 

Commun. Theor. Phys. 72 (2020) 025101 (8pp) https://doi.org/10.1088/1572-9494/ab6182

# Complementarity between micro-micro and micro-macro entanglement in a Bose–Einstein condensate with two Rydberg impurities

**Zhen Li, Yu Han and Le-Man Kuang**[1]

Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education, Department of Physics and Synergetic Innovation Center for Quantum Effects and Applications, Hunan Normal University, Changsha 410081, China

E-mail: lmkuang@hunnu.edu.cn

Received 4 October 2019, revised 10 November 2019
Accepted for publication 15 November 2019
Published 22 January 2020

**Abstract**

We theoretically study complementarity between micro-micro and micro-macro entanglement in a Bose–Einstein condensate with two Rydberg impurities. We investigate quantum dynamics of micro-micro and micro-macro entanglement in the micro-macro system. It is found that strong micro-macro entanglement between Rydberg impurities and the BEC can be generated by the use of initial micro-micro entanglement between two Rydberg impurities, which acts as the seed entanglement to create micro-macro entanglement. We demonstrate a curious complementarity relation between micro-micro and micro-macro entanglement, and find that the complementarity property can be sustained to some extent even though in the presence of the BEC decoherence.

Keywords: complementarity, micro-macro entanglement, micro-micro entanglement, a Bose–Einstein condensate, Rydberg impurities

(Some figures may appear in colour only in the online journal)

## 1. Introduction

Quantum entanglement is a distinctive feature of quantum mechanics that lies at the core of many new quantum applications in the emerging science of quantum information. In particular, it is an open challenge for fundamental physics that reveal the quantum entanglement of a microscopic-macroscopic system [1–14]. This endeavor could contribute to challenge the observability of quantum features at the macroscopic level, which is one of the very fascinating open problems in quantum physics. The difficulties inherent in such a question are manifold, and they are related not only to quantum decoherence induced by the surrounding environment [15–25], but also to a measurement precision sufficient to observe quantum effects at such macroscales.

On the other hand, complementarity is one of the most characteristic properties of quantum mechanics [26–28], which distinguishes the quantum world from the classical one. Quantum entanglement in composite quantum systems is a uniquely quantum resource with no classical counterpart. An interesting problem to ask is whether quantum entanglement can be incorporated into the principle of complementarity. Some authors have explored this question and obtained some interesting results, such as the complementarities between distinguishability and entanglement [29], between coherence and entanglement [30], and between local and nonlocal information [31], etc. Additionally, some complementarity relations in multi-qubit pure systems are also observed, such as the relationships between multipartite entanglement and mixedness [32, 33], and between the single-particle properties and the $n$ bipartite entanglements [34]. In this paper, we add the complementarity between micro-micro and micro-macro entanglement into the complementarity list on quantum mechanics.

A Bose–Einstein condensate (BEC) with Rydberg impurities [35–39] presents a totally new platform to study micro-micro and micro-macro entanglement where microscopic impurities meet a macroscopic matter, the BEC. As the interaction among Rydberg atoms can be tailored by electric fields and microwave fields [40, 41] while the BEC allows for an extremely

[1] Author to whom any correspondence should be addressed.

precise control of interatomic interactions by manipulating $s$-wave scattering length [42, 43], they provide a new tool for many-body quantum physics of hybrid quantum systems [44], consisting of microscopic-macroscopic systems.

In this paper, we study complementarity between micro-micro and micro-macro entanglement in a Bose–Einstein condensate with two Rydberg impurities. We investigate quantum dynamics of micro-micro and micro-macro entanglement in the impurities-doped BEC system. It is found that strong micro-macro entanglement between Rydberg impurities and the BEC can be generated by the use of initial micro-micro entanglement between two Rydberg impurities, which acts as the seed entanglement to create micro-macro entanglement. It is shown that the micro-macro system under our consideration exhibits not only micro-micro entanglement collapse and revival (ECR) between two Rydberg impurities, but also micro-macro ECR between Rydberg impurities and the BEC. We demonstrate a curious complementarity relation between the micro-micro and micro-macro entanglement, and find that micro-micro entanglement can perfectly transfer into the micro-macro entanglement.

The remainder of this paper is organized as follows. In section 2, we introduce the impurities-doped BEC model consisting of the BEC and two Rydberg impurities. We obtain an analytical solution of the impurities-doped BEC model. In section 3, we investigate quantum dynamics of micro-micro entanglement between two Rydberg impurities, and the influence of the BEC decoherece. It is shown that the ECR of micro-micro entanglement between two Rydberg impurities exists and the BEC decoherence suppresses the revival of micro-micro entanglement. In section 4, we demonstrate the complementarity between micro-micro and micro-macro entanglement by calculating micro-macro entanglement, and obtain an exact complementarity between micro-micro and micro-macro entanglement. It is indicated that the complementarity property can be sustained to some extent even though in the presence of the BEC decoherence. Finally, section 5 is devoted to some concluding remarks.

