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## PAPER

## Optimal phase sensitivity by quantum squeezing based on a Mach–Zehnder interferometer

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## Abstract

A novel scheme for the enhancement of phase sensitivity based on a Mach–Zehnder interferometer (MZI) and intensity detection is proposed. With the input of bright entangled twin beams from four wave mixing (FWM), the phase sensitivity can beat shot noise limit (SNL) and approach Heisenberg limit. This scheme is special due to that only one of bright entangled twin beams enters into the MZI and the other one is employed for measurement. In addition, by altering the parametric strength of FWM and the implementation of maximum quantum squeezing, the optimal phase sensitivity can reach sub-SNL. Optical intensity depletion of photon detectors and internal intensity depletion of the MZI are also discussed. The scheme displays that by employing external resources, while one input of the MZI is an vacuum beam, the phase sensitivity still can beat SNL.

## 1. Introduction

As the basis of precision measurement applications, an optical interferometer, such as a Mach–Zehnder interferometer (MZI), is the commonly used tool for the phase measurement. When the inputs are one coherent beam and one vacuum beam or two coherent beams [1, 2], the phase sensitivity based on a MZI is limited to shot noise limit (SNL)  $\frac{1}{\sqrt{N}}$ . For a given measurement scheme, the phase sensitivity can be calculated by error propagation formula. In order to get the higher phase sensitivity, a lot of research groups proposed that by employing quantum resources, the phase sensitivity can reach sub-SNL or approach Heisenberg limit (HL) [3], and the usual quantum resources contain: N00N states [4], twins Fock states [5], and squeezed states [6, 7].

Generally, the interferometers consist of three constructions. 1: the input (classical state and quantum state); 2: the interferometer scheme. For example, a MZI is made up of a beam splitter (BS); an SU(1,1) interferometer [8, 9] contains an optical parametric amplification or a four wave mixing (FWM) process [10–12] and a hybrid interferometer consists of a BS and a FWM; 3: detection method [13]. The useful detection may contain intensity sum or difference detection [14], homodyne detection [15, 16], parity measurement [17]. However, it is impossible to get the optimal phase sensitivity by employing all the detection methods. Fortunately, Braunstein and Caves introduced the quantum Fisher information (QFI) which is independent on the method [18]. For a given input state and interferometer, the ultimate precision bounds of the phase sensitivity is given by the quantum Cramer–Rao bound (QCRB) which is  $\Delta^2\phi \geq \frac{1}{F_Q}$  and  $F_Q$  is the QFI [19–25].

Even though QCRB is much easier without considering the detection method, the question is that when the detection method needs another beam, the local beam in homodyne detection, the QFI is different. It was firstly proposed by Jarzyna *et al* who want to give the QFI when the interferometer is considered with and without an external phase reference [23]. Gong *et al* found that the QFI is different in an SU(1,1) interferometer when the phase shift is in upper arm or lower arm [22]. This problem was solved by You *et al* who showed that the phase average method can amend the QFI [25]. In addition, Takeoka *et al* claimed that for the MZI, if one of the inputs

is the vacuum, one cannot beat the SNL by any nonclassical input from the other port [24]. This view also was confirmed by You *et al* on an SU(1,1) interferometer [25]. Furthermore, Takeoka *et al* thought that the view is only true when both the arms have the unknown phase. If the unknown phase is only in one arm, in fact, by using external resource at the input or detection, the phase sensitivity can beat SNL [24]. However, a detailed scheme by employing external resources for the phase sensitivity is missing. So an unique scheme based on a MZI with an vacuum beam as one input with external resources is displayed in this paper. This is special due to the employment of bright entangled twin beams with only one entering into the intensity detection.

The paper is organized as follows: section 2 introduces the scheme where the inputs are bright entangled twin beams and the detection method is intensity difference. Different from the previous schemes, only one of the twin beams enters into the MZI while the other one enters into the detector directly. In section 3, the factors which have a effect on the phase sensitivity including parametric strength and photon number are discussed. In addition, the optimal phase sensitivity can be realized with the implementation of the maximum quantum squeezing. In section 4, the intensity depletion on the sensitivity are also studied. When the detection method needs uncounted resources, the results are summarized in section 5.

