 Communications in Theoretical Physics

Commun. Theor. Phys. **72** (2020) 035502 (5pp) https://doi.org/10.1088/1572-9494/ab5fb3

Alfvén waves in temperature anisotropic Cairns distributed plasma

S Ayaz, Imran A Khan, Z Iqbal and G Murtaza

Salam Chair, Department of Physics, GC University Lahore Katchery Road, 54000 Lahore, Pakistan

E-mail: iali361@gmail.com

Received 23 September 2019, revised 6 November 2019
Accepted for publication 8 November 2019
Published 21 February 2020

Abstract

The dispersion relations and Landau damping of Alfvén waves in kinetic and inertial limits are studied in temperature anisotropic Cairns distributed plasma. In the case of kinetic Alfvén waves (KAWs), it is found that the real frequency is enhanced when either the electron perpendicular temperature or the non-thermal parameter Λ increases. For inertial Alfvén waves (IAWs), the real frequency is slightly affected by the electron temperature anisotropy and Λ. Besides the real frequency, the damping rate of KAWs is reduced when the electron perpendicular temperature or Λ increases. In the case of IAWs, the temperature anisotropy and Λ either enhance or reduce the damping rate depending upon the perpendicular wavelength. These results may be helpful to understand the dynamics of KAWs and IAWs in space plasmas where the non-Maxwellian distribution of particles are routinely observed.

Keywords: Cairns distribution, kinetic Alfvén waves, inertial Alfvén waves

(Some figures may appear in colour only in the online journal)

1. Introduction

Alfvén waves are low frequency electromagnetic waves, first discovered by Hannes Alfvén [1]. When the perpendicular wavelength of these waves is comparable to the ion gyro-radius, they are called kinetic Alfvén waves (KAWs) [2]. Hasegawa was the first person [3, 4] whose pioneering work on KAWs is still a standard reference in space plasma. Later on, Lysak and Lotko [5] discussed these waves in Maxwellian distributed plasmas. Then Bashir *et al* [6] extended the work of Lysak and Lotko by considering bi-Maxwellian distribution function. They showed that the KAWs stand modified due to acoustic effects arising from temperature anisotropy. Various mechanisms have been proposed for the generation of these low frequency waves (for details, see [7–10]). More recently, Barik *et al* [11] studied the generation of KAWs by ion beam and velocity shear in the Earth's magnetosphere. From the application point of view, KAWs play a pivotal role in the heating processes in corona, magnetosphere, ionosphere etc [12]. These waves accelerate the charged particles in plasma through Landau damping mechanism [13].

Kinetic limit is not the only regime in which Alfvén waves exist. If the perpendicular wavelength of the Alfvén waves becomes comparable to electron inertial length, rather than the ion gyro-radius, then these waves are termed as inertial Alfvén waves (IAWs). The inertial limit of these waves was first reported by Goertz and Boswell [14]. It is believed that inertial Alfvén waves are responsible for accelerating the electrons parallel to the magnetic field in the polar or auroral zone regions [15–17]. An important feature of IAWs is that they suit well in earth's ionosphere and the edge regions of laboratory plasmas [18].

It should be noted that in all the above references, both the KAWs and IAWs were studied either in Maxwellian or bi-Maxwellian distributed plasmas. But realistic plasma particles show deviations from Maxwellian and bi-Maxwellian distribution functions because plasma systems, most of the time, are out of thermal equilibrium due to the presence of high energetic non-thermal particles [19–25]. To date, different types of velocity distribution functions have been considered to study wave phenomena [6, 21, 24, 26, 27]. Among these distributions, Cairns distribution function, first proposed by Cairns *et al* [19] has recently attracted the attention of many researchers ([27–29] and references therein). The Cairns distribution function has been applied to study nonlinear properties of electrostatic waves and the effects of energetic electrons on the nonlinear ion acoustic structures [28, 30, 31]. Recently, the Whistler instability has been discussed in Cairns distributed plasma, where it is shown that the non-thermality significantly affects the growth rate of the wave [32]. Hence, Cairns distribution may serve as a good theoretical model for the family of non-thermal space plasmas.

