Journal of Physics: Conference Series **1457** (2020) 012006 doi:10.1088/1742-6596/1457/1/012006

Approximation of the maximally flat filter by using Bézier curve with an exponential function

B Purahong[1], I Kanjanasurat[2], P Sithiyopasakul[1], V Chutchavong[1] and C Pintavirooj[2]

[1] Department of Computer Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand
[2] Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand

corresponding author's e-mail: yuna001@gmail.com

Abstract. This paper presents a design of filter by using Bézier curve with an exponential function. This paper used the advantage of The Bézier curve which had ability for approximation and an exponential function which had the adaptable parameters of the polynomial. It can adjust the characteristic of frequency response for the best performance. The simulation results of various setting show the frequency response, step response. The comparison of response between the Bernstein filter and Butterworth filter in order two show that the rise time of Bernstein filter better than Butterworth filter and Bézier curve filter has not overshoot. Furthermore, the stability Nyquist criterion has been used to guarantee the stability of the transfer function.

1. Introduction

The analog filter is widely used in signal processing such as communication which noise-canceling or medical device which acquired a biosignal [1,2]. It requires the accuracy of the signal and offers the flexibility of the attenuation and frequency adjustment. The excellent characteristics of a filter are high attenuation rate and low passband ripple [3]. The Bernstein polynomial was first used to approximate filter by L. Rajagpoal [4]. It was a maximally flat FIR filter. David Baez-Lopez converted the Bernstein analog filter to the digital filter and created an IIR filter. the Bernstein filter was used to solver for mean curvature regularized models by Yuanhao Gong [5]. This study showed that the filter was magnitude faster than traditional solvers. Moreover, Chien-Cheng Tseng presented the study of power line interference removal in ECG using bernstein-polynomial-based FIR notch filter [6]. It was showed another way to use the Bernstein filter in biomedical application. The mathematical which similar the Bernstein polynomial is Bézier curve. It widely used in computer graphics. Mohammad Mahdi el at. [7] had development of a direct time integration method based on Bezier curve and 5th-order Bernstein basis function. It showed the similar of mathematical model. It had a stability analysis, numerical dispersion and dissipation of proposed method.

As mentioned above, the advantages of the Bézier curve are adaptability by varying parameters and flexibility by changed mathematical model. The advantages of this paper are the step response of the Bézier curve has an excellent characteristic better than the Butterworth filter and are adaptability by varying two parameters.

This paper is divided into four parts. The first part discusses the background and research target. The second part explains the design of a mathematical model. Next section presents the result such as frequency response, Step response, and Stability of filter. Last is a conclusion and discussion.

2. Methodology

The Bézier curve [8] in order 2 and order 3 was presented in equation (1) and equation (2), respectively.

$$B(t) = P_1 + (1-t)^2 (P_0 - P_1) + t^2 (P_2 - P_1) \tag{1}$$

$$B(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2 (1-t) P_2 + t^3 P_3 \tag{2}$$

where P is a control point and t is positive integers which $0 \le t \le 1$

A mathematical model of Bézier curve can be applied to approximate the desired function. An exponential function is used approximation and defined as equation (3)

$$f(x) = 1 / a^{\alpha t} \tag{3}$$

using Bézier curve estimate equation (3). Characteristic of the transfer function is shown in equation (4)

$$H(f;t) = \frac{1}{B(t)} = \frac{1}{1/a^{\alpha t}} \tag{4}$$

Thus, the approximation of the desired transfer function for a low pass filter is presented by equation (5).

$$H(f;a,\alpha,t) = 1/(a^{\alpha/2} + (1-t)^2 (1-a^{\alpha/2}) + t^2 (a^{\alpha} - a^{\alpha/2}) \tag{5}$$

the second order of Bézier curve approximation function is shown in equation (6).

$$H(f;a,\alpha,t) = \frac{1}{(a^{\alpha} - 2a^{\alpha/2} + 1)t^2 + 2(a^{\alpha/2} - 1)t + 1} \tag{6}$$

Consideration of the design of Bézier curve approximate function by the adjustable parameter is modifying the property of frequency response.

3. Implementation and Results

The results were divided into three parts. Firstly, the frequency response of filter was calculated by the Bode diagram. It showed a change of magnitude and phase. Secondly, an arbitrary dynamic system was determined by step response. It can be represented by the following quantities related to its time behavior. Last, Stability was proved by Nyquist stability criteria. Moreover, each part showed a response of different parameter.