## 2. Model

The transformation of a FWM is  $\hat{a}_1 = \cosh r \hat{a}_0 + \sinh r \hat{b}_0^\dagger$ , and  $\hat{b}_1 = \sinh r \hat{a}_0^\dagger + \cosh r \hat{b}_0$ .  $r$  is the parametric strength of FWM.  $\hat{a}_0$ ,  $\hat{b}_0$ ,  $\hat{a}_0^\dagger$  and  $\hat{b}_0^\dagger$  are annihilation and creation operators for the two input modes, respectively.  $\hat{a}_1$ ,  $\hat{b}_1$ ,  $\hat{a}_1^\dagger$  and  $\hat{b}_1^\dagger$  are annihilation and creation operators after FWM. For BS, the relationship is  $\hat{a}_2 = \sqrt{T} \hat{a}_1 + i\sqrt{R} \hat{b}_0$ , and  $\hat{b}_2 = i\sqrt{R} \hat{a}_1 + \sqrt{T} \hat{b}_0$ .  $T$  and  $R$  are transmissivity and reflectivity of intensities, and they are equal to  $\frac{1}{2}$ .  $\hat{a}_2$ ,  $\hat{b}_2$ ,  $\hat{a}_2^\dagger$  and  $\hat{b}_2^\dagger$  are annihilation and creation operators after BS. So the total transform of the operators in the scheme is given by

$$\begin{aligned}\hat{a}_{\text{out}} &= m_1 \hat{a}_1 + m_2 \hat{b}_0, \\ \hat{a}_{\text{out}}^\dagger &= m_3 \hat{a}_1^\dagger + m_4 \hat{b}_0^\dagger.\end{aligned}\quad (1)$$

And  $m_1 = \left(\frac{1}{2}\sqrt{T_2 T_4} e^{i\phi} - \frac{1}{2}\sqrt{T_3 T_4}\right)$ ,  $m_2 = \left(\frac{1}{2}\sqrt{T_2 T_4} i e^{i\phi} + \frac{1}{2}\sqrt{T_3 T_4}\right)$ ,  $m_3 = \left(\frac{1}{2}\sqrt{T_2 T_4} e^{-i\phi} - \frac{1}{2}\sqrt{T_3 T_4}\right)$ ,  $m_4 = \left(-\frac{1}{2}\sqrt{T_2 T_4} e^{-i\phi} - \frac{1}{2}\sqrt{T_3 T_4}\right)$ .  $\hat{a}_{\text{out}}$  and  $\hat{a}_{\text{out}}^\dagger$  are the annihilation and creation operators after the  $T_4$  from output,  $\phi$  is the phase shift. The FWM process is employed for the generation of bright entangled twin beams. After the FWM, the probe beam enters into a MZI as figure 1 shows. For the conjugate beam, it directly enters into the photon detector. The intensity of a probe beam after the gain of FWM is  $\cosh^2 rN + \sinh^2 r$ , and the intensity of a conjugate beam is  $\sinh^2 rN + \sinh^2 r$ .  $N$  is the photon number of the input coherent beam before FWM. For the input of the coherent beam before FWM, it is  $\langle \hat{a}_0 \rangle = \langle \hat{a}_0^\dagger \rangle = \sqrt{N}$ . The intensity of a probe beam after the MZI is

$$\langle I_a \rangle = \left( \frac{1}{4} T_2 T_4 + \frac{1}{4} T_3 T_4 - \frac{1}{2} T_4 \sqrt{T_2 T_3} \cos \phi \right) (\cosh^2 rN + \sinh^2 r). \quad (2)$$

With the intensity difference detection, the detection signal intensity is  $\langle I_- \rangle = \left( \frac{1}{4} T_2 T_4 + \frac{1}{4} T_3 T_4 - \frac{1}{2} T_4 \sqrt{T_2 T_3} \cos \phi \right) (\cosh^2 rN + \sinh^2 r) - T_1 (\sinh^2 rN + \sinh^2 r)$ . If the phase shift  $\phi = 0$  and there are no optical depletion in the scheme, the intensity of a probe beam after MZI is 0 which is even lower than the intensity of a conjugate beam. If  $\phi = \pi$ , the intensity is same to that of input which is larger than that of the conjugate beam.

