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# Research of the relay-linear control of an electro-hydraulic servo drive on FESTO learning system

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**Abstract.** The task of a relay-linear control algorithm for an electro-hydraulic servo drive (EHSD) was considered. This algorithm provides reconfiguration of the state components feedback coefficients at the very initial stage of the control object movement, when the optimized control action does not depend on these coefficients. A FESTO hydraulic learning system combined with Matlab+Simulink Desktop Real Time was used for realization of the developed algorithm. Experimental results are included.

## 1. Introduction

One of the classical control theory task for a servo system is to follow out some reference signal that changes in time stepwise (some square wave for example). This task can be solved via several methods: as some relay law according to the L.S. Pontryagin maximum principle [1–8], or as some state feedback law [9]. There are various methods of relay-linear control [10–13]. A control law in such methods is a phase state function. The function describes some relay control law that changes during control object motion to some state feedback law nearby an origin of phase coordinates to exclude self-oscillations. N.A. Kuznetsov proposed to use specific EHSD relay-linear control law in his article [14]. The control law (that defines a hydraulic servo valve (HSV) control signal) changes from a relay law to state components feedback when a phase point of controlled hydraulic drive reachs specially built switching surface. Besides that, a relay law consists of a single constant-sign time-interval segment during an initial phase of EHSD step response. Figure 1 shows experimental results for such relay-linear control of the EHSD assembled on a hydraulic research system. (The research system based on the hydraulic learning system FESTO is described in [15]).

An experimental series have been produced to analyze the EHSD step response for a given step values. The EHSD reactions have been compared for the case of a state components feedback [9,15] and for the case of relay-linear control by the method [14]. It turned out that the feedback coefficients could be configured so that the relay-linear control can not significantly improve a quality of the obtained transients (a transient time minimization was considered). This can be explained by the fact that in this case the relay portion of the relay-linear control (Fig.1) coincides with the saturation zone of the control signal supplied to the HSV of the EHSD in the proportional feedback control case.

Figure 2 shows a block-diagram of a simple nonlinear model of the EHSD open loop (control object), realized on the hydraulic research system [15]. In the given bock-diagram the speed and position saturation of a HSV spool and a saturation of the control signal to the HSV are considered.

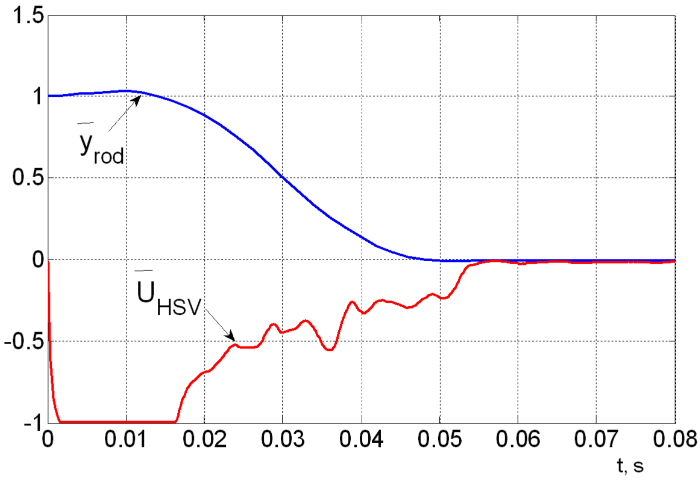


**Fig. 1.** The EHSD step response and hydraulic servo valve control signal (the signals are normalized, a required step input is M=0.2V) in a case of relay-linear control method described in [14].

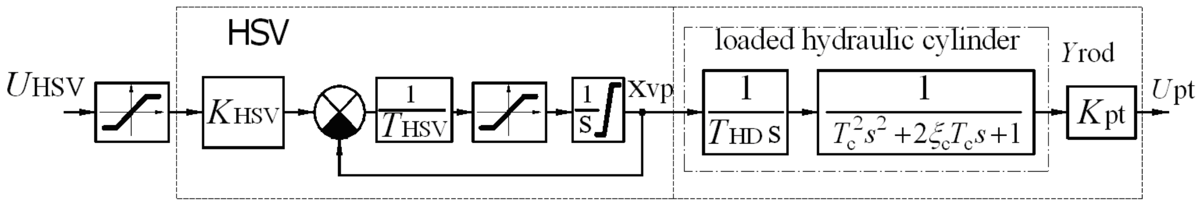


**Fig. 2.** The EHSD mathematical model block-diagram: $K_{HSV}$ — the HSV gain; $T_{HSV}$ — the HSV time constant; $T_{HD}$ — the hydraulic drive time constant; $T_c$ — the time constant of the loaded hydraulic cylinder; $\xi_c$ — the hydraulic cylinder damping coefficient; $K_{pt}$ — the potentiometric feedback sensor gain; $Y_{rod}$ — the position of the hydraulic cylinder rod; $U_{pt}$ — the rod position feedback signal.

