

Susceptibilities of strongly interacting matter in a finite volume

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Abstract

We investigate possible finite-volume effects on baryon number susceptibilities of strongly interacting matter. Assuming that a hadronic and a deconfined phase both contribute to the thermodynamic state of a finite system due to fluctuations, it is found that the resulting shapes of the net-baryon number distributions deviate significantly from the infinite volume limit for a given temperature T and baryochemical potential μ_B . In particular, the constraint on color-singletness for the finite quark–gluon phase contribution leads to a change of the temperature dependence of the susceptibilities in finite volumes. According to the model, the finite-volume effect depends qualitatively on the value of μ_B .

Keywords: heavy-ion collisions, baryon number susceptibility, quark–gluon plasma, phase transition

(Some figures may appear in colour only in the online journal)

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1. Introduction

In recent years considerable attention has been devoted to event-by-event fluctuations and correlations of conserved quantities in relativistic heavy-ion collisions (see [1] for an overview of the beam energy scan program at RHIC and the theoretical background). One important goal of this endeavour is to experimentally probe the phase structure of strongly interacting matter and possibly identify its critical point. Direct comparisons of measured net-baryon cumulants with the corresponding thermodynamic susceptibilities from lattice QCD calculations [2, 3] appear very promising in this respect. These observables are believed to provide rather robust signatures of the underlying thermodynamics in the case of heavy-ion experiments. However, in order to relate experimental measurements and theoretical equilibrium properties, several complications have to be considered: one practical problem is the finite phase space acceptance and the limited efficiency of any real experiment which may obscure relevant signatures in the data. Necessary corrections have been addressed theoretically, e.g. in [4–7]. A fundamental and obvious issue is the fact that conserved charges do not fluctuate globally. Therefore, one has to restrict the statistical analysis to a part of the total phase space of the heavy-ion reaction. The acceptance window of such an analysis must be chosen sufficiently small, so that the observables can be considered as reflecting a subsystem in thermodynamic contact with a heat bath. Only then, the grand canonical ensemble can be applied to the theoretical description of this subsystem. The potential influence of a finite volume of the source has been explored in [8–11]. At the same time, however, the subsystem must be large enough to allow the relevant particle correlations to show in the first place, see e.g. [12]. Finally, one has to take into account initial state (e.g. volume) fluctuations in heavy-ion experiments, which overlay the possibly critical fluctuations under investigation [13].

In the following, we want to draw the attention to an aspect, which has not been addressed yet: even under ideal circumstances and with complete control over all the beforementioned issues, a heavy-ion experiment can only probe finite volumes of strongly interacting matter, while lattice QCD calculations generally refer to the infinite volume case. In a finite volume, the partition function of the system is subject to additional constraints which may lead to changes in the thermodynamic properties like the baryon number susceptibilities. Firstly, in a finite volume we expect fluctuations of the phase composition that are suppressed in the infinite volume case: i.e. admixtures of unfavorable macroscopic configurations of the system can be realized with finite probability and can have an effect on the thermodynamics of the system. Secondly, when the finite hadronic system of a heavy-ion reaction is forced to undergo a phase transition to a deconfined phase by the collision dynamics, we expect the finite quark–gluon plasma phase to be suppressed—as compared to the case of infinite matter—due to the requirement of color-singletness, i.e. an explicit volume dependence of the quark–gluon plasma equation of state. In this letter, we present an exploratory study of these effects and their implications with respect to net-baryon fluctuations for scenarios with different baryochemical potentials.

2. The model

We extend the model proposed in [14] to allow for the investigation of finite-volume effects on baryon number susceptibilities as function of temperature and chemical potential. The basic assumption of the schematic model is a first order phase transition between a hadronic

phase and a quark–gluon plasma phase, which is justified for sufficiently high values of the baryochemical potential⁶. In a finite volume, it presumes coexistence of the two phases due to fluctuations: as a consequence of the finite (fixed) volume of the total system, any macroscopic configuration \mathbf{x} contributes with a probability $p(\mathbf{x}) \sim \exp[-\Phi(\mathbf{x})/T]$, where $\Phi(x)$ is the grand canonical potential of the system [15]. In the present simplified set-up, the macroscopic configuration is the composition of the total system in terms of the partial volumes of the individual phases, which are assumed to be microscopically uncorrelated, i.e. the partition function factorizes. In this picture, all intensive thermodynamic quantities of the total macroscopic system are given as expectation values based on the weight of all possible configurations.