So far, to the best of our knowledge, Cairns distribution incorporating temperature anisotropy (or in other words, bi-Cairns distribution) has not been employed for the study of KAWs and IAWs. The aim of this paper is to examine how this distribution influences the dispersion and damping relations of KAWs and IAWs. The plan of the manuscript is as follows. In section 2, we derive the generalized dispersion and damping of KAWs and IAWs. In section 3, we present results and their discussion. Finally in section 4, we conclude the paper.

2. Mathematical formalism

To derive the dispersion and damping relations of KAWs and IAWs, we use Vlasov–Maxwell set of equations. Following Bashir *et al* [6], the general dispersion relation of Alfvén waves can be written as

$$D = (\epsilon_{xx} - n_{||}^2)\epsilon_{zz} - \epsilon_{xx} n_{\perp}^2. \tag{1}$$

Here, $n_{||,\perp} = ck_{||,\perp}/\omega$ is the parallel/perpendicular refractive index, ϵ_{xx} and ϵ_{zz} are the elements of permittivity tensors, given as

$$\epsilon_{xx} = 1 + \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega} \int v_\perp \mathrm{d}^3v \sum_{n=-\infty}^{\infty} \left(\frac{n^2}{\mu^2} J_n^2(\mu)\right) \times \frac{\left(1 - \frac{k_{||}v_{||}}{\omega}\right)\frac{\partial f_{0\alpha}}{\partial v_\perp} + \frac{k_{||}v_\perp}{\omega}\frac{\partial f_{0\alpha}}{\partial v_{||}}}{\omega - k_{||}v_{||} - n\Omega_\alpha} \tag{2}$$

and

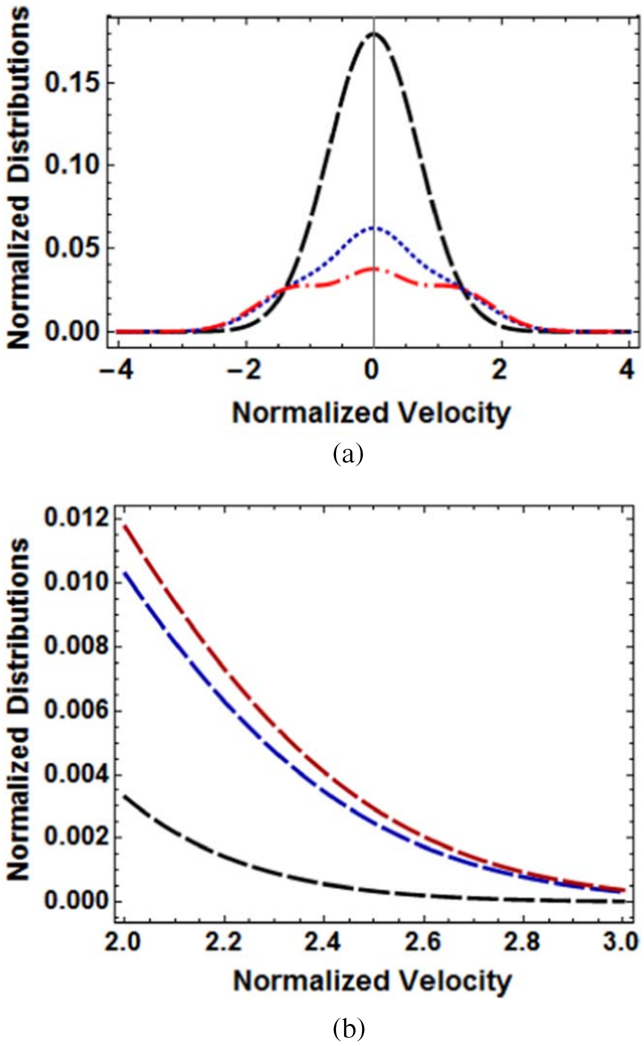
$$\epsilon_{zz} = 1 + \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega} \int v_\perp \mathrm{d}^3v \sum_{n=-\infty}^{\infty} \left(J_n^2(\mu)\frac{v_{||}}{v_\perp}\right) \times \frac{\frac{n\Omega_\alpha}{\omega}\frac{v_{||}}{v_\perp}\frac{\partial f_{0\alpha}}{\partial v_\perp} + \left(1 - \frac{n\Omega_\alpha}{\omega}\right)\frac{\partial f_{0\alpha}}{\partial v_{||}}}{\omega - k_{||}v_{||} - n\Omega_\alpha} \tag{3}$$

respectively.