A study on the mathematical model (Fig.2) for the state components feedback law and for a time-dependent relay control law was produced. The same ITAE criterion was used for obtaining the state components feedback coefficients and for defining the switching times of the control signal to the HSV respectively. The results of the numerically researches are shown in Figure 3.

In this paper, we consider a case in which the EHSD parameters determined by its inertial load are not known in advance and the drive with state components feedback follows some required position step input. The EHSD parameters are unchanged during the transition time. The idea of the synthesized controller is based on the two features. Firstly, as a rule, a HSV control signal enters a saturation zone in an initial phase of the EHSD working out the step in our case. In the initial phase an EHSD phase trajectory is determined by its parameters only, including the load's parameters. Secondly, the EHSD parameters determine optimized coefficients of the state components feedback. Therefore, we will first apply to the HSV the maximum possible control action (according to a relay law), with simultaneous analysis of the motion of the actuator in the phase space. As a result of this analysis, the feedback coefficients, optimal for the current EHSD parameters, are selected in real time and the control is switched from relay to linear with the selected coefficients. In common case, the drive may continue to move in the HSV control signal saturation mode after switching to linear feedback control and will come out of saturation mode naturally. That is, we do not determine the operating time of the controller in the saturation mode. We use this time to determine and set during it the optimal state feedback coefficients, depending on the current parameters of the control object.

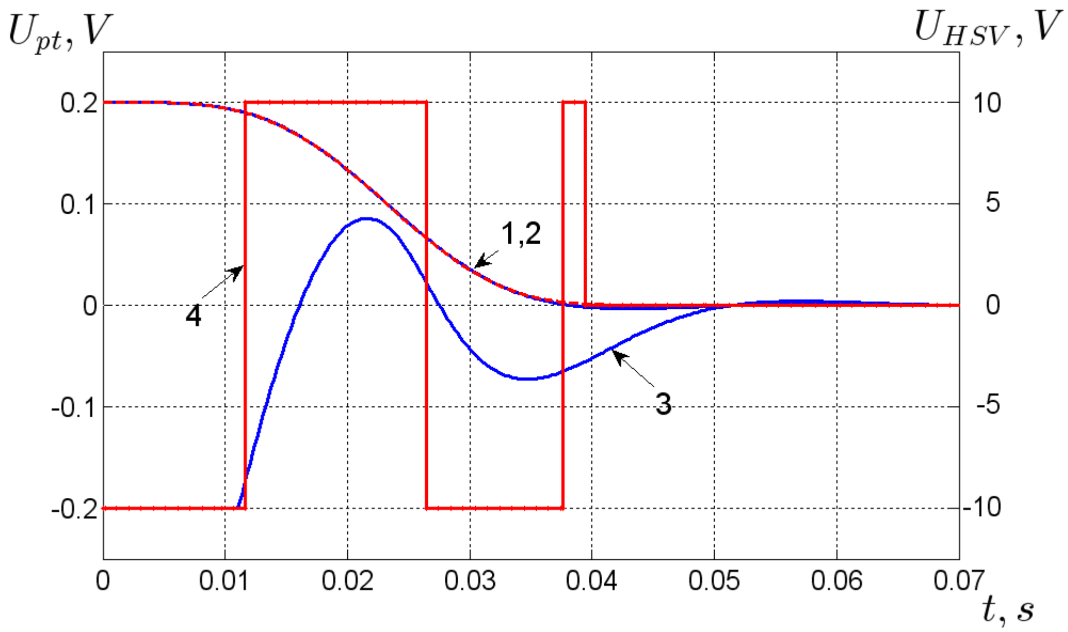


**Fig. 3.** The EHSD transients $U_{pt}$ (the lines 1,2) and corresponding to them normalized control signals $\overline{U}_{\text{HSV}}$ (the lines 3,4), optimized according to ITAE criterion, for the case of the state components feedback control (the lines 1,3) and for the case of the time-dependent relay control (the lines 2,4).

**Relay-linear control realization on FESTO hydraulic learning system**

Research of the developed relay-linear control were carried out on the EHSD physical model [15] and on its mathematical model. A scheme of the research system — the EHSD physical model (Fig. 4) is shown in Figure 5.