The equations of state of the subsystems are a relativistic ideal quantum gas of non-strange hadron resonances (with eigenvolume correction) on the one hand, and a relativistic ideal quantum gas of two massless quark flavors and gluons confined in an MIT bag on the other hand.

For further details we refer the reader to the appendix.

3. Susceptibilities of baryon number in the finite system

The susceptibilities of the baryon number in the finite system are calculated as

$$\chi_i^B = -\frac{\partial^i \hat{\phi}}{\partial \hat{\mu}_B^i}, \quad (1)$$

from the dimensionless density of the grand canonical potential $\hat{\phi} = \Phi(T, \mu_B, V)V^{-1}T^{-4}$, where $\hat{\mu}_B = \mu_B/T$ is the reduced baryochemical potential.

The first order susceptibility is proportional to the expectation value of the net-baryon number,

$$\chi_1^B(V)VT^3 = \langle N_B \rangle. \quad (2)$$

For $\mu_B = 0$ this quantity has a value of zero, which, of course, is reflected by the model outcome. Let us consider the second order susceptibility, which is proportional to the variance of the net-baryon number:

$$\chi_2^B(V)VT^3 = \sigma_B = \langle (\delta N_B)^2 \rangle. \quad (3)$$

Figure 1 shows this quantity as a function of temperature at $\mu_B = 0$ for different system volumes. The quark–gluon plasma phase is constructed without color-singlet and zero-momentum constraint, i.e. there is no explicit volume dependence. The model calculations show a softening and broadening of the phase transition which is more pronounced for small volumes. As was discussed in [14], this can be understood as a consequence of the admixture of the ‘unfavorable’ phase in the finite system at any given temperature. Although suppressed exponentially, the presence of the quark–gluon phase below the critical temperature T_C^∞ and the presence of the hadronic phase above T_C^∞ has a finite probability (T_C^∞ marks the first order phase transition for infinite matter). The effect presented in figure 1 should in principle be relevant in any realistic model-treatment of strongly interacting matter in finite volumes. The basic infinite matter equations of state of the phases used in our model and the resulting

⁶ For $\mu_B = 0$, it is not in accordance with lattice QCD calculations. However, we expect that the qualitative behavior of the addressed effects will also be present—although less pronounced—in the case of a crossover.

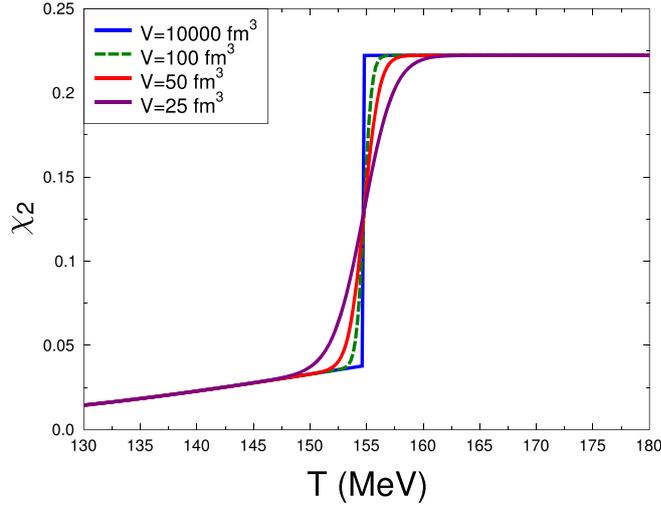


Figure 1. Second order baryon number susceptibilities χ_2^B as function of temperature at $\mu_B = 0$. For the individual phases the equations of state without explicit volume effects are employed.

equation of state of the two-phase system are of course only schematic approximations to study the finite size effects, as can be seen from the comparison with lattice QCD results⁷. However, the model calculation gives an indication of how strong the deviation from a given baseline—the true properties of infinite matter—can be due to the finite volume. In this respect one would conclude from figure 1 that the finite volume effect considered here should in principle affect the observed slope of the crossover curve of $\chi_2^B(T)$. But the effect seems too small to be relevant for the analyses of heavy-ion experiments at present.