In the above equations, $\omega_{p\alpha} = \sqrt{4\pi n_o e^2/m_\alpha}$ is the plasma frequency, $\Omega_\alpha = eB_0/m_\alpha c$ is the gyro-frequency, $J_n(\mu)$ is the Bessel function with argument $\mu = k_\perp v_\perp/\Omega_\alpha$, and $f_{0\alpha}$ is the unperturbed bi-Cairns distribution function given as

$$f_{0\alpha} = \frac{1}{\left(1 + \frac{15}{4}\Lambda\right)\pi^{3/2} v_{T\perp\alpha}^2 v_{T||\alpha}} \left[1 + \Lambda\left(\frac{v_\perp^2}{v_{T\perp\alpha}^2} + \frac{v_{||}^2}{v_{T||\alpha}^2}\right)^2\right] \exp\left(-\frac{v_{||}^2}{v_{T||\alpha}^2} - \frac{v_\perp^2}{v_{T\perp\alpha}^2}\right), \tag{4}$$

where $v_{T\perp,||\alpha} = \sqrt{2T_{\perp,||\alpha}/m_\alpha}$ is the perpendicular/parallel thermal velocity of α-species, where α shows electrons or ions and $\Lambda \geqslant 0$ is the spectral index that control the non-thermal features of the plasma system. By putting $\Lambda = 0$ in equation (4), bi-Maxwellian distribution is retrieved. It is difficult to visualize equation (4) due to the temperature difference in different directions. So, for the purpose of simplicity and illustrations, we plot the isotropic Cairns distribution function as shown in figure 1.



(a)

(b)

Figure 1. Normalized distributions versus normalized parallel velocity. On the horizontal and vertical axes, we considered $v/v_{T\alpha}$ and $v_{T\alpha}^3 f_{0\alpha}$, respectively. (a) The following values of Λ are used: $\Lambda = 0$ (black curve), $\Lambda = 0.5$ (blue curve) and $\Lambda = 1$ (red curve) (b). Tails of the distribution functions with the same values of Λ.

Using $f_{0\alpha}$ in equations (2) and (3), and executing the parallel and perpendicular integrations in ϵ_{xx} and ϵ_{zz} under the assumptions of low β-plasma and low frequency limit ($\omega \ll \Omega_i$), we get

$$\epsilon_{xx} = 1 + \frac{1}{1 + \frac{15}{4}\Lambda}\left[\frac{c^2}{v_A^2}\left(\frac{1 - \Gamma_0(\lambda_i)}{\lambda_i}\right) - \frac{c^2k_{||}^2}{\omega^2}\chi_1 + \frac{c^2}{v_A^2}\Lambda\left(\frac{1}{4} + \frac{11}{4}\chi_1\right)\right] \tag{5}$$

and

$$\epsilon_{zz} = 1 - \frac{1}{1 + \frac{15}{4}\Lambda}\left[\frac{\Gamma_0(\lambda_e)}{2k_{\|}^2\lambda_{De}^2}Z'(\zeta_{oe}) + \frac{\Gamma_0(\lambda_i)}{2k_{\|}^2\lambda_{Di}^2}Z'(\zeta_{oi}) - \frac{\omega_{pi}^2}{\omega^2}\chi_2 - \frac{15\Lambda}{4}\frac{\omega_{p_i}^2}{\omega^2}\chi_2\right], \quad (6)$$

where $v_A = B_0/\sqrt{4\pi n_0 m_i}$ is the Alfvén speed; c is the speed of light; $\lambda_{e,i} = k_{\perp}^2\rho_{e,i}^2$ with $\rho_{e,i} = v_{T\perp e,i}/2\Omega_{e,i}$ is the electron/ion gyro-radius; $\lambda_{De,i} = v_{Te,i}/2\omega_{pe,i}$ is electron/ion Debye length; $c_s = \sqrt{T_{\|e}/m_i}$ is the acoustic speed; $Z'(\zeta_{e,i})$ is the derivative of the plasma dispersion function with respect to its argument $\zeta_{e,i} = \omega/k_{\|}v_{Te,i}$, $\Gamma_0(\lambda_{e,i}) = \exp(-\lambda_{e,i})I_n(\lambda_{e,i}) \approx 1 - \lambda_{e,i} + \frac{3}{4}\lambda_{e,i}^2$ for small $\lambda_{e,i}$ and χ_1 and χ_2 represent thermal anisotropies given by the following expressions