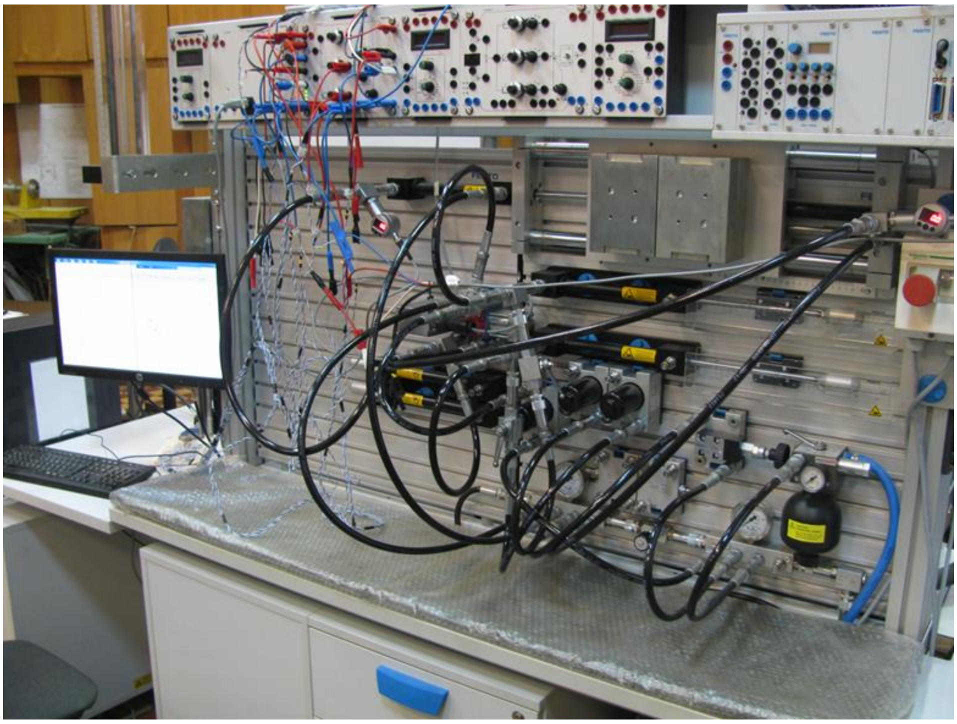

**Fig. 4.**The hydraulic research system.

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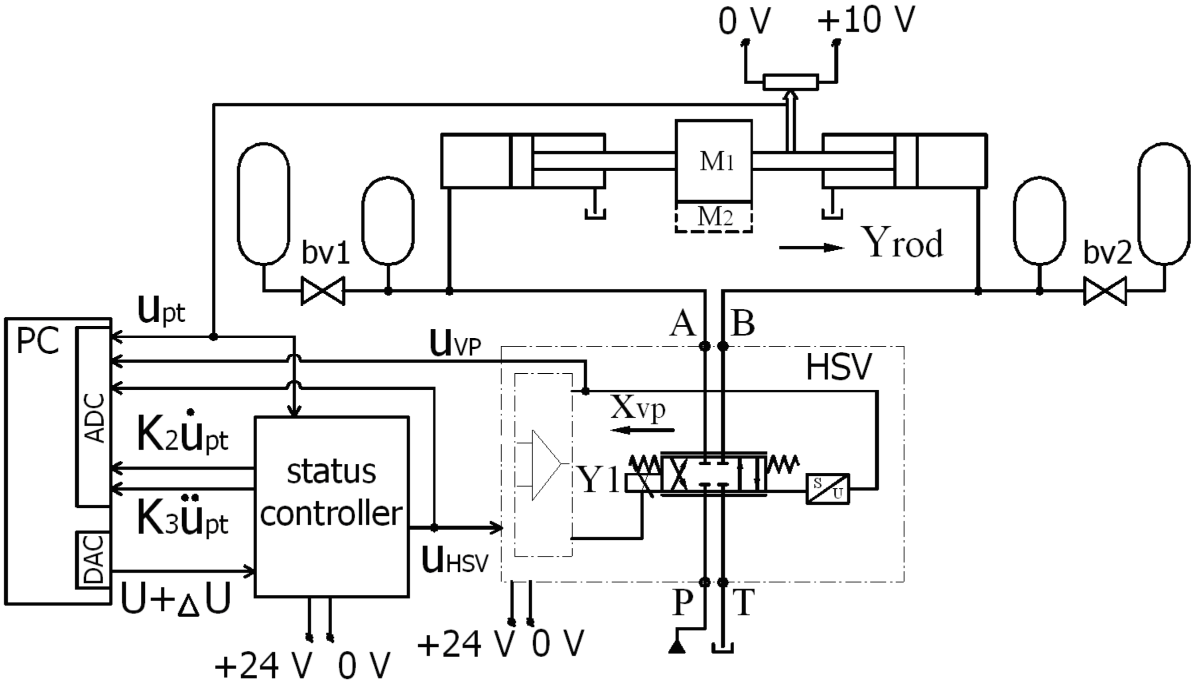


**Fig. 5.** The scheme of the research system for studying the EHSD with relay-linear control: bv1, bv2 — the ball valves; $U_{\text{HSV}}$ — the HSV control signal (in effect a control piston setpoint position value); $\text{U}_{\text{VP}}$ — the measured position of the HSV control piston; $M_1 = 27.5\,\text{kg}$, $M_2 = 10\,\text{kg}$, — the masses of the inertial load (main and additional, respectively). Status controller — the unit, shortly described below.