Figure 2 (top) shows the same quantity as figure 1 for the two-phase model, now explicitly taking into account the color-singlet constraint in the quark–gluon equation of state. The picture now changes completely. The softening of the step in the second order susceptibility is reduced, while the temperature of the sudden rise is shifted drastically with decreasing volume. Both effects are a consequence of the fact that a smaller volume of the quark–gluon phase gives rise to larger grand canonical potential density, corresponding to a smaller pressure. For a given temperature the quark–gluon phase is thus less favorable (more suppressed) in a finite volume as compared to the infinite volume limit. The effective critical temperature T_C^{eff} which can be thought of as the temperature where the effective number of degrees of freedom increases substantially, is strongly volume dependent (as was shown in [14]; see [16] for qualitatively contrary findings within a different phenomenological model).

The suppression of small quark–gluon droplets—due to the color-singlet constraint—also implies that quark–gluon admixtures relevant for the total system stem predominantly from relatively large droplets at $T \geq T_C^{\text{eff}}$. Any contribution of the quark–gluon phase to the total equation of state must therefore be a relatively strong contribution. This is why the softening of the phase-transition is reduced. As a result, one finite-size effect cancels the other finite-size effect.

⁷ For $\mu_B = 0$, lattice QCD does not exhibit a discontinuity of $\chi_2^B(T)$ for infinite matter as our simple model, but rather a smooth transition.

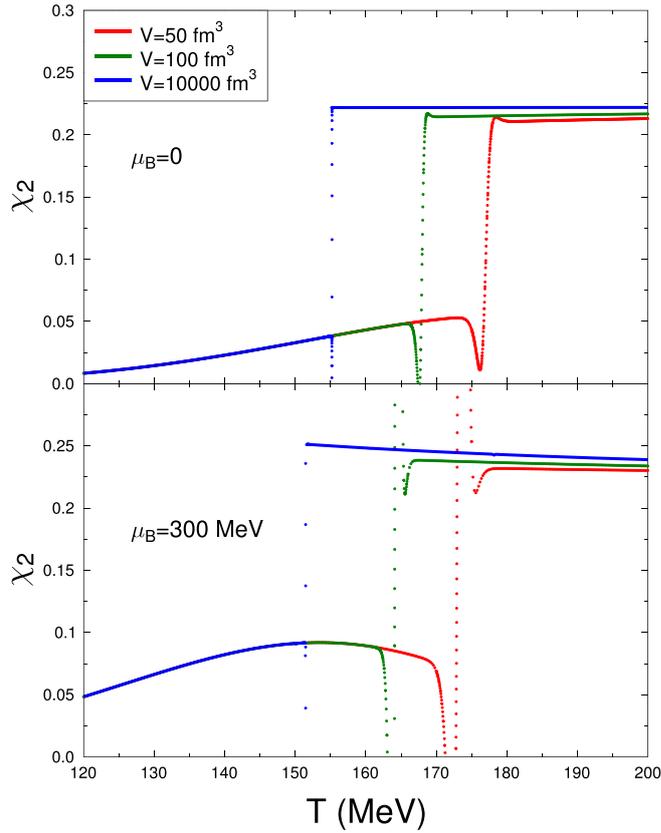


Figure 2. Second order baryon number susceptibilities χ_2^B as function of temperature for different system volumes at $\mu_B = 0$ (top) and $\mu_B = 300$ MeV (bottom). For the QGP phase the equation of state with explicit volume effects is employed.