For the definition of visibility, it is

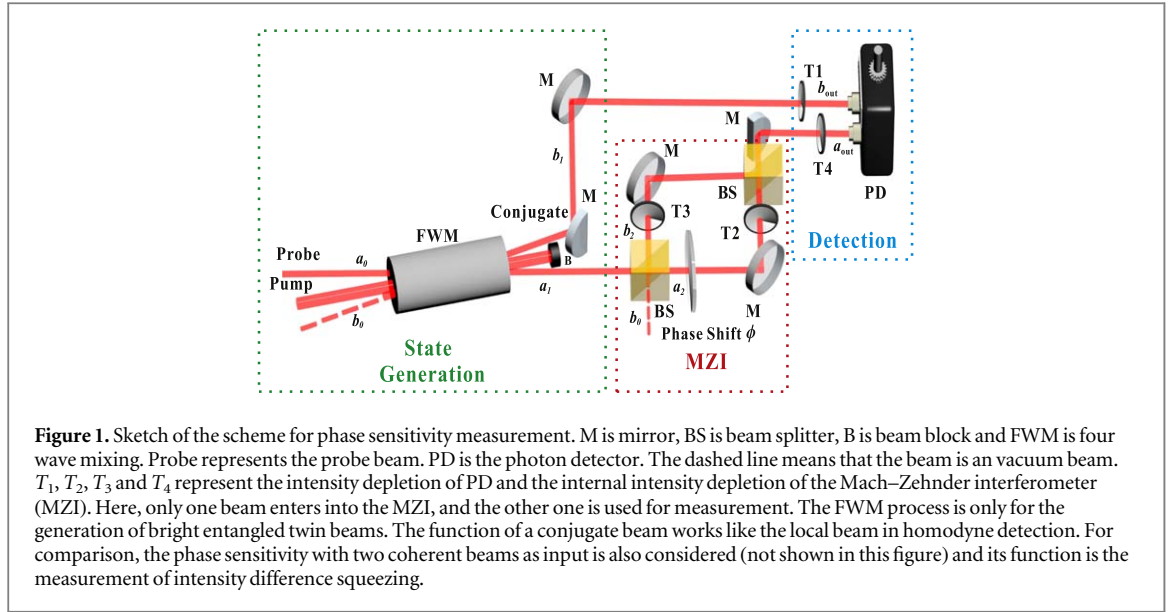
$$V = \frac{I_-^{\max} - I_-^{\min}}{I_-^{\max} + I_-^{\min}}. \quad (3)$$

$I_-^{\max}$  and  $I_-^{\min}$  is the maximum and minimum value of  $|\langle I_- \rangle|$ . So  $V = \frac{\cosh^2 rN + \sinh^2 r}{\cosh^2 rN + \sinh^2 r} = 1$  without intensity depletion, which represents that even this scheme is different from the previous interferometer, it still can realize the process of an interferometer.

The sensitivity  $\Delta \phi$  as the uncertainty in estimating a phase shift  $\phi$  is

$$\Delta \phi = \frac{\sqrt{\Delta^2 \hat{I}_-}}{|\langle \partial_\phi \hat{I}_- \rangle|}. \quad (4)$$

The variance of the measurement is  $\Delta^2 \hat{I}_- = |\langle \hat{I}_-^2 \rangle - \langle \hat{I}_- \rangle^2|$ . Take the bright entangled twin beams into calculation, the sensitivity can be written as



$$\Delta\phi_Q = \frac{\sqrt{\Delta^2 \hat{I}_{-Q}}}{|(\partial_\phi \langle \hat{I}_{-} \rangle)|_Q}. \quad (5)$$

Then

$$\left| \frac{\delta \langle \hat{I}_{-} \rangle}{\delta \phi} \right|_Q = \left| \frac{1}{2} \sqrt{T_2 T_3 T_4} \sin(\phi) (\cosh^2 r N + \sinh^2 r) \right|, \quad (6)$$

and

$$\begin{aligned} \Delta^2 \hat{I}_{-Q} = & |(m_3 m_1 \cosh^2 r - T_1 \sinh^2 r)^2 N + \sinh^2 r \cosh^2 r (m_3 m_1 - T_1)^2 N \\ & + m_2 m_3 \cosh^2 r \sinh r (m_3 m_1 - T_1) N + m_1 m_2 m_3 m_4 \cosh^2 r N \\ & + \sinh r \cosh^2 r (m_3 m_1 - T_1) m_1 m_4 N + 2 m_1 m_2 m_3 m_4 \sinh^2 r \\ & + \sinh^2 r \cosh^2 r (m_3 m_1 - T_1)^2|. \end{aligned} \quad (7)$$