$$\chi_1 = \frac{c_s^2}{v_A^2}\left[\left(\frac{T_{\perp i}}{T_{\| i}} - 1\right)\frac{T_{\| i}}{T_{\perp e}} + \left(\frac{T_{\perp e}}{T_{\| e}} - 1\right)\right]$$

and

$$\chi_2 = \frac{T_{\perp e}}{T_{\perp i}}\left(\frac{T_{\| e}}{T_{\perp e}} - 1\right) + \left(\frac{T_{\| i}}{T_{\perp i}} - 1\right)$$

respectively.

2.1. KAWs in Kinetic limit

In the kinetic limit, $\zeta_e \ll 1$ and $\zeta_i \gg 1$, so we take $Z'(\zeta_e) \approx -2 - 2\zeta_e \mathrm{i}\sqrt{\pi}\mathrm{e}^{-\zeta_e^2}$ for electrons and $Z'(\zeta_i) \approx \zeta_i^{-2} - 2\zeta_i \mathrm{i}\sqrt{\pi}\mathrm{e}^{-\zeta_i^2}$ for ions. Generally ω is complex i.e. $\omega = \omega_r + \mathrm{i}\gamma$ with $\gamma \ll \omega_r$. With these assumptions, and after substitution of equations (5) and (6) in (1), we obtain

$$\frac{\omega_r^2}{k_{\|}^2 v_A^2} = \left[\left(\frac{1 + \frac{15}{4}\Lambda + \chi_1}{\left(1 + \Lambda\left(\frac{1}{4} + \frac{11}{4}\chi_1\right)\right)}\right)\left(1 + \frac{3}{4}k_{\perp}^2\rho_i^2\right) + \left(1 + \frac{15}{4}\Lambda\right)\left(\frac{T_e}{T_i}k_{\perp}^2\rho_i^2 - \frac{c_s^2}{v_A^2}k_{\perp}^2\rho_i^2\chi_2\right)\right] \quad (7)$$

and

$$\frac{\gamma}{k_{\|}v_A} = -\frac{P}{Q - R - S}\sqrt{\pi}\left[1 + \left(\frac{T_e}{T_i}\right)^{3/2}\sqrt{\frac{m_i}{m_e}}\mathrm{e}^{-\zeta_{oi}^2}\right], \quad (8)$$

where

$$P = \frac{\omega^2}{k_{\|}^2 v_A^2}\frac{v_A}{v_{T\| e}}\frac{1}{2}\left[\frac{\omega^2}{k_{\|}^2 v_A^2}\left\{1 + \Lambda\left(\frac{1}{4} + \frac{11}{4}\chi_1\right)\left(1 + \frac{3}{4}k_{\perp}^2\rho_i^2\right)\right\} - \left(1 + \frac{15}{4}\Lambda + \chi_1\right)\left(1 + \frac{3}{4}k_{\perp}^2\rho_i^2\right)\right],$$

$$Q = \frac{2\omega^2}{k_{\|}^2 v_A^2}\left[1 + \Lambda\left(\frac{1}{4} + \frac{11}{4}\chi_1\right)\left(1 + \frac{3}{4}k_{\perp}^2\rho_i^2\right)\right] - \left[\left(1 + \frac{15}{4}\Lambda + \chi_1\right)\left(1 + \frac{3}{4}k_{\perp}^2\rho_i^2\right)\right],$$

$$R = \left(1 + \frac{15}{4}\Lambda\right)\left[1 + \Lambda\left(\frac{1}{4} + \frac{11}{4}\chi_1\right) \times \left(1 + \frac{3}{4}k_{\perp}^2\rho_i^2\right)\right]\frac{c_s^2}{v_A^2}\left(\frac{c^2k_{\perp}^2}{\omega_{pe}^2}\frac{m_i}{m_e} - k_{\perp}^2\rho_i^2\chi_2\right),$$

and

$$S = \left(1 + \frac{15}{4}\Lambda + \chi_1\right)\left(1 + \frac{3}{4}k_{\perp}^2\rho_i^2\right)$$

Equations (7) and (8) show how the dispersion and damping rate of KAW are modified due to the presence of non thermal parameter Λ and temperature anisotropy. Note that by putting $\Lambda = 0$ in equations (7) and (8), the results of [33] are retrieved.