The influence of the explicit finite-size effect of the quark–gluon equation of state on the second order baryon number susceptibility is certainly not negligible according to the model and should be addressed in analyses of experimental observables. This is especially true if the reaction volume as a hidden parameter implicitly changes in a series of measurements, e.g. in centrality dependence, while only T or μ_B is supposed to vary.

Both model calculations of, with and without explicit color-singlet constraint of the quark–gluon plasma equation of state, approach the value $2/9$ expected for an ideal gas of massless quarks and gluons with two quark flavors.

In figure 2 (bottom), the influence of a finite baryochemical potential is depicted. The qualitative behavior with respect to system size variations appears to be similar to the $\mu_B = 0$ case, i.e. one observes a significant shift of the effective critical temperature T_C^{eff} with decreasing volume. The transition region around T_C^{eff} is characterized by strong variations of χ_2^B in a narrow temperature range, which is much more pronounced at finite μ_B than at $\mu_B = 0$. For a given volume, the discontinuity of χ_2^B as a function of T occurs at lower temperatures at finite μ_B as compared to $\mu_B = 0$. This reflects the decreasing T_C with increasing μ_B already present in the infinite matter case.

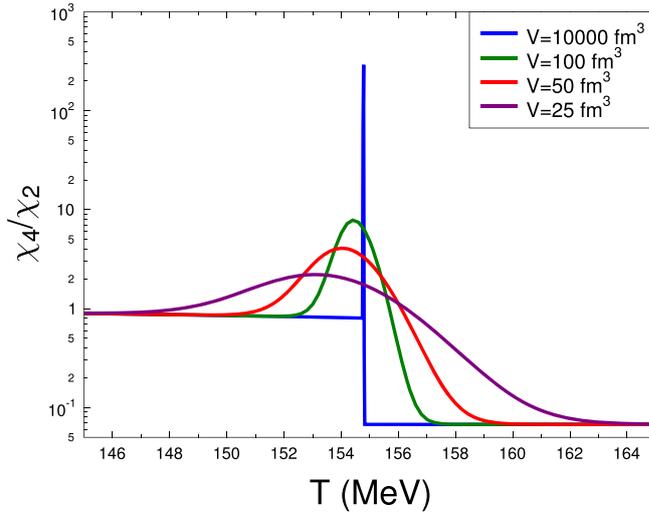


Figure 3. Fourth to second order baryon number susceptibility ratio χ_4^B/χ_2^B as function of temperature at $\mu_B = 0$. For the QGP phase the equation of state without explicit volume effects is employed.

Another quantity currently discussed extensively is the ratio of the fourth to second order susceptibility, which is connected to the excess kurtosis

$$\kappa_B = \frac{\langle(\delta N_B)^4\rangle}{\langle(\delta N_B)^2\rangle} - 3, \quad (4)$$

via

$$\frac{\chi_4^B}{\chi_2^B} = \kappa_B \sigma_B^2. \quad (5)$$

This quantity is of particular interest since it is experimentally observable, while there also exist lattice QCD results to compare with [1, 2]. Moreover, the volume and temperature terms cancel out, when the ratio of the susceptibilities is used. Figure 3 shows the fourth to second order baryon number susceptibility ratio χ_4^B/χ_2^B as function of temperature at $\mu_B = 0$ for the two-phase model. Again, we consider first the scenario without color singlet-constraint for the quark–gluon phase. The two-phase model exhibits a strong divergence of the susceptibility ratio with increasing volume at the critical temperature (of infinite matter) T_C^∞ . This translates to extreme net-baryon number fluctuations on an event-by-event basis in a small temperature range. However, the critical behavior exhibited by this model scenario comes along with the assumption of a first order phase transition for the infinite matter limit, which is, as was stated above, not characteristic of strongly interacting matter according to lattice QCD at $\mu_B = 0$. In any case, the divergence of χ_4^B/χ_2^B is damped for smaller volumes according to the model calculations. This is plausible since the absolute effect of the system fluctuating between the two phases (or the maximum correlation length, respectively) is limited by the finite system size.