Here, Q means that the input beams are quantum beams. Usually, for comparison, two coherent beams with the same intensities of the quantum beams are used to replace the probe and conjugate beams. The phase sensitivity of two coherent beams as input can be expressed as

$$\Delta\phi_C = \frac{\sqrt{\Delta^2 \hat{I}_{-C}}}{|(\partial_\phi \langle \hat{I}_{-} \rangle)|_C}. \quad (8)$$

So

$$\begin{aligned} \Delta^2 \hat{I}_{-C} = & |(m_1^2 m_3^2 + m_1 m_2 m_3 m_4) (\cosh^2 r N + \sinh^2 r) \\ & + T_1^2 (\sinh^2 r N + \sinh^2 r)|, \end{aligned} \quad (9)$$

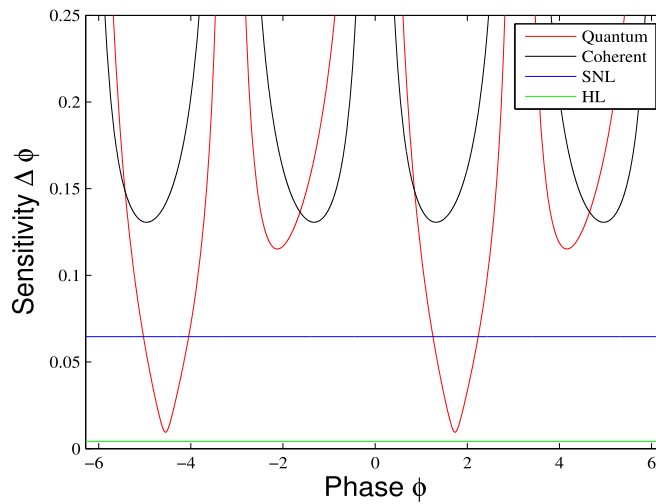
and

$$\left| \frac{\delta \langle \hat{I}_{-} \rangle}{\delta \phi} \right|_C = \left| \frac{1}{2} \sqrt{T_2 T_3 T_4} \sin(\phi) (\cosh^2 r N + \sinh^2 r) \right|. \quad (10)$$

According to equations (6) and (10), the bright entangled twin beams and two coherent beams have the same slope  $|(\partial_\phi \langle \hat{I}_{-} \rangle)|$ , which means the bright entangled twin beams can get better phase sensitivity than the two coherent beams if  $\sqrt{\Delta^2 \hat{I}_{-Q}} < \sqrt{\Delta^2 \hat{I}_{-C}}$ .

### 3. Phase sensitivity and quantum squeezing

In this section, the optimal phase sensitivity with bright entangled twin beams, two coherent beams, SNL and HL are compared. For the MZI in the scheme, the SNL is  $1/\sqrt{N_{a1}}$  and the HL is  $1/N_{a1}$  where  $N_{a1} = \cosh^2 r N + \sinh^2 r$  is the total photon number inside the interferometer. Here, the conjugate beam works like the local beam in homodyne detection and the photon number of the conjugate beam is not taken into counted. The function of the conjugate beam is also little different from the local beam. For the conjugate beam, it only provides the power. The local beam