3.1 Frequency response

The proposed method can be applied to the various filter. The family curve of frequency response by a varying "a" parameter of Bézier curve filter was shown in figure 1. This parameter is a base of the exponential function. The attenuation of magnitude response was increased by increasing "a" parameter. Furthermore, a modification of the phase in the same frequency-shifted left and more by increasing the value of "a" parameter.
Figure 2 shows the frequency response of the Bézier curve in low pass filter with α parameter. The attenuation of the magnitude response that fixed order 2, a=2, and varied alpha was increased by increasing α parameter. Moreover, the phase was move left by rising α.

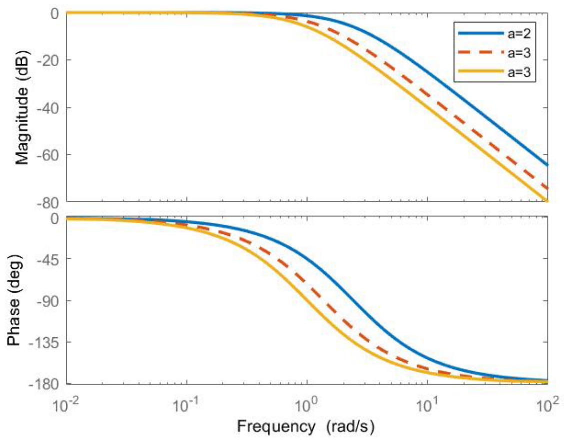

Figure 1. The comparison frequency response of low pass filter with a=2, a=3, and a=4.

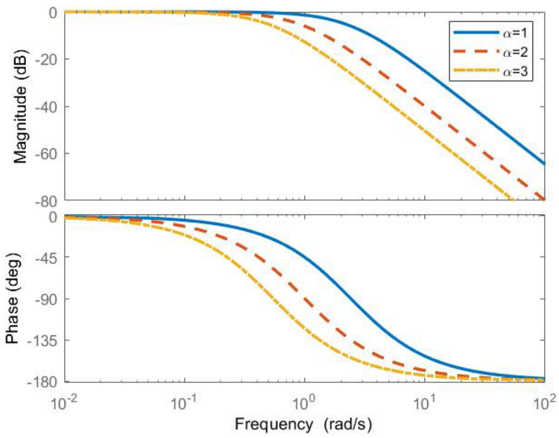

Figure 2. The comparison frequency response of low pass filter with α=1, α=2, and α=3.

The order symbolizes the power of Bézier curve. The graph in figure 3, which set a=2 and α=1 was shown a frequency response of Bézier curve filter. The magnitude response in high order had a high attenuation. Furthermore, the increase in phase is similar to the attenuation of magnitude response, which is the result of an increase in orders.

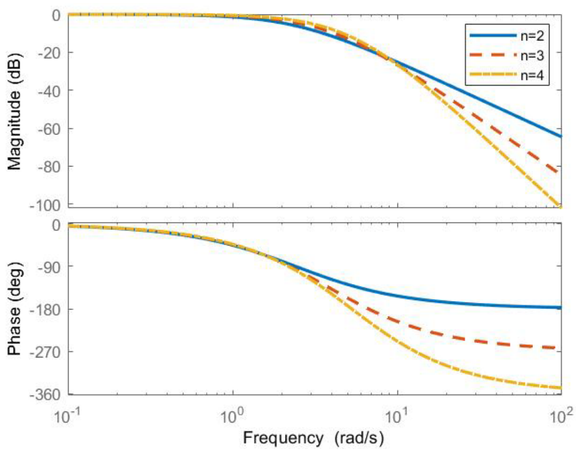

Figure 3. The comparison frequency response of low pass filter with n=2, n=3, and n=4.

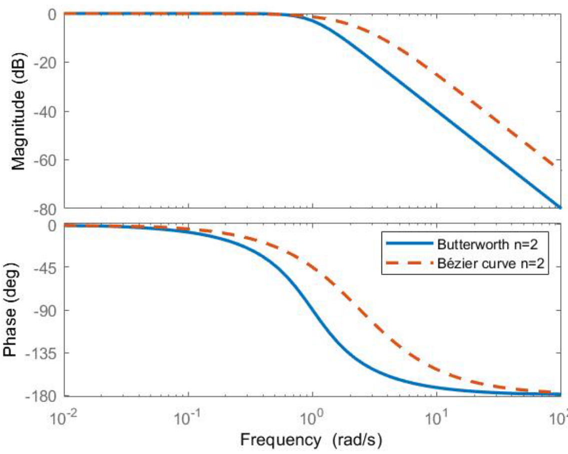

Figure 4. The comparison frequency response of Butterworth and Bézier curve.

Butterworth filter which a type of filter was compared with Bézier curve filter in figure 4. The graph is shown a comparison frequency response of the Butterworth filter and Bézier curve filter in order n=2. Bézier curve filter set a=2 and α=1.