2.2. KAWs in the inertial limit

In the inertial limit $\zeta_{e,i} \gg 1$, we take $Z'(\zeta_e) \approx \zeta_e^{-2} - 2\zeta_e \mathrm{i}\sqrt{\pi}\mathrm{e}^{-\zeta_e^2}$ and $Z'(\zeta_i) \approx \zeta_i^{-2}$. For ions, the imaginary term is very small, that is why we neglected it. Carrying out the same procedure as above, we obtain

$$\frac{\omega_r^2}{k_{\|}^2 v_A^2} = \frac{1 + \frac{15}{4}\Lambda + \chi_1 + \left(1 + \frac{15}{4}\Lambda\right)\frac{c^2k_{\perp}^2}{\omega_{pe}^2}\chi_1}{\left[1 + \Lambda\left(\frac{1}{4} + \frac{11}{4}\chi_1\right)\right]\left(1 + \left(1 + \frac{15}{4}\Lambda\right)\frac{c^2k_{\perp}^2}{\omega_{pe}^2}\right)} \quad (9)$$

and

$$\frac{\gamma}{k_{\|}v_A} = -\frac{v_A^3}{v_{T\| e}^3}\frac{\omega^4}{k_{\|}^4 v_A^4} \times \frac{\left[1 + \frac{15}{4}\Lambda + \chi_1 - \frac{\omega^2}{k_{\|}^2 v_A^2}\left\{1 + \Lambda\left(\frac{1}{4} + \frac{11}{4}\chi_1\right)\right\}\right]}{1 + \frac{15}{4}\Lambda + \chi_1 + \left(1 + \frac{15}{4}\Lambda\right)\frac{c^2k_{\perp}^2}{\omega_{pe}^2}\chi_1} \times \sqrt{\pi}\mathrm{e}^{-\zeta_{0e}^2} \quad (10)$$

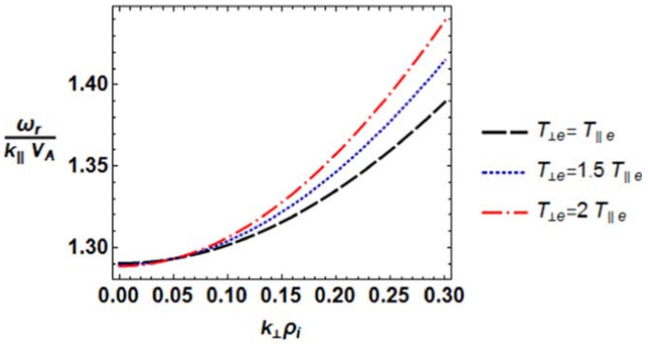
Equations (9) and (10) are modified due to temperature anisotropy and Cairns distribution parameter Λ. Putting $\Lambda = 0$ in equations ((9) and (10), the result of bi-Maxwellian distribution is retrieved [34].

3. Results and discussion

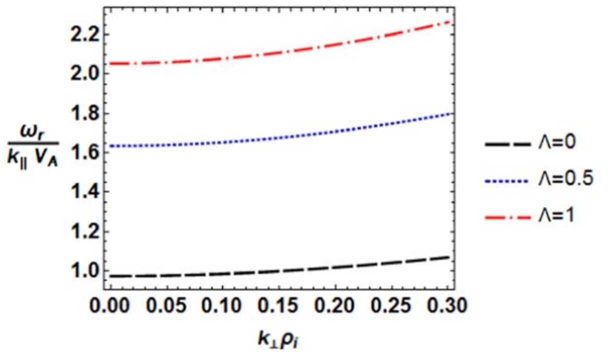
The real frequency ω_r and damping rate γ of KAWs have been graphically analyzed for plasma sheet boundary layer at altitude of 4–6 R_E where the following parameters are appropriate [35–37]: $B_0 = 4.0 \times 10^{-7}$ T, $k_{\|} = 1.0 \times 10^{-10}$ cm^{-1}, $T_{\| i} = T_{\perp i} = 20$ KeV, $n_{0e} = n_{0i} = 1$ cm^{-3}.

With the choice of these parameters, the real frequency of KAWs is enhanced for large perpendicular wavenumber when $T_{\perp e} > T_{\| e}$ (figure 2(a)). The real frequency also increases when Λ increases (figure 2(b)).

