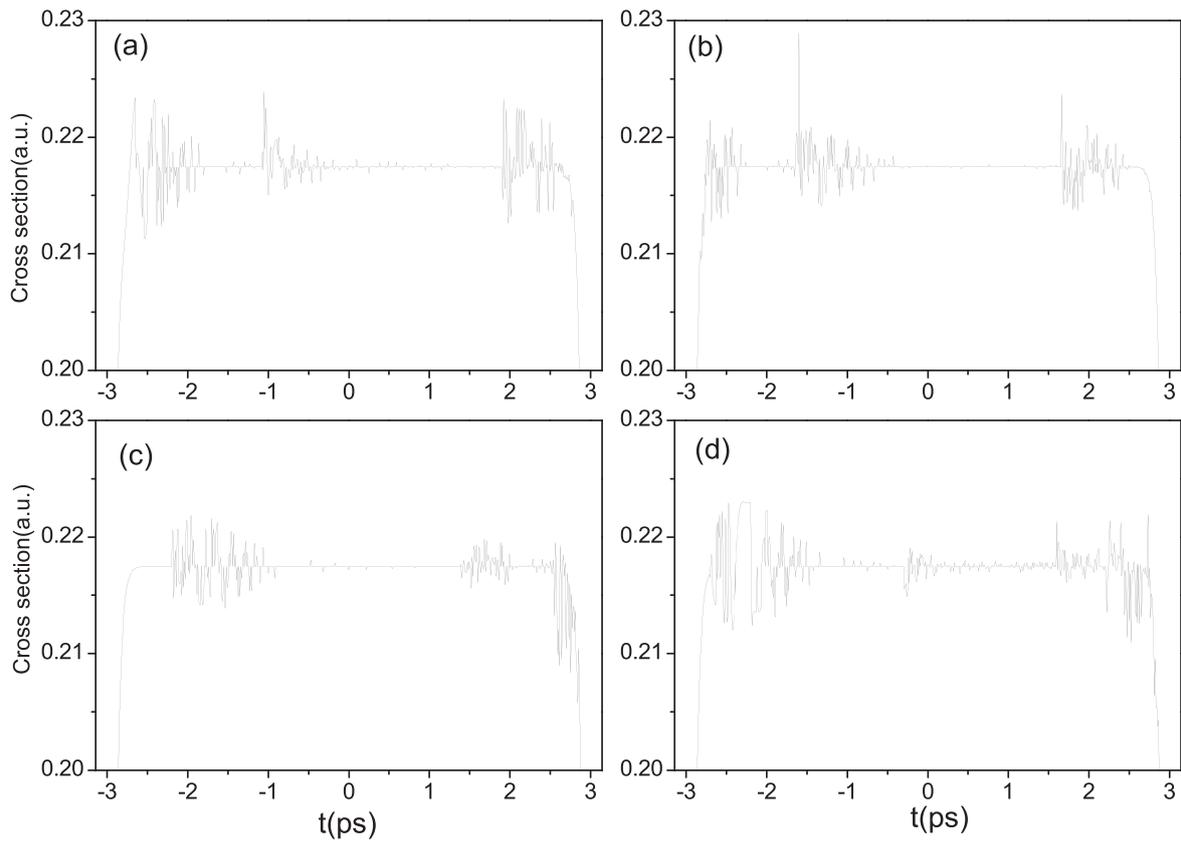


**Figure 7.** Dependence of the  $t-t_i$  map for the detached electron's closed orbit on the initial phase in the few-cycle laser field. Suppose  $A_0 = 0.5$ ,  $\omega = 3.63 \times 10^{-5}$  a.u.,  $N = 2$ . The initial phase in the laser field are as follows: (a)  $\alpha = \pi/4$ ; (b)  $\alpha = \pi/2$ ; (c)  $\alpha = 3\pi/4$ ; (d)  $\alpha = \pi$ . Different types of closed orbit are denoted by different lines.