3.2 Step response

The step response is the time behavior of a system. It changes output zero to one in a short time. In research, there are three factors which consider the behavior of time. Firstly, the rise time, it is the time taken by a signal to grow from a low value to a high value. Secondly, settling time, it is determined by the time reaches and stay within a range of a certain percentage of the final value. Last is an overshoot, the occurrence of a signal or function exceeding its target.

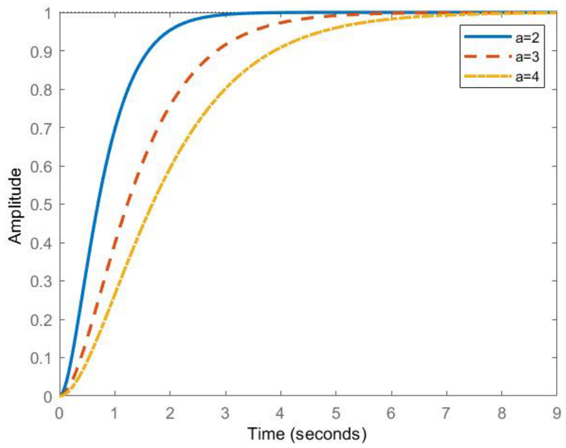

Figure 5. The comparison step response of low pass filter with a=2, a=3, and a=4.

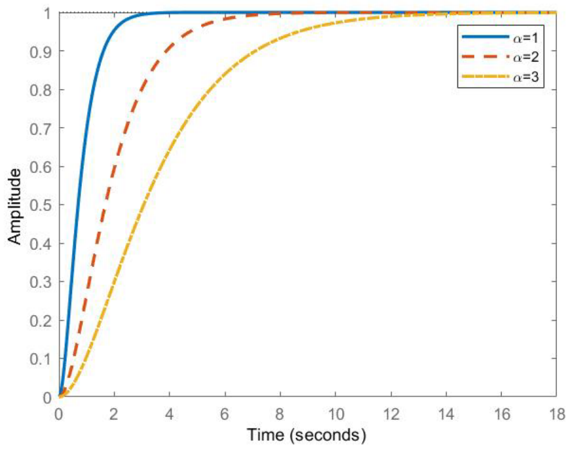

Figure 6. The comparison step response of low pass filter with α=1,α =2, and α=3.

Figure 5 shows a step response of the Bézier curve filter with n=2, α=1, and vary a. Rise time was increased from 1.39 second to 3.35 second by increase a=2 to a=4. A settling time was risen from 2.41 second to 5.83 seconds by increase a=2 to a=4. An overshoot was zero.
Figure 6 presents a step response of the Bézier curve with n=2, a=2, and modify α parameter. Rise time was increased from 1.39 second to 6.13 second by increase α=1 to α =3. A settling time was scaled from 2.41 second to 10.66 seconds by increase α =1 to α =3. Moreover, an overshoot was zero.

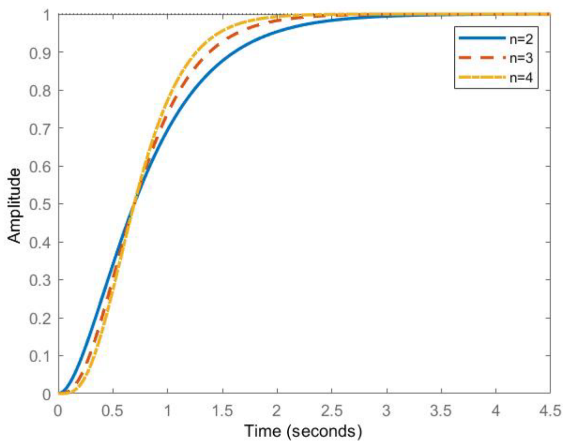

Figure 7. The comparison step response of low pass filter with n=2, n=3, and n=4.

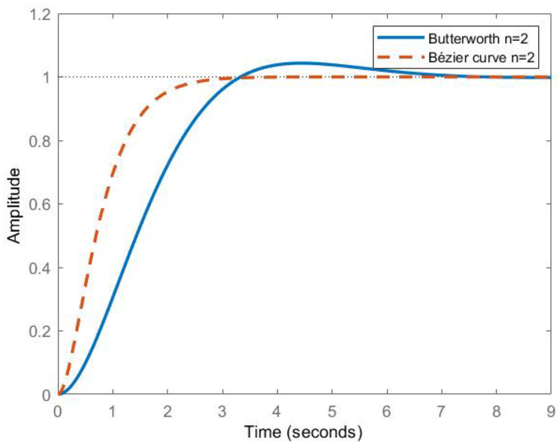

Figure 8. The comparison of step response between Butterworth and Bézier curve.