Figure 3 shows the damping rate of KAWs. It is found that for large perpendicular wave-number, the damping rate (in magnitude) decreases when $T_{\perp e} > T_{\| e}$ and increases when $\Lambda > 0$ (figures 3(a) and (b)).

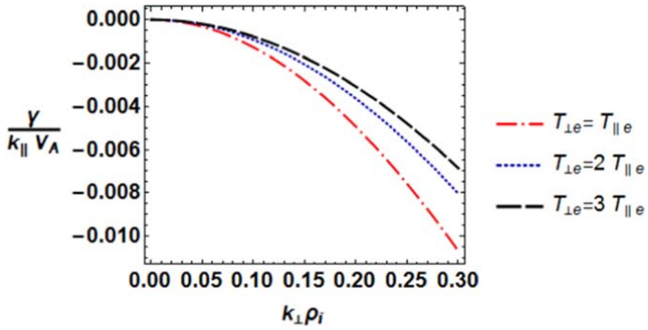


(a)

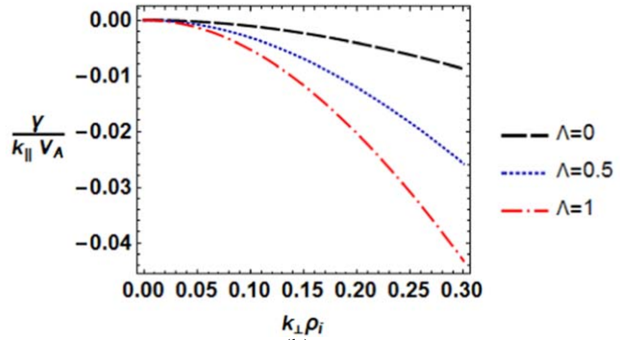


(b)

Figure 2. Normalized frequency of kinetic Alfvén waves in anisotropic Cairns distributed plasma (a) $\Lambda = 0.2$. (b) $T_{\perp e}/T_{\parallel e} = 2$.



(a)



(b)

Figure 3. Normalized damping rate of kinetic Alfvén waves in Cairns distributed plasma. (a) $\Lambda = 0.2$. (b) $T_{\perp e}/T_{\parallel e} = 2$.

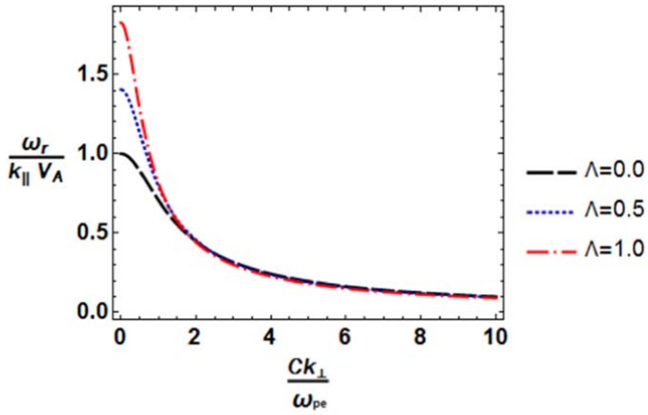


Figure 4. Normalized frequency of inertial Alfvén waves in anisotropic Cairns distributed plasma. The temperature ratio is $T_{\perp e}/T_{\parallel e} = 2$.

The results suggest that the existence of temperature anisotropy $T_{\perp e} > T_{\parallel e}$ or the non-thermal particles with $\Lambda > 0$ enables the wave to damp either at slow or fast rate. This further implies that the waves can heat the plasma particles over both small and large distances.

When the first term in equation (8) dominates, the electrons will be heated; when the second term dominants, then the ions heating will occur.