## 2. Model Hamiltonian

The microscopic-macroscopic system under our consideration consists of a BEC and two localized Rydberg impurity atoms immersed in the BEC, as shown in figure 1. The two separated Rydberg impurities are frozen in place and they interact with each other via a repulsive van der Waals interaction [40, 41]. The relevant internal level structure for each Rydberg atom is given by the atomic ground state $|0\rangle$ and the excited Rydberg state $|1\rangle$, which form a two-level system. The Hamiltonian of two Rydberg impurities in the absence of the external laser field [41] is given by

$$H_R = \frac{\omega}{2}(\sigma_z^1 + \sigma_z^2) + J\sigma_z^1\sigma_z^2, \tag{1}$$

where $\omega$ is the transition frequency between two internal states of each Rydberg impurity atom, the second term accounts for the van der Waals interaction between the Rydberg impurities with the coupling strength $J = C_6/R^6$ where $R$ is the distance

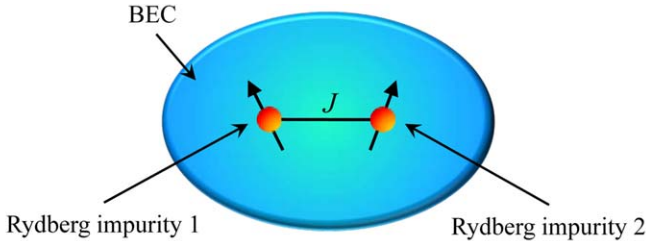


**Figure 1.** Schematic diagram of two Rydberg impurities immersed in a Bose-Einstein condensate. Here $J$ denotes the van der Waals interaction between two Rydberg impurities.

between two localized Rydberg impurities, $C_6 \propto \bar{n}^{11}$ with $\bar{n}$ being the principal quantum number of the Rydberg excitation. We have set $\hbar = 1$ in Hamiltonian (1) and throughout the paper.

The Hamiltonian of a BEC confined in a trapping potential is given by

$$H_B = \int dx \Psi^\dagger(x)\left[-\frac{1}{2m}\nabla^2 + V(x) + \frac{U}{2}\Psi^\dagger(x)\Psi(x)\right]\Psi(x), \tag{2}$$

where $\Psi(x)$ is the annihilator field operator of the BEC at the position $x$, $V(x)$ is the external trapping potential, $U$ is the inter-atomic interaction strength, and $m$ is the mass of an atom. Assuming that the BEC is trapped in a deep potential, we can use the single-mode approximation $\Psi(x) \approx a\,\phi(x)$ to describe the BEC with $a$ and $\phi(x)$ being the annihilation operator and the mode function of the condensate, respectively. Then, the Hamiltonian of the BEC (2) can be written as the following Kerr-interaction form

$$H_B = \omega_b a^\dagger a + \chi a^\dagger a^\dagger aa, \tag{3}$$

where the mode frequency $\omega_b$ and the coupling constant $\chi$ are defined by

$$\omega_b = \int dx\left[-\frac{1}{2m}|\nabla\phi(x)|^2 + V(x)|\phi(x)|^2\right], \tag{4}$$

$$\chi = \frac{U}{2}\int dx|\phi(x)|^4. \tag{5}$$

The two Rydberg impurities interact with the BEC via coherent collisions. The impurity-BEC interaction Hamiltonian can be described as

$$H_I = \frac{\lambda}{2}(\sigma_z^1 + \sigma_z^2)a^\dagger a, \tag{6}$$

and $\lambda$ is the interaction strength.

Hence, the Hamiltonian of the total system including the two Rydberg impurities and the BEC is given by

$$H = H_R + H_B + H_I, \tag{7}$$

which is a diagonal Hamiltonian with the following eigenvalues and eigenstates

$$\begin{aligned} E_{ijn} = {} & \frac{1}{2}\omega[(-1)^i + (-1)^j] + (-1)^{i+j}J \\ & + \frac{1}{2}\lambda[(-1)^i + (-1)^j]n + \omega_b n + \chi n(n-1), \end{aligned} \tag{8}$$

$$|\psi\rangle_{ijn} = |ijn\rangle, \tag{9}$$

where $|ijn\rangle = |i\rangle \otimes |j\rangle \otimes |n\rangle$ with $|i\rangle(|j\rangle)$ is an eigenstate of $\sigma_z^i(\sigma_z^j)$ with $i = 0,1$ $(j = 0,1)$, and $|n\rangle$ is a Fock state with $n = 0, 1, 2,\cdots,\infty$.

In what follows, we will use the total Hamiltonian (7) to study quantum dynamics of micro-micro and micro-macro entanglement in the impurity-doped BEC system. We will demonstrate the complementarity between micro-micro and micro-macro entanglement, and uncover the physical mechanism behind the micro-micro and micro-macro complementarity in the dynamical evolution.

## 3. Micro-micro entanglement between two Rydberg impurities

In this section, we want to explore dynamical characteristics of micro-micro entanglement between two Rydberg impurities and show the existence of entanglement collapse and revival (ECR) phenomenon for the two Rydberg impurities in the dynamical evolution. Concretely, we investigate micro-micro entanglement between the two Rydberg impurities when the two Rydberg impurities are initially in a Bell-type state while the BEC is initially in a coherent state and an arbitrary pure state, respectively.

### *3.1. The coherent state case*

We assume that the two Rydberg impurities initially are a Bell-type state $(\cos\theta|00\rangle + \sin\theta|11\rangle)$ with $|0\rangle$ and $|1\rangle$ denoting the ground state and excited Rydberg state of each Rydberg impurity, respectively, and the initial state of the BEC is a coherent state $|\alpha\rangle = \exp(-|\alpha|^2/2)\sum_{n=0}^{\infty}\alpha^n/\sqrt{n!}\,|n\rangle$. Then, the initial state of the micro-macro system under our consideration can be written as

$$|\psi(0)\rangle = (\cos\theta|00\rangle + \sin\theta|11\rangle) \otimes |\alpha\rangle. \tag{10}$$