"Heavy" and "Light" EHSD physical models (that are named as H and L respectively) are realized on FESTO hydraulic research system. Two models require different optimized feedback coefficients while working out some step signal from a given range of amplitudes. The state components feedback controller of the investigated EHSD is realized by the unit (Part Number 162253), which is a part of FESTO hydraulic learning system. Its feedback coefficients are set manually using the unit potentiometers (Fig.6).

The studied relay-linear control algorithm implies a change in the state components feedback coefficients during the EHSD operation. To do this, an external computer produce dynamic adjustment of the control signal U, providing the required change in the state components feedback coefficients. An additive $\Delta U$ to the signal *U,* supplied to the input of the status controller, (Fig. 7) is calculated as a function of:

$$\Delta U = f_{\Delta U}\left(U, U_{pt}, \dot{U}_{pt}, \ddot{U}_{pt}, {}^{H}\bar{K}, \bar{K}\right).$$

Let us call a proposed control principle (Fig. 7) as a state components feedback with dynamic correction. State components feedback coefficients $\left[{}^{H}K_1 \quad {}^{H}K_2 \quad {}^{H}K_3\right]$ are tuned on the status controller as time-invariable set (fig.6, 7). Let's write the following form for the status controller:

$$U_{HSV} = f\left(U + \Delta U, U_{pt}\right) =$$

$$= \text{K}_{\text{amp}} \cdot {}^{H}\text{K}_1 \cdot \left[\text{U} + \Delta\text{U}\right] - \text{K}_{\text{amp}} \cdot \left({}^{H}\text{K}_1 + {}^{H}\text{K}_2 \cdot s + {}^{H}\text{K}_3 \cdot s^2\right) \cdot U_{pt}, (1)$$

where $\Delta U$ — is an additive to the U signal supplied to the input of the status controller (fig.6,7). The aim of the $\Delta U$ adding is to obtain the required control law with state components feedback:

$$U_{HSV} \overset{\Delta}{=} K_{amp} \cdot K_1 \cdot U - K_{amp} \cdot \left(K_1 + K_2 \cdot s + K_3 \cdot s^2\right) \cdot U_{pt} \quad (2)$$