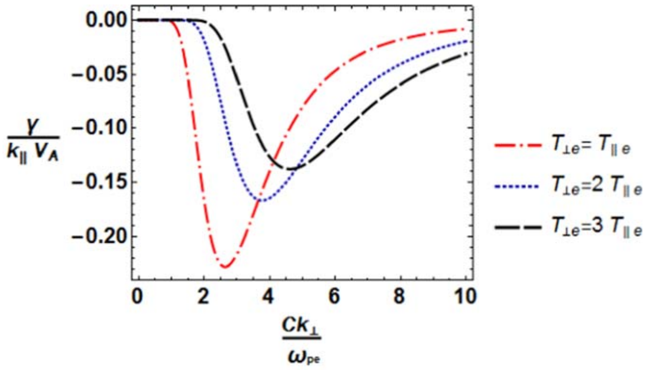
In order to evaluate the real frequency and damping rate of IAW, we use the following parameters that are relevant for auroral zone region at altitude of 1700 km [16]; $B_0 = 2.9 \times 10^{-5}$ T, $v_A = 2.1 \times 10^8\,\mathrm{cm\,s^{-1}}$, $n_0 = 6100\,\mathrm{cm^{-3}}$ and $T_{\perp i} = T_{\parallel i} = 0.5$ eV.

It is found that the real frequency of IAWs increases with Λ, when the perpendicular wavelength is relatively small (figure 4). But the real frequency is not affected by the temperature anisotropy. The reason is the following. We consider low $\beta\,(=c_s^2/v_A^2)$ case i.e. $\beta \ll 1$. Under this condition, the anisotropy does not significantly affect the real frequency of IAWs (see equation (9)).

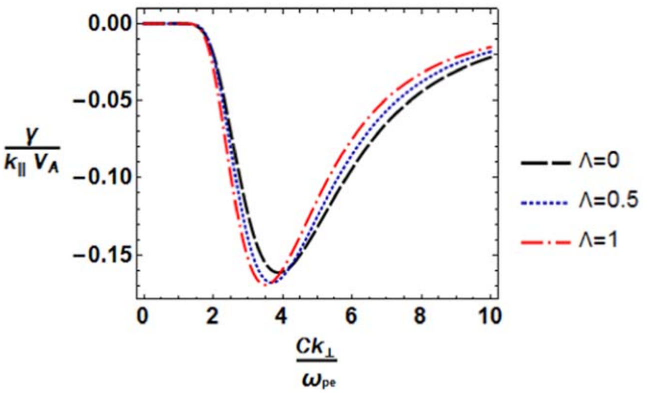
Figure 5 illustrates the damping of IAWs in Cairns distributed plasmas. In both the cases $T_{\perp e} > T_{\parallel e}$ and Λ, the damping rate is either increased or decreased depending upon the perpendicular wave number. The change with $T_{\perp e} > T_{\parallel e}$ is substantial, whereas the change with Λ is small. Unlike KAWs, IAWs can only heat the electrons due to the fact that only electrons can effectively resonate with the wave.

As we have seen that both KAWs and IAWs remain damped in the presence of temperature anisotropy and non-thermal parameter Λ. But as we know the Cairns distribution function can have positive slope which may possess free energy to make the wave unstable. However, as shown in figure 1, for those values of Λ which we have selected in the paper, the distribution function does not show positive slope anywhere in the region where the resonance points of KAWs and IAWs lie. The resonance points of KAWs is shifted towards the tail of electrons distribution due to the dynamics of ions. In the case of IAWs, the resonance point also lies in the tail but very far away.

The other free energy source, temperature anisotropy, also only changes the number of resonance particles but does not make the wave unstable because we have considered low $\beta\,(=c_s^2/v_A^2) \ll 1$.



(a)



(b)

Figure 5. Normalized damping rate of inertial Alfvén waves in anisotropic Cairns distributed plasma. (a) $\Lambda = 0.2$. (b) $T_{\perp e}/T_{\|e} = 0.5$.

4. Conclusion

In this paper, we have studied the real frequencies and damping rates of KAWs and IAWs in temperature anisotropic Cairns distributed plasma. The kinetics limit has been analyzed in plasma sheet boundary layer while the inertial limit in the auroral zone regions. We have found that temperature anisotropy and the non-thermal parameter Λ significantly influence KAWs and IAWs. The real frequency of KAWs is enhanced with both the temperature anisotropy and Λ at larger perpendicular wave-number regions. However, the real frequency of IAWs is not considerably affected by the temperature anisotropy but slightly affected by Λ. Furthermore, in the case of KAWs, the temperature anisotropy and Λ enhance the damping rate. On the other hand, the damping rate of IAWs either increases or decreases depending upon the values of the perpendicular wave-number. These results may find interesting applications because both KAWs and IAWs are predominant sources of energy transport in space plasmas. Moreover, the present work can be extended to more realistic situations by considering other effects such as the density gradients, magnetic field gradients, or a combination thereof.