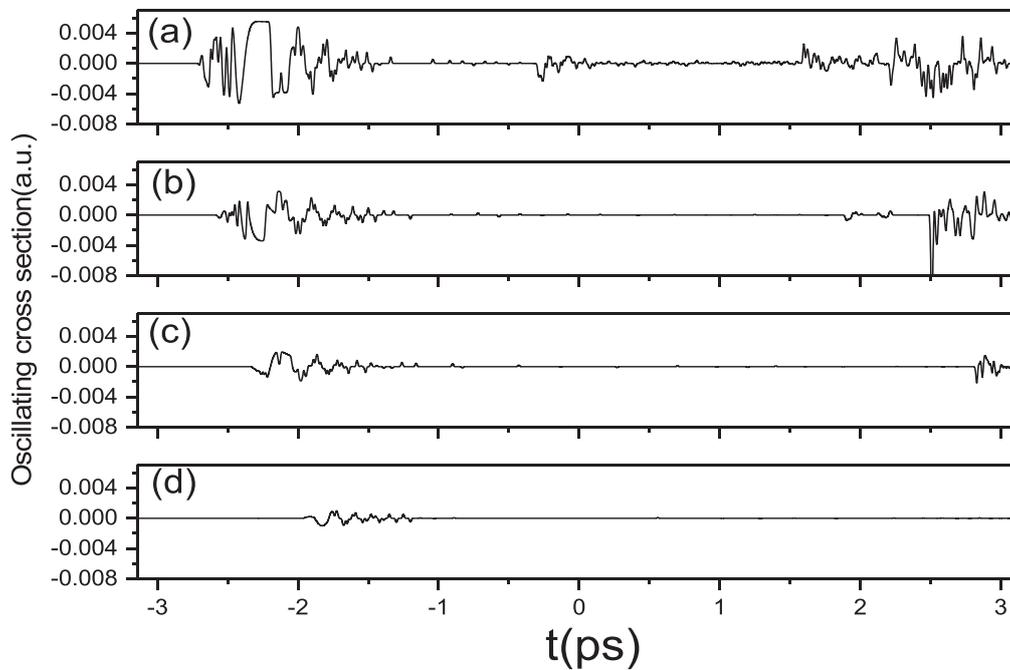
Figure 8 shows the dependence of the photodetachment cross section of  $\text{Cl}^-$  ion on the initial phase in the laser field. It is found as the initial phase  $0 < \alpha \leq \pi/2$  (figures 8(a) and (b)), the oscillation in the photodetachment cross section is mainly caused by the up closed orbit, followed by the down orbit. However, as  $\pi/2 < \alpha \leq \pi$ , the contribution of the down closed orbit to the photodetachment cross section plays the main role, followed by the up closed orbit, as we show in figures 8(c) and (d). The contribution of the other types of closed orbit only has a small modulation on the total cross section. In addition, although the phase difference shown in figures 8(d) and 3(b) is  $\pi$ , the photodetachment cross section is totally the same.

Finally, we discuss how the optical cycle  $N$  in the laser field affects the photodetachment cross section of  $\text{Cl}^-$  ion. Suppose the peak value of the vector potential  $A_0 = 0.5$  a.u., the frequency of the laser field  $\omega = 3.63 \times 10^{-5}$  a.u. and the initial phase  $\alpha = 0$ . The results are shown in figure 9. For clarity, we only calculate the oscillating part  $\sigma_{osc}(E, t)$  in the photodetachment cross section. Figure 9(a) is the oscillating cross section with the optical cycle  $N = 2$ . We find the oscillatory structures are expanded in a large region, and the cross section gets rather complicate. As we increase the