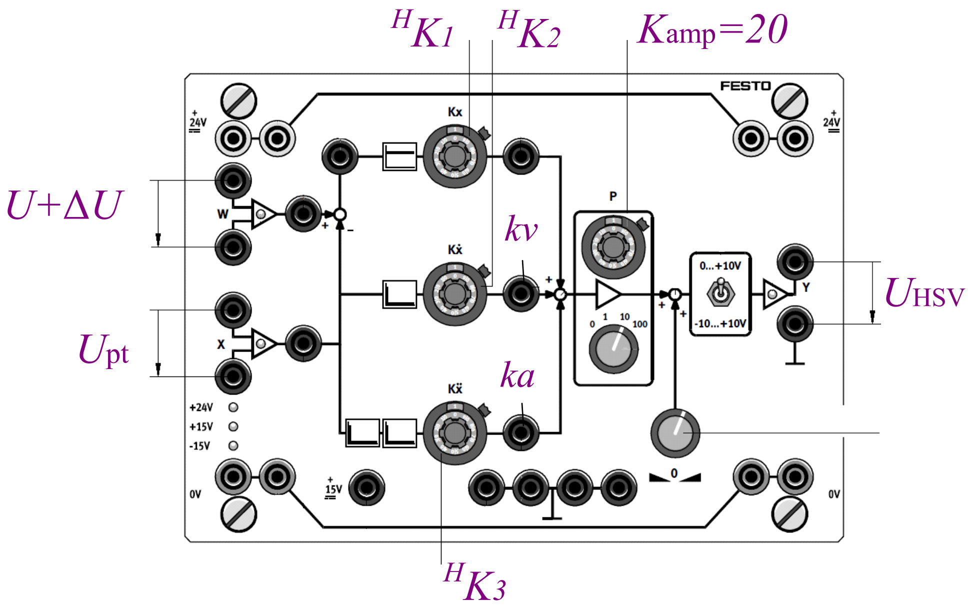


**Fig. 6.**Status controller front panel.

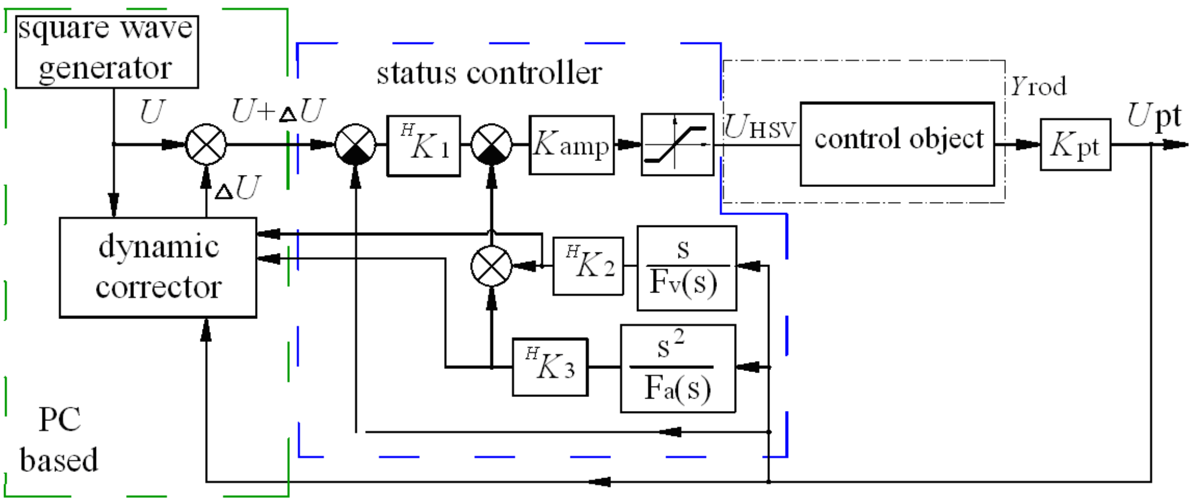


**Fig. 7.** A block scheme shows an adjustment of the state components feedback coefficients by the control signal dynamic correction: $F_v(s), F_a(s)$ — the low pass filters for derivative and double derivative computing.

The required law of $\Delta U$ is determined by the equation:

$$[\Delta U] = \frac{\left(K_1 - {}^{H}K_1\right) \cdot U + \left[\left({}^{H}K_1 - K_1\right) + \left({}^{H}K_2 - K_2\right) \cdot s + \left({}^{H}K_3 - K_3\right) \cdot s^2\right] \cdot U_{pt}}{{}^{H}K_1} \quad (3)$$

Let's introduce notations: $kv = {}^{H}\mathrm{K}_2 \cdot \dot{U}_{pt}$, $ka = {}^{H}\mathrm{K}_3 \cdot \ddot{U}_{pt}$ — signals on the corresponding outputs of the status controller unit (Fig. 6), and we get the expression for the additive $\Delta U$:

$$[\Delta U] = \begin{bmatrix} \dfrac{{}^{H}\mathrm{K}_1 - \mathrm{K}_1}{{}^{H}\mathrm{K}_1} & \dfrac{{}^{H}\mathrm{K}_2 - \mathrm{K}_2}{{}^{H}\mathrm{K}_1} & \dfrac{{}^{H}\mathrm{K}_3 - \mathrm{K}_3}{{}^{H}\mathrm{K}_1} \end{bmatrix} \cdot \begin{bmatrix} U_{pt} - U \\ \dfrac{kv}{{}^{H}K_2} \\ \dfrac{ka}{{}^{H}K_3} \end{bmatrix} \quad (4)$$

Figure 8 shows experimental transients of the hydraulic cylinder rod and the signals supplied to the input of the status controller unit when working out the step action U=0.7 V in two modes of the EHSD controller operation. The first mode: the coefficients of the state components feedback loops K =[1.64 0.00325 0.00016] are set by the potentiometers on the status controller unit. The second mode: ${}^{H}$K =[1.58 0.0154 0.000353] are set the same way on the unit and the resulting feedback coefficients K are obtained by adding signal $\Delta U$ (4) to the input signal U. The input of the status controller is their sum.