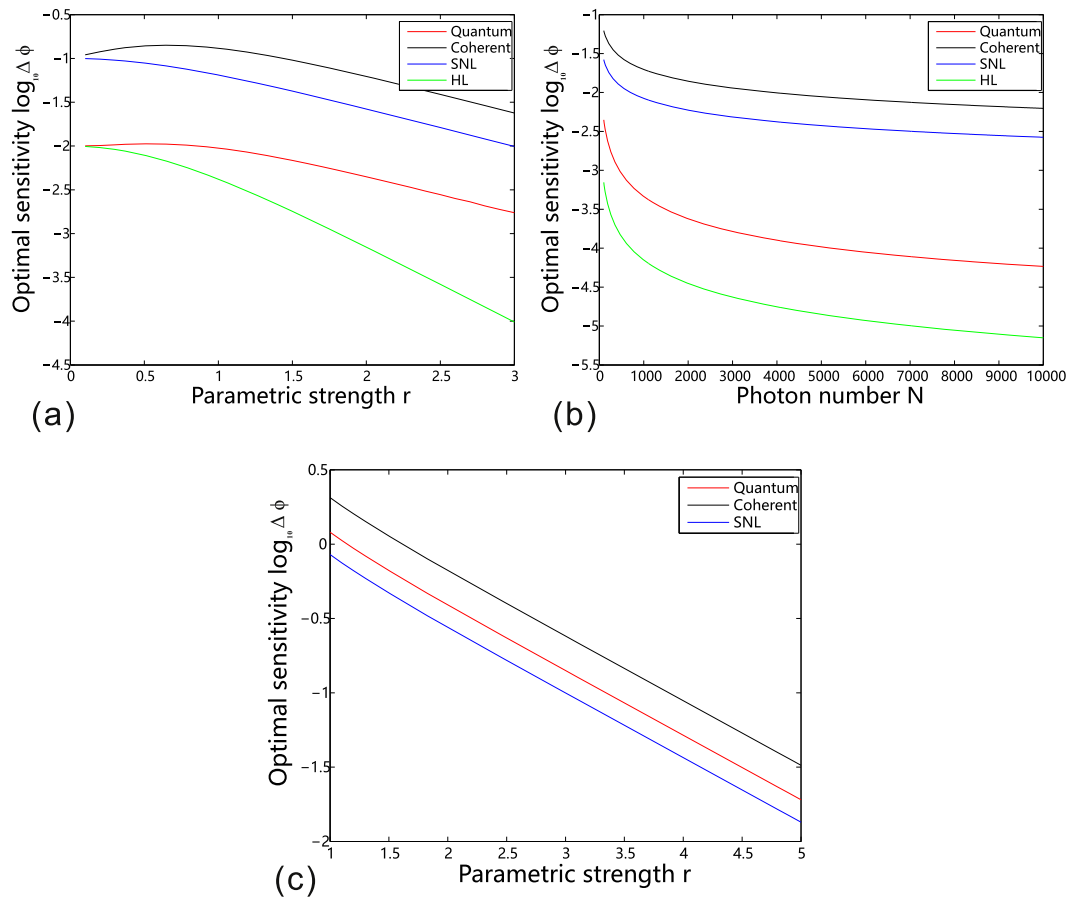
**Figure 2.** The phase sensitivity  $\Delta\phi$  of the scheme with ‘Quantum’ means the twins beams generated from four wave mixing. ‘Coherent’ means two coherent beams. ‘SNL’ and ‘HL’ mean shot noise limit and Heisenberg limit. Parametric strength  $r = 1$  and  $N = 100$  without depletion.

offers the phase reference and power. Figure 2 shows the phase sensitivity versus the phase shift. The optimal phase sensitivity with bright entangled twin beams can beat SNL and approach HL while the minimum value of phase sensitivity means the optimal phase sensitivity. For two coherent beams, it even cannot reach SNL. In addition, for the bright entangled twin beams and two coherent beams, they can get the optimal phase sensitivity at different phase points. This method in fact uses the external resources. According to [24], for a MZI, when one input is a vacuum beam, the phase shift cannot beat SNL while the other input is a quantum or classical beam. This view is only true without the external resources or two phase estimations are compared. However, this scheme is for the only one phase estimation with external resources, which does not broke the conclusion from [24]. Meanwhile, because the conjugate beam in the system works for measurement like the local beam in homodyne detection, the photon number of the external beam is not counted.