References

[1] Alfvén H 1942 *Nature* **150** 405
[2] Hasegawa A and Chen L 1976 *Phys. Fluids* **19** 1924
[3] Hasegawa A 1976 *J. Geophys. Res.* **81** 5083
[4] Hasegawa A and Chen L 1975 *Phys. Rev. Lett.* **35** 370
[5] Lysak R L 1996 *J. Geophys. Res.* **101** 5085
[6] Bashir M F, Iqbal Z, Aslam I and Murtaza G 2010 *Phys. Plasmas* **17** 102112
[7] Lakhina G S 1990 *Astrophys. Space Sci.* **165** 153
[8] Lakhina G S 2008 *Adv. Space Res.* **41** 1688
[9] Thompson B J and Lysak R L 1996 *J. Geophys. Res.: Space Phys.* **101** 5359
[10] Nose M, Iyemori T, Sugiura M, Slavin J A, Holfman R A, Winningham J D and Sato N 1998 *J. Geophys. Res.* **103** 17587
[11] Barik K C, Singh S V and Lakhina G S 2019 *Phys. Plasmas* **26** 022901
[12] Cramer N F 2001 *The Physics of Alfven Waves* (New York: Wiley)
[13] Sharma R P and Kumar S 2010 *J. Plasma Phys.* **76** 239
[14] Goertz C K and Bosewell R W 1979 *J. Geophys. Res.* **84** 7239
[15] Shukla N, Mishra R, Varma P and Tiwari M S 2008 *Plasma Phys. Control. Fusion* **50** 025001
[16] Andersson L, Wahlund J E, Clemmons J, Gustavsson B and Eliasson L 2002 *Ann. Geophys.* **20** 139
[17] Chaston C C, Salem C, Bonnell J W, Carlson C W, Ergun R E, Strangeway R J and McFadden J P 2008 *Phys. Rev. Lett.* **100** 175003
[18] Vincena S, Gkelman W and Maggs J 2004 *Phys. Rev. Lett.* **93** 105003
[19] Cairns R A, Mamum A A, Bingham R, Boström R, Dendy R O, Nairn C M C and Shukla P K 1995 *Geophys. Res. Lett.* **22** 2709
[20] Khan I A and Murtaza G 2018 *Plasma Sci. Technol.* **20** 035302
[21] Khan I A, Iqbal Z, Naim H and Murtaza G 2018 *Phys. Plasmas* **25** 082111
[22] Khan I A, Iqbal Z and Murtaza G 2019 *Eur. Phys. J. Plus* **134** 80
[23] Khan I A, Khokhar T H, Shah H A and Murtaza G 2019 *Physica* A **535** 122385
[24] Azra K, Iqbal Z and Murtaza G 2018 *Commun. Theor. Phys.* **69** 699
[25] Watt C E J and Rankin R 2007 *J. Geophys. Res.* **112** 04214
[26] Qureshi M N S, Shi J K and Ma S Z 2005 *Phys. Plasmas* **12** 122902
[27] Bara D, Djebli M and Doumaz D B 2014 *Laser Part. Beams* **32** 391
[28] Verheest F and Pillay S R 2008 *Phys. Plasmas* **15** 013703
[29] Baluku T K and Hellberg M A 2011 *Plasma Phys. Control. Fusion* **53** 095007
[30] Mamun A 1997 *Phys. Rev.* E **55** 1852
[31] Wang B J and Hau L N 2015 *Plasma Phys. Control. Fusion* **57** 095012
[32] Malik M U, Masood W and Mirza A M 2017 *Phys. Plasmas* **24** 102120
[33] Naim H, Bashir M F and Murtaza G 2014 *Phys. Plasmas* **21** 032120
[34] Naim H and Murtaza G 2013 *Phys. Lett.* A **377** 2348
[35] Wygant J R *et al* 2000 *J. Geophys. Res.* **105** 18675
[36] Wygant J R, Keiling A, Cattell C A, Lysak R L, Temerin M, Mozer F S, Kletzing C A, Scudder J D, Streltsov V, Lotko W and Russell C T 2002 *J. Geophys. Res.* **107** 1201
[37] Shukla N, Mishra R, Varma P and Tiwari M S 2008 *Plasma Phys. Control. Fusion* **50** 025001