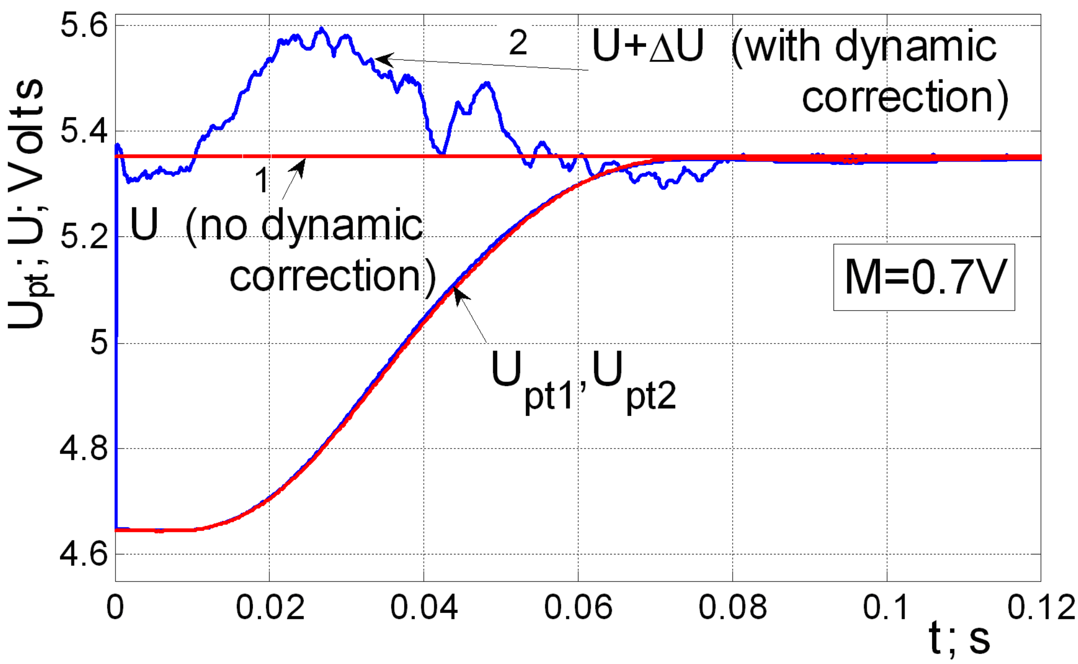


**Fig. 8.** The EHSD 0.7V step response and status controller input signal for the two cases: 1 — NO dynamic correction, 2 — WITH the dynamic correction (Figure shows experimental results for the EHSD physical L — model).

The resulting EHSD block-diagram with synthesized relay-linear controller is shown in Figure 9.

Figure 10 shows typical control signal U (Fig. 7) during some step change of desirable position of the hydraulic cylinder rod (a square wave with superimposed relay control). At time $t_1$ the control object phase-point reaches a switching hypersurface. The hypersurface defines switching from relay to linear control. The state components feedback coefficients are assigned as a result of location analysis of the point. Also, at time $t_1$ the feedback control is realized with selected coefficients by using the status controller with the dynamic correction (Fig.7).