Consistent with figure 1, the ratio of fourth to second order susceptibility drops from ≈ 1 (expected for an ideal hadron gas) to $\approx 2/(3\pi^2)$ (expected for a gas of free, massless u/d quarks and gluons) in a relatively small temperature range around T_C^∞ even for small total

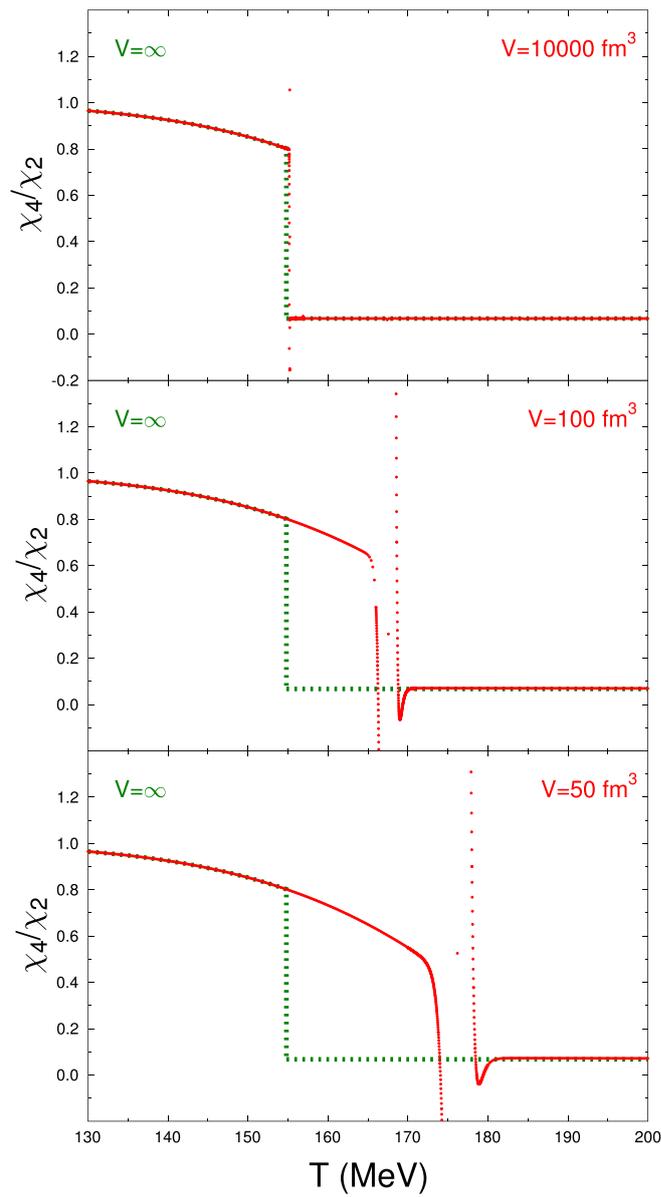


Figure 4. Fourth to second order baryon number susceptibility ratio χ_4^B/χ_2^B as function of temperature at $\mu_B = 0$ for different volume sizes. For the QGP phase the equation of state with explicit volume effects is employed. The green dots show the infinite volume limit of the model.

volumes. This picture changes as soon as a more realistic ansatz for the quark–gluon phase is chosen, i.e. the color-singlet constraint is preserved in the partition function.

Figure 4 shows the fourth to second order baryon number susceptibility ratio χ_4^B/χ_2^B as function of temperature at $\mu_B = 0$ with the color singlet-constraint for the quark–gluon phase. In line with figure 2 we observe a temperature shift of the sudden decrease of the ratio for