optical cycle  $N$ , the laser field can diminish the oscillatory structures in the oscillating cross section. For example, in figure 9(b),  $N = 3$ , the oscillatory structures in the middle region is nearly disappeared. As  $N = 5$  (figure 9(d)), the oscillatory structure in the cross section is only limited in a small region, and the oscillating amplitude in the cross section gets decreased. The reason is as follows: with the increase of the optical cycle  $N$ , the oscillating amplitude in the vector potential  $A(t)$  becomes decreased. Correspondingly, the laser field intensity becomes weakened. As the electron emitted from the ion source, it will take quite a long time to return back to the origin. Thus the evolution time of each type of closed orbit becomes longer. From equation (29), we find the evolution time  $\tau = t - t_i$  plays an important role in the semiclassical amplitude. The longer the evolution time is, the smaller the semiclassical amplitude becomes. Therefore, the interference effect between the returning electron wave with the initial outgoing wave gets weakened with the increase of the optical cycle  $N$ . This figure suggests that the optical cycle  $N$  in the laser field is an important parameter for controlling the photodetachment of negative ion.



**Figure 8.** Dependence of the time-dependent photodetachment cross section of  $\text{Cl}^-$  ion on the initial phase in the laser field. Suppose  $A_0 = 0.5$ ,  $\omega = 3.63 \times 10^{-5}$  a.u.,  $N = 2$ . The initial phase in the laser field are as follows: (a)  $\alpha = \pi/4$ ; (b)  $\alpha = \pi/2$ ; (c)  $\alpha = 3\pi/4$ ; (d)  $\alpha = \pi$ .



**Figure 9.** Dependence of the oscillating photodetachment cross section of  $\text{Cl}^-$  ion on the optical cycle  $N$  in the laser field. Suppose  $A_0 = 0.5$ ,  $\omega = 3.63 \times 10^{-5}$  a.u.,  $\alpha = 0$ . The optical cycle  $N$  in the laser field are as follows: (a)  $N = 2$ ; (b)  $N = 3$ ; (c)  $N = 4$ ; (d)  $N = 5$ .

## 5. Conclusions

We have extended the time-dependent COT to study the photodetachment of  $\text{Cl}^-$  ion in the few-cycle laser field. Firstly, we put forward an analytical formula for calculating the time-dependent photodetachment cross section of  $\text{Cl}^-$  ion in the laser field. This formula is suitable to all the halide negative ions. Then we found out all the closed orbit of the detached electron by solving the Hamiltonian canonical equations. The calculation results suggest that the photodetachment cross section of  $\text{Cl}^-$  ion in the laser field is highly sensitive to variation in (i) the peak value of the vector potential  $A_0$ , (ii) the frequency  $\omega$ , (iii) the initial phase  $\alpha$ , and (iv) the optical cycle of the laser field  $N$ . Calculated photodetachment cross section predicts the oscillatory behavior expected from quantum interference effects between the returning electron wave with the initial outgoing wave. It has been noted that as the peak value of the vector potential increases at fixed frequency and initial phase, the strength of the laser field gets increased, the period of the detached electron's closed orbit becomes shortened, the interference effect of the returning electron wave with the initial outgoing wave gets stronger, which makes the oscillatory structure in the photodetachment cross section becomes much more complicated. On the other hand, if we increase the frequency of the laser field, the number of the closed orbit increased, and the photodetachment cross section becomes complex as well. Analysis of the  $t-t_i$  map of the detached electron reveals a remarkably correlation with the initial phase in the laser field. It has been observed that the photodetachment cross section for the initial phase  $\alpha$  is identical to that for  $\pi - \alpha$ . In addition, the photodetachment cross section of this system is also found to be sensitive to variation of the optical cycle number  $N$  in the laser field. In this paper, we considered the  $\text{Cl}^-$  ion as a single electron system and did not consider the re-scattering effect. Future directions for applications and extensions include extending this model to incorporate the re-scattering mechanism in order to consider the high-energy regime. It would also be very interesting to extend this method to study the photodetachment of other halide negative ions, such as  $\text{F}^-$  ion,  $\text{Br}^-$  ion, etc This work provides a general framework for the experimentalists to coherent control of the photodetachment dynamics of negative ion in combing with a few-cycle laser field.

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