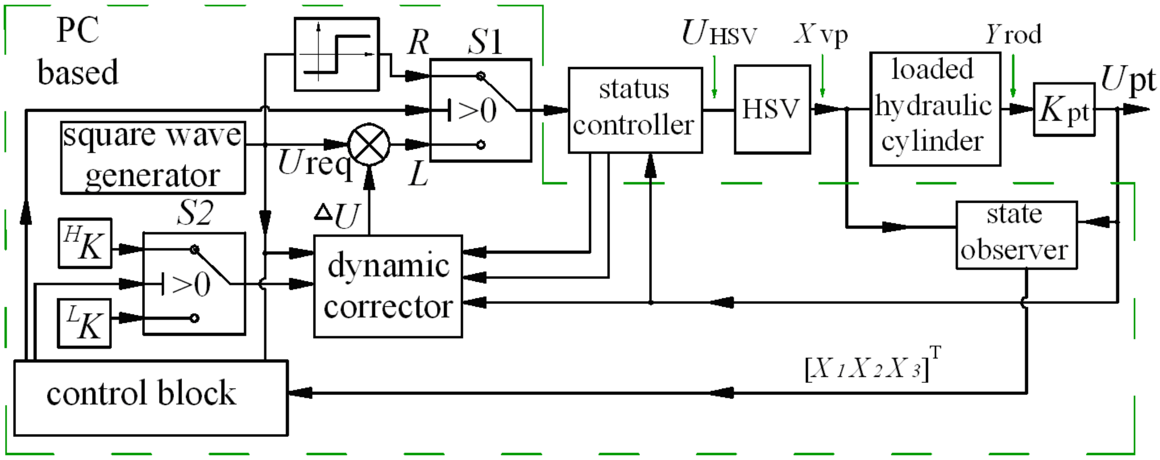


**Fig. 9.**The EHSD block-diagram with synthesized relay-linear controller.

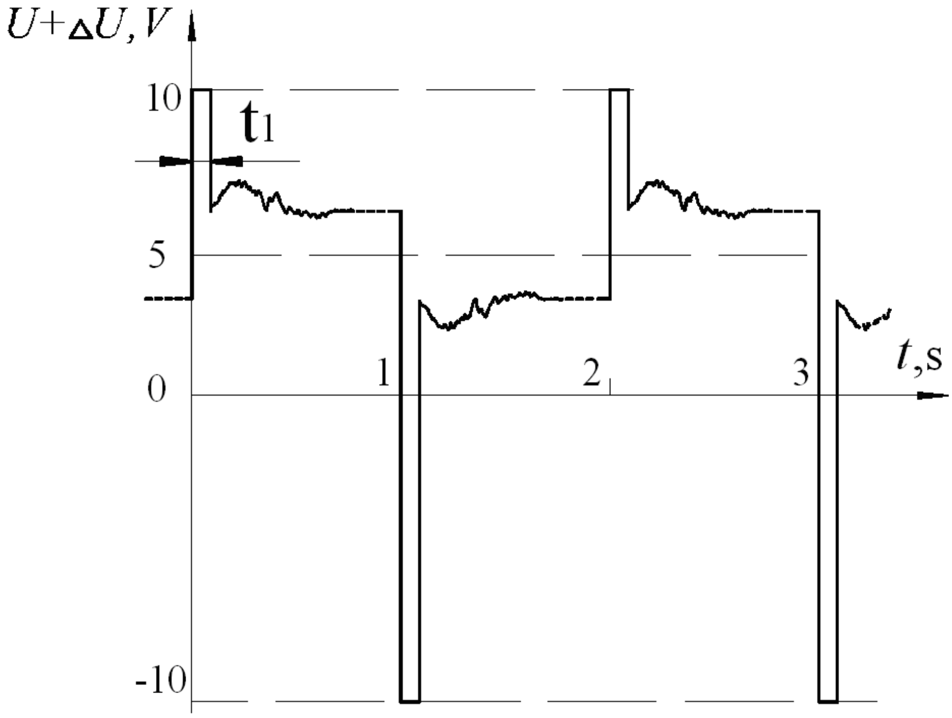


**Fig. 10.** Status controller input signal $U+\Delta U$ during relay-linear control.

**Analysis of the relay-linear control work**

We use the approach described in [15] to physically simulate the operation of loaded hydraulic drives on FESTO hydraulic learning system. Two physical models with different parameters were formed for the experimental study of the relay-linear control on the experimental complex described above. Next, we will denote them: L — is less loaded model and H — is more loaded. There are two differences between these physical models. Firstly, the different masses of the inertial load, moved by the hydraulic cylinders ($M_1$=27 kg and $M_1$+$M_2$=37 kg, respectively) are used. Secondly, the different additional volumes connected to the piston cavities of the hydraulic cylinders (0.00006m$^3$ and 0.000185m$^3$, respectively) are used (Fig.5).

We will conduct studies on two physical models (L and H) with two sets of state components feedback coefficients $K_L$ and $K_H$ optimized for their respective models. Optimized coefficients of the state components feedback for each physical model were determined in advance. The relay-linear control algorithm described in this article has a task to select the "correct" coefficients from a set of previously defined. The phase trajectory of the control object, which is used to select the "correct" set of the feedback coefficients, is estimated in real time using a state observer [9, 15], implemented in our experimental complex.

Figure 11 shows experimentally determined phase trajectory segments of the control object in physical models L and H, when working out the step action U=0.7 V.