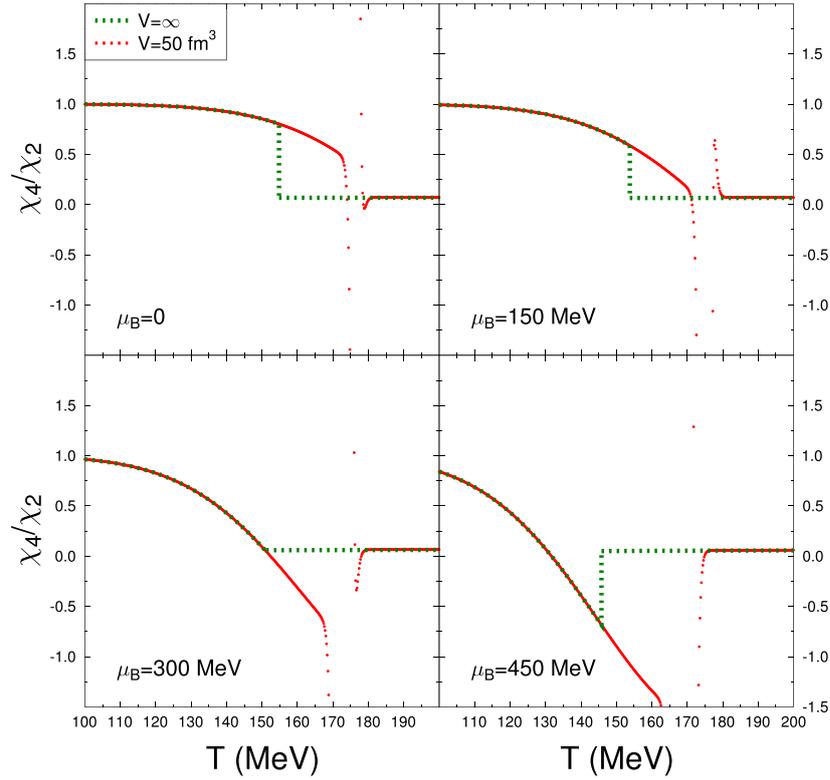


Figure 5. Fourth to second order baryon number susceptibility ratio χ_4^B/χ_2^B as function of temperature at different values of baryochemical potential μ_B . For the QGP phase the equation of state with explicit volume effects is employed. The results for system size of $V = 50 \text{ fm}^3$ (red dots) are compared with the infinite volume limit (green dots).

small system volumes. The total system seems to behave as if it was infinite matter, however with a higher critical temperature marking a first order phase transition. For a system size of $V = 100 \text{ fm}^3$ there is a temperature range above T_C^∞ between $155 \text{ MeV} < T < 165 \text{ MeV}$, where the observable susceptibility ratios reflect the properties of a ‘superheated’ hadron gas, not that of a quark–gluon plasma. For even smaller volumes of $V = 50 \text{ fm}^3$, this region extends to $T \approx 175 \text{ MeV}$. Moreover, according to the model, the sudden change of the thermodynamic bulk properties in a relatively small temperature range around T_C^{eff} gives rise to a near-divergent behavior, i.e. very large negative and positive values of χ_4^B/χ_2^B . Note that for a typical volume encountered at RHIC energies ($V \approx 1000 \text{ fm}^3$ in one unit of rapidity) the finite-size effect is still very weak, and the effective critical temperature is shifted only by $\Delta T_C^{\text{eff}} \approx 2 \text{ MeV}$. Thus, for such a volume a direct comparison to IQCD is reliable.

Figure 5 shows the fourth to second order baryon number susceptibility ratio χ_4^B/χ_2^B as function of temperature for different values of μ_B for $V = 50 \text{ fm}^3$ in comparison to infinite matter. We consider the case with color singlet-constraint for the quark–gluon phase. In the infinite volume case, finite baryochemical potentials change the susceptibility ratio as a function of temperature as compared to a system at $\mu_B = 0$. This is mainly due to the hadronic phase which shows strongly reduced values of χ_4^B/χ_2^B with increasing baryochemical potential, while the susceptibility ratio of the pure quark–gluon phase is only

weakly dependent on μ_B . The critical temperature according to the two-phase model is lowered from $T_C^\infty(\mu_B = 0) \approx 155$ MeV to $T_C^\infty(\mu_B = 450 \text{ MeV}) \approx 147$ MeV. At $T_C^\infty(\mu_B)$, the susceptibility ratio of the two-phase system switches from the hadron gas to the quark–gluon plasma value. Thus, generally, it exhibits a sudden increase or decrease of its value, depending on μ_B . Only for $\mu_B = 300$ MeV, the values of $\chi_4^B/\chi_2^B(T_C^\infty)$ happen to be the same for the hadron and the quark–gluon phase, leading to a smooth transition. At higher values of μ_B , infinite matter exhibits a negative susceptibility ratio in a certain temperature range below T_C^∞ , because the values of the hadronic phase are sufficiently low.