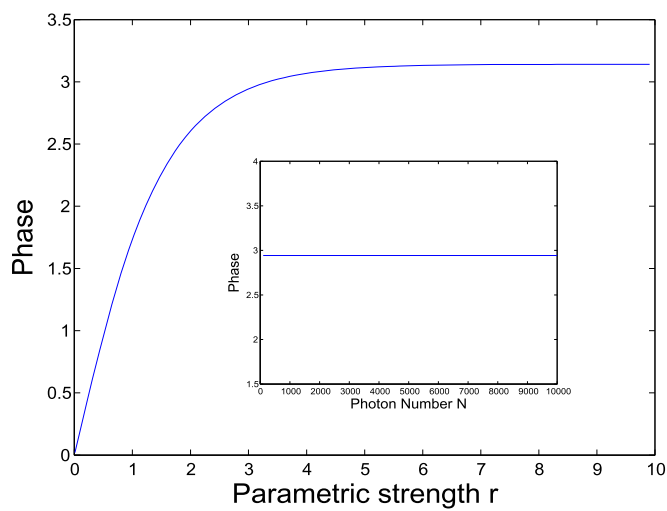
Next, the factors which have impacts on the optimal phase sensitivity are discussed. In figure 3, with the increase of parametric strength  $r$  of FWM and photon number  $N$ , the optimal sensitivities of bright entangled twin beams and two coherent beams are all better. From figure 3(a), when  $r$  is approaching 0 and  $N = 100$ , the optimal phase sensitivity of bright entangled twin beams is approaching HL while the optimal sensitivity of the two coherent beams can approach SNL. With the input of two coherent beams, only one of them enters into the interferometer and the other input is the vacuum beam, the optimal phase is worse than SNL in this case, which is corresponding to [24]. And the optimal sensitivity of bright entangled twin beams cannot reach HL while  $r$  is much bigger. However, if  $r$  is 0, the scheme will reduce to an ordinary one where only a coherent beam enters into the MZI and there are no extra resources. Then phase sensitivity will be limited to SNL. From figure 3(b), the optimal phase sensitivity of twin beams, two coherent beams, HL and SNL are all tending to be better with the increase of photon number  $N$ . If  $N = 0$ , the bright entangled twin beams will reduce to the two-mode squeezed vacuum beams. At this time, even parametric strength  $r$  varies, for both quantum and classical beams, the phase sensitivities are all worse than SNL as figure 3(c). The total photon number in the interferometer is  $N_{\text{two-mode}} = \sinh^2 r$ . Considering the photon number is more than or equal to 1, in figure 3(c), the  $r$  is assumed to be more than 1. Furthermore, the difference between figure 3(a) from figure 3(b) is that with the increase of photon number  $N$ , the phase sensitivities seem to be the constant values and it is opposite for parametric strength  $r$ .

In addition, by varying parametric strength  $r$ , the optimal phase sensitivity can be realized at different phase points which is shown as figure 4. When  $r$  is approaching 0, the optimal phase sensitivity can be realized at the point where the phase shift is approaching 0. The phase shift is approaching  $\pi$  with the increase of parametric strength. So the optimal phase sensitivity always can be realized at any point by altering parametric strength which will make the process much easier. Meanwhile, the optimal phase point is not related to photon number as shown in the inserted figure 4. In order to achieve the optimal phase sensitivity, only parametric strength  $r$  needs to be taken into account.

From equations (6) and (10), the bright entangled twin beams and coherent beams have the same  $(|\frac{\delta \langle \hat{I}_- \rangle}{\delta \phi}|)$ . So the smaller  $\Delta^2 \hat{I}_-$ , the better phase sensitivity. The definition of intensity difference squeezing according to [26] is  $\text{SD} = \frac{\text{Var}(\hat{N}_a - \hat{N}_b)_Q}{\text{Var}(\hat{N}_a - \hat{N}_b)_C}$ . Then SD can be written as