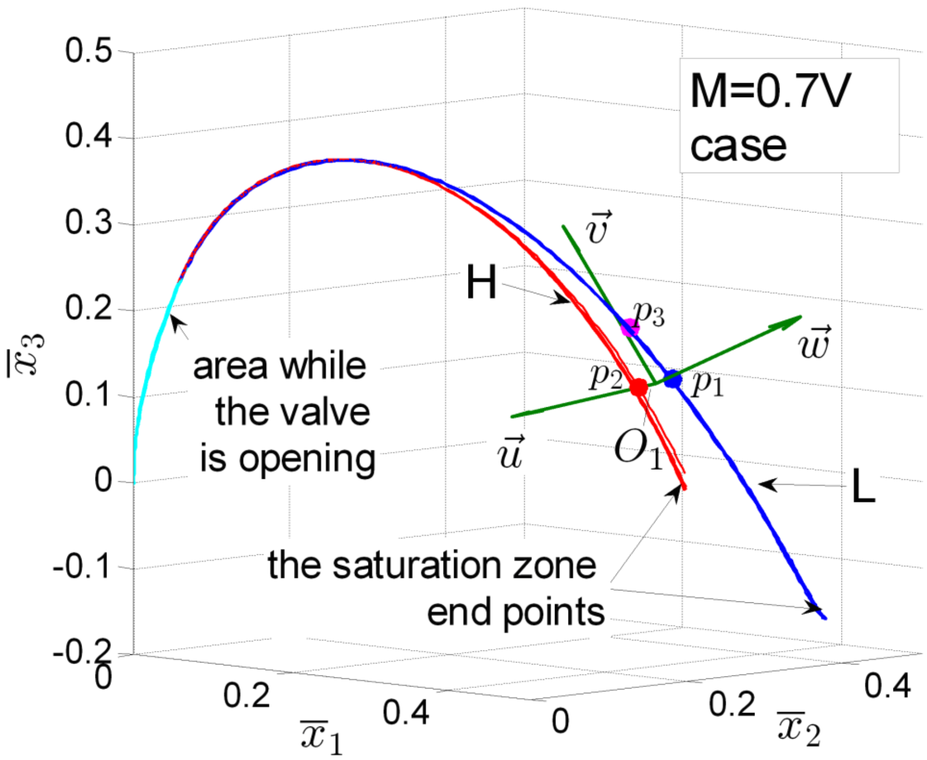


**Fig. 11.** The experimental phase trajectories while control signal $U_{HSV}$ is in the saturation zone.

Figure 11 shows the parts of the phase trajectories, which are limited by the condition, that the signal $U_{HSV}$ (Fig. 7, 10) is in the saturation zone only $(\overline{x}_1(t)=x_1(t)-x_1(0)$, $\overline{x}_2(t)=x_2(t)/40$, $\overline{x}_3(t)=x_3(t)/3000)$.Each trajectory shown in Figure 11 includes a motion phase trajectory section due to a relay control (before the intersection with a hyperplane $O_1uw$) and a motion phase trajectory section with a linear control with the state components feedback coefficients optimized for respective model (after that intersection). The HSV is in the saturation zone in the both sections.

The control block (Fig. 9) performs the following functions:

1 in the event of a step required input action, it switches the operation mode of the controller (s1) to relay control;

2 it tracks the phase trajectory of the control object;

3 when a phase point of controlled hydraulic drive reaches hyperplane O1uw (Fig.11):

3.1 it analyzes the position of the intersection point in the $O_1UVW$ coordinate system;

3.2 it assigns the state components feedback coefficients (s2);

3.3 it switches back the operation mode of the controller (s1) to linear control with the selected coefficients.

One can judge the expediency and efficiency of switching to a linear control with "correctly" assigned state components feedback coefficients by the transients shown in Figure 12.

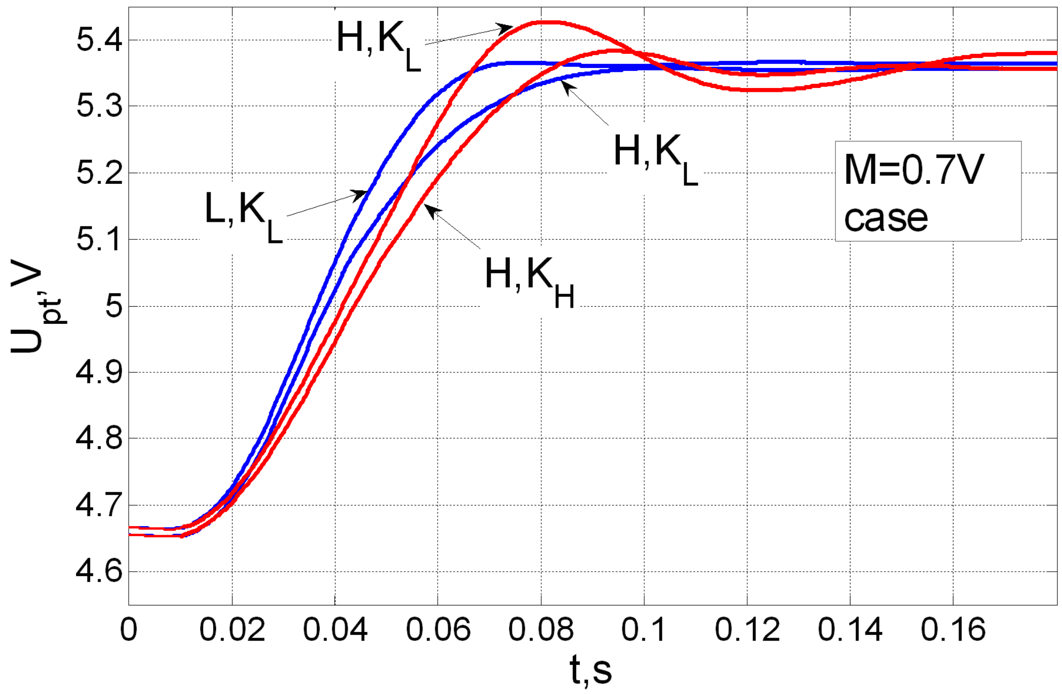


**Fig. 12.**The experimental step responses for the two EHSD physical models L, H and two status controller coefficients $K_L$ and $K_H$.

The obtained transients for two physical models L and H are shown in different colors. For each model, optimized state components feedback coefficients $K_L$ and $K_H$ were previously determined. Each physical model was studied with both sets of feedback coefficients.

Solid/dashed lines shows the transients for the EHSD physical models, working with the "correct" feedback coefficients and the feedback coefficients configured for a different model respectively.

## Conclusions

Two EHSD physical models with different inertial load parameters were realized on the research system based on the hydraulic learning system FESTO. These physical models require significantly different optimized state components feedback coefficients. The relay-linear algorithm was implemented and researched, which provides reconfiguration of the state components feedback coefficients at the very initial stage of the control object movement, when the optimized control action does not depend on these coefficients.

The experimental complex developed on the basis of FESTO hydraulic learning system with the use of Matlab+Simulink Desktop Real Time allows to carry out the study of linear and nonlinear EHSD control.

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