Now we consider a system of size $V = 50 \text{ fm}^3$. At a finite, but moderate baryochemical potential of $\mu_B = 150$ MeV, one recognizes a reduction of the susceptibility ratio in the temperature range of the ‘superheated’ hadron gas, $T_C^\infty < T < T_C^{\text{eff}}$, as compared to the $\mu_B = 0$ case. Still, the value of χ_4^B/χ_2^B is higher in the finite system as compared to infinite matter. In the same temperature range (where the quark–gluon contribution is suppressed), χ_4^B/χ_2^B drops to negative absolute values at $\mu_B = 300$ MeV. Here, the finite volume creates an effect contrary to the one predicted for small baryochemical potentials, as the observable susceptibility ratios are considerably lower than for infinite matter. At $\mu_B = 450$ MeV, this effect is even more pronounced.

4. Summary

We have presented the first study on a novel finite size effect on baryon number susceptibilities. According to our schematic model, the expected change of the reaction volume in heavy-ion collisions at different beam energies and centralities should lead to non-trivial effects concerning χ_4^B/χ_2^B even though one naively expects a canceling of the (average) volume dependence in these ratios. In order to make connections between experimental observables and the actual thermodynamic properties of strongly interacting matter, one has to be very careful with respect to the finite volumes of the reactions under investigation. In real experiments one does not probe the phase diagram in the shape of the classic $T - \mu$ plane but in the $T - \mu - V$ space. From our model we infer that the effective equation of state of strongly interacting matter in a finite volume is necessarily different from the infinite matter equation of state (which can be theoretically explored with lattice QCD calculations).

We concede that the results presented here provide only a first exploratory study within a simplified model and can hardly be considered as quantitative predictions. However, they point to a possibly significant complication in the analyses of experimental observables which are aimed at comparisons with lattice QCD predictions. It seems that more theoretical work in this respect needs to be done.

Acknowledgments

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Appendix A. The grand canonical potential of the two-phase system

As described in [14], it is assumed that the partition function of the total system factorizes into the partition functions of the two individual phases for fixed ξ , where ξ characterizes the macroscopic configuration: the volume of the hadronic phase and the quark–gluon phase are $V_h = \xi V$ and $V_q = (1 - \xi) V$, respectively. The grand canonical potential Φ of the total system in configuration ξ is then given by

$$\begin{aligned} \Phi_\xi(T, \mu_B, V) = & [\varphi_h(T, \mu_B, \xi V)\xi \\ & + \varphi_q(T, \mu_B, (1 - \xi)V)(1 - \xi)]V, \end{aligned} \quad (\text{A1})$$

where φ_h and φ_q are the densities of the grand canonical potential of the individual phases⁸.

The normalized probability for the total system being in configuration ξ must then be

$$p(\xi; T, \mu_B, V) = \frac{\exp[-\Phi_\xi(T, \mu_B, V)/T]}{\int_0^1 \exp[-\Phi_\xi(T, \mu_B, V)/T] d\xi}. \quad (\text{A2})$$

Any intensive thermodynamic quantity of the total system $A_{\text{tot}}(T, \mu_B, V)$ —including the density of the grand canonical potential φ_{tot} —can then be expressed as

$$\begin{aligned} A(T, \mu_B, V) = & \int_0^1 p(\xi; T, \mu_B, V) [A_h(T, \mu_B, \xi V)\xi \\ & + A_q(T, \mu_B, (1 - \xi)V)(1 - \xi)] d\xi. \end{aligned} \quad (\text{A3})$$