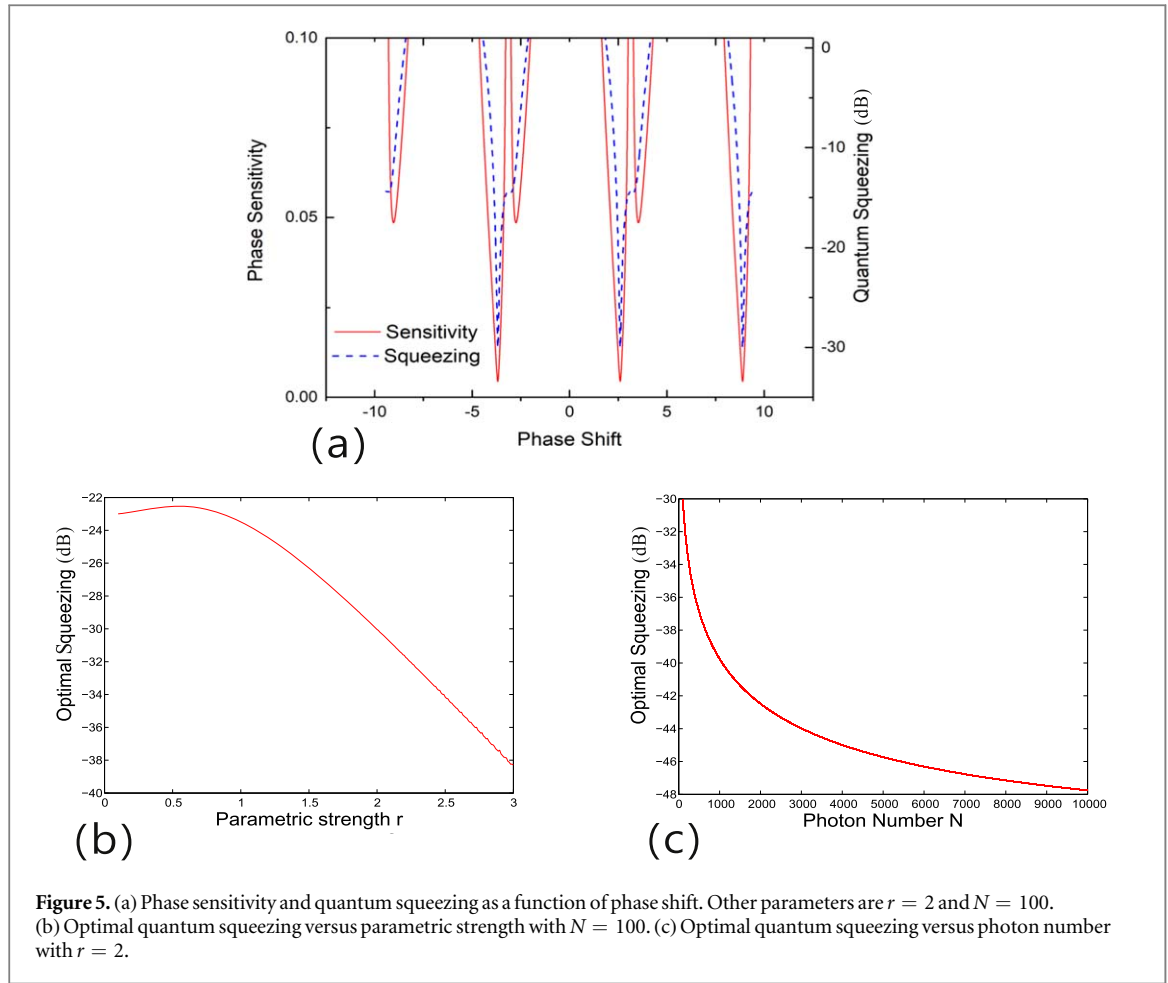


**Figure 3.** The optimal phase sensitivity with twin beams and two coherent beams versus (a) parametric strength  $r$  with photon number  $N = 100$  and (b) photon number with parametric strength  $r = 2$  and (c) parametric strength  $r$  with photon number  $N = 0$ ,  $T_1 = T_2 = T_3 = T_4 = 1$ . For better comparison,  $\log_{10} \Delta \phi$  are used to show the optimal phase sensitivity.



**Figure 4.** The phase where optimal phase sensitivity can be got versus parametric strength  $r$  with  $N = 100$ . The inset is the phase where optimal phase sensitivity is achieved versus photon number  $N$  with  $r = 3$  and there are no depletion.

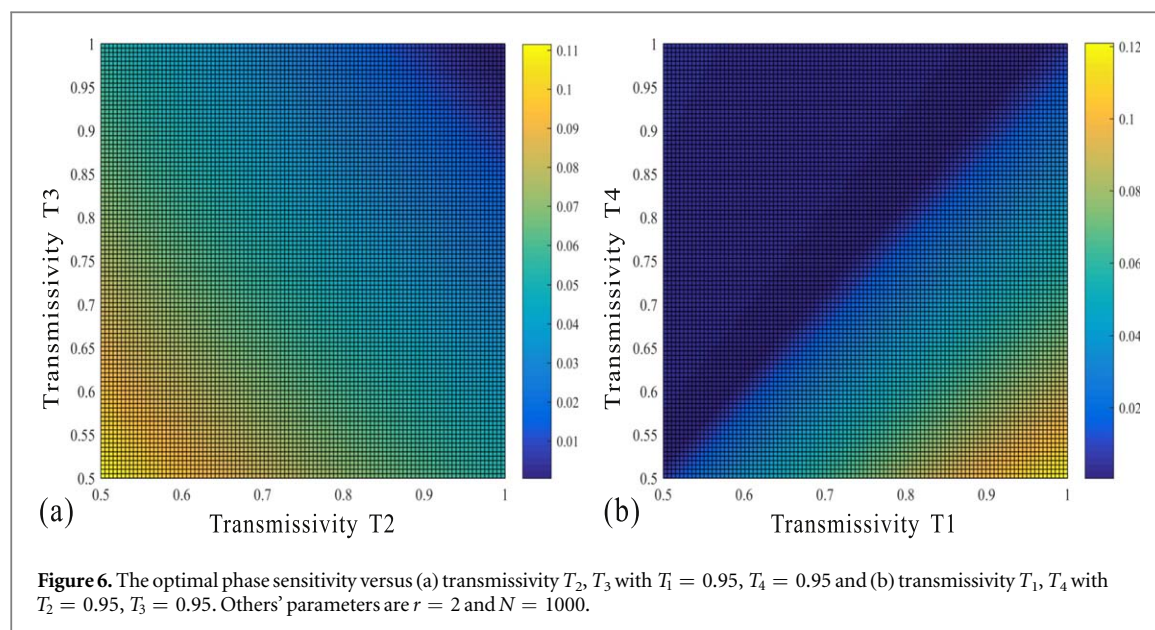
$$SD = \frac{\Delta^2 \hat{I}_{-Q}}{\Delta^2 \hat{I}_{-C}}, \quad (11)$$