Figure 7 displays a step response of the Bézier curve filter with a=2, α=1, and adjusted order n. Rise time was decreased from 1.39 second to 0.93 seconds by increase n=2 to n=4. A settling time was dropped from 2.41 second to 1.71 seconds by increase n=1 to n=3. Also, a zero overshoot.
Figure 8 is a comparison step response both Butterworth and Bézier curve in the order n=2. A rise time of Butterworth had 2.14 second. Settling time was 5.96 second. On the other hand, the step response of the Bézier curve filter had a rise time 1.39 second and settling time 2.41 second. An overshoot of Bézier curve was zero. However, Butterworth was 4.32 second.

3.3 Nyquist stability

Nyquist stability is a graphical technique for determining the stability of system [9]. Figure 9 shows the Nyquist diagram, which is the curve of the order change. All the curves rotated in a counterclockwise direction and not pass the unstable point. A lower-order (n=2) had a short radius and expanded in high order.

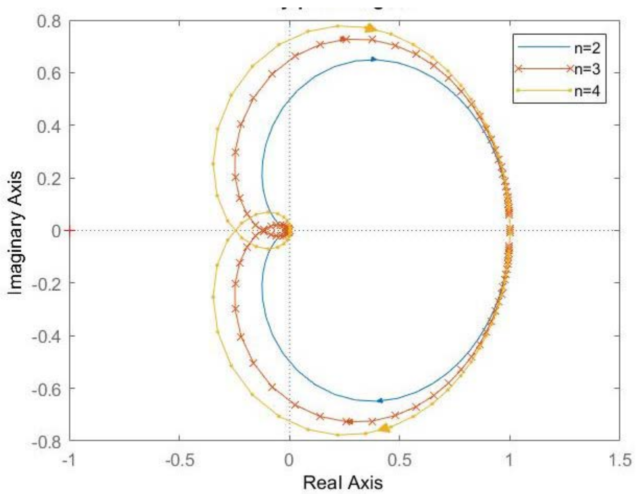

Figure 9 The family curve of the Nyquist diagram of low pass filter in order n=2, n=3, and n=4.

4. Conclusions

This paper presented a filter design by using the Bézier curve with the exponential function. The Bézier curve filter has three adjustable parameters for control a characteristic of frequency response. Firstly, increasing a parameter can increase the attenuation of the filter and increase the rise time of step response. Secondly, α parameter similar a parameter. The increased value of the parameter raised the attenuation and rose time. Last is order, it can increase the attenuation of magnitude response and decrease a rise time. Bézier curve filter was compared with the Butterworth filter. At the same order (n=2), Butterworth's attenuation is better than Bézier curve filter. The step response of the Bézier curve filter has an excellent characteristic better than a Butterworth filter, and the filter is stable.
In the future, the filter will be applied to a circuit and clean noise in the signal. Also, it can use in image processing for the noise cancelling.

5. References

[1] Mostafa Shanawaz S, Horta N, Ravelo-García G A and Morgado-Dias F 2018 Analog active filter design using a multi objective genetic algorithm *International Journal of Electronics and Communications* p 83
[2] Tseng C and Lee S 2017 Power line interference removal in ECG using bernstein-polynomial-based FIR notch filter *IEEE 6th Global Conference on Consumer Electronics (GCCE)* p 1
[3] Bensouici T, Charef A, Assadi I2018 A new approach for the design of fractional delay by an FIR filter *ISA Transactions* p 73
[4] Rajagpoal L and Roy S D 1987 Design of maximally-flat FIR filters using the Bernstein polynomial *IEEE Transactions on Circuits and Systems* p 1587
[5] Gong Y 2016 Bernstein filter: A new solver for mean curvature regularized models *IEEE International Conference on Acoustics, Speech and Signal Processing* pp. 1701-1705
[6] Tseng C and Lee S 2017 Power line interference removal in ECG using bernstein-polynomial-based FIR notch filter *Global Conference on Consumer Electronics* p 1

[7] Mohammad Mahdi Malakiyeh, Saeed Shojaee, Saleh Hamzehei Javaran 2018 Development of a direct time integration method based on Bezier curve and 5th-order Bernstein basis function *Computers & Structures* p15
[8] Winkel R 2015 On a generalization of Bernstein polynomials and Bézier curves based on umbral calculus (II): de Casteljau algorithm *Computer Aided Geometric Design* p1
[9] Svarc I 2014 Contributions to the application of Popov and circle criterion for stability analysis *International Carpathian Control Conference (ICCC)* p 562