Appendix B. The grand canonical potential of the hadronic phase

The model equation of state of the hadronic phase is constructed as an ideal relativistic quantum gas of experimentally established non-strange baryon and meson resonances up to masses of 2 GeV. We calculate the density of the grand canonical potential (which equals the negative pressure) as

$$\varphi_h = -\sum_i \frac{g_i}{6\pi^2} \int_0^\infty \frac{dp}{E_i} \frac{p^4}{\exp[(E_i - \mu_i)/T] \pm 1}, \quad (\text{B1})$$

where ‘+’ stands for fermions and ‘−’ for bosons, g_i denotes the degeneracy of particle species i . $E_i = \sqrt{p^2 + m_i^2}$ is the energy of particle species i and μ_i its chemical potential.

In order to take into account repulsive short-range interactions (or eigenvolumes) of the hadrons, all thermodynamic quantities are corrected by the Hagedorn factor $1/(1 + \epsilon/4B)$ [17], where ϵ is the energy density of point particles and B is the bag constant (its value is chosen consistent with appendix C). Note that φ_h is a function of T and μ_B only, it does not depend on the subsystem’s volume V .

Appendix C. The grand canonical potential of the quark–gluon plasma phase

The model equation of state of a color-singlet quark–gluon plasma of volume V , temperature T and quark-chemical potential $\mu_q = \mu_B/3$ has been derived in [18]. The deconfined phase is

⁸ For $\mu_B = 0$ —as was the case in [14]—the grand canonical potential Φ can be replaced by the free energy F in all basic formulas of the model.

thought of as a gas of non-interacting quarks and gluons in a cavity, held together by a phenomenological vacuum pressure B . The color neutrality and total momentum constraints on all many-particle states involved are accounted for with a group-theoretical projection method: starting with a Fock space representation of free quarks and gluons, a group-integration over the generating function (using an appropriate Haar measure) effectively projects out a canonical partition function with respect to color and momentum quantum numbers. The method consistently takes into account the discreteness of the single-particle states and thus includes average shell effects in quark–gluon plasma droplets of finite size. In the case of two flavors of massless quarks and fixed total momentum of zero the resulting grand canonical partition function reads (according to [18]):

$$Z(T, R, \mu_q) = \frac{1}{2} \sqrt{\frac{1}{3}} \pi C^{-4} D^{-3/2} \exp(-BV/T) Z_0, \quad (\text{C1})$$

where Z_0 is the unprojected partition function including shell corrections. It is given by

$$\ln Z_0(T, R, \mu_q) = X - Y, \quad (\text{C2})$$

with

$$\begin{aligned} X &= \pi^2 VT^3 \times \left[\frac{37}{90} + \left(\frac{\mu_q}{\pi T} \right)^2 + \frac{1}{2} \left(\frac{\mu_q}{\pi T} \right)^4 \right] \\ Y &= \pi RT \left[\frac{38}{9} + 2 \left(\frac{\mu_q}{\pi T} \right)^2 \right]. \end{aligned} \quad (\text{C3})$$

The parameters for the finite-size corrections in (C1) are given by

$$D = 2X - \frac{1}{3}Y \quad (\text{C4})$$

and

$$C = 2VT^3 \left[\frac{4}{3} + \left(\frac{\mu_q}{\pi T} \right)^2 \right] + \frac{20}{3\pi} RT, \quad (\text{C5})$$

where $R = \left(\frac{3V}{4\pi} \right)^{1/3}$ is the radius of a spherical droplet. The phenomenological bag pressure has been fixed to the value $B^{1/4} = 215$ MeV in order to recover a critical temperature of $T_C^\infty \approx 155$ MeV at $\mu_B = 0$ within the simple model (for infinite volumes), which matches the current estimates of the chiral transition temperature from lattice calculations (see [2] and references therein).

The density of the grand canonical potential of the quark–gluon plasma phase is then calculated from the grand canonical partition function as $\varphi_q(T, \mu_q, V) = -T/V \ln Z$. This quantity depends explicitly on the subsystem's volume V .

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