Making use of the Hamiltonian of the total system (7), we can get the state of the system at time $t \geqslant 0$

$$|\psi(t)\rangle = [\cos\theta|00\rangle \otimes |\varphi_0(t)\rangle + \sin\theta|11\rangle \otimes |\varphi_1(t)\rangle], \tag{11}$$

where $|\varphi_0(t)\rangle$ and $|\varphi_1(t)\rangle$ are two generalized coherent states

$$|\varphi_0(t)\rangle = \mathrm{e}^{-\frac{|\alpha|^2}{2}}\sum_{n=0}^{\infty}\mathrm{e}^{\mathrm{i}t\theta_0(n)}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \tag{12}$$

$$|\varphi_1(t)\rangle = \mathrm{e}^{-\frac{|\alpha|^2}{2}}\sum_{n=0}^{\infty}\mathrm{e}^{\mathrm{i}t\theta_1(n)}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \tag{13}$$

where we have introduced two running frequencies

$$\theta_0(n) = \lambda n - \chi n(n-1) + \omega - \omega_b n - J, \tag{14}$$

$$\theta_1(n) = -\lambda n - \chi n(n-1) - \omega - \omega_b n - J. \tag{15}$$

From equation (9) it is easy to get the reduced density operator of the two Rydberg impurities

$$\rho_R(t) = \cos^2\theta|00\rangle\langle 00| + \sin^2\theta|11\rangle\langle 11| + \xi(t)|00\rangle\langle 11| + \xi^*(t)|11\rangle\langle 00|, \tag{16}$$

where $\xi(t)$ is defined by

$$\xi(t) = \frac{1}{2}\sin 2\theta\exp[2\mathrm{i}\omega t - |\alpha|^2(1 - \mathrm{e}^{2\mathrm{i}\lambda t})]. \tag{17}$$

We can use quantum concurrence [45] to measure the amount of entanglement for an arbitrary quantum state of the two Rydberg impurities. The concurrence of an arbitrary quantum state of two qubits with a density operator $\rho_R(t)$ [45] is given by

$$\mathcal{C}_1 = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{18}$$

where the $\lambda_i$ $(i = 1, 2, 3, 4)$ are the square roots of the eigenvalues in descending order of the operator $R = \rho_R(t)(\sigma_y^1 \otimes \sigma_y^2)\rho_R^*(t)(\sigma_y^1 \otimes \sigma_y^2)$ with $\sigma_y$ being the Pauli operator in the computational basis. It ranges from $\mathcal{C}_1 = 0$ for a separable state to $\mathcal{C}_1 = 1$ for a maximally entangled state.

If the time-dependent density matrix of a two-qubit system can be expressed as

$$\rho(t) = \begin{pmatrix} w(t) & 0 & 0 & z(t) \\ 0 & x(t) & 0 & 0 \\ 0 & 0 & x(t) & 0 \\ z^*(t) & 0 & 0 & y(t) \end{pmatrix}, \tag{19}$$

one finds that the concurrence corresponding to this state is given by [46–48]

$$\mathcal{C}_1(t) = \max\{0, 2|z(t)| - 2x(t)\}. \tag{20}$$

For the quantum state of two Rydberg impurities $\rho_R(t)$ given by equation (16), we have $x(t) = 0$ and $z(t) = \xi(t)$. Hence we can obtain the quantum concurrence between two Rydberg impurities with the following expression

$$\mathcal{C}_1(t) = \mathcal{C}_1(0)\exp\{|\alpha|^2[\cos(2\lambda t) - 1]\}, \tag{21}$$

where $\mathcal{C}_1(0)$ is the initial quantum concurrence between two Rydberg impurities with the following expression

$$\mathcal{C}_1(0) = |\sin(2\theta)|. \tag{22}$$

From equation (21) we can see that the entanglement dynamics between two Rydberg impurities depends on the initial entanglement $\mathcal{C}_1(0)$, the initial-state parameter of the BEC, and the impurity-BEC interaction strength $\lambda$. It is independent of interaction between two Rydberg impurities $J$ and inter-atomic interaction in the BEC $\chi$. Obviously, without the initial entanglement between two Rydberg impurities, i.e. $\mathcal{C}_1(0) = 0$, entanglement dynamics does not exist between two Rydberg impurities. In fact, the initial entanglement between two Rydberg impurities $\mathcal{C}_1(0)$ is the maximal entanglement amount of two Rydberg impurities in the process of dynamic evolution. On the other hand, equation (21) indicates that the entanglement dynamics between two Rydberg impurities is periodic with the evolution period $T = \pi/\lambda$, which is determined only by the impurity-BEC coupling

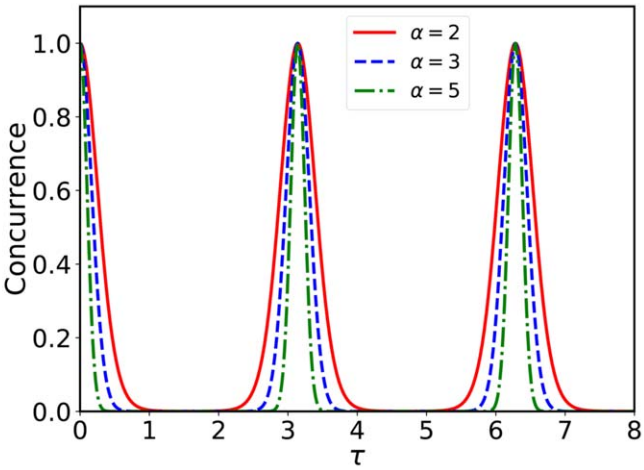


**Figure 2.** Micro-micro entanglement between two Rydberg impurities denoted by the concurrence versus the scaled time $\tau$ for different values of the initial-state parameter $\alpha$ with $\alpha = 2$, 3, and 5, when the two Rydberg impurities are initially in the Bell state with $\theta = \pi/4$.

strength $\lambda$. The stronger the impurity-BEC coupling strength is, the faster the entanglement oscillation in the dynamic evolution.

Equation (21) indicates that one can control the micro-micro entanglement dynamics between two Rydberg impurities by changing the amount of the initial micro-micro entanglement, the initial-state parameter of the BEC, and the impurity-BEC interaction strength. In figure 2, we plot the evolution of the concurrence between two Rydberg impurities with respect to the scaled time $\tau = \lambda t$ for different values of the initial-state parameter of the BEC $\alpha = 2$, 3 and 5, when the two Rydberg impurities are initially in a Bell state with $\mathcal{C}_1(0) = 1$, i.e. $\theta = \pi/4$. Figure 2 indicates that the concurrence of two Rydberg impurities exhibits periodical collapses and revivals during the time evolution. The initial concurrence can be completely revived during one evolution period. The peak value of the revivals is the initial amount of the concurrence. It is interesting to note that the collapse and revival velocity of the concurrence can be manipulated by changing the initial-state parameter of the BEC $\alpha$. From figure 2 we can see that the collapse/revial velocity of the concurrence speeds up with the increase of the initial-state parameter $\alpha$.

### *3.2. The arbitrary pure state case*

We now study quantum dynamics of the micro-micro entanglement when the initial state of the BEC is an arbitrary pure state $|\varphi(0)\rangle$ while the two Rydberg impurities initially are a Bell-type state $(\cos\theta|00\rangle + \sin\theta|11\rangle)$. Then, the initial states of the micro-macro system under our consideration can be written as

$$|\psi(0)\rangle = (\cos\theta|00\rangle + \sin\theta|11\rangle) \otimes |\varphi(0)\rangle, \tag{23}$$

Making use of the Hamiltonian of the total system (7), we can find that at time $t \geqslant 0$ the state of the system becomes

$$|\psi(t)\rangle = \cos\theta|00\rangle \otimes |\varphi_0'(t)\rangle + \sin\theta|11\rangle \otimes |\varphi_1'(t)\rangle, \tag{24}$$

where

$$|\varphi_0'(t)\rangle = \mathrm{e}^{\mathrm{i}t\theta_0'(\hat{n})}|\varphi(0)\rangle, \tag{25}$$

$$|\varphi_1'(t)\rangle = \mathrm{e}^{\mathrm{i}t\theta_1'(\hat{n})}|\varphi(0)\rangle, \tag{26}$$

where we have introduced two running frequencies

$$\theta_0'(\hat{n}) = \omega - J - (\omega_b - \lambda)\hat{n} - \chi\hat{n}(\hat{n} - 1), \tag{27}$$

$$\theta_1'(\hat{n}) = -\omega - J - (\omega_b + \lambda)\hat{n} - \chi\hat{n}(\hat{n} - 1). \tag{28}$$

From equation (24), it is easy to obtain the reduced density operator of two Rydberg impurities at time $t$

$$\begin{aligned}\rho_R'(t) = &\cos^2\theta|00\rangle\langle 00| + \sin^2\theta|11\rangle\langle 11| \\ &+ \xi'(t)|00\rangle\langle 11| + \xi'^*(t)|11\rangle\langle 00|,\end{aligned} \tag{29}$$

where $\xi'(t)$ is defined by

$$\xi'(t) = \frac{1}{2}\sin 2\theta\langle\varphi_1'(t)|\varphi_0'(t)\rangle. \tag{30}$$

The reduced density operator of two Rydberg impurities (29) can be expressed as the form of equation (19) with $x(t) = 0$ and $z(t) = \xi'(t)$. Hence from equation (20) we can obtain the quantum concurrence of the quantum state $\rho_R'(t)$ with the following expression,

$$\begin{aligned}\mathcal{C}_1(t) &= \mathcal{C}_1(0)|\langle\varphi_1'(t)|\varphi_0'(t)\rangle|, \\ &= \mathcal{C}_1(0)|\langle\varphi(0)|\exp(2\mathrm{i}t\lambda\hat{n})|\varphi(0)\rangle|,\end{aligned} \tag{31}$$

where we have used the following orthogonal relation

$$\langle\varphi_1'(t)|\varphi_0'(t)\rangle = \langle\varphi(0)|\exp[2\mathrm{i}t(\omega + \lambda\hat{n})]|\varphi(0)\rangle. \tag{32}$$

From equation (31), we can see that the entanglement dynamics between two Rydberg impurities depends on the initial entanglement $\mathcal{C}_1(0)$, the initial state of the BEC $|\varphi(0)\rangle$, and the impurity-BEC interaction strength $\lambda$, even though the BEC is initially an arbitrary pure state.

### *3.3. The decoherence influence*

No quantum system is totally isolated with respect to its environment. Interactions between a quantum system and its environment cause quantum decoherence. Here we take into account the influence of the BEC decoherence on entanglement dynamics of two Rydberg impurities. The dynamic evolution of the density operator of totally system can be described by the zero-temperature master equation in the Born-Markov approximation

$$\dot{\rho} = -\frac{\mathrm{i}}{\hbar}[H, \rho] + \mathcal{L}(\rho), \tag{33}$$

where the superoperator $\mathcal{L}(\rho)$ is given by

$$\mathcal{L}(\rho) = \kappa\left(a\rho a^\dagger - \frac{1}{2}a^\dagger a\rho - \frac{1}{2}\rho a^\dagger a\right), \tag{34}$$

where $\kappa$ is the decay factor of the BEC, and $a$ is the BEC annihilation operator.

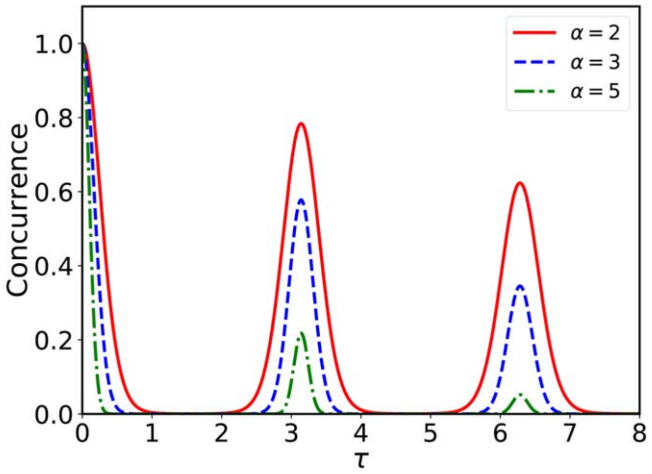


**Figure 3.** Two Rydberg atom concurrence versus the scaled time $\tau$ with different BEC initial coherent state parameter $\alpha = 2$, 3, and 5. The BEC decoherence rates is assumed $\kappa = 0.02$.

We now numerically study the influence of BEC decoherence on entanglement dynamics of two Rydberg impurities by the use of the master equation (33). We focus on the investigation of effects of the BEC decoherence with the assistance of the Qu-TIP SoftWare Package [49]. In figure 3 we plot the dynamic evolution of the quantum concurrence of two Rydberg impurities with respect to the scaled time $\tau$ when the BEC decay factor $\kappa = 0.02$, and two Rydberg impurities are initially in the Bell state $(|00\rangle + |11\rangle)/\sqrt{2}$ while the BEC is initially in a coherent state $|\alpha\rangle$ for different values of the initial-state parameter of the BEC $\alpha = 2$, 3, and 5, respectively. From figure 3 we can see that the decoherence suppresses concurrence revivals and degrades the peaks of the concurrence in the dynamic evolution. The larger the value of the initial-state parameter of the BEC, the faster the concurrence peak decay. Comparing figure 3 with figure 2, we can find that decoherence does not change the positions of the concurrence peaks, i.e. the time points at which the concurrence peaks appear in the processes of the time evolution for different the initial-state values of the BEC. However, the concurrence revivals will asymptotically disappear in the long time evolution.

## 4. Complementarity relation between micro-macro and micro-micro entanglement

In this section, we study the complementarity between micro-macro and micro-micro entanglement in the Rydberg impurities-doped BEC system during the dynamic evolution. To do so, we need to calculate micro-macro entanglement between two Rydberg impurities and the BEC. In what follows, we will demonstrate that an exact complementarity relation exists between micro-micro and micro-macro entanglement when two Rydberg impurities are initially in a Bell-type state while the BEC is initially in a coherent state and an arbitrary pure state, respectively.

### *4.1. The coherent state case*

When the two Rydberg impurities are initially in a Bell-type state $(\cos\theta|00\rangle + \sin\theta|11\rangle)$ and the BEC is initially in a coherent state $|\alpha\rangle$, at time $t$ the quantum state of the Rydberg impurities-doped BEC is given by equation (11), which is a micro-macro entangled state with two components. For a entangled pure state consisting of two components in the following form

$$|\psi(t)\rangle = \frac{1}{N}[\mu|\eta\rangle \otimes |\gamma\rangle + \nu|\xi\rangle \otimes |\delta\rangle], \tag{35}$$

where $|\eta\rangle$ and $|\xi\rangle$ are normalized states of the first subsystem, $|\gamma\rangle$ and $|\delta\rangle$ are normalized states of the second subsystem with complex $\mu$ and $\nu$. The normalization constant $N$ is given by

$$N = \sqrt{|\mu|^2 + |\nu|^2 + 2\mathrm{Re}(\mu^*\nu p_1 p_2^*)}, \tag{36}$$

where we have introduced two overlapping functions

$$p_1 = \langle\eta|\xi\rangle, \quad p_2 = \langle\delta|\gamma\rangle. \tag{37}$$

For the two-component entangled state given by equation (35), the quantum concurrence is given by [23, 50]

$$\mathcal{C}_2 = \frac{2|\mu||\nu|}{N^2}\sqrt{(1 - |p_1|^2)(1 - |p_2|^2)}. \tag{38}$$

For the micro-macro entangled state (11), we have $N = 1$, $\mu = \cos\theta$ and $\nu = \sin\theta$, two overlapping functions are given by

$$p_1 = 0, \quad p_2 = \exp\{2\mathrm{i}\omega t + |\alpha|^2(\mathrm{e}^{2\mathrm{i}\lambda t} - 1)\}. \tag{39}$$

Making use of equation (38) and (39), we can obtain the concurrence of the micro-macro entangled state (11) with the following expression

$$\mathcal{C}_2(t) = \mathcal{C}_1(0)\sqrt{1 - \exp\{2|\alpha|^2[\cos(2\lambda t) - 1]\}}, \tag{40}$$

where $\mathcal{C}_1(0)$ is the initial micro-micro entanglement between two Rydberg impurities given by equation (21).

Combining the concurrence of the micro-micro entanglement given by equation (21) and the expression of the micro-macro entanglement given by equation (40), we can find the following complementarity relation between the micro-micro and the micro-macro entanglement

$$\mathcal{C}_1^2(t) + \mathcal{C}_2^2(t) = \mathcal{C}_1^2(0). \tag{41}$$

Equation (41) indicates entanglement transfer between impurities and impurities-BEC. The decreasing of $\mathcal{C}_1(t)$ will cause the increasing of $\mathcal{C}_2(t)$. Inversely, the increasing of $\mathcal{C}_1(t)$ will cause the decreasing of $\mathcal{C}_2(t)$.

The concurrence between Rydberg impurities and BEC is shown in figure 4 for different initial-state parameters of the BEC $\alpha$. Comparing figure 2 with figure 4, one can easily find the entanglement between Rydberg impurities and BEC will strengthen when the entanglement of two Rydberg impurities becoming weak even almost disappear. Then, the entanglement of two Rydberg impurities will revive along with time evolution. At the same time, entanglement between Rydberg impurities and BEC will weaken until it almost disappears. This phenomenon is a good description entanglement transfer

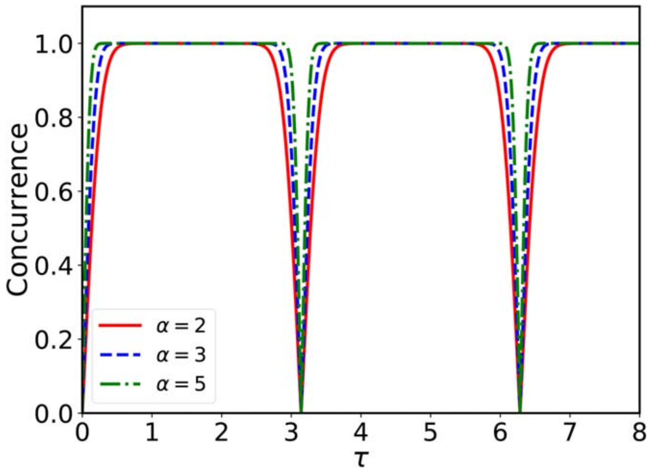


**Figure 4.** Micro-macro entanglement between two Rydberg impurities and the BEC denoted by the concurrence versus the scaled time $\tau$ for different values of the initial-state parameter $\alpha$ with $\alpha = 2$, 3, and 5, when the two Rydberg impurities are initially in the Bell state with $\theta = \pi/4$.

between two Rydberg impurities and Rydberg impurities-BEC. The transfer of entanglement indicated that the quantum coherence or information transfer between two Rydberg impurities and Rydberg impurities-BEC.

In addition, no matter what we adjust $\lambda$ or $\alpha$, the concurrence of two Rydberg impurities will return to the initial value after entanglement collapse for different coupling strength $\lambda$ or parameter $\alpha$. The reason for this phenomenon is that we do not consider decoherence of the system.

### 4.2. The arbitrary pure state case

When the two Rydberg impurities initially are a Bell-type state $(\cos\theta|00\rangle + \sin\theta|11\rangle)$ and the BEC is initially in an arbitrary pure state $|\varphi(0)\rangle$, at time $t$ the quantum state of the Rydberg impurities-doped BEC is given by equation (24), which is also a micro-macro entangled state with two components in the form of equation (35).

Making use of equation (35)–(37), for the micro-macro entangled state (24), we obtain $N = 1$, $\mu = \cos\theta$ and $\nu = \sin\theta$, and two overlapping functions given by

$$p_1 = 0, \quad p_2 = \langle\varphi_0'(t)|\varphi_1'(t)\rangle. \tag{42}$$

Hence, it is easy to get the concurrence of the micro-macro entangled state (24)

$$C_{Mi-Ma} = C_{Mi-Mi}(0)\sqrt{1 - |\langle\varphi_0'(t)|\varphi_1'(t)\rangle|^2}, \tag{43}$$

where $C_{Mi-Mi}(0) = |\sin(2\theta)|$ is the initial micro-micro entanglement of the Rydberg impurities-doped BEC system.

Combining the concurrence of the micro-micro entanglement given by equation (31) and the expression of the micro-macro entanglement given by equation (43), we can find the following complementarity relation between the micro-micro entanglement and the micro-macro entanglement

$$\mathcal{C}^2_{Mi-Ma}(t) + \mathcal{C}^2_{Mi-Mi}(t) = \mathcal{C}^2_{Mi-Mi}(0). \tag{44}$$

Above complementarity relation between the micro-micro and micro-macro entanglement is the key result obtained in the present paper. Equation (44) indicates not only the curious complementarity relation between the micro-micro and micro-macro entanglement, but also reveals the physical mechanism for the generation of the micro-macro entanglement. From equation (44) we can see that the initial micro-micro entanglement $\mathcal{C}^2_{Mi-Mi}(0)$ is the seed entanglement to produce the micro-macro entanglement under our consideration. Without the initial seed entanglement, the micro-macro entanglement cannot be generated in the system. On the other hand, from equation (44) we also see that the micro-macro entanglement is produced through transferring from the micro-micro system of two Rydberg impurities to the micro-macro system consisting of the Rydberg impurities and the BEC. The maximal amount of the micro-macro entanglement cannot be beyond the amount of the seed entanglement in the time evolution of the dynamics.

### 4.3. The decoherence influence

We now take into account the influence of the BEC decoherence on the complementarity between micro-micro and micro-macro entanglement in the micro-macro system. In the presence of the BEC decoherence the micro-macro system is generally in a mixed state in the dynamic evolution. The characterization of entanglement for such a kind of bipartite system of large dimensions in mixed states is one of unsolved fundamental problems of quantum foundations [51]. The quantum concurrence method used in previous sections does not apply to the micro-macro system in mixed states. A simple computable measure of entanglement for high-dimension bipartite systems in mixed states is the negativity [52, 53] that can be computed effectively for any mixed state of an arbitrary bipartite system. The negativity for a bipartite system described by a density operator $\rho$ is given by the sum of the absolute values of the negative eigenvalues of the partially transposed density matrix $\rho^{\mathrm{pT}}$,

$$N = \frac{1}{2}\sum_j (|\lambda_j| - \lambda_j), \tag{45}$$

where the $\lambda_j$ is the eigenvalues of $\rho^{\mathrm{pT}}$ and for a density operator of the micro-macro system $\rho = \sum_{klmn} p_{klmn}|k\rangle\langle l| \otimes |m\rangle\langle n|$ the partial transposed is given by $\rho^{pT} = \sum_{klmn} p_{klmn}|k\rangle\langle l| \otimes |n\rangle\langle m|$.

The dynamic evolution of the total density operator of the micro-macro system under our consideration can be described by the zero-temperature master equation, equation (33), in the Born-Markov approximation. Making use of equation (33), we can obtain not only the total density operator of the micro-macro system, but also the reduced density operator of the two impurities. Then, we calculate their negativity to measure the micro-micro and micro-macro entanglement under BEC decoherence. In figure 5 we have plotted the dynamic evolution of the negativity to describe micro-macro and micro-micro entanglement with respect to the scaled time $\tau$ when two Rydberg impurities are initially in the Bell state

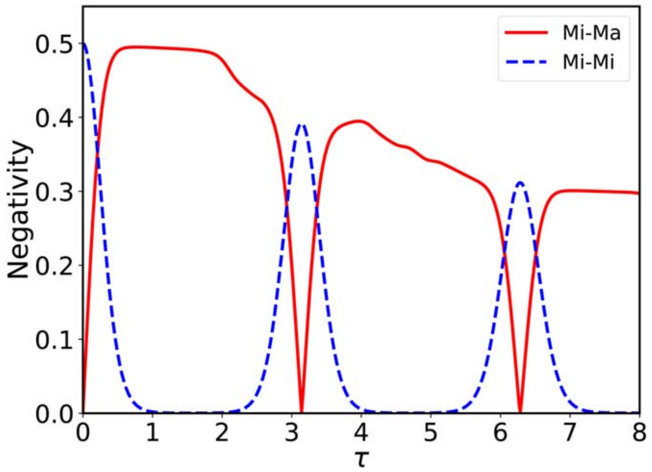


**Figure 5.** Micro-macro entanglement (red solid line) between two Rydberg impurities and the BEC denoted by the negativity versus the scaled time $\tau$, micro-micro entanglement (blue dashed line) between two Rydberg impurities denoted by the negativity versus the scaled time $\tau$, when the two Rydberg impurities is initially in the Bell state with $\theta = \pi/4$ and initial state parameter of the BEC $\alpha = 2$. The BEC decoherence rates is assumed $\kappa = 0.02$.

$(|00\rangle + |11\rangle)/\sqrt{2}$ while the BEC is initially in a coherent state $|\alpha\rangle$ with $\alpha = 2$ for the BEC decay factor $\kappa = 0.02$. In figure 5 the solid and dashed lines denote the negativity for micro-macro and micro-micro entanglement, respectively. From figure 5 we can see that the complementarity relation between the micro-micro and micro-macro entanglement in the micro-macro system is generally preserved in the time evolution process even though in the presence of the BEC decay. In particular, we can find that the peaks of the micro-micro negativity exactly correspond to the dips of the micro-macro negativity although the BEC decoherence suppresses the entanglement creation.

## 5. Conclusions

In this paper, we have studied the complementarity between micro-micro and micro-macro entanglement based on a micro-macro system, which consists of two microscopic Rydberg impurities and the macroscopic BEC. We investigate dynamics of quantum entanglement in a Bose–Einstein condensate (BEC) system with two Rydberg impurities. It is found that strong micro-macro entanglement between Rydberg impurities and the BEC can be generated by the use of initial micro-micro entanglement between two Rydberg impurities, which acts as the seed entanglement to create micro-macro entanglement. It is shown that the micro-macro system exhibits not only micro-micro entanglement collapse and revival (ECR) between two Rydberg impurities, but also micro-macro ECR between Rydberg impurities and the BEC. We point out the possibility of controlling the ECR through changing key parameters of the impurities-doped BEC system such as initial-state parameters and Rydberg impurities-BEC coupling strength, and uncover the physical mechanism behind the ECR phenomenon. It should be mentioned that this quantum ECR effect reveals the quantum nature of the BEC matter-wave field. It is similar to the quantum collapse and revival effect of the Rabi oscillations in the Jaynes–Cummings model, which reflects the quantum nature of the optical field. We demonstrate a curious complementarity relation between the micro-micro entanglement and the micro-macro entanglement, and find that micro-micro entanglement can perfectly transfer into the micro-macro entanglement. We have numerically studied the effect of the BEC decoherence on the complementarity. It is indicated that the complementarity property can be sustained to some extent even though in the presence of the BEC decay. Our results not only cast a new light on the complementarity in quantum mechanics, but also provide a route for understanding and controlling quantum entanglement in micro-macro quantum systems.

## Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grants Nos 11775075, 11434011, and 11935006.

## References

[1] Wallraff A, Schuster D I, Blais A, Frunzio L, Huang R-S, Majer J, Kumar S, Girvin S M and Schoelkopf R J 2004 *Nature* **431** 162
[2] Liao J Q, Huang J F, Liu Y X, Kuang L M and Sun C P 2009 *Phys. Rev.* A **80** 014301
[3] Aoki T, Dayan B, Wilcut E, Bowen W P, Parkins A S, Kippenberg T J, Vahala K J and Kimble H J 2006 *Nature* **443** 671
[4] Zipkes C, Palzer S, Sias C and Köhl M 2010 *Nature* **464** 388
[5] Schmid S, Härter A and Denschlag J H 2010 *Phys. Rev. Lett.* **105** 133202
[6] Gigan S, Böhm H R, Paternostro M, Blaser F, Langer G, Hertzberg J B, Schwab K C, Bäuerle D, Aspelmeyer M and Zeilinger A 2006 *Nature* **444** 67
[7] Arcizet O, Cohadon P-F, Briant T, Pinard M and Heidmann A 2006 *Nature* **444** 71
[8] Kleckner D and Bouwmeester D 2006 *Nature* **444** 75
[9] Liao J Q, Law C K, Kuang L M and Nori F 2015 *Phys. Rev.* A **92** 013822
[10] Liao J Q, Huang J F and Tian L 2016 *Phys. Rev.* A **93** 033853
[11] Liao J Q and Tian L 2016 *Phys. Rev. Lett.* **116** 163602
[12] Liao J Q, Cheung H K and Law C K 2012 *Phys. Rev.* A **85** 025803
[13] Martini F D, Sciarrino F, Vitelli C and Cataliotti F S 2010 *Phys. Rev. Lett.* **104** 050403
[14] Marshall W, Simon C, Penrose R and Bouwmeester D 2003 *Phys. Rev. Lett.* **91** 130401
[15] Bruno N, Martin A, Sekatski P, Sangouard N, Thew R T and Gisin N 2013 *Nat. Phys.* **9** 545
[16] Martini F D, Sciarrino F and Vitelli C 2008 *Phys. Rev. Lett.* **100** 253601
[17] Spagnolo N, Vitelli C, Paternostro M, Martini F D and Sciarrino F 2011 *Phys. Rev.* A **84** 032102
[18] Song Y J and Kuang L M 2019 *Ann. Phys.* **531** 1800423
[19] Yuan J B and Kuang L M 2013 *Phys. Rev.* A **87** 024101
[20] Tan Q S, Yuan J B, Jin G R and Kuang L M 2017 *Phys. Rev.* A **96** 063614
[21] Yuan J B, Xing H J, Kuang L M and Yi S 2017 *Phys. Rev.* A **95** 033610

[22] Yuan J B, Lu W J, Song Y J and Kuang L M 2017 *Sci. Rep.* **7** 7404
[23] Kuang L M and Zhou L 2003 *Phys. Rev.* A **68** 043606
[24] Kuang L M, Chen Z B and Pan J W 2007 *Phys. Rev.* A **76** 052324
[25] Peng Z H, Ding J H, Zhou Y, Ying L L, Wang Z, Zhou L, Kuang L M, Liu Y X, Astafiev O V and Tsai J S 2018 *Phys. Rev.* A **97** 063809
[26] Bohr N 1928 *Nature* **121** 580
[27] Feynman R P, Leifhton R B and Sands M 1965 *The Feynamn Lectures of Physics* vol 3 (Reading: Addison-Wesley)
[28] Scully M O, Englert B-G and Walther H 1991 *Nature* **351** 111
[29] Bose S and Home D 2002 *Phys. Rev. Lett.* **88** 050401
[30] Saleh B E A, Abouraddy A F, Sergienko A V and Teich M C 2000 *Phys. Rev.* A **62** 043816
[31] Oppenheim J, Horodecki K, Horodecki M, Horodecki P and Horodecki R 2003 *Phys. Rev.* A **68** 022307
[32] Jaeger G, Sergienko A V, Saleh B E A and Teich M C 2003 *Phys. Rev.* A **68** 022318
[33] Tessier T E 2005 *Phys. Lett.* **18** 107
[34] Peng X H, Zhu X W, Suter D, Duuter J F, Liu M and Gao K 2005 *Phys. Rev.* A **72** 052109
[35] Heidemann R, Raitzsch U, Bendkowsky V, Butscher B, Löw R and Pfau T 2008 *Phys. Rev. Lett.* **100** 033601
[36] Schmidt R, Sadeghpour H R and Demler E 2016 *Phys. Rev. Lett.* **116** 105302
[37] Camargo F *et al* 2018 *Phys. Rev. Lett.* **120** 083401
[38] Mukherjee R, Ates C, Li W and Wüster S 2015 *Phys. Rev. Lett.* **115** 040401
[39] Płodzień M, Lochead G, de Hond J, van Druten N J and Kokkelmans S 2017 *Phys. Rev.* A **95** 043606
[40] Vogt T, Viteau M, Chotia A, Zhao J, Comparat D and Pillet P 2007 *Phys. Rev. Lett.* **99** 073002
[41] Büchler H P, Micheli A and Zoller P 2007 *Nat. Phys.* **3** 726
[42] Timmermans E, Tommasini P, Hussein M and Kerman A 1999 *Phys. Rep.* **315** 199
[43] Bloch I, Dalibard J and Zwerger W 2008 *Rev. Mod. Phys.* **80** 885
[44] Smith M S 2016 *Phys. Rev. Lett.* **117** 100001
[45] Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245
[46] Yu T and Eberly J H 2007 *Quantum. Inform. Comput.* **7** 459
[47] Santos M F, Milman P, Davidovich L and Zagury N 2006 *Phys. Rev.* A **73** 040305(R)
[48] Werlang T and Rigolin G 2010 *Phys. Rev.* A **81** 044101
[49] Johansson J R, Nation P D and Nori F 2012 *Comput. Phys. Commun.* **183** 1760
Johansson J R, Nation P D and Nori F 2013 *Comput. Phys. Commun.* **184** 1234
[50] Rungta P, Bužek V, Caves C M, Hillery M and Milburn G J 2001 *Phys. Rev.* A **64** 042315
[51] Aolita L, de Melo F and Davidovich L 2015 *Rep. Prog. Phys.* **78** 042001
[52] Vidal G and Werner R F 2002 *Phys. Rev.* A **65** 032314
[53] Życzkowski K, Horodecki P, Sanpera A and Lewenstein M 1998 *Phys. Rev.* A **58** 883