which represents that, for the bright entangled twin beams, where there is quantum squeezing, there is a better phase sensitivity. The squeezing degree of intensity difference can be described as  $10\log_{10}SD$ . And the optimal squeezing exists where the phase sensitivity is the optimal. It is clear that the maximum quantum squeezing can be more than 25 dB while the optimal sensitivity is less than 0.02 according to figure 5(a). This characteristic makes this scheme adaptable and it has been employed for the measurement of plasmonic sensing [26]. Figure 5(b) shows the optimal squeezing versus  $r$ . Overall, while  $r$  becomes higher, the optimal squeezing degree is superior and can reach more than 30 dB. While the maximum squeezing in the current experiment is less than 10 dB, it is due to the depletion of FWM as [27]. In addition, as the trend of sensitivity versus parametric strength and photon number, the optimal quantum squeezing also increases with the increase of  $r$  and  $N$  which is displayed in figures 5(b) and (c).

#### 4. Depletion on the phase sensitivity

In this part, the intensity depletion which play a negative effect on the phase sensitivity are discussed. For simplification, only the condition that transmissivity is more than 0.5 are counted: the optical intensity depletion of photon detectors and the internal intensity depletion of the MZI [28, 29]. In the current technology, the efficiency of photon detectors can be more than 0.95 or even more. For the interferometer, the internal efficiency can reach more than 0.8. However, in order to better show the optimal phase sensitivity, the condition that  $T_2 = 0.5$  and  $T_3 = 0.5$  is followed. From figure 6(a), the optimal phase sensitivity can be achieved when both  $T_2$  and  $T_3$  are approaching perfect without depletion. The optimal phase sensitivity even can be better than 0.01. And the transmissivity  $T_2$  and  $T_3$  play the same roles on the sensitivity. For example, when  $T_2 = 0.6$ ,  $T_3 = 0.7$  or  $T_2 = 0.7$ ,  $T_3 = 0.6$ , the optimal phase sensitivity is the same. So even the MZI is not perfect, the phase sensitivity can be better with higher transmissivity. From figure 6(b), the phase sensitivity always can reach optimal while  $T_1 = 0.5$ . Moreover, it also can be realized while  $T_4$  approaches 1. So the higher photon detector efficiency, the better phase sensitivity. For the whole scheme and current experiment technology, the optimal phase sensitivity can be realized while the intensity depletion of the photon detector and internal intensity depletion of a MZI are less than 0.05.



## 5. Conclusion

In conclusion, this paper has shown that by the employment of quantum squeezing beams, the phase sensitivity can beat SNL and approach HL in a MZI. Different from others' interferometers, only one of twin beams enters into the MZI and the other one is employed for measurement. While the external resources are employed with the inputs being a vacuum beam and a quantum beam for a MZI, the phase sensitivity really can beat SNL and approach HL by only the implementation of maximum quantum squeezing. The role of the conjugate beam in the scheme is similar to the local beam in homodyne detection and the photon number of a conjugate beam is also not counted when the SNL and HL are employed for contrast. Moreover, the intensity depletion of photon detectors and internal intensity depletion of the MZI are discussed and the phase sensitivity still can be optimal while the intensity depletion in a MZI are same. Due to the easy realization and function of boost from twin beams, this scheme offers a new method for the enhancement of phase sensitivity and will be very useful in LIGO, VIRGO and many other precision measurements, such as the remote sensing and enhanced quantum imaging [30